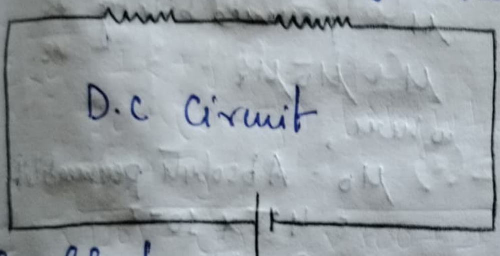
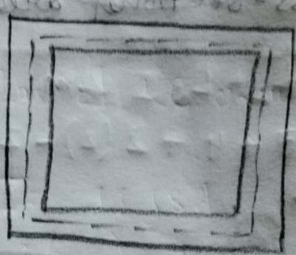
Module - 3 : Magnetic Circuits

① Introduction to Magnetic Circuits
② Analogy between Electric v/s Magnetic circuits
③ Series and Parallel magnetic circuits
④ Series and Parallel magnetic circuits with dot conventions
⑤ Self and Mutual Inductance
⑥ Applications of Magnetic Circuits
- DC Machines (motors and generators)
- Transformers

Introduction to Magnetic Circuits :-

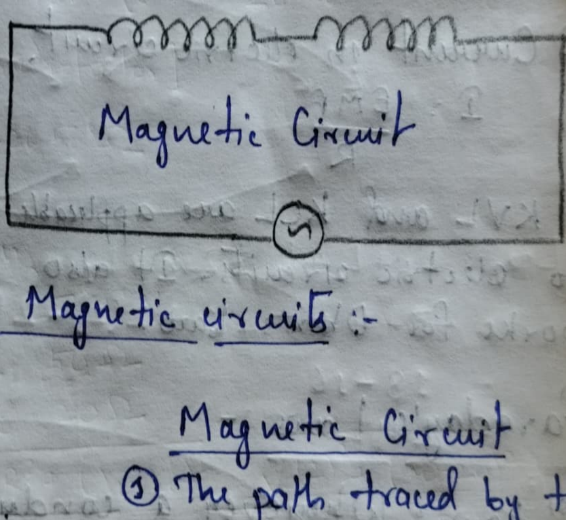




$R = \frac{\rho l}{a}$ [$\rho = 10^{-8}$ to 10^{-6} (good conductor)]

$R = \frac{EMF}{I}$

$\phi = \frac{MMF}{Reluctance} = \frac{N \times I}{S_T}$



Analogy between Electric and Magnetic circuits :-

Electric Circuit	Magnetic Circuit
① The path traced by the current is known as electric circuit.	① The path traced by the flux is known as magnetic circuit.
② Electromotive Force (EMF) is the driving force in the electric circuit.	② Magnetomotive force (MMF) is the driving force in magnetic circuit.
③ There is a current flowing through the electric circuit, which is measured in Ampere, used symbol is 'I'.	③ The flux is flowing through the magnetic circuit, which is measured in weber, used symbol is 'ϕ'.

④ A floor of electrons decide the current in the conductor.

⑤ Resistance opposes the flow of current. Unit of resistance is 'ohm', used symbol is 'R'.

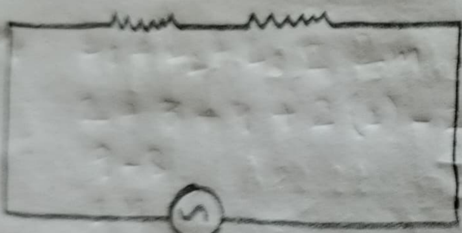
$$R = \rho \frac{l}{a}$$

where,

ρ = resistivity

l = length of conductor

a = Cross-sectional area



⑥ Current in electric circuit,

$$I = \frac{EMF}{R}$$

⑦ KVL and KCL are applicable to electric circuits. It also works for Ohm's Law.

④ Magnetic lines decide the flux in the circuit.

⑤ Reluctance opposes the flow of magnetic lines. The symbol used is 'S'.

$$S = \frac{l}{\mu a}$$

where,

l = length of coil

a = cross-sectional area

μ = permeability

$\mu = \mu_0 \mu_r$

where,

μ_0 = Absolute permeability $= 4\pi \times 10^{-7}$

μ_r = Relative permeability

⑥ Flux in magnetic circuit $= \frac{MMF}{S} = \frac{NI}{S_T}$

⑦ Kirchoff's flux and MMF laws are applicable in magnetic circuits.

Faraday's Laws :-

Ist Law : Whenever a conductor cuts a magnetic field, an EMF is induced with the coil.

IInd Law : EMF is directly proportional to the rate of change of flux linkages.

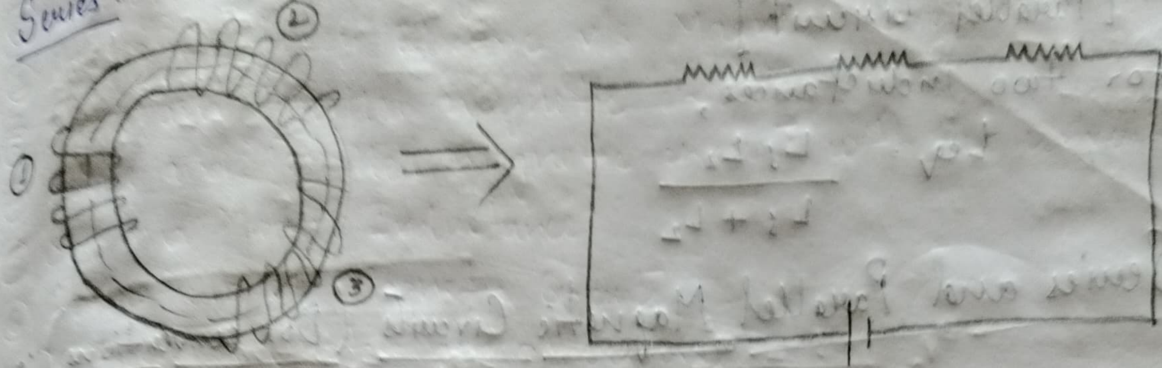
$$e = -\frac{d\phi}{dt} \quad [\text{Lenz law}]$$

where,

$-ve$ sign represents the Lenz law. Magnitude of EMF is in reverse direction.

Series and Parallel Magnetic Circuits :-

Series :-



$$S_T = S_1 + S_2 + S_3$$
$$= \frac{l_1}{\mu_0 \mu_r a_1} + \frac{l_2}{\mu_0 \mu_r a_2} + \frac{l_3}{\mu_0 \mu_r a_3}$$

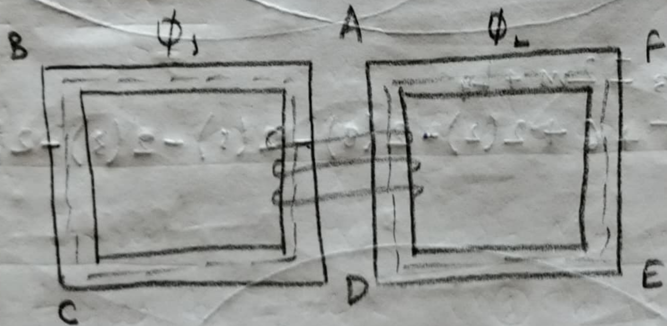
$$\phi = \frac{MMF}{S_T} = \frac{NI}{S_1 + S_2 + S_3}$$

① $MMF = \phi(S_1 + S_2 + S_3) = \phi S_1 + \phi S_2 + \phi S_3$

② $MMF = HL = \frac{B}{\mu}$

where,
- H = magnetising force
- B = magnetic flux density

Parallel :-

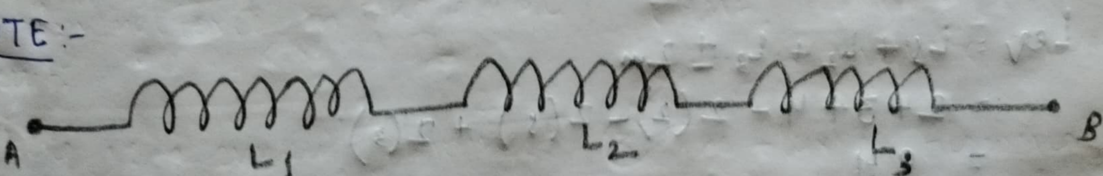


$$\phi = \phi_1 + \phi_2$$

ABCDA — $MMF = \phi_1 S_1 + \phi_c S_c$

AFEDA — $MMF = \phi_2 S_2 + \phi_c S_c$

NOTE :-

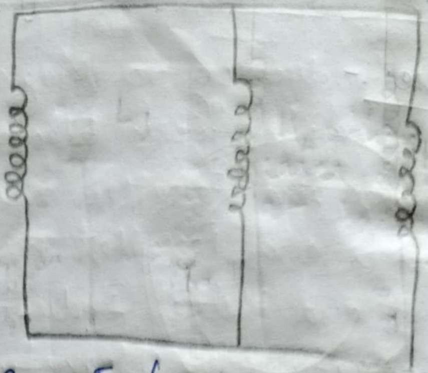


$$L_{eq} = L_1 + L_2 + L_3$$

[Series circuit]

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$

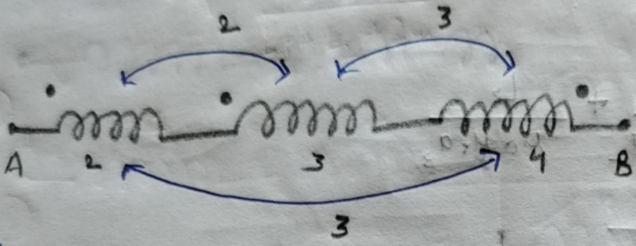
[Parallel Circuit]



For two inductances,

$$L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$$

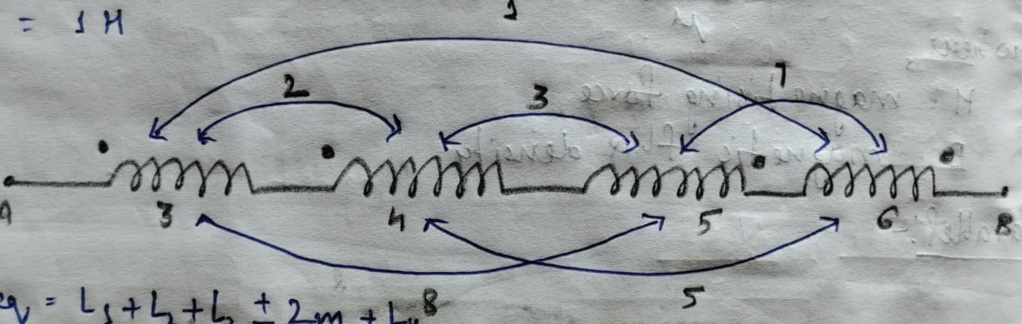
Series and Parallel Magnetic Circuits (Dot conventions):



Dot - Dot : '+'ve
Dot - coil : '-'ve
Coil - Dot : '-'ve
Coil - Coil : '+'ve

$L_{eq} = L_1 + L_2 + L_3 \pm 2m$
$= 2 + 3 + 4 + 2(2) - 2(3) - 2(3)$
$= 9 - 8$
$= 1H$

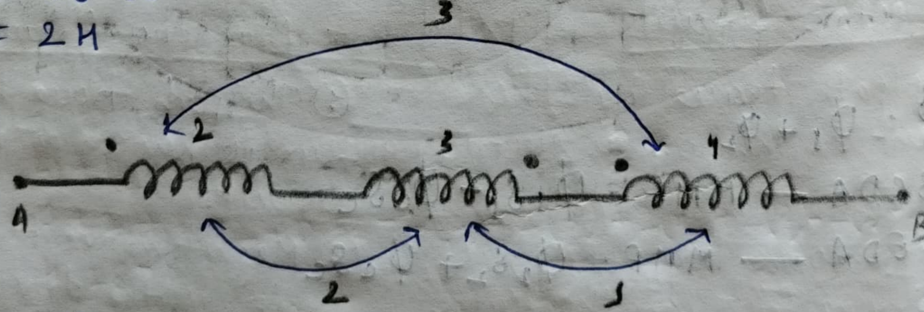
Q)



$L_{eq} = L_1 + L_2 + L_3 \pm 2m + L_4$
$= 3 + 4 + 5 + 6 + 2(2) - 2(8) - 2(1) - 2(3) - 2(5) + 2(7)$
$= 18 - 16$
$= 2H$

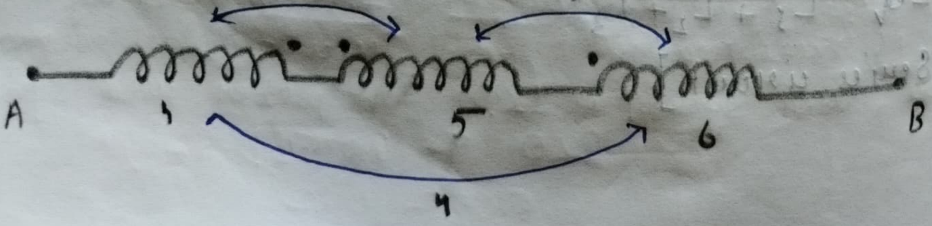
H.W Q

Q)



$L_{eq} = L_1 + L_2 + L_3 \pm 2m$
$= 2 + 3 + 4 - 2(2) - 2(1) + 2(3)$
$= 9H$

Q)



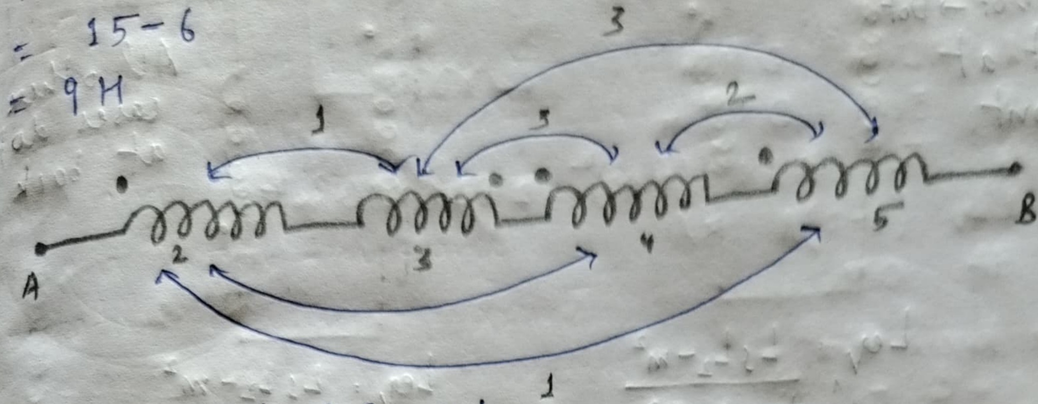
$L_{eq} = L_1 + L_2 + L_3 \pm 2m$

$= 4 + 5 + 6 - 2(2) + 2(3) - 2(4)$

$= 15 - 6$

$= 9\,H$

Q)

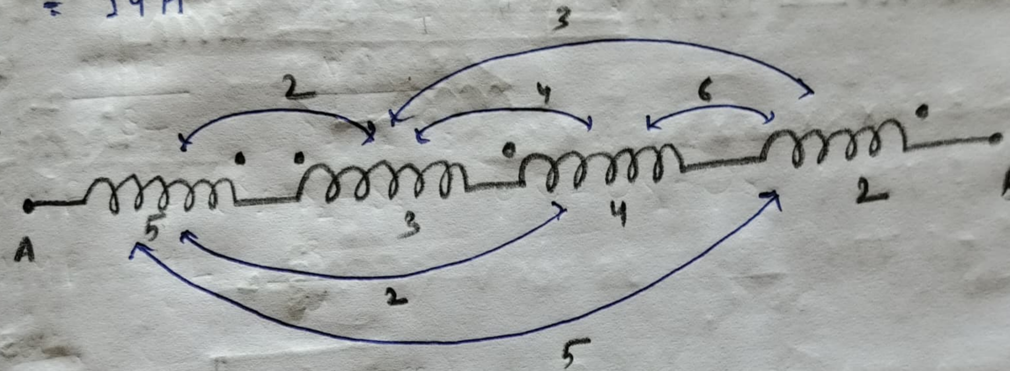


$L_{eq} = L_1 + L_2 + L_3 \pm 2m + L_4$

$= 2 + 3 + 4 + 5 - 2(1) + 2(2) + 2(1) + 2(2) - 2(3) - 2(1)$

$= 14\,H$

Q)



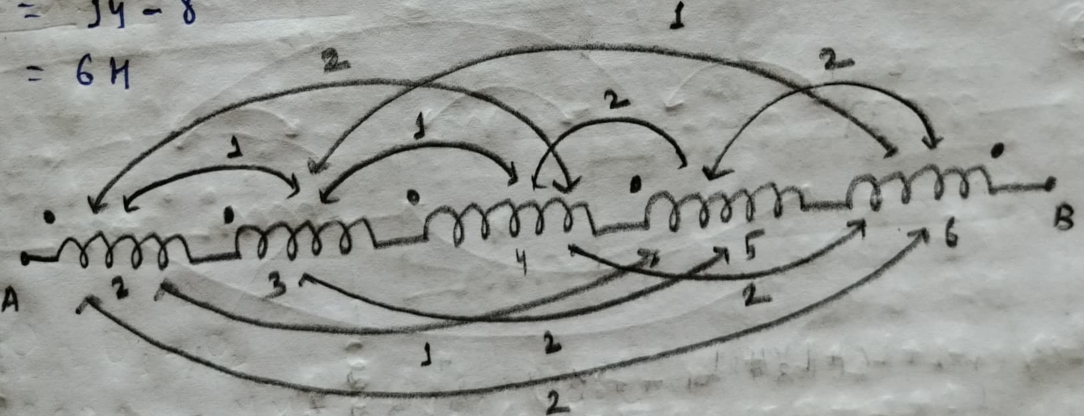
$L_{eq} = L_1 + L_2 + L_3 + L_4 \pm 2m$

$= 5 + 3 + 4 + 2 - 2(2) + 2(4) - 2(6) - 2(2) + 2(5) - 2(3)$

$= 14 - 8$

$= 6\,H$

Q)



$L_{eq} = L_1 + L_2 + L_3 + L_4 + L_5 \pm 2m$

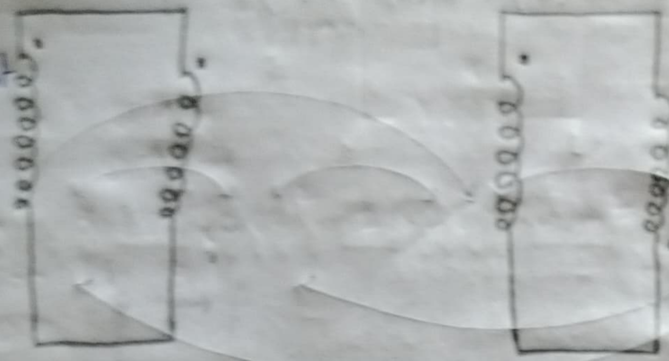
$= 2 + 3 + 4 + 5 + 6 + 2(1) + 2(2) + 2(1) - 2(2) + 2(1) + 2(2) - 2(1) + 2(2) - 2(2) - 2(2)$

$= 20 + 4$

$= 24\,H$

Parallel Circuits with Dot conventions

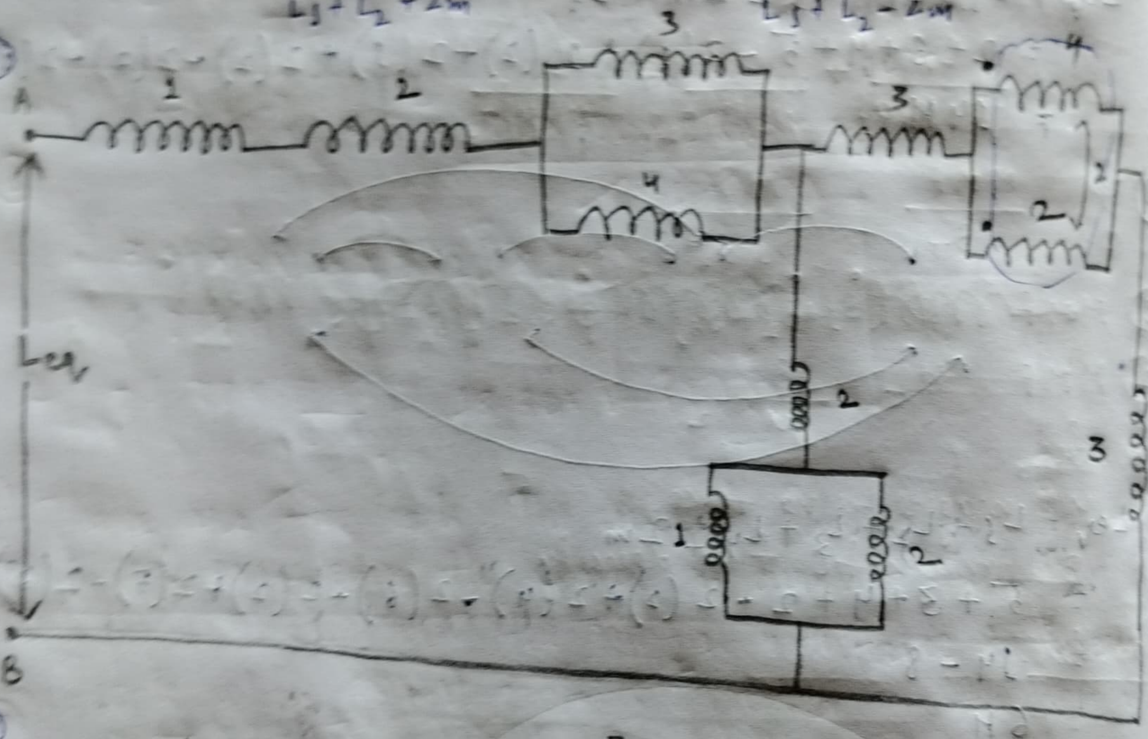
Dot Dot → Dot
Dot at front



All other cases dot at back

$$L_{eq} = \frac{L_1 L_2 - m^2}{L_1 + L_2 + 2m} \qquad L_{eq} = \frac{L_1 L_2 - m^2}{L_1 + L_2 - 2m}$$

①



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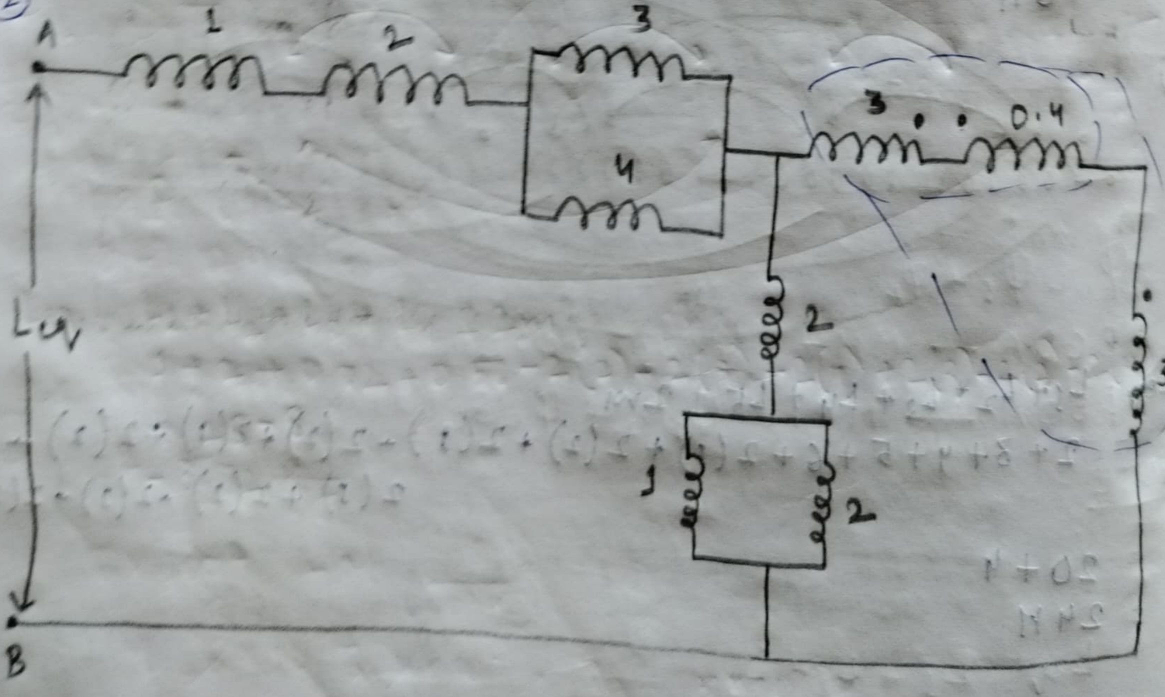

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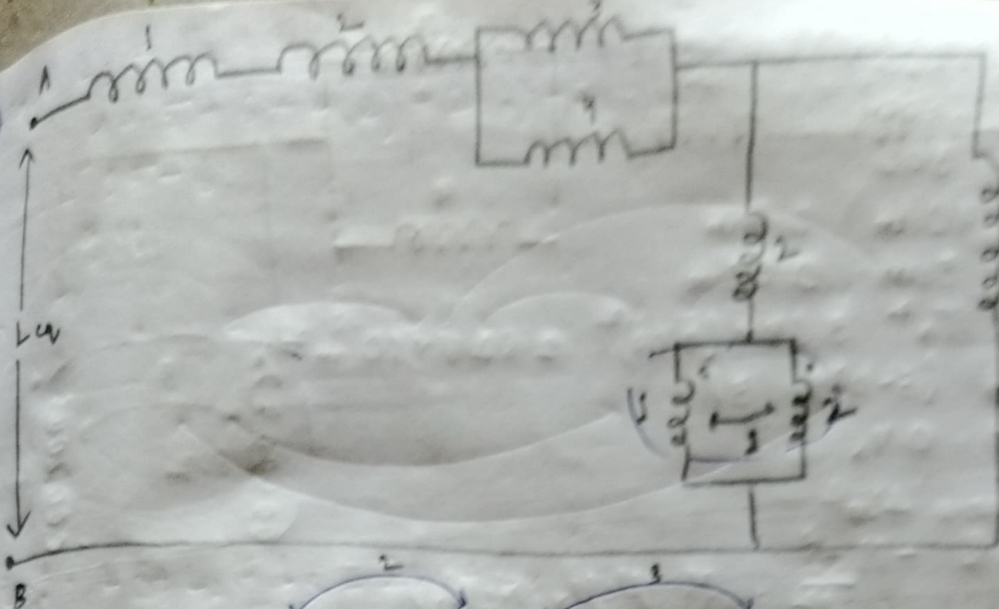
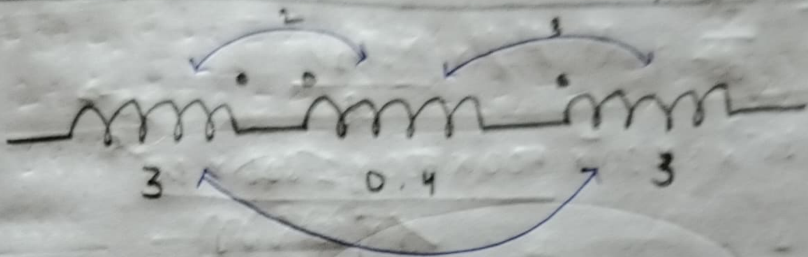




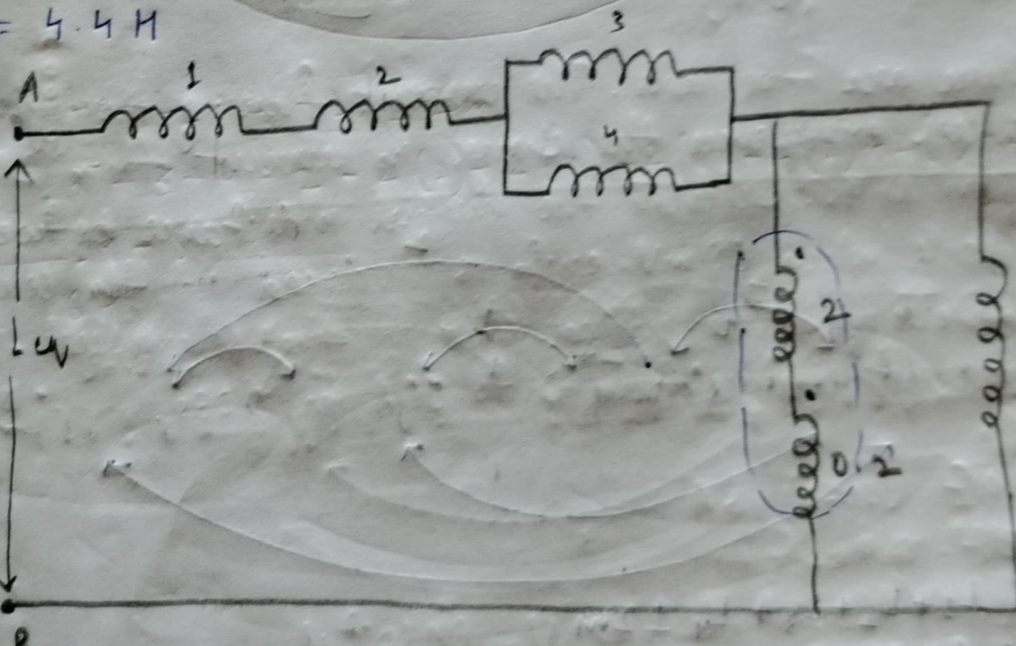
$= L_1 + L_2 + L_3 \pm 2m$

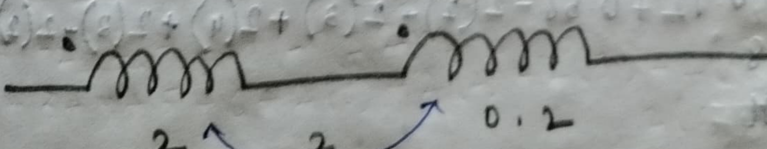
$= 3 + 0.4 + 3 - 2(2) - 2(2) + 2(3)$

$= 3.4 + 3 - 2$

$= 4.4\ H$

4





$= L_1 + L_2 \pm 2m$

$= 2 + 0.2 + 2(2)$

$= 6.2\ H$

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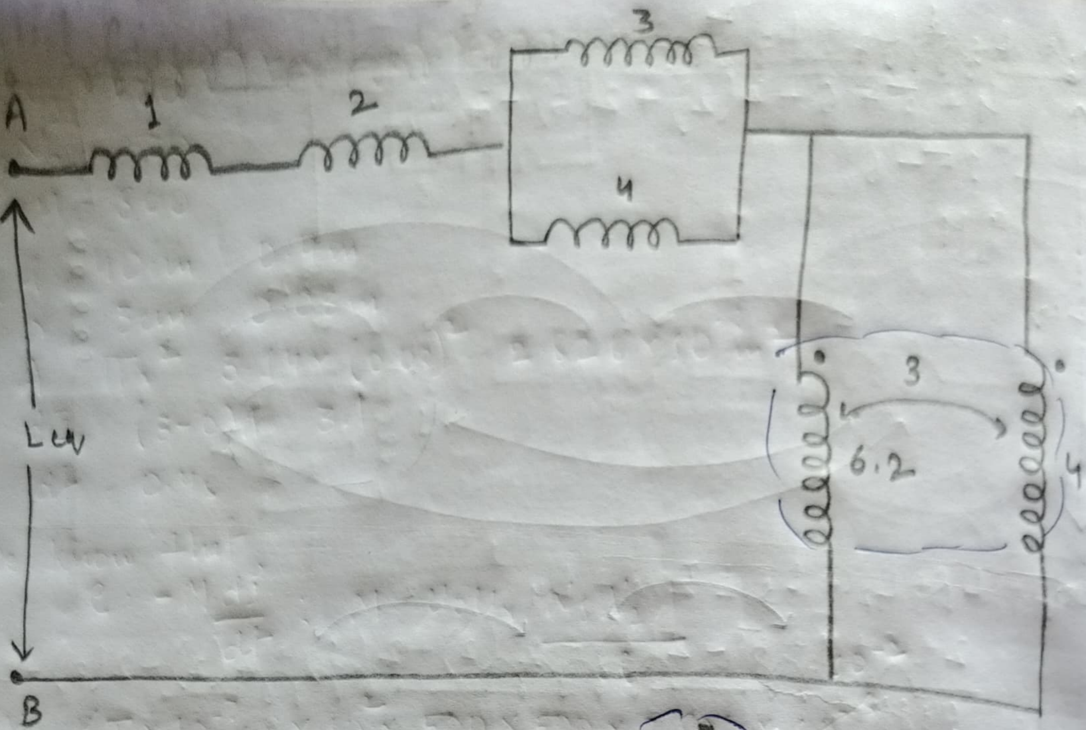

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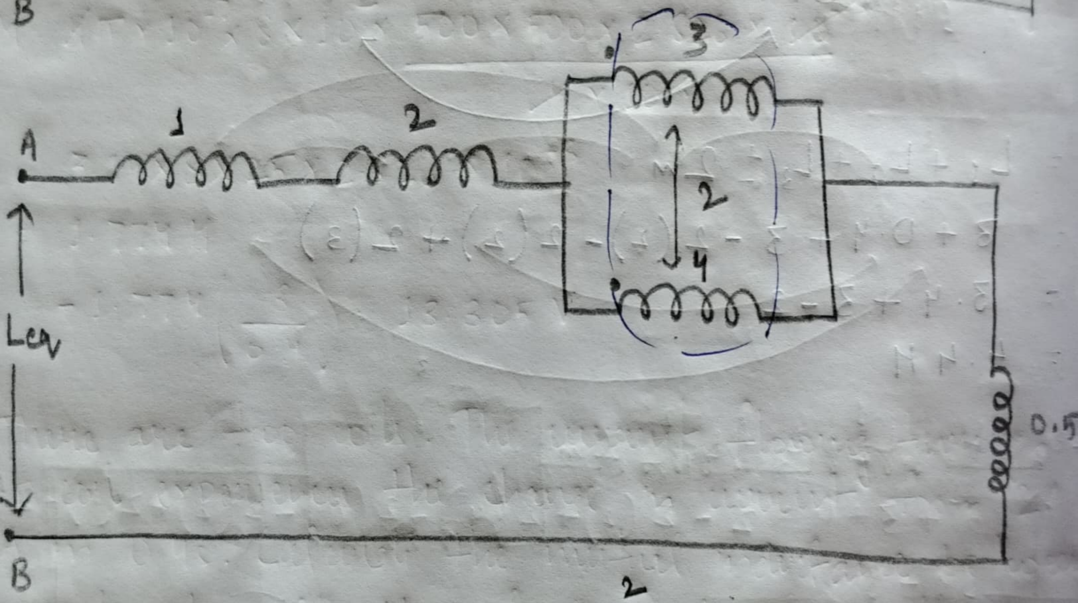

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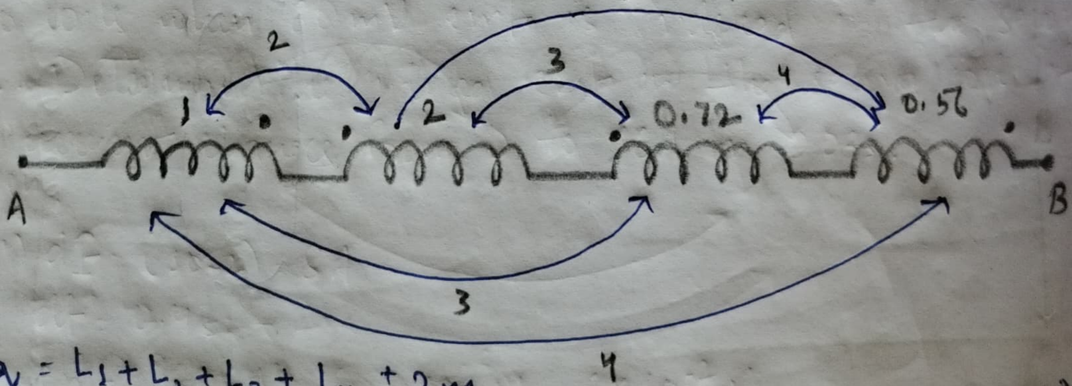


$$L_{eq} = L_1 + L_2 + L_3 + L_4 \pm 2M$$
$$= 1 + 2 + 0.72 + 0.56 - 2(2) - 2(3) + 2(4) + 2(3) - 2(2) - 2(4)$$
$$= 4.28 - 8$$
$$= -3.72\ H$$

Self Inductance :-

The change in current going through a coil produces an emf in the same coil.

$\phi \propto I$

$\phi = LI$

$\uparrow$ self inductance

$$e = -\frac{d\phi}{dt} = -\frac{d(LI)}{dt}$$

$$e = -L\frac{dI}{dt}$$

And,

$$L = \frac{N \times \phi}{I}$$

Mutual Inductance :-

The change in current flowing through the primary coil produces an emf in the secondary coil.

$\phi \propto I$

$\phi = MI$

$\uparrow$ Mutual inductance

$$e = -M.\frac{dI}{dt}$$

$$M = \mu_0 \mu_r \frac{N_1 N_2 A}{l}$$

Energy stored in the coil $= \frac{1}{2} LI^2$

Q) Magnetic circuit having two coils. The number of primary and secondary turns are 500. Their relative permeability is 800. The length of the coil is 40cm. The radius of the coil is 3cm. The change in current is from 0 to 3A. Calculate the induced emf for the change in current for the time of 0.4s.

Given,

$N_1 = 500$

$N_2 = 500$

$\mu = 800$

$l = 40cm = 0.4m$

$r = 3cm = 0.03m$

$A = \pi r^2 = 3.14 \times (0.03)^2 = 2.826 \times 10^{-3} m^2$

$dI = (3-0)A = 3A$

$dt = 0.4s$

We know that,

$$e = -M\frac{dI}{dt} \; ; \; M = \mu_0 \mu_r \frac{N_1 N_2 A}{l}$$

$$M = \cancel{4}\pi \times 10^{-7} \times 8 \times 10^{2} \times \frac{500 \times 500 \times 2.826 \times \cancel{10^{-3}}^{10^{-2}}}{\cancel{4} \times \cancel{10^{-1}}}$$

$$= 3.14 \times 8 \times 25 \times 2.826 \times 10^{-3}$$

$$= 1.774 \, H$$

$$e = -1.774\left(\frac{3}{0.4}\right) = 13.305 \, V$$

Q) There are two coils. The current flowing through the first coil experiences the change in current from 2A to 10A in 0.4s. Calculate the mutual inductance between two coils when 60 mV emf is induced in the second coil. Determine the induced emf in the second coil if the current changes from 0 to 16A in 0.03s.

Given,

(i) $dI = (10-2) = 8A$

$dt = 0.4s$

$emf = 60mV = 60 \times 10^{-3} V = 6 \times 10^{-2} V$

We know that,

$$e = -M\frac{dI}{dt}$$

$$6 \times 10^{-2} = -M\left(\frac{8}{0.4}\right)$$

$$M = \cancel{6}^{3} \times 10^{-2} \times \cancel{4} \times 10^{-1} \times \frac{1}{\cancel{8}} = 3 \times 10^{-3} H$$

(ii) And,

$$e = 3 \times 10^{-3}\left(\frac{12}{0.03}\right) = \cancel{3} \times 10^{-3} \times \frac{12}{\cancel{3} \times 10^{-2}}$$

$$e = 1.2 \, V$$

Q) In a magnetic circuit, the no. of turns in the coil is 400. The length of the coil (L_c) is 50 cm. The length of the area (l_g) is 1 mm. The cross-sectional area of A_c and A_g is 15 cm². The relative permeability is 3000. The current flowing through a coil is 1 A. Find out:

i) flux
ii) flux density
iii) self inductance of the coil

NOTE :- The relative permeability in area is 1.

Given,

$N = 400$

$l_c = 50\ cm = 0.5\ m$

$l_g = 1\ mm = 10^{-3}\ m$

$A_c = A_g = 15\ cm^2 = 15 \times 10^{-4}\ m^2$

$\mu_r = 3000$

$I = 1\ A$

$$S_c = \frac{L_c}{\mu_0 \mu_r A_c} = \frac{0.5 \times 10^4}{4\pi \times 10^{-7} \times 3000 \times 15} = 8.84 \times 10^4$$

$$S_g = \frac{l_g}{\mu_0 \mu_r A_g} = \frac{10^{-3} \times 10^4}{4\pi \times 10^{-7} \times 1 \times 15}$$

$$S_T = S_c + S_g = 6.18 \times 10^5$$

(i) $$\phi = \frac{MMF}{S_T} = \frac{N \times I}{S_T} = \frac{400 \times 1}{6.18 \times 10^5} = 64.7 \times 10^{-5}$$

(ii) $$B_g = \frac{Flux}{A_g} = \frac{64.7 \times 10^{-5}}{15 \times 10^{-4}} = 0.43$$

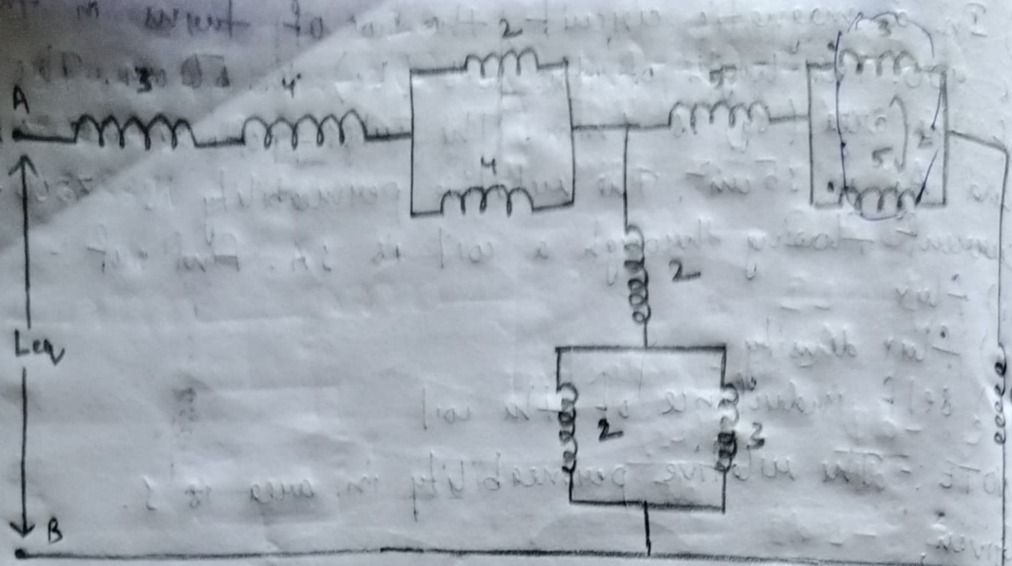
(iii) $$L = \frac{N\phi}{I} = 400 \times 64.7 \times 10^{-5} = 25880 \times 10^{-5} \simeq 0.26\ H$$

Applications of Magnetic Circuit :-

1. Transformers
2. DC machines (motors and generators)
3. MRI (medical applications)
4. Loudspeakers
5. Earphones
6. Electric meters
7. Computers
8. Audio video systems
9. Electromagnetics
10. Solenoids
11. Paramagnetism
12. Ferromagnetism
13. mass spectrometry
14. Credit and debit cards

H.W (16)

Q) (1)



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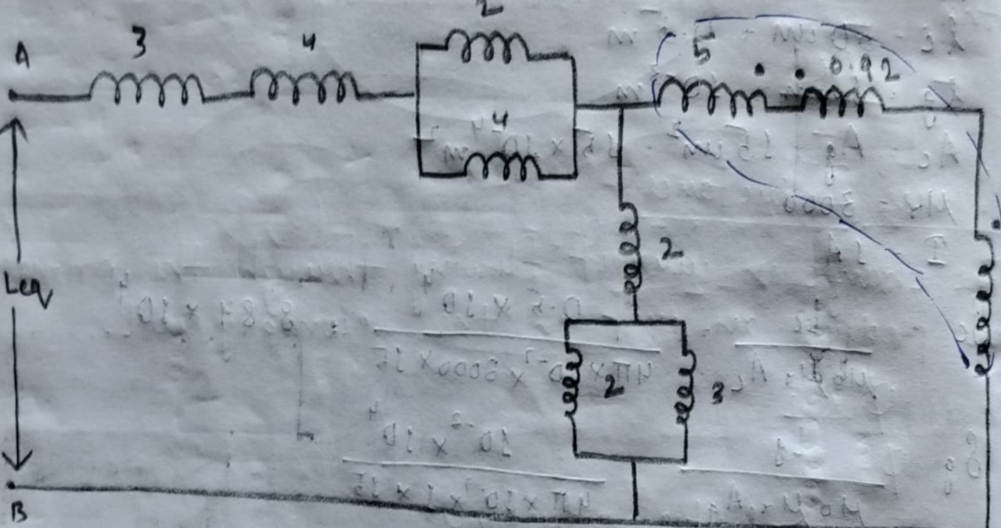
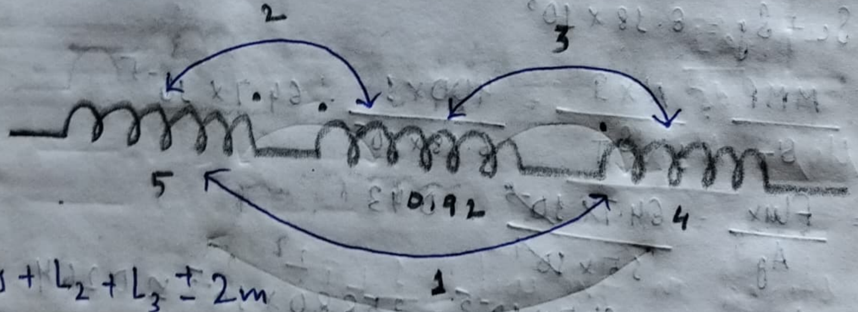


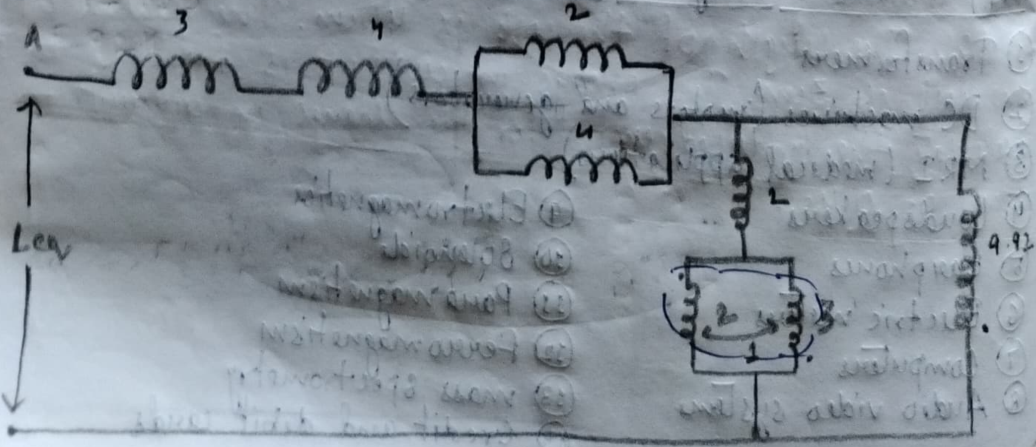


$= L_1 + L_2 + L_3 \pm 2m$

$= 5 + 0.92 + 4 - 2(2) + 2(3) - 2(1)$

$= 9.92\,H$

(3)



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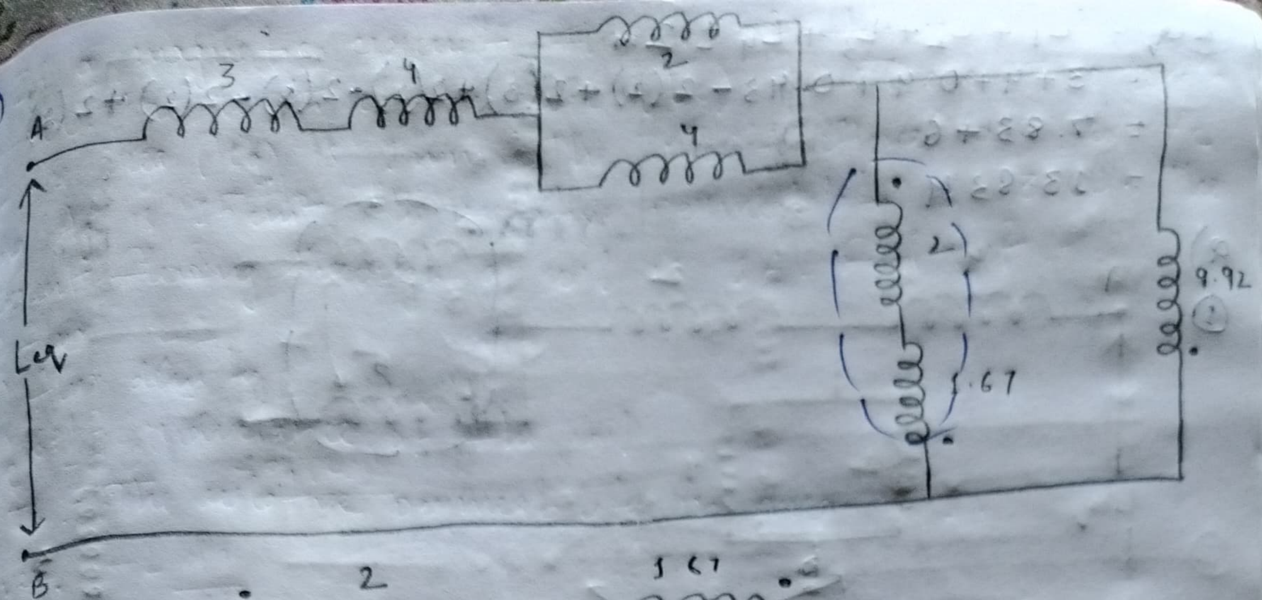
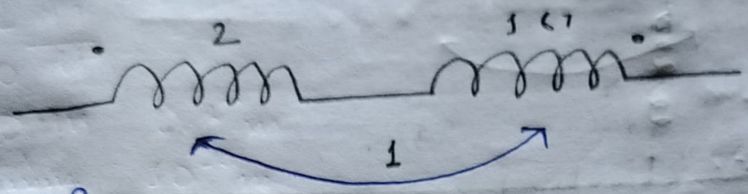


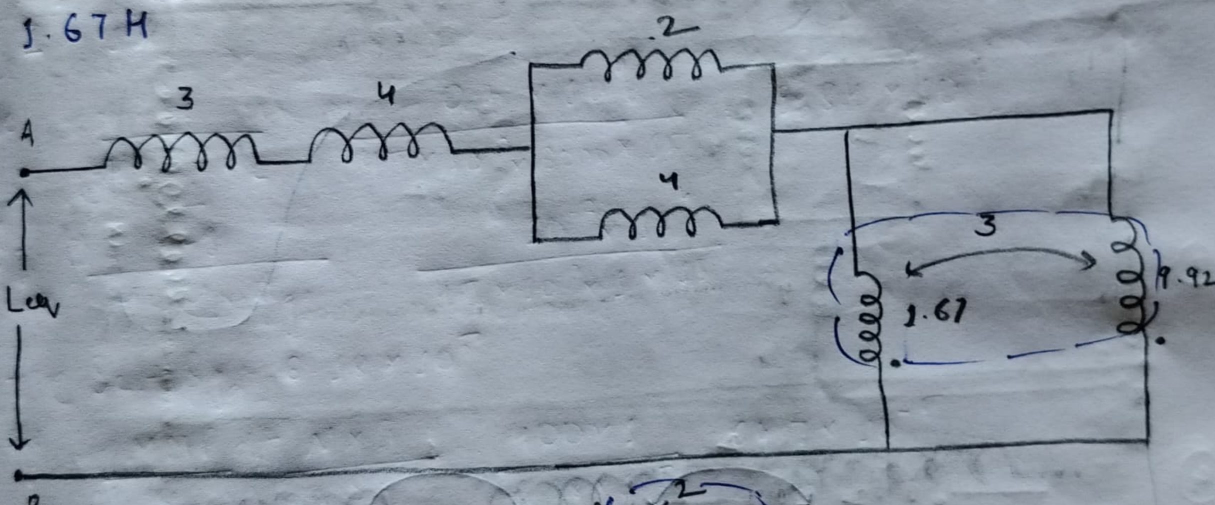


$= L_1 + L_2 \pm 2m$

$= 2 + 1.67 - 2(1)$

$= 1.67\,H$

(5)



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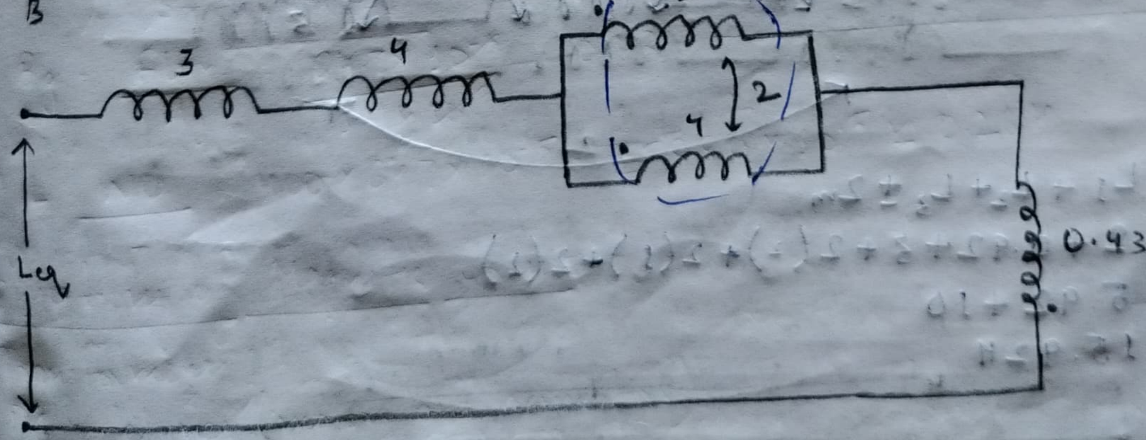

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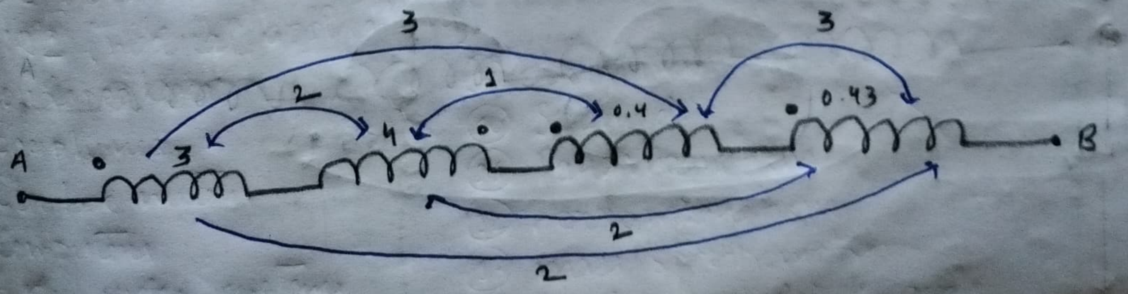
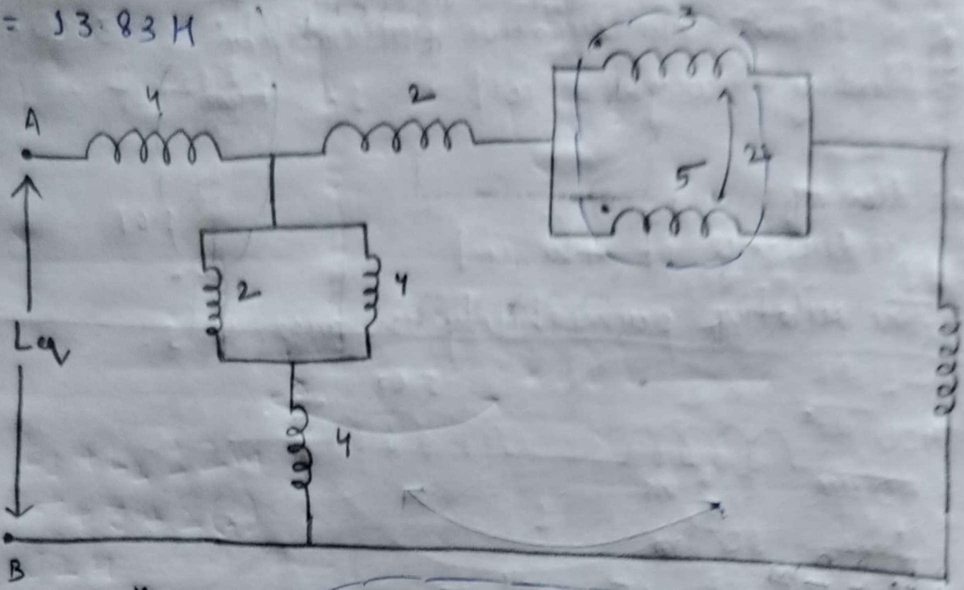


$$L_{eq} = L_1 + L_2 + L_3 + L_4 \pm 2m$$
$$= 3 + 4 + 0.4 + 0.43 - 2(1) + 2(3) + 2(2) - 2(1) - 2(2) + 2(3)$$
$$= 7.83 + 6$$
$$= 13.83\,H$$

9)

(1)



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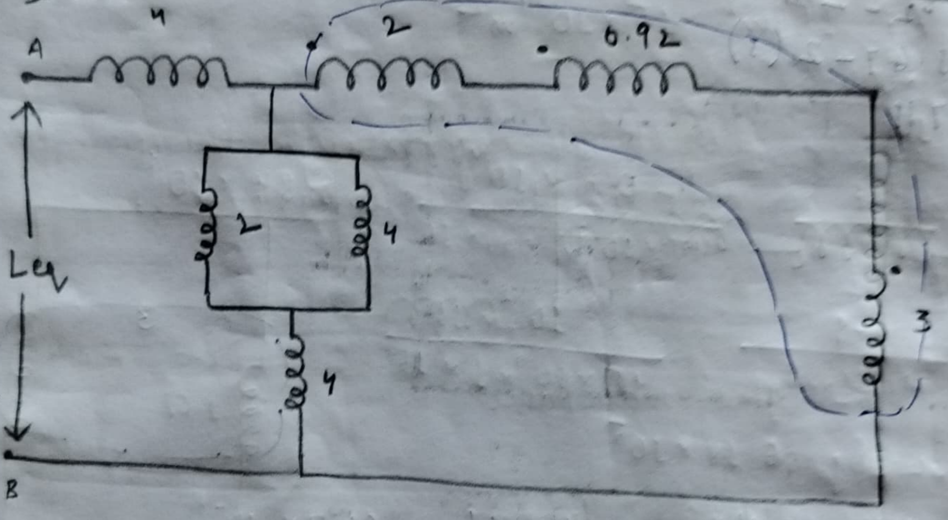
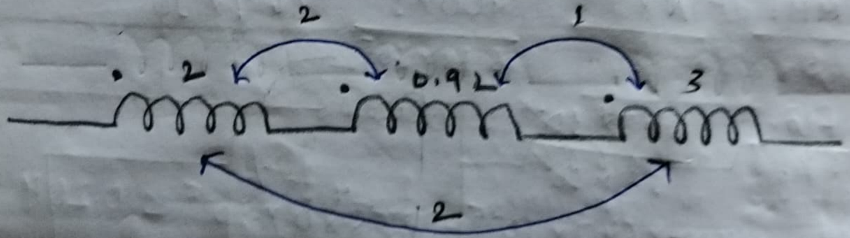
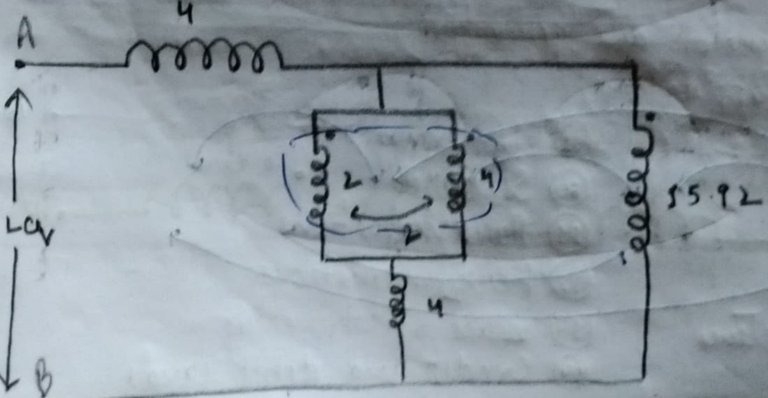


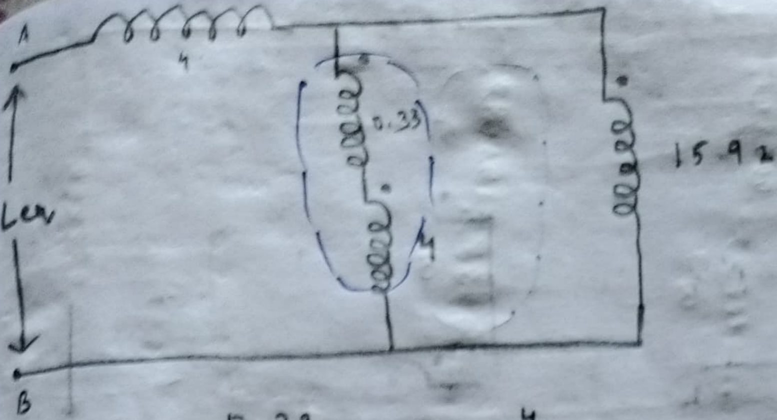


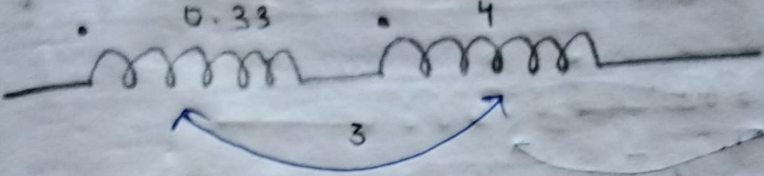
$$= L_1 + L_2 + L_3 \pm 2m$$
$$= 2 + 0.92 + 3 + 2(2) + 2(1) + 2(2)$$
$$= 5.92 + 10$$
$$= 15.92\,H$$

(3)



④



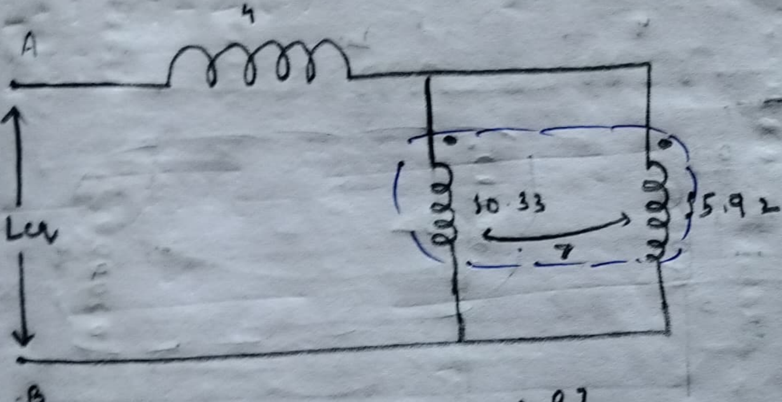


$= L_1 + L_2 \pm 2m$

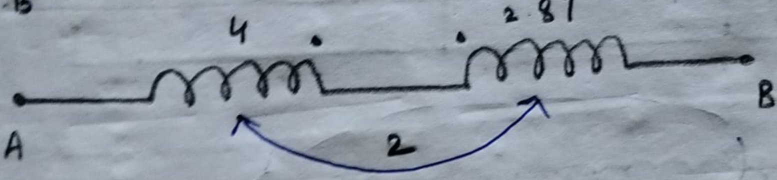
$= 0.33 + 4 + 2(3)$

$= 10.33\ H$

⑤



⑥



$L_{eq} = L_1 + L_2 \pm 2m$

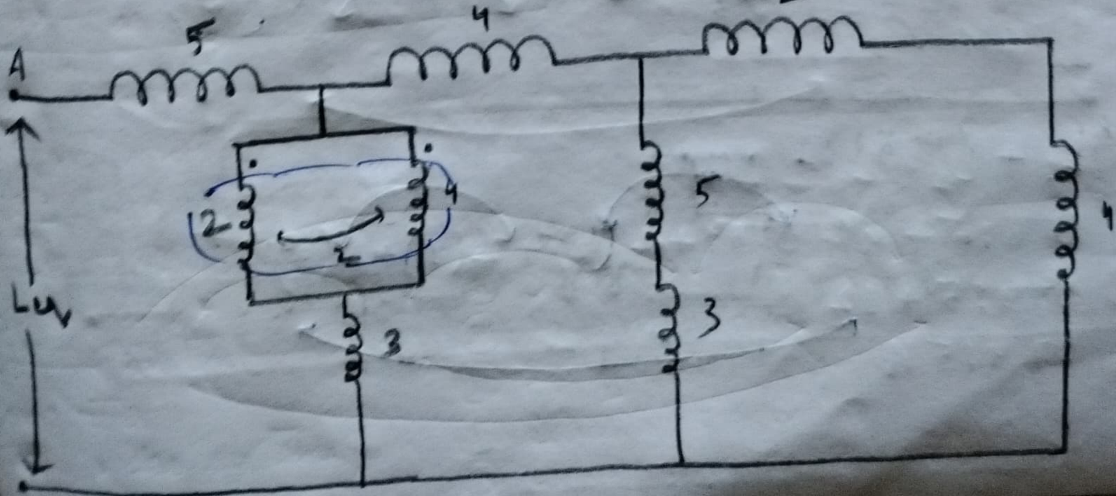
$= 4 + 2.87 - 2(2)$

$= 6.87 - 4$

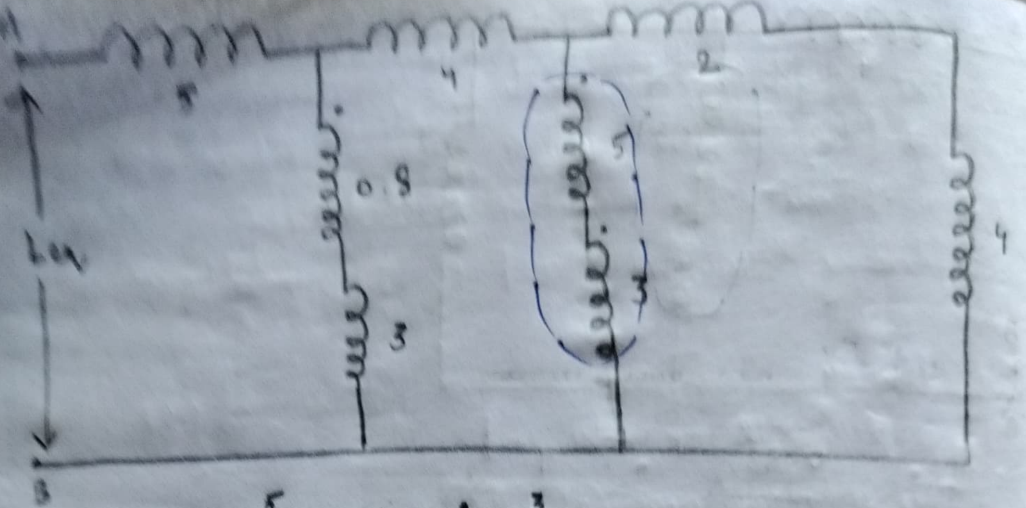
$= 2.87\ H$

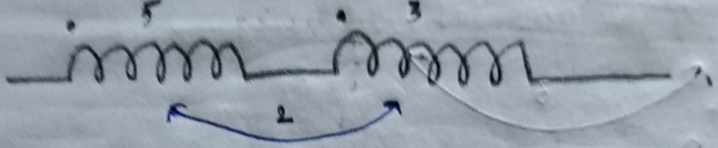
9)

⑦



(2)



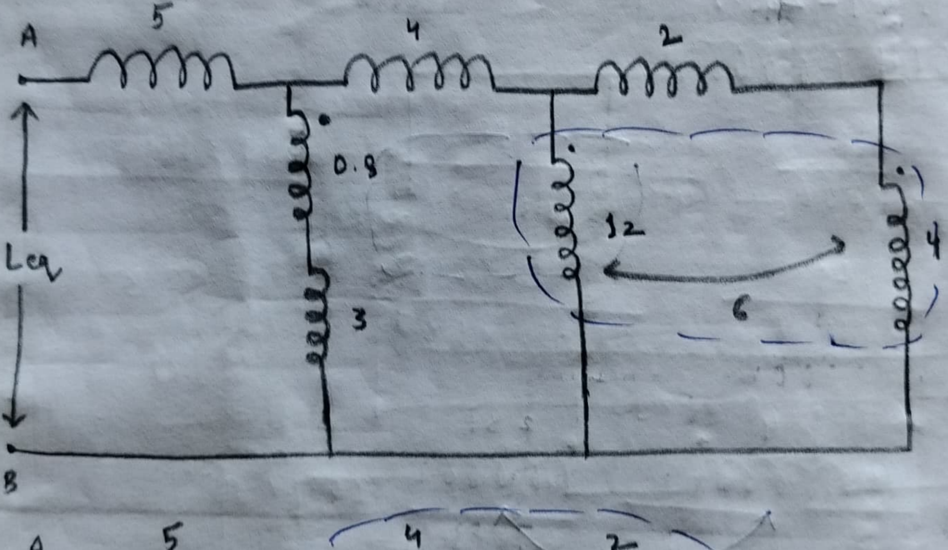


$= L_1 + L_2 \pm 2M$

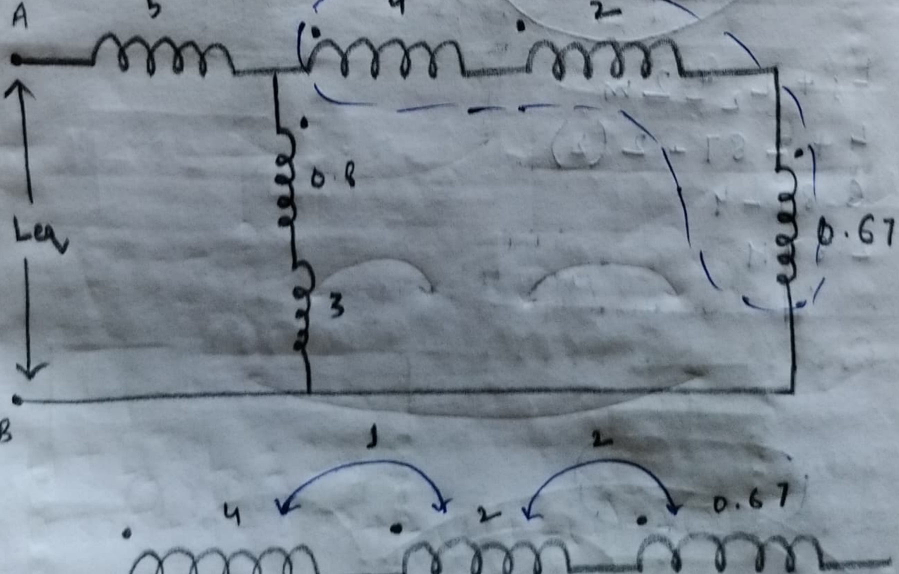
$= 5 + 3 + 2(2)$

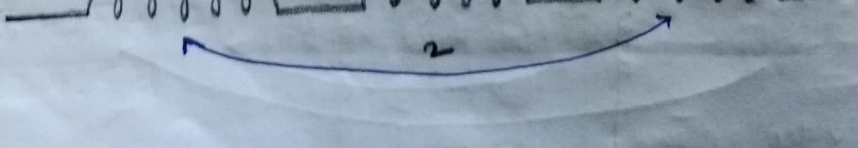
$= 12\ H$

(3)



(4)





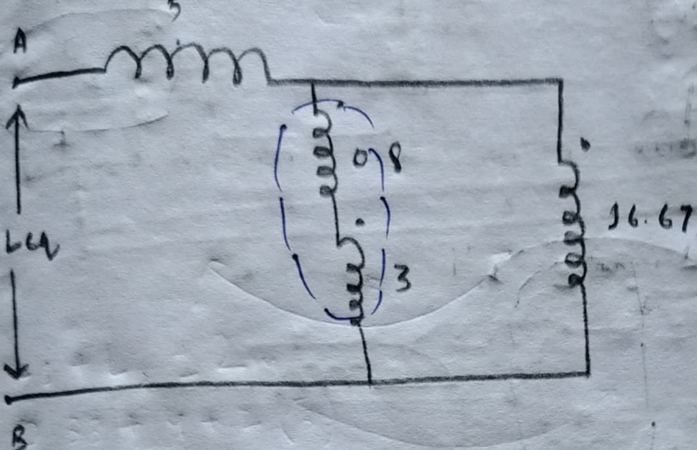
$= L_1 + L_2 + L_3 \pm 2m$

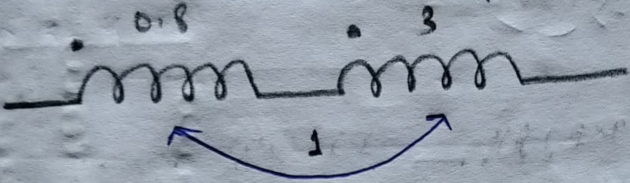
$= 4 + 2 + 0.67 + 2(1) + 2(2) + 2(2)$

$= 6.67 + 10$

$= 16.67\,H$

⑤



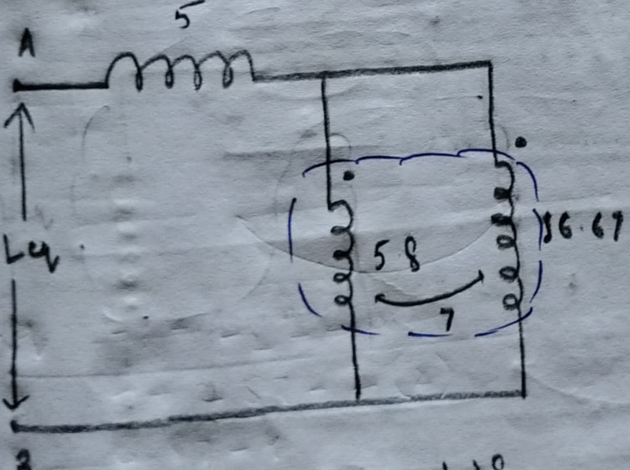


$= L_1 + L_2 \pm 2m$

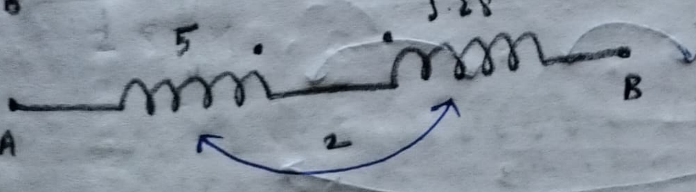
$= 0.8 + 3 + 2(1)$

$= 5.8\,H$

⑥



⑦



$L_{eq} = L_1 + L_2 \pm 2m$

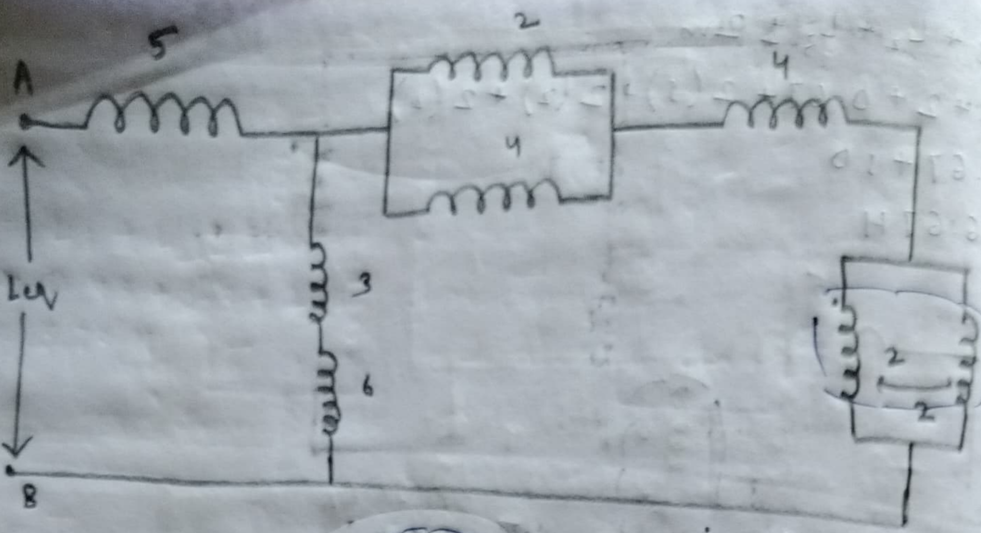
$= 5 + 1.28 - 2(2)$

$= 6.28 - 4$

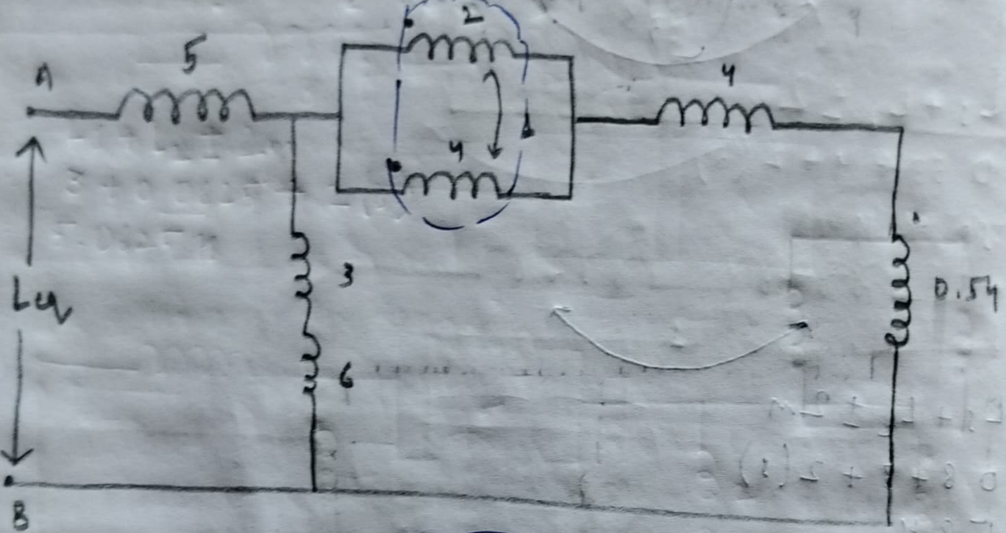
$= 2.28\,H$

9)

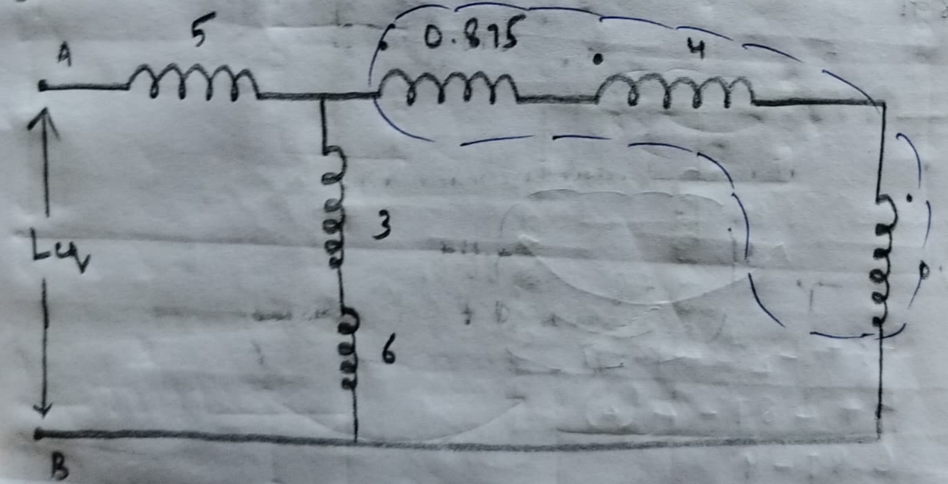
(1)

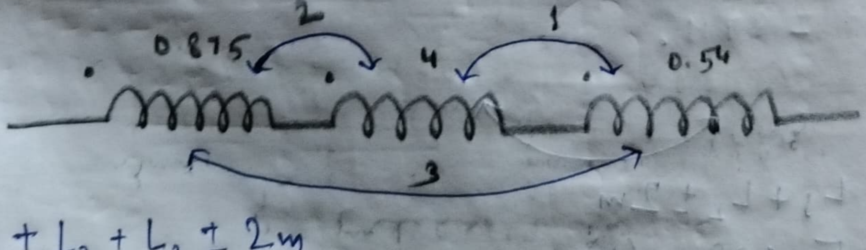


(2)



(3)





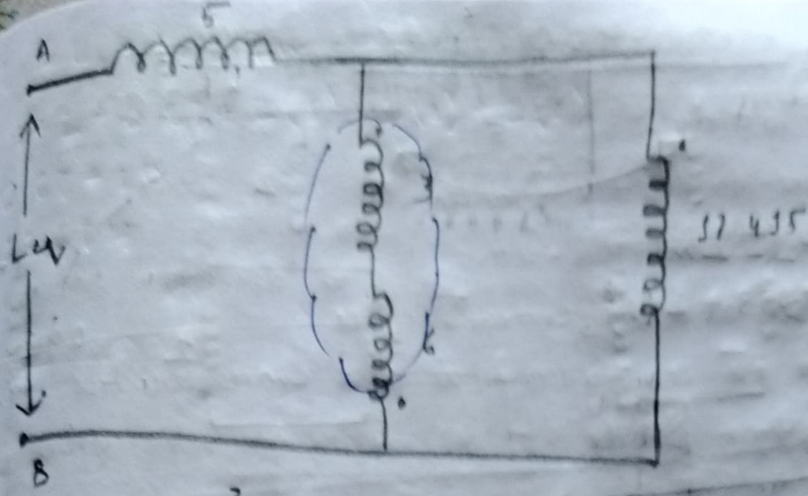
$= L_1 + L_2 + L_3 \pm 2m$

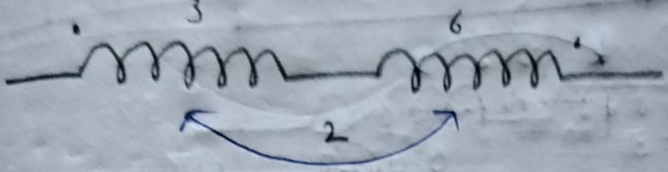
$= 0.875 + 4 + 0.54 + 2(2) + 2(1) + 2(3)$

$= 5.415 + 12$

$= 17.415\,H$

(4)



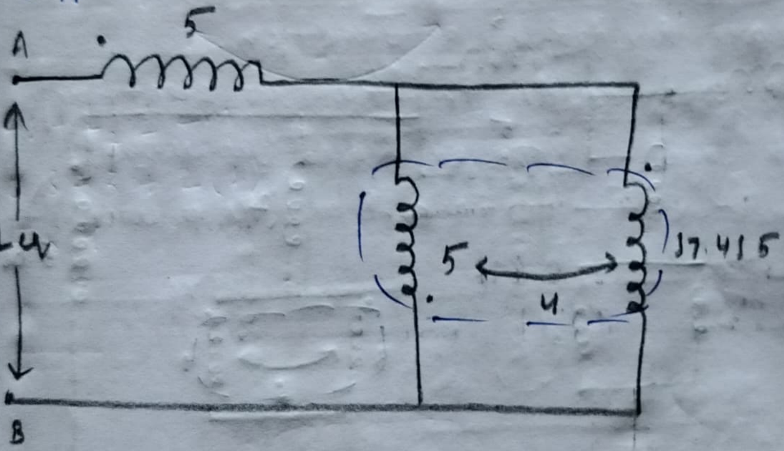


$= L_1 + L_2 \pm 2m$

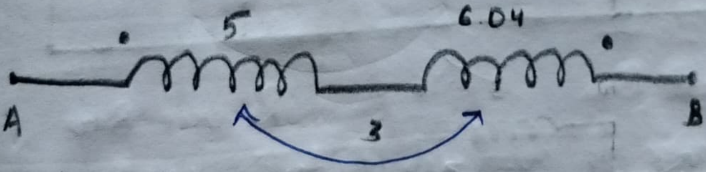
$= 3 + 6 - 2(2)$

$= 5\ H$

(5)



(6)

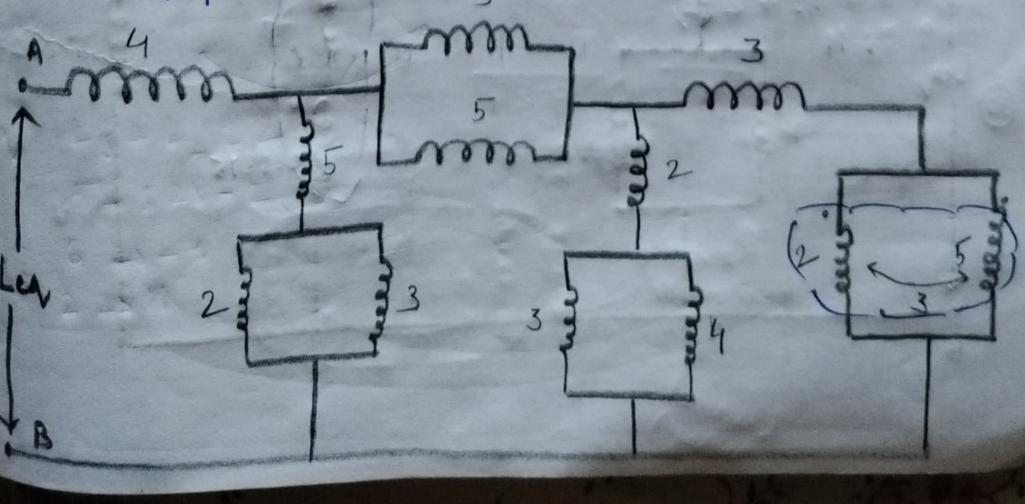


$L_{eq} = L_1 + L_2 \pm 2m$

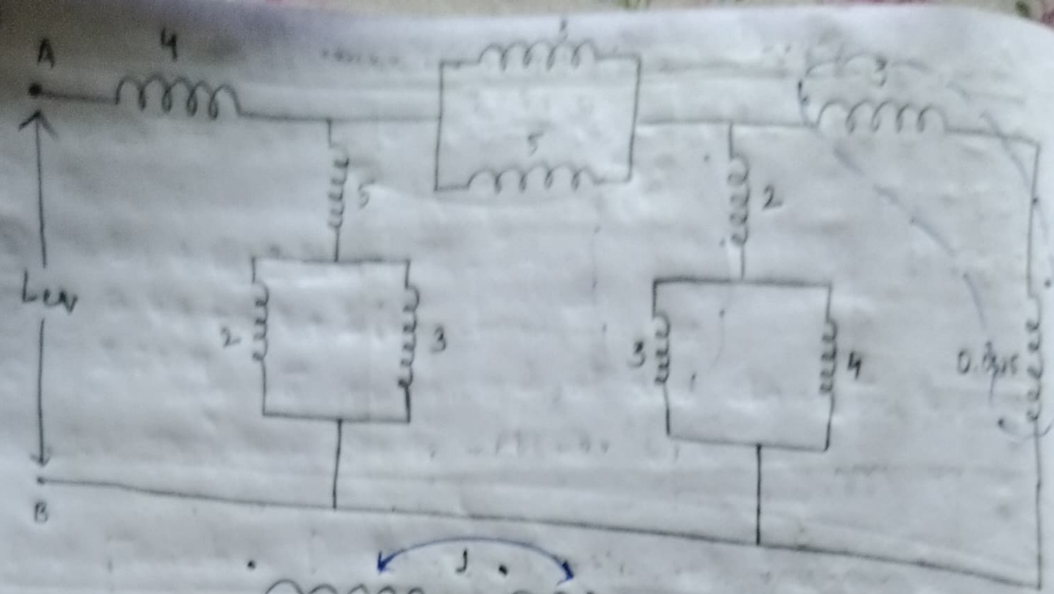
$= 5 + 6.04 - 2(3)$

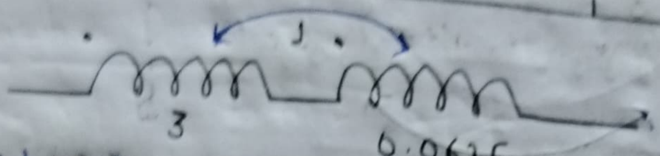
$= 5.04\ H$

(7)



②



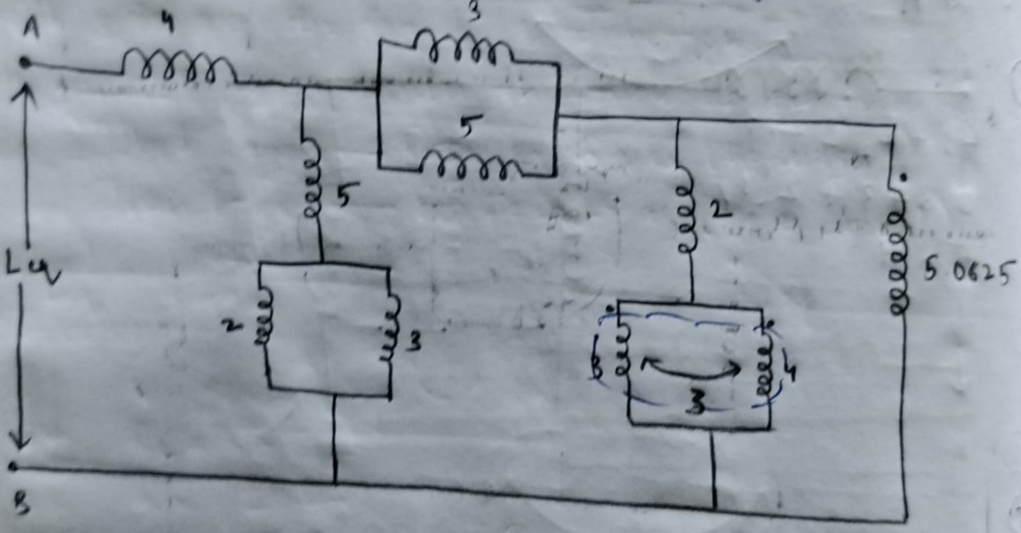


$= L_1 + L_2 \pm 2m$

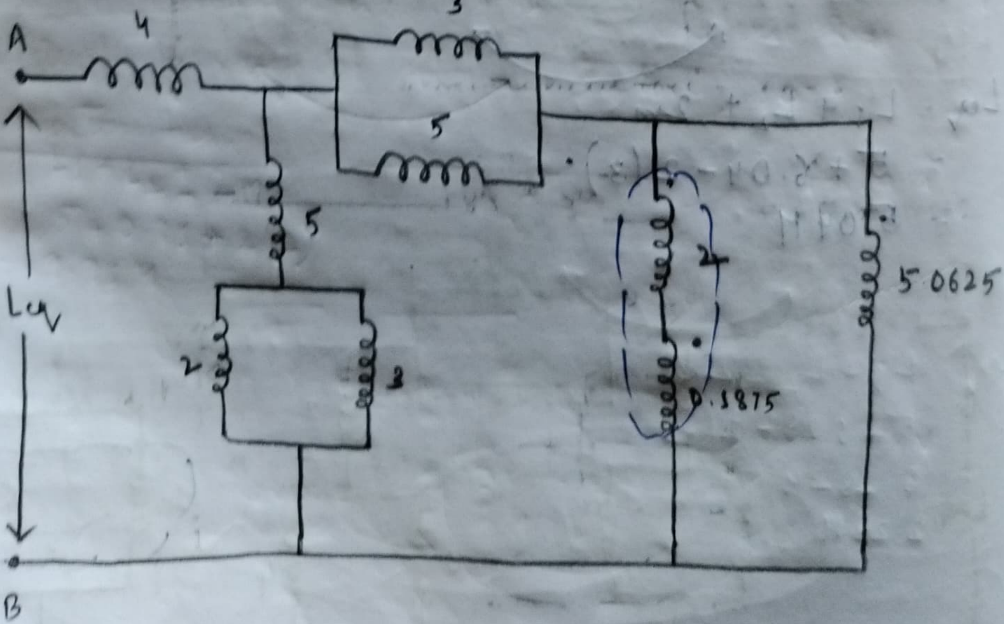
$= 3 + 0.0625 + 2(1)$

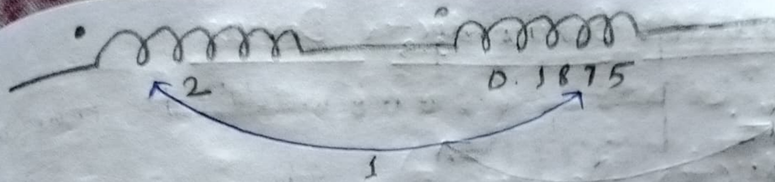
$= 5.0625\ H$

③



④



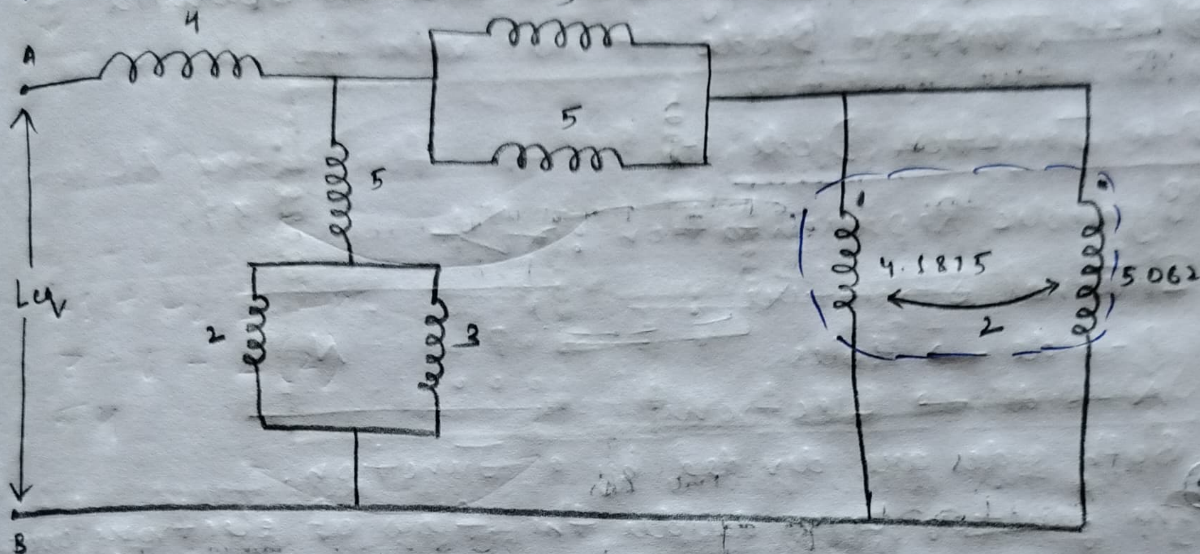


$= L_1 + L_2 \pm 2m$

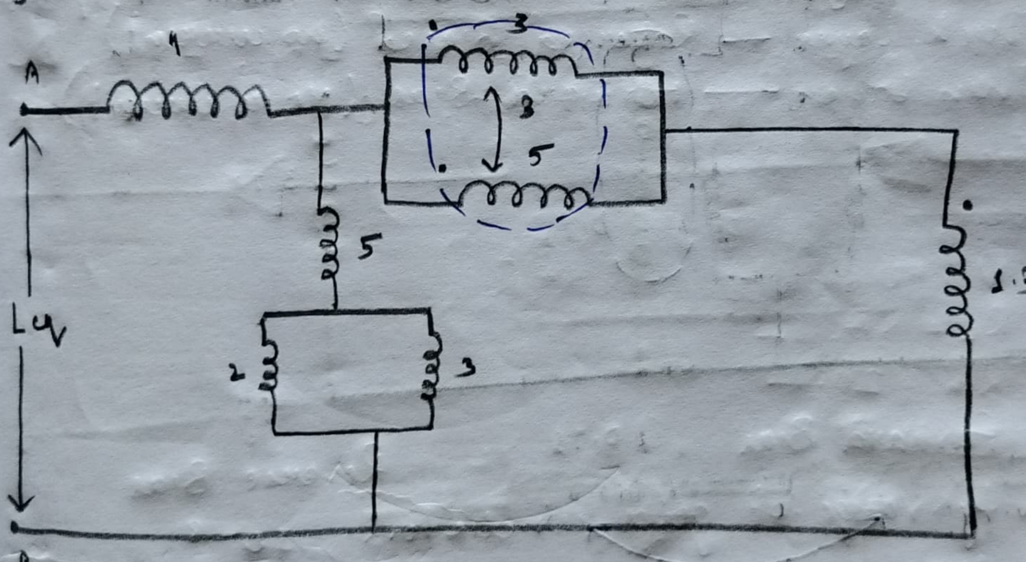
$= 2 + 0.1875 + 2(1)$

$= 4.1875\ H$

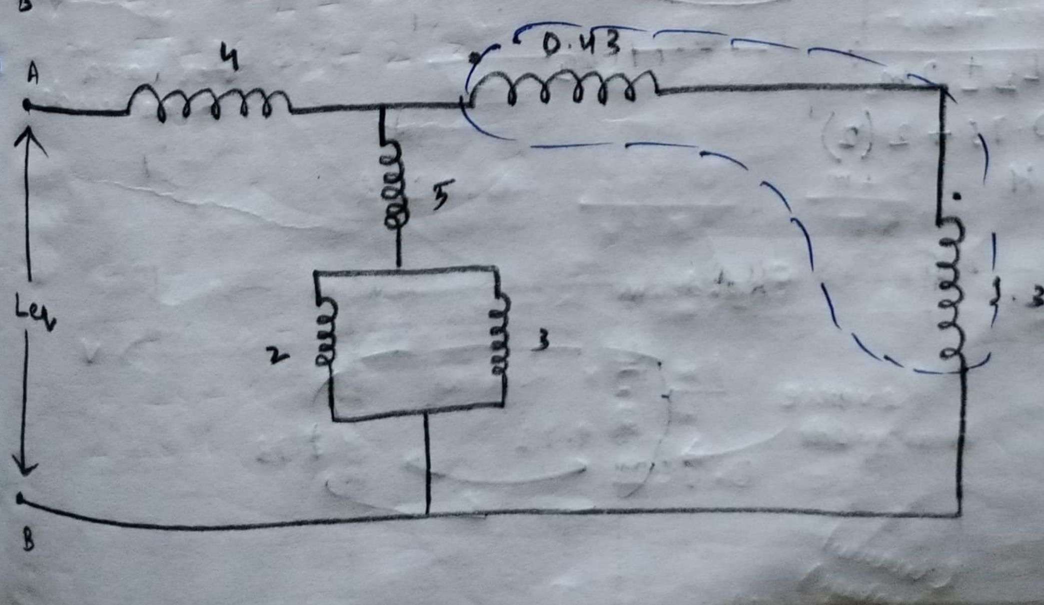
5



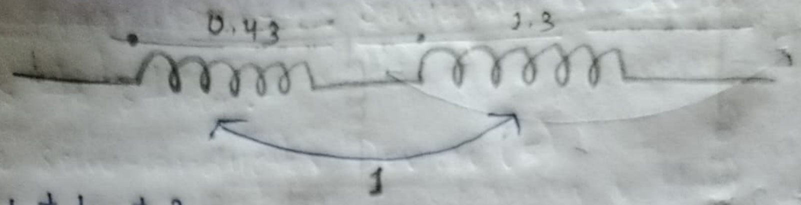
6



7



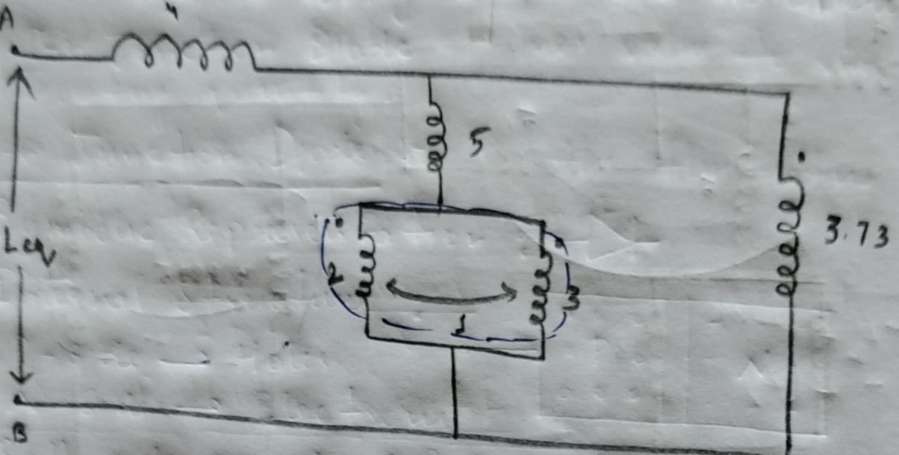
8



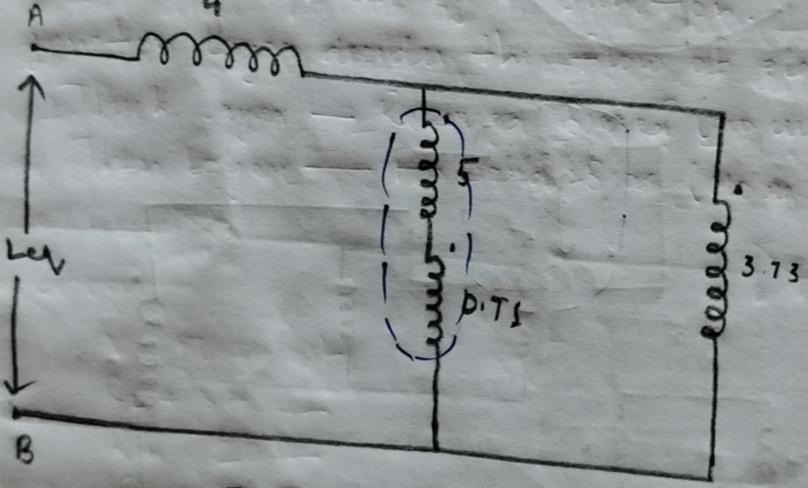
$= L_1 + L_2 \pm 2m$

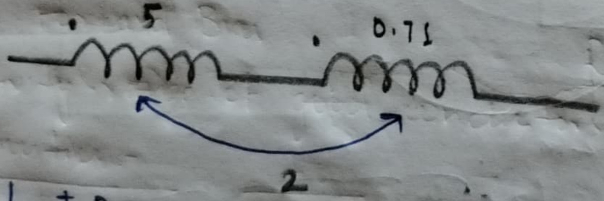
$= 0.43 + 1.3 + 2(1)$

$= 3.73\ H$



9



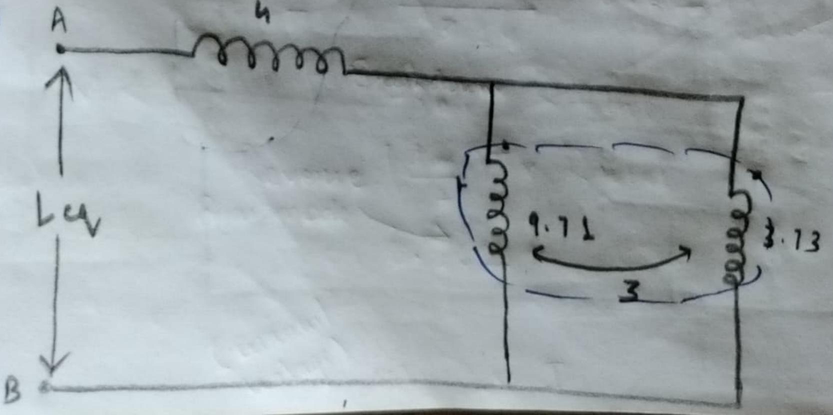


$= L_1 + L_2 \pm 2m$

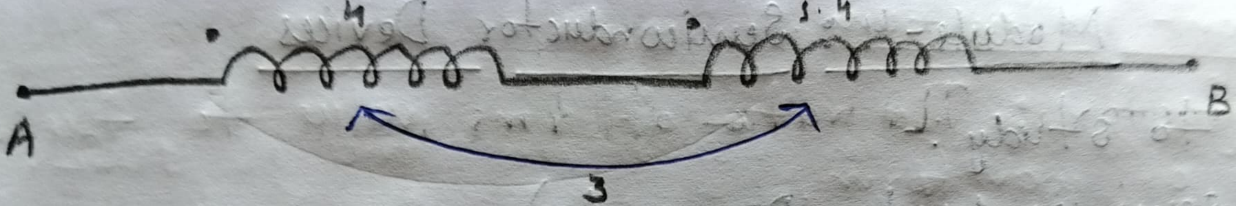
$= 5 + 0.75 + 2(2)$

$= 9.75\ H$

10



(11)



$$L_{eq} = L_1 + L_2 \pm 2M$$
$$= 4 + 1.4 + 2(3)$$
$$= 5.4 + 6$$
$$= 11.4\ H$$