Optimal Fault-Tolerant Configurations of Control Moment Gyros

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Abstract

This study presents an optimization method for fault-tolerant configurations of single-gimbal control moment gyros (CMGs) aimed at maximizing satellite attitude controllability under actuator failure. The method optimizes CMG configurations to enlarge the angular momentum envelope, ensuring attitude stabilization even with reduced functionality. The approach models the optimization as a min-max problem inspired by Thomson's problem, distributing gimbal axis orientations to maximize orthogonality and angular momentum volume. Extensions include weighting optimizations for reliability-based constraints, underactuated conditions, and specific mission requirements, such as zero-momentum states and directional preferences. The simulation results show that this method works better than the standard pyramid configuration, especially when some actuators fail. It offers a straightforward and effective way to design reliable satellite attitude control systems that can handle failures and adapt to different conditions.

Contents

1. Introduction

Control Moment Gyroscopes (CMGs) play a key role in controlling the orientation of satellites because they can generate strong torques with minimal energy use. However, one of the biggest challenges with CMGs is ensuring that satellites remain operational even when some of these components fail. This is crucial because the reliability of these actuators directly affects how long a satellite can perform its mission successfully.

When some CMGs fail, the satellite's ability to control its orientation is reduced. This is known as an "underactuated" condition, where fewer control inputs are available than needed. To address this, engineers often rely on specific CMG arrangements, like the pyramid configuration, which is widely used because it provides a large range of motion under normal conditions. But when one or more CMGs stop working, the pyramid configuration doesn't perform as well, leaving room for improvement.

Recent research has focused on finding better ways to arrange CMGs to ensure the satellite can still function effectively even if some fail. This involves rethinking how the CMG axes are oriented to maximize the angular momentum, which is the key to controlling the satellite's movement. The goal is to make the system more resilient and adaptable, especially for longer missions where failures are more likely.

In this study, we introduce a new method for arranging CMGs that's inspired by a classic physics problem called Thomson's problem. In this problem, points are evenly distributed on the surface of a sphere to minimize the forces between them. We apply a similar idea to the CMG axes by treating them like points on a sphere and arranging them to minimize interference and maximize effectiveness. This method ensures the satellite can maintain control even if some CMGs fail.

What makes this approach special is its flexibility. It can be adjusted to meet specific mission needs, such as favoring certain directions for control or taking into account how likely each CMG is to fail. Simulations show that this new arrangement works better than the traditional pyramid configuration, especially when dealing with failures. This makes it a promising solution for designing reliable and adaptable satellite control systems.

The rest of this paper explains the details of this method and compares its performance with traditional designs. We also explore how it can be adapted for different missions and challenges, showing its potential to improve the reliability of future satellites.

2. Preliminaries

2.1 CMG Configuration

A CMG configuration comprising N CMGs is defined by the orientations of the gimbal axes. An arbitrary direction of the gimbal axis of the i^{th} CMG ($i = 1, 2, \ldots, N$) is defined as:

$$\mathbf{g}_{g,i} = \begin{bmatrix} \cos\alpha_i \sin\beta_i \\ \sin\alpha_i \sin\beta_i \\ \cos\beta_i \end{bmatrix}$$

where α_i and β_i are azimuth and elevation angles, respectively.

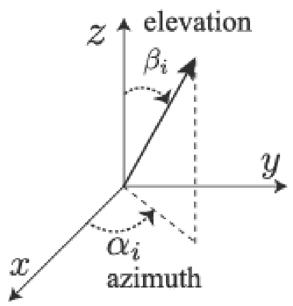

Figure 2.1: Azimuthal and Elevation angle.

The spin direction $\mathbf{g}_{s,i}$ of a rotor is orthogonal to the gimbal vector $\mathbf{g}_{g,i}$ and is described as follows:

$$\mathbf{g}_{s,i} = \begin{bmatrix} -\sin\alpha_i \\ \cos\alpha_i \\ 0 \end{bmatrix}$$

The angular momentum by each gimbal will be on the $\mathbf{g}_{s,i} - \mathbf{g}_{t,i}$ plane, where $\mathbf{g}_{t,i} = \mathbf{g}_{g,i} \times \mathbf{g}_{s,i}$. Thus, $\mathbf{h}_i$ can be defined as:

$$\mathbf{h}_i = \mu_i \begin{bmatrix} \mathbf{g}_{s,i} & \mathbf{g}_{t,i} \end{bmatrix} \begin{bmatrix} \cos\delta_i \\ \sin\delta_i \end{bmatrix}$$

where μ_i and δ_i denote the magnitude of the angular momentum and the gimbal angle, respectively.

The total angular momentum of N CMGs, $\mathbf{H}$, is given by the following expressions:

$$\mathbf{H} = \sum_{i=1}^{N} \mathbf{h}_i = \sum_{i=1}^{N} \mu_i \begin{bmatrix} \mathbf{g}_{s,i} & \mathbf{g}_{t,i} \end{bmatrix} \begin{bmatrix} \cos\delta_i \\ \sin\delta_i \end{bmatrix}$$

This can be further simplified into:

$$\mathbf{H} = \begin{bmatrix} \mathbf{G}_1 & \mathbf{G}_2 & \dots & \mathbf{G}_N \end{bmatrix} \begin{bmatrix} \cos\delta_1 \\ \sin\delta_1 \\ \vdots \\ \sin\delta_N \end{bmatrix} \implies \mathbf{H} = \mathbf{J}_N \begin{bmatrix} \cos\delta_1 \\ \sin\delta_1 \\ \vdots \\ \sin\delta_N \end{bmatrix}$$

Where:

$$\mathbf{G}_i = \begin{bmatrix} \mu_i \mathbf{g}_{s,i} & \mu_i \mathbf{g}_{t,i} \end{bmatrix}, \quad \mathbf{J}_N = \begin{bmatrix} \mathbf{G}_1 & \mathbf{G}_2 & \dots & \mathbf{G}_N \end{bmatrix}$$

2.2 Momentum Envelope of N CMGs

The angular momentum envelope of N CMGs is similar to how robotic arms calculate their range of movement. It represents the range of angular momentum the system can achieve. The size of this envelope, which essentially measures how effective the CMGs are at controlling the satellite's movement is defined by

$$\det(\mathbf{J}_N \mathbf{J}_N^\top)$$

In this study, we look at how the system performs with different numbers of CMGs, especially in situations where only a few are working. Typically, the system is designed to work with two to four CMGs, as controlling the satellite with just one CMG is almost impossible except in rare cases. The goal is to optimize the system to make the best use of the available CMGs, even if some fail, by maximizing the cost function defined by:

$$J = w_4 \det(\mathbf{J}_4 \mathbf{J}_4^\top) + w_3 \det(\mathbf{J}_3 \mathbf{J}_3^\top) + w_2 \det(\mathbf{J}_2 \mathbf{J}_2^\top)$$

where w_i $(i = 2, 3, 4)$ are positive weights

Although the optimization process can be done using tools like MATLAB, it has some challenges. One major issue is determining the initial weights and correctly aligning the configuration with the satellite's body frame. The current approach focuses only on maximizing the overall angular momentum, which means the solution might still be effective even if the configuration is rotated. However, this can lead to unexpected results, especially if the angular momentum along critical axes is too small, limiting the satellite's ability to control itself in those directions.

To address this, it's important to maximize the angular momentum in line with the satellite's specific structure and inertia. The next section introduces a method that allows the optimization to consider specific directions and constraints, making the configuration more practical and reliable for real-world applications.

3. Different Methods of Optimization

3.1 Energy Potential Method Based on Thomson's Problem

This study proposes an optimization method based on solving Thomson's problem. Thomson's problem addresses the optimization of equally distributing multiple points on the surface of a unit sphere. In this problem, each point is treated as a point charge that interacts with other charges through Coulomb forces, based on their relative distances. By adjusting the positions of the point charges according to their interaction forces, the configuration eventually converges to one with minimum interaction energy.For equally charged particles, this minimum energy configuration results in an equal distribution of the charges across the sphere's surface.

Similar to how Thomson's problem is solved, the optimization of a CMG configuration involves representing the two ends of each gimbal axis as points on the surface of a sphere. Since there are N gimbal axes, this results in 2N points, or "charges," on the sphere. These charges are arranged evenly across the sphere's surface to minimize the interaction energy between them, ensuring an optimal and balanced configuration for the CMGs.

The Coulomb force acting between the i^{th} and j^{th} charges is described as:

$$f_{ij} = A_i A_j \frac{\mathbf{r}}{\|\mathbf{r}\|^2}$$

where A_i and A_j are the magnitudes of each charge, and $\mathbf{r} = \mathbf{r}_i - \mathbf{r}_j$ for position vectors $\mathbf{r}_i$ and $\mathbf{r}_j$.

The total force acting on the i^{th} charge is:

$$f_i = \sum_{i \neq j}^{2N} f_{ij}$$

To confine the charges to the surface of the sphere, the Coulomb forces are projected onto the sphere as:

$$f_{i,\text{proj}} = -(\mathbf{f}_i \times \mathbf{r}_i) \times \mathbf{r}_i$$

A virtual damping effect is added to avoid divergence, and the interaction force acting on the i^{th} charge becomes:

$$F_i = f_{i,\text{proj}} - c_i \dot{\mathbf{r}}_i$$

There are $2N$ charges on the unit sphere, each interacting with the others. Each charge pair at the endpoints of each gimbal axis must be symmetrically located with respect to the origin. This position constraint is mathematically described as:

$$\mathbf{r}_i = -\mathbf{r}_{i+N} \quad (i = 1, 2, \ldots, N)$$

The symmetry reduces the computational burden, as only N charges require direct motion calculations using Coulomb forces. For this analogously we have to find the gimbal axis orientations α_i and β_i $(i = 1, \ldots, N)$ for N CMGs that minimize the maximum value of the inner products of the gimbal axis vectors, that is,

$$\min \left[\max_{i<j} (g_i^T g_j) \right]$$

where $i < j$ indicates all possible pairs of the gimbal axes.

3.2 Modifications of the Proposed Optimization Method

3.2.1 Weighting Along a Body-Fixed Frame

The proposed method minimizes Coulomb potential, allowing weighting in a body-fixed frame, unlike numerical optimization. Weighting along a body-fixed axis is achieved by adding fixed external charges apart from the unit sphere, which interact with surface charges. For example, positioning fixed charges along the x_b axis moves surface charges away from x_b, making gimbal axes non-parallel to x_b. As a result, angular momentum vectors align along x_b, increasing angular momentum in that direction. Using fixed external charges enables an optimal CMG configuration with enhanced angular momentum along a desired axis.

3.2.2 Weighting According to Number of Available CMGs and Reliability

One possible criterion to determine the weights in this study considers a reliability-based weighting that uses statistical reliability data of actuators. The Weibull distribution is widely used for modeling the reliability of systems based on collected failure data. A Weibull probability density function is expressed as:

$$f(t, \beta, \theta) = \frac{\beta}{\theta} \left(\frac{t}{\theta} \right)^{\beta-1} \exp \left[- \left(\frac{t}{\theta} \right)^{\beta} \right]$$

The reliability function $R(t)$ of a random variable T represents the probability that T is greater than a certain time t, i.e., the probability that the system or component survives beyond time t. Mathematically, this is defined as:

$$R(t) = P(T > t)$$

The cumulative distribution function (CDF) $F(t)$ represents the probability that T is less than or equal to t, i.e.,

$$F(t) = P(T \leq t)$$

Therefore, the reliability function $R(t)$ is:

$$R(t) = 1 - F(t)$$

Derivation of Reliability Function $\mathbf{R(t)}$

For finding $F(t)$, we integrate $f(t)$ from 0 to t:

$$F(t) = \int_0^t f(t', \beta, \theta)\, dt' = \int_0^t \frac{\beta}{\theta} \left(\frac{t'}{\theta}\right)^{\beta-1} \exp\left[-\left(\frac{t'}{\theta}\right)^{\beta}\right] dt'$$

Thus, the integral simplifies into:

$$F(t) = 1 - \exp\left(-\left(\frac{t}{\theta}\right)^{\beta}\right)$$

Hence, $R(t) = 1 - F(t)$:

$$R(t) = 1 - \left(1 - \exp\left(-\left(\frac{t}{\theta}\right)^{\beta}\right)\right) = \exp\left(-\left(\frac{t}{\theta}\right)^{\beta}\right)$$

Thus, the reliability function for a Weibull distribution is:

$$R(t, \beta, \theta) = \exp\left(-\left(\frac{t}{\theta}\right)^{\beta}\right)$$

Now let's explain how this can be applied to our problem with an example.

Suppose we are planning a mission with a lifetime of 30 years. The goal is to optimize the placement of four CMGs to maximize the system's reliability over its expected lifetime. This means configuring the CMGs in a way that not only maximizes performance when all four CMGs are operational but also ensures the system remains controllable if one or two CMGs fail.

- **R(0)**: Initial reliability when all four CMGs are working.
- **R(5)**: Reliability after five years, relevant for the case when only three CMGs are expected to be operational.
- **R(20)**: Reliability after 20 years, relevant for the case when two CMGs might still be operational.

Now, by effectively adjusting the importance of each configuration (four-, three-, or two-CMG) at different times:

- **Four-CMG Configuration:** Weighted by $R(0)$ to emphasize performance at the start of the mission.
- **Three-CMG Configuration:** Weighted by $R(5)$ to reflect the system's reliability after five years, in case one CMG fails.
- **Two-CMG Configuration:** Weighted by $R(20)$, prioritizing system functionality even if two CMGs fail after 20 years.

3.3 Zero Momentum Condition

A zero-momentum condition occurs when the net angular momentum of the CMG system is zero. This means the angular momentum vectors of all operational CMGs cancel each other out, leaving the satellite in a stable, stationary orientation. Each CMG generates angular momentum in a specific direction based on its gimbal angle. When all CMGs are functioning, their angular momentum vectors can be controlled to counteract each other, achieving the zero-momentum state.These configurations are designed to keep the satellite as stable as possible, even if it's not possible to completely cancel out all movement. They may involve sharing the workload among the remaining working CMGs, focusing on controlling the most important directions, or using other systems like reaction wheels to assist with stability.

4. Results

Both four and three CMG cases (Thomson's Problem)

Weights used : $w_1 = 1, w_2 = 1, w_3 = w_4 = 2$

	g_2	g_3	g_4	g_5
α_i (degrees)	-35.5817	123.1275	-110.8036	16.3646
β_i (degrees)	104.0802	174.6545	81.1653	65.2555

Table 4.1: Gimbal axis orientations for both four- and three- CMG cases

$$\text{Determinant of } J_4J_4^T : 18.7930$$
$$\text{Determinant of } J_3J_3^T : 7.7920$$
$$\text{Determinant of } J_2J_2^T : 1.9501$$
$$\text{Total Cost Function Value: } 55.1200$$

This result indicates that the derived CMG configuration is optimized not only for the four-CMG configuration but also for the three-CMG configuration, whereas the pyramid configuration is optimized only for the four-CMG configuration. In fact, the respective four- and three-CMG volumes of the momentum envelope are $\det(J_4J_4^T) = 18.79$ and $\det(J_3J_3^T) = 7.79$ for the optimal CMG configuration and $\det(J_4J_4^T) = 18.96$ and $\det(J_3J_3^T) = 7.41$ (based on standard configuration) for the pyramid configuration.

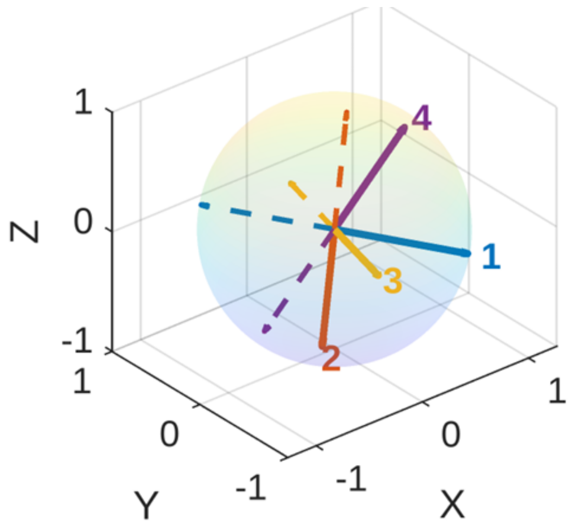

Figure 4.1: Optimal gimbal axes for both four- and three- CMG cases.

Based on weighting along the x_b axis

	g_1	g_2	g_3	g_4
α, deg	106.4	-159.3	36.8	108.8
β, deg	67.1	34.3	41.2	126.3

Table 4.2: Optimal CMG configuration based on weighting along the x_b axis

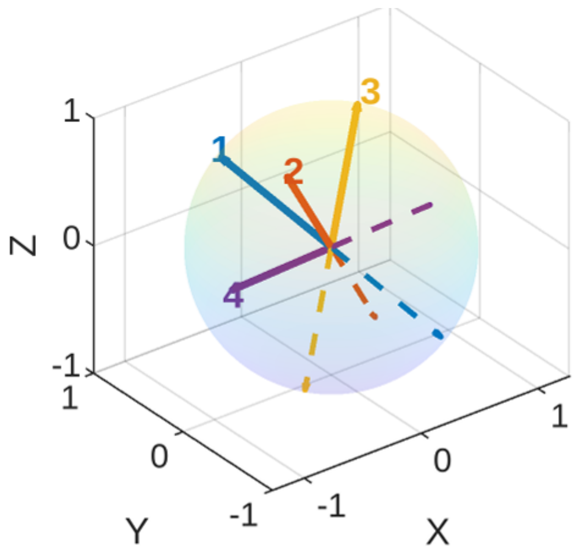

Figure 4.2: Optimal gimbal axes based on weighting along the x_b axis.

Based on Reliability

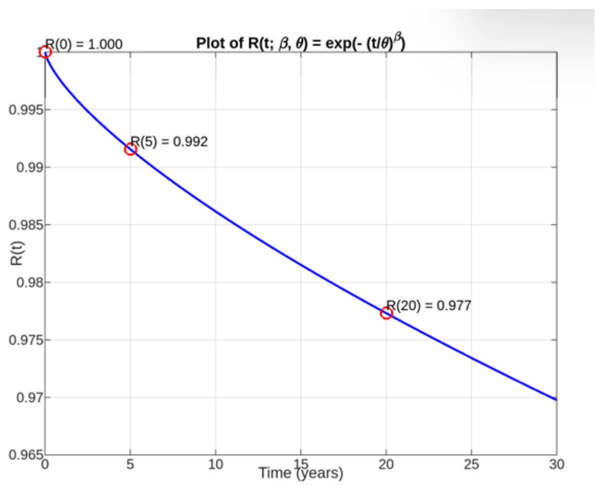

Figure 4.3: Gyro reliability as function of time ($beta = 0.7182$ and $theta = 3831$)

As an example, we consider a four-CMG configuration that is optimized with consideration for three- and two-CMG configurations (i.e., one and two CMG failures, respectively) with an expected lifetime $t_f = 30$. In this case, the weights for the optimization are chosen according to the reliability in the early phase. Here, we choose them according to $R(0)$ for four CMGs, $R(5)$ for three CMGs, and $R(20)$ for two CMGs, which are applied to the magnitudes of the point charges as

$$(w_1, w_2, w_3, w_4) = \big(R(0) + R(5) + R(20),\ R(0) + R(5) + R(20),\ R(0) + R(5),\ R(0)\big)$$

From the graph

$$(w_1, w_2, w_3, w_4) = (2.969,\ 2.969,\ 1.992,\ 1.000).$$

	g_1	g_2	g_3	g_4
α, deg	-105.9232	-34.6072	34.6454	85.8490
β, deg	16.1586	104.1907	65.1489	100.2938

Table 4.3: Optimal gimbal axes orientation based on reliability.

$$\begin{aligned}\text{Determinant of } J_4J_4^T &: 18.7874\\ \text{Determinant of } J_3J_3^T &: 7.7945\\ \text{Determinant of } J_2J_2^T &: 1.9556\\ \text{Total Cost Function Value} &: 83.2044\end{aligned}$$

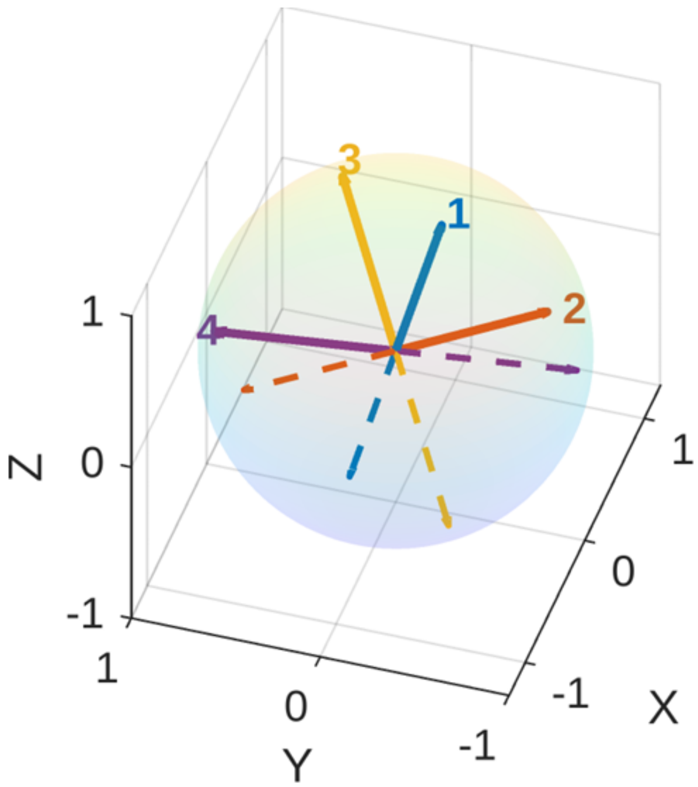

Figure 4.4: Optimal gimbal axes based on reliability.

Zero Momentum Condition

This arrangement ensures zero-momentum condition for a five-CMG configuration, divided into two and three CMGs. Constraints are applied to the azimuth angles of three gimbal axes, with one fixed along the y-axis and two constrained at $\alpha_2 = 210°$ and $\alpha_3 = 330°$. The remaining two axes have no constraints. The angular momentum vectors $h_1 = \left[-1.0, 0.0, 0.0\right]^T$, $h_2 = \left[0.5, -0.87, 0.0\right]^T$, and $h_3 = \left[0.5, 0.87, 0.0\right]^T$ satisfy zero momentum, while the other two CMGs balance each other. This demonstrates that adding azimuth constraints optimizes the configuration for zero momentum.

	g_1	g_2	g_3	g_4	g_4
α, deg	90.0	-150.0	-30.0	150.0	-30.0
β, deg	90.0	90.0	18	45.2	77.8

Table 4.4: Optimal CMG configuration satisfying the zero-momentum condition,

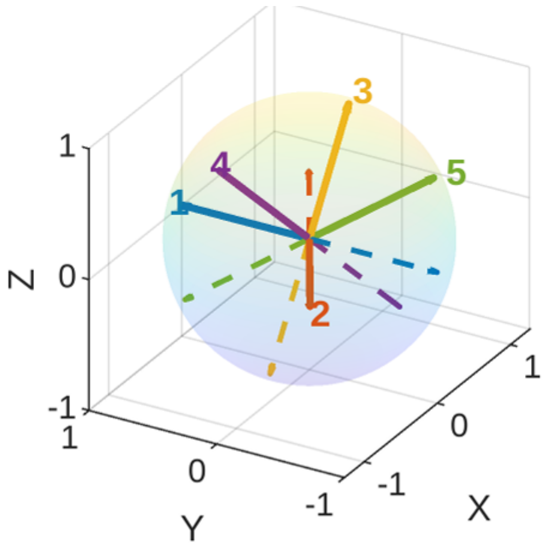

Figure 4.5: Optimal CMG axes satisfying zero momentum condition.

5. Conclusion

This study proposes an intuitive and effective approach to designing configurations for single-gimbal control moment gyroscopes (CMGs), which are essential for controlling a satellite's orientation. The method ensures that the satellite remains operational even if some CMGs fail, making it both efficient and fault-tolerant. By treating the directions of CMG axes as points on a sphere and minimizing the interaction between them, the optimal arrangement is determined. This technique allows for flexibility in prioritizing performance factors, such as maximizing angular momentum or improving system reliability, depending on the mission requirements. It can also accommodate specific constraints, such as achieving a zero-momentum state when needed. Notably, the approach eliminates the need for initial guesses, streamlining the design process for engineers. This makes it a practical and robust solution for creating CMG configurations that balance optimal performance with reliability, ensuring satellites remain controllable in various scenarios.

References

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