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Speaker: 

Fractional-Order Complementary Filters for Sensor Applications

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cost
EUROPEAN COOPERATION IN SCIENCE & TECHNOLOGY

Fractional
COST Action CA15225; fractional-systems.eu

H.F.R.I.
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Background

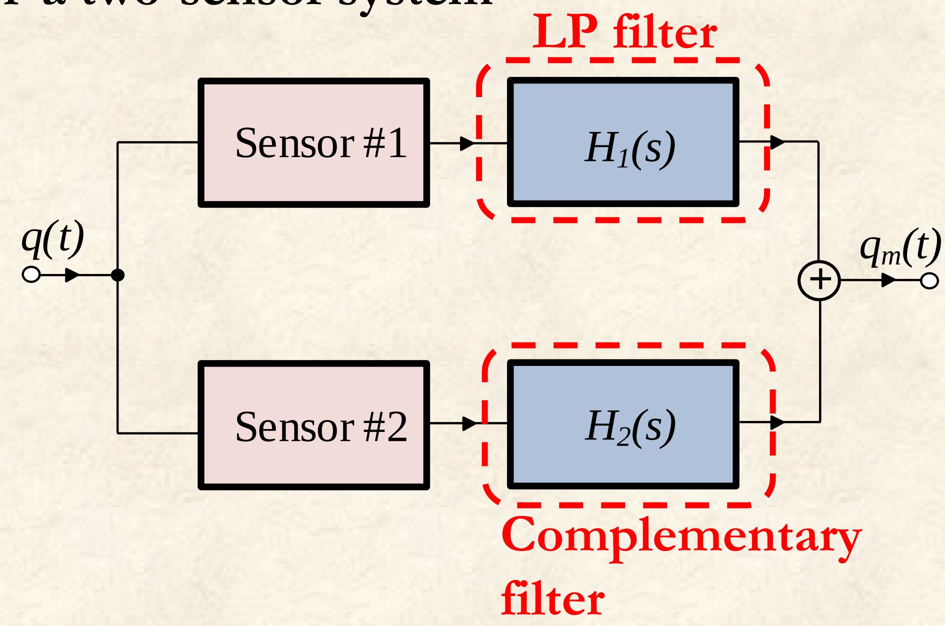
- Complementary Filters in Sensors

General Atomics MQ-9 Reaper

Modern drone

Background

- **FBD of a two-sensor system**



✓ Frequency-domain specifications → **cut-off frequency** of the LP filter

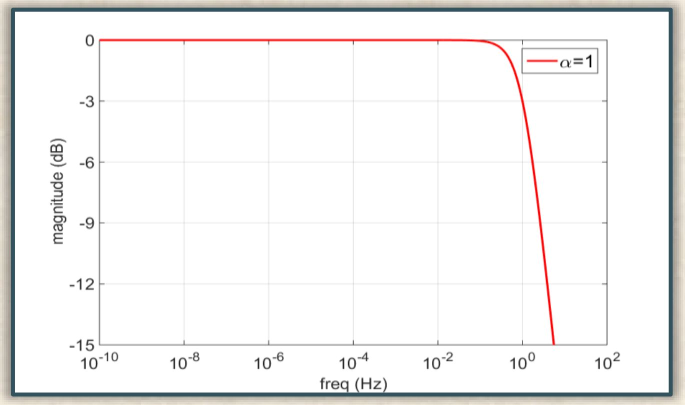
$$\boxed{H_1(s) + H_2(s) = k}$$

Background

- Importance of FO filters

  - Slope: $\boxed{-6 \cdot (n + \alpha) dB/oct}$

    integer   fractional $(0 < \alpha < 1)$

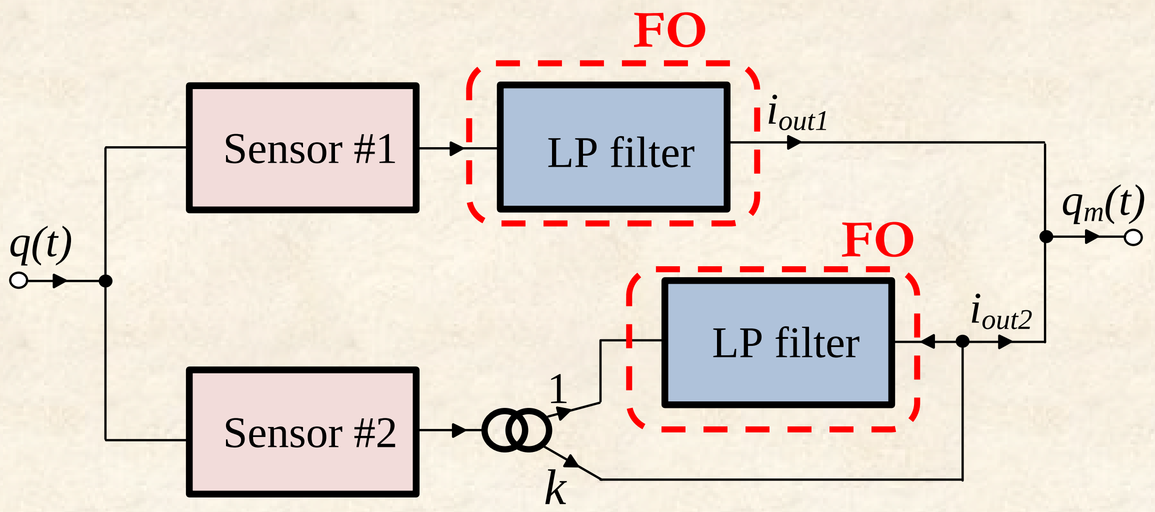


✓ More precise control of the attenuation gradient

✓ Scaling of time-constants, allowing extremely large time-constants

Background

- **FBD of a two-sensor system**



$$\boxed{H_1(s) + H_2(s) = k}$$

Background

- **FO LP filter of order α**

$$\boxed{H_1(s) = \frac{k}{(\tau s)^{\alpha} + 1}} \qquad 0 < \alpha < 1$$

- **Pole frequency:** $\boxed{\omega_0 = \frac{1}{\tau}}$

- **Cut-off frequency:**

$$\boxed{\omega_{c,lp} = \omega_0 \left[\sqrt{1 + \cos^2\left(\frac{\alpha\pi}{2}\right)} - \cos\left(\frac{\alpha\pi}{2}\right)\right]^{1/\alpha}}$$

- **Slope:** $\boxed{-6 \cdot \alpha \; dB/Oct.}$

Background

- **FO Complementary filter of order α**

$$\boxed{H_2 = k - H_1(s) = k\frac{(\tau s)^{\alpha}}{(\tau s)^{\alpha}+1}} \qquad 0<\alpha<1$$

- **Pole frequency:** $\boxed{\omega_0 = \frac{1}{\tau}}$
- **Cut-off frequency:**

$$\boxed{\omega_{c,comp} = \omega_0\left[\sqrt{1+\cos^2\left(\frac{\alpha\pi}{2}\right)}+\cos\left(\frac{\alpha\pi}{2}\right)\right]^{1/\alpha}}$$

- **Slope:** $\boxed{+6\cdot\alpha\ dB/Oct.}$

Background

- FO LP filter of order $1 + \alpha$

$$H_1(s) = \frac{k_1}{(\tau s)^{1+\alpha} + k_3(\tau s)^{\alpha} + k_2}$$

$$0 < \alpha < 1$$

$$(k = k_1/k_2)$$

- **Pole frequency:** $\boxed{\omega_0 = \frac{1}{\tau}}$
- **Cut-power frequency:** $\boxed{\left|H_1(\omega_{h,lp})\right| = 0.707k}$
- **Slope:** $\boxed{-6 \cdot (1 + \alpha)\ dB/Oct.}$

Background

- FO Complementary filter of order $1+\alpha$

$$\boxed{H_2 = k - H_1(s) = k\frac{(\tau s)^{1+\alpha} + k_3(\tau s)^{\alpha}}{(\tau s)^{1+\alpha} + k_3(\tau s)^{\alpha} + k_2}}$$

$$0 < \alpha < 1$$

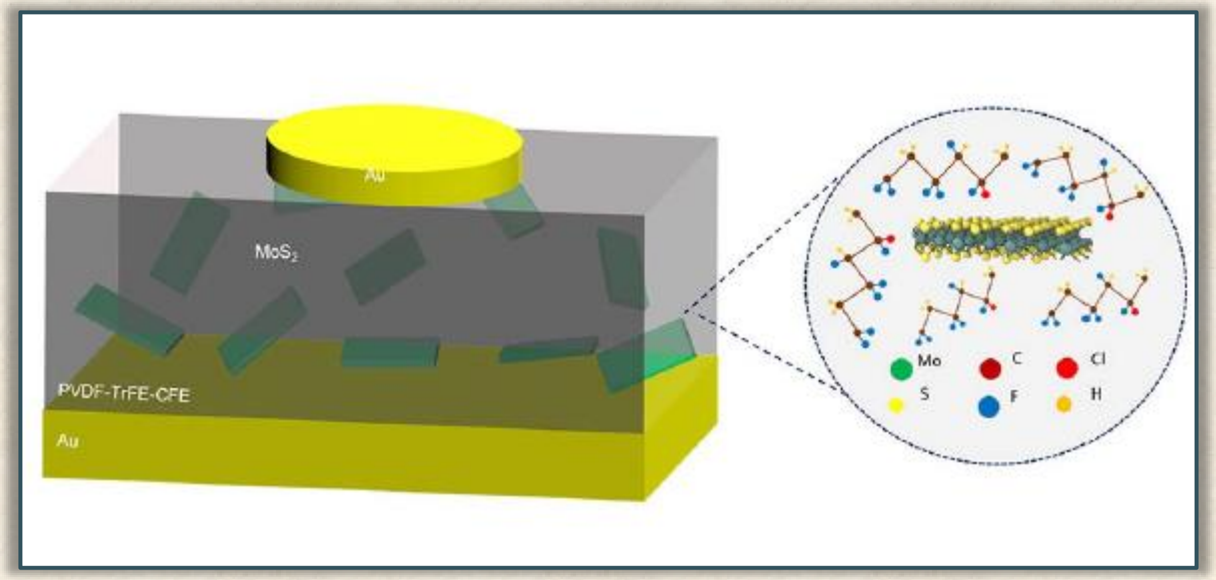
$$(k = k_1/k_2)$$

- **Pole frequency:** $\boxed{\omega_0 = \frac{1}{\tau}}$
- **Cut-off frequency:** $\boxed{\left|H_1\left(\omega_{h,lp}\right)\right| = 0.707k}$
- **Slope:** $\boxed{+6 \cdot (1+\alpha)\ dB/Oct.}$

Background

❑ Implementation of FO filters

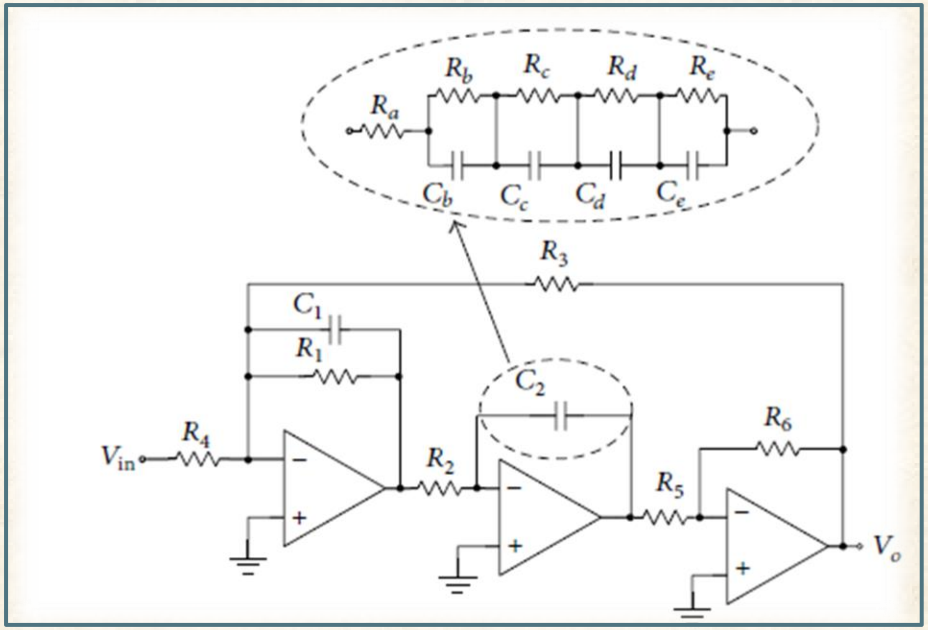
✓ Fractional-order capacitor (CPE) with molybdenum-disulfide polymer composite developed in KAUST



Commercial unavailability

Background

- **Implementation of FO filters**
  - ✓ Approximation by appropriate RC networks



☹ Absence of electronic tuning

Background

- **Implementation of FO filters**

  - Approximation by integer-order transfer functions around a center frequency

    - ✓ Various approximation tools: Continued Fraction Expansion (CFE), Oustaloup, Matsuda etc.

order α

$$H(s) \cong \frac{A_n s^n + A_{n-1} s^{n-1} + \cdots + A_1 s + A_0}{s^n + B_{n-1} s^{n-1} + \cdots + B_1 s + B_0}$$

Background

- **Implementation of FO filters**

  - Approximation by integer-order transfer functions around a center frequency

    - ✓ Various approximation tools: Continued Fraction Expansion (CFE), Oustaloup, Matsuda etc.

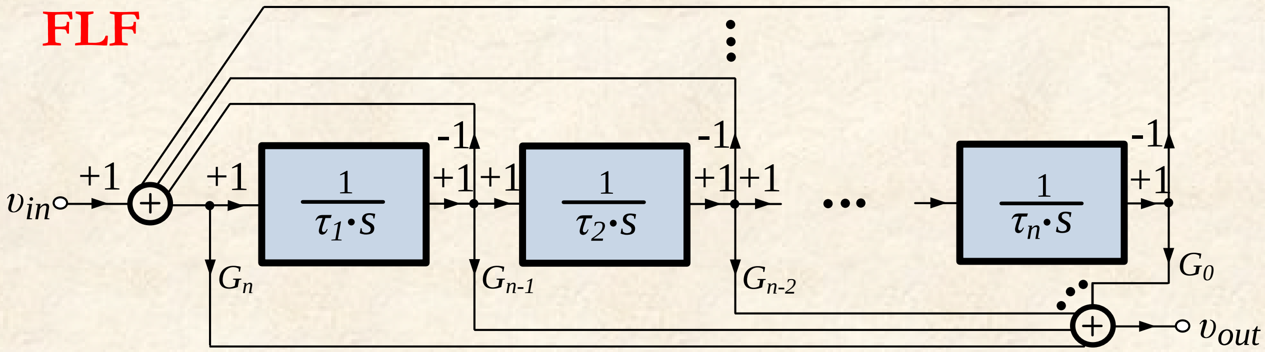
order $1+\alpha$

$$\boxed{H(s) \cong \frac{A_{n+1}s^{n+1} + A_n s^n + \cdots + A_1 s + A_0}{s^{n+1} + B_n s^n + \cdots + B_1 s + B_0}}$$

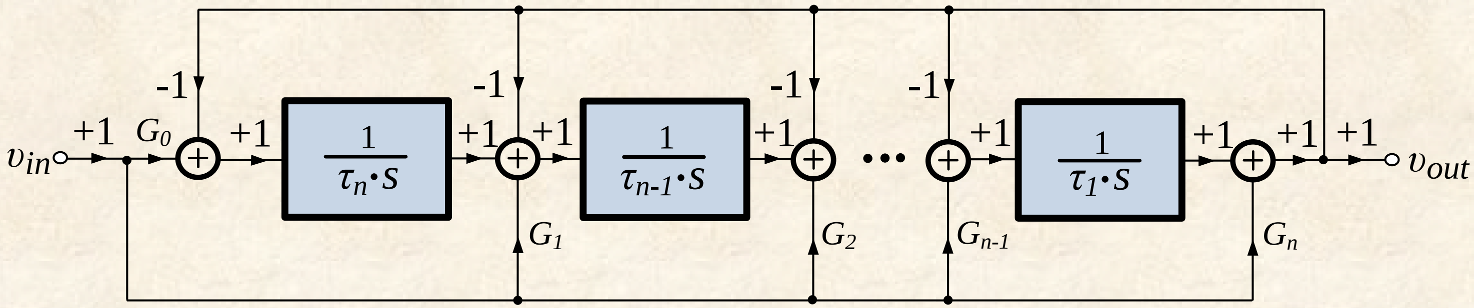
Background

- **Implementation of FO filters** **(order α)**
  - Multi-feedback structures

FLF



IFLF

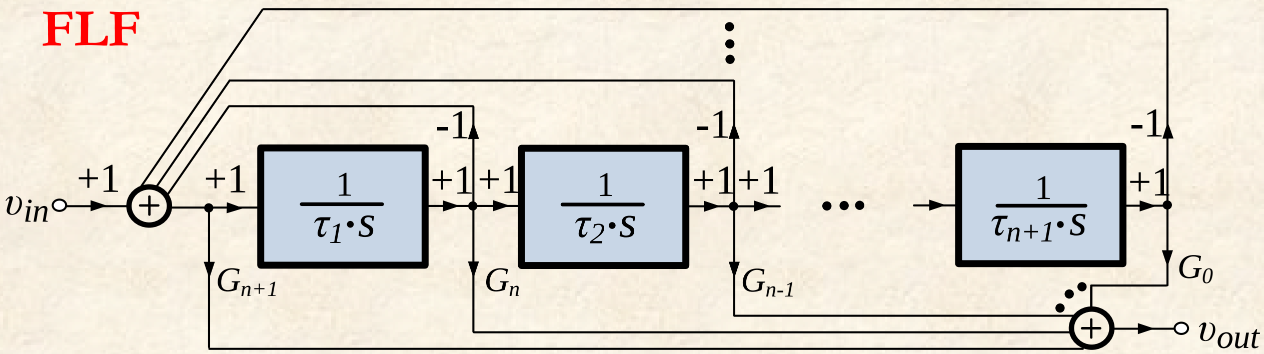


Background

❑ Implementation of FO filters (order $1+\alpha$)

- ➢ Multi-feedback structures

FLF



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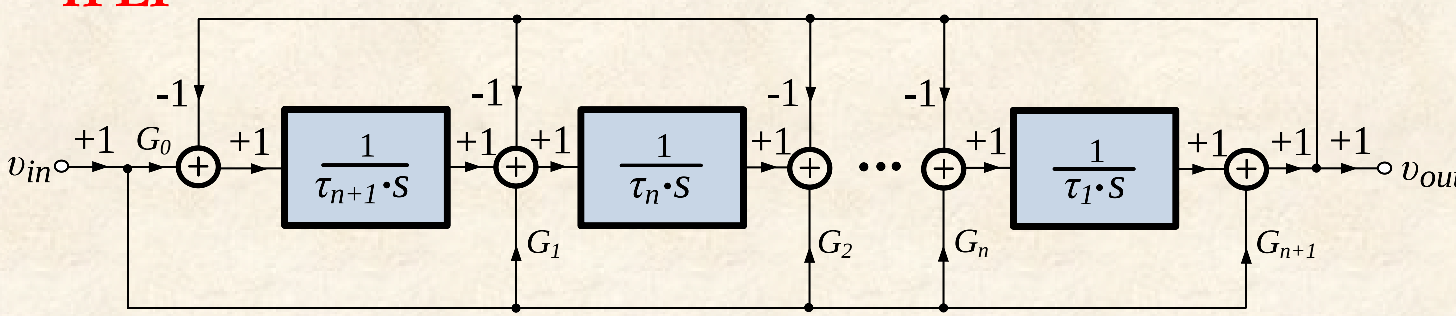
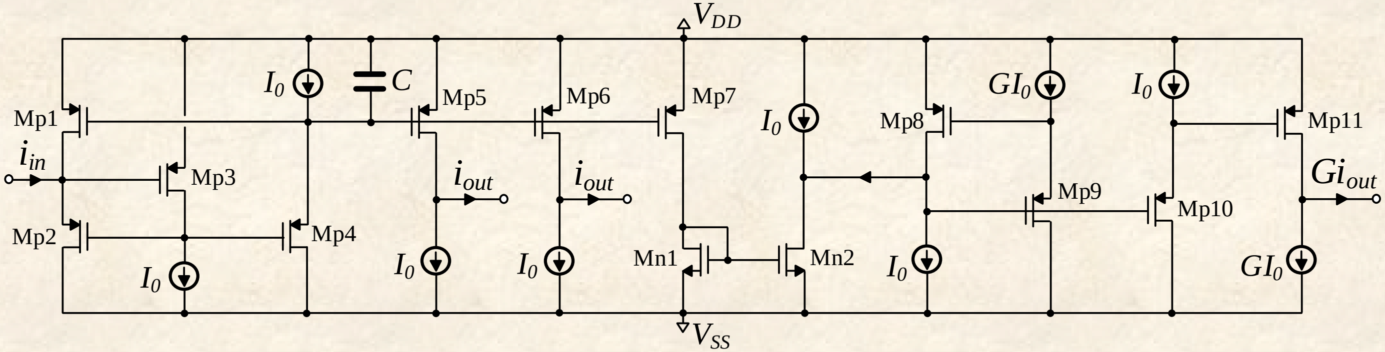


Circuit implementation

- **Log-domain lossless integrator**



$$H_{int}(s) = \frac{1}{\tau s}$$

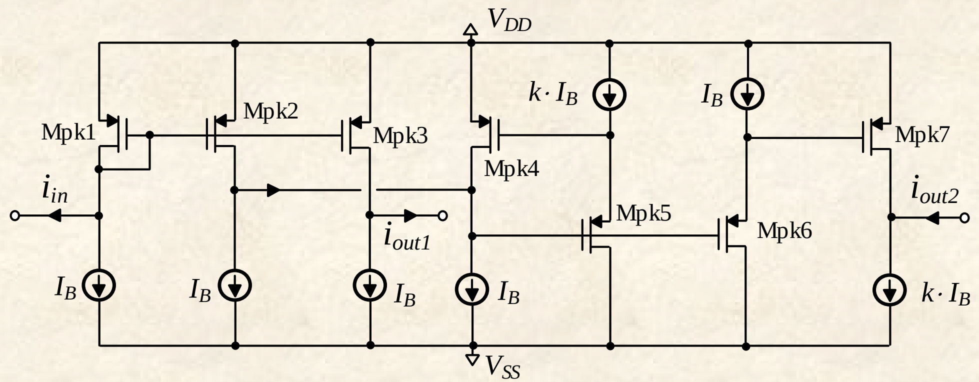
$$\tau = \frac{nCV_T}{I_0}$$

Electronic tunability of the time-constant & the scaling factor

Large signal current-voltage characteristic

Circuit implementation

Multiple-output current-mirror



$$i_{out1} = i_{in}$$

$$i_{out2} = k \cdot i_{in}$$

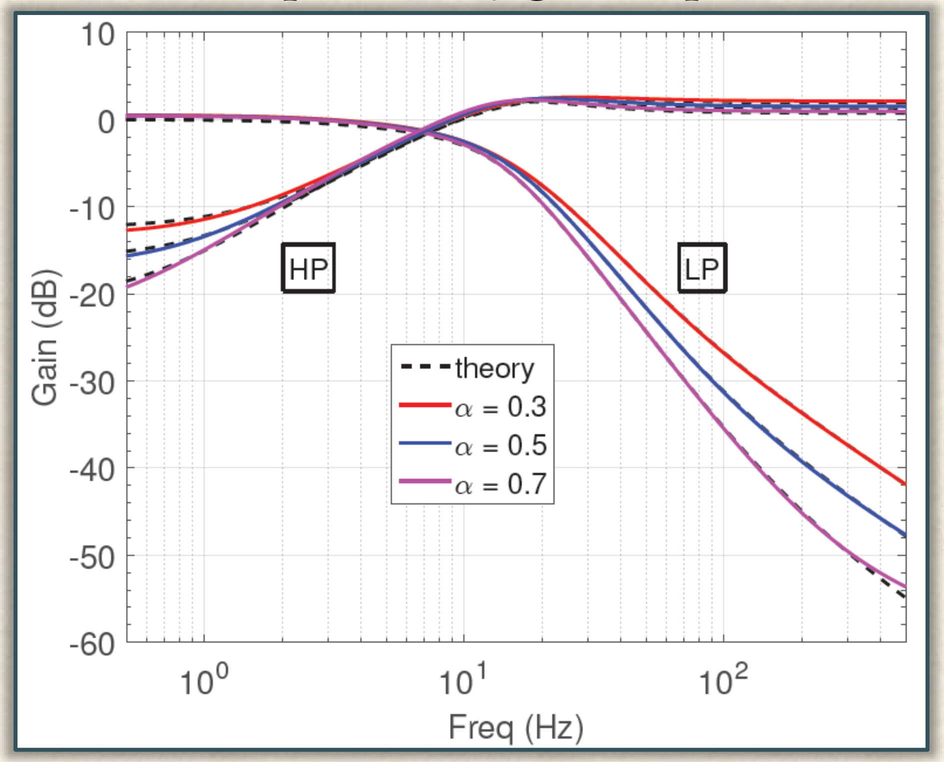
Electronic adjustment of the scaled output

Fully electronic control of the system

Simulation results

- **AMS 0.35μm CMOS process**
  - $V_{DD} = -V_{SS} = 0.75V$

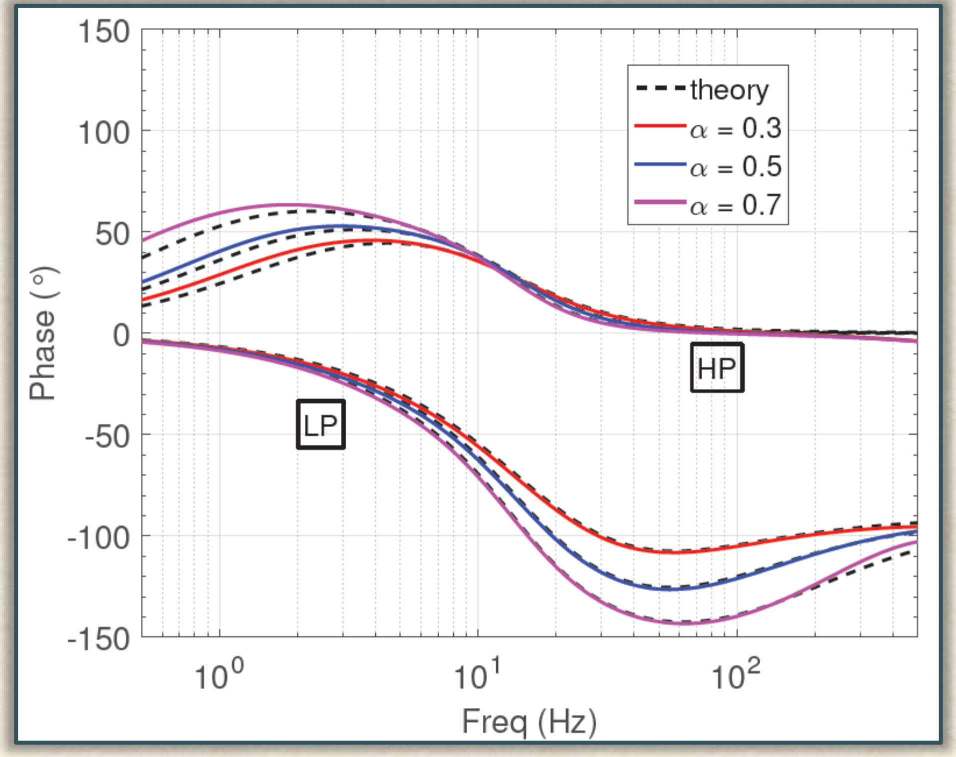
FO complementary gain responses



Simulation results

- **AMS 0.35μm CMOS process**
  - $V_{DD} = -V_{SS} = 0.75V$

FO complementary phase responses



Simulation results

Performance characteristics of the FO Complementary Filters

Variable	α=0.3	α=0.5	α=0.7
$f_{-3dB}(Hz)$	11 (10.7) 5.4 (5.7)	10.9 (10.7) 5.6 (5.8)	10.2 (10.1) 5.2 (5.4)
$\angle H(f_{-3dB})(^o)$	−59.7 (−58.3) 44.9 (44)	−67 (−66) 49.5 (48.8)	−72.1 (−71) 54 (53)
$\sigma[f_{-3dB}]\ (Hz)$	0.09 0.08	0.12 0.06	0.11 0.06
$\sigma[\angle H(f_{-3dB})]\ (^o)$	0.4 1	0.6 1.2	0.6 1.3

Conclusions

- ✓ Implementation of novel FO complementary filters for the first-time in literature
- ✓ Implementation of FO complementary filters using only LP filters and a gain stage
- ✓ Log-domain lossless integrators for the implementation of the required multi-feedback structures
- ✓ Electronic tuning of the frequency characteristics and the scaling factors of the complementary filters

Future work

- ✓ Partial fraction decomposition tool for the implementation of the LP filter, reducing the number of MOS transistors and spread of values
- ✓ Use of other active elements which compose the integrators
- ✓ Other sensor applications, including orientation estimation in UAVs, accelerometers, gyroscopes and motion measurement

THANK YOU...!!!