

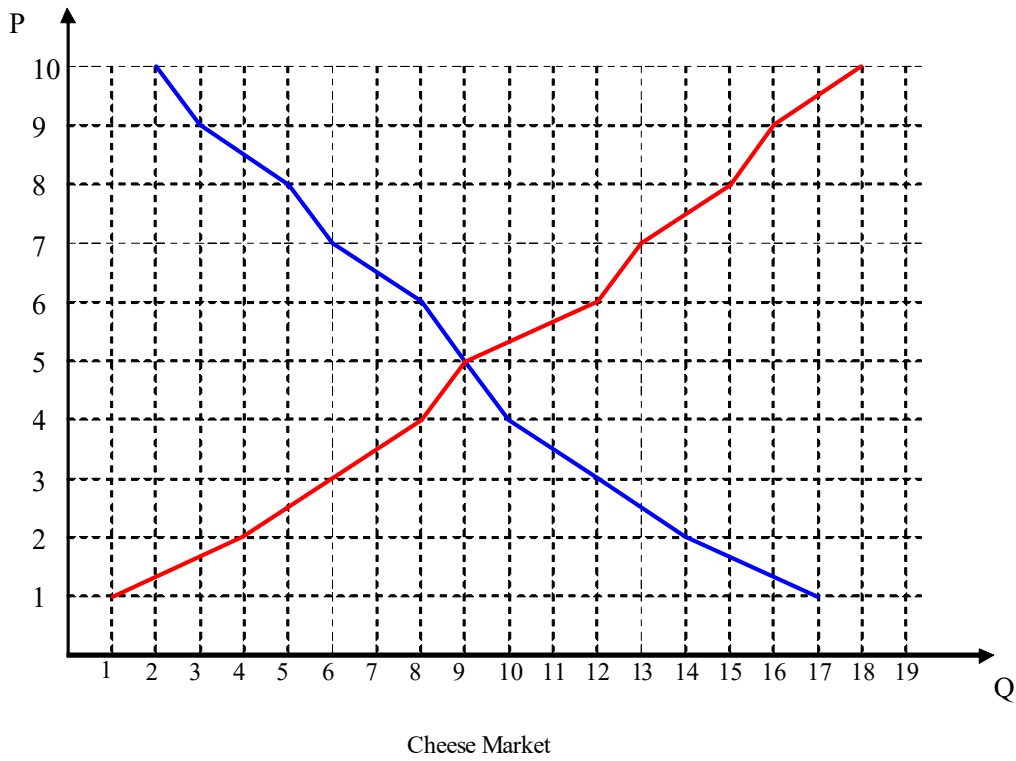
Economics for International Affairs

Gonalo Fonseca

EXERCISE # 2 - SOLUTIONS

(Question 1) Consumer & Producer Surplus. Plot the following in the graph below (or on a side-sheet, if you find it easier):

Cheese	Demand for Cheese	Supply of Cheese
\$10	2	18
\$9	3	16
\$8	5	15
\$7	6	13
\$6	8	12
\$5	9	9
\$4	10	8
\$3	12	6
\$2	14	4
\$1	17	1



(A) What is the market-clearing price and quantity?

The market-clearing price is \$5 and the quantity is 9.

(B) Assuming the actual price settles at the market-clearing price, calculate the Total Consumer's Surplus.

[Hint: As the demand curve isn't a straight line in this example, you can *not* use the triangle area formula. Rather, you must figure out the area piecemeal, rectangle by rectangle. Filling in the following worksheet table can help you out (remember to ignore negative entries!) I give you two of the entries, you figure out the rest]

The consumer surplus is the area of the 'triangle' under the demand curve. Now, some of you might be tempted to just apply the triangle area formula: $1/2 \times \text{base} \times \text{height} = 1/2 \times 9 \times 5 = 22.5$. But this is *wrong*. The demand curve, as we see above, is *not* a straight line, so it is not a triangle, properly speaking, and the formula doesn't work. We need to figure out the area *explicitly*.

We do so by figuring out the consumer surplus at *every price* and then adding up the total. That's what the table asks you to do.

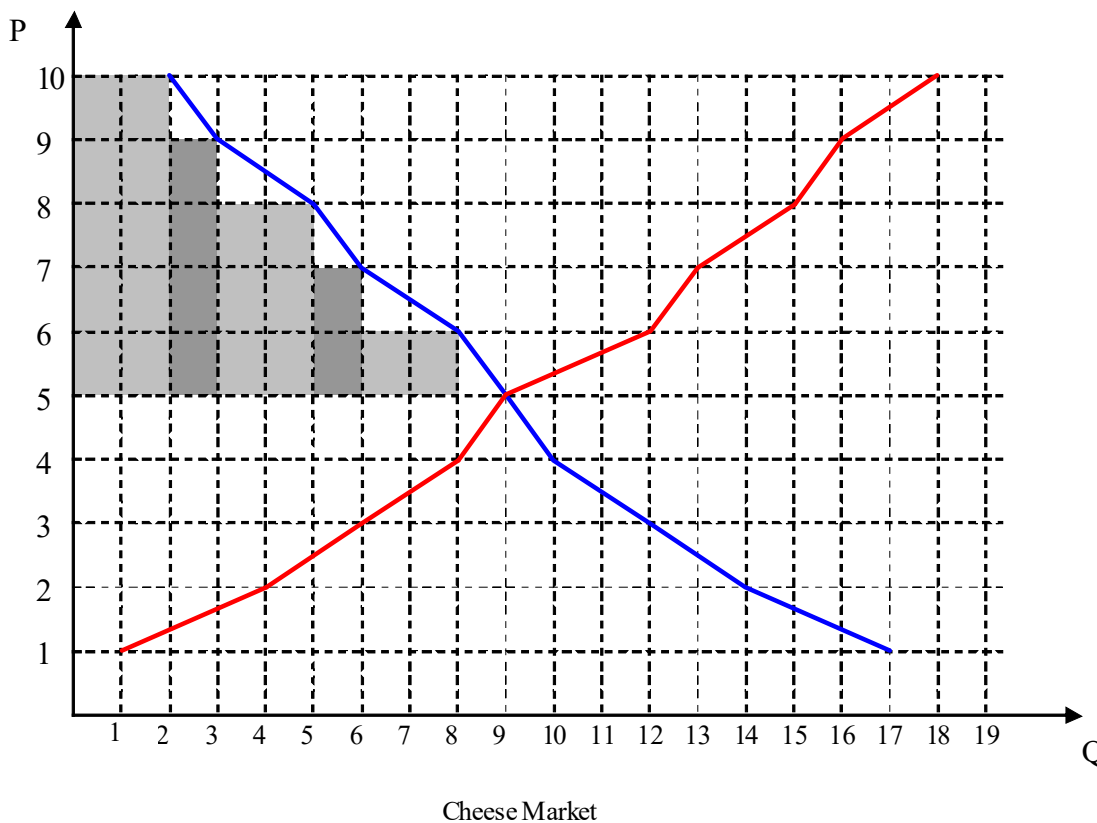
We see that the demand curve tells us that *if* the price of cheese was \$10, consumers are willing to buy 2 pounds of cheese (or, equivalently, two consumers are willing to buy a pound of cheese each). But since the market price is actually \$5, that means those two consumers are reaping a consumer surplus of \$5 (= what they're willing to pay (\$10) minus what they actually pay (\$5)). Since two consumers are enjoying this surplus, then that is $\$5 \times 2 = \10 .

If the price of cheese was \$9, consumers are willing to buy 3 pounds of cheese (or equivalently three consumers will buy a pound of cheese each). So the additional surplus generated at the price of \$9 is \$4 (= what they're willing to pay (\$9) minus what they actually pay (\$5)). However, remember that the first two pounds of cheese would have been bought already at \$10. So, at \$9, there is really only one additional consumer. Or put another way, the first two consumers enjoy a surplus of \$5, the third consumer enjoys a surplus of \$4.

If the price of cheese was \$8, demand is now 5 pounds. So the surplus for a consumer at this price is \$3 (= \$8 - \$5). However, again, at this price, only two extra consumers come into the market that weren't there before; the first three consumers would have been willing to buy cheese before, at higher prices: so the first two consumers are really enjoying a surplus of \$5, the third consumer a surplus of \$4, and only the fourth & fifth consumers get a surplus of \$3 each. So the extra surplus generated here is $\$3 \times 2 = \6 .

Does the logic of the sequence make sense?

Intuitively, we want to approximate the area of the 'triangle' under the curve by means of little consumer surplus rectangles that look like the following:



The first rectangle on the extreme left is the consumer surplus enjoyed by the first two consumers. The next rectangle is the surplus enjoyed by the third consumer. The next rectangle is the surplus of the fourth & fifth consumers, and so on. If we figure out the area of each of these little rectangles, we will have the total consumer surplus.

The area of a rectangle, as you know from grade school, is merely $\text{base} \times \text{height}$.

You should immediately tell that for any of the drawn rectangles, the height is the surplus enjoyed and the base is the number of *extra* (or additional) consumers drawn in at that price.

So for the first rectangle, height = \$5 (surplus enjoyed) and base = 2 (first two consumers), so the area of that rectangle is $\$5 \times 2 = \10 .

For the second rectangle, height = \$4 (surplus enjoyed) and base = 1 (one extra consumer, the third guy), so the area is $\$4 \times 1 = \4 .

For the third rectangle, height = \$3 (surplus enjoyed) and base = 2 (two extra consumers, the fourth & fifth guys), so the area is $\$3 \times 2 = \6

For the fourth rectangle, height = \$2 (surplus enjoyed) and base = 1 (one extra consumer, the sixth guy), so the area is $\$2 \times 1 = \2

For fifth rectangle, height = \$1 (surplus enjoyed) and base = 2 (two extra consumers, the seventh and eight guys), so the area is $\$1 \times 2 = \2 .

Finally, for the sixth rectangle, which has no area, the height = \$0 (no surplus enjoyed) and base is 1 (one extra consumer, the ninth and final one), so the area is $\$0 \times 1 = \0 .

So the area of all the rectangles added together is:

$$\text{Total Consumer Surplus} = \$10 + \$4 + \$6 + \$2 + \$2 + \$0 = \$24.$$

That's all there's to it.

To guide you in this construction, I gave you table to work it out. It seems most of figured out the heights (surplus gained) at every price properly, but many of you messed up the 'base' (extra demand at that price, or additional consumers brought into the market at that price). It should have read like this:

Price of Cheese	Demand for Cheese	Height	Base	Area = Height \times Base
		↓ Surplus gained (diff. from equil. price)	↓ No. of extra demand at that price ¹	↓ Consumer's Surplus (surplus gained \times extra demand)
\$11	0	\$6	0	$\$6 \times 0 = 0$
\$10	2	\$5	2	$\$5 \times 2 = \10
\$9	3	\$4	1	$\$4 \times 1 = \4
\$8	5	\$3	2	$\$3 \times 2 = \6
\$7	6	\$2	1	$\$2 \times 1 = \2
\$6	8	\$1	2	$\$1 \times 2 = \2
\$5	9	\$0	1	$\$0 \times 1 = 0$
\$4	10	-	-	-
\$3	12	-	-	-
\$2	14	-	-	-
\$1	17	-	-	-
\$0	19	-	-	-
				Total Consumer's Surplus = <u>\$24</u>

¹ By 'extra demand' I mean the number of *additional* consumers brought into the market at that price, e.g. if demand at price \$20 is 10 and demand at price \$19 is 13, then the 'extra demand' at \$19 is 3, or if you want, three additional consumers are brought into the cheese market when the price is dropped from \$20 to \$19

Negative entries - that is, all entries below \$5 - are ignored, since Consumer surplus is only the area *above* the market price.

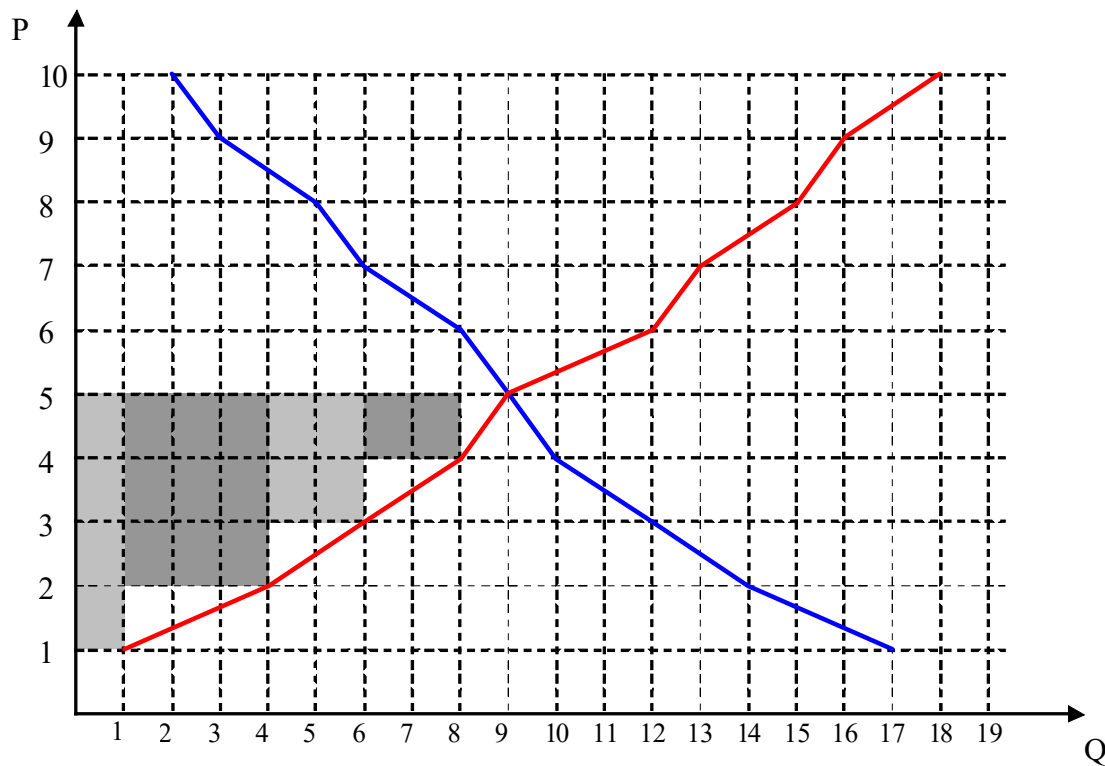
That's all there's to it.

[For those in the know, the usual mathematical method of calculating the area under a general curve (not necessarily a straight line) is a branch of mathematics known as *integral calculus*. The logic behind 'integration' happens to be *exactly* the same one we use here: figuring out the areas of little rectangles that approximately give us the shape of the curve!]

(C) Now calculate the Total Producer's Surplus.

[Hint: Again, fill the following worksheet table below the equilibrium price. Again, I give you some entries, you figure out the rest]

The same logic is used. This time we're finding the area above the supply curve and below the market price of \$5. That is, we need to figure out the areas of the little rectangles below:



Cheese Market

Starting from below, if the price was \$1, there is only one cheese supplier was willing to supply. So the surplus he gains is \$4 (difference from market price). That's area of the rectangle on the very left, so $\$4$ (surplus gained) \times 1 (number of suppliers) = \$4.

If the price was \$2, four cheese-makers would supply cheese. But the first was already willing to supply at \$1. So the price of \$2 only brings in *three* additional cheese-makers to the market. Each of these additional producers makes \$3 in consumer surplus. So the area of the next rectangle is \$3 (surplus gained) \times 3 (number of additional suppliers) = \$6.

and so on. Using the worksheet:

		Height ↓	Base ↓	Area = Height \times Base ↓
Price of Cheese	Supply of Cheese	Surplus gained (diff. from equil. price)	No. of extra supply at that price	Producer's Surplus (surplus gained \times extra supply)
\$11	19			
\$10	18	-	-	-
\$9	16			
\$8	15			
\$7	13			
\$6	12			
\$5	9	\$0	1	$\$0 \times 1 = 0$
\$4	8	\$1	2	$\$1 \times 2 = 2$
\$3	6	\$2	2	$\$2 \times 2 = 4$
\$2	4	\$3	3	$\$3 \times 3 = 9$
\$1	1	\$4	1	$\$4 \times 1 = 4$
\$0	0	\$5	0	$\$5 \times 0 = 0$
				Total Producer's Surplus = <u>19</u>

(D) Who is making more welfare gains at the equilibrium price, Consumers or Producers?

Straightforward: at the market price of \$5, consumer surplus is \$24 and producer surplus is \$19. Both are making welfare gains from trading with each other at \$5, but the gains accruing to the consumers (\$24) are greater than that of the producer (\$19).

(E) Suppose the government introduces a \$4 sales tax on cheese. Calculate the new market-clearing equilibrium (note: there will be a price wedge!):

Price consumers pay: \$7
 Price producers receive: \$3
 Market-clearing quantity traded : 6

A \$4 excise tax on the market for cheese is growing to create a \$4 difference between the prices consumers pay and what producers receive.

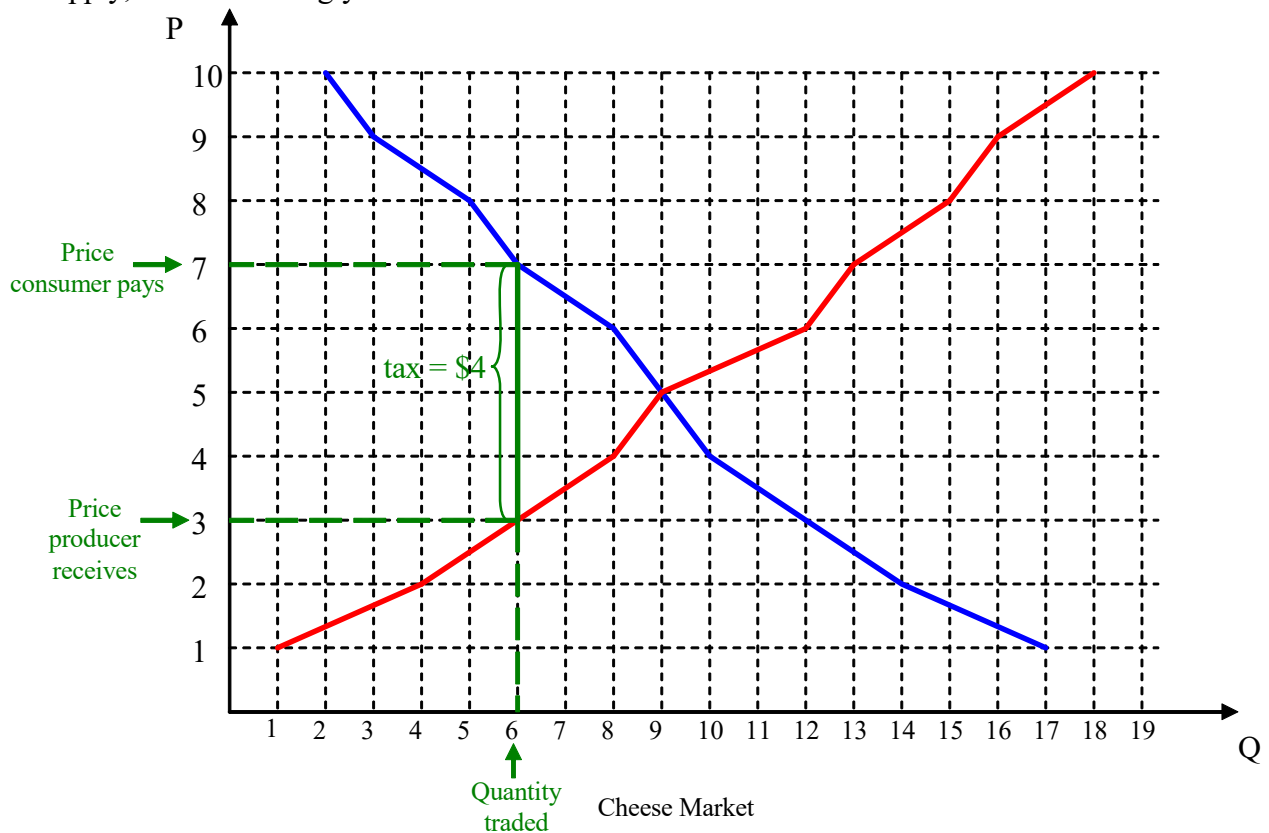
We were at \$5 before, with consumers demanding 9 and producers producing 9..
Intuitively, a \$4 sales tax would raise the price consumers face from \$5 to \$9. But at \$9 demand for cheese is merely 3 units, while the supply of cheese (at price producers receive, \$5) is still 9 units. We have an excess supply of cheese. So this won't work. Markets aren't clearing, which means the prices aren't settled.

So we have to consider other possibilities - essentially pick *pairs* of prices, one for consumers, another for producers, with a \$4 difference, and check if the market clears. Here are some candidates:

\$9 for consumers , \$5 for producers - \$4 tax is met, but markets don't clear ($D = 3, S = 9$)
\$8 for consumers, \$4 for producers - \$4 tax is met, but markets don't clear ($D = 5, S = 8$)
\$7 for consumers, \$3 for producers - \$4 tax is met *and* markets clear ($D = 6, S = 6$)
\$6 for consumers, \$2 for producers - \$ tax is met, but markets don't clear ($D = 8, S = 4$)

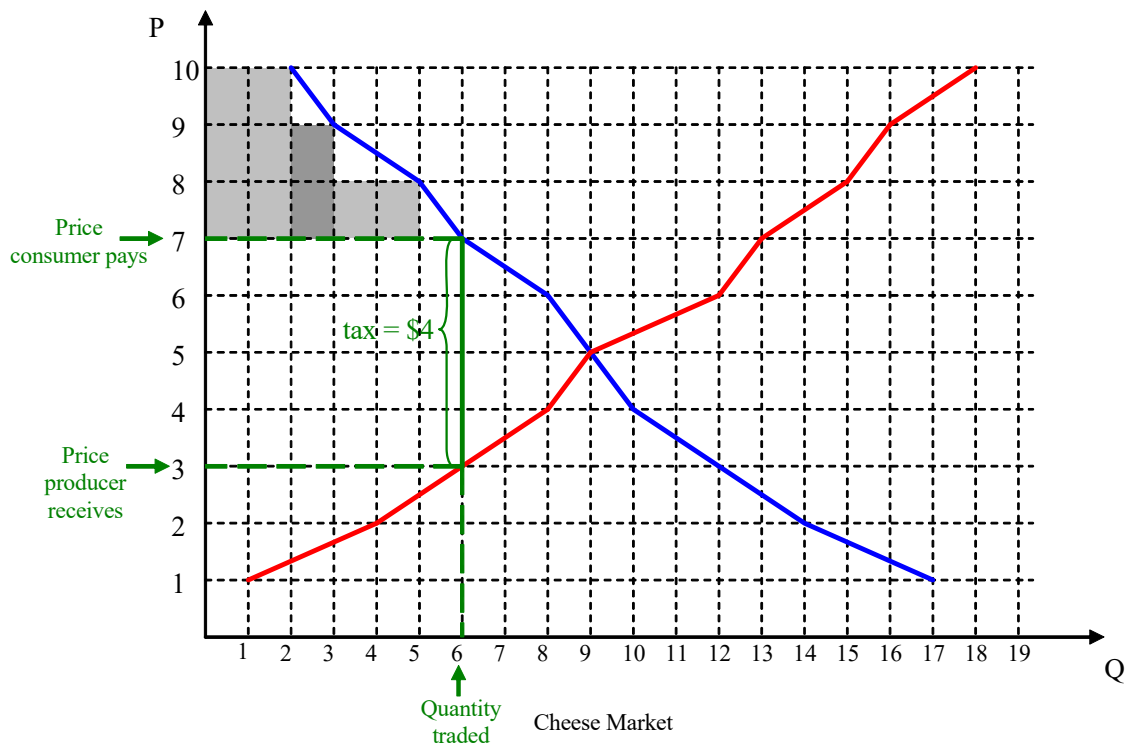
So it seems the only candidate for a solution is where consumers face price \$7, producers receive \$3, government takes the \$4 in between, and the amount traded is 6 units of cheese.

Diagrammatically, this is just "fitting a vertical wedge" of size \$4 between the demand and & supply, until it fits snugly.



(F) Calculate the new consumer surplus:

With the excise tax, the consumer surplus is smaller. So now we have to calculate things again. The consumer's surplus, remember, is defined as the difference between what consumers are willing to pay and what they actually do pay. Since, with the tax, consumers are now paying \$7 for cheese, the consumer's surplus is the area under the demand curve and above \$7. It is shown below



Calculating is the same way as before, except with \$7 as our new reference point for the surplus gained. So, using the table:

Price of Cheese	Demand for Cheese	Surplus gained (diff. from consumer price)	No. of extra demand	Consumer's Surplus
\$11	0	\$4	0	$\$4 \times 0 = 0$
\$10	2	\$3	2	$\$3 \times 2 = \6
\$9	3	\$2	1	$\$2 \times 1 = \2
\$8	5	\$1	2	$\$1 \times 2 = \2
\$7	6	\$0	1	$\$0 \times 1 = \0
\$6	8			
\$5	9			
\$4	10			

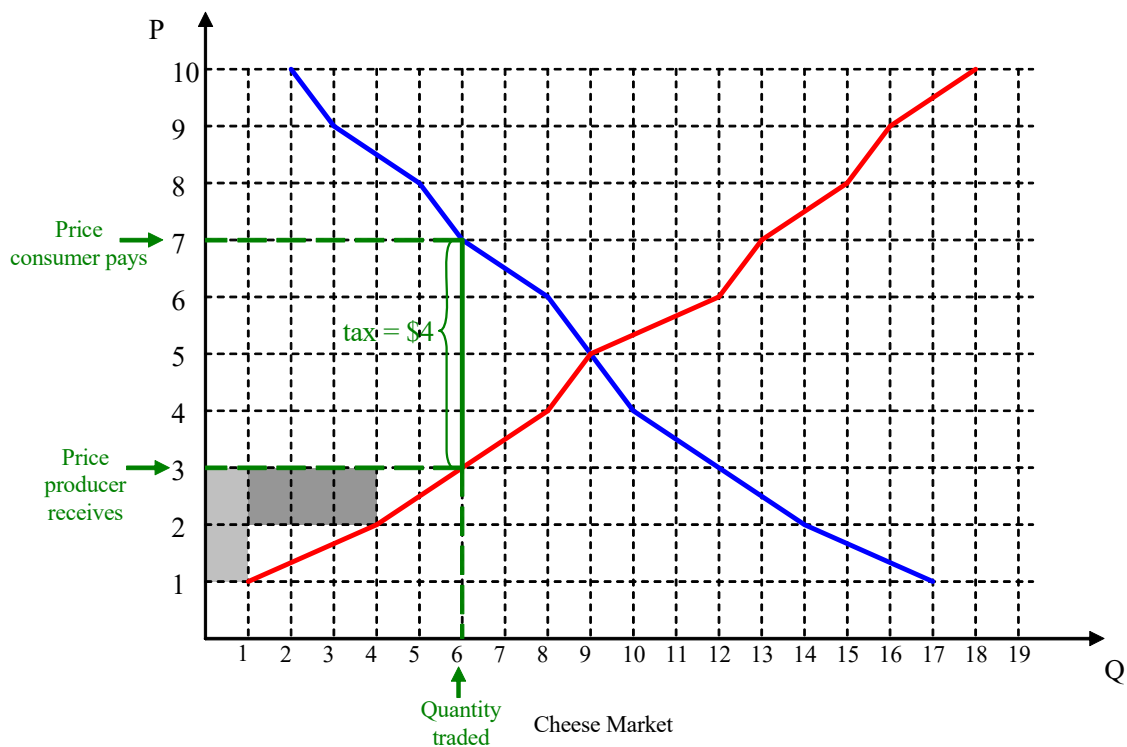
\$3	12			
\$2	14			
\$1	17	-	-	-
				New Consumer's Surplus = <u>10</u>

By how much has consumer surplus declined as a result of the tax? 14

They had a consumer's surplus of 24 before, now they have a surplus of 10. So they lost 14 (or fell by 58.33%).

(G) Calculate the new producer surplus:

With the excise tax, the producer surplus is smaller. The producer's surplus is defined as the difference between what producers are willing to accept and what they actually receive. With the tax, producers receive \$3 for cheese, so the producers' surplus is the area below \$3 and above the supply curve, as shown below



Calculating is the same way as before, except with \$7 as our new reference point for the surplus gained. So, using the table:

Price of Cheese	Supply of Cheese	Surplus gained (diff. from equil. price)	No. of extra supply at that price	Producer's Surplus (surplus gained \times extra supply)
\$10	18	-	-	-
\$9	16			
\$8	15			
\$7	13			
\$6	12			
\$5	9			
\$4	8			
\$3	6	\$0	2	$\$0 \times 2 = 0$
\$2	4	\$1	3	$\$3 \times 1 = 3$
\$1	1	\$2	1	$\$2 \times 1 = 2$
\$0	0	\$3	0	$\$3 \times 0 = 0$
				New Producer's Surplus = <u>5</u>

By how much has producer's surplus declined as a result of the tax? 14

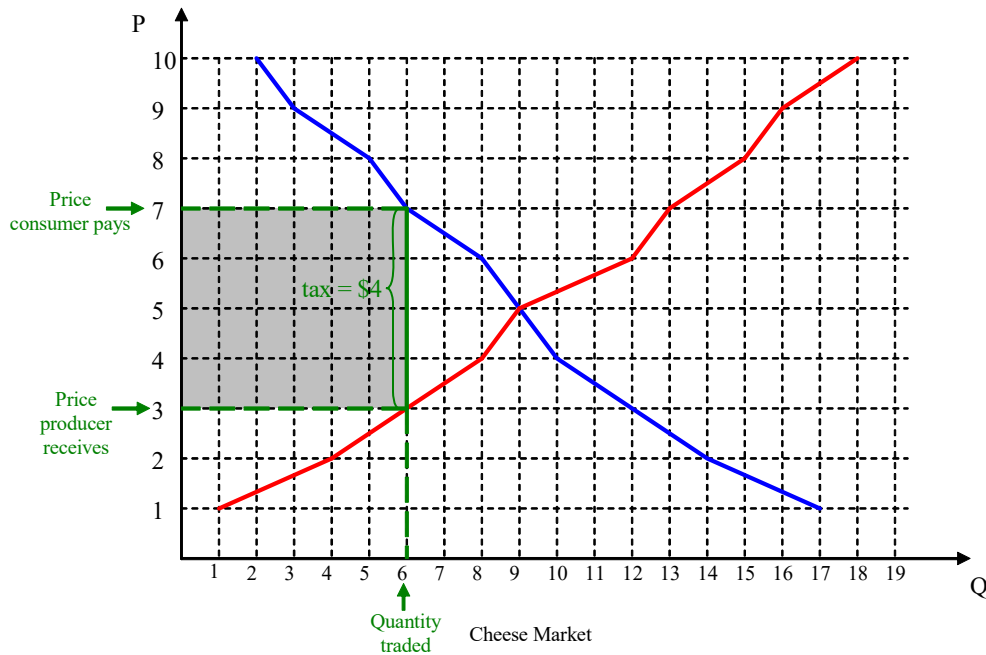
There was a producers surplus of 19 before, now they have a surplus of 5. So they lost 14 (or fell by 73.68%).

(H) Who has borne the brunt of the sales tax, consumers & producers? How about in proportional terms? (i.e. loss as a % of their previous surpluses)

Consumers lost \$14 in consumers' surplus (or 58.33% of what they had before), whereas producers lost \$14 in producers' surplus (or 73.68%). Although they seem to lose the same dollar amount (14), in percentage terms, the brunt is being borne by the producer.

(I) How much revenue has the government collected from the sales tax?

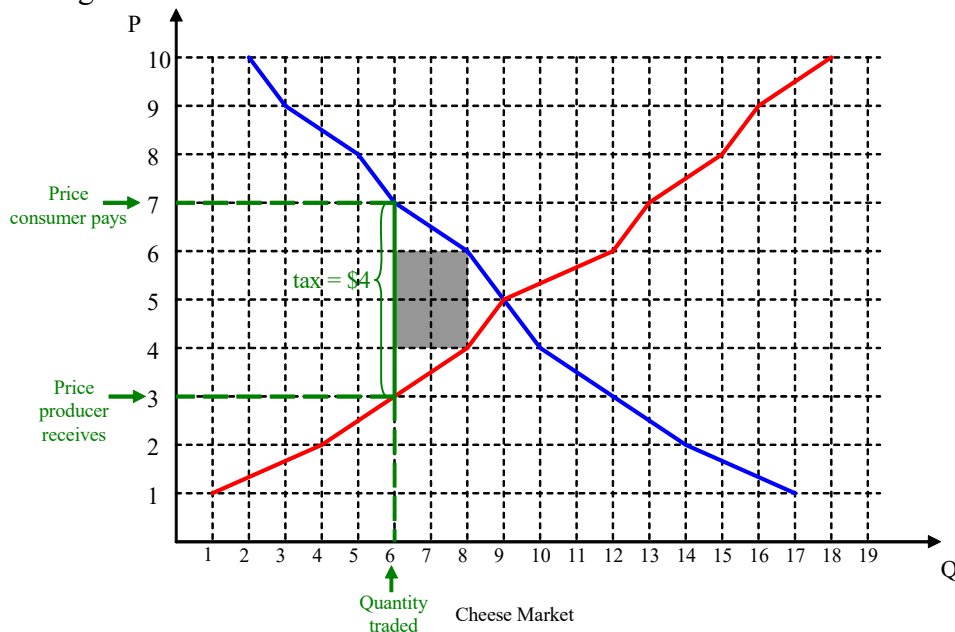
Government has collected a \$4 tax on sales of six units. That is $\$4 \times 6 = \24 collected in tax. Diagrammatically, that is given as the area of the square below:



(J) What is the size of the "deadweight loss"?

The "deadweight loss" is defined as the loss to both consumers & producers surplus that is *not* recouped by the government in taxes.

This is easy to calculate directly. From the tax, consumers lost \$14 in consumers' surplus, producers lost \$14 in producers' surplus, so the total loss was \$28. The government reaped \$24 in taxes. So the deadweight loss is $28 - 24 = 4$. This is shown by the area of the "triangle" below.



(Question 2) Welfare with Algebra

The surpluses calculated above was done the tiresome way (price by price, etc.). But if we have *linear* demand and supply curves, then we can figure it out by using the triangle area formula.

$$\text{area of triangle} = \frac{1}{2} \times \text{base} \times \text{height}$$

Suppose supply and demand for lamps are governed by the following

$$Q^d = -2P + 270$$

$$Q^s = 3P - 30$$

Part I - Free Market

(A) What is the equilibrium price and equilibrium quantity?

$$P = \underline{\$60} \qquad Q = \underline{150}$$

Answer: as usual, just equate:

$$Q^d = Q^s$$

$$-2P + 270 = 3P - 30$$

so:

$$5P = 300$$

then equilibrium price:

$$P = 300/5 = 60$$

Equilibrium quantity can be figured out simply as (using the supply curve equation):

$$Q = 3(60) - 30 = 180 - 30 = 150.$$

(B) What is the Consumers' Surplus?

[Hint: Because the demand and supply curves are linear, we can use the triangle formula. For Consumers' Surplus, base = quantity exchanged, and height = vertical intercept of demand curve minus equilibrium price. To find vertical intercept of demand curve set $Q^d = 0$ in demand curve, then solve for P. The resulting P from

that operation will be the vertical intercept. Subtract equilibrium P from that number to get the "height" of the CS triangle)

$$\text{Consumers' Surplus} = \frac{1}{2} \times \underline{150} \times \underline{\$75} = \underline{\$5,625}$$

Answer: As we saw, quantity exchanged = 150. Vertical intercept of the demand curve can be found from the demand formula when we set $Q^d = 0$. So:

$$0 = -2P + 270$$

so:

$$2P = 270$$

so $P = 270/2 = 135$. So vertical intercept is 135.

Since equilibrium $P = 60$, then height of triangle = $135 - 60 = 75$.

The Consumers' Surplus is consequently:

$$CS = \frac{1}{2} \times 150 \times 75$$

$$CS = \$5,625.$$

(C) What is the Producers' Surplus?

[Hint: for Producers' Surplus, base = quantity exchanged, and height = equilibrium price *minus* vertical intercept of supply curve. To find vertical intercept of supply curve set $Q^s = 0$ in demand curve equation, then solve for P. The resulting P from that operation will be the vertical intercept of supply curve. Subtract that from the equilibrium P to get the "height" of the PS triangle.]

$$\text{Producers' Surplus} = \frac{1}{2} \times \underline{150} \times \underline{\$50} = \underline{\$3,750}$$

As noted, vertical intercept of the supply curve can be found from the supply formula when we set $Q^s = 0$. So:

$$0 = 3P - 30$$

so:

$$3P = 30$$

so $P = 30/3 = 10$. So vertical intercept is 10.

Since equilibrium $P = 60$, then height of triangle $= 60 - 10 = 50$.

The Producers' Surplus is consequently:

$$PS = \frac{1}{2} \times 150 \times 50$$

$$PS = \$3,750.$$

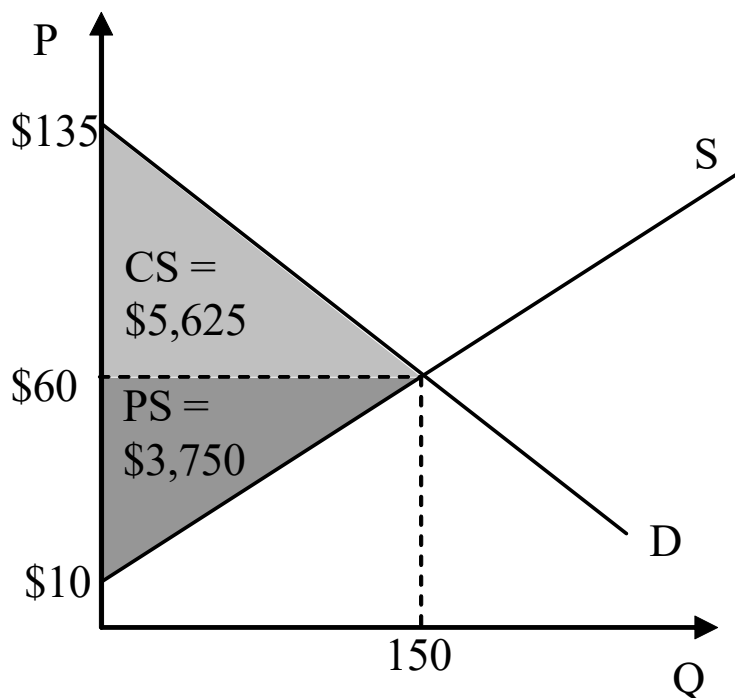
(D) Who is making more gains from trade? Consumers or producers?

Consumers' surplus was $CS = \$5,625$.

Producers' surplus was $PS = \$3,750$

So evidently consumers are making greater gains than producers.

To summarize what you have discovered diagrammatically:



Part II - with Tax

Suppose the government imposes a \$10 sales tax on lamps.

(E) What is the equilibrium price for consumers, price for producers and equilibrium quantity?

[Hint: This is a little tricky. To solve it remember that it is still true that $Q^d = Q^s$. But now there is a \$10 "wedge" in prices. So adjust the formulas so:

$$Q^d = -2(\text{Price for Consumers}) + 270$$
$$Q^s = 3(\text{Price for Producers}) - 30$$

where Price for Consumers = Price for Producers + \$10 tax. Or, letting P^p be the producers price, then:

$$Q^d = -2(P^p + 10) + 270$$
$$Q^s = 3(P^p) - 30$$

Go from here.]

$$\text{Price to consumers} = \underline{66}$$

$$\text{Price to producers} = \underline{56}$$

$$\text{Quantity exchanged} = \underline{138}$$

Answer: The equilibrium condition remains the same:

$$Q^d = Q^s$$

So:

$$-2(P^p + 10) + 270 = 3(P^p) - 30$$

opening up the brackets (since $-2 \times 10 = -20$):

$$-2 P^p - 20 + 270 = 3 P^p - 30$$

and moving things around:

$$270 - 20 + 30 = 3 P^p + 2 P^p$$

or:

$$280 = 5P^p$$

so:

$$P^p = 280/5 = 56$$

So price for producers is 56. We know the price for consumers (P^c) will be $P^p + 10$, or:

$$P^c = 66$$

Finally, there is the issue of quantity. Here we have to be careful, since price for producers enters the supply function and price for consumers enters the demand function. So it is not interchangeable. Calculating:

$$Q^d = -2 P^c + 270$$

$$Q^s = 3 P^p - 30$$

So using the demand curve and the price to consumers ($P^c = 66$):

$$Q^d = -2 (66) + 270 = -132 + 270 = 138$$

or, using the supply curve and the price to producers ($P^p = 56$)

$$Q^s = 3 (56) - 30 = 168 - 30 = 138$$

yielding the same quantity (as expected).

(F) What is the new Consumer's Surplus after tax?

[Hint: use the triangle formula again. Consumers' Surplus = $\frac{1}{2} \times \text{base} \times \text{height}$. Base = new quantity. Height = vertical intercept of demand minus the price to *consumers*. Vertical intercept is the same as what you figured out in part B]

$$\text{Consumers' Surplus} = \frac{1}{2} \times \underline{138} \times \underline{\$69} = \underline{\$4,761}$$

We know the new base = 138, the quantity produced. The issue is height. We saw before that the vertical intercept of the demand curve was 135. With the new tax, we saw in the previous question that the price to consumers is now 66. So the "height" of the new triangle is $135 - 66 = 69$

$$CS = \frac{1}{2} \times 138 \times 69 = \$4,761.$$

Evidently less than before (\$5,625.)

(G) What is the new Producers' Surplus after tax?

[Hint: again triangle formula. Height = price to producers *minus* vertical intercept of supply curve. Vertical intercept is the same as what you figured out in part C]

$$\text{Producers' Surplus} = \frac{1}{2} \times \underline{138} \times \underline{\$46} = \underline{\$3,174}$$

Base = 138. We saw before the vertical intercept of the supply curve was 10. With the tax, the price to producers is 56. So the height of the new PS triangle is $56 - 10 = 46$. So:

$$PS = \frac{1}{2} \times 138 \times 46 = \$3,174.$$

evidently less than before (PS = \$3,750.)

(H) How much does the government take in revenue?

Revenues to government = $\$10 \times 138 = \$1,380$ in taxes.

(I) What is the "deadweight loss" from the \$10 sales tax on lamps?

The following scribble pad might help:

CS before tax = \$5,625

CS after tax = \$4,761

Loss in CS = \$864

PS before tax = \$3,750

PS after tax = \$3,174

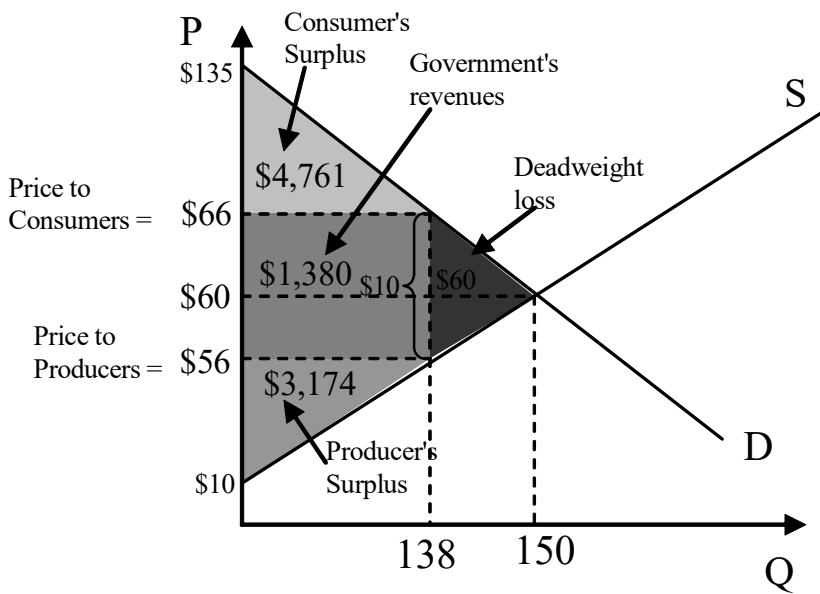
Loss in PS = \$576

Total surplus lost (= CS Loss + PS Loss) = \$1440

Government revenues = \$1380

Deadweight loss (= Total Surplus Loss - Government revenues) = \$60.

Or, summarized diagrammatically:



(Question 3) Cost-Benefit Analysis Suppose the Port Authority considers a proposition to build another bridge between New York and New Jersey. They consider selling lifetime permits for people to cross the bridge. Suppose the potential clientele for the bridge divides into three parts, commuters, shoppers and tourists. 500,000 commuters, who would use it every day, would each be willing to pay \$10 for such a pass; 1 million shoppers, who would use it only occasionally, would be willing to pay \$5; tourists wouldn't be willing to pay anything at all, in fact, they would actually demand a slight payment to go out of their way to cross it.

(A) Suppose no price is charged for the permits, what is the number of people who will cross the bridge?

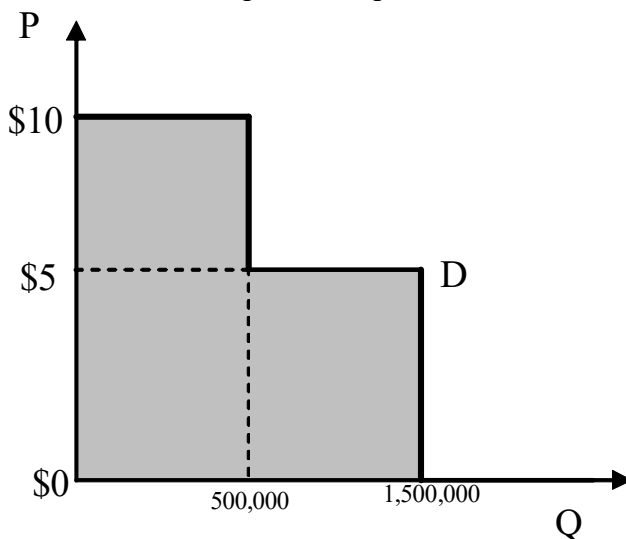
At \$10, the demand is for 500,000 passes (commuters). At \$5, an additional 1 million passes are demanded (shoppers) for a total of 1.5 million. Since tourists would rather not cross unless you paid them, then at \$0, the total demand for passes remains 1.5 million.

If price of permits is \$0, then the number of people who will want to cross the bridge is 1.5 million.

(B) How much would they be willing to pay? (i.e. what is the total consumers surplus if permits are free?)

Each of the 500,000 commuters was willing to pay \$10, thus their consumer surplus (if passage is free) is \$5,000,000. Each of the 1 million shoppers was willing to pay \$5. So their consumer surplus (if passage is free) would be \$5,000,000. So the total consumer surplus if passage is free is \$10,000,000. (\$10 million).

We can envision it as the area under the demand curve, where the demand curve is like a staircase and the equilibrium price is \$0:



(C) If the bridge cost \$9,999,999 to build, should it be built? What if it costs \$10,000,001?

If the bridge costs less than the money value of the satisfaction it brings (that is, willingness to pay), it should be built. Otherwise it should not, as more resources will be wasted than satisfaction created. So, the bridge *will* be built if it costs \$9,999,999 (a \$1 gain in public satisfaction), while it should *not* be built if it costs \$10,000,001 (a loss of \$1).

Cost-benefit analysis of public projects must have a *net public benefit*. If you are willing to build things where the total cost exceeded the total benefit, then you might as well indulge in projects such as leveling the Rocky Mountains to turn it into farmland and using the dirt to fill in the Great Lakes and turn them into ski resorts.

(Question 4) Cost-Benefit Analysis Again Suppose Port Authority is thinking about expanding a particular road to add another lane. Currently, that road sees 3,000 trips per hour, each trip taking around 50 minutes. Adding another lane will increase the number of trips to 4,000 per hour, each trip taking around 30 minutes. People being the busy type, let's presume they value their time at ten cents per minute (\$0.10 per minute).

(A) Calculate the social benefits (in \$ terms per hour) of adding another lane.

[*Hint:* You need to calculate separately the \$ saved on trips that would have been taken anyway *and* the \$ consumer surplus on the new trips generated. For the latter part, use the triangle formula, i.e. consumer surplus on new trips = $\frac{1}{2} \times (\text{value of time saved} \times \text{new trips})$.]

Benefit from trips that would have been taken anyway: \$6000
Benefit from new trips generated: \$1000
Total Benefit: \$7000

The answer was basically already given in the clues.

If the value of time is \$0.10 per minute, then the cost of a 50-minute trip is \$5.
With a new lane, the length of a trip is reduced to 30 minutes, so the cost of a trip is now \$3.

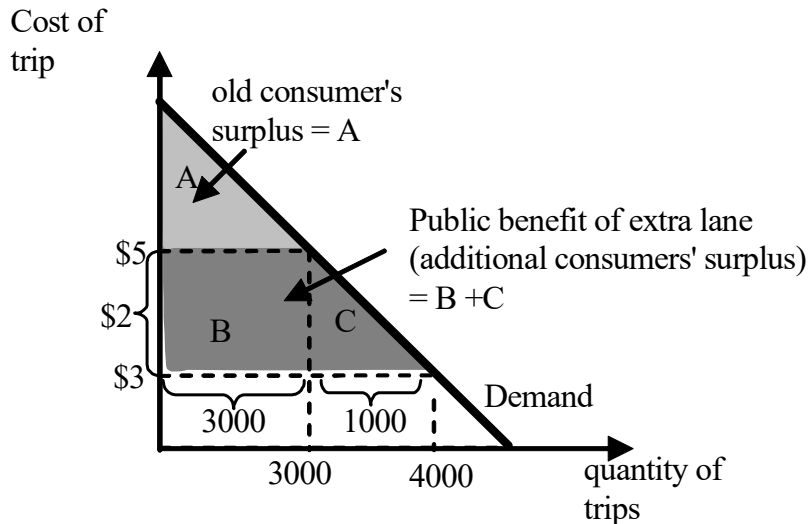
Before the addition of the lane, 3,000 people were taking the trip and incurring costs of \$5 each. With the lane, these 3,000 are now only paying \$3. So they are *saving* \$2 each. So the benefit of the lane to those who would have taken the trip = \$2 saved \times 3,000 trips = \$6,000 dollars saved.

But we also noted the addition of the lane induces people who *weren't* taking the trip before to start taking it. That is, an additional 1,000 people will take the trip now where they wouldn't take it before. What are their savings?

This is the trickier bit to calculate. We can't say they are saving \$2 on the trip - because when the price was \$5, they *weren't* taking the trip. In other words, they didn't value the trip

as worth \$5, so a reduction in price to \$3 *isn't* saving them \$2. They might have valued the trip at, say, \$4.5, in which case they are saving only \$1.50. Others might have valued the trip at only \$3.50, in which case they are now only saving \$0.50.

How do we calculate the total benefit to this group that didn't cross before? In the hint, I proposed you use the formula of $\frac{1}{2} \times (\text{value of time saved} \times \text{new trips})$. The reason for this formula can be defended diagrammatically.



Notice in the figure above, that *before* the addition of the lane, a trip cost \$5 and the quantity of trips taken was 3,000. So the old consumer's surplus was the lightly-shaded triangle A. *After* the addition of the lane, the price of a trip is \$3 and the quantity of trips taken is 4,000. So the *new* consumer's surplus is the larger triangle A+B+C. A was already there; it is the darkly-shaded part B+C that is the *additional* consumer's surplus that adding the lane has generated.

What I'm asking is for you to calculate the *extra* consumer's surplus - that is the area of rectangle B *plus* the area of triangle C.

Notice that B is a rectangle of height \$2 and length 3,000. So the area of B = $\$2 \times 3000 = \$6,000$. This we've already calculated: it is the benefit to the 3,000 people who would have taken the trip when it was \$5.

It is the area of C that is the tricky bit we want to capture - the benefit to the people who *weren't* taking the trip before - those that value it only at \$4.5 or \$3.5, etc. How do we calculate this area?

Well, notice that C looks like a triangle with height \$2 and base 1,000. By the triangular area formula, the area of C = $\frac{1}{2} \times \text{height} \times \text{base}$. Which is exactly what I proposed to you in the hint $\frac{1}{2} \times (\text{value of time saved} \times \text{new trips})$

$$\frac{1}{2} \times (\$2 \times \$1,000) = \frac{1}{2} \times \$2,000 = \$1,000.$$

So area of C, the amount saved by people who weren't taking the trip before, is \$1,000.

(Of course, to use the triangle area formula, the demand curve must be a straight line and that is by no means guaranteed (as we saw with the earlier consumers' surplus example). But by proposing the formula, I'm assuming that is the case. In general, again notwithstanding our more precise cheese example, using the triangle formula is a good approximation for rough public cost-benefit analysis).

In sum, the additional benefit of the extra lane is \$6000 to those who would have taken the trip anyway (area B) and \$1,000 to those who are now induced take the trip (area C), so the total benefit of the lane = $B + C = \$6,000 + \$1,000 = \$7,000$.

So the extra lane generates \$7,000 of public benefit. And that's how you value public projects in the real world.

[Actually, in the real world you'd have to calculate the benefit per hour, per day, per week, per year, over the years of lifetime of the road (discounted accordingly the further away it is in time) and subtract all the calculated maintenance costs. A little more complicated, but the basic idea is the same.]

(B) The State government refuses to pay for the construction of the extra lane from general taxes. As a result, the Port Authority decides to introduce a \$1 toll on the road to pay for it. That toll will *reduce* the anticipated traffic of the expanded road from 4,000 to 3,500 trips per hour. Now calculate the social benefits from adding another lane *and* a \$1 toll.

Assuming the Port Authority has the welfare of drivers in mind, is it worth it? Why or why not?

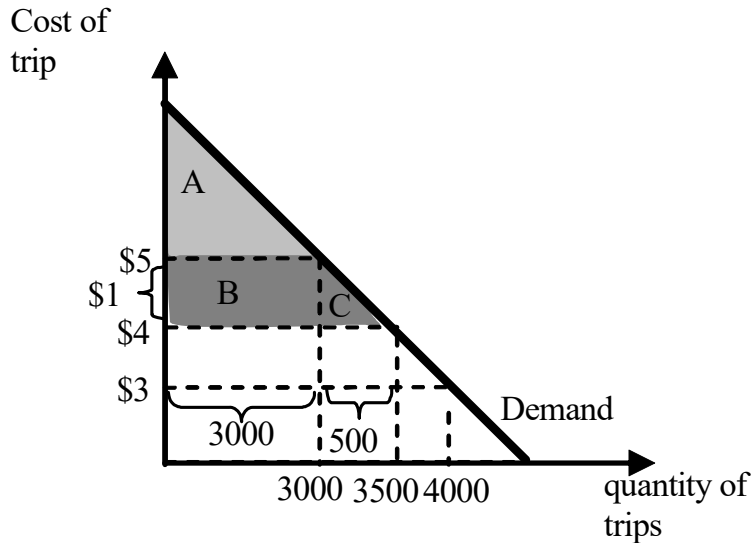
Remember that before the lane, the cost a trip was \$5. With the lane, the length of a trip is 30 minutes, or \$3 *plus* the \$1 toll, so the cost a trip is now \$4.

The 3,000 people taking the trip before are paying \$4, thus saving only \$1 each. So the benefit to those who would have taken the trip anyway is $\$1 \text{ saved} \times 3,000 \text{ trips} = \$3,000$ dollars saved.

For the remainder, use the same logic and triangle area formula as before. Only 500 new trips are induced. They save \$1 on the trip, so:

$$\frac{1}{2} \times (\$1 \times \$500) = \$250.$$

So the total benefit of the extra lane is \$3,250.



(C) Finally, suppose that the Port Authority's construction unit comes up with an estimate that requires the toll must be \$2.50 to pay for the lane. Calculate the benefit now. Is it worth it? Why or why not?

Before the lane, the cost a trip was \$5. With the lane, the length of a trip is 30 minutes, or \$3 *plus* the \$2.50 toll, so the cost a trip is now \$5.50. So a trip actually costs *more* than it did before. Not only will no new drivers be induced (if they weren't willing to pay \$5 before, they certainly aren't willing to pay \$5.50 now), but some of the earlier 3000 drivers who were willing cross before will no longer be willing to cross.

In other words, by expanding the lane and adding the \$2.50 lane, you haven't increased crossings, you've reduced them. And those still willing to cross will be taking home less consumers' surplus. The costs of the extra lane exceed the social benefits. So no, it's not worth it.