# PERFORMANCE ANALYSIS OF WAVELET AND FOURIER TRANSFORMS APPLIED TO NON-STATIONARY VIBRATION DATA

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**Abstract**

Machine condition monitoring is one of the most important and a highly demanding field and had captured the attention of majority of researchers working on efficient fault detection techniques. Early fault detection in machine condition monitoring not only ensures the smooth operation of the machinery but also reduces the maintenance cost. Frequency domain analysis is an effective tool for earlier fault detection techniques. To analyze a signal's frequency domain properties, the Discrete Time Fourier Transform is a useful tool. For a certain block length, there is a particular time and frequency resolution. In case of non stationary signals the Wavelet Transform may also be used which does not have a constant time and frequency resolution for a particular block length. A comparative performance analysis of both techniques using vibration data is presented on the basis of results to show the efficiency and scope of each technique. The analysis is based on advantages and disadvantages of both the techniques over each other to analyze the stationary and non-stationary vibration signals.



**Keywords:** Fourier Transform, Wavelet Transform, Condition Monitoring

**Acknowledgement**

First of all we would like to thanks Almighty Allah who gave us the strength, wisdom and capability of doing this work. We show our bundle of thanks to Mr. Imran Khan for his guidance, feedback and support throughout our thesis work. We appreciate him for giving us his valuable time in guiding us in sorting out issues related with our thesis by his technical expertise. We are indebted to our professors, colleagues and seniors for their support and help on issues for understanding some key points on condition monitoring and research papers. There is lot of appreciation for our respected parents for their support on each and every step to complete our studies in Sweden.

Muhammad Shahzad and Aziz Muhammad
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# Contents

# List of Figures

# Chapter 1

# 1 Introduction

## 1.1 Background

Smooth machinery operation is essential in every industry. The operation of the machinery is constantly monitored to detect faults at earlier stages, thus reducing both maintenance and operation costs. Vibration signals are directly related to a machine's structural dynamics (behaviour of a machine on the basis of its design and manufacturing when subject to some action) and properties of excitation sources etc., therefore, oftenly used by engineers for an effective indication of machine failure [1][2][3]. Most of the faults in the machinery may be observed as an impulse or discontinuity in the monitored vibration signals [2].

Vibration signals produced by an operating machine may be characterized as stationary or non stationary. A signal is said to be stationary when its statistical properties do not depend on time like a simple sine wave shown in figure 1 and non stationary signals are those whose statistical properties do change with time like speech signal shown in figure 2 [2].

Different mathematical techniques or transforms may be used for the analysis of vibration signals. Discrete Fourier Transform may provide detailed information about each frequency component present in a signal.

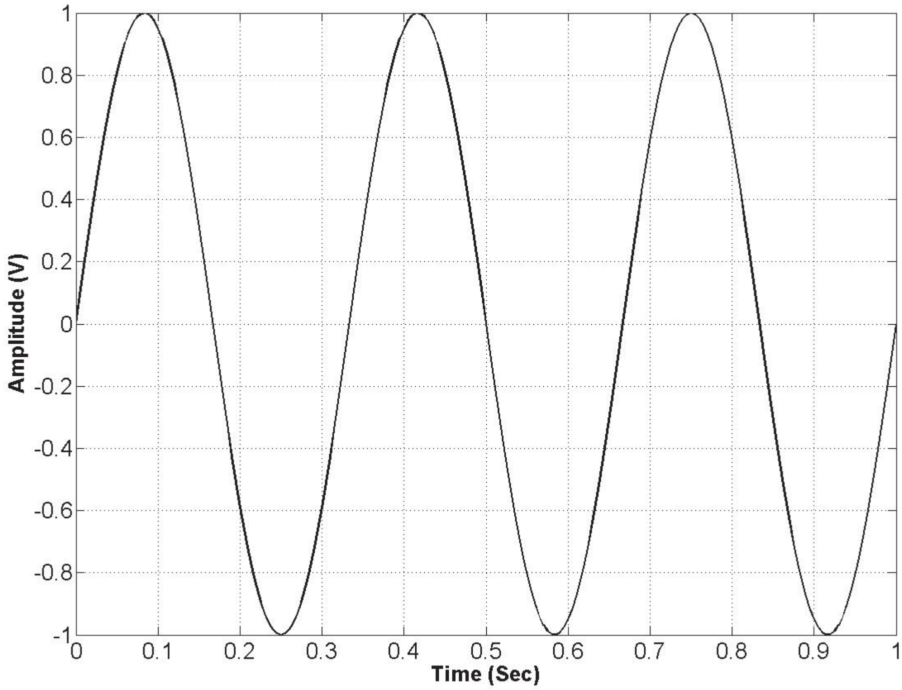


Figure 1: A stationary signal in time domain.

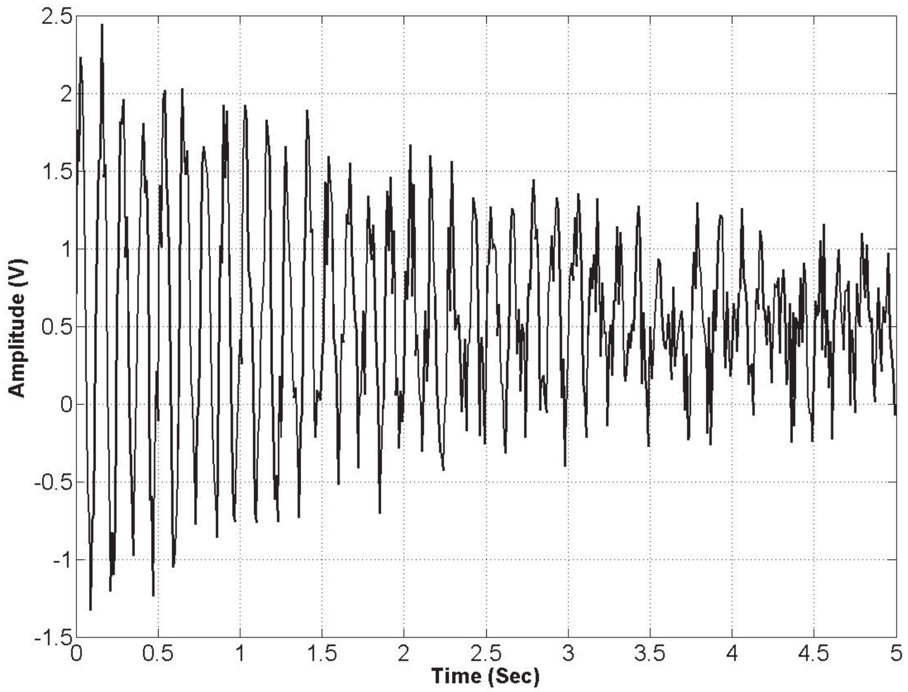


Figure 2: A non-stationary signal in time domain.

Time information may not be important in case of stationary signals but it may be important in case of non stationary signals [4]. To illustrate this point two different sinusoid signals having the frequencies of 20 and 80 Hz and with magnitude 1 Volt have been produced. Based on those two signals two new signals have been produced see figures 3 and 4. Matlab standard function auto power spectrum is used as spectrum analyzer for the signals shown in figure 1 and figure 2. For noisy signals, PSD is efficient function for spectrum analysis [2].
The Welch single-sided power spectral density estimator is given by [5]:

$$PSD_{xx}(k) = \frac{2}{F_s L \sum_{n=0}^{N-1}(w(n))^2} \sum_{l=0}^{L-1} \left| \sum_{n=0}^{N-1} x_l(n) w(n) \mathrm{e}^{-j2\pi kn/N)} \right|^2, 0 < k < N/2-1 \tag{1}$$

Where Fs is the sampling frequency, L is the number of periodograms and N is the length of the periodogram.

A Flattop window is used with block length equal to the length of the signal. A window dependant scaling factor is used to obtain the correct magnitude of the spectrum for each periodic component.

$$A_w = \frac{2}{(\sum_0^{N-1} W(n))^2} \tag{2}$$

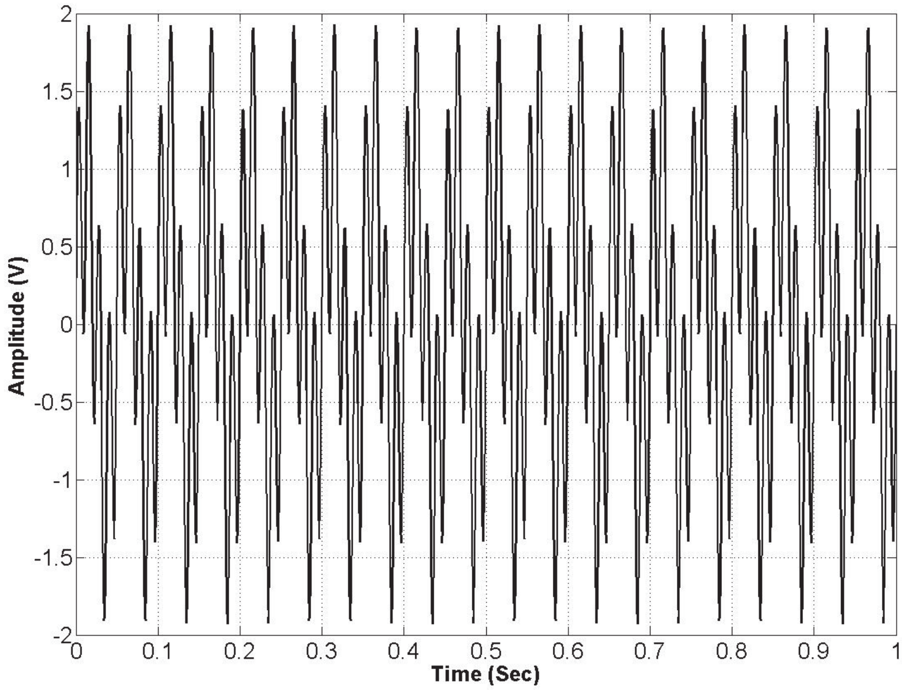


Figure 3: Time domain signal having two frequency components at all time (20Hz and 80Hz).

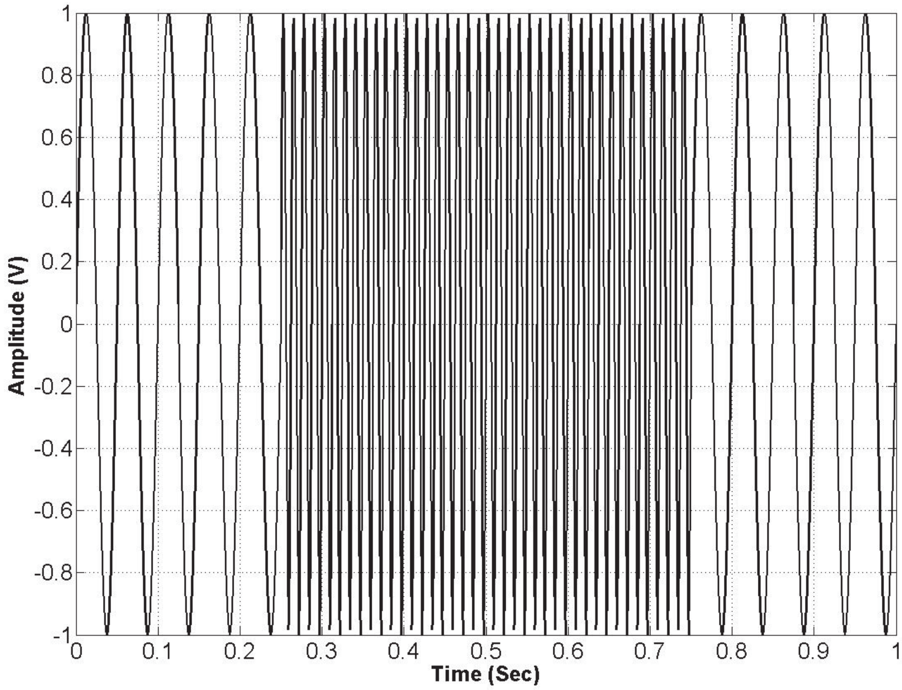


Figure 4: Time domain signal having two frequency components at different time (20Hz, 80Hz and 20Hz).

Figure 5 and 6 are the estimates of the spectra for the time signals shown in figures 3 and 4 respectively. From these two figures it is noticed that although the time signal in figure 3 has two frequency components at all time and figure 4 has them in different times, their frequency spectrum contain energy at same frequencies.

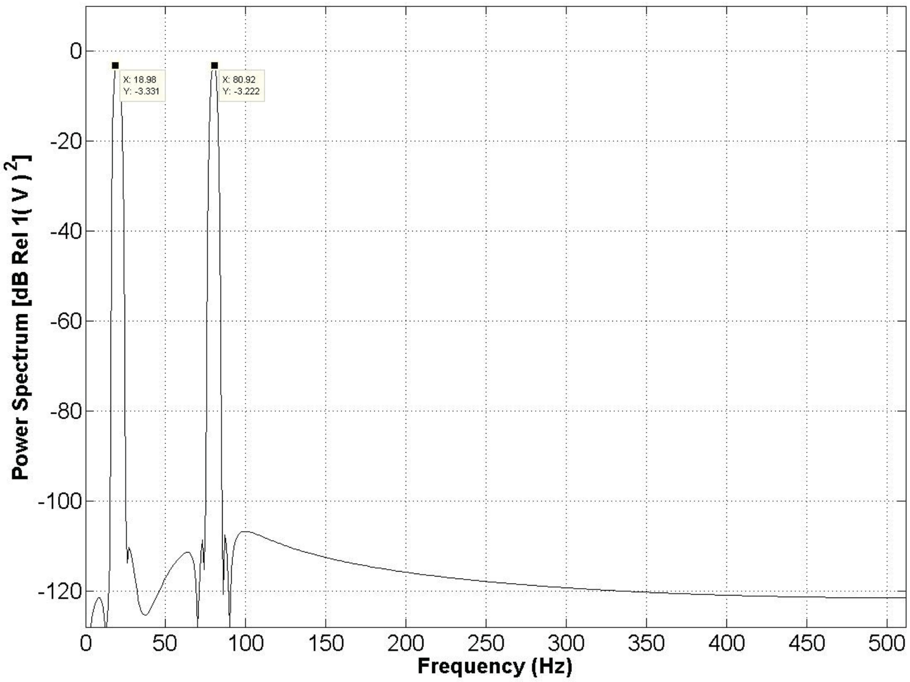


Figure 5: Power Spectrum of the signal having two frequency components at all time shown in figure 3.

The spectrogram for the time domain signals in figures 3 and 4 are shown in figures 7 and 8 respectively. Different window lengths have been tried and the window length of 128 with an overlap of 100 were selected for the spectrogram estimation. The spectrogram is a 3D representation of the spectra along with a time information on one of axis. Two parallel lines in figure 7 represent the frequency content of the signal in figure 3 which last for whole time period, an indication that two frequencies are present all the time. This is what one expect to see in the spectrum after the STDFT application. Figure 8 shows the STDFT spectrum of the signal having two frequencies at different time interval shown in figure 4. It is clear that two frequency components lie at their respective time of occurrence.

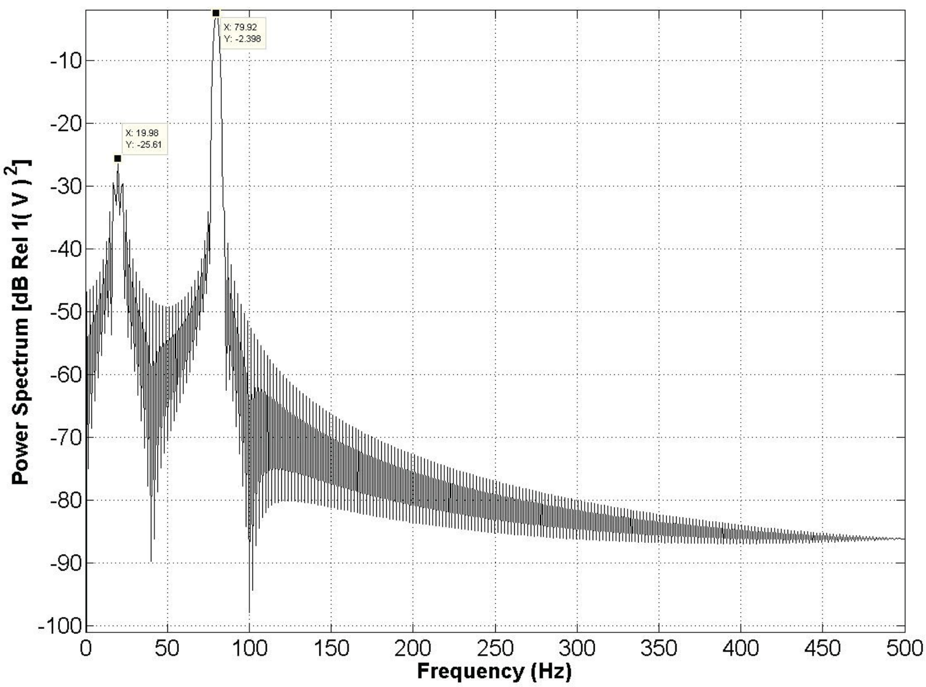


Figure 6: Power Spectrum of the signal having two frequency components at different time shown in figure 4.

The Wavelet Transform may be applied to the time domain signals in figure 3 and 4 in the form of scalogram plots and resulting spectra are shown in figure 9 and 10. The Scalogram plot is the time frequency representation of a signal when using wavelet transform [6]. The plots have been made using the wavelet tool box in MATLAB. Scale values on vertical axes of figures 9 and 10 are inversely proportional to the frequency. Frequency value corresponding to each scale value can be computed by the following MATLAB command.

$$F = scal2frq(A,'wname', DELTA); \tag{3}$$

Here $F$ represents the pseudo-frequencies corresponding to the scales given by $A$, $DELTA$ is the sampling period (seconds) and $wname$ is the wavelet function used which is Morlet wavelet in this case.
The Morlet wavelet is given by; [7]

$$\psi(t) = e^{-(\beta^2 t^2)/2} \cos(\pi t) \tag{4}$$

Where $\beta$ is less than 1 and $t$ is the time.

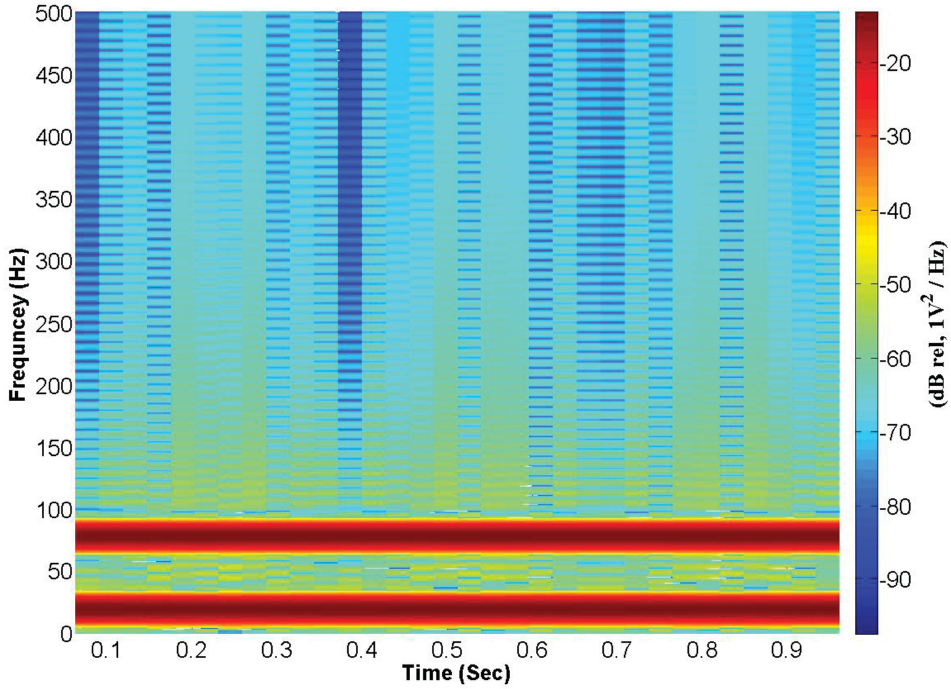


Figure 7: Spectrogram of the signal having two frequency components at all the time (20Hz and 80Hz).

The scalogram can provide the time-frequency features of a signal and is a useful method for fault diagnostics at the early developing stage. If $W_\psi^x(a,b)$ represents the Wavelet Transform of a signal x(t) in (a,b) plane, the scalogram is a measure of the energy distribution $E_x$ over time shift "$b$" and scaling factor "$a$" of the signal [8].

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |W_\psi^x(a,b)|^2 \frac{dadb}{a^2} \tag{5}$$

If instead of the scaling factor $a$ the frequency value $f = 1/a$ is used, the value $f$ is only the real frequency if $\omega_0 = 2\pi$. It follows with $da = \frac{da}{df}df =$ -$\frac{1}{f^2}df$

$$E_x = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |W_\psi^x(f,b)|^2 df\, db \tag{6}$$

It is possible to divide this total energy into an energy density over time and over frequency. This is achieved by one integration over frequency or time. The energy density over time is defined by;

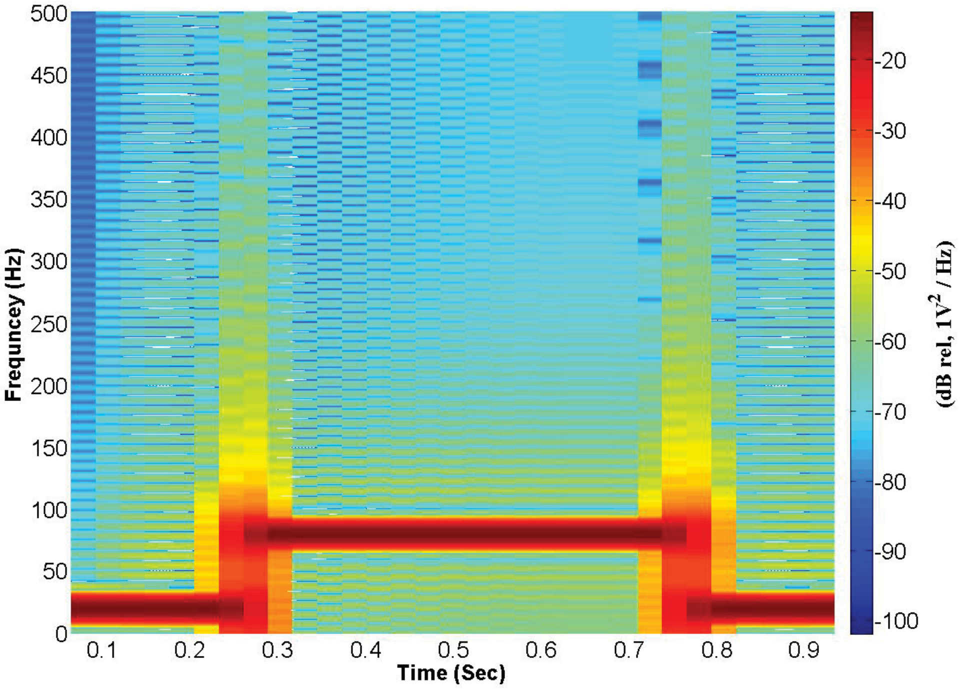


Figure 8: Spectrogram of the signal having two frequency components at different time (20Hz, 80Hz and 20Hz).

$$E_t(b) = \int_{-\infty}^{\infty} |W_{\psi}^{x}(f,b)|^2 df \tag{7}$$

The energy density over frequency, or the energy density spectrum is defined by;

$$E_f(f) = \int_{-\infty}^{\infty} |W_{\psi}^{x}(f,b)|^2 db \tag{8}$$

The wavelet scalogram has been widely used for the analysis of non-stationary signal, and the scalogram can be seen as a spectrum with constant relative bandwidth. In MATLAB, Scalogram has been implemented using Wavelet toolset.

One multi-color line and light white line underneath it in figure 9 represent the low and high frequencies respectively occurring all the time which can be computed by the MATLAB function given in equation 1. However these frequencies occur at different times as one expect and clearly shown in figure

10. For the sake of clarity, scalogram plots and their respective time domain signals are presented in the same plot.

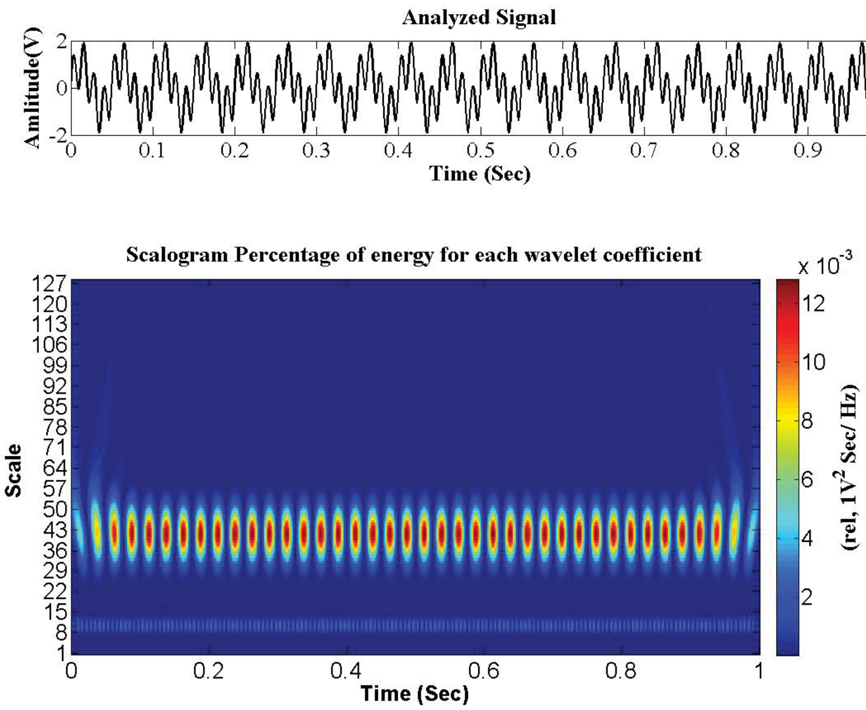


Figure 9: Scalogram of the signal having two frequency components at all time using Wavelet Transform (20Hz and 80Hz).

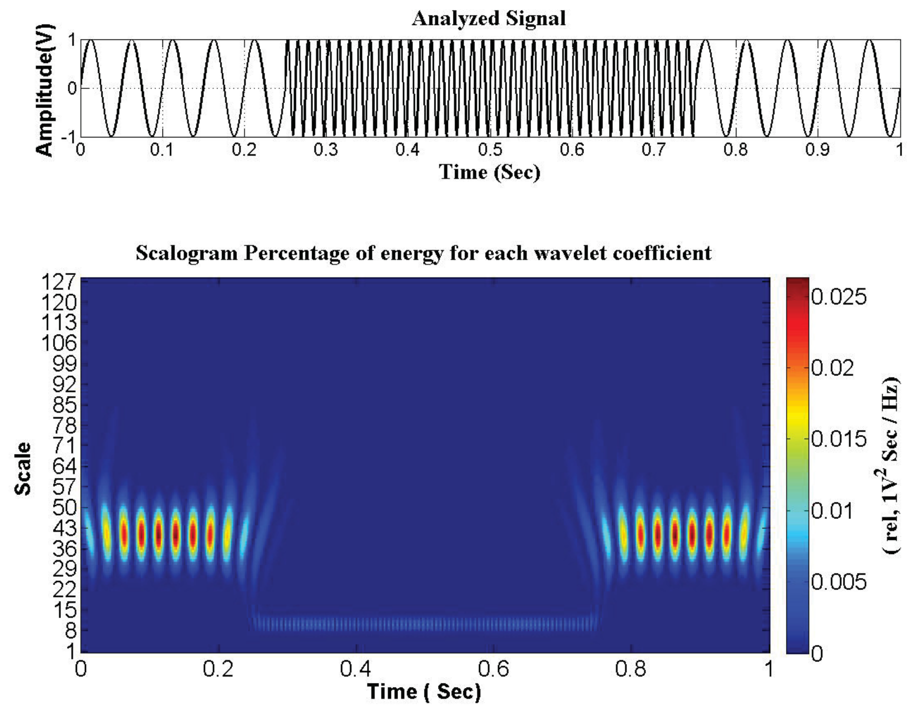


Figure 10: Scalogram of the signal having two frequency components at different time using Wavelet Transform (20Hz, 80Hz and 20Hz).

By comparing figures 8 and 10, it follows that the spectrogram and scalogram provides similar information concerning the non-stationary signal.

## 1.2 Research Questions

The aim of this thesis is to make comparison between Discrete Fourier Transform (DFT) and Wavelet method for vibration data analysis. The research questions for this thesis are

- Performance analysis of Wavelet Transform and Discrete Fourier Transform when dealing with the non-stationary vibration signals.
- What is the significance of using Wavelet Transform instead of Discrete Fourier Transform (DFT) for the analysis of non-stationary vibration signals?

## 1.3 Motivation and Objectives

In Short Time Discrete Fourier Transform (STDFT), a fixed length window is used. This approach is suitable for non-stationary vibration signals [4]. The length of the windowing function determines whether there is good frequency resolution i.e. frequency components close together are separated or good time resolution the time at which frequencies change is clearly indicated. A large window size gives better frequency resolution but poor time resolution and a narrower window size gives good time resolution but poor frequency resolution [4].

Time-Frequency information for a signal can also be obtained using Wavelet Transform which is based on the concept of Multi Resolution Analyses (MRA) [9]. In MRA as its name reveals, a signal is analyzed at different frequency bands with different time resolution. MRA is developed to produce good frequency resolution and poor time resolution at low frequencies and vice versa at high frequencies [7].

The motivation for this research work is to make comparative analysis to the scope and accuracy of Signal Processing transform techniques discussed above when different types of vibration signals are under considerations. A record of vibration data measured on a gearbox with constant shaft rotation speed is used as a reference in this report, See chapter 4 for details. A brief history of DFT, STDFT and Wavelet Transform is presented in chapter 2. Chapter 3 presents a brief study of machine and gearbox vibration with reference to their origin. While chapter 4 and chapter 5 concerns the Discrete Fourier and Wavelet Transform of the vibration data measured on a general industrial gearbox.

# Chapter 2

# 2 DFT, STDFT and Wavelet Transform

This chapter covers a brief introduction to the Discrete Fourier Transform, Short Time Discrete Fourier Transform (STDFT), Spectrogram and Wavelet Transform. It also explains the shortcomings in the rather conventional fourier transform techniques and how these shortcomings are covered with the aid of STDFT and Wavelet Analysis for a variety of signals [4].

## 2.1 FFT as Basic Signal Analyzing Tool

Fourier Transform (FT) is widely used in signal processing in terms of Fast Fourier Transform (FFT) as it is a powerful tool that allows one to analyze a particular signal in the frequency domain. The idea of fourier transform was first put forth by the French mathematician and physicist Jean Baptiste Joseph Fourier [10].
If $f(t)$ is a time domain function(signal) then the frequency spectrum $F(f)$ of this function using Fourier transform, assumed that it exists is given by,

$$F(f) = \int_{-\infty}^{\infty} f(t)e^{-i2\pi ft}dt \tag{9}$$

Fourier Transform is a remarkable platform as it tells what frequencies are present in the signal [11]. As a filter's frequency response can be obtained by taking fourier transform of its impulse response so fourier transform can also be used in designing of filters [6]. Fourier analysis has a family of mathematical techniques based on decomposing signals into different complex frequency components [6]. A signal can be either continuous or discrete and periodic or aperiodic in time. Different combinations of these attributes generate the four categories of the signal. Aperiodic-Continuous signals are continuous time signals like exponential and the Gaussian curve. Fourier way of representing this kind of the signals if it exists is simply called the Fourier Transform. Periodic-Continuous signals are normally sine waves, square waves, and any waveform that is repeated after regular interval of time from negative to positive infinity. This version of the fourier transform is called the Fourier Series. In Digital Signal Processing as the signals where the signals are digitized the Discrete Fourier Transform (DFT) is extensively used [6]. The Fast Fourier Transform (FFT) is basically an algorithm which realizes the DFT in an efficient computational way by reducing the number of multiplications and additions involved. The FFT was first introduced by J. W. Cooley and J. W. Tukey in 1960s [12]. The DFT of a signal x(n) is given by;

$$X(k) = \sum_{n=0}^{N-1} x(n)\mathrm{e}^{-j(\frac{2\pi}{N})nk}, 0 \leq k \leq N-1 \tag{10}$$

Where $N$ is the length of the signal, $n$ is the discrete time(n*Ts), where Ts is the sampling time interval, $X(k)$ is the DFT of signal $x(n)$ and k/n represent the normalized discrete frequencies. The Welch single-sided power spectral density estimator is given by [5]:

$$PSD_{xx}(k) = \frac{2}{F_s L \sum_{n=0}^{N-1}(w(n))^2} \sum_{l=0}^{L-1} \left| \sum_{n=0}^{N-1} x_l(n)w(n)\mathrm{e}^{-j2\pi kn/N)} \right|^2, 0 < k < N/2-1 \tag{11}$$

Where Fs is the sampling frequency, L is the number of periodograms and N is the length of the periodogram.

## 2.2 Time-Frequency Domain Analysis of the Signal

Discrete Fourier Transform has been used as a basic signal analysis tool for decades as it provides spectral characteristics of a signal. Information concerning the occurrence of frequency components vs time in a signal may be important to identify, in certain applications e.g. in fault analysis. To enable that, the idea of Time-Frequency domain representation of the signals has been developed [10].

### 2.2.1 Short Time Fourier Transform

In principle there are two basic approaches to analyze a non-stationary vibration signal in the time domain and in the frequency domain simultaneously. One approach is to split a non-stationary vibration signal at first into segments in the time domain by proper selection of a window function and then to carry out a Discrete Fourier Transform on each of these segments separately. The second approach is with the aid of Wavelet Transform method. The Short Time Discrete Fourier Transform is one of the most straightforward approaches for performing time-frequency analysis and might help to understand the concept of time-frequency analysis. The Short Time Fourier Transform is given as [13].

$$STFT(\tau, \omega) = \int_{-\infty}^{\infty} f(t)w(t-\tau)\mathrm{e}^{-j\omega t}dt \tag{12}$$

Where $x(t)$ is a time domain signal and $w(t-\tau)$ is window function.
The STDFT is generally produced with the aid of Fast Fourier Transform (FFT).

$$X(k, lD) = \sum_{n=0}^{M-1} x(n)w(n-lD)\mathrm{e}^{-j2\pi\frac{k}{N}(n-lD)} \tag{13}$$

Here D is delay, $l$ is any integer, n is the discrete time on which sampled signal has some values. Here w(n) is a discrete time window suitable for the particular analysis defined as:

$$w(n) = \left\{ \begin{array}{ll} w(n), & 0 \leq n < N \\ 0, & \text{Otherwise} \end{array} \right\} \tag{14}$$

The Spectrogram estimate for a signal x(n) may be produced as:

$$PSD_{xx}(k, lD) = \frac{2}{F_s \sum_{n=0}^{N-1}(w(n))^2} \left| \sum_{n=0}^{M-1} x(n)w(n-lD)\mathrm{e}^{-j2\pi\frac{k}{N}(n-lD)} \right|^2, 0 < k < N/2-1 \tag{15}$$

Spectrogram function is used in MATLAB to apply STDFT. Spectrogram divides the signal automatically into segments and then apply fft.

The Short Time Fourier Transform (STFT), also called the windowed Fourier Transform or the sliding window transform, segments the time-domain signal into several disjointed or overlapped blocks by multiplying the signal with a window function as shown in figure 11. After that, Discrete Fourier Transform is applied to each block. Because each block corresponds to different time intervals, the resulting STDFT indicates the spectral content of the signal at each time interval. When sliding window is moved, spectral content of the signal are obtained over different time intervals. Therefore, the STDFT is a function of time and frequency that indicates how the spectral content of a signal evolve over time. The magnitudes of the STDFT coefficients form a magnitude time-frequency spectrum, and the phases of the STDFT coefficients form a phase time-frequency spectrum [11]. While the STFT compromise between time and frequency information can be useful [11]. However, many signals require a more flexible approach like one where the window size can be varied to determine more accurately either time or frequency [11]. One class of such signals are the non-stationary signals where a variable window size is needed to cover the abrupt changes in a signal.

### 2.2.2 Wavelet and Multi-resolution Analysis

In the previous section the analysis of a non-stationary signal was discussed using STDFT and Spectrogram. The Wavelet Transform may also be used

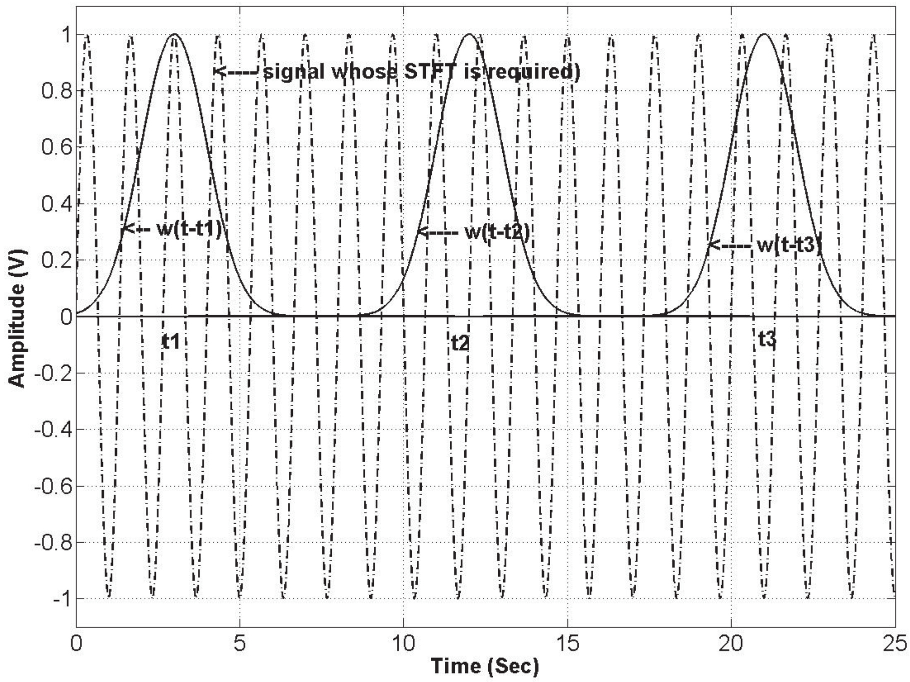


Figure 11: Windowed time domain signal.

for the analysis of non-stationary vibration signals. Wavelet analysis allows the use of long time intervals where more precise low-frequency information is needed, and shorter regions where high-frequency information is needed [14]. Mathematically wavelet transform is the inner product of the signal with a function called wavelet; [15]

$$C(a,b;f(t),\psi(t)) = \frac{1}{\sqrt{a}}\int_{-\infty}^{\infty} f(t)\psi(\frac{t-b}{a})\,\mathrm{d}t \tag{16}$$

Where $a$ and $b$ are the scale and translation (time) parameters respectively and $\psi$(t) is the mother wavelet. The term wavelet represents a small wave and the scale is functionally related to frequency. If $a$>1, the signal is dilated and if $a$<1 signal is compressed. The parameter $b$ is translation parameter which determines the location of wavelet along the time axis [16].
Due to the compression and expansion of the wavelet, the WT performs a time scale decomposition of the signal f(t), thus f(t) might basically work as a set of weighted scaled wavelet functions [4]. Wavelets are functions that can be used to decompose signals. Unlike sinusoids, which are symmetric, smooth, and regular, wavelets can be symmetric or asymmetric, sharp or

smooth, regular or irregular [4].
There are different types of wavelets and the selection of a particular wavelet depend upon a particular feature to be extracted from a signal. For example Morlet wavelet has impulse like response so it is used in detection of transients in the signal (to be discussed in detail in chapter 5). The family of wavelets includes Haar, Dubachies, Morlet, Gussian wavelets etc [17]. Traditionally, the prototype function (wavelet function) is called a mother wavelet. The scaling and shifting of the wavelets determine how the mother wavelet dilates and translates along the time axis. The wavelet transform may be categorized as two classes, the continuous wavelet transform and the discrete wavelet transform. The discrete wavelet transform (DWT) is used for both signal analysis and processing, such as noise reduction, data compression, peak detection etc. Wavelet signal processing is e.g. suitable for non-stationary signals, whose spectral content changes over time [9].

### 2.2.3 Advantages of Wavelet Analysis

The properties of wavelets and the flexibility to select wavelets make e.g wavelet signal processing useful for extraction of different signal features. The Wavelet transform can represent signals sparsely, capture the transient features of signals, and enable signal analysis at multiple resolutions [9]. Because wavelets are flexible in shape and may have short time duration, the wavelet transform may capture transient features precisely [9].

## 2.3 Illustration

This section attempts to illustrate the usefulness of the Wavelet Transform in machine condition monitoring. A signal with the unit Volts is generated according to,

$$X = 40cos(2\pi tF1) + 4sin(2\pi tF2) - 10\delta(t-5) - 20\delta(9-t) \tag{17}$$

Here 0<t<1024, F1 = 4 Hz and F2 = 2 Hz.

$\delta$(t) is delta function which in this case induce discontinuity. Changes are induced in the signal at the time 5 and 9 seconds as shown in figure 12. Transient or abrupt changes are introduced in the signal deliberately at the time 5 and 9 second. This illustration is made just to show which method (DFT, STDFT or Wavelet Analysis) may better pin point transients or abrupt changes in frequency in a signal.

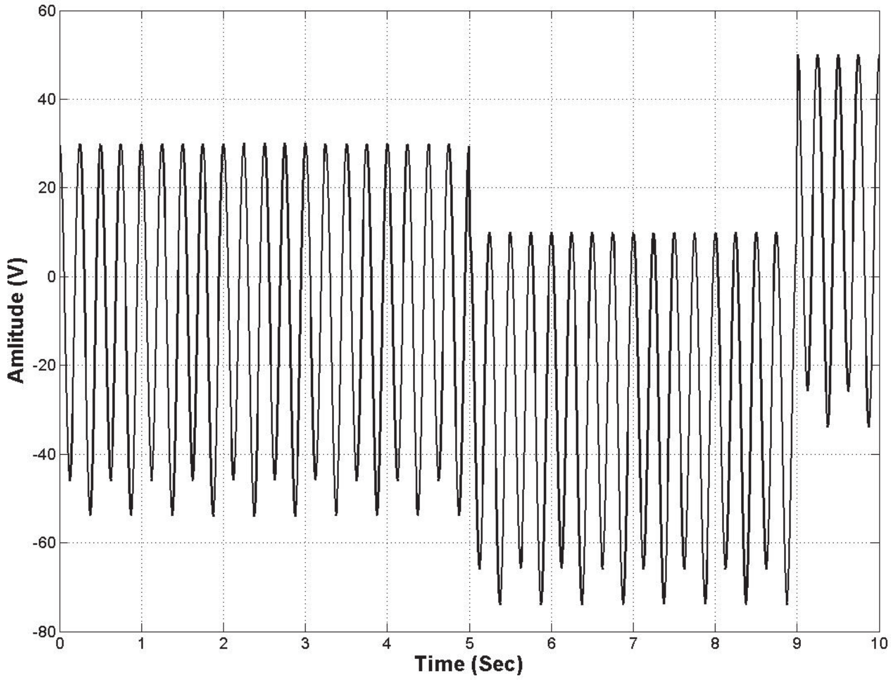


Figure 12: Time signal used in the illustration of DFT and Wavelet Analysis.

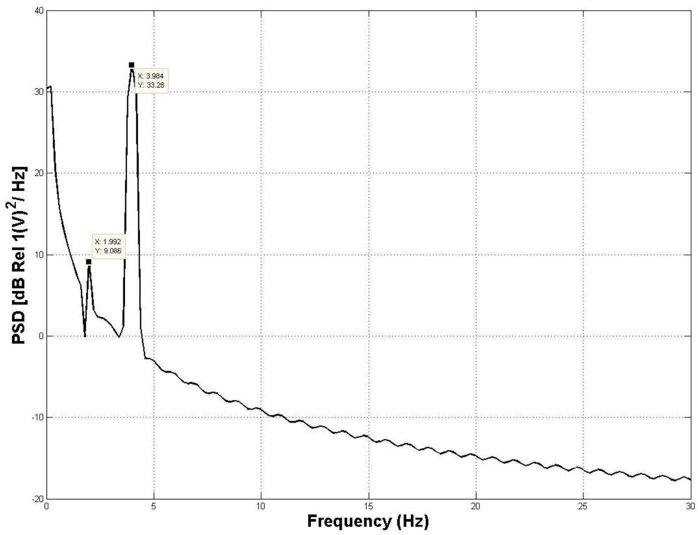


Figure 13: PSD of the signal in example.

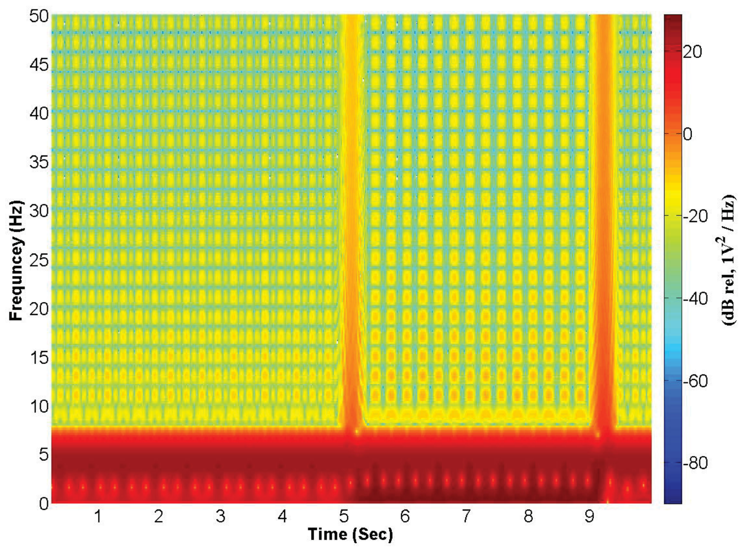


Figure 14: Spectrogram of the signal in example.

Spectrogram, Mesh and scalogram plots of the Wavelet Transform of the test signal are shown in figure 14, 15 and 16 respectively. Mesh plot shows that there are high coefficients values at 5 and 9 seconds in the form of high towers. These high coefficient values represents abrupt change in signal.

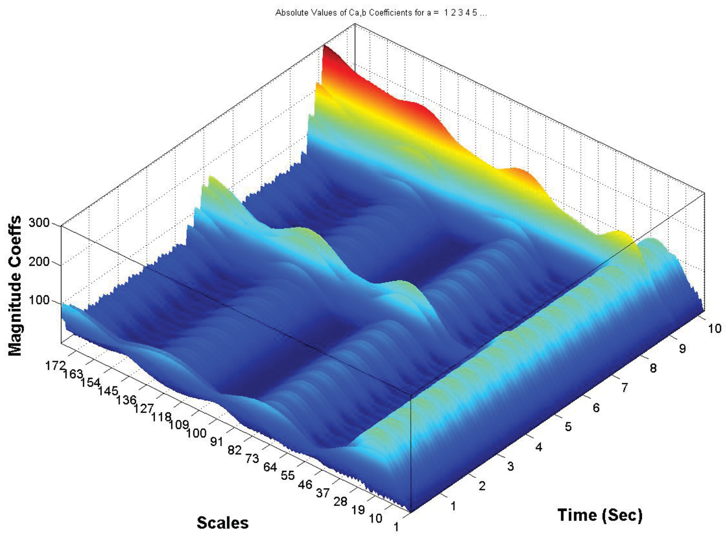


Figure 15: Mesh plot of Wavelet Transform of the test signal.

In figure 16, change in colour intensity at 5 and 9 seconds represents the abrupt changes in test signal. Scalogram and contour plots in Wavelet Analysis are normally used to enable observation of non-stationary behaviour in vibration signals. A scalogram is basically the square magnitude of the Wavelet Transform [18].

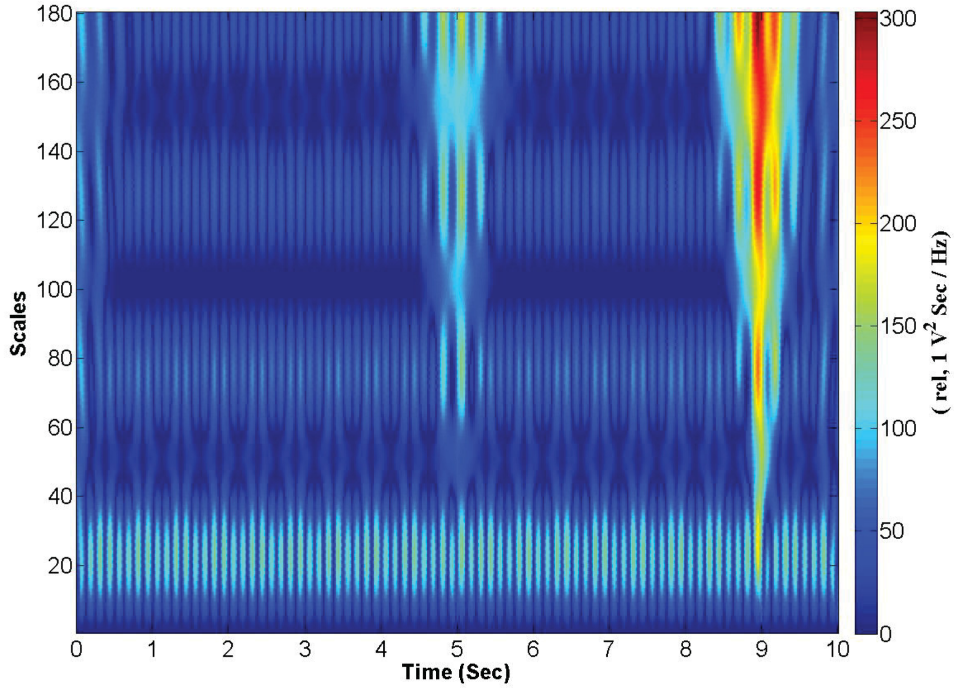


Figure 16: Scalogram of signal to show abrupt change in signal.

# Chapter 3

## 3 Effect of gear dynamics on the vibration and causes of the machine vibration

This chapter is divided into two sections. The first section discusses a brief relationship between gear dynamics and vibration produced and the second section covers the main causes of the machine vibration.

### 3.1 Effect of gear dynamics on the vibration

Gears are widely used in a variety of industries. It is one of the many common mechanisms for motion and/or power transmission from one shaft to another. The geometrical characteristics of gear teeth affect the dynamics and vibration of the gear system, motivating the research on gear dynamics. Many studies have focused on analyzing vibration by developing dynamic models of gear systems which provide understanding of the work status of gear systems and establish a basis for fault diagnosis of gear systems [19]. Dynamic modeling of gear vibration can provide not only the vibration generation mechanisms in gear transmission, but also the dynamic properties of various types of gear faults. With such models, we may find the vibrational behavior of the vibration sources in gear transmission under normal or faulty conditions [19]. Further detail about such models can be found in [19].

### 3.2 Causes of rotating machinery vibrations

This section includes the general study of the factors directly involved in causing machine vibrations. The origin of the vibrations of the machine during operations must be comprehended well enough to better analyze the mechanical vibrations of the machine. Rotating machines specially the asynchronous motors are excited by various phenomena which may cause rotor vibrations. The excitation could be either mechanical excitation (mechanical unbalance) or electromagnetic excitation of different frequencies caused by the rotor eccentricity [20]. An eccentricity between the rotor and the stator magnetic field causes additional air gap eccentricity fields by modulation of the main flux distribution. Such eccentricity fields cause mainly radial magnetic forces on the rotor, which influence the rotor dynamics. Similarly, the design and manufacturing of the machine play an important role in understanding the behavior of the machine vibration and a comprehensive study is presented in [21].

# Chapter 4

## 4 FFT and STDFT Analysis

This chapter starts with a general discussion on analysis methods for different kinds of faults like unbalance, misalignment, looseness etc. in a rotating machine and then discusses the results obtained using Fourier Transform techniques for the vibration analysis. DFT and STDFT is applied to the vibration data taken from the generic industrial gearbox and is involved in a discussion concerning some of the major faults that may occur in such gearboxes.

### 4.1 Fault analysis method of rotating machine

Rotating industrial machines are prone to different kinds of mechanical faults like unbalance, misalignment, looseness in the rotating shaft [15]. A fault measurement methodology based on DFT analysis discussed in [22] is applied to analyze the results in this thesis. In this fault measurement approach, the vibration spectrum during normal (unfault) operation is characterized by a tone located at the shaft rotating frequency, $F_\omega$ followed by a number of harmonics whose amplitudes are generally lower than one third of $F_\omega$ amplitude as shown in figure 17 [22].

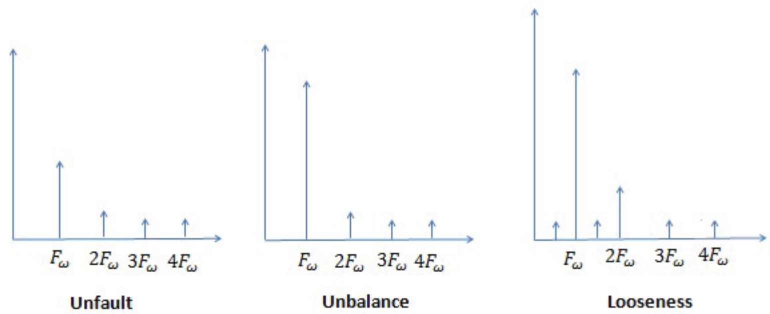


Figure 17: Quantitative vibration spectrum analysis in unfaulty condition and in presence of some mechanical faults.

A shaft unbalance causes a high intensity radial vibration at $F_\omega$, with an amplitude depending on the stiffness in the direction of analysis. The effect on the spectrum is a remarkable increase in the amplitude of the tone at

$F_\omega$. A misalignment is the main cause of in the radial vibration frequency equal to $2F_\omega$. In this case, the tone amplitude at 2 $F_\omega$ usually exceeds 75% of the amplitude of the $F_\omega$ tone, and even of 150% in case of serious damage. A mechanical looseness in the bearing cap or support is characterized by a large number of harmonics of the shaft frequency $F_\omega$. Other types of looseness involve the support of the whole machine; in this case the effect is simply an increase of the $F_\omega$ tone amplitude [22].

## 4.2 FFT and STDFT analysis of the vibration from a generic industrial gearbox

For all the machines, the vibration signal is a composite signal of a fundamental frequency and several other harmonics which is due to the shaft speed plus random noise [23]. Generally, any fault in a rotating system causes energy to be increased in the vibration signal measured on it. Vibration measured on a faulty rotating machinery and plotted in the time domain, the information about such faults may not be obvious from them but frequency domain analysis e.g, DFT may reveal information regarding faults by plotting the DFT Magnitude Vs Frequency [18].

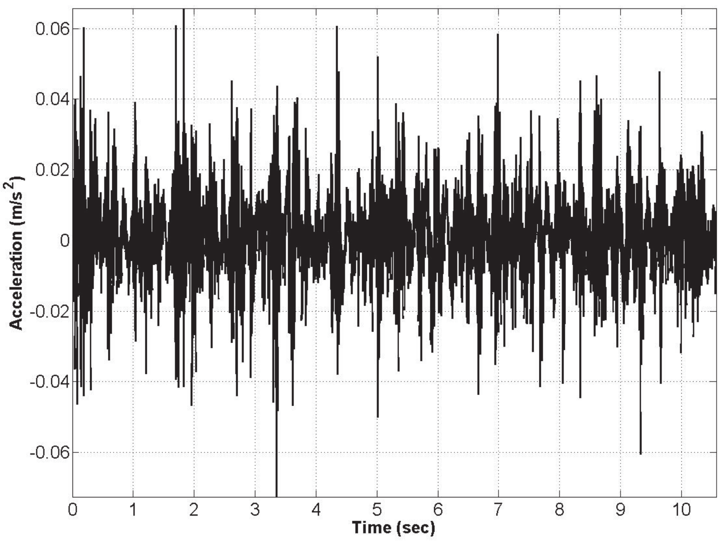


Figure 18: A gearbox vibration signal.

Figure 18 is time domain plot of vibration measured on generic industrial gearbox having a shaft rotational speed of 30 Hz or 1800 rpm. Data were recorded with a 66,666.67Hz sampling rate from accelerometers mounted on both the input and output shaft retaining plates on the gearbox [24]. Matlab standard function PSD is used for estimating the Power Spectral Density. Hanning window is used with length of $2^{17}$ and data block length of $1.5*2^{21}$, so number of averages taken are 24.
There are number of frequencies other than the fundamental frequency present in the vibration signal related to the health of the machine. The PSD of a baseline (healthy) vibration signal shows high amplitude at shaft frequency and other frequencies are normally have lower amplitude relative to the peak of fundamental frequency [22][25]. By analyzing vibration signal measured on a gearbox faults like unbalance, misalignment and looseness etc. may be detected.

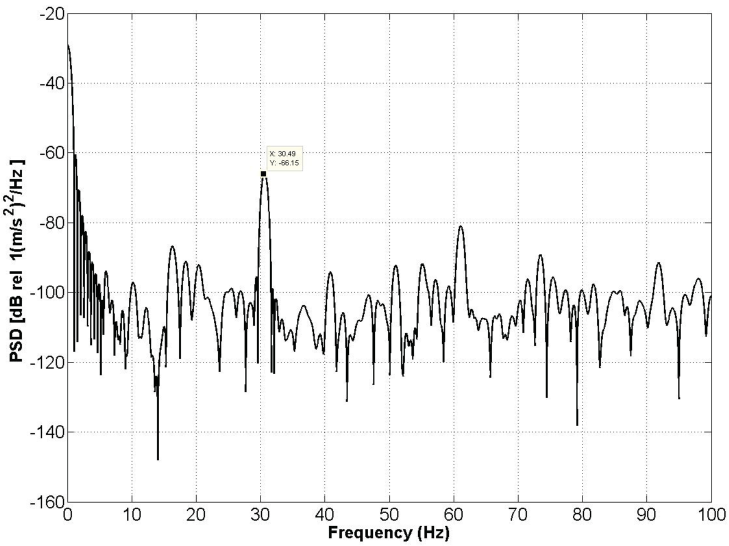


Figure 19: Power Spectral Density of vibration measured on a healthy gearbox.

Faults like unbalanace and looseness in the shaft are better pin pointed by DFT, however, for the sake of comparison with STDFT, spectrogram plots are also presented. High color intensity at 30 Hz in figure 20 shows the shaft speed but it is not very clear due to fixed window length.

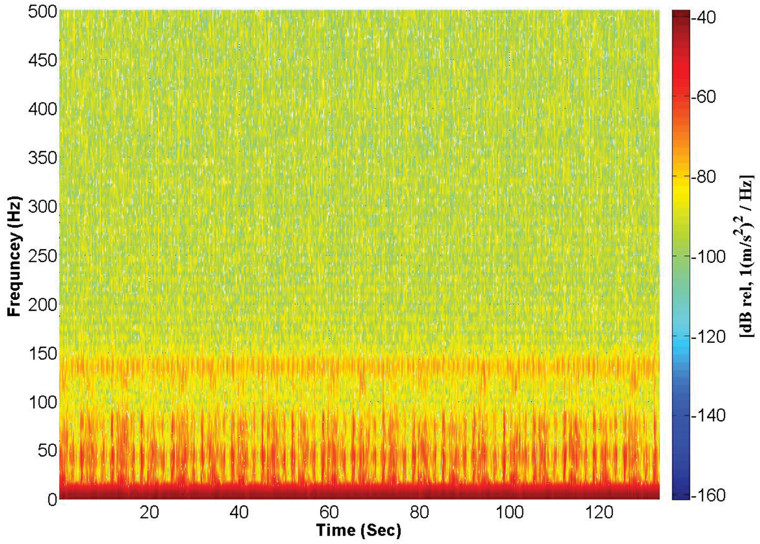


Figure 20: Spectrogram of vibration measured on a healthy gearbox.

### 4.2.1 Unbalance

Figure 19 shows the baseline or healthy spectrum of the vibration from a healthy gearbox. If a gearbox shaft is unbalanced, the vibration spectrum for a gear box will display a higher amplitude at the shaft frequency as compared to the baseline frequency shown in the figure 20 [22].

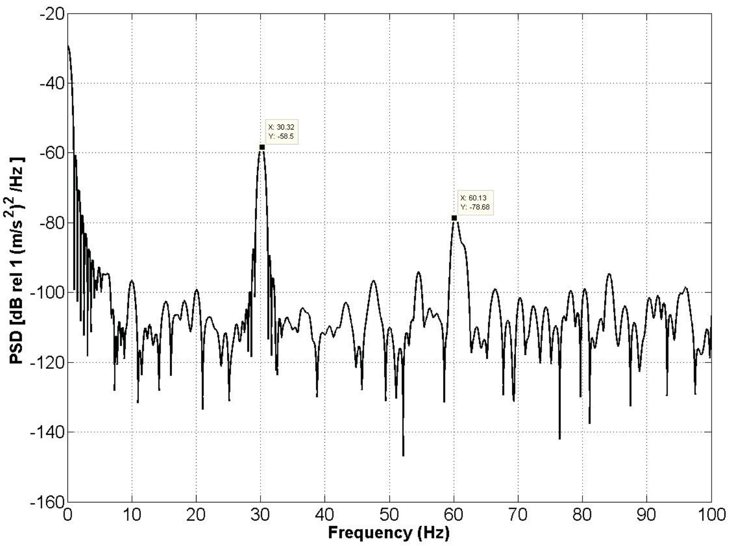


Figure 21: Power Spectral Density of vibration measured on a gearbox with unbalance in the shaft.

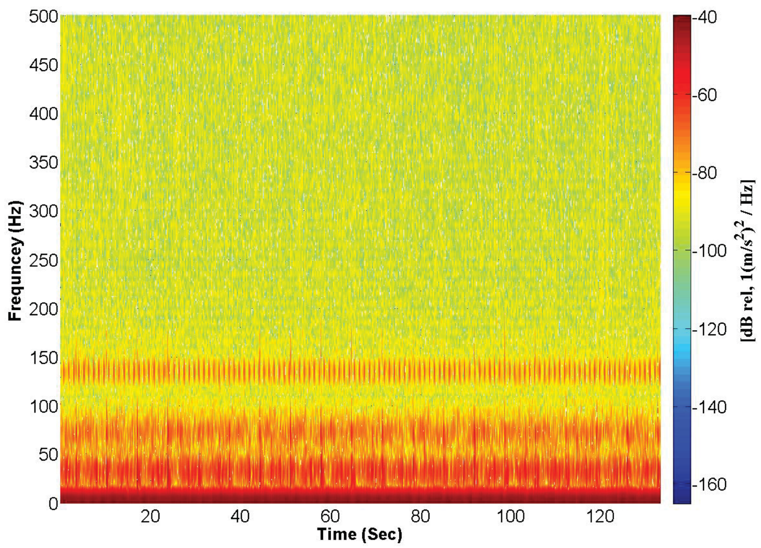


Figure 22: Spectrogram of vibration measured on a gearbox with unbalance in the shaft.

### 4.2.2 Looseness

Looseness of a rotating shaft is characterized by the presence of large number of harmonics and subharmonics around the shaft frequency. In Figure 23

PSD of the vibration measured on the gearbox with shaft looseness is shown.

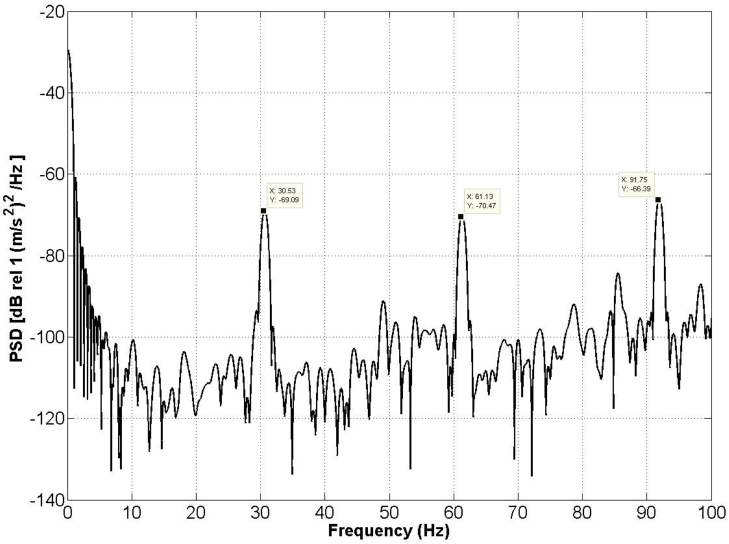


Figure 23: Power Spectral Density of vibration measured on the gearbox with shaft looseness.

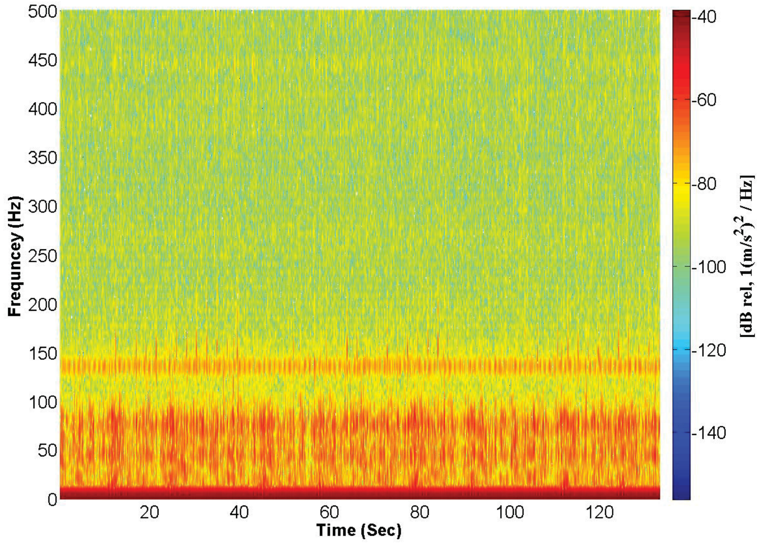


Figure 24: Spectrogram of vibration measured on a gearbox with looseness in the shaft.

## 4.3 Conclusion

To address non-stationarities in vibration signals, imposed by machinery faults, the direct application of DFT may not be relevant. Alternatives that are more appropriate to approach such signals with might be the Short Time Discrete Fourier Transform (STDFT) and the Discrete Wavelet Transform [26][27].

# Chapter 5

# 5 Wavelet Analysis

This chapter discusses a brief overview of the Wavelet Transform and its different modes of application in terms of scalogram and contour plots [16]. Wavelet Transform is applied on the vibration data from general industrial gearbox and its scalogram and contour plots are shown in this chapter.
The Wavelet Transform of signal produces both time and frequency information of the signal simultaneously[26]. The basic idea of wavelet is that it analyzes the signal with respect to a scale. Mathematically wavelet transform is the inner product of the signal with a function called wavelet [15]. Wavelet function has certain properties which should be satisfied by any function to qualify for wavelet function. Wavelet must, [28]

- be oscillatory
- Have appropriate length in time
- have an average value of zero

Wavelet-Transform of a signal is an interesting and relatively new tool. Like in Fourier Transform, sinusoids are used as the basis functions, wavelet transform is the decomposition of a signal where an orthonormal family of basis functions may be applied. Unlike a sine wave, a wavelet has its energy concentrated in time. Sinusoids are useful in analyzing periodic and time-invariant phenomena, while wavelets are well suited for the analysis of transient, time-varying signals. However for sampled signals non-staionarities may conveniently be studied with the aid of the DFT via FFT.

## 5.1 Types of wavelet transform

There are two types of wavelet transform which are discussed below.

### 5.1.1 Continuous wavelet transform

Continuous wavelet transform is is so called because it is obtained by continuously comparing the signal with a wavelet at different positions and scale. Mathematically, Continuous Wavelet Transform is given by [29].

$$C(a,b;f(t),\psi(t)) = \int_{-\infty}^{\infty} f(t)\frac{1}{\sqrt{a}}\psi(\frac{t-b}{a})\,\mathrm{d}t \tag{18}$$

### 5.1.2 Discrete wavelet transform

The DWT is derived from the discretization of continuous wavelet transform CWT (a, b) by replacing of scaling and translation parameters by integer power of $2^{\frac{m}{2}}$. It is called dyadic grid. Actually, DWT is determined on a discretized grid of 'm' scales and 'n' discrete time where k is the discrete dialation parameter. Mathematical expression for DWT is given as [30].

$$DWT(m,n) = 2^{\frac{m}{2}} \sum\nolimits_k f(k)\psi(2^m, k-n), \tag{19}$$

Here $k = 0$ to $n-1$.

## 5.2 Different types of wavelet function

Different choices available for the selection of mother wavelet, in fault detection process are Morlet, Gaussian and Haar wavelets etc [16]. A trial and error technique is used to find which wavelet gives best match [31][32].

### 5.2.1 Morlet Wavelet

Morlet wavelet is one of the most commonly used non-orthogonal wavelets. It is very useful for detecting impulsive or short duration patterns in signal [15]. The Morlet wavelet is given by; [7]

$$\psi(t) = e^{-(\beta^2 t^2)/2} \cos(\pi t) \tag{20}$$

Where $\beta$ is less than 1 and $t$ is the time.
It is a symmetrical exponentially decaying cosine function. It resembles with an impulse and that is why it is used for impulse isolation in fault detection techniques [33].

### 5.2.2 Haar Wavelet

Any discussion of wavelets begins with Haar, the first and simplest. The Haar Wavelet is discontinuous, and resembles a step function. It represents the same wavelet as Daubechies db1 [34]. The Haar wavelet family for $t \in$ [0,1] is defined as follows [35].

$$h_i(t) = \left\{ \begin{array}{lll} 1, & for & t \in (\frac{k}{m}, \quad \frac{k+0.5}{m}) \\ -1, & for & t \in (\frac{k+0.5}{m}, \quad \frac{k+1}{m}) \\ 0 & & \text{elsewhere} \end{array} \right\} \tag{21}$$

Integer $m = 2^j$, where j= (0,1, 2..., J), indicates the level of the wavelet. K = 0,1, 2,....., m-1 is the translation parameter. Maximal level of resolution is J. The index $i$ is calculated according the formula $i$ = m+k+1 in case of minimal values m = 1, k = 0. The maximal value of $i$ is $i$ = 2M = $2^{J+1}$. It is assumed that the value $i = 1$ corresponds to the scaling function for Which $h_i = 1$ in [0,1].

### 5.2.3 Maxican Hat Wavelet

This wavelet has no scaling function and is derived from a function that is proportional to the second derivative function of the Gaussian probability density function [34].

$$\psi(t) = (\frac{2}{3}\pi^{-1/4})(1 - x^2)e^{-x^2/2} \tag{22}$$

where $x$ is the sampled signal and $t$ is time.

## 5.3 Mesh Plot

A Mesh plot is a 3-D spectral representation of a signal in Time-Frequency domain or Time-Scale domain. In this section, mesh plots of the gearbox vibration data are discussed.

### 5.3.1 Mesh plot of a Wavelet Transform of vibration measured on a general industrial gearbox

Figure 25 shows a 3-D plot of a Wavelet Transform of vibration data measured on a gearbox. In Figure 21 it may be observed that the magnitude of wavelet coefficients are high at high scales which corresponds to low frequencies,. However the values of the coefficients display low values for low scales and thus at high frequencies.

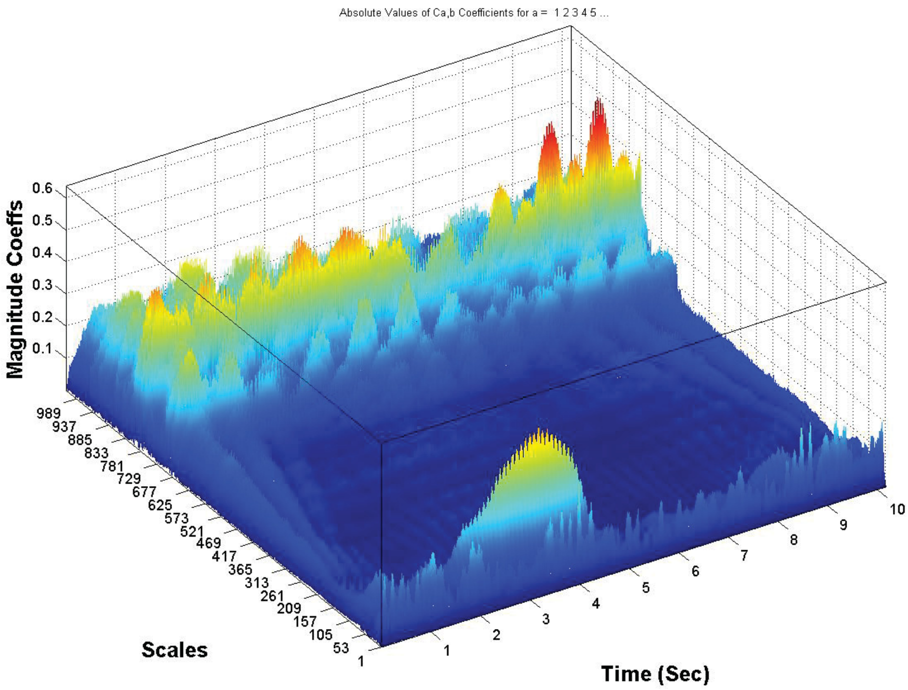


Figure 25: Mesh plot of the Wavelet Transform of vibration signal measured on a generic industrial gearbox.

## 5.4 Scalogram Plot

A scalogram for the wavelet of a vibration signal measured on the gearbox is shown in figure 26. The Scalogram of the signal is basically given by the magnitude squared of the wavelet transform. The contribution of signal energy at any scale $a$ and location $b$ is given by [36][37].

$$E(a,b) = |CWT(a,b)|^2 \tag{23}$$

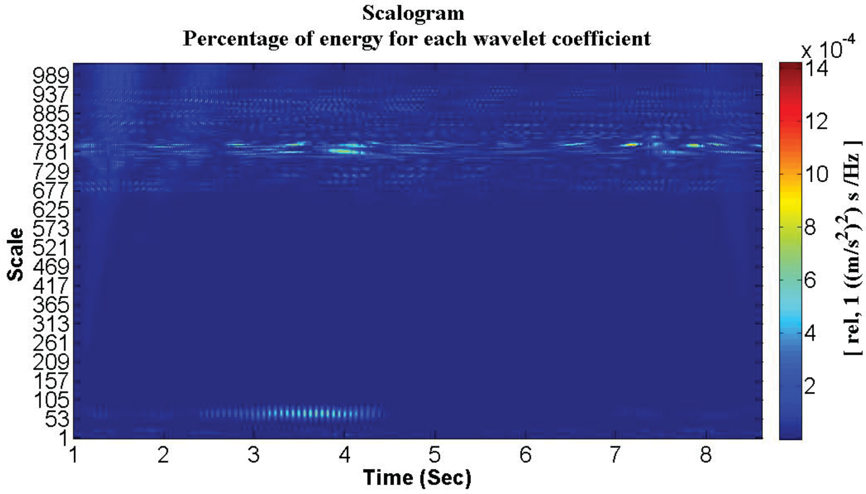


Figure 26: Scalogram plot of the Wavelet Transform of vibration signal measured on general industrial gearbox.

## 5.5 Contour Plot

The contour plot, see figure 23, is another application of Wavelet Transform like Scalogram. The Contour plot shows, where the magnitude of the wavelet Transform dominate in time-frequency plane [18]. Figure 27 shows that the CWT coefficients have high amplitude in scale 760 to 790 (shown by high color intensity) at all the time.

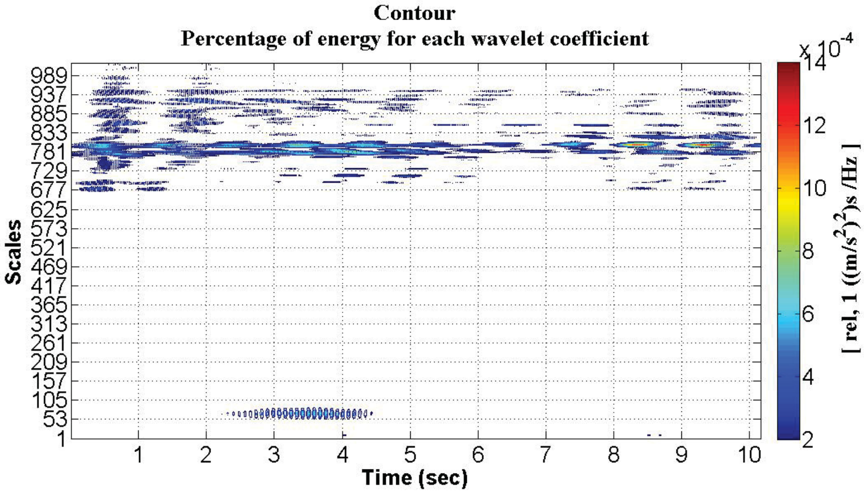


Figure 27: Contour plot of Wavelet Transform of vibration signal measured on generic industrial gearbox

# Chapter 6

# 6 Conclusion and Future Work

## 6.1 Conclusion

In this Masters thesis report, a comparative analysis of Fourier and wavelet transform is made in context of machine's condition monitoring. Both the transforms are carried out on vibration signal measure on a general industrial gearbox and also a number of signals generated in Matlab have been considered. Different machine faults are discussed using different transform techniques. Machine faults like unbalance, misalignment and looseness in the shaft may be addressed with the aid of FFT in terms of Power Spectra and Power Spectral Densities. To address non-stationarities of of signals the DFT and in particular the STDFT and the DWT may be utilized . STDFT uses a fixed window support during the whole transform process. Wavelet transform uses variable window size. Different applications of the wavelet transform like scalogram plot and the contour plot are used to clearly capture the changes in a signals frequency content.

## 6.2 Future Work

Machine condition monitoring can be done using different kinds of data like current, torque, vibration etc. In this thesis work, fourier and wavelet based transform techniques are used to extract different features from vibration data. Wavelet and multiresolution analysis techniques of signal transformation are already in practice in other signal processing fields like acoustics, digital image processing, etc. As morlet wavelet has been used as mother wavelet in this thesis, similar kind of work can be done using other wavelets to compare the efficiency in terms of quality of the features to be extracted. Work can also be done in expanding the available wavelets options to maximize its applications.

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