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INDEPENDENT COMPONENT ANALYSIS AND K-MEANS CLUSTERING FOR DAMAGE DETECTION IN STRUCTURAL SYSTEMS

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Abstract

This paper introduces a new approach to detect structural damage using independent component analysis combined with a *k*-mean clustering algorithm. Starting with the vibration history measurements for a simulated multi-degree of freedom mass-spring-damper system with breakable springs and dampers, Independent Component Analysis is used to identify the dominant underlying dynamics as well as the mixing matrix. Such a representation captures the essential structure of the vibration data and reduces the volume of the data available to a scale that allows the use of a clustering method such as K-mean clustering to separate the damaged from the healthy states.

1.0 Introduction

Structural damage detection using measured vibration data is becoming increasingly important, not only for preventing catastrophic failures but also for uninterrupted operation and prolonged service life. Detailed surveys of damage detection methods are given in [1-3]. Generally speaking, damage detection techniques can be classified according to the type of measured data on which they are based. Structures often have complex frequency spectrums, when a component of the structure wears or breaks up, a frequency component in the spectrum will change. In fact, each fault in a structure produces vibrations with distinctive characteristics that can be measured and compared with reference ones in order to perform the fault detection and diagnosis.

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Vibration monitoring systems require storing a large amount of data. Vibrations are often measured using multiple sensors mounted on different parts of the structure. The analysis of this large amount of data can be tedious and sensitive to errors. Further, depending on the sensor position, large deviations or noise may occur during measurement. Due to this problem in dealing with large data sets and the effect of noise in the measured signals, intelligent compression of the vibration data may aid in data management for fault diagnostics purpose. Independent Component Analysis (ICA) can be used to find a structure in large amount of multivariate data. ICA can be used to compress measurements from several sensors into smaller amount of signal combinations – statistically independent component of measurements – that could indicate faults in the structure.

In recent years, techniques based on Principal Component Analysis (PCA) with statistical process control have been used to condense vibration data [4]. Worden and Manson [5] implemented visualization and dimension reduction with PCA for damage detection. More recently, Zang et al. [6] used the Independent Component Analysis (ICA) technique to extract the dynamic features from measured vibration time histories. Examples of a four degree of freedom (DOF) un-damped system and a three DOF system with viscous damping show that the “measured” time domain data represented with a linear combination of dominant statistical independent components and the mixing matrix capture the essential dynamic characteristics. The main benefit of using ICA instead of PCA for data reduction is the availability of higher-order statistical data that can be used during the damage detection process. Back [7] applied ICA to multivariate financial time series such as a portfolio of stocks and the results were compared to those obtained using traditional PCA. It was demonstrated that the overall stock price can be reconstructed accurately using a small number of ICs, in contrast to poor reconstruction performance using PCA. Biswal and Ulmer [8] separated multiple signal components present in functional MRI data sets, and obtained better results using ICA rather than PCA.

This paper starts by giving a summary of ICA and compares the main characteristics of ICs versus the traditional principal components obtained from PCA. Then ICA is performed for the vibration data obtained from the simulation of a MDOF system with viscous damping in order to extract the first few independent components. Finally, the mixing matrix obtained from the ICA is used along with a *K*-mean clustering technique in order to identify the failed states versus the healthy states in the MDOF system. It will be shown that the exposures of the different vibration signals to the first few ICs are enough to differentiate between failed and safe states for the structure.

2.0 Independent Component Analysis (ICA)

Independent component analysis ICA, is a method for multi-channel signal processing to separate mixed signals. ICA has a major advantage over Principal Component Analysis (PCA) in that the former exploits higher order statistics and has no restriction on its transformation, whereas the latter exploits only second order statistics and is restricted to orthogonal transformation. ICA is based on the following mixing model

$$\{Y(t)\} = [A]\{S(t)\} \quad , \quad t = 1,\ldots,m \tag{1}$$

And the corresponding de-mixing model

$$\{Y(t)\} = [W]\{X(t)\}, \qquad t = 1,...,m \tag{2}$$

Where $\{S(t)\} = (s_1(t),....,s_n(t))^T$, $\{X(t)\} = (x_1(t),....,x_n(t))^T$ and $\{Y(t)\} = (y_1(t),....,y_n(t))^T$ denote the source signal, the mixed signals (measured) and the de-mixed signals respectively, $[A]$ and $[W]$ denote the $n \times n$ mixing matrix and de-mixing matrix respectively. The mixing and de-mixing models are written in matrix forms as $[X(t)] = [A][S(t)]$ and $[Y(t)] = [W][X(t)]$ where $[S(t)],[X(t)]$ and $[Y(t)]$ denote the $n \times m$ matrices whose *t*-th columns are $s(t), x(t)$ and $y(t)$ respectively. In other words, ICA interprets the observed data $[X(t)]$ as a mixture of statistically independent (or as independent as possible) variables $[S(t)]$, called independent components or sources. The ICA basis vectors (the columns of the mixing matrix [*A*]) are referred to as ICA modes.

In the presence of non-Gaussian data, the uncorrelated principal components obtained through PCA are not statistically independent and this represents a limitation for the representation of the original data using those principal components as PCA will be unable to reconstruct the original data. Statistical independence and non-Gaussianity are the guiding principles of ICA [9]. Independence is a much stronger property than uncorrelatedness, because higher-order statistics carry additional information about the distribution. Two random variables are independent if the value from any one of them can not be inferred from the value of the other one.

The ICA algorithms normally find the independent components of a data set my minimizing or maximizing some measure of independence. Cardoso [10] gave a review of the solution to the ICA problem using various information theoretic criteria, such as mutual information, negentropy, maximum entropy and infomax, as well as the maximum likelihood approach. In this paper, we will use the fixed point algorithm [11] based on maximizing the kurtosis which is one of the simplest statistical indicators for measuring the non-Gaussianity of a random variable, it is zero for a Guassian distribution. The fixed-point algorithm is the most suited for handling raw time-domain data and has excellent convergence properties. The algorithm will now be described briefly.

The first step in identifying the independent components is to pre-whiten the measured data vector $\{X\}$ using a linear transformation to produce a vector $\{\tilde{X}\}$ whose elements are mutually uncorrelated and all have unit variance. A singular value decomposition of the covariance matrix of the measured data matrix $[C] = E[\{X\{t\}\}\{X(t)\}^T]$ yields,

$$[C] = [\Psi][\Sigma][\Psi]^T \tag{3}$$

Where $[\Sigma] = diag(\sigma_1, \sigma_2, ..., \sigma_n)$ is a diagonal matrix of singular values, and $[\Psi]$ is the associated singular vector matrix. Then the vector $\{\tilde{X}\}$ can be expressed as,

$$\{\tilde{X}(t)\} = [\Sigma]^{-1/2}[\Psi]^T\{X(t)\} \tag{4}$$

An advantage of using a SVD-based technique is the possibility of noise reduction by discarding singular values smaller than a specified threshold.

The second step is to employ the fixed-point algorithm. Define the de-mixing matrix *[W]* that transforms the measured data vector $\{X(t)\}$ to a vector $\{Y(t)\}$, such that all elements $y_i(t)$ are both mutually uncorrelated and have unit variance. Independent random variables are always uncorrelated, and hence ensuring the transformed variables are uncorrelated reduces the number of free parameters available and simplifies the problem. The fixed-point algorithm then determines [*W*] by maximizing the absolute value of the kurtosis of $\{Y(t)\}$. The vector $\{Y(t)\}$ has the properties required for the independent components and thus,

$$\{\hat{S}(t)\} = \{Y(t)\} = [W]\{\tilde{X}(t)\} \qquad (5)$$

From Equation (4), $E[\{\tilde{X}(t)\}\{\tilde{X}(t)\}^T] = I$, and by definition we require $E[\{Y(t)\}\{Y(t)\}^T] = I$. Hence, from Equation (5),

$$[W][W]^T = I \qquad (6)$$

And thus [W] is an orthogonal matrix.

If we consider only one source signal at a time, the problem of estimating the de-mixing matrix [*W*] can be somewhat simplified. From Equation (5),

$$\hat{s}_i(t) = y_i(t) = \{w_i\}^T\{\tilde{X}(t)\} \qquad (7)$$

Where $\{w_i\}^T$ denotes the i^{th} row of [*W*]. Using the deflation approach [12] , [*W*] may be estimated on a row-by-row basis, where each independent component is estimated separately. To estimate M independent components, the algorithm must run M times. To ensure that a different independent component is obtained every time, a simple orthogonalising projection must be used inside the loop. Such a projection is possible because the rows of the de-mixing matrix [*W*] are orthonormal from equation (6). Hence the independent components are estimated by projecting the current solution vector $\{w_i\}$ onto the space orthogonal to the previously found rows of the de-mixing matrix [*W*].

The kurtosis of the estimated signal $y_i(t)$ is defined as

$$kurt(y_i(t)) = E[y_i^4(t)] - 3(E[y_i^2(t)])^2 \qquad (8)$$

Kurtosis may be viewed as a normalized fourth-order moment of a signal and has the linearity properties $kurt(x_1 + x_2) = kurt(x_1) + kurt(x_2)$ and $kurt(\alpha x_1) = \alpha^4 kurt(x_1)$, where α is a scalar. For a Gaussian distribution the kurtosis is zero, and thus kurtosis is used as a measure of the non-Gaussianity of a signal. The key to ICA is maximizing the non-Gaussianity and thus maximizing the absolute value of kurtosis [13].

From Equations (6) – (8), since the variance of $y_i(t)$ is unity,

$$kurt(y_i(t)) = kurt(\{w_i\}^T \{\tilde{X}(t)\}) = E[(\{w_i\}^T \{\tilde{X}(t)\})^4] - 3 \qquad (9)$$

And the ith de-mixing vector $\{w_i\}$ may be obtained by maximizing the kurtosis of $y_i(t)$ using the gradient descent algorithm [13].

Following the determination of the de-mixing matrix [W] and the vector of independent components $\{S\}$, the mixing matrix [A], may be estimated using the orthogonality of [W] and $[\Psi]$, as

$$[A] = [\Psi][\Sigma]^{1/2}[W]^T \qquad (11)$$

Therefore using the ICA algorithm, the time response data obtained from different sensors can be transformed into a linear mixture of 4-th order statistically independent components. The original data can be reconstructed using the mixing matrix [A] as,

$$\hat{x}_i(t) = \sum_{j=1}^{M} a_{ij} s_j(t) + \bar{x}_i \quad ; \quad i = 1,2,\ldots,N \quad ; \quad j = 1,2,\ldots,M \tag{12}$$

Where $\bar{x}_i$ denotes the mean response of the ith sensor. The matrix [A] is called the mixing matrix; it represents the relationship between the measured responses (inputs) and the independent components (outputs). Thus [A] maybe viewed as a transformation matrix between the time domain data and the characteristic dynamic features in the data. For very lightly damped, undamaged structures this transformation will be closely related to the most important modes of a structure, where the relative importance depends on the frequency range of interest. For damaged structures, the independent components will also account for the effect of the damage on the dynamic response, and are therefore sensitive to the presence, location and severity of the damage. The components of the mixing matrix [A] can be used to determine if the measured sensor vibration data reflect damage in the structure. In this study, the *k*-means clustering method for pattern recognition will be used along with two Independent Components to recognize the damaged states of a multi-degree of freedom system with breakable springs and dampers. In the next section, a summary of the *k*-means clustering algorithm is presented.

3.0 *k*-Means Clustering Algorithm

Data clustering is a common technique for statistical data analysis, which is used in many fields, including machine learning, data mining, pattern recognition and image analysis. Clustering is the classification of similar objects into different groups, or more precisely, the patterning of a data set into subsets (clusters), so that the data in each subset share some common trait, often proximity according to some defined distance measure. Machine learning typically regards data clustering as a form of unsupervised learning.

Data clustering algorithms can be hierarchical or partitional. Hierarchical algorithms find successive clusters using previously established clusters, whereas partitional algorithms determine all clusters at once. The *k*-means clustering algorithm is one of the typical partitional data clustering algorithms.

The *k*-means algorithm is a variant of the expectation maximization algorithm in which the goal is to determine the *k*-means of data generated from Gaussian distributions. It assumes that the object attributes form a vector space. The objective is to minimize total intra-cluster variance, or, the function

$$V = \sum_{i=1}^{k} \sum_{j \in S_i} \left| x_j - \mu_i \right|^2 \tag{13}$$

Where there are *k* clusters $S_i, i = 1,2,...,k$ and μ_i is the centroid or mean of al the points $x_j \in S_i$. The *k*-means algorithm assigns each point to the cluster whose centroid is nearest. The center is the average of all the points in the cluster. That is, its coordinates are the arithmetic mean for each dimension separately over all the points in the cluster. The algorithm can be summarized as follow:

1. randomly generate *k* clusters and determine the clusters centers
2. assign each point to the nearest cluster center
3. recomputed the new cluster centers, and
4. repeat until some convergence criterion is met (usually that the assignment hasn't changed).

The main advantages of this algorithm are its simplicity and speed which allows it to run on large data sets. Its disadvantage is that it does not yield the same result with each run, since the resulting clusters depend on the initial random assignments. It maximizes inter-cluster variance, but does not ensure that the result has a global minimum variance. For more details on the k-means clustering procedure the reader is referred to Macqueen [14].

5. The MDOF structural model with breakable springs and dampers

The Multi-degree of freedom spring-mass-damper system shown in Figure (1) is used to investigate the combined ICA/Clustering damage detection method suggested in this paper. The system has two masses, two springs and two dampers. Four different damage scenarios will be used in this study along with the base healthy response in order to identify the first two independent components of this system. The equations of motion for the two-degree-of-freedom system are given by:

$$m_1\ddot{x}_1 + (c_1 + c_2)\dot{x}_1 + (k_1 + k_2)x_1 - c_2\ddot{x}_2 - k_2 x_2 = 0 \quad (14)$$

$$m_2\ddot{x}_2 + c_2\dot{x}_2 + k_2 x_2 - c_2\dot{x}_1 - k_2 x_1 = f(t) \quad (15)$$

Where m_1, m_2 are the system masses, c_1, c_2 are the viscous damping coefficients, and k_1, k_2 are the two total stiffness coefficients. The base healthy displacement response under impulse excitation as measured at the first mass is shown in Figure (2). Next, the stiffness of the first spring is reduced by 70% to model possible damage. The response in this first damaged scenario along with the healthy response is shown in Figure (2). Next, the stiffness of the first spring is reset to its healthy value and the damping coefficient of the first damper, c_1 is reduced by 50%, the response to the impulse input is measured at the first mass and shown in Figure (2) along with the healthy baseline response. These two scenarios are repeated one more time using the spring and damper connected to the second mass (k_2, c_2) and the displacement obtained from all five scenarios (one healthy and four damaged) are shown in Figure (2). The damage scenarios are summarized in Table 1.

5.1 ICA for data reduction

The 5 time histories obtained in the previous section corresponding to one healthy state and 4 damaged states were used to form a 5x1000 matrix, where 1000 is the number of points in each time history. Such a large matrix can not be used directly with a *k*-means clustering algorithm and the ICA procedure described earlier is used for data reduction. The five independent components extracted from the displacement date are shown in Figure (3). Only the first two independent components are used in the clustering step. The resulting reduced mixing matrix [*A*] (5 x 2) is then used in the *k*-means clustering algorithm to differentiate between the healthy and the damaged states.

6.0 Results

In this section, the mixing matrix is used along with the *k*-means clustering algorithm to detect the damaged states in the MDOF previously described. K-means clustering will be applied using only the loadings to the first two independent components. More ICs can be used in the clustering algorithm, but using only the first two components will allow the graphical representation of the clusters along with the damaged and healthy conditions. As previously indicated, the obtained ICs are not ordered, but are obtained using a successive reduction procedure. In order to define a specific order for the 5 obtained independent components, we start by ranking them in descending order according to the maximum magnitude of the displacement in each IC. The 5 independent components extracted are shown in descending order in Figure (3). Only the first two will be used in the *k*-means clustering procedure. Figure (4) shows the loadings to both the first and second ICs as obtained from the mixing matrix. The *k*-mean clustering algorithm was then executed for segmenting the 5 data points (each representing one scenario) into two clusters; the two obtained clusters are shown in Figure (4) verifying that the first two independent components were sufficient to differentiate between damaged and healthy conditions for the MDOF system.

7.0 Conclusions

The results demonstrate that data reduction from time domain data using independent component analysis, followed by *k*-mean clustering for damage detection provide a suitable methodology for structural health monitoring. Independent component analysis is a powerful tool in decomposing signals into not only uncorrelated, but independent components. The corresponding mixing matrix represents the dynamic characteristics of the structure and the effect of damage which enables us to use it for pattern recognition using k-mean clustering. The suggested procedure has the advantage of not only reducing the size of the data set we have to work with, but also provides robustness relative to noise contamination. Further more, the procedure for obtaining the ICs takes into consideration higher order statistics which makes them more suitable for damage identification than traditional Principal Components. Another major advantage of this procedure is that it only needs the vibration response to be measured and not the excitation force causing the response, which makes this procedure more adequate for online industrial applications. The number of ICs selected for the clustering algorithm determines the size of the reduced data used in the clustering and how much information is contained in the reconstructed signals. In the simple example presented in this study, two ICs were sufficient to identify the damaged states, but in general using too few components will lose some characteristics of the data that the k-mean clustering algorithm could find useful in the detection phase. More work need to be done to determine the optimum number of ICs to use in the characterization phase. Other more complicated structural systems (frames, beams, plates) will be considered in the future.

8.0 References

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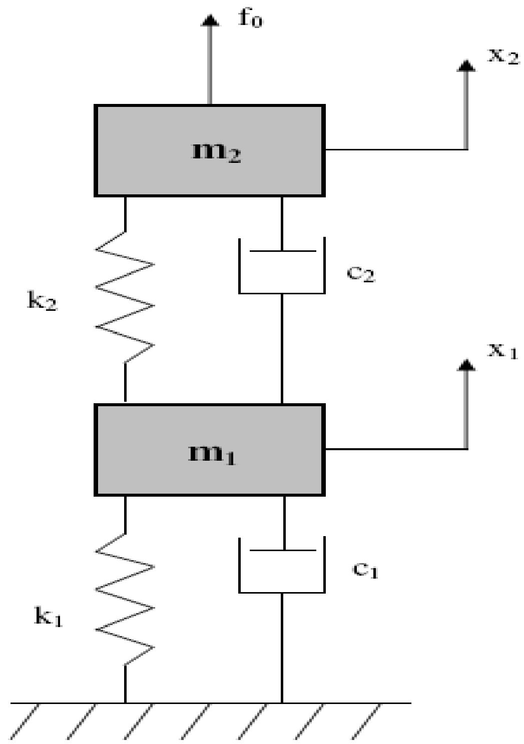

Figure (1): Multi-Degree of Freedom system with breakable springs and dampers.

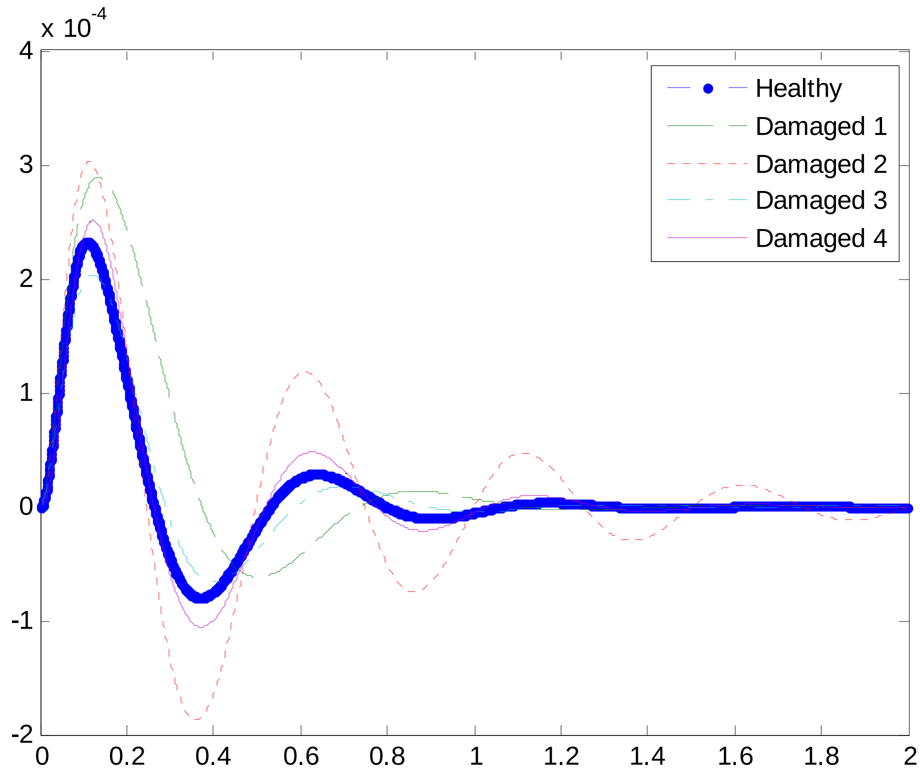

Figure (2): Displacement response of mass m_1 under different conditions

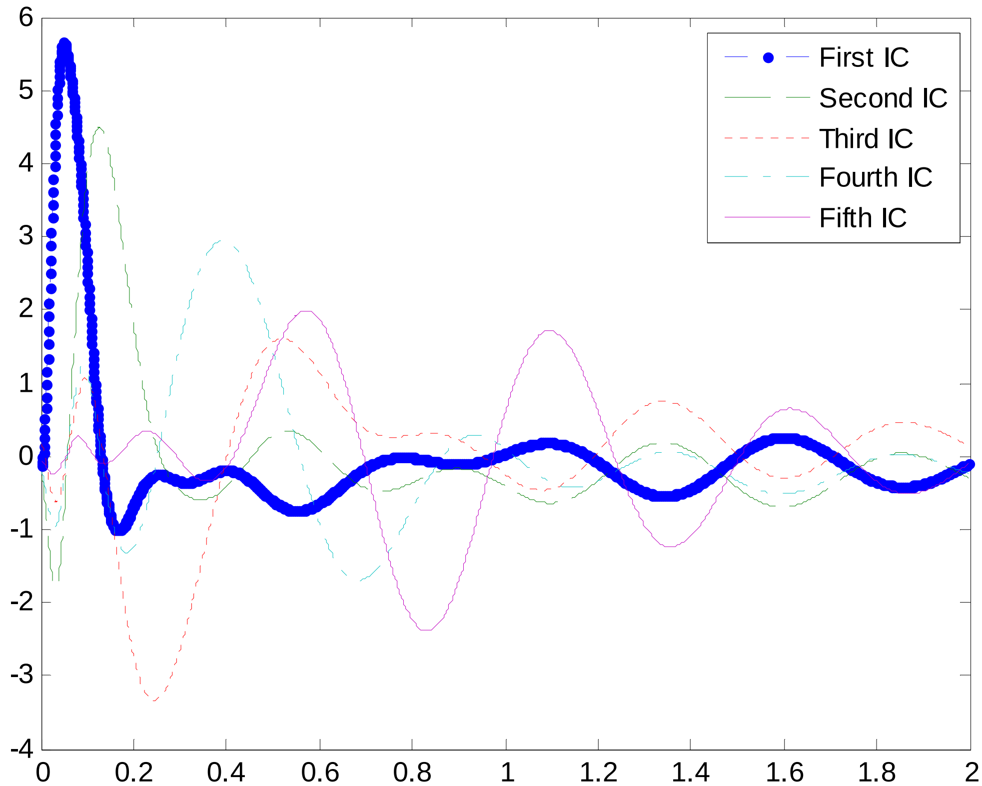

Figure (3): Five Extracted Independent Components

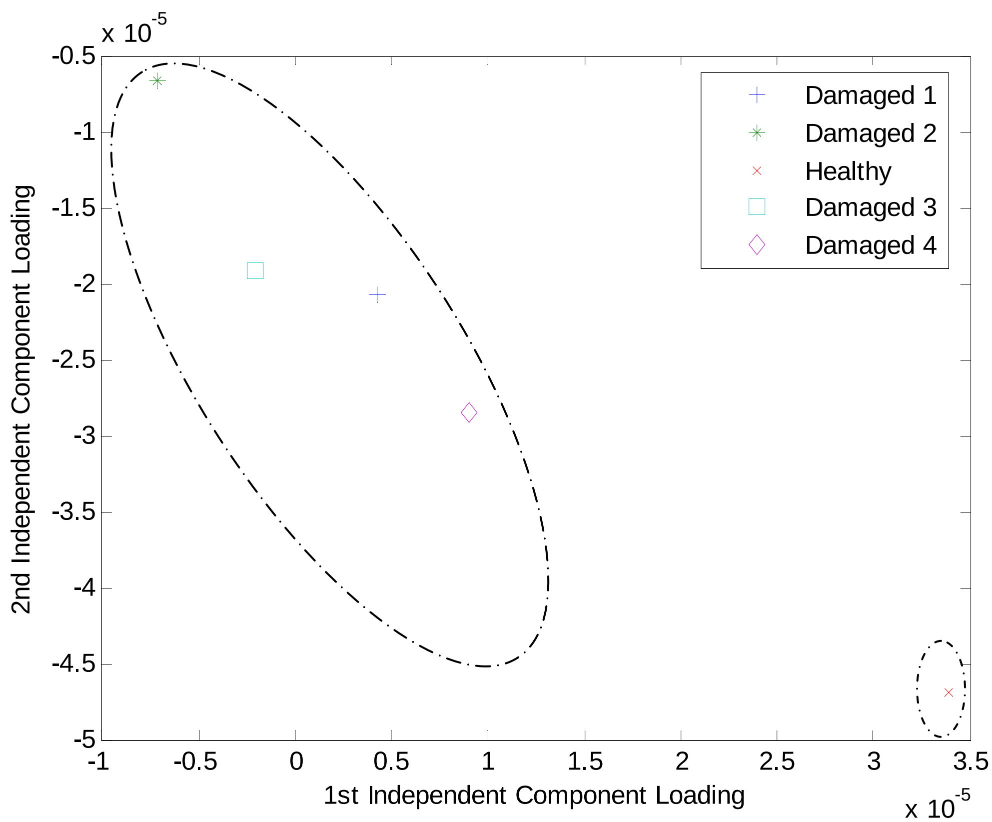

Figure (4): Pattern Recognition Using k-means clustering and the first 2 Independent Components

Damage Cases	Description	Condition (H: Healthy, D: Damaged)
Case 1	Baseline	H
Case 2	Break Spring 1	D
Case 3	Break Damper 1	D
Case 4	Break Spring 2	D
Case 4	Break Damper 2	D

Table (1): Damage Scenarios