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Natural Landmark Extraction in 2D Laser Data based on Local Curvature Scale for Mobile Robot Navigation

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***Abstract*—This paper proposes a novel algorithm for natural landmark extraction from 2D laser scan data. The algorithm permits to extract several types of landmarks such as dominant points, edges, center of curve segments and virtual corners. This is achieved by using a new geometrical feature identification approach based on a novel concept of local scale called local curvature scale. Intuitively, this approach is computationally efficiencies due to needless of the construction of scale space map which is required in the other methods based on scale space theory. Experimental results show that the algorithm is efficient to detect landmarks for semi-structured environments.**

I. Introduction

LOCALIZATION is a fundamental ability required of an autonomous mobile robot. Relative localization employs the proprioceptive sensors such as odometers, inertial sensors and gyros to estimate the pose of robot on the base of a start position, whereas dead-reckoning induces errors which increase without bounds. Measurement of known landmarks by exteroceptive sensors such as laser rangefinders and cameras can also be used to update the robot's pose. Lots of navigation systems use artificial beacons to realize localization, but the approach needs the environment to be changed. It may not be realistic in some applications such as exploration of unknown environment.

Natural landmark-based localization methods, which utilize typical natural structures of environment to achieve a similar performance as artificial beacons, have become increasingly popular in the past decades. Such methods require that natural landmarks can be robustly detected from sensor data. Hence, it is decisive for the success of these methods to choose a correct type of natural landmark and external sensor, and develop a fast and reliable algorithm of extracting landmarks from a large set of noisy and uncertain sensor data [1].

Due to the reliability and accuracy of laser rangefinders, they are employed to extract the features of local environment commonly. In structured environments, walls and corners are always used as landmarks. However, in unstructured environments, such landmarks are sparse and can be detected infrequently. Therefore, some virtual corners [1], curvature extrema points [2], or tree-like objects [3] play the role of natural landmarks.

Generally, pattern recognition concepts and procedures can be applied for extracting these landmarks in laser data. Some of the early efforts in this direction have focused on extracting line features, such as Hough transform [4], fuzzy clustering [5] and Kalman filter [3], [6]. These methods analyze only one type feature. Curvature, on the contrary, allows identifying several types of geometrical features. Moreover, features extracted from curvature are invariant to view-point [7]. However, with the limit of angle resolution of laser rangefinder and the noise of measurement processing, curvature estimation in laser data is still a challenging task. There are few methods that simultaneously meet the acquirement of easy, fast and robust to noise [1].

According to the fact that objects in the world appear in different ways depending on the scale of observation [8], it naturally leads us to conclude that coarse scale features are more robust and reliable to noise than fine scale features to obtain landmarks. Thus, the scale space theory, which has been developed by the computer vision community to analyze the multi-scale nature of real-world images, has been applied to estimate the curvature and detect features in laser scan data. In [2], an iterative curvature scale space approach, which convolves the curve descriptor with a Gaussian kernel and smoothing at different levels of scale, is proposed to extract stable corners from laser data. In [9], a modified adaptive smoothing algorithm is presented for laser data and applied to extract lines within a scale space framework. More recently in [10], an adaptive smoothing algorithm with a model based mask, which conforms to the concept of anisotropic diffusion and smoothes laser data in multi-scale space, is proposed so that dominant features are extracted. These methods based on multi-scale theory can remove high frequency noise and extract robust landmarks from laser data.

The above-mentioned methods belong to Global scale approaches which process the data at each of various fixed scales and combine the results. One problem of these methods comes to construct complete scale space map which is expensive computation, and determination of the proper scale for extracting features is also difficult [11]. Alternatively, Local scale approaches define the largest homogeneous region at each point and treat these points as fundamental units, thus the construction of scale space map is needless. In [11], a fire-new concept of local scale based on curvature, called Local Curvature Scale or *C-Scale*, is proposed to extract features in the digital boundary. And in [12], a shape descriptor based on C-Scale is developed to define mathematical landmarks with different levels of detail, and illustrate the descriptor is very reliable and robust for digital

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effect and noise.

In this paper, a novel natural landmark extraction algorithm based on C-Scale is proposed to analyze the 2D laser data, and it consists of four procedures: laser data pre-processing, breakpoint detection, feature identification and landmark selection. This algorithm can simultaneously detect several types of landmarks such as dominant points, edges, center of curve segments and virtual corners.

The principal contributions of this paper are as follows: 1) the underlying idea of C-Scale is modified to suit with the asymmetry property of laser data; and 2) a natural landmark extraction algorithm based on the modified C-Scale is proposed to extract landmarks from laser data. In the rest of this paper, we summarize the theory of C-Scale in Section 2. In Section 3, we show the laser data pre-processing. In Section 4, we describe how to extract natural landmarks using C-Scale. Experimental results are shown in Section 5. Conclusions and future work are stated in Section 6.

II. Local curvature scale

Local curvature scale, or *C-Scale*, is a novel concept of local scale proposed by pattern recognition community to modeling sharp in digital boundary. It is different from previous methods of curvature estimation and can be directly applied to digital boundaries without requiring prior approximations of the boundary, giving robust and accurate results at different levels of detail by considering the local morphometric scale of the object [11]. In this section, we first summarize the basic C-Scale theory, and then present a modified C-Scale version for laser data.

Let $\mathfrak{B} = \{b_i | i = 1, \cdots, K\}$ define a discrete boundary, and b_i is the boundary element. C-Scale segment $C(b_i)$ associated with b_i is defined as *"the largest connected set of points of $\mathfrak{B}$ symmetrically situated with respect to b_i such that the distance d of any of these points from a line connecting the two end points of the connected set is within a fixed value t "*(see Fig.1a) [11]. Thus, $C(b_i)$ can be determined by

$$C(b_i) = \{b_{i-t}, \cdots, b_i, \cdots, b_{i+t} | t = sup\ (Dist_i^n < D_{max}^{scale} : n \in [1, \cdots, \lfloor (K-1)/2 \rfloor])\}, \quad (1)$$

where $Dist_i^n$ denotes the distance of b_i to the chord of $C(b_i)$, which is the straight line segment connecting the end points b_{i-n} and b_{i+n} of $C(b_i)$, and D_{max}^{scale} is a threshold used to control the level of detection detail. The C-Scale value $C_c(b_i)$ is the chord length corresponding to $C(b_i)$, and the arc length $C_a(b_i)$ at b_i corresponding to $C(b_i)$ can be determined from $C_c(b_i)$ by assuming that $C(b_i)$ represents a local circular arc. Let b_i, b_{i-t}, b_{i+t} be three points on a circle with radius r and center o (Fig.1a), using the geometrical properties of circle and triangles, the radius of the osculating circle at b_i can be calculated by

$$r = \frac{4s^2 + C_c(b_i)^2}{8s}, \quad (2)$$

where s represents the distance between the mid-point of the arc and the mid-point of the chord. Then, the relation between the chord length $C_c(b_i)$ and the arc length $C_a(b_i)$ can be expressed as

$$C_a(b_i) = 2r \cdot tan^{-1}\left(\frac{C_c(b_i)}{2(r-s)}\right). \quad (3)$$

The arc length $C_a(b_i)$ is an indirect indicator of the curvature at b_i, if it is large, it indicates low curvature at b_i, and if it is small, it indicates high curvature [11].

For digital boundary, the boundary elements are assumed to follow a Uniform distribution, viz. it assumes that the distances of each element between its neighbors are equal approximately. It is therefore reasonable to detect the C-Scale segment with symmetrical way in order to control the scale and remove detail and noise. However, for laser data, the range value of each scan point is related with the shape of scanning surface and the bearing angle of laser pulse, the distances of scan point between its neighbors are unequal, especially when the incidence angle of laser pulse is closing to 90°. If the symmetrical method is used to determine the C-Scale segment in laser data, the determined segment will not be able to satisfy the principle of fitting a local circular, and the above equations will be unsuitable. So, in this paper, a modified C-Scale is presented to meet the asymmetry property of laser data. The new principle of determining $C(b_i)$ can be expressed as

$$C(b_i) = \{b_{i-b}, \cdots, b_i, \cdots, b_{i+e} | \underset{\langle b,e\rangle \in op(Dist_i^{b,e},\ D_{max}^{scale})}{arg\ min} (\|b_{i-b} - b_i\| - \|b_i - b_{i+e}\|)\}, \quad (4)$$

where $op(Dist_i^{be},\ D_{max}^{scale})$ is a function which is employed to detect all the ordered pair $\langle b, e\rangle$ that subject to $Dist_i^{b,e} < D_{max}^{scale}$ && $(Dist_i^{b+1,e} \geq D_{max}^{scale}\ ||\ Dist_i^{b,e+1} \geq D_{max}^{scale})$, and $Dist_i^{b,e}$ denotes the distance of b_i to straight line segment connecting the end points b_{i-b} and b_{i+e} of $C(b_i)$. Let b_i, b_{i-b}, b_{i+e} be three points located on a circle (see Fig.1b), the radius of the circle can be calculated by

$$r = \frac{\|b_{i-b} - b_i\| \cdot \|b_i - b_{i+e}\|}{2s}. \quad (5)$$

Then, the arc length $C_a(b_i)$ can be expressed as

$$C_a(b_i) = 2r \cdot sin^{-1}\left(\frac{C_c(b_i)}{2r}\right). \quad (6)$$

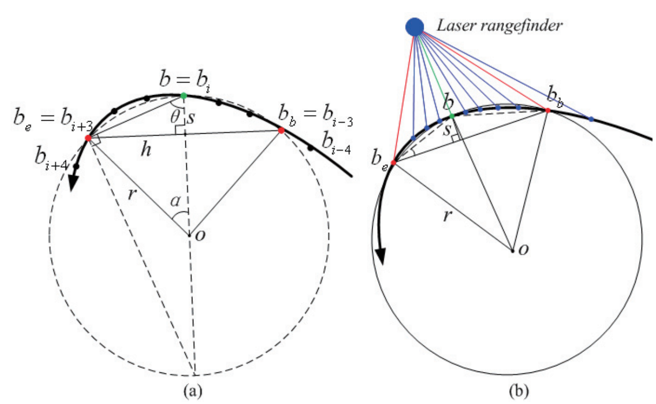

Fig.1. (a) C-Scale definition; (b) Modified C-Scale for laser data.

As shown in Fig.2a, a simulated frame of laser data is used to illustrate the modified C-Scale. This laser scan data is constructed from lines and circular arcs, we apply the C-Scale representation method with $D_{max}^{scale} = 3.5$. This parameter controls the level of detail, and $D_{max}^{scale} \in [3,5]$ works well for all tested laser data by trial and error, which can preserve appropriate details and ward against noise at the same time. The basic $C(b_i)$ and modified $C(b_i)$ at each scan point are represented in Fig.2b. We observe that the modified version can represent the underlying idea of C-Scale precisely, and decrease the influence of asymmetry of laser data obviously. In evidence, large $C(b_i)$ corresponds to low curvature, whereas small $C(b_i)$ indicates high curvature. The points of valleys in $C(b_i)$ represent high curvature points or corners, and the peaks in $C(b_i)$ correspond to middle points of line segments or inflection points [11]. By detecting these valleys and peaks, the characteristic points of laser data can be detected.

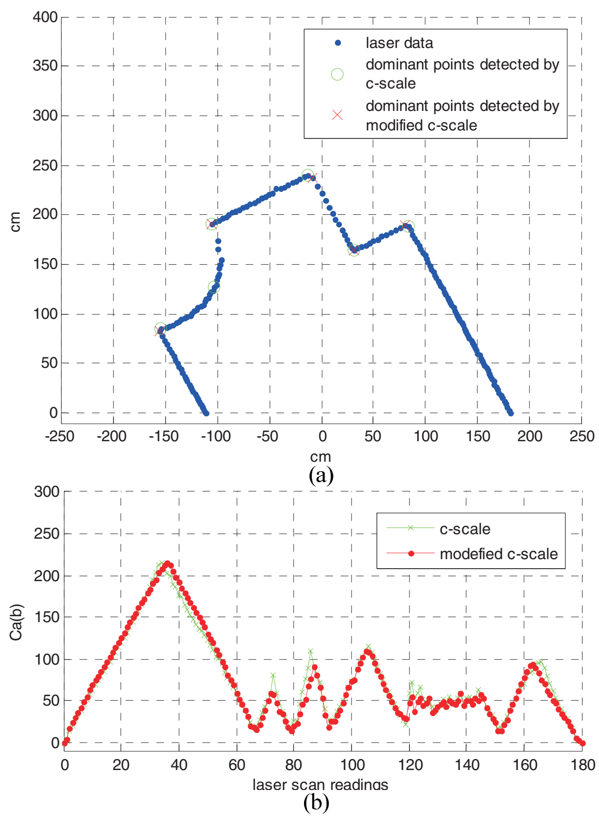

Fig.2. (a) A 2D laser data formed by lines and circular arcs; (b) The value $C_a(b_i)$ calculated by basic and modified C-Scale for the laser data.

III. Laser Data Pre-processing

The typical scan data of 2D laser rangefinder can provide an range image of the local environment, which is stored in a local polar form $\mathcal{R}_p = \{(\varphi_i, r_i) | i = 1, \ldots, N\}$, where r_i is the measured distance of an obstacle to the sensor at direction φ_i, and N is the number of scan points related with the field of vision FOV and the given angular resolution $\Delta\varphi$ by $N = FOV/\Delta\varphi$. Another equivalent representation is the local Cartesian form $\mathcal{R}_c = \{(x_i^L, y_i^L) | i = 1, \ldots, N\}$, with $x_i^L = r_i cos\varphi_i$ and $y_i^L = r_i sin\varphi_i$.

A. Motion Correction

The scanning period of laser rangefinder is not zero. It means that the range images may be warped during the scanning period. Therefore, it is necessary to perform motion correction for laser data before feature extraction [1], [5].

Let $\mathbf{P}_i^S = (x_i^G, y_i^G, \theta_i^G)$ be the laser rangefinder's absolute position when the i-th scan reading acquired, then, $\mathbf{P}_1^S$ and $\mathbf{P}_N^S$ represent the laser's poses at first and last readings acquired respectively. When the i-th scan reading acquisition, the local coordinate frame has been displaced $\Delta\mathbf{P}_i^S = \mathbf{P}_i^S - \mathbf{P}_1^S$ from $\mathbf{P}_1^S$. If we want to recover the coordinates $(\hat{x}_i^L, \hat{y}_i^L)$ of the i-th scan point when the laser is on $\mathbf{P}_1^S$, the displacement of sensor is $\Delta\mathbf{P}_i^S = (\Delta x_i^s, \Delta y_i^s, \Delta\theta_i^s)$ taken into account as

$$\begin{pmatrix} \hat{x}_i^L \\ \hat{y}_i^L \end{pmatrix} = \begin{pmatrix} cos\Delta\theta_i^s & sin\Delta\theta_i^s \\ -sin\Delta\theta_i^s & cos\Delta\theta_i^s \end{pmatrix} \cdot \begin{pmatrix} x_i^L + \Delta x_i^s \\ y_i^L + \Delta y_i^s \end{pmatrix}. \quad (7)$$

Equation (7) indicates that it is necessary to know the laser's relative displacement $\Delta\mathbf{P}_i^S$ but its absolute pose $\mathbf{P}_i^S$ at i-th scan point [1]. Here, a well calibrated odometry is used to estimate the translational velocity $\boldsymbol{v}_t = (v_t^x, v_t^y)$ and rotational velocity ω_t of robot platform when the laser is on $\mathbf{P}_1^S$. The linear interpolating estimation of velocity may be precise because the scan period is very short. Thus, $\Delta\mathbf{P}_i^S$ are estimated by

$$(\Delta x_i^s \quad \Delta y_i^s \quad \Delta\theta_i^s) = (v_t^x \quad v_t^y \quad \omega_t) \cdot \frac{i}{N \cdot Fr_s}, \quad (8)$$

where Fr_s is the scanning frequency of the laser rangefinder.

B. Laser Data Filtering

The laser scan data is noisy but not proportional spacing, in order to enhance its quality, two fast filters are used to modify the laser data [7].

The circular way of laser rangefinder emitted the laser beams leads to the scan points close to the sensor are denser. It is time-consuming for detecting C-Scale segment to use these verbose scan points. Thus, the reduction filter is first used to discard the closer scan points if their distance are less than a threshold D_{max}^{reduce}. The laser scan data may occur salt and pepper noise which arises from reflection at glass surfaces, or at edges where the laser beam hits two surfaces. An extend median filter is used to remove the outliers by replacing them with the median value of the surrounding points iff the distance between both is larger than a fixed threshold D_{max}^{median}. In our experiments, the parameters of these filters are set as $D_{max}^{reduce} = 2$cm, $D_{max}^{median} = 200$cm.

IV. Natural Landmark Extraction via C-Scale

In this section, a landmark extraction algorithm based on C-Scale is proposed to extract the variety of landmarks from the filtered laser data. Particularly, the algorithm provides four types of features: dominant points, edges, center of curve segments and virtual corners. Aiming at extracting these landmarks, a breakpoint detector is first used to segment the laser data into *groups*. And then, a feature identification method based on C-Scale is applied to this semi-segmentation *group* to segment it into more typically *clusters* which associate with different structures, and the geometrical features are identified from these *clusters*. Finally, the natural landmarks are selected form these candidate features by using a process of landmark selection.

A. Breakpoint Detection

Breakpoints are remarkable scan discontinuities due to change of surface being scanned by the laser rangefinder, which can serve as a sign of semi-segmentation. Breakpoints are detected by making inferences about the possible presence of discontinuities in a sequence of valid laser data [5]. The basic criterion of detection is based on the distance between two consecutive points. An adaptive method [5] is applied to determine the segment boundaries in this paper, which treats two consecutive scan readings belong to the different segments if

$$\|r_{i+1} - r_i\| > r_i \cdot \frac{sin\Delta\varphi}{sin\,(\gamma - \Delta\varphi)} + 3\sigma_r\,, \qquad (9)$$

where $\Delta\varphi$ is the angular resolution of laser, γ is a constant parameter which represents the worst acceptable angle of incidence, and σ_r is the residual variance. In our experiments, these parameters are set as $\Delta\varphi = 1°$, $\sigma_r = 5cm$, and $\gamma = 10°$. Furthermore, the group whose members are less than 10 is not taken into account in the following process.

B. Feature identification

Each group based on breakpoints always consists of two or more different structures, it isn't sufficient to identify features except edges. Here, for each valid group, we employ the modified C-Scale to develop an approach to identify the geometrical features. The proposed approach includes the following seven steps:

Step1. Calculating the C-Scale value for each group. For each reading b_i of the processing group, the C-Scale segment $C(b_i)$ is determined by principle (4), and the arc length $C_a(b_i)$ is calculated via (5) and (6).

Step2. Segmenting the group by detection the valleys of $C_a(b_i)$. We convolve $C_a(b_i)$ with a discretized version of the derivative of the Gaussian with a fixed variance $\sigma = 1.5$, and then, the zero-crossing points of this convolved signal is applied to find the valleys of $C_a(b_i)$. The two endpoints of group are also treated as the first and last valley. These valleys segment the semi-segmentation group into more typically clusters which associate with different structures of environment. The cluster of which members are less than 5 will be neglected.

Step3. Calculating the index ci for each cluster. This index is used to decide whether a cluster is associated to a line or a circle. It is defined as

$$ci = \frac{\bar{C}_a(b_i)}{max\,\{C_a(b_i)\}}\,, \qquad (10)$$

where $\bar{C}_a(b_i)$ is the mean of $C_a(b_i)$ of the disposed cluster. If $ci \approx 1$, the distribution of the arc length $C_a(b_i)$ is almost even, which indicates the estimated radius of the circle are equal approximately, and the segment can be considered as a circular arc. Alternatively, an approximate isosceles triangle of $C_a(b_i)$ can symptomize a line segment, and it is equal to ci ≈ 0.5.

Step4. Detecting dominant points. Dominant points result from the change of surface being scanned or the change in the orientation of the scanned surfaces (e.g. corners) which associate with curvature extrema [11]. Thus, they correspond to valleys in $C_a(b_i)$ which are not associated with laser scan discontinuities.

Step5. Detecting edges. Edges are defined as breakpoints associated with free end-points of plane surfaces. Therefore, they are the set of breakpoints which locate at endpoints of a line segment, and the breakpoint is nearer to the robot than the other breakpoint defined by the same discontinuity [1].

Step6. Estimating center of curve segments. Center of curve segments are decided by the scan of curve surfaces (e.g. trees, pillars). For each cluster with $ci > 0.7$, and its members are greater than 10. We consider these clusters associate to a circle segment, and their center coordinates (x_c, y_c) and the radius r_c can be estimated by means of a least squares fitting method.

Step7. Estimating virtual corners. Virtual corners are defined by the intersection of two lines corresponding to detected line segments. For the clusters with $ci \in [0.4, 0.6]$ and their members are greater than 5, they will be considered as line segments and the parameters in a slope-intercept form are estimated by using the least squares fitting method. Then, the crossing points of the arbitrary two lines are treated as virtual corners.

C. Landmark selection

Not all these features identified though the above process are suitable as landmarks for robot navigation. The natural landmarks must be selected carefully from these candidate features. The dominant points are located in one of the scan readings, failing to identify the correct corners in the laser data will lead to large errors especially when the corners are distant from the robot. On account of a portion of the dominant points and some virtual corners represent the same corners; the corners which located at two mutually perpendicular lines will be instead by virtual corners. Meanwhile, those virtual corners which locate at a very near position (their distance are less than 5cm in our experiments) will be fused into one virtual corner with mean coordinates. Moreover, the virtual corners which associate to two lines with an included angle is larger than 30° will be selected as landmarks so that the errors of their positions are reduced. For the circle segments, their center will be selected as landmarks if the estimated radii are larger than 10cm. Other features that can't satisfy the above conditions will be treated as landmarks directly.

V. Experimental results

The natural landmark extraction algorithm proposed above has been applied to a navigation system of mobile robot Pioneer3-DX, which is equipped with a laser SICK LMS200. The rangefinder fixed in front of the robot has a view of 180°

and an angular resolution of 1°. The experiments operate in an indoor environment with smooth ground. And C++ and Matlab are integrated to implement all procedures of the algorithm.

Here, we provide two typical examples to show the effectiveness of the proposed algorithm. The first example is based on a simulated laser scan data shown in Fig.2a; it aims to provide evidence on the use of the modified C-Scale and the performance of the proposed feature identification method. This laser data presents five line segments and a circle arc. Fig.3a shows the identified features including dominant points, centers of curve segments and virtual corners, meanwhile the fitting lines and fitting circle are also marked. Fig.3b presents the arc length $C_a(b_i)$ associated to the laser data, and the detected valleys and the ci indexes of each cluster have been marked on the figure. It can be noted that all interested features have been identified correctly.

The second example is based on a real-world laser scan data that acquired in an office-like environment without moving persons; it aims to test the complete algorithm including the all processes of laser data pre-processing, breakpoint detection, feature identification and landmark selection. Fig.4 illustrates the result of this experiment. Fig.4a shows the raw scan data and indexes the semi-segmentation groups, as well as the extracted natural landmarks are marked. Fig.4b shows all identified features, ignored groups, fitting lines and fitting circle. Fig.4c illustrates the values of $C_a(b_i)$ associated to the filtered laser data, and the detected valleys and the ci indexes of each cluster are also been marked. From the final selected landmarks that have been marked with red stars on Fig.4a, we can note that all defined landmarks have been detected correctly. These experiments verify the capability of the proposed algorithm to extract several kinds of landmarks from laser data.

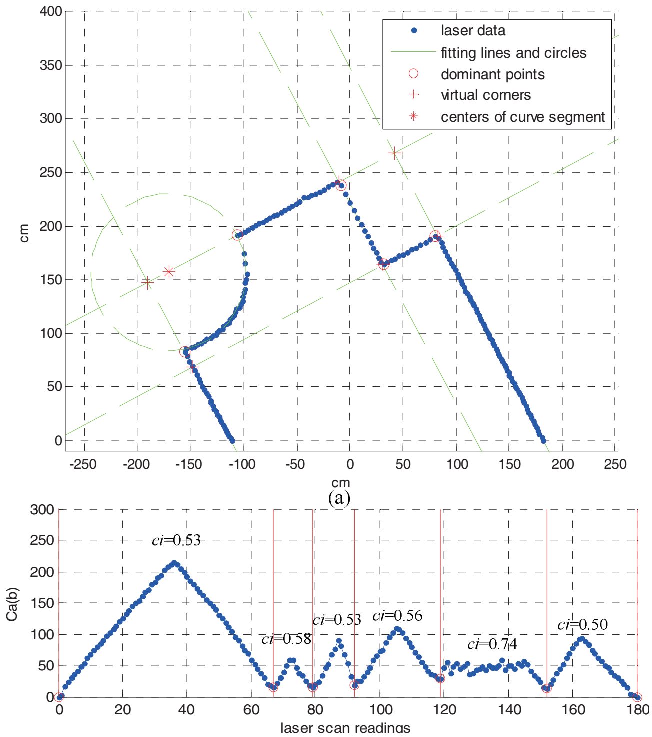

Fig.3. (a) The identified features from a simulated laser data as shown in Fig.2a; (b) The arc length $C_a(b_i)$ associated to this laser data.

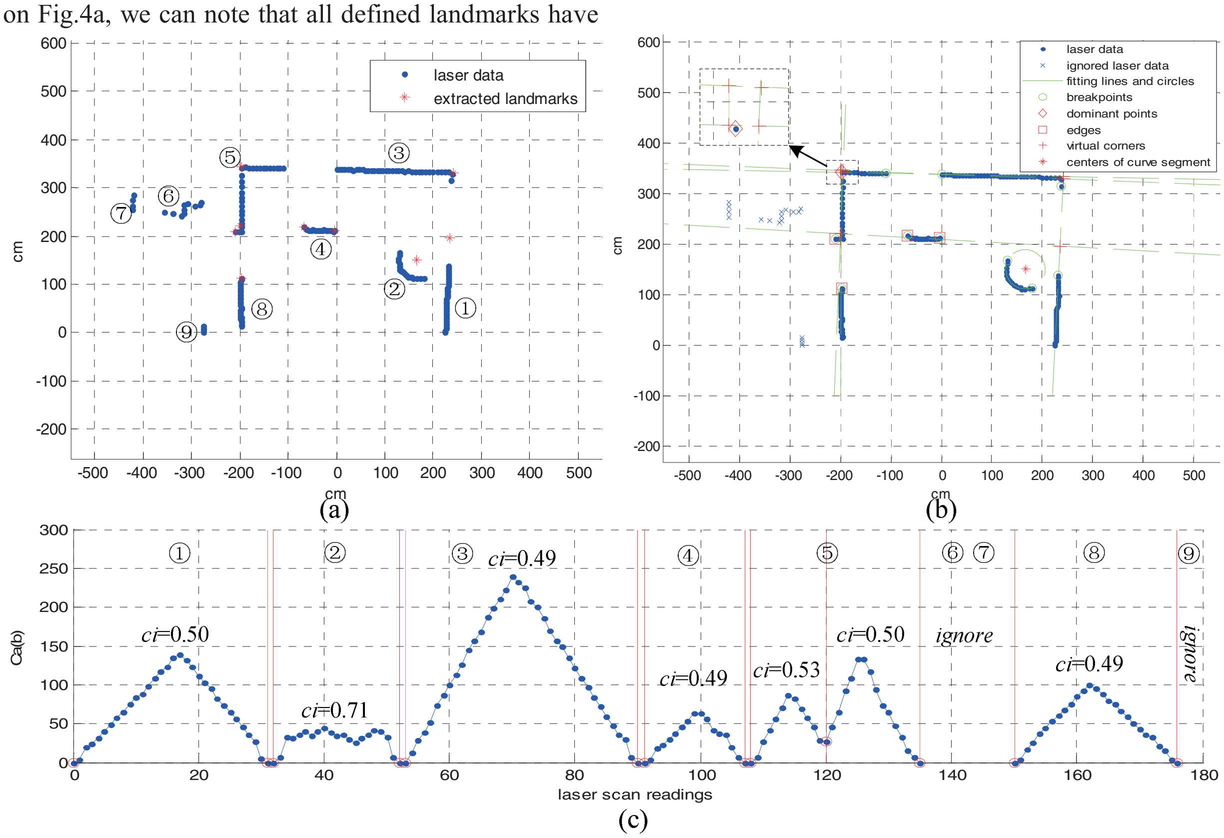

Fig.4. (a) Segmentation of the laser scan and the extracted landmark; (b) identified features; (c) the arc length $C_a(b)$ associated to (a).

VI. CONCLUSION

In this paper, a novel algorithm for natural landmark extraction from 2D laser scan data is presented. Based on a new concept of local scale called local curvature scale, or C-Scale, the proposed algorithm can extract several types of landmark such as dominant points, edges, center of curve segments and virtual corners robustly. In contrast with the previous methods based on scale space theory, this algorithm is intuitively faster due to the construction of scale space map is unneeded. The effectiveness of the modified C-Scale and the proposed landmark extraction algorithm are illustrated in an experiment, the results verify our algorithm works well.

In future work, we will compare the proposed algorithm to other methods for natural landmark extraction in order to evaluate the speed, correctness and robustness of these algorithms. Moreover, we plan to develop a more reliable version of the algorithm and integrate it with a relative localization algorithm aiming at developing an autonomous mobile robot for operating at unknown semi-structured environment.

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