

This file describes all performance evaluation metrics used in the simulations. In real experiments, terminal error is the only metric, since continuous ground truth of robot poses cannot be obtained. T-test is applied to every pairwise comparison based on all trajectory templates to check if it is statistically significance that is implied by the p-value (less means more significance). The same abbreviations PO, RO, SLAM, TS and VS+ as the paper are used in the outcomes. There are totally five metrics to evaluate the trajectory tracking methods: average lateral error, terminal error, normalized path difference area, angular normalized control effort and control smoothness.

## 1 Average Lateral Error (ALE) (reported in the paper)

Average lateral error is the 2-norm of the perpendicular distance  $\tilde{y}$  to the robot heading direction averaged over entire time  $T$  of the trajectory.

$$\text{ALE} = \sqrt{\frac{\|\tilde{y}\|_2^2}{T}} = \sqrt{\frac{\int_0^T \tilde{y}(t)^2 dt}{T}} \quad (1)$$

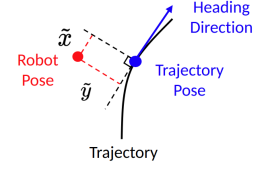


Figure 1: Lateral Error  $\tilde{y}$

### 1.1 Short Distance Simulation Outcomes

Table 1: Short distance simulation ALE outcomes.

(a) Sim ALE (cm)					(b) p-values of pairwise comparisons	
Seq.	PO	SLAM	TS	VS+	PO vs SLAM	<1e-5
SS	0.70	0.84	0.88	x	PO vs TS	2e-3
SWT	0.71	1.35	1.23	x	SLAM vs TS	8.4e-4
SST	0.55	1.64	0.82	x		
STS	0.86	1.68	0.91	x		
STT	0.75	1.21	0.96	x		
<b>Avg.</b>	0.71	1.34	<b>0.96</b>	x		

### 1.2 Long Distance Simulation Outcomes

Table 2: Long distance simulation ALE outcomes.

(a) Sim ALE (cm)					(b) p-values of pairwise comparisons	
Seq.	PO	SLAM	TS	TS+PO	PO vs SLAM	<1e-5
LRU	0.53	3.88	4.00	1.47	PO vs TS	<1e-5
LLU	0.86	8.21	5.18	1.61	PO vs TS+PO	5e-3
LST	1.13	5.03	3.00	2.03	SLAM vs TS	1.4e-3
LZZ	1.06	7.54	5.90	5.04	SLAM vs TS+PO	<1e-5
<b>Avg.</b>	0.90	6.17	<b>4.52</b>	2.54	TS vs TS+PO	2.1e-3

## 2 Terminal Error (TE) (reported in the paper)

Terminal error is the distance between robot's stopped pose  $\mathbf{p}_{rob}(t_{end})$  to the final stopped position  $\mathbf{p}_{traj}(t_{end})$  of the desired trajectory.

$$TE = ||\mathbf{p}_{rob}(t_{end}) - \mathbf{p}_{traj}(t_{end})||_2 \quad (2)$$

## 2.1 Short Distance Simulation Outcomes

Table 3: Short distance simulation TE outcomes.

(b) Sim TE (cm)				(b) p-values of pairwise comparisons	
Seq.	PO	SLAM	TS	<b>PO vs SLAM</b>	2.4e-3
SS	1.06	1.53	2.05	<b>PO vs TS</b>	1.8e-1
SWT	1.24	2.23	2.87	<b>SLAM vs TS</b>	5.6e-3
SST	2.07	3.33	2.47		
STS	1.97	4.14	2.18		
STT	4.34	4.99	2.42		
<b>Avg.</b>	2.14	3.24	<b>2.40</b>		

## 2.2 Long Distance Simulation Outcomes

Table 4: Long distance simulation TE outcomes.

(a) Sim TE (cm)					(b) p-values of pairwise comparisons	
Seq.	PO	SLAM	TS	TS+PO	<b>PO vs SLAM</b>	1.4e-3
LRU	8.57	10.66	6.42	4.15	<b>PO vs TS</b>	1.6e-1
LLU	5.54	29.48	15.75	4.02	<b>PO vs TS+PO</b>	4.2e-1
LST	6.01	7.60	1.76	6.61	<b>SLAM vs TS</b>	8.2e-3
LZZ	7.83	9.28	9.00	12.21	<b>SLAM vs TS+PO</b>	1.8e-3
<b>Avg.</b>	6.99	14.26	<b>8.23</b>	6.75	<b>TS vs TS+PO</b>	1.8e-1

## 2.3 Short Distance Real Experiment Outcomes

Table 5: Short distance real experiment TE outcomes.

(b) Real TE (cm)				(b) p-values of pairwise comparisons	
Seq.	RO	SLAM	TS	<b>RO vs SLAM</b>	4.6e-3
SS	4.5	8.9	4.9	<b>RO vs TS</b>	<1e-5
SWT	6.8	10.4	4.9	<b>SLAM vs TS</b>	<1e-5
SST	8.0	13.1	5.7		
STS	13.3	11.9	5.0		
STT	9.5	10.5	3.5		
<b>Avg.</b>	8.4	10.9	<b>4.8</b>		

## 2.4 Long Distance Real Experiment Outcomes

Table 6: Long distance real experiment TE outcomes.

(b) Real TE (cm)				(b) p-values of pairwise comparisons	
Seq.	RO	SLAM	TS	<b>RO vs SLAM</b>	2.3e-1
LS	8.2	14.1	10.9	<b>RO vs TS</b>	1.3e-1
LT	12.8	11.8	6.8	<b>SLAM vs TS</b>	1.2e-1
<b>Avg.</b>	10.5	13.0	<b>8.9</b>		

## 3 Normalized Path Difference Area (NPDA)

Normalized path difference area (NPDA) [1,2] is a way to evaluate the shape similarity between the real and desired path. It is the difference area  $A$  enclosed by two paths and divided by the

length  $L$  of desired path,  $\text{NPDA} = A/L$ .

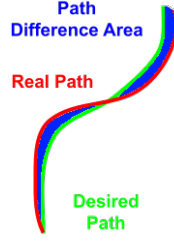


Figure 2: The method to compute path difference area

### 3.1 Short Distance Simulation Outcomes

Table 7: Short distance simulation NPDA outcomes.

(a) Sim NPDA (cm)				(b) p-values of pairwise comparisons	
Seq.	PO	SLAM	TS	PO vs SLAM	<1e-5
SS	0.61	0.71	0.72	PO vs TS	2.2e-2
SWT	0.63	1.12	0.88	SLAM vs TS	8.4e-4
SST	0.51	1.47	0.59		
STS	0.73	1.33	0.75		
STT	0.66	0.93	0.80		
<b>Avg.</b>	0.63	1.11	<b>0.75</b>		

### 3.2 Long Distance Simulation Outcomes

Table 8: Long distance simulation NPDA outcomes.

(a) Sim NPDA (cm)					(b) p-values of pairwise comparisons	
Seq.	PO	SLAM	TS	TS+PO	PO vs SLAM	<1e-5
LRU	0.50	3.28	4.23	1.29	PO vs TS	<1e-5
LLU	0.84	9.35	6.03	1.42	PO vs TS+PO	1.1e-2
LST	1.10	7.50	8.94	2.54	SLAM vs TS	1e-1
LZZ	0.93	10.31	4.54	7.60	SLAM vs TS+PO	2.4e-3
<b>Avg.</b>	0.84	7.61	<b>5.94</b>	3.21	TS vs TS+PO	1.1e-2

## 4 Angular Normalized Control Effort

The angular control effort evaluates the necessary amount of angular energy for trajectory tracking tasks. Since the trajectories may have different lengths, the control effort is normalized by the length  $L$ .

$$\text{Angular Normalized Control Effort} = \frac{\int_0^T \omega(t)^2 dt}{L} = \frac{\sum_{i=0}^{N_{end}} \omega(t_i)^2 dt_i}{L} \quad (3)$$

## 4.1 Short Distance Simulation Outcomes

Table 9: Short distance simulation angular normalized control efforts.

(a) Sim Angular Normalized Control Effort				(b) p-values of pairwise comparisons	
Seq.	PO	SLAM	TS	<b>PO vs SLAM</b>	4.5e-1
SS	0.00	0.00	0.01	<b>PO vs TS</b>	2.6e-1
SWT	0.00	0.00	0.02	<b>SLAM vs TS</b>	2.2e-1
SST	0.07	0.06	0.13		
STS	0.11	0.10	0.13		
STT	0.09	0.09	0.11		
<b>Avg.</b>	0.05	0.05	0.08		

## 4.2 Long Distance Simulation Outcomes

Table 10: Long distance simulation angular normalized control efforts.

(a) Sim Angular Normalized Control Effort					(b) p-values of pairwise comparisons	
Seq.	PO	SLAM	TS	TS+PO	<b>PO vs SLAM</b>	4.9e-1
LRU	0.0193	0.0196	0.0211	0.0207	<b>PO vs TS</b>	4.4e-1
LLU	0.0246	0.0233	0.0285	0.0265	<b>PO vs TS+PO</b>	4.6e-1
LST	0.0766	0.0766	0.0801	0.0800	<b>SLAM vs TS</b>	4.4e-1
LZZ	0.0279	0.0279	0.0297	0.0289	<b>SLAM vs TS+PO</b>	4.6e-1
<b>Avg.</b>	0.0371	0.0369	0.0399	0.0390	<b>TS vs TS+PO</b>	4.8e-1

## 5 Angular Control Smoothness

The angular control smoothness is indicated by the time differentiated control signal norm. Larger means less smoothness.

$$\text{Angular Control Smoothness} = \frac{\sum_{i=0}^{N_{end}} \left| \frac{\omega(t_{i+1}) - \omega(t_i)}{t_{i+1} - t_i} \right|}{N_{end}} \quad (4)$$

### 5.1 Short Distance Simulation Outcomes

Table 11: Short distance simulation angular control smoothness.

Seq.	PO	SLAM	TS
SS	0.001	0.006	0.458
SWT	0.024	0.025	0.849
SST	0.294	0.288	1.975
STS	0.254	0.265	1.750
STT	0.296	0.287	1.468
<b>Avg.</b>	0.174	0.174	1.300

### 5.2 Long Distance Simulation Outcomes

Table 12: Long distance simulation angular control smoothness.

Seq.	PO	SLAM	TS	TS+PO
LRU	0.167	0.164	0.508	0.474
LLU	0.123	0.124	0.481	0.394
LST	0.199	0.194	0.746	0.739
LZZ	0.113	0.111	0.525	0.463
<b>Avg.</b>	0.151	0.148	0.565	0.518

## References

- [1] Y. Mao, H. Zhong, X. Xiao, and X. Li, “A segment-based trajectory similarity measure in the urban transportation systems,” *Sensors*, vol. 17, no. 3, p. 524, 2017.
- [2] H. Su, S. Liu, B. Zheng, X. Zhou, and K. Zheng, “A survey of trajectory distance measures and performance evaluation,” *The VLDB Journal*, vol. 29, no. 1, pp. 3–32, 2020.