Module-2 : AC Circuits and Magnetic Circuits

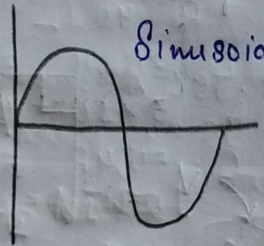
① AC Circuit Analysis :-
(i) Fundamentals of AC Circuit
(ii) R, RL, RC, LC and RLC Circuits
(iii) Series and Parallel Circuits
(iv) Phasor Diagrams
(v) Resonance in AC Circuit

② Magnetic Circuits :-
(i) Analogy between electric and magnetic circuits
(ii) Series and Parallel magnetic circuits
(iii) Self and Mutual Inductance
(iv) Magnetic circuits Applications
- DC motors
- Generators
- Transformers

Fundamentals of AC Circuit :-

DC

AC

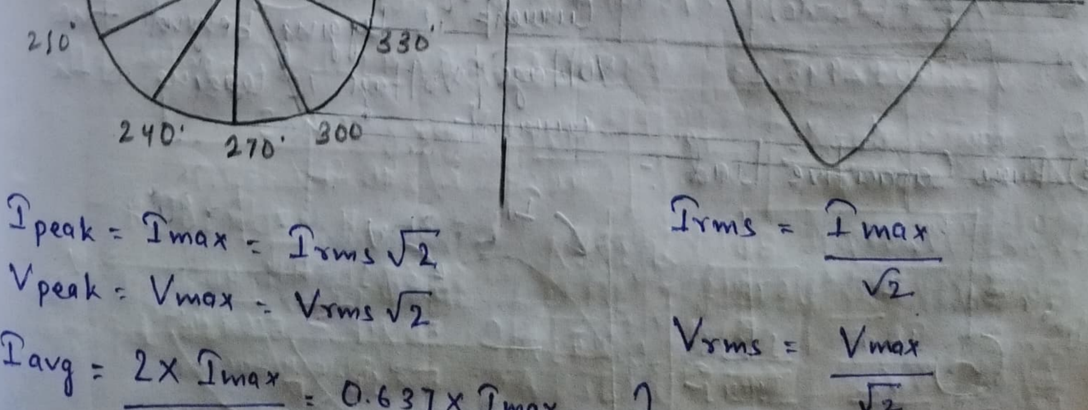


Sinusoidal

$f = 50\,Hz$ (standard Indian household)

$T = \frac{1}{f} = \frac{1}{50} = 0.02\,s$

Time Period



$I_{peak} = I_{max} = I_{rms}\sqrt{2}$

$V_{peak} = V_{max} = V_{rms}\sqrt{2}$

$I_{rms} = \frac{I_{max}}{\sqrt{2}}$

$V_{rms} = \frac{V_{max}}{\sqrt{2}}$

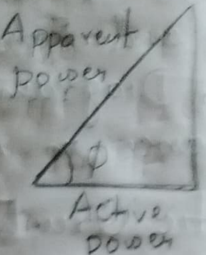
$I_{avg} = \frac{2 \times I_{max}}{\pi} = 0.637 \times I_{max}$

$V_{avg} = \frac{2 \times V_{max}}{\pi} = 0.637 \times V_{max}$

$\frac{2}{\pi} = 0.637$

Power :-

- The first power is Active Power (useful power).
- The second power is Reactive Power (useless power)
- The third power is Apparent Power.



Mathematically,

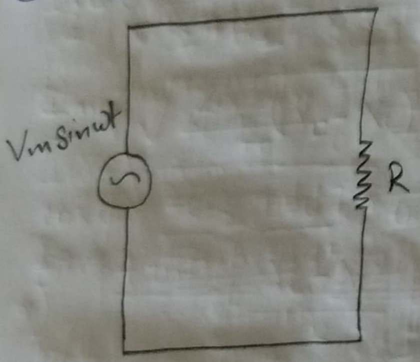
Reactive Power (Q) = $V \times I \times \sin\phi$

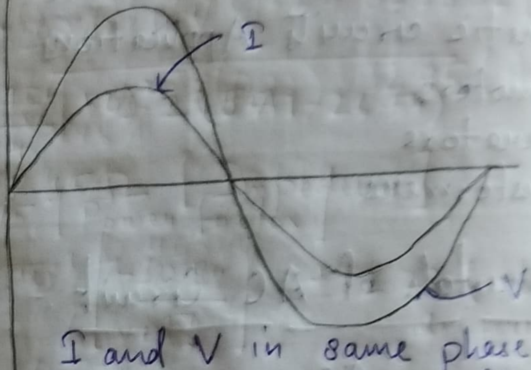
Apparent Power (S) = $\sqrt{P^2 + Q^2}$

Active Power (P) = $VI \times \cos\phi = I^2R$

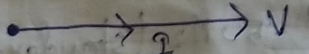
AC Circuit : Types of Loads :-

① Pure Resistive Load :-



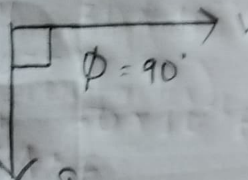


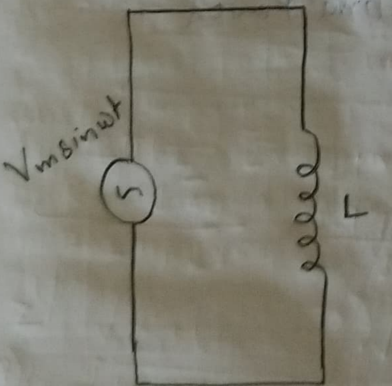
I and V in same phase
Magnitude can be different

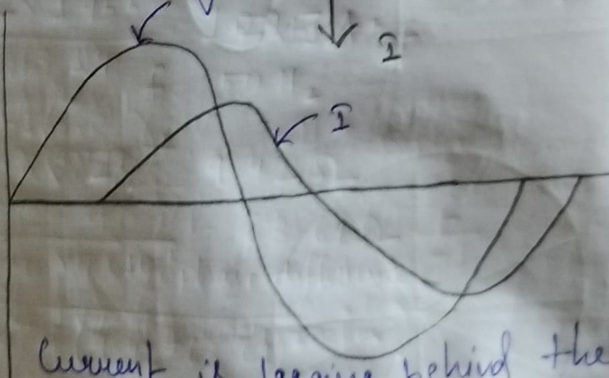


Power Factor (P.F) = 1
(Unity)

② Pure Inductive Load :-

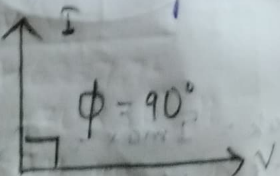


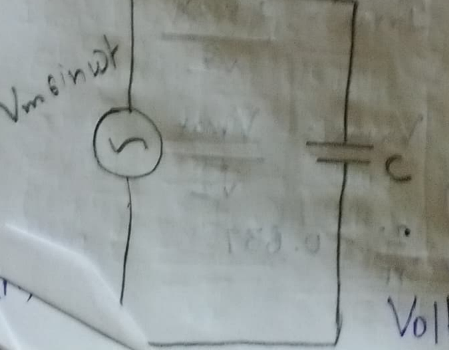


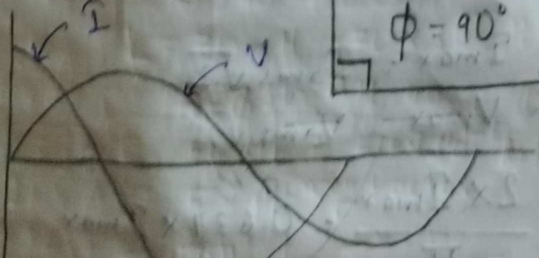


Current is lagging behind the voltage / Voltage is leading with current

③ Pure Capacitive Load :-







Current is leading with the voltage
Voltage is lagging behind current

(4) Resistive - Inductive [RL-circuit] :-
Current lags with the voltage | Voltage leads with current

(5) Resistive - Capacitive [RC-circuit] :-
Current leads with the voltage | Voltage lags with current

(6) Resistive - Inductive - Capacitive [RLC-circuit] :-
If $X_L > X_C$: Lagging power factor
If $X_C > X_L$: Leading power factor

$X_L = \omega L = 2\pi f L$
$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$
$\omega = 2\pi f$

NOTE :-
$I_{max} = \frac{V_{max}}{Z}$
$\cos\phi = \frac{R}{Z}$ = Power factor

(i) In R-L circuit :-
$Z\angle\theta = R + jX_L$
$Z = \sqrt{R^2 + X_L^2}$
$\theta = \tan^{-1}\left[\frac{X_L}{R}\right]$

(ii) In RC-circuit :-
$Z\angle\theta = R - jX_C$
$Z = \sqrt{R^2 + X_C^2}$
$\theta = \tan^{-1}\left[\frac{-X_C}{R}\right]$

(iii) In RLC-circuit :-
$Z\angle\theta = R + j(X_L - X_C)$
$Z = \sqrt{R^2 + (X_L - X_C)^2}$
$\theta = \tan^{-1}\left[\frac{X_L - X_C}{R}\right]$

In case of Parallel AC Circuit [RLC] :-

$$Z = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}}$$

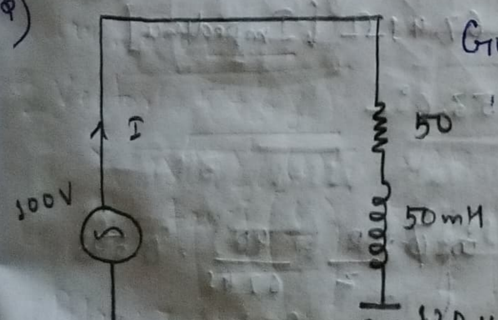
when the given AC circuit has resistors, inductors and capacitors parallely connected.

RC-circuit : $Z = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_C}\right)^2}}$

RL-circuit : $Z = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L}\right)^2}}$

Parallel Circuit

Q)



Given,
$V_{max} = 100V$
$R = 50\Omega$
$L = 50mH = 50\times10^{-3}H$
$= 5\times10^{-2}H$
$C = 120\mu F = 120\times10^{-6}F$
$= 12\times10^{-5}F$
$f = 50 Hz$

$X_L = 2\pi f L = 2 \times 3.14 \times 50 \times 5 \times 10^{-4} = 15.7\ \Omega$

$X_C = \dfrac{1}{2\pi f C} = \dfrac{1}{2 \times 3.14 \times 50 \times 12 \times 10^{-5}} = 26.54\ \Omega$

$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(50)^2 + (15.7 - 26.54)^2}$

$= \sqrt{2500 + 117.5} = 51.15\ \Omega$ [Impedance]

$\theta = \tan^{-1} \left| \dfrac{X_L - X_C}{R} \right| = \tan^{-1} \left| \dfrac{-10.84}{50} \right| = 12.23°$

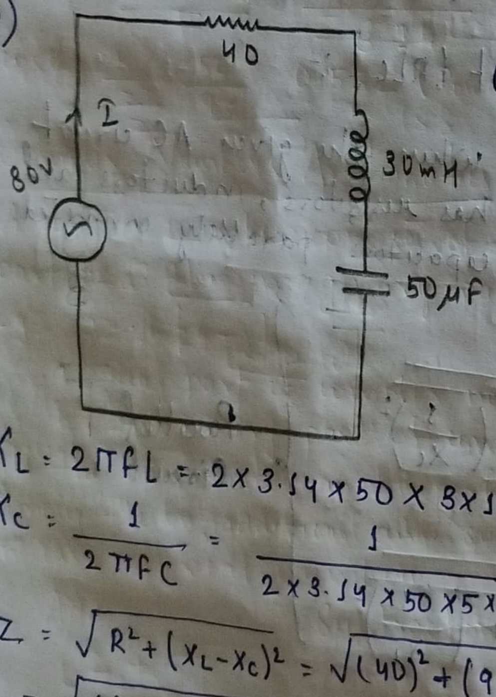
$I_{max} = \dfrac{V_{max}}{Z} = \dfrac{100}{51.15} = 1.955\ A$

$I_{rms} = \dfrac{I_{max}}{\sqrt{2}} = \dfrac{1.955}{1.41} = 1.386\ A$

$I_{avg} = \dfrac{2 \times I_{max}}{\pi} = \dfrac{2 \times 1.955}{3.14} = 1.295\ A$

$\cos\phi = \dfrac{R}{Z} = \dfrac{50}{51.15} = 0.9775$ [Power factor]

a)



Given,

$V_{max} = 80V$

$R = 40\ \Omega$

$L = 30mH = 30 \times 10^{-3} H$
$= 3 \times 10^{-2} H$

$C = 50 \mu F = 50 \times 10^{-6} F$
$= 5 \times 10^{-5} F$

$f = 50 Hz$

$X_L = 2\pi f L = 2 \times 3.14 \times 50 \times 3 \times 10^{-2} = 9.42\ \Omega$

$X_C = \dfrac{1}{2\pi f C} = \dfrac{1}{2 \times 3.14 \times 50 \times 5 \times 10^{-5}} = 63.69\ \Omega$

$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(40)^2 + (9.42 - 63.69)^2}$

$= \sqrt{1600 + 2945.23} = \sqrt{4545.23} = 67.41\ \Omega$ [Impedance]

$\theta = \tan^{-1} \left| \dfrac{X_L - X_C}{R} \right| = \tan^{-1} \left| \dfrac{-54.27}{40} \right| = 53.6°$

$I_{max} = \dfrac{V_{max}}{Z} = \dfrac{80}{67.41} = 1.186\ A$

$I_{rms} = \dfrac{I_{max}}{\sqrt{2}} = \dfrac{1.186}{1.41} = 0.841\ A$

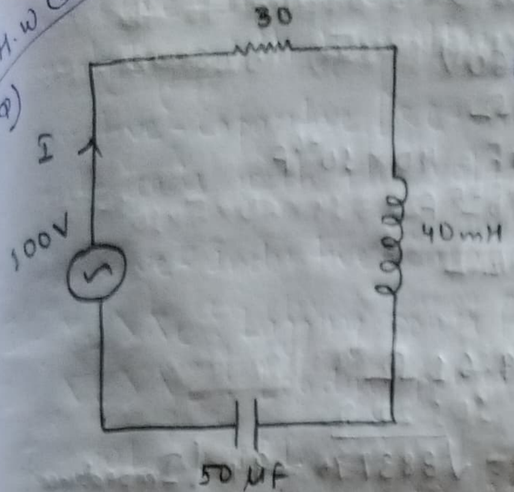
$\cos\phi = \dfrac{R}{Z} = \dfrac{40}{67.41} = 0.593$

power factor

$I_{avg} = \dfrac{2 \times I_{max}}{\pi} = \dfrac{2 \times 1.186}{3.14} = 0.755\ A$

H.W (6)

a)



Given,

$V_{max} = 100V$

$R = 30\Omega$

$L = 40mH = 40\times10^{-3}H$

$= 4\times10^{-2}H$

$C = 50\mu F = 50\times10^{-6}F$

$= 5\times10^{-5}F$

$f = 50Hz$

$X_L = 2\pi fL = 2\times3.14\times50\times4\times10^{-2} = 12.56\Omega$

$X_C = \frac{1}{2\pi fC} = \frac{1\times10^5}{2\times3.14\times50\times5} = 63.69\Omega$

$Z = \sqrt{R^2+(X_L-X_C)^2} = \sqrt{(30)^2+(12.56-63.69)^2}$

$= \sqrt{900+2614.27} = \sqrt{3514.27} = 59.281\Omega$ [Impedence]

$\theta = \tan^{-1}\left|\frac{X_L-X_C}{R}\right| = \tan^{-1}\left|\frac{-51.13}{40}\right| = 59.581^\circ$

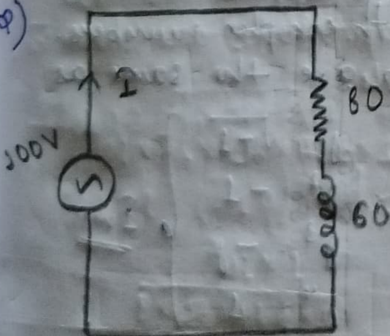
$I_{max} = \frac{V_{max}}{Z} = \frac{100}{59.281} = 1.686\,A$

$I_{rms} = \frac{I_{max}}{\sqrt{2}} = \frac{1.686}{1.41} = 1.195\,A$

$I_{avg} = \frac{2\times I_{max}}{\pi} = \frac{2\times1.686}{3.14} = 1.073\,A$

$\cos\phi = \frac{R}{Z} = \frac{30}{59.281} = 0.506$ [Power factor]

b)



Given,

$V_{max} = 100V$

$R = 80\Omega$

$L = 60mH = 60\times10^{-3}H$

$= 6\times10^{-2}H$

$f = 50Hz$

$X_L = 2\pi fL = 2\times3.14\times50\times6\times10^{-2} = 18.84\Omega$

$Z = \sqrt{R^2+X_L^2} = \sqrt{(80)^2+(18.84)^2} = \sqrt{6755} = 82.18\Omega$ [Impedence]

$\theta = \tan^{-1}\left[\frac{X_L}{R}\right] = \tan^{-1}\left[\frac{18.84}{80}\right] = \tan^{-1}(0.23) = 12.95^\circ$

$I_{max} = \frac{V_{max}}{Z} = \frac{100}{82.18} = 1.216\,A$

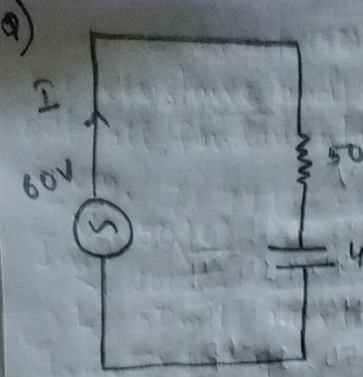
$I_{avg} = \frac{2\times I_{max}}{\pi} = \frac{2\times1.216}{3.14}$

$I_{avg} = 0.774\,A$

$I_{rms} = \frac{I_{max}}{\sqrt{2}} = \frac{1.216}{1.41} = 0.862\,A$

$\cos\phi = \frac{R}{Z} = 0.9734$ (power factor)

9)



Given,
$V_{max} = 60V$
$R = 50\Omega$
$C = 40\mu F = 40 \times 10^{-6} F$
$= 4 \times 10^{-5} F$
$f = 50 Hz$

$X_c = \frac{1}{2\pi f C} = \frac{1 \times 10^5}{2 \times 3.14 \times 50 \times 4} = 79.61\,\Omega$

$Z = \sqrt{R^2 + X_c^2} = \sqrt{(50)^2 + (79.61)^2} = \sqrt{8837.75} = 94$ [Impedance]

$\theta = \tan^{-1}\left[\frac{X_c}{R}\right] = \tan^{-1}\left[\frac{79.61}{50}\right] = \tan^{-1}(1.5922) = 57.868°$

$I_{max} = \frac{V_{max}}{Z} = \frac{60}{94} = 0.6328 A$

$I_{rms} = \frac{I_{max}}{\sqrt{2}} = \frac{0.6328}{1.41} = 0.448 A$

$I_{avg} = \frac{2 \times I_{max}}{\pi} = \frac{2 \times 0.6328}{3.14} = 0.403 A$

$\cos\phi = \frac{R}{Z} = \frac{50}{94} = 0.532$ (Power factor)

Complex Numbers Arthametics :-

A complex number consists of two parts namely, the real part and the imaginary part

Eg :- $a + ib \mid a - jb$

a = Real part of complex no.
b = Imaginary part of complex no.

All the operations can be performed with complex numbers. Among them, addition and subtraction are the same as in case of real numbers.

Addition :-

$$\begin{array}{r} 2 + 5i \\ 3 + 4i \\ \hline 5 + 9i \end{array}$$

Subtraction :-

$$\begin{array}{r} 5 + 5i \\ 2 + 2i \\ \hline 3 + 3i \end{array}$$

$$\left.\begin{array}{l} j = \sqrt{-1} \\ j^2 = -1 \\ j^3 = -j \\ j^4 = 1 \end{array}\right\} \text{Imp}$$

Multiplication :-

$(5+4j)(6+3j) = 30 + 24j + 15j + 12j^2$
$= 30 + 39j - 12$
$= 18 + 39j$

$(6+4j)(3+10j) = 18 + 60j + 12j + 40j^2$
$= 18 + 72j + 40j^2$
$= -22 + 72j$

a) $\frac{6+4j}{3+10j} \times \frac{(3-10j)}{(3-10j)} = \frac{(6+4j)(3-10j)}{9-100j^2}$

$= \frac{18-60j+12j-40j^2}{9+100} = \frac{58-48j}{109} = \frac{58}{109} - \frac{48j}{109} = 0.5 - 0.4j$

Vector to Polar form :-

Vector : $2+3j$

magnitude $= r = \sqrt{(2)^2+(3)^2} = \sqrt{4+9} = \sqrt{13} = 3.60$

$\theta = \tan^{-1}\left(\frac{3}{2}\right) = 56.30°$ [Phase angle]

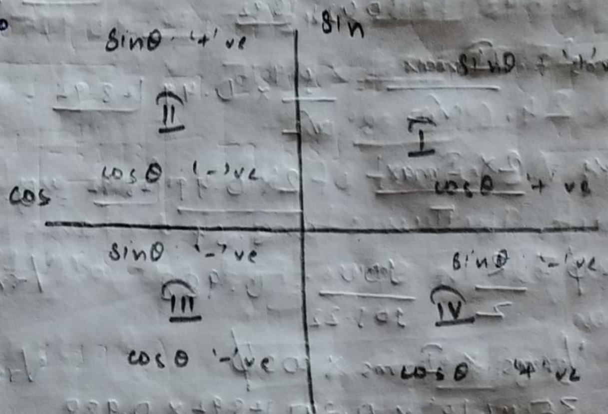
Polar : $3.60 \angle 56.30°$

Vector : $5+6j$

$r = \sqrt{(5)^2+(6)^2} = \sqrt{25+36} = \sqrt{61} = 7.81$

$\theta = \tan^{-1}\left(\frac{6}{5}\right) = 50.19°$

Polar : $7.81 \angle 50.19°$



Polar to Vector form :-

Vector form : $a+jb$

Polar : $5 \angle 120°$

Magnitude (a) = 5

Phase angle (θ) : $120°$

$a = r\cos\theta$
$= r\cos 120°$
$= r\cos(180° - 120°)$
$= r(-\cos 60°)$
$= 5 \times \frac{1}{2}$
$= 2.5$

$b = r\sin\theta$
$= r\sin 120°$
$= r\sin(180° - 120°)$
$= r\sin 60°$
$= 5 \times \frac{\sqrt{3}}{2}$
$= 4.33$

Vector : $2.5 + 4.33j$

Polar : $7.8 \angle 50.19°$

$a = r\cos\theta$
$= r\cos(50.19°)$
$= 7.8 \times 0.64$
$= 5$

$b = r\sin\theta$
$= r\sin(50.19°)$
$= 7.8 \times 0.768$
$= 6$

Vector : $5+6j$

Polar : $3.6 \angle 56.3$

$a = r\cos\theta$
$= r\cos(56.3)$
$= 3.6 \times 0.554$
$= 2$

$b = r\sin\theta$
$= r\sin(56.3)$
$= 3.6 \times 0.832$
$= 3$

Vector : $2+3j$

An AC circuit 50V at an angle 0° is applied to the circuit. The resistance and inductance are connected in series. Find out all fundamentals.

Given,

$V = 50\angle 0°$

$R = 100\,\Omega$

$L = 50\,mH = 50 \times 10^{-3}\,H = 5 \times 10^{-2}\,H$

$X_L = 2\pi f L = 2 \times 3.14 \times 50 \times 5 \times 10^{-2} = 15.7\,\Omega$

$Z = \sqrt{R^2 + X_L^2} = \sqrt{(100)^2 + (15.7)^2} = 101.22\,\Omega$

$\theta = \tan^{-1}\left(\frac{15.7}{100}\right) = 8.92°$

$Z = 101.22\angle 8.92°$ [ZLθ]

$I_{max} = \frac{V}{Z} = \frac{50\angle 0°}{101.22\angle 8.92} = 0.49\angle +8.92\,A$

$I_{rms} = \frac{I_{max}}{\sqrt{2}} = \frac{1}{\sqrt{2}} \times 0.49\angle +8.92 = 0.347\angle +8.92\,A$

$I_{avg} = \frac{2 \times I_{max}}{\pi} = \frac{2 \times 0.49\angle +8.92}{3.14} = 0.288\angle +8.92\,A$

$\cos\phi = \frac{R}{Z} = \frac{100}{101.22} = 0.988$

$V_{rms} = \frac{V_{max}}{\sqrt{2}} = \frac{50\angle 0°}{\sqrt{2}}$

$V_{rms} = 35.46\angle 0°\,V$

$P = V_{rms} \times I_{rms} \times \cos\phi$

$= 35.46\angle 0° \times 0.347\angle +8.92 \times 0.988$

$= 12.3\angle +8.92 \times 0.988$

$= 12.1524\angle +8.92\,W$

9) An AC circuit of 100V at an angle 0° is applied to the circuit. The resistance, inductance and capacitance are connected in series. Find out all the fundamentals.

Given,

$R = 20\,\Omega$

$V = 100\angle 0°$

$L = 25\,mH = 25 \times 10^{-3}\,H$

$C = 90\,\mu F = 90 \times 10^{-6}\,F = 9 \times 10^{-5}\,F$

$X_L = 2\pi f L = 2 \times 3.14 \times 50 \times 25 \times 10^{-3} = 7.85\,\Omega$

$X_C = \frac{1}{2\pi f C} = \frac{1 \times 10^5}{2 \times 3.14 \times 50 \times 9} = 35.38\,\Omega$

$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(20)^2 + (7.85 - 35.38)^2} = \sqrt{1158} = 34\,\Omega$

$\theta = \tan^{-1}\left|\frac{X_L - X_C}{R}\right| = \tan^{-1}\left|\frac{7.85 - 35.38}{20}\right| = \tan^{-1}(1.3765) = 53.9°$

$Z\angle\theta = 34\angle 53.9°\,\Omega$

$I_{max} = \frac{V}{Z} = \frac{100\angle 0°}{34\angle 53.9} = 2.941\angle +53.9°\,A$

$I_{rms} = \frac{I_{max}}{\sqrt{2}} = \frac{1}{\sqrt{2}} \times 2.945 \angle +53.9^\circ = 2.085 \angle +53.9\ A$

$I_{avg} = \frac{2 \times I_{max}}{\pi} = \frac{2 \times 2.945}{3.14} \angle +53.9^\circ = 1.873 \angle +53.9^\circ\ A$

$\cos\phi = \frac{R}{Z} = \frac{20}{34} = 0.588^\circ$ $V_{rms} = \frac{V_{max}}{\sqrt{2}} = \frac{1}{\sqrt{2}} \times 100 \angle 0^\circ$

$V_{rms} = 71 \angle 0^\circ\ V$

$P = V_{rms} \times I_{rms} \times \cos\phi$

$= 71 \angle 0^\circ \times 2.085 \angle +53.9^\circ \times 0.588$

$= 148.035 \angle +53.9^\circ \times 0.588$

$= 87.04458 \angle +53.9^\circ\ W$

H.W 7

Q) An AC circuit 80V at an angle 0° is applied to the circuit. The resistance and capacitance are connected in series. Find out all fundamentals.

Given,

$V = 80 \angle 0^\circ$

$R = 80\Omega$

$C = 100\mu F = 100 \times 10^{-6} F$

$= 10^{-4} F$

$X_c = \frac{1}{2\pi f C} = \frac{1 \times 10^4}{2 \times 3.14 \times 50} = \frac{10^4}{314}$

$= 31.84\Omega$

$Z = \sqrt{R^2 + X_c^2} = \sqrt{(80)^2 + (31.84)^2} = \sqrt{7413.78} = 86.1\Omega$

$\theta = \tan^{-1}\left[\frac{-X_c}{R}\right] = \tan^{-1}\left(\frac{31.84}{80}\right) = \tan^{-1}(0.398) = -21.7^\circ$

$Z\angle\theta = 86.1 \angle 21.7^\circ\ \Omega$

$I_{max} = \frac{V}{Z} = \frac{80 \angle 0^\circ}{86.1 \angle 21.7^\circ} = 0.929 \angle +21.7^\circ\ A$

$I_{rms} = \frac{I_{max}}{\sqrt{2}} = \frac{1}{\sqrt{2}} \times 0.929 \angle +21.7^\circ = 0.658 \angle +21.7^\circ\ A$

$I_{avg} = \frac{2 \times I_{max}}{\pi} = \frac{2 \times 0.929}{3.14} \angle +21.7^\circ = 0.591 \angle +21.7^\circ\ A$

$\cos\phi = \frac{R}{Z} = \frac{80}{86.1} = 0.929$ $V_{rms} = \frac{V_{max}}{\sqrt{2}} = \frac{80 \angle 0^\circ}{\sqrt{2}}$

$V_{rms} = 56.73 \angle 0^\circ$

$P = V_{rms} \times I_{rms} \times \cos\phi$

$= 56.73 \angle 0^\circ \times 0.658 \angle +21.7^\circ \times 0.929$

$= 34.678 \angle +21.7^\circ\ W$

Q) An AC circuit 60V at an angle 30° is applied to the circuit. The resistance and inductance are connected in series. Find out all fundamentals.

Given,

$V = 60 \angle 30^\circ$

$R = 70\Omega$

$L = 50mH = 50 \times 10^{-3} H = 5 \times 10^{-2} H$

$X_L = 2\pi f L = 2 \times 3.14 \times 50 \times 5 \times 10^{-2}$

$= 15.7\Omega$

$Z = \sqrt{R^2 + X_L^2} = \sqrt{(70)^2 + (15.7)^2} = \sqrt{5146.5} = 71.74\ \Omega$

$\theta = \tan^{-1}\left[\frac{X_L}{R}\right] = \tan^{-1}\left(\frac{15.7}{70}\right) = \tan^{-1}(0.224) = 12.62^\circ$

$Z\angle\theta = 71.74\angle 12.62^\circ\ \Omega$

$I_{max} = \frac{V}{Z} = \frac{60\angle 30^\circ}{71.74\angle 12.62^\circ} = 0.836\angle 17.38^\circ\ A$

$I_{rms} = \frac{I_{max}}{\sqrt{2}} = \frac{1}{\sqrt{2}} \times 0.836\angle 17.38^\circ = 0.593\angle 17.38^\circ\ A$

$I_{avg} = \frac{2 \times I_{max}}{\pi} = \frac{2 \times 0.836}{3.14}\angle 17.38^\circ = 0.532\angle 17.38^\circ\ A$

$\cos\phi = \frac{R}{Z} = \frac{70}{71.74} = 0.975$, $V_{rms} = \frac{V_{max}}{\sqrt{2}} = \frac{1}{\sqrt{2}} \times 60\angle 30^\circ$

$V_{rms} = 42.55\angle 30^\circ$

$P = I_{rms} \times V_{rms} \times \cos\phi$
$= 0.593\angle 17.38^\circ \times 42.55\angle 30^\circ \times 0.975$
$= 25.23\angle 47.38^\circ \times 0.975$
$= 24.6\angle 47.38^\circ\ W$

Q) An AC circuit of 100V at an angle 45° is applied to the circuit. The resistance, inductance and capacitance are connected in series. Find out all the fundamentals.

Given,

$V = 100\angle 45^\circ$
$R = 60\Omega$
$L = 40mH = 40 \times 10^{-3} H = 4 \times 10^{-2} H$
$C = 120\mu F = 120 \times 10^{-6} F = 12 \times 10^{-5} F$

$X_L = 2\pi f L = 2 \times 3.14 \times 50 \times 4 \times 10^{-2} = 12.56\ \Omega$

$X_C = \frac{1}{2\pi f C} = \frac{1 \times 10^5}{2 \times 3.14 \times 50 \times 12} = \frac{1 \times 10^5}{3768} = 26.54\ \Omega$

$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(60)^2 + (12.56 - 26.54)^2} = \sqrt{3795.44} = 61.6\ \Omega$

$\theta = \tan^{-1}\left|\frac{X_L - X_C}{R}\right| = \tan^{-1}\left|\frac{12.56 - 26.54}{60}\right| = \tan^{-1}(0.233) = 13.11^\circ$

$Z\angle\theta = 61.6\angle 13.11\ \Omega$

$I_{max} = \frac{V}{Z} = \frac{100\angle 45^\circ}{61.6\angle 13.11^\circ} = 1.623\angle 31.89^\circ\ A$

$I_{rms} = \frac{I_{max}}{\sqrt{2}} = \frac{1}{\sqrt{2}} \times 1.623\angle 31.89^\circ = 1.151\angle 31.89^\circ\ A$

$I_{avg} = \frac{2 \times I_{max}}{\pi} = \frac{2 \times 1.151}{3.14}\angle 31.89^\circ = 0.733\angle 31.89^\circ\ A$

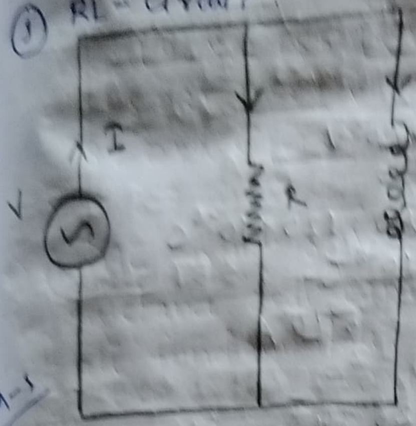
$\cos\phi = \frac{R}{Z} = \frac{60}{61.6} = 0.974$ $V_{rms} = \frac{V_{max}}{\sqrt{2}} = \frac{1}{\sqrt{2}} \times 100\angle 45^\circ$

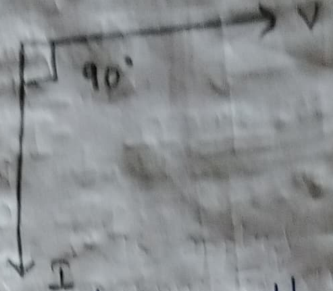
$V_{rms} = 70.92\angle 45^\circ$

$P = V_{rms} \times I_{rms} \times \cos\phi$
$= 70.71\angle 45^\circ \times 1.151\angle 31.89^\circ \times 0.974$
$= 81.67\angle 76.89^\circ \times 0.974$
$= 79.5\angle 76.89^\circ$ (W)

AC - Parallel Circuits

① RL - Circuit





Current lags with the voltage by 90°.

M-1

$I\angle\theta = I_R + I_L$
$I\angle\theta = I_R - j I_{XL}$

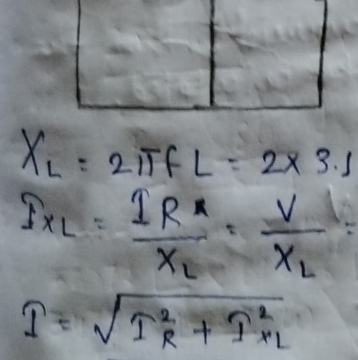
$I = \sqrt{I_R^2 + (-I_{XL})^2}$
$I = \sqrt{I_R^2 + I_{XL}^2}$

$\theta = \tan^{-1}\left[\frac{-I_{XL}}{I_R}\right]$

M-2

$Z = \frac{Z_1 Z_2}{Z_1 + jZ_2}$ = Impedance $\left[I = \frac{V}{Z}\right]$

NOTE: Voltage always remains at 0°.



Given;
$V = 75\angle 0^\circ$ V
$R = 10\,\Omega$
$L = 30\text{mH} = 30\times 10^{-3}$ H
$= 3\times 10^{-2}$ H

$I_R = \frac{V\angle 0}{R\angle 0} = \frac{75\angle 0^\circ}{10\angle 0^\circ} = 7.5\angle 0^\circ$ A

$X_L = 2\pi f L = 2\times 3.14\times 50\times 3\times 10^{-2} = 9.42\,\Omega$

$I_{XL} = \frac{IR}{X_L} = \frac{V}{X_L} = \frac{75\angle 0^\circ}{9.42\angle 90^\circ} = 7.96\angle -90^\circ$ A [Current lags by 90° — RL circuit]

$I = \sqrt{I_R^2 + I_{XL}^2}$
$= \sqrt{(7.5\angle 0^\circ)^2 + (7.96\angle 90^\circ)^2}$
$= \sqrt{56.25\angle 0^\circ + 63.36\angle 180^\circ}$
$= \sqrt{56.25 - 63.36}$ [Vector form]
$= \sqrt{-7.11} = 7.11$ A [Polar form]

$a + bj \mid a' + b'j$
$a = 56.25\cos 0^\circ$; $b = 56.25\sin 0^\circ$
$a = 56.25$; $b = 0$
$a' = 63.36\cos\pi$
$a' = -63.36$; $b' = \sin\pi = 0$
$\sqrt{(-7.11)^2 + 0^2} = 7.11$ (positive value taken)

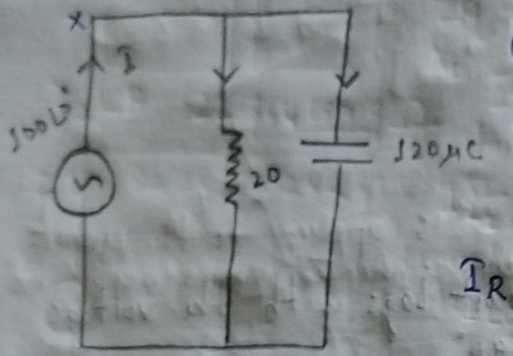
$\tan^{-1}\left(-\frac{X_L}{R}\right) = \tan^{-1}\left(-\frac{9.42}{10}\right) = \tan^{-1}(-0.942) = -43.29°$

$I\angle\theta = 7.11\angle 43.29°$ A

$Z = \frac{V\angle\theta}{I\angle\theta} = \frac{75\angle 0°}{7.11\angle 43.29°} = \frac{75}{7.11}\angle 0 + 43.29° = 10.55\angle 43.29°\ \Omega$

Total Impedance of the circuit.

② RC-Circuit :-



Given,

$V = 100\angle 0°$ V

$R = 20\ \Omega$

$C = 120\ \mu C = 120 \times 10^{-6}$ C

$= 12 \times 10^{-5}$ C

$I_R = \frac{100\angle 0°}{20\angle 0°} = 5\angle 0°$ A

$X_C = 2\pi f C = 2 \times 3.14 \times 50 \times 12 \times 10^{-5} = 26.54\ \Omega$

$I_{XC} = \frac{V}{X_C} = \frac{100\angle 0°}{26.54\angle 90°} = 3.76\angle -90°$ [Current leads by 90° — RC-circuit]

$I = \sqrt{I_R^2 + I_{X_C}^2}$

$= \sqrt{(5\angle 0°)^2 + (3.76\angle -90°)^2}$

$= \sqrt{25\angle 0° + 14.13\angle -180°}$

$= \sqrt{25 - 14.13}$ [Vector form]

$= \sqrt{10.87} = 3.297$ A — Same answer for polar form.

$a + bj \mid a' + b'j$

$a = 25\cos 0° = 25$; $b = 25\sin 0 = 0$

$a' = 14.13\cos\pi = -14.13$;

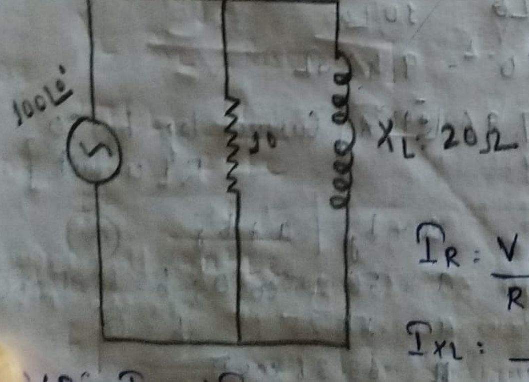
$b' = -\sin\pi = 0$

$\theta = \tan^{-1}\left(\frac{X_C}{R}\right) = \tan^{-1}\left(\frac{26.54}{20}\right) = \tan^{-1}(1.327) = 53°$

$I\angle\theta = 3.297\angle 53°$ A

$Z = \frac{V\angle\theta}{I\angle\theta} = \frac{100\angle 0°}{3.297\angle 53°} = 22.748\angle -53°\ \Omega$

Q)



Given,

$V = 100\angle 0°$ V

$R = 10\ \Omega$

$X_L = 20\ \Omega$

$I_R = \frac{V\angle\theta}{R\angle\theta} = \frac{100\angle 0°}{10\angle 0°} = 10$ A

$I_{XL} = \frac{V}{X_L} = \frac{100\angle 0°}{20\angle 90°} = 5\angle -90°$ A

$I\angle\theta = I_R - jI_{XL}$

$= 10\angle 0° - 5j$ [Magnitude in case of RL-circuit]

$I = \sqrt{(10)^2 + (5)^2} = \sqrt{100 + 25} = \sqrt{125} = 11.18$ A

$\theta = \tan^{-1}\left(-\frac{5}{10}\right) = \tan^{-1}\left(\frac{-1}{2}\right) = \tan^{-1}(0.5) = -26.56^\circ$

$\cos\theta = \cos(-26.56^\circ) = \cos(26.56^\circ) = 0.89$ [Power factor]

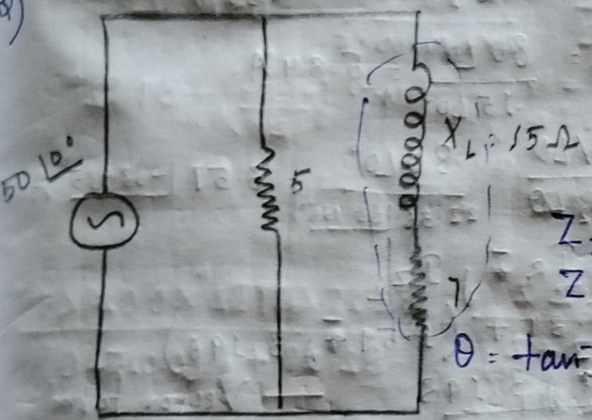
$Z = \frac{V\angle\theta}{I\angle\theta} = \frac{100\angle 0^\circ}{11.18\angle -26.56^\circ} = 8.94\angle 26.56^\circ\ \Omega$

[Verified!]

$Z_1 = 10\angle 0^\circ$; $Z_2 = 20\angle 90^\circ$

$Z = \frac{Z_1 Z_2}{Z_1 + jZ_2} = \frac{10\angle 0^\circ \times 20\angle 90^\circ}{10 + (0 + j20)} = \frac{200\angle 90^\circ}{10 + j20} = \frac{200\angle 90^\circ}{22.36\angle 63.4^\circ} = 8.94\angle 26.52$

Q)



Given,

$V = 50\angle 0^\circ$ V

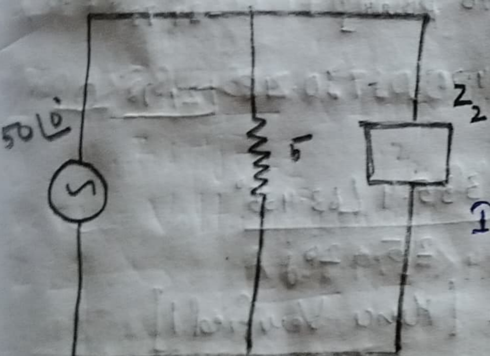
$X_L = 15\Omega$

$Z_2 = \sqrt{R^2 + (X_L)^2} = \sqrt{(7)^2 + (15)^2}$

$Z_2 = \sqrt{49 + 225} = \sqrt{274} = 16.55\Omega$

$\theta = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{15}{7}\right) = 64.9^\circ$

$Z_2 = 16.55\angle 64.9^\circ\ \Omega$



$I_R = \frac{50\angle 0^\circ}{5\angle 0^\circ} = 10$ A

$I_{Z_2} = \frac{V\angle\theta}{Z_2\angle\theta} = \frac{50\angle 0^\circ}{16.55\angle 64.9^\circ} = 3.02\angle -64.9^\circ$

$I_{Z_2} = 1.28 - j2.7$ [Vector form]

$I\angle\theta = I_R + jI_{Z_2}\ (I_R)$

$= 10 + 1.28 - j2.7$

$= 11.28 - j2.7$

$I = \sqrt{(11.28)^2 + (2.7)^2} = \sqrt{127.2384 + 7.29} = \sqrt{184.5284} = 11.59$ A

$\theta = \tan^{-1}\left(\frac{-2.7}{11.28}\right) = -13.46^\circ$

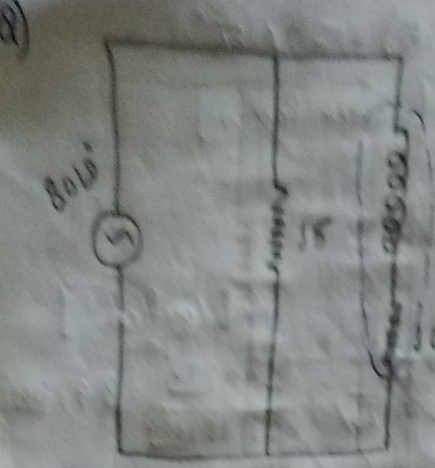
$I\angle\theta = 11.59\angle -13.46^\circ$ A

$Z = \frac{V\angle\theta}{I\angle\theta} = \frac{50\angle 0^\circ}{11.59\angle -13.46^\circ} = 4.31\angle 13.46^\circ\ \Omega$

$Z = \frac{Z_1 Z_2}{Z_1 + jZ_2} = \frac{5\angle 0^\circ \times 16.55\angle 64.9^\circ}{5 + (7 + j15)} = \frac{82.75\angle 64.9^\circ}{12 + j15}$

$= \frac{82.75\angle 64.9^\circ}{19.2\angle 51.34^\circ} = 4.31\angle 13.46^\circ\ \Omega$ [Hence Verified!]

(8)



Given,

$V = 80\angle 0°\ V$

$X_L = 20\ \Omega$

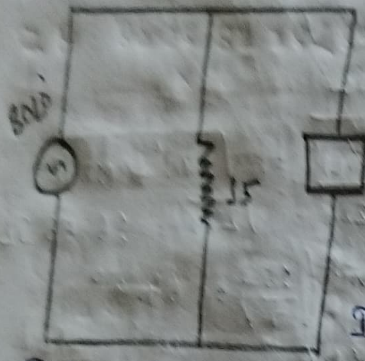
$Z_2 = \sqrt{R^2 + X_L^2} = \sqrt{(20)^2 + (10)^2}$

$Z_2 = \sqrt{400 + 100} = \sqrt{500} = 22.36\Omega$

$\theta = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{20}{10}\right) = 63.435$

$Z_2\angle\theta = 22.36\angle 63.435°\ \Omega$

$I_R = \frac{80\angle 0°}{15\angle 0°} = 5.34A$



$I_{Z_2} = \frac{V\angle\theta}{Z_2\angle\theta} = \frac{80\angle 0°}{22.36\angle 63.435°} = 3.57\angle -63.435°\ A$

$I\angle\theta = I_R \mp j I_{Z_2}$

$= 5.34 + (1.59 \mp j3.193)$

$= 6.93 \mp j3.193$ — Vector form

$I = \sqrt{(6.93)^2 + (3.193)^2} = 7.63$; $\theta = \tan^{-1}\left(\frac{-3.193}{6.93}\right) = -24.73°$

$I\angle\theta = 7.63\angle -24.73°\ A$ [Again converted to polar]

$Z = \frac{V\angle\theta}{I\angle\theta} = \frac{80\angle 0°}{7.63\angle -24.73°} = \frac{80}{7.63}\angle 0° - (-24.73°) = 10.48\angle 24.73°\ \Omega$

$Z = \frac{Z_1 Z_2}{Z_1 + jZ_2} = \frac{15\angle 0° \times 22.36\angle 63.435°}{15 + (10 + 20j)} = \frac{335.4\angle 63.435°}{25 + 20j}$

$= \frac{335.4\angle 63.435°}{32.055\angle 38.66°} = 10.48\angle 24.73°\ \Omega$ [Hence Verified!]

Resonance :-

When the input frequency of the AC circuit gives zero phase difference between voltage and current, then the resonance of the given AC circuit can be calculated.

Conditionally,

$X_L = X_C$ [Resonance]

OR, $2\pi f L = \frac{1}{2\pi f C}$

OR, $f^2 = \frac{1}{4\pi^2 LC}$

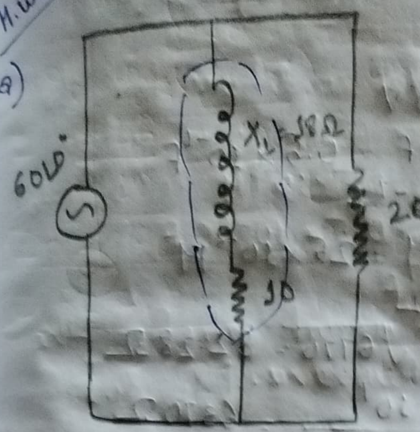
Thus,

$f = \frac{1}{2\pi\sqrt{LC}}$ rad [In radians]

$f = \frac{1}{\sqrt{LC}}$ Hz [In Hertz]

H.W ③

a)



Given,

$V = 60\angle 0^\circ \, V$

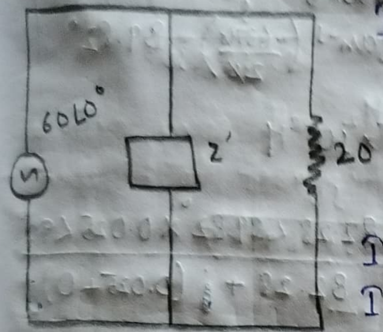
$X_L = 18\,\Omega$

$Z' = \sqrt{R^2 + X_L^2} = \sqrt{(10)^2 + (18)^2}$

$Z' = \sqrt{100+324} = \sqrt{424} = 20.59\,\Omega$

$\theta = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}(1.8) = 60.94^\circ$

$Z'\angle\theta = 20.59\angle 60.94^\circ\,\Omega$



$I_R = \frac{60\angle 0^\circ}{20\angle 0^\circ} = 3A$; $I_{Z'} = \frac{60\angle 0^\circ}{20.59\angle 60.94^\circ} = 2.914\angle -60.94$ A

$I\angle\theta = I_R - jI_{Z'}$ [Inductor representation]

$= 3 + (1.41 - 2.54j)$

$= 4.41 - 2.54j$

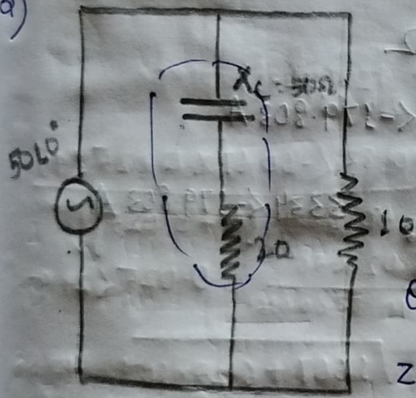
$I = \sqrt{(4.41)^2 + (2.54)^2} = \sqrt{19.44 + 6.45} = \sqrt{25.89}$

$I = 5.08\,A$

$\theta = \tan^{-1}\left(\frac{-2.54}{4.41}\right) = \tan^{-1}(-0.576) = -29.94^\circ$; $I\angle\theta = 5.08\angle -29.94^\circ$ A

$Z = \frac{V\angle\theta}{I\angle\theta} = \frac{60\angle 0^\circ}{5.08\angle -29.94^\circ} = 11.81\angle +29.94^\circ\,\Omega$

a)



Given,

$V = 50\angle 0^\circ\, V$

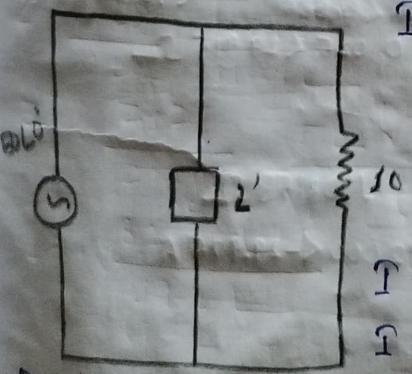
$X_C = 50\Omega$

$Z' = \sqrt{R^2 + X_C^2} = \sqrt{(20)^2 + (50)^2} = \sqrt{400+250}$

$Z' = \sqrt{650} = 25.49\,\Omega$

$\theta = \tan^{-1}\left(\frac{-X_C}{R}\right) = \tan^{-1}\left(\frac{-50}{20}\right) = -68.19^\circ$

$Z'\angle\theta = 25.49\angle -68.19^\circ\,\Omega$



$I_R = \frac{50\angle 0^\circ}{10\angle 0^\circ} = 5A$; $I_{Z'} = \frac{50\angle 0^\circ}{25.49\angle -68.19^\circ} = 1.96\angle 68.19^\circ$ A

$I\angle\theta = I_R + jI_{Z'}$

$= 5 + (0.72 + j1.82)$

$= 5.72 + j1.82$

$I = \sqrt{(5.72)^2 + (1.82)^2} = \sqrt{32.71 + 3.31}$

$I = \sqrt{36.02} = 6A$ (Approx.)

$\theta = \tan^{-1}\left(\frac{1.82}{5.72}\right) = \tan^{-1}(0.31) = 17.22^\circ$

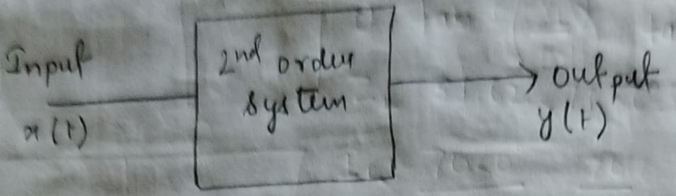
$I\angle\theta = 6\angle 17.22^\circ$

$Z = \frac{V\angle\theta}{I\angle\theta} = \frac{50\angle 0^\circ}{6\angle 17.22^\circ} = 8.34\angle -17.22^\circ\,\Omega$

Natural and Forced Responses in 2nd Order System :-

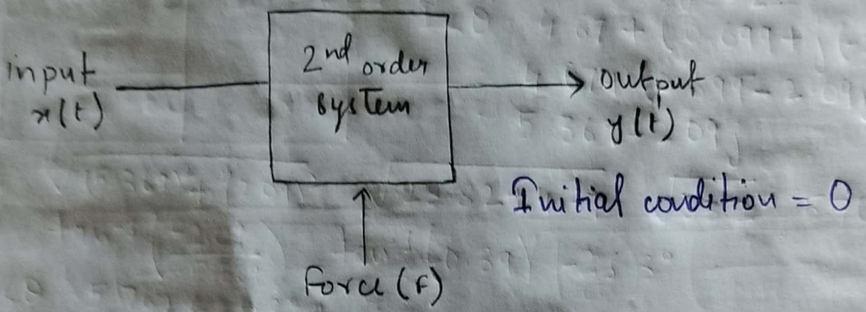
① **Natural Response** : When certain initial conditions for the input is set. It is a system response with some initial conditions, without any external force and $x(t)$ is set to be zero. $x(t)$ is nothing but the input.

Symbolic Representation : $y_n(t)$



② **Forced Response** : It is a system response with some external force ($y_p(t)$) with all the initial condition set to be zero.

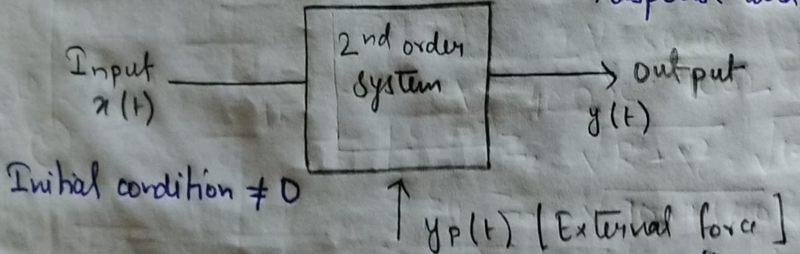
Symbolic Representation : $y_f(t)$



③ **Total Response** : Natural Response + Forced Response

Symbolic Representation : $y_t(t)$

$y_t(t) = y_n(t) + y_f(t)$ Addition of Natural Response and forced Response.



$$\frac{d^2}{dt^2} y(t) + 5\frac{d}{dt} y(t) + 6y(t) = -\frac{d}{dt} x(t) + 4x(t)$$

$x(t) = e^{-t}u(t)$

← unit response = 1

(1) Natural Response :-

$y(0) = 3$; $\frac{d}{dt} y(0) = 0$

$$\frac{d^2}{dt^2} y(t) + 5\frac{d}{dt} y(t) + 6y(t) = 0$$

$\lambda^2 + 5\lambda + 6 = 0$ $[y(t) = \lambda]$

$\lambda^2 + 3\lambda + 2\lambda + 6 = 0$

$\lambda(\lambda+3) + 2(\lambda+3) = 0$

$(\lambda+2)(\lambda+3) = 0$

$\lambda = -2 \mid \lambda = -3$

Homogeneous solutions :-

$y_n(t) = C_1 e^{-2t} + C_2 e^{-3t}$

$y(0) = 3 \Rightarrow C_1 + C_2 = 0$ ——— (I)

$\frac{d}{dt} y(0) = 0 \Rightarrow -2C_1 - 3C_2 = 0$

$2C_1 + 3C_2 = 0$ ——— (II)

Solving eq^n (I) and (II) we get,

$C_1 = 9$ and $C_2 = -6$

(II) Forced Response :-

$x(t) = e^{-t}(1) = e^{-t}$ [Given]

$y_p(t) = ke^{-t}$ [External force acting]

$\frac{d}{dt} y_p(t) = -ke^{-t}$; $\frac{d^2}{dt^2} y_p(t) = ke^{-t}$

$\frac{d}{dt} x(t) = -e^{-t}$

$ke^{-t} + 5(-ke^{-t}) + 6(ke^{-t}) = -e^{-t} + 4e^{-t}$

$ke^{-t} - 5ke^{-t} + 6ke^{-t} = 3e^{-t}$

$2ke^{-t} = 3e^{-t}$

$k = \frac{3}{2} = 1.5$

Forced Response : $y_n(t) + y_p(t)$

$= C_1 e^{-2t} + C_2 e^{-3t} + 1.5e^{-t}$

$y(0) = 0 \Rightarrow \frac{d}{dt} y(0) = 0$

Thus,

$C_1 + C_2 + 1.5 = 0$ ——— (III)

Now,

$\frac{dy}{dt} = -2e^{-2t} C_1 - 3e^{-3t} C_2 - 1.5e^{-t}$

$\frac{d}{dt} y(0) = -2C_1 - 3C_2 - 1.5 = 0$ ——— (IV)

Solving eq^n (III) and (IV),

we get, $C_1 = -3$ and $C_2 = 1.5$

$y_f(t) = -3e^{-2t} + 1.5e^{-3t} + 1.5e^{-t}$

And,

$y_t(t) = y_n(t) + y_f(t)$

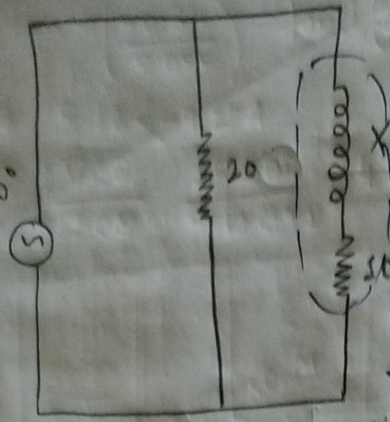
$= 9e^{-2t} - 6e^{-3t} - 3e^{-2t} + 1.5e^{-3t} + 1.5e^{-t}$

$= 6e^{-2t} - 4.5e^{-3t} + 1.5e^{-t}$ [Total Response]

H.W 8

Contd.

Q)



Given,

$V = 70\angle 0° V$

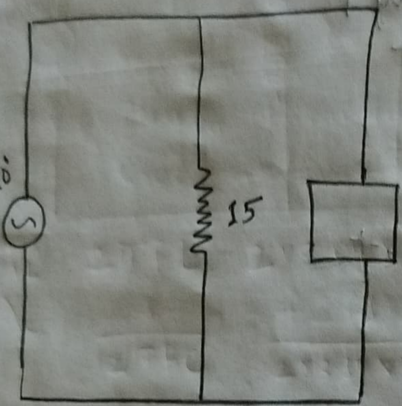
$X_L : 30\Omega$ $X_L : 30\Omega$

$Z_2 = \sqrt{R^2 + X_L^2} = \sqrt{(10)^2 + (30)^2}$

$Z_2 = \sqrt{100 + 900} = \sqrt{1000} = 31.62\Omega$

$\theta = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{30}{10}\right) = 71.56°$

$Z_2 = 31.62 \angle 71.56° \Omega$



$I_R = \frac{70\angle 0°}{15\angle 0°} = \frac{70}{15} = 4.67 A$

$I_{Z_2} = \frac{V\angle\theta}{Z_2\angle\theta} = \frac{70\angle 0°}{31.62\angle 71.56°} = 2.21\angle -71.56° A$

Z_2

$I\angle\theta = I_R + j I_{Z_2}$

$I\angle\theta = 4.67 + (0.699 + j(-2.09))$

$= 4.67 + (0.699 - 2.09j)$

$= 5.369 - 2.09j$

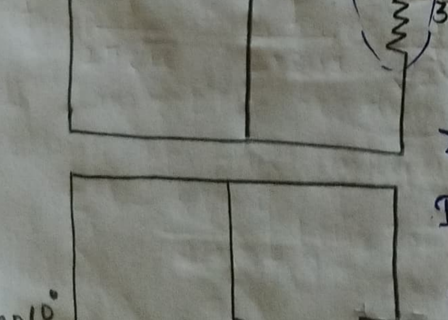
$I = \sqrt{(5.369)^2 + (2.09)^2} = \sqrt{28.82 + 4.37} = \sqrt{33.2} = 5.762 A$

$\theta = \tan^{-1}\left(\frac{-2.09}{5.369}\right) = \tan^{-1}(-0.39) = -21.3°$

$I\angle\theta = 5.762 \angle -21.3° A$

$Z = \frac{V\angle\theta}{I\angle\theta} = \frac{70\angle 0°}{5.762\angle -21.3°} = 12.148 \angle +21.3° \Omega$

Q)



Given,

$V = 80\angle 0°$

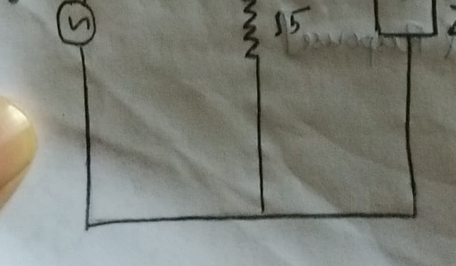
$X_C = 40\Omega$ $X_C = 40\Omega$

$Z_2 = \sqrt{R^2 + X_C^2} = \sqrt{(30)^2 + (40)^2}$

$Z_2 = \sqrt{900 + 1600} = \sqrt{2500} = 50\Omega$

$\theta = \tan^{-1}\left(\frac{-X_C}{R}\right) = \tan^{-1}\left(\frac{-40}{30}\right) = -53.13°$

$Z_2\angle\theta = 50\angle -53.13° \Omega$



$I_R = \frac{80\angle 0°}{15\angle 0°} = \frac{80}{15} = 5.34 A$

$I_{Z_2} = \frac{V\angle\theta}{Z_2\angle\theta} = \frac{80\angle 0°}{50\angle -53.13°} = 1.6\angle 53.13° A$

Z_2

$I\angle\theta = I_R + I_{Z_2}$

$I\angle\theta = 5.34 + (0.96 + j1.28)$

$= 6.3 + j1.28$

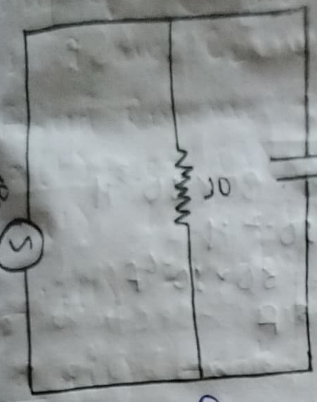
$I = \sqrt{(6.3)^2 + (1.28)^2} = \sqrt{39.69 + 1.64} = \sqrt{41.33} = 6.43\ A$

$\theta = \tan^{-1}\left(\frac{1.28}{6.3}\right) = \tan^{-1}(0.203) = 11.475^\circ$

$I\angle\theta = 6.43\angle 11.475^\circ\ A$

$Z\angle\theta = \frac{V\angle\theta}{I\angle\theta} = \frac{80\angle 0^\circ}{6.43\angle 11.475^\circ} = 12.44\angle -11.475^\circ\ \Omega$

Q)



Given,

$R = 10\Omega$

$X_C = 20\Omega$

$V = 100\angle 0^\circ V$

$I_R = \frac{V\angle\theta}{R\angle\theta} = \frac{100\angle 0^\circ}{10\angle 0^\circ} = 10\ A$

$I_{X_C} = \frac{V}{X_C} = \frac{100\angle 0^\circ}{20\angle -90^\circ} = 5\angle 90^\circ\ A$

$I\angle\theta = I_R + j I_{X_C}$

$= 10 + (0 + 5j)$ [Magnitude in case of RC circuit]

$= 10 + 5j$

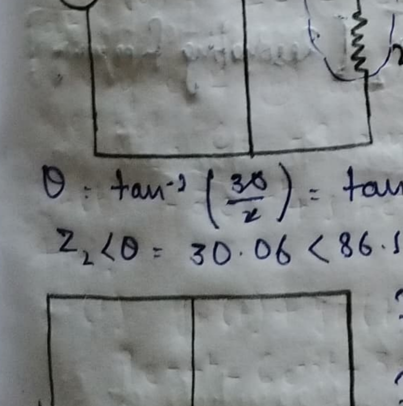
$I = \sqrt{(10)^2 + (5)^2} = \sqrt{100 + 25} = \sqrt{125} = 11.18\ A$

$\theta = \tan^{-1}\left(\frac{5}{10}\right) = \tan^{-1}\left(\frac{1}{2}\right) = \tan^{-1}(0.5) = 26.56^\circ$

$\cos\theta = \cos(26.56^\circ) = 0.89$ [power-factor]

$Z = \frac{V\angle\theta}{I\angle\theta} = \frac{100\angle 0^\circ}{11.18\angle 26.56^\circ} = 8.94\angle -26.56^\circ\ \Omega$

Q)



Given,

$V = 25\angle 0^\circ V$

$X_L = 50\Omega$

$X_C = 20\Omega$

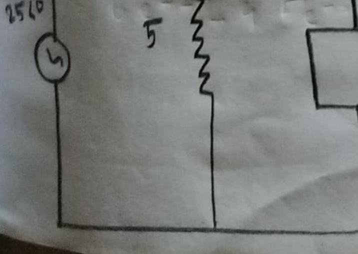
$Z_2 = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(2)^2 + (50-20)^2}$

$Z_2 = \sqrt{4 + 900} = \sqrt{904} = 30.06\ \Omega$

$\theta = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{50-20}{2}\right)$

$\theta = \tan^{-1}\left(\frac{30}{2}\right) = \tan^{-1}(15) = 86.19^\circ$

$Z_2\angle\theta = 30.06\angle 86.19^\circ\ \Omega$



$I_R = \frac{25\angle 0^\circ}{5\angle 0^\circ} = \frac{\cancel{25}}{\cancel{5}} = 5\ A$

$I_{Z_2} = \frac{V\angle\theta}{Z_2\angle\theta} = \frac{25\angle 0^\circ}{30.06\angle 86.19^\circ} = 0.83\angle -86.19^\circ\ A$

$I\angle\theta = I_R + j I_{Z_2}$

$I\angle\theta = 5 + (0.055 - 0.82j)$

$= 5.055 - j0.82$

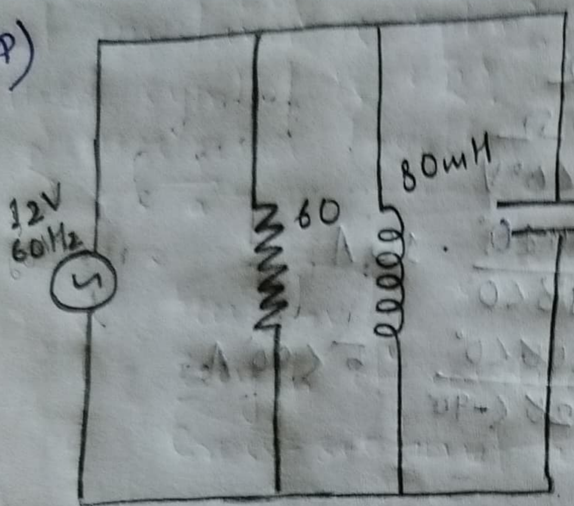
$I = \sqrt{(5.055)^2 + (0.82)^2} = \sqrt{25.55 + 0.67} = \sqrt{26.22} = 5.12\,A$

$\theta = \tan^{-1}\left(\frac{-0.82}{5.055}\right) = \tan^{-1}(-0.162) = -9.2^\circ$

$I\angle\theta = 5.12\angle -9.2^\circ A$

$Z\angle\theta = \frac{V\angle\theta}{I\angle\theta} = \frac{25\angle 0^\circ}{5.12\angle -9.2^\circ} = 4.88\angle 9.2^\circ\,\Omega$

9)



Given,

$V = 12V$

$f = 60Hz$

$L = 80mH = 80 \times 10^{-3} H$

$= 8 \times 10^{-2} H$

$C = 30\mu F = 30 \times 10^{-6} F$

$= 3 \times 10^{-5} F$

$R = 60\Omega$

$X_L = 2\pi f L = 2 \times 3.14 \times 60 \times 8 \times 10^{-2} = 30.16\,\Omega$

$X_C = \frac{1}{2\pi f C} = \frac{1}{2 \times 3.14 \times 60 \times 3 \times 10^{-5}} = 88.42\,\Omega$

$Z = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}} = \frac{1}{\sqrt{\left(\frac{1}{60}\right)^2 + \left(\frac{1}{30.16} - \frac{1}{88.42}\right)^2}}$

$= \frac{1}{\sqrt{\frac{1}{3600} + \left(\frac{1}{30.16} - \frac{1}{88.42}\right)^2}} = 36.39\,\Omega$

$I_L = \frac{V}{X_L} = \frac{12}{30.16} = 0.3978\,A$

$I = \frac{V}{Z} = \frac{12}{36.39} = 0.3297\,A$

$I_C = \frac{V}{X_C} = \frac{12}{88.42} = 0.1357\,A$

$I_R = \frac{V}{R} = \frac{12}{60} = \frac{1}{5} = 0.2\,A$

$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2 \times 3.14} \times \frac{1}{\sqrt{24 \times 10^{-7}}} = 102.7\,Hz$ [Resonating frequency]