HomeWorks-01.b

Introduction

1. Networks Everywhere

List three different real networks and state the nodes and links for each of them.

2. Your Interest

Tell us of the network you are personally most interested in. Address the following questions:

3. What are its nodes and links?
4. How large is it?
5. Can be mapped out?
6. Why do you care about it?

7. Impact

In your view what would be the area where network science could have the biggest impact in the next decade? Explain your answer.

Graph Theory

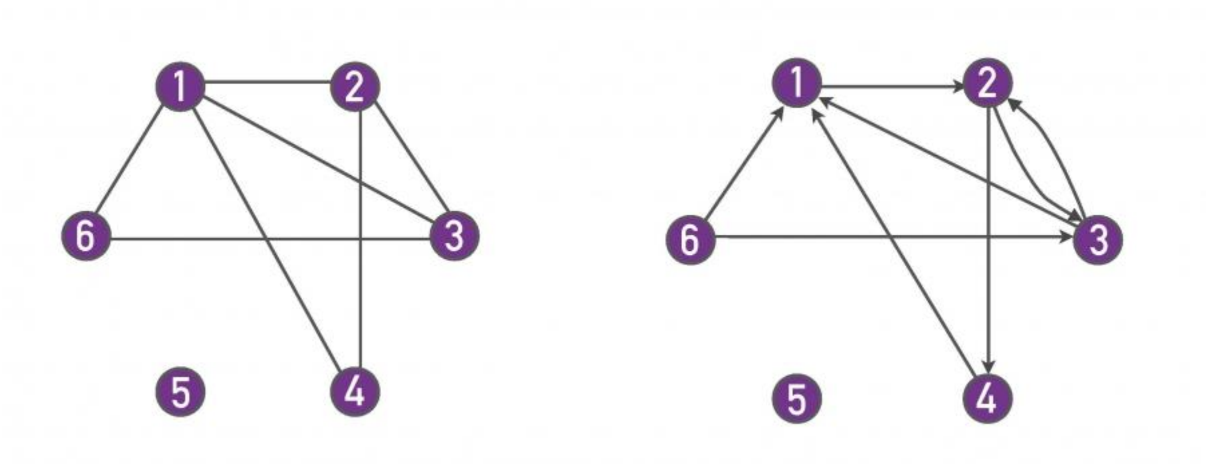
- **Matrix Formalism**

Let A be the *NxN* adjacency matrix of an undirected unweighted network, without self-loops. Let 1 be a column vector of *N* elements, all equal to 1. In other words 1 = (1, 1, ..., 1)^T , where the superscript *T* indicates the *transpose* operation. Use the matrix formalism (multiplicative constants, multiplication row by column, matrix operations like transpose and trace, etc, but avoid the sum symbol Σ) to write expressions for:

- The vector k whose elements are the degrees k_i of all nodes i = 1, 2,..., N.
- The total number of links, L, in the network.
- The number of triangles T present in the network, where a triangle means three nodes, each connected by links to the other two (Hint: you can use the trace of a matrix).
- The vector k_{nn} whose element i is the sum of the degrees of node i's neighbors.
- The vector k_{nnn} whose element i is the sum of the degrees of node i's second neighbors.

- **Graph Representation**

The adjacency matrix is a useful graph representation for many analytical calculations. However, when we need to store a network in a computer, we can save computer memory by offering the list of links in a $Lx2$ matrix, whose rows contain the starting and end point i and j of each link. Construct for the networks (a-left) and (b-right) in Image:



Graph Representation

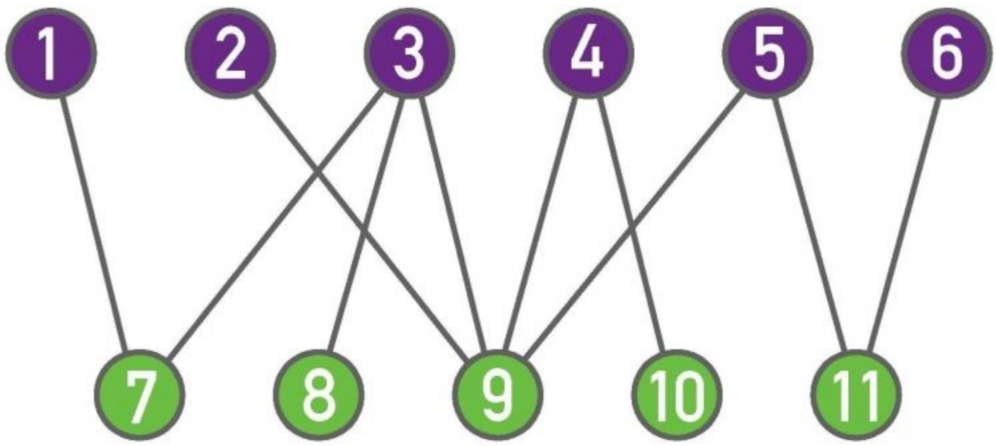
- Undirected graph of 6 nodes and 7 links.
- Directed graph of 6 nodes and 8 directed links.
- The corresponding adjacency matrices.
- The corresponding link lists.
- Determine the average clustering coefficient of the network shown in Image a
- If you switch the labels of nodes 5 and 6 in Image a, how does that move change the adjacency matrix? And the link list?
- What kind of information can you not infer from the link list representation of the network that you can infer from the adjacency matrix?
- In the (a) network, how many paths (with possible repetition of nodes and links) of length 3 exist starting from node 1 and ending at node 3? And in (b)?
- With the help of a computer, count the number of cycles of length 4 in both networks.

- Degree, Clustering Coefficient and Components
- Consider an undirected network of size N in which each node has degree $k = 1$. Which condition does N have to satisfy? What is the degree distribution of this network? How many components does the network have?
- Consider now a network in which each node has degree $k = 2$ and clustering coefficient $C = 1$. How does the network look like? What condition does N satisfy in this case?

- **Bipartite Networks**

Consider the bipartite network of Image below

- Construct its adjacency matrix. Why is it a block-diagonal matrix?
- Construct the adjacency matrix of its two projections, on the purple and on the green nodes, respectively.
- Calculate the average degree of the purple nodes and the average degree of the green nodes in the bipartite network.
- Calculate the average degree in each of the two network projections. Is it surprising that the values are different from those obtained in point (c)?



Bipartite network

Bipartite network with 6 nodes in one set and 5 nodes in the other, connected by 10 links.

- Bipartite Networks - General Considerations
- Consider a bipartite network with N_1 and N_2 nodes in the two sets.
- What is the maximum number of links L_{max} the network can have?
- How many links cannot occur compared to a non-bipartite network of size $N = N_1 + N_2$?
- If $N_1 \ll N_2$, what can you say about the network density, that is the total number of links over the maximum number of links, L_{max}?
- Find an expression connecting N_1, N_2 and the average degree for the two sets in the bipartite network, $\langle k_1 \rangle$ and $\langle k_2 \rangle$.