Case 1 (2026): Migration & Default Risk (3 questions)

Overview In this case study, we analyze the one-period credit risk (migration & default) for a portfolio of bonds. We will examine how credit rating changes affect portfolio value under different concentration levels and correlation scenarios.

1. Migration & Default Assumptions

Assume the following one period migration & default transition matrix holds

Rating	AAA	AA	A	BBB	BB	B	CCC	D (default)
AAA	91.115%	8.179%	0.607%	0.072%	0.024%	0.003%	0.000%	0.000%
AA	0.844%	89.626%	8.954%	0.437%	0.064%	0.036%	0.018%	0.021%
A	0.055%	2.595%	91.138%	5.509%	0.499%	0.107%	0.045%	0.052%
BBB	0.031%	0.147%	4.289%	90.584%	3.898%	0.708%	0.175%	0.168%
BB	0.007%	0.044%	0.446%	6.741%	83.274%	7.667%	0.895%	0.926%
B	0.008%	0.031%	0.150%	0.490%	5.373%	82.531%	7.894%	3.523%
CCC	0.000%	0.015%	0.023%	0.091%	0.388%	7.630%	83.035%	8.818%

Table 1: One Period Migration & Default Transition Matrix

2. The Rating Model

Assume that a one factor Merton model holds which drives rating transition, i.e.:

$$X_i = \sqrt{\rho} \cdot Y + \sqrt{1-\rho} \cdot \epsilon_i$$

Where X_i denotes the asset return of issuer i, Y is the common factor (standard normal), ϵ_i is the idiosyncratic factor (standard normal), and ρ represents the asset correlation coefficient.

3. Bond Valuations

The market values of the (zero-coupon) bonds are provided below for two scenarios:

1. **Current Value (t=0):** The current market value of the bond.
2. **Forward Value (t=1):** The market value of the bond at the end of the period, after migration.

Rating	Current Value (t=0)	Forward Value (t=1)
AAA	99.40	99.50
AA	98.39	98.51
A	97.22	97.53
BBB	92.79	92.77
BB	90.11	90.48
B	86.60	88.25
CCC	77.16	77.88
D (default)		60.00

Table 2: 5-year Zero-Coupon Bond Values

4. Portfolio Definitions

We assume two distinct portfolios, each with a total market value of €1500 mln:

- **Portfolio I (Investment Grade):** 60% AAA, 30% AA, 10% BBB.
- **Portfolio II (Junk):** 60% BB, 35% B, 5% CCC.

Question 1 (Concentrated portfolio)

In this section, we analyze a **concentrated portfolio model.**

Please use the following assumptions to calculate your portfolio metrics.

- **Portfolio Allocation (Pro Rata)**
  Assume the total invested amount (€1500 mln) is allocated across rating classes according to the specific weightings of the portfolio. Example for Portfolio I:
  - AAA Class: $60\% \cdot €1500 \text{ mln} = € 900 \text{ mln}$
  - AA Class: $30\% \cdot €1500 \text{ mln} = € 450 \text{ mln}$
  - BBB Class: $10\% \cdot €1500 \text{ mln} = € 150 \text{ mln}$
- **Issuer Assumptions**
  Assume there is only one single issuer per rating class. This means the entire capital allocated to a specific rating class is invested in just one issuer.

Apply this same logic to the other rating classes (AA, BBB) and to Portfolio II.

Model Validation

Before generating your results, you must validate that your simulation is calibrated correctly.

1. **Default Threshold Check:** Analytically calculate the specific Z-score threshold required for a BBB-rated issuer to migrate to Default and verify that your code uses this exact threshold.
2. **Convergence Check:** Run your simulation for Portfolio II ($\rho = 33\%$) three times with different seeds. Observe the fluctuation in the 99.5% VaR. If the instability is unacceptable for a risk model, increase N until stable. Report your final N, the observed range and justify your precision choice.

Required Calculations

For the portfolios defined above and asset correlations $\rho \in \{0, 33\%, 66\%, 100\%\}$, perform the following calculations:

- **Expected Portfolio Value:** Compute the expected (average) portfolio value.
- **Risk Metrics:** Compute the 90% and 99.5% Value-at-Risks (VaR) and Expected Shortfall (ES).

Single Issuer per rating	Rho	Expected Value	90% VaR	99.5% VaR	90% ES	99.5% ES
Portfolio I	0%					
Portfolio I	33%					
Portfolio I	66%					
Portfolio I	100%					
Portfolio II	0%					
Portfolio II	33%			Check 😊		
Portfolio II	66%					
Portfolio II	100%					

Question 2 (Diversified portfolio)

In this section, we are moving to a **diversified portfolio model**.

Please use the following assumptions to recalculate your portfolio metrics.

- **Portfolio Allocation (Pro Rata)**
  Assume the total invested amount (€1500 mln) is allocated across rating classes according to the specific weightings of the portfolio. Example for Portfolio I:
  - AAA Class: $60\% \cdot €1500 \text{ mln} = € 900 \text{ mln}$
  - AA Class: $30\% \cdot €1500 \text{ mln} = € 450 \text{ mln}$
  - BBB Class: $10\% \cdot €1500 \text{ mln} = € 150 \text{ mln}$
- **Issuer Assumptions**
  Instead of a single issuer per class, assume there are now 100 distinct issuers per rating class. The capital allocated to each rating class is divided equally among these issuers. Example Calculation (Portfolio I, AAA Class):
  - Total Allocation to AAA Class: €900 mln
  - Number of Issuers: 100
  - Investment per Issuer: $\frac{€900 \text{ mln}}{100 \text{ issuers}} = €9 \text{ mln per issuer}$

Apply this same logic to the other rating classes (AA, BBB) and to Portfolio II.

Required Calculations

For the portfolios defined above and asset correlations $\rho \in \{0, 33\%, 66\%, 100\%\}$, perform the following calculations:

- **Expected Portfolio Value:** Compute the expected (average) portfolio value.
- **Risk Metrics:** Compute the 90% and 99.5% Value-at-Risks (VaR) and Expected Shortfall (ES).

100 Issuers per rating	Rho	Expected Value	90% VaR	99.5% VaR	90% ES	99.5% ES
Portfolio I	0%					
Portfolio I	33%					
Portfolio I	66%					
Portfolio I	100%					
Portfolio II	0%					
Portfolio II	33%					
Portfolio II	66%					
Portfolio II	100%					

Question 3 (Analysis)

Please answer the following questions and explain your reasoning:

1. **Comparison:** Compare the results between Portfolio I and Portfolio II.
   What drives the difference in risk profile?
2. **Correlation:** Explain the differences in results for increasing levels of ρ.
   Why does correlation impact the "Single Issuer" and "100 Issuers" cases differently.
3. **Tail Fatness:** For a standard normal distribution, the ratio of 99.5% ES / 99.5% VaR is approximately 1.125. Calculate this ratio for your Portfolio II results. Is your ratio higher or lower? Explain what this implies about the shape of the credit loss distribution compared to the underlying asset returns.