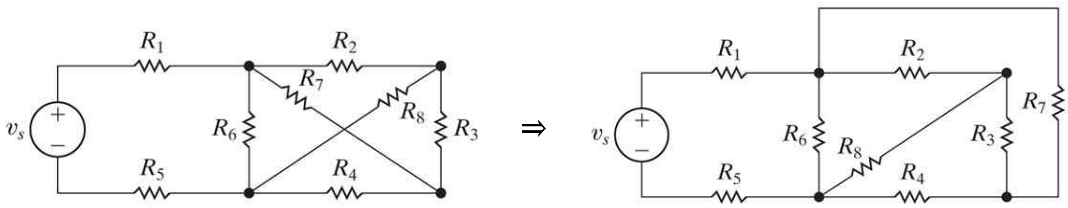
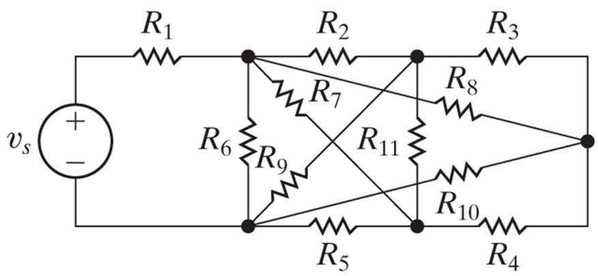
Chap.4 Techniques of Circuit Analysis

§4.1 Terminology

● planar circuit



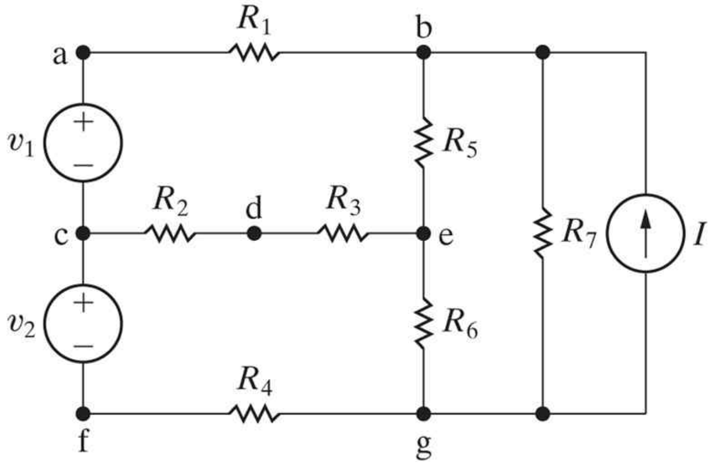
● nonplanar circuit



● terms for describing circuits

Name	Definition
node	A point where two or more circuit elements join
essential node	A node where three or more circuit elements join
path	A trace of adjoining basic elements with no elements included more than once
branch	A path that connects two nodes
essential branch	A path which connects two essential nodes without passing through an essential node
loop	A path whose last node is the same as the starting node
mesh	A loop that does not enclose any other loops
planar circuit	A circuit that can be drawn on a plane with no crossing branches

(Example) nodes, paths, branches, loops, meshes

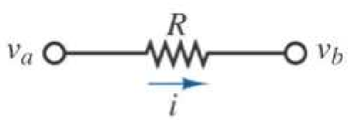


§4-2 Introduction to the Node-Voltage Method

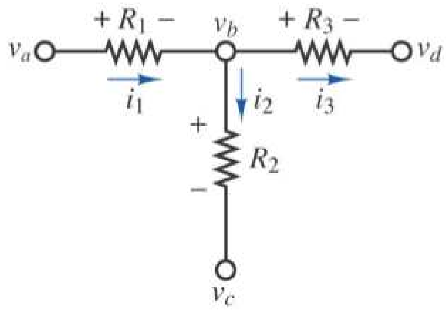
Node Voltage Method

- Select one of the node as the reference node (usually ground).
- Define the voltage at each node as "independent variables".
- Use the Ohm's law to determine the current along the branch between nodes.
- Apply Kirchoff's Current Law (KCL) at each node.

$$i = \frac{v_a - v_b}{R}$$

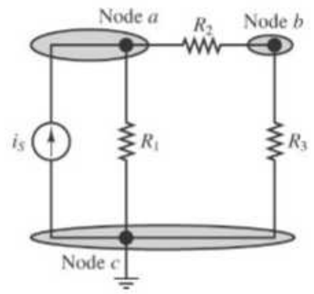


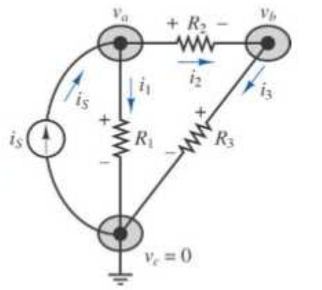
$$\frac{v_a - v_b}{R_1} - \frac{v_b - v_c}{R_2} - \frac{v_b - v_d}{R_3} = 0$$



Steps

1. Select a reference node (usually ground). This node usually has most elements connected to it. All other nodes are referenced to this node.
2. Define the remaining n-1 node voltages as the independent or dependent variables. If the nodes connected to any voltage source, it is a dependent variable.
3. Apply KCL at each node with independent variable (voltage), expressing each current in terms of the adjacent node voltages.





Node Voltage Method: Example I

Step 1:
Choose node c as the reference (ground).

Step 2:
Select variables v_a and v_b for nodes a and b

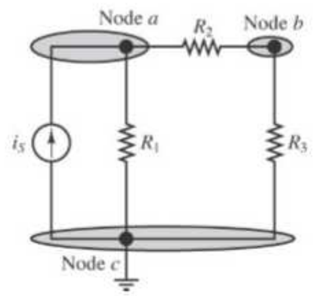
Step 3:
At node a, $i_S - i_1 - i_2 = 0$ $\quad$ $i_S - \dfrac{v_a}{R_1} - \dfrac{v_a - v_b}{R_2} = 0$

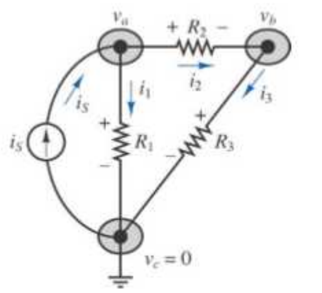
At node b, $i_2 - i_3 = 0$ $\quad$ $\dfrac{v_a - v_b}{R_2} - \dfrac{v_b}{R_3} = 0$

Step 4:

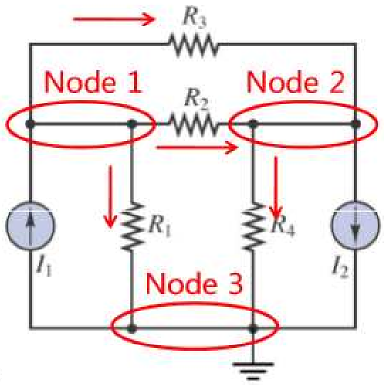
$$\left(\frac{1}{R_1} + \frac{1}{R_2}\right) v_a + \left(-\frac{1}{R_2}\right) v_b = i_S$$

$$\left(\frac{1}{R_2}\right) + \left(-\frac{1}{R_2} - \frac{1}{R_3}\right) v_b = 0$$





Node Voltage Method: Example II



Step 1:
Choose node 3 as the reference (ground).

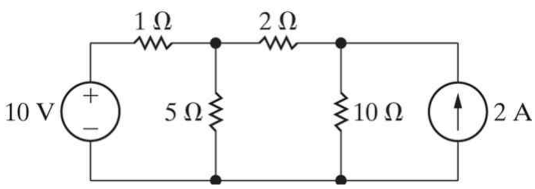
Step 2:
Select variables v_1 and v_2 for nodes 1 and 2.

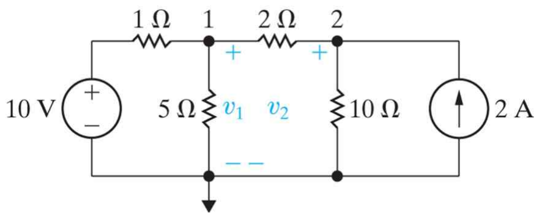
Step 3:
At node 1, $I_1 - \frac{v_1}{R_1} - \frac{v_1 - v_2}{R_2} - \frac{v_1 - v_2}{R_3} = 0$

At node 2, $\frac{v_1 - v_2}{R_2} + \frac{v_1 - v_2}{R_3} - \frac{v_2}{R_4} - I_2 = 0$

Step 4:
$$\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)v_1 + \left(-\frac{1}{R_2} - \frac{1}{R_3}\right)v_2 = I_1$$
$$\left(\frac{1}{R_2} + \frac{1}{R_3}\right)v_1 + \left(-\frac{1}{R_2} - \frac{1}{R_3} - \frac{1}{R_4}\right)v_2 = I_2$$

(Example III)





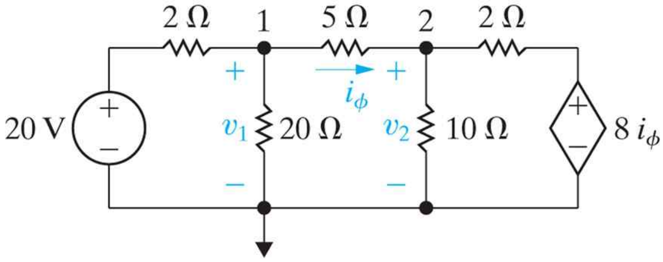
$$\text{node 1 :}\ \frac{v_1 - 10}{1} + \frac{v_1}{5} + \frac{v_1 - v_2}{2} = 0$$

$$\text{node 2 :}\ \frac{v_2 - v_1}{2} + \frac{v_2}{10} - 2 = 0$$

$$\therefore\ v_1 = \frac{100}{11} = 9.09\ \mathrm{V},\quad v_2 = \frac{120}{11} = 10.91\ \mathrm{V}$$

§4.3 The Node-Voltage Method and Dependent Sources

(Example) The node-voltage method with dependent sources



node 1 : $\dfrac{v_1-20}{2}+\dfrac{v_1}{20}+\dfrac{v_1-v_2}{5}=0$

node 2 : $\dfrac{v_2-v_1}{5}+\dfrac{v_2}{10}+\dfrac{v_2-8i_\phi}{2}=0$

\# unknowns: 3. We need one more equation.

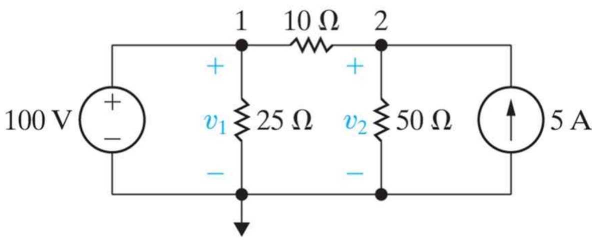
3rd equation : $i_\phi=\dfrac{v_1-v_2}{5}$

$\therefore\ v_1=16\,\mathrm{V},\quad v_2=10\,\mathrm{V},\ i_\phi=1.2\,\mathrm{A}$

§4.4 The Node-Voltage Method: Some Special Cases

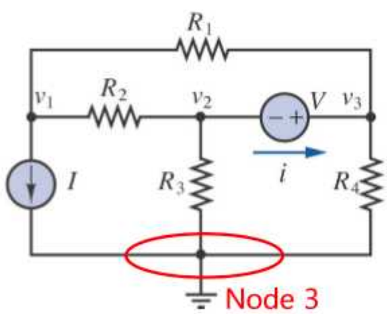
- A circuit with a known node voltage

※ $v_1=100\,\mathrm{V}$, we need only one equation.

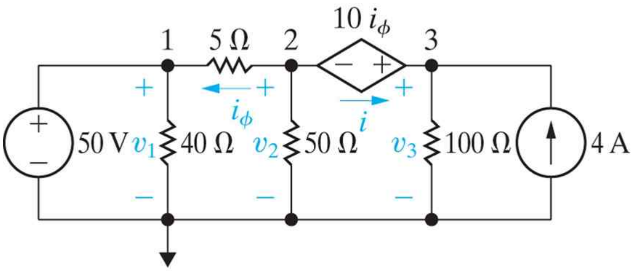


- The Concept of a Supernode

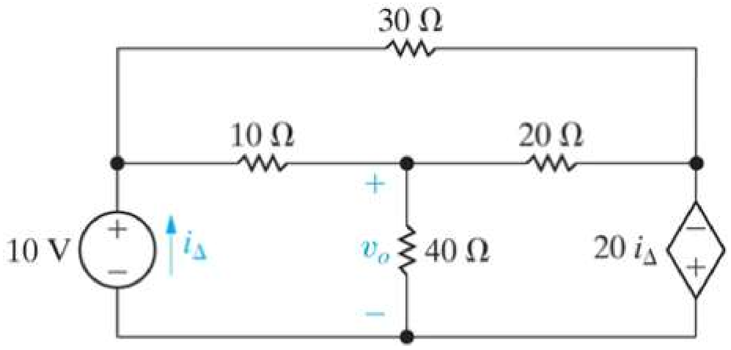
(A circuit with a voltage source connected between nodes)



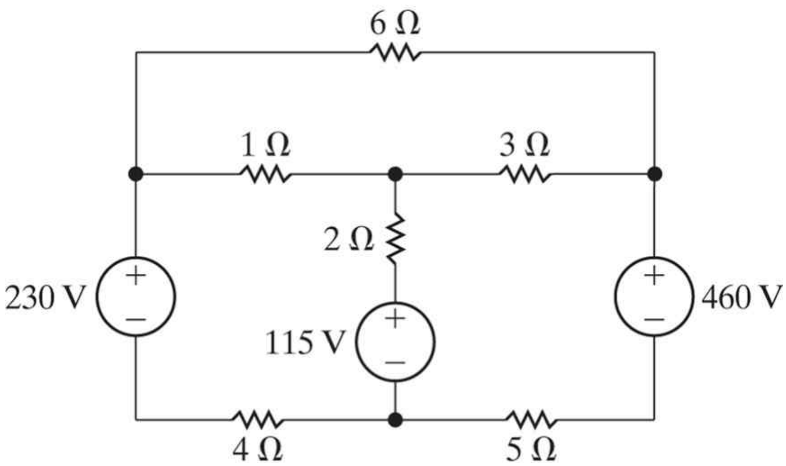
(Example) Supernode



(Example) Find v_o using the node-voltage method.

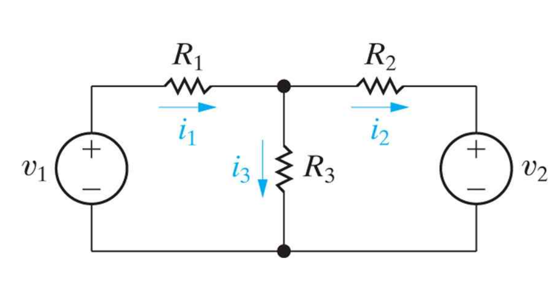


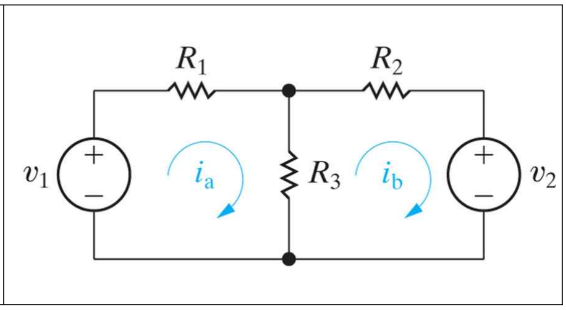
(Example)



§4.5 Introduction to the Mesh-Current Method

(※ It works only with PLANAR circuits.)



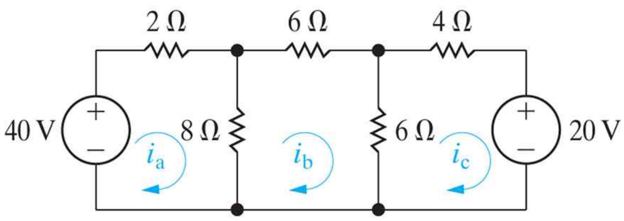


$$i_3 = i_1 - i_2,\quad i_1 = i_a,\quad i_2 = i_b,\quad i_3 = i_a - i_b$$

$$v_1 = R_1 i_1 + R_3 i_3 = R_1 i_1 + R_3(i_1 - i_2) = (R_1 + R_3)i_1 - R_3 i_2 = (R_1 + R_3)i_a - R_3 i_b$$

$$-v_2 = R_2 i_2 - R_3 i_3 = R_2 i_2 - R_3(i_1 - i_2) = (R_2 + R_3)i_2 - R_3 i_1 = (R_2 + R_3)i_b - R_3 i_a$$

(Example) Using the mesh-current method



$$-40 + 2i_a + 8(i_a - i_b) = 0$$

$$8(i_b - i_a) + 6i_b + 6(i_b - i_c) = 0$$

$$6(i_c - i_b) + 4i_c + 20 = 0$$

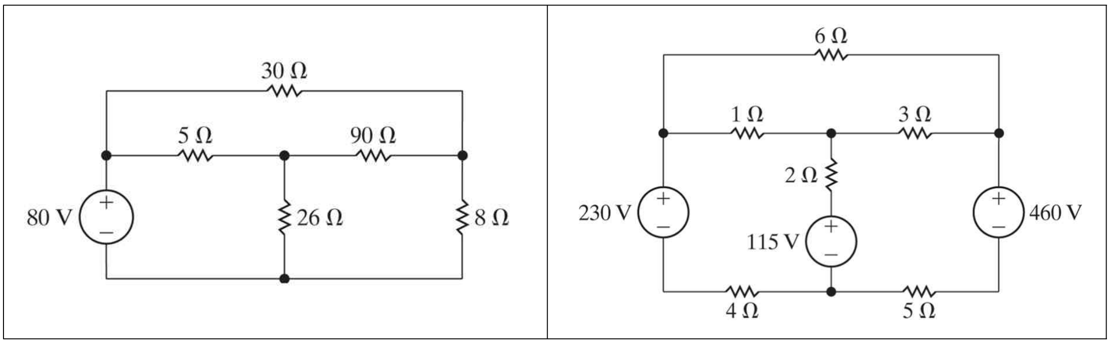
Finally,

$$\begin{aligned} (2+8)i_a - 8i_b \qquad\qquad &= 40 \\ -8i_a + (8+6+6)i_b - 6i_c &= 0 \\ -6i_b \qquad + (6+4)i_c &= -20 \end{aligned}$$

we can represent those equations using matrix

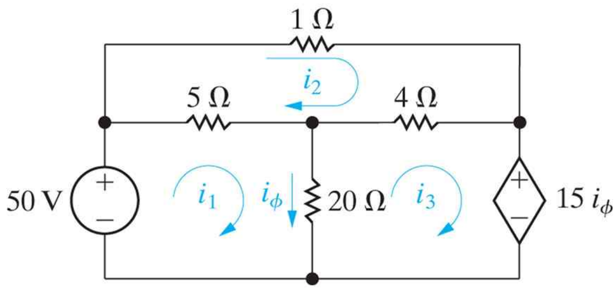
$$\begin{bmatrix} 10 & -8 & 0 \\ -8 & 20 & -6 \\ 0 & -6 & 10 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \begin{bmatrix} 40 \\ 0 \\ -20 \end{bmatrix}$$

(Example) node-voltage method vs. mesh-current method



§4.6 The Mesh-Current Method and Dependent Sources

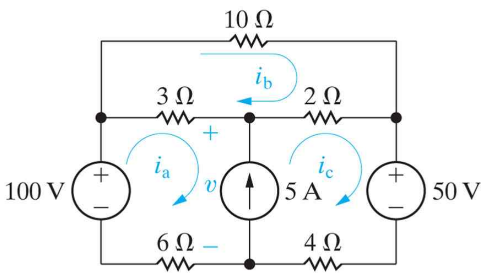
(Example) the mesh-current method with dependent sources



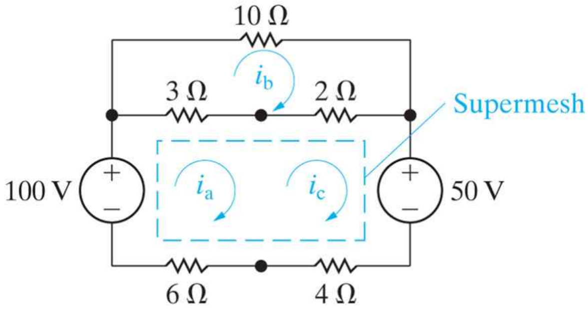
$$50 = 5(i_1 - i_2) + 20(i_1 - i_3)$$
$$0 = 5(i_2 - i_1) + 1i_2 + 4(i_2 - i_3)$$
$$0 = 20(i_3 - i_1) + 4(i_3 - i_2) + 15i_\phi$$
$$i_\phi = i_1 - i_3$$

§4.7 The Mesh-Current Method: Some Special Cases

- A branch includes a current source

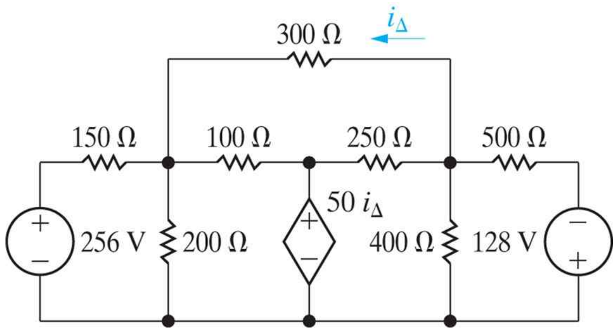
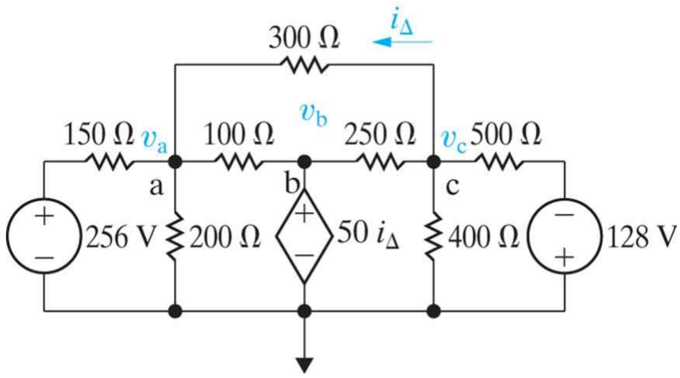
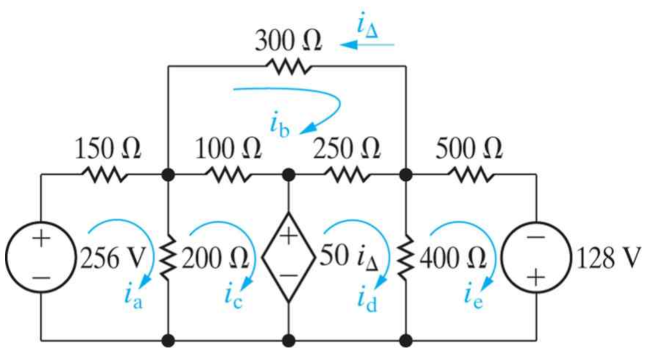


● the concept of a supermesh

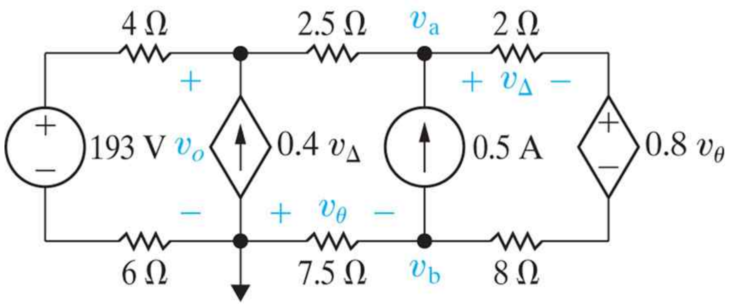


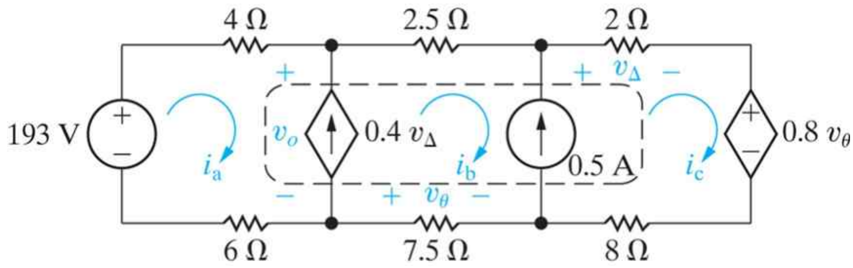
§4.8 Node-Voltage Method Versus Mesh-Current Method

(Example)

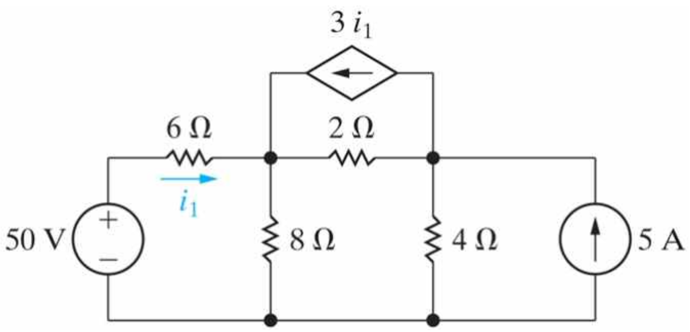
Given Circuit	
Node-Voltage Method	
Mesh-Current Method	

(Example) Node-Voltage Method Versus Mesh-Current Method

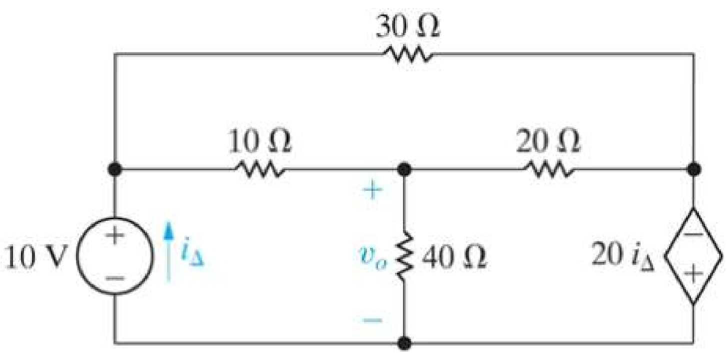




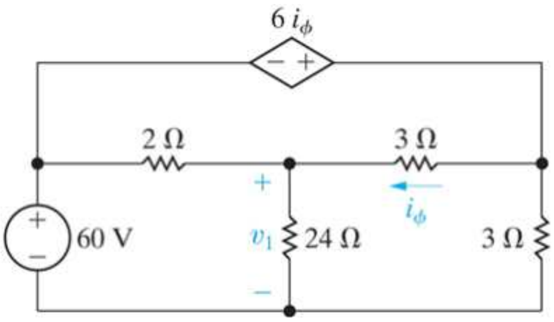
(Example)



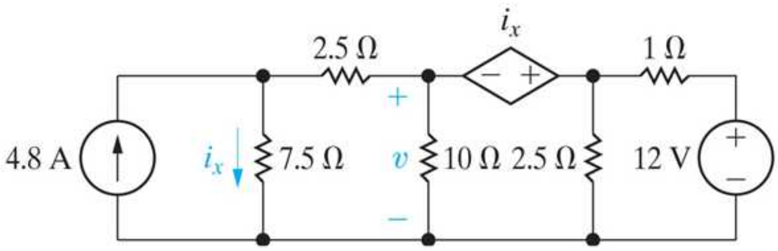
(Example)



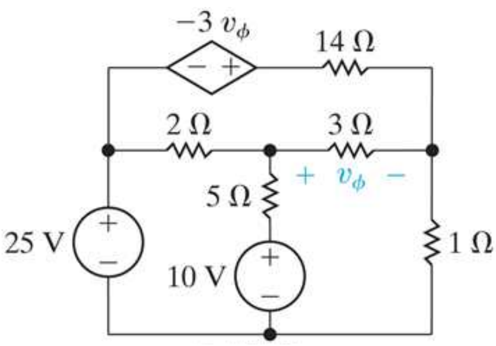
(Example)



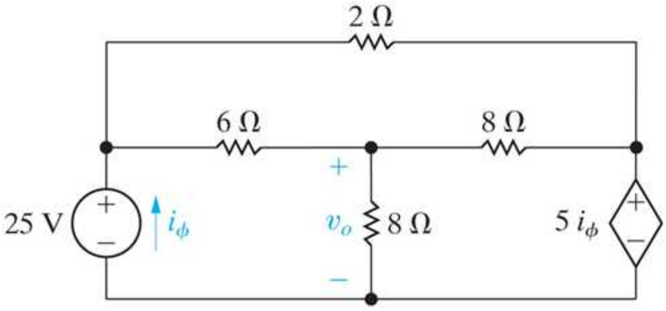
(Example)



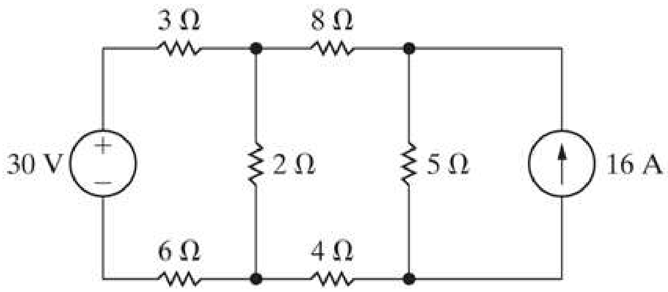
(Example)



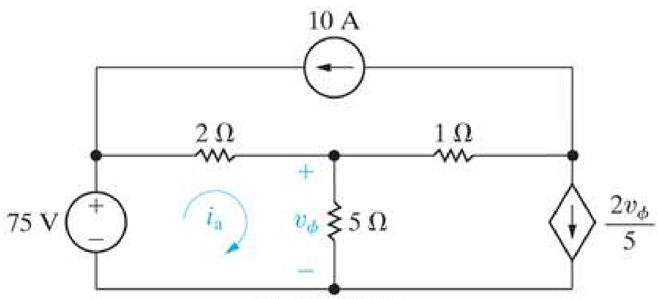
(Example)



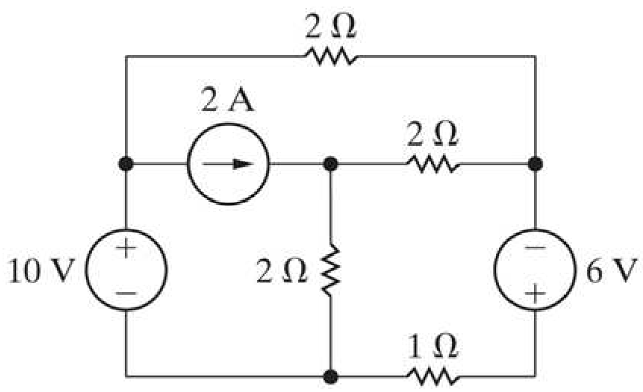
(Example)



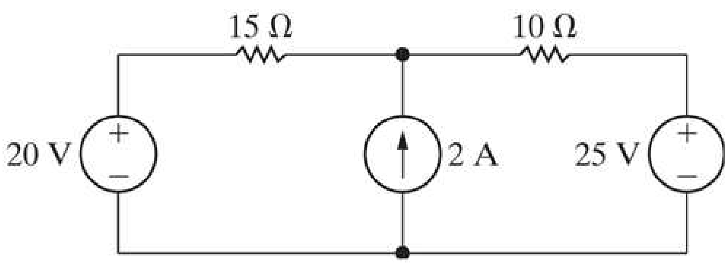
(Example)



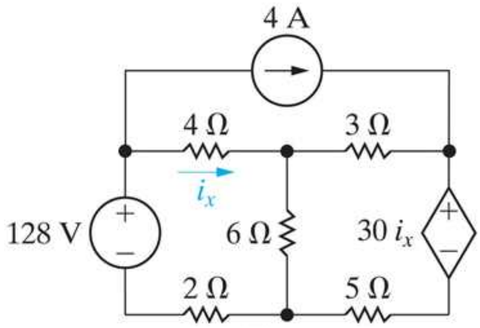
(Example)



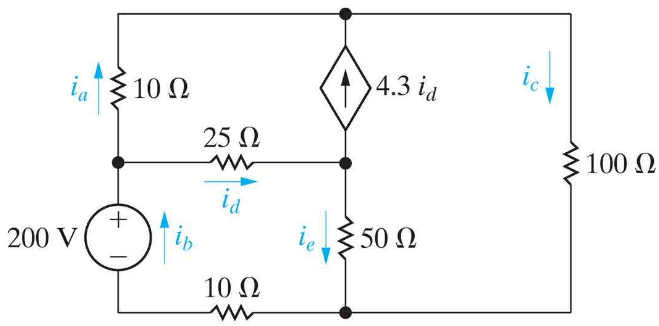
(Example)



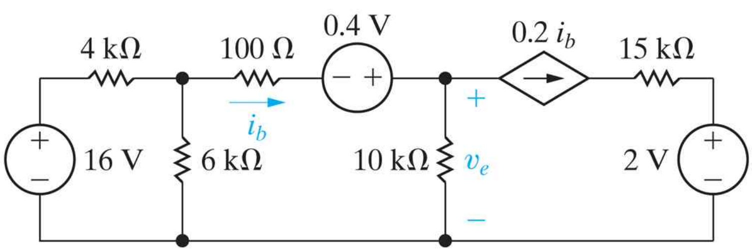
(Example)



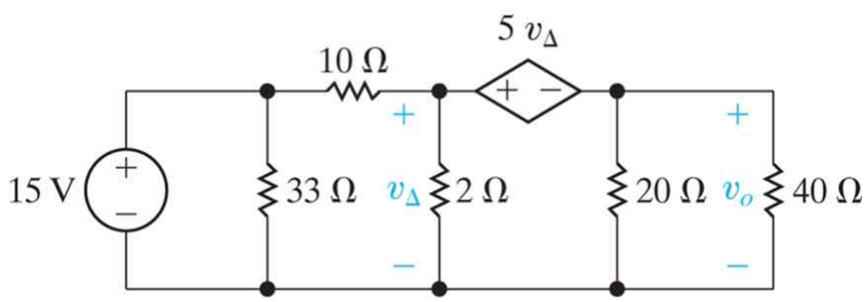
(Example) Find i_a, i_b, i_c, i_d and i_e.



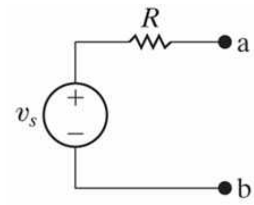
(Example) Find v_e and i_b .



(Example) Find v_o .

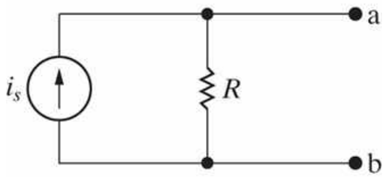


§4-9 Source Transformations

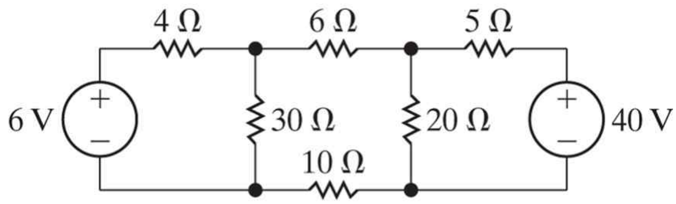


⇔

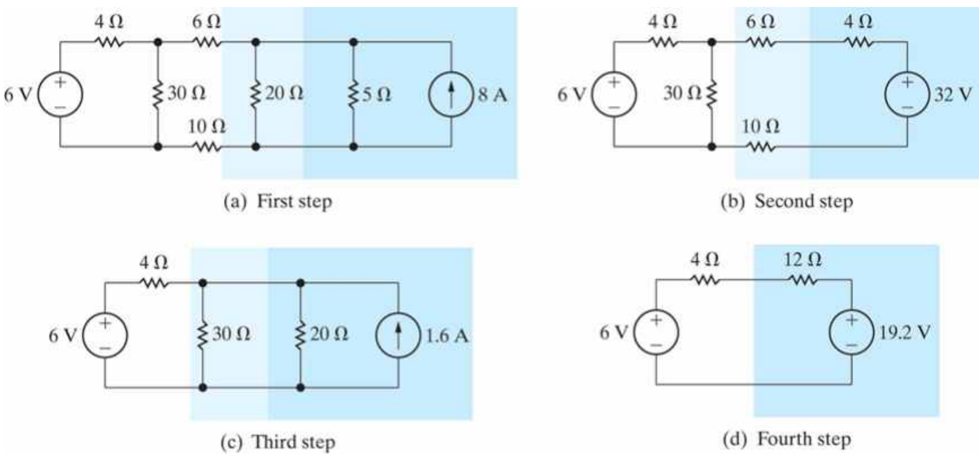
$$i_s = \frac{v_s}{R}$$



(Example) Find the current through 4Ω resistor.



- source transforming



(a) First step

(b) Second step

(c) Third step

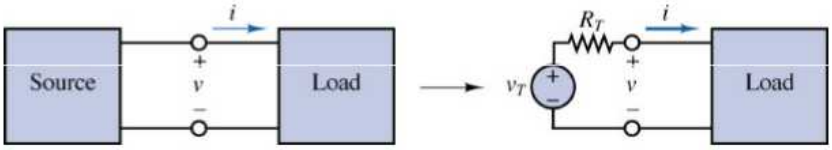
(d) Fourth step

§4.10 Thevenin and Norton Equivalent Circuits

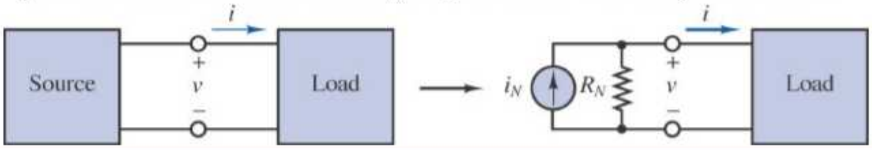
Thevenin and Norton Equivalent Circuits

- The source circuits can be viewed as the equivalent circuits with either voltage source + R (serially connected) or current source + R (parallelly connected).

- Thevenin Theorem
  When viewed from the load, any network composed of ideal voltage and current sources, and of linear resistors, may be represented by an equivalent circuit consisting of an ideal voltage source v_T in series with an equivalent resistance R_T.

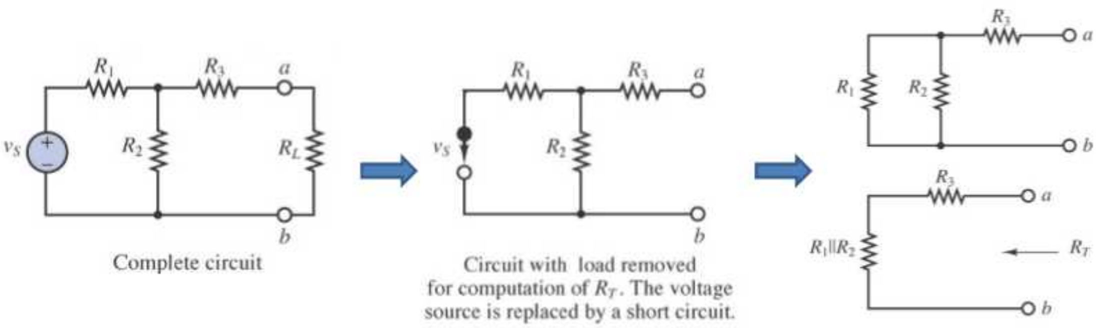


- Norton Theorem
  When viewed from the load, any network composed of ideal voltage and current sources, and of linear resistors, may be represented by an equivalent circuit consisting of an ideal current source i_N in parallel with an equivalent resistance R_N.



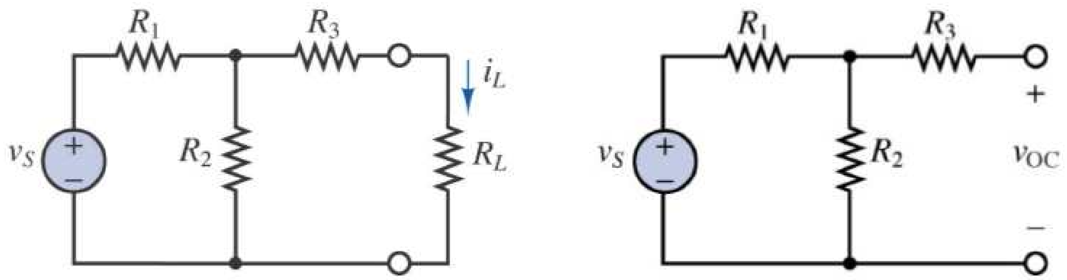
How to Determine Norton or Thevenin Equivalent Resistance?

- The Thevenin and Norton equivalent resistances are one and same quantity.
- We obtain it by the following steps:
  1. Remove the load
  2. Zero all independent voltage and current sources.
  3. Compute total resistance between load terminals with the load removed.



How to Compute the Thevenin Voltage?

- The equivalent (Thevenin) source voltage is equal to the "open-circuit voltage" present at the load terminals with the load removed.
- Procedure:
    1. Remove the load (load terminal: open circuited)
    2. Apply either node voltage method or mesh current method to solve the open circuit voltage (v_{oc}).
    3. The Thevenin voltage is $v_T = v_{oc}$.



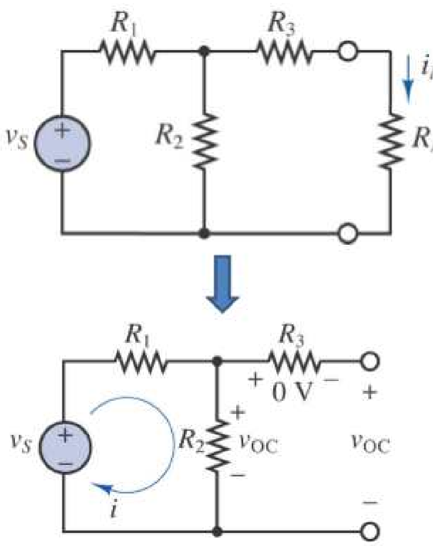
Computation of the Thevenin Voltage: Example I

Step 1:
Remove load R_L.

Step 2:
No current is flown along R_3 (open circuit).
This structure is same as the voltage divider.

$$v_{OC} = \frac{R_2}{R_1 + R_2} v_S$$

Step 3:
Thevenin voltage is $v_T = v_{OC} = \dfrac{R_2}{R_1 + R_2} v_S$



* Thevenin equivalent resistance is

$$R_T = (R_1 \parallel R_2) + R_3$$

Thus, the current i_L through the load in the original circuit is

$$i_L = \frac{v_T}{R_T + R_L} = \frac{1}{\left[(R_1 \parallel R_2) + R_3\right] + R_L} \cdot \frac{R_2}{R_1 + R_2} v_S$$

How to Compute the Norton Current?

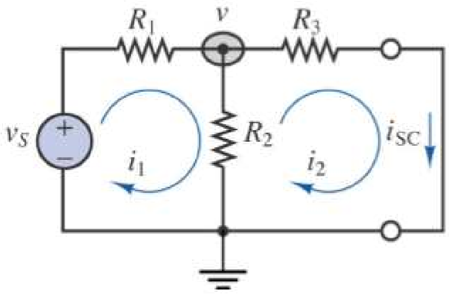
- The equivalent (Norton) source current is equal to the "short-circuit current" that would flow if the load were replaced by a short circuit.
- Procedure:
  1. Replace the load with a short circuit.
  2. Apply either node voltage method or mesh current method to solve the short circuit current (i_{sc}).
  3. The Norton current is $i_N = i_{sc}$.

Mesh 1: $v_S - R_1 i_1 - R_2(i_1 - i_2) = 0$

Mesh 2: $-R_2(i_2 - i_1) - R_3 i_2 = 0$

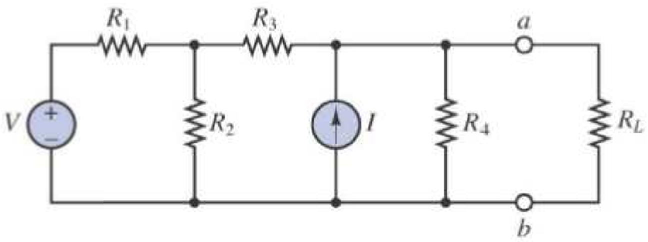
Compute the current i_2.
Then,

$$i_N = i_{SC} = i_2$$

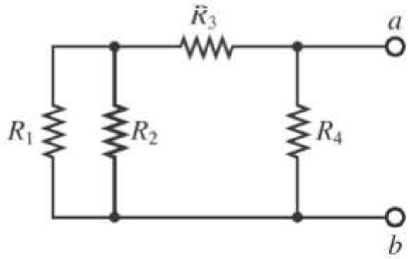


Norton (Thevenin) Equivalent Resistance: Example I

1. Remove the load
2. Zero all independent sources
   V → short circuit
   I → open circuit
3. Compute the total resistance



$$R_T = [(R_1 \parallel R_2) + R_3] \parallel R_4$$



● Summary: How to get Thevenin and Norton equivalent circuits

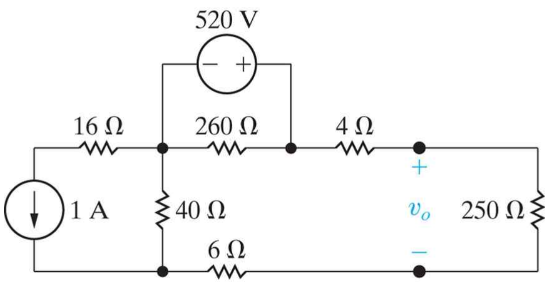
case	Network A	Thevenin & Norton
I	independent sources and resistors	Find two of v_{oc} , i_{sc} , and R_{TH}. Calculate one by $v_{oc} = R_{TH}\, i_{sc}$.
II	independent sources, dependent sources and resistors	Find v_{oc} and i_{sc} . Calculate R_{TH} by $R_{TH} = v_{oc} / i_{sc}$.

(※ $v_{oc} = v_{TH}$ in the Thevenin equivalent circuits)

(※ $i_{sc} = i_N$ in the Norton equivalent circuits)

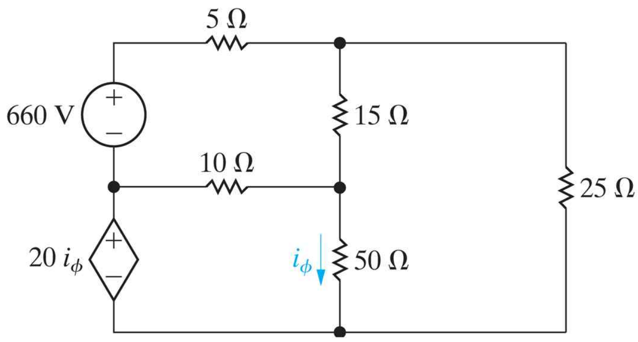
<u>Examples</u>

(Case I) Find the power consumed at 250Ω using Thevenin circuit.



(answers: v_{oc} = v_{TH} = 480 V, i_{sc}= 9.6 A, R_{TH} = 50 Ω)

(Case II) Find the power consumed at 25Ω using Thevenin circuit.



(answers: v_{oc} = v_{TH} = 660V, i_{sc} = 132A. ∴ $R_{TH} = v_{oc} / i_{sc}$ = 5Ω)

(Example) Find v_o using Thevenin theorem.

Problem

Compute the load current i by the Thévenin equivalent method in the circuit of Figure 3.49.

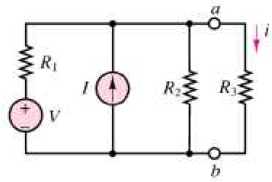


Figure 3.49

Solution

Known Quantities: Source voltage, resistor values.

Find: Load current i.

Schematics, Diagrams, Circuits, and Given Data: $V = 24$ V; $I = 3$ A; $R_1 = 4\ \Omega$; $R_2 = 12\ \Omega$; $R_3 = 6\ \Omega$.

Assumptions: Assume the reference node is at the bottom of the circuit.

Analysis: We first compute the Thévenin equivalent resistance. According to the method proposed earlier, we zero the two sources by shorting the voltage source and opening the current source. The resulting circuit is shown in Figure 3.50. We can clearly see that $R_T = R_1 \| R_2 = 4\|12 = 3\ \Omega$.

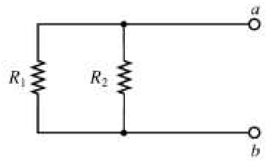


Figure 3.50

Following the Thévenin voltage Focus on Methodology box, first we remove the load and label the open-circuit voltage v_{OC}. The circuit is shown in Figure 3.51. Next, we observe that since v_b is equal to the reference voltage (i.e., zero), the node voltage v_a will be equal, numerically, to the open-circuit voltage. In this circuit, a single nodal equation is required to arrive at the solution:

$$\frac{V - v_a}{R_1} + I - \frac{v_a}{R_2} = 0$$

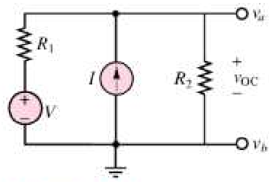


Figure 3.51

Substituting numerical values, we find that $v_a = v_{OC} = v_T = 27$ V.

Finally, we assemble the Thévenin equivalent circuit, shown in Figure 3.52, and reconnect the load resistor. Now the load current can be easily computed to be

$$i = \frac{v_T}{R_T + R_L} = \frac{27}{3+6} = 3\text{ A}$$

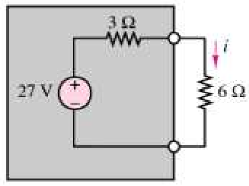
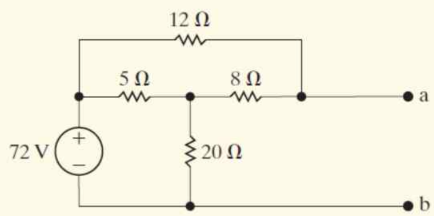


Figure 3.52 Thévenin equivalent

Comments: It may appear that the calculation of load current by the Thévenin equivalent method leads to more complex calculations than, say, node voltage analysis (you might wish to try solving the same circuit by node analysis to verify this). However, there is one major advantage to equivalent circuit analysis: Should the load change (as is often the case in many practical engineering situations), the equivalent circuit calculations still hold, and only the (trivial) last step in the above example needs to be repeated. Thus, knowing the Thévenin equivalent of a particular circuit can be very useful whenever we need to perform computations pertaining to any load quantity.

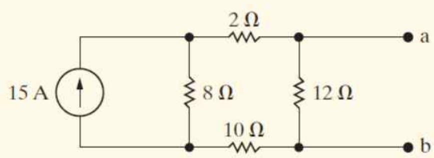
(Examples)

4.16 Find the Thévenin equivalent circuit with respect to the terminals a,b for the circuit shown.



Answer: $V_{ab} = V_{Th} = 64.8$ V, $R_{Th} = 6\ \Omega$.

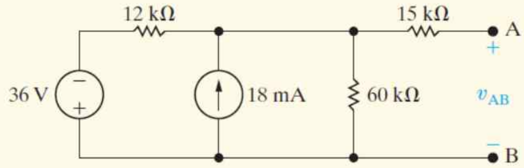
4.17 Find the Norton equivalent circuit with respect to the terminals a,b for the circuit shown.



Answer: $I_N = 6$ A (directed toward a), $R_N = 7.5\ \Omega$.

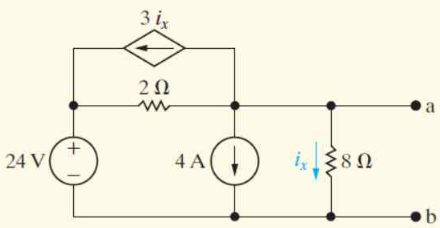
4.18 A voltmeter with an internal resistance of 100 kΩ is used to measure the voltage v_{AB} in the circuit shown. What is the voltmeter reading?

Answer: 120 V.



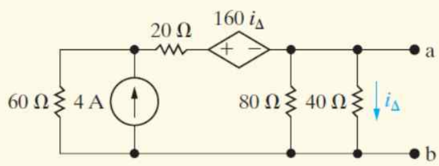
4.19 Find the Thévenin equivalent circuit with respect to the terminals a,b for the circuit shown.

Answer: $V_{Th} = v_{ab} = 8$ V, $R_{Th} = 1\ \Omega$.

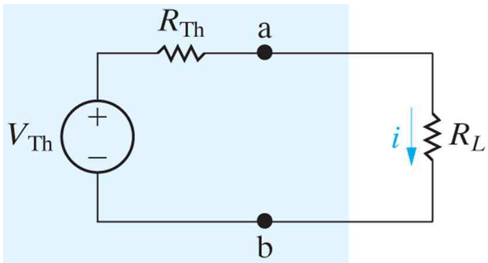


4.20 Find the Thévenin equivalent circuit with respect to the terminals a,b for the circuit shown. (*Hint:* Define the voltage at the left-most node as v, and write two nodal equations with V_{Th} as the right node voltage.)

Answer: $V_{Th} = v_{ab} = 30$ V, $R_{Th} = 10\ \Omega$.



§4.12 Maximum Power Transfer



$$p_L = i^2 R_L = \left(\frac{V_{TH}}{R_{TH}+R_L}\right)^2 R_L$$

Find the maximum value of p_L

$$\frac{dp_L}{dR_L} = V_{TH}^2 \left[\frac{(R_{TH}+R_L)^2 - R_L \cdot 2(R_{TH}+R_L)}{(R_{TH}+R_L)^4}\right] = 0$$

$$\therefore\ (R_{TH}+R_L)^2 - R_L \cdot 2(R_{TH}+R_L) = 0$$

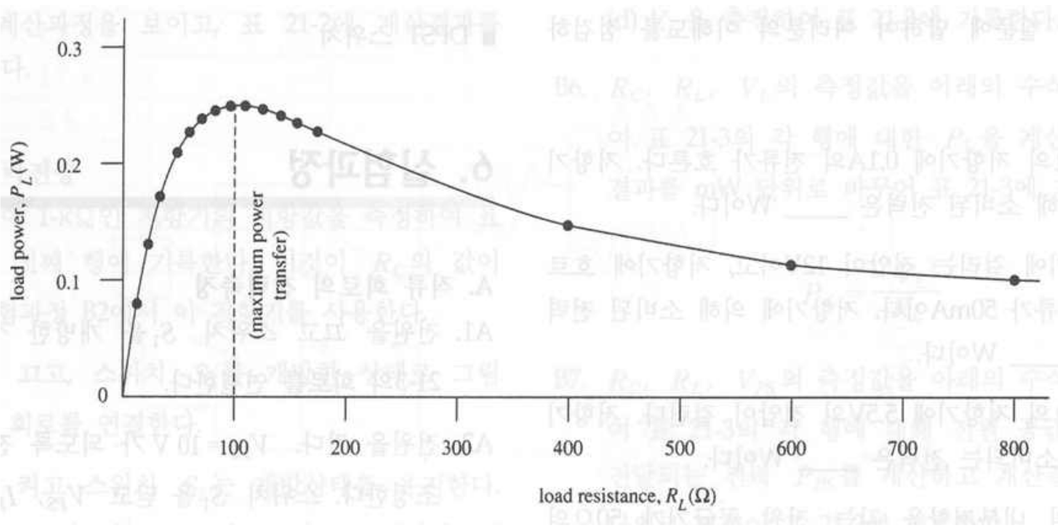
$$\therefore\ R_L = R_{TH}$$

(※ When $R_L = R_{TH}$, the power consumed at R_L is maximum.

$$p_{L\ \max} = \frac{V_{TH}^2 R_L}{(2R_L)^2} = \frac{V_{TH}^2}{4R_L}$$

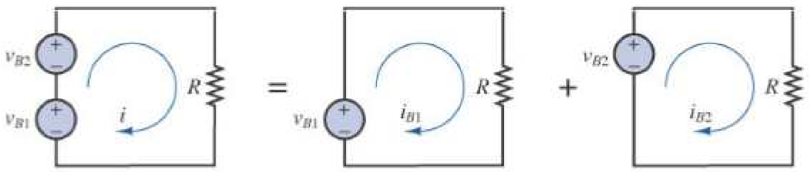
- power transfer to the load

v_{TH}	R_{TH}	R_L	v_L	i	p_L	remarks
10	100	0	0	0.1	0	short
10	100	20	1.666	0.0833	0.1387	
10	100	40	2.857	0.0714	0.2039	
10	100	60	3.750	0.0625	0.2343	
10	100	80	4.444	0.0555	0.2466	
10	100	100	5.000	0.05	0.25	max power transfer
10	100	120	5.454	0.0454	0.2476	
10	100	140	5.833	0.0416	0.2426	
10	100	200	6.666	0.0333	0.2219	
10	100	1,000	9.091	0.0091	0.0827	
10	100	10,000	9.901	0.00099	0.00980	
10	100	100,000	9.990	0.000099	0.000998	
10	100	∞	10	0	0	open



§4.13 The Principle of Superposition

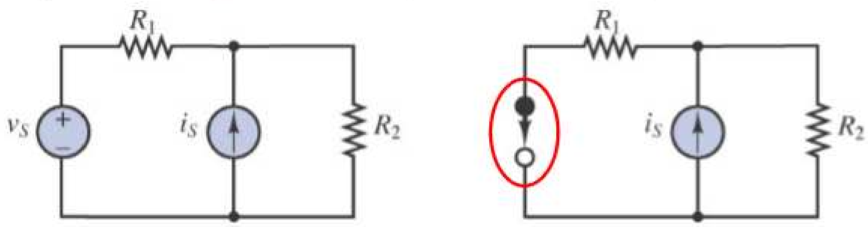
- In LINEAR* circuit that contains N sources, each branch voltage and current is the sum of N voltages and currents (i.e. sum of the results from N independent sources).
- Each set of these voltages and currents can be computed by setting only one (voltage or current) source ON and all others ZERO (i.e. OFF) and solving the circuit containing that single source.



The net current through R is the sum of the individual source currents: $i = i_{B1} + i_{B2}$.

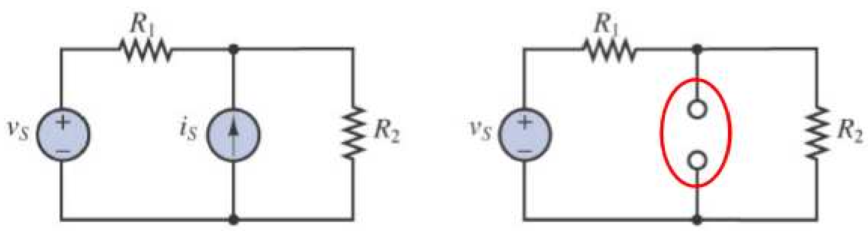
* Linearity of system
  - Additivity : f (x+y) = f (x) + f (y)
  - Homogeneity : f (αx) = α f(x)

- Setting the Voltage source equal to zero : replace w/ a short circuit



A circuit — The same circuit with $v_S = 0$

- Setting the Current source equal to zero : replace w/ a open circuit



A circuit — The same circuit with $i_S = 0$

The Principle of Superposition: Example I

- Circuit a (voltage source $V_G \rightarrow$ zero)

KCL at node 1 $\quad I_B - \frac{1}{R_B} v_{R,a} - \frac{1}{R_G} v_{R,a} - \frac{1}{R} v_{R,a} = 0$

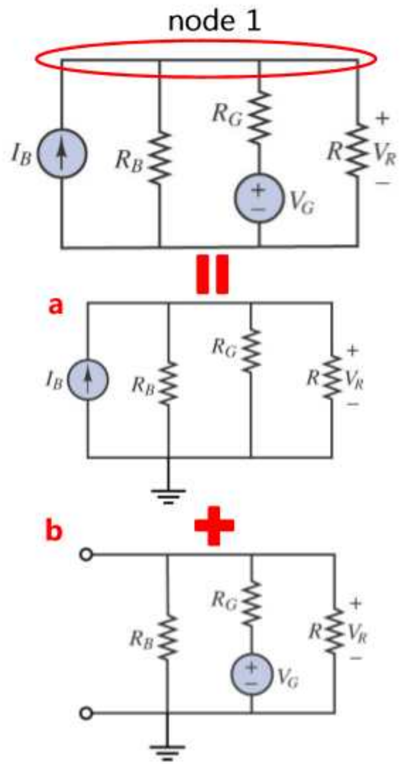
$$\left(\frac{1}{R_B} + \frac{1}{R_G} + \frac{1}{R} \right) v_{R,a} = I_B$$

- Circuit b (current source $I_B \rightarrow$ zero)

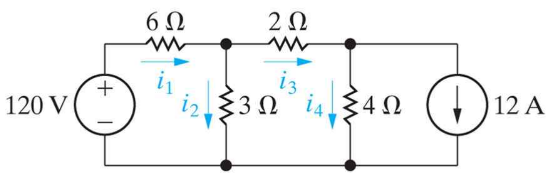
KCL at node 1 $\quad -\frac{1}{R_B} v_{R,b} - \frac{1}{R_G} (v_{R,b} - V_G) - \frac{1}{R} v_{R,b} = 0$

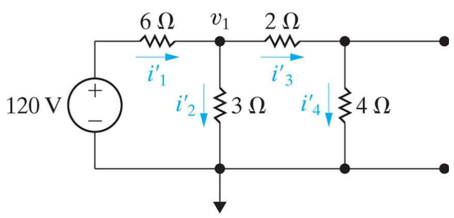
$$\left(\frac{1}{R_B} + \frac{1}{R_G} + \frac{1}{R} \right) v_{R,b} = \frac{1}{R_G} V_G$$

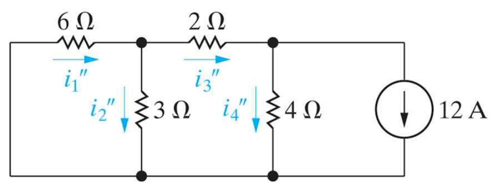
Total voltage is obtained by the superposition of two voltage values. $\Rightarrow$ $v_R = v_{R,a} + v_{R,b}$



(Example) Superposition







$$i_1 = i_1' + i_1'' \ , \quad i_2 = i_2' + i_2'' \ , \quad i_3 = i_3' + i_3'' \ , \quad i_4 = i_4' + i_4''$$

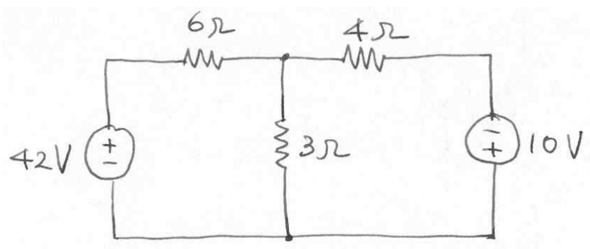
and $v_1 = v_1' + v_1''$

※ Superposition is a very important concept for a linear system, but superposition principle is not widely used to analyze a multi-source circuit, since we need to analyze the circuit several times.

● Summary

<u>node-voltage method</u>,
mesh-current method,
<u>Thevenin equivalent circuit</u>,
superposition,
maximun power transfer

(Example) Find the voltage across the 3Ω resistor. [answer: 6V]



(1) Ohm's Law, KVL, KCL
(2) Node-voltage method
(3) mesh-current method
(4) superposition method
(5) source transformation
(6) Thevenin (or Norton) equivalent circuit