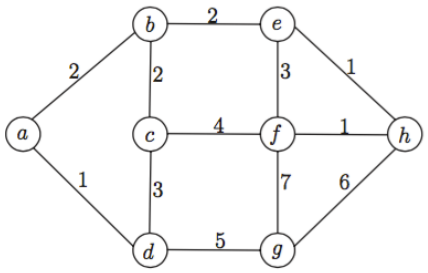
Discrete Mathematics
Practice Class 7
26-03-2024

Problem 1. Consider the weighted graph



(i) Use Dijkstra's algorithm to find the shortest path and its weight between the vertex a and the others.

(ii) Introduce the graph in Magrada using **Graphic Mode**. Check the result of **(i)** with Magrada (Menu **Algorithms**, Option **Dijkstra**).

Problem 2. It is known the iteration m = 5 of the Floyd-Warshall's algorithm:

$$\Omega^{(5)} \equiv \begin{array}{c} \\ e \\ d \\ c \\ b \\ a \end{array} \begin{array}{c} \begin{array}{ccccc} e & d & c & b & a \end{array} \\ \begin{pmatrix} \infty & 2 & 2 & 5 & 6 \\ \infty & \infty & \infty & 4 & 4 \\ \infty & \infty & \infty & 3 & 8 \\ \infty & \infty & \infty & \infty & 5 \\ 6 & 8 & 8 & 11 & 12 \end{pmatrix} \end{array} \qquad \theta^{(5)} \equiv \begin{array}{c} \\ e \\ d \\ c \\ b \\ a \end{array} \begin{array}{c} \begin{array}{ccccc} e & d & c & b & a \end{array} \\ \begin{pmatrix} - & e & e & c & d \\ - & - & - & d & d \\ - & - & - & c & b \\ - & - & - & - & b \\ a & e & e & c & d \end{pmatrix} \end{array}$$

(a) Complete the Floyd-Warshall's algorithm.
(b) Using the matrices obtained form Floyd-Warshall's method, identify the shortest path and its weight from vertex a to b and from b to c

Problem 3. A computer network connects 5 points A, B, C, D and E. The connections can be represented using an undirected weighted graph, where the weights assigned to the edges represent the time in milliseconds needed to transmit a word from one point to another. We need to know the minimum transmission time routes between each pair of points. Express the general solution for any pair of points using a matrix. Calculate, in particular, the route for the connection from A to C and the time necessary for this connection. On the other hand, it is known that the links from point B fail sometimes. When this failure occurs a message can not pass through point B. Calculate the minimum transmission time alternative route to use it, from A to C, in the case that this failure takes place.

$$\Omega \equiv \begin{array}{c|ccccc} & A & B & C & D & E \\ \hline A & \infty & 1 & \infty & 3 & 8 \\ B & 1 & \infty & \infty & 2 & 1 \\ C & \infty & \infty & \infty & 4 & 2 \\ D & 3 & 2 & 4 & \infty & \infty \\ E & 8 & 1 & 2 & \infty & \infty \end{array}$$

Problem 4. A computer network connects 5 points A, B, C, D and E. The connections can be represented using an undirected weighted graph, where the weights assigned to the edges represent the time in milliseconds needed to transmit a word from one point to another. We need to know the minimum transmission time routes between each pair of points. Express the general solution for any pair of points using a matrix. Calculate, in particular, the route for the connection from A to C and the time necessary for this connection. On the other hand, it is known that the links from D fail sometimes. When this failure occurs a message can not pass through point D. Calculate the minimum transmission time alternative route to use it, from A to C, in the case that this failure takes place. It is known the penultimate iteration (m = 5) of the Floyd-Warshall algorithm.

$$\Omega^{(5)} \equiv \begin{array}{c} \\ E \\ A \\ C \\ B \\ D \end{array} \begin{array}{c} \begin{array}{ccccc} E & A & C & B & D \end{array} \\ \begin{pmatrix} 2 & 6 & 1 & 5 & 3 \\ 6 & 2 & 5 & 1 & 2 \\ 1 & 5 & 2 & 4 & 2 \\ 5 & 1 & 4 & 2 & 1 \\ 3 & 2 & 2 & 1 & 2 \end{pmatrix} \end{array} \qquad \Theta^{(5)} \equiv \begin{array}{c} \\ E \\ A \\ C \\ B \\ D \end{array} \begin{array}{c} \begin{array}{ccccc} E & A & C & B & D \end{array} \\ \begin{pmatrix} C & B & E & C & E \\ C & B & B & A & B \\ C & B & E & C & C \\ C & B & B & A & B \\ D & B & D & D & B \end{pmatrix} \end{array}$$