Optimal Fault-Tolerant Configurations of Control Moment Gyros

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Abstract

This study presents an optimization method for fault-tolerant configurations of single-gimbal control moment gyros (CMGs) aimed at maximizing satellite attitude controllability under actuator failure.

- Optimizes CMG configurations to enlarge the angular momentum envelope.
- Ensures stabilization even with reduced functionality.
- Inspired by Thomson's problem for distributing gimbal axis orientations.

Introduction

- CMGs generate strong torques with minimal energy, crucial for satellite orientation control.
- Challenges arise in underactuated conditions when some CMGs fail.
- Pyramid configuration is widely used but suboptimal under failures.
- This study proposes a novel configuration inspired by Thomson's problem, optimizing gimbal axes for resilience and adaptability.

CMG Configuration: Overview

Definition: A CMG configuration with N CMGs is defined by the gimbal axis orientations.

Gimbal Axis Vector:

$$\mathbf{g}_{g,i} = \begin{bmatrix} \cos\alpha_i \sin\beta_i \\ \sin\alpha_i \sin\beta_i \\ \cos\beta_i \end{bmatrix}$$

Spin Direction:

$$\mathbf{g}_{s,i} = \begin{bmatrix} -\sin\alpha_i \\ \cos\alpha_i \\ 0 \end{bmatrix}$$

Torque Plane Direction:

$$\mathbf{g}_{t,i} = \mathbf{g}_{g,i} \times \mathbf{g}_{s,i}$$

Angular Momentum of Each Gimbal

Angular Momentum Vector:

$$\mathbf{h}_i = \mu_i \begin{bmatrix} \mathbf{g}_{s,i} & \mathbf{g}_{t,i} \end{bmatrix} \begin{bmatrix} \cos\delta_i \\ \sin\delta_i \end{bmatrix}$$

Total Angular Momentum:

$$\mathbf{H} = \sum_{i=1}^{N} \mathbf{h}_i = \begin{bmatrix} \mathbf{G}_1 & \mathbf{G}_2 & \dots & \mathbf{G}_N \end{bmatrix} \begin{bmatrix} \cos\delta_1 \\ \sin\delta_1 \\ \vdots \\ \sin\delta_N \end{bmatrix}$$

Where:

$$\mathbf{G}_i = \begin{bmatrix} \mu_i \mathbf{g}_{s,i} & \mu_i \mathbf{g}_{t,i} \end{bmatrix}, \quad \mathbf{J}_N = \begin{bmatrix} \mathbf{G}_1 & \mathbf{G}_2 & \dots & \mathbf{G}_N \end{bmatrix}$$

$$\mathbf{H} = \mathbf{J}_N \begin{bmatrix} \cos\delta_1 \\ \sin\delta_1 \\ \vdots \\ \sin\delta_N \end{bmatrix}$$

Momentum Envelope of N CMGs

Definition: The angular momentum envelope represents the range of angular momentum achievable by the system.

Envelope Size:

$$\det(\mathbf{J}_N \mathbf{J}_N^\top)$$

Cost Function for Optimization:

$$J = w_4 \det(\mathbf{J}_4 \mathbf{J}_4^\top) + w_3 \det(\mathbf{J}_3 \mathbf{J}_3^\top) + w_2 \det(\mathbf{J}_2 \mathbf{J}_2^\top)$$

Where w_i $(i = 2, 3, 4)$ are positive weights prioritizing larger configurations.

Challenges in Momentum Optimization

Key Challenges:

- Determining initial weights for optimization.
- Aligning configuration with the satellite's body frame.
- Maximizing angular momentum in critical directions.

Solution: Optimize angular momentum in line with the satellite's structure and inertia to ensure reliable and practical performance.

Energy Potential Method

Thomson's Problem: Evenly distribute $2N$ gimbal axis endpoints on a sphere to minimize interaction forces. **Coulomb Force:**

$$f_{ij} = A_i A_j \frac{\mathbf{r}}{\|\mathbf{r}\|^2}$$

Total force on the i^{th} charge:

$$f_i = \sum_{i \neq j}^{2N} f_{ij}$$

Confining charges to the sphere:

$$f_{i,\text{proj}} = -(\mathbf{f}_i \times \mathbf{r}_i) \times \mathbf{r}_i$$

Symmetry constraint:

$$\mathbf{r}_i = -\mathbf{r}_{i+N} \quad (i = 1, 2, \ldots, N)$$

Gimbal Axis Optimization

Objective: To find gimbal axis orientations α_i and β_i ($i = 1, \ldots, N$) for N CMGs that minimize the maximum value of the inner products of the gimbal axis vectors.

Mathematical Formulation:

$$\min \left[\max_{i<j} ({g_i}^T g_j) \right]$$

Explanation:

- ${g_i}^T g_j$: Inner product of the gimbal axis vectors g_i and g_j.
- $i < j$: Indicates all possible pairs of gimbal axes.
- Minimize the maximum value of these inner products to achieve optimal spacing.

Weighted Body Frame Optimization

Objective: Prioritize angular momentum along specific axes. Add fixed external charges to the sphere:

- Moves surface charges away from fixed charges.
- Aligns angular momentum vectors along desired axes.

Example:

$$\text{Adding fixed charges along } x_b \implies \text{Enhanced momentum along } x_b.$$

Reliability-Based Weighting

Objective: To determine weights for CMG configurations based on reliability, using statistical data and the Weibull distribution.

Weibull Distribution:

$$f(t, \beta, \theta) = \frac{\beta}{\theta} \left(\frac{t}{\theta}\right)^{\beta-1} \exp\left[-\left(\frac{t}{\theta}\right)^{\beta}\right]$$

Parameters:

- t: Time of operation
- θ: Scale parameter (characteristic life)
- β: Shape parameter (failure rate behavior)

Reliability Function $R(t)$

Definition: The reliability function represents the probability that a system or component survives beyond a given time t:

$$R(t) = P(T > t)$$

Related Functions:

- Cumulative Distribution Function (CDF):

$$F(t) = P(T \leq t)$$

- Relationship between $R(t)$ and $F(t)$:

$$R(t) = 1 - F(t)$$

Derivation of Reliability Function $R(t)$

Step 1: Integrate the PDF over time:

$$F(t) = \int_0^t f(t', \beta, \theta)\, dt'$$

Substitute $f(t, \beta, \theta)$:

$$F(t) = \int_0^t \frac{\beta}{\theta} \left(\frac{t'}{\theta} \right)^{\beta - 1} \exp\left[-\left(\frac{t'}{\theta} \right)^{\beta} \right] dt'$$

Simplification of Reliability Function $R(t)$

Step 2: Solve the Integral:

$$F(t) = 1 - \exp\left(-\left(\frac{t}{\theta}\right)^{\beta}\right)$$

Step 3: Define $R(t)$:

$$R(t) = 1 - F(t)$$

Substitute $F(t)$:

$$R(t) = \exp\left(-\left(\frac{t}{\theta}\right)^{\beta}\right)$$

Final Expression for Reliability:

$$R(t, \beta, \theta) = \exp\left(-\left(\frac{t}{\theta}\right)^{\beta}\right)$$

Applications of Reliability-Based Weighting

Use in CMG Weighting:

- Assign weights to configurations based on reliability at different mission times.
- Examples:
  - $R(0)$: Initial reliability with all CMGs operational.
  - $R(5)$: Reliability after 5 years, for scenarios with 3 operational CMGs.
  - $R(20)$: Reliability after 20 years, for scenarios with 2 operational CMGs.

Weight Formula:

$$w_i = R(0) + R(5) + R(20) \quad \text{(for 4-CMG configuration, as an example)}$$

Zero Momentum Condition

Definition: Zero momentum occurs when the net angular momentum of the CMG system is zero.

Key Points:

- Achieved by counteracting angular momentum vectors of all operational CMGs.
- Ensures a stable, stationary satellite orientation.
- Requires precise control of gimbal angles to balance angular momentum.
- Useful for maintaining stability during critical phases of a mission.

Additional Strategies:

- Share the workload among operational CMGs.
- Use reaction wheels for additional stabilization if needed.

Results: Four and Three CMGs (Thomson's Problem)

Objective:

- Optimize CMG configurations to maximize the momentum envelope.
- Weights used: $w_1 = 1, w_2 = 1, w_3 = w_4 = 2.$

Key Determinants:

$$\begin{aligned} \text{Determinant of } J_4 J_4^\top &: \ 18.7930 \\ \text{Determinant of } J_3 J_3^\top &: \ 7.7920 \\ \text{Determinant of } J_2 J_2^\top &: \ 1.9501 \\ \text{Total Cost Function Value:} &: \ 55.1200 \end{aligned}$$

Gimbal Axis Orientations

Optimized Gimbal Axis Orientations:

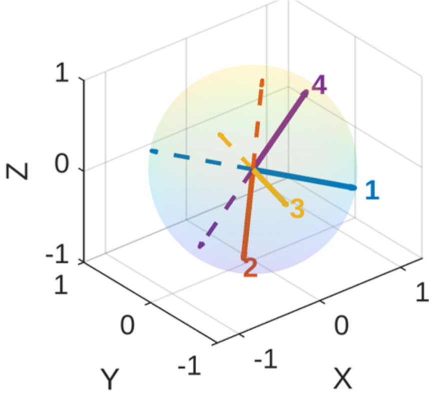
	g_2	g_3	g_4	g_5
α_i (degrees)	-35.5817	123.1275	-110.8036	16.3646
β_i (degrees)	104.0802	174.6545	81.1653	65.2555

Table: Gimbal axis orientations for both four- and three-CMG cases.

Optimized Gimbal Axes Plot

Visualization:

- The optimized configuration is superior to the pyramid configuration in both four- and three-CMG scenarios.
- Provides larger momentum envelope in underactuated conditions.



Results: Weighting Along the x_b Axis

Objective:

- Enhance angular momentum along the x_b axis by optimizing CMG configuration.
- Use external charges to influence gimbal orientations favorably.

Key Highlights:

- Angular momentum vectors align more effectively along the x_b axis.
- Improves satellite control in critical mission directions.

Optimized CMG Orientations

Optimal Gimbal Orientations:

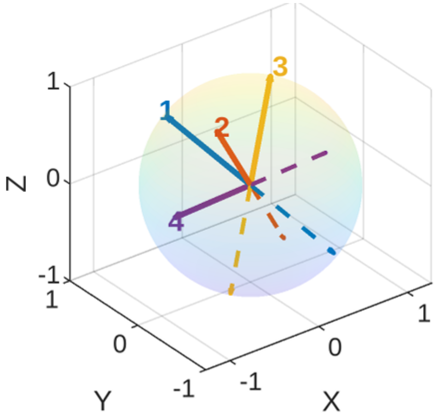
	g_1	g_2	g_3	g_4
α, deg	106.4	-159.3	36.8	108.8
β, deg	67.1	34.3	41.2	126.3

Table: Optimal CMG configuration based on weighting along the x_b axis.

Visualization of Results

Key Observations:

- The optimized gimbal orientations maximize angular momentum efficiency along the x_b axis.
- Ensures better control in body-fixed frames with prioritized directions.



Reliability-Based Optimization: Overview

Objective:

- Optimize CMG configurations considering reliability over time.
- Account for failures in four-, three-, and two-CMG scenarios.

Reliability Function:

$$R(t, \beta, \theta) = \exp\left(-\left(\frac{t}{\theta}\right)^{\beta}\right)$$

Parameters:

- $\beta = 0.7182$: Shape parameter.
- $\theta = 3831$: Scale parameter (characteristic life).

Reliability-Based Weighting

Weight Formulation:

$$(w_1, w_2, w_3, w_4) = \big(R(0)+R(5)+R(20),\ R(0)+R(5)+R(20),\ R(0)+R(5),\ R(0)\big)$$

Weights Derived:

$$(w_1, w_2, w_3, w_4) = (2.969,\ 2.969,\ 1.992,\ 1.000)$$

Key Observations:

- Reliability is prioritized for early mission phases (higher weights for four-CMG scenarios).
- Adjusts to handle reduced configurations as CMGs fail over time.

Reliability Function Visualization

Reliability vs. Time:

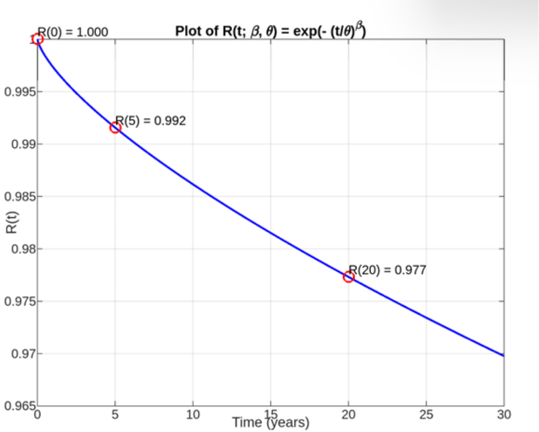


Figure: Gyro reliability as a function of time ($\beta = 0.7182, \theta = 3831$).

Optimized Gimbal Orientations

Optimal Gimbal Axes Based on Reliability:

	g_1	g_2	g_3	g_4
α, deg	-105.9232	-34.6072	34.6454	85.8490
β, deg	16.1586	104.1907	65.1489	100.2938

Table: Optimal gimbal axes orientation based on reliability.

Key Determinants and Plot

Determinants of Momentum Envelope:

$$\begin{aligned} \text{Determinant of } J_4 J_4^\top &: 18.7874 \\ \text{Determinant of } J_3 J_3^\top &: 7.7945 \\ \text{Determinant of } J_2 J_2^\top &: 1.9556 \\ \text{Total Cost Function Value:} &: 83.2044 \end{aligned}$$

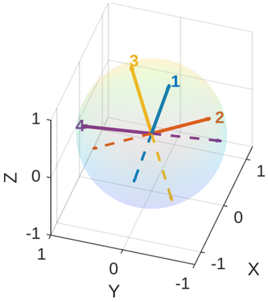


Figure: Optimal gimbal axes based on reliability.

Zero Momentum Condition

Objective: Achieve zero net angular momentum in a five-CMG configuration for stability.

Key Details:

- Configuration divided into two and three CMGs.
- Constraints:
  - One gimbal fixed along y-axis.
  - Azimuth angles of two gimbals constrained at $\alpha_2 = 210^\circ$ and $\alpha_3 = 330^\circ$.
  - Remaining two axes left unconstrained.
- Angular momentum vectors:

$$h_1 = \left[-1.0, 0.0, 0.0\right]^T, \ h_2 = \left[0.5, -0.87, 0.0\right]^T, \ h_3 = \left[0.5, 0.87, 0.0\right]^T$$

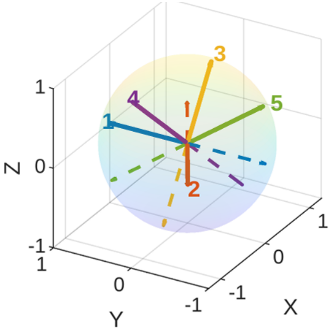
Results: Zero Momentum Condition

Optimized Gimbal Orientations:

	g_1	g_2	g_3	g_4	g_5
α, deg	90.0	-150.0	-30.0	150.0	-30.0
β, deg	90.0	90.0	18.0	45.2	77.8

Table: Optimal CMG configuration satisfying zero-momentum condition.

Visualization:



Conclusion

- Proposed methods ensure fault-tolerant and efficient CMG configurations.
- Adaptable to mission-specific requirements like reliability and zero momentum.
- Optimization techniques reduce computational complexity and enhance resilience.

References

- http://dx.doi.org/10.2514/1.G001249