Exercices

Exercices:

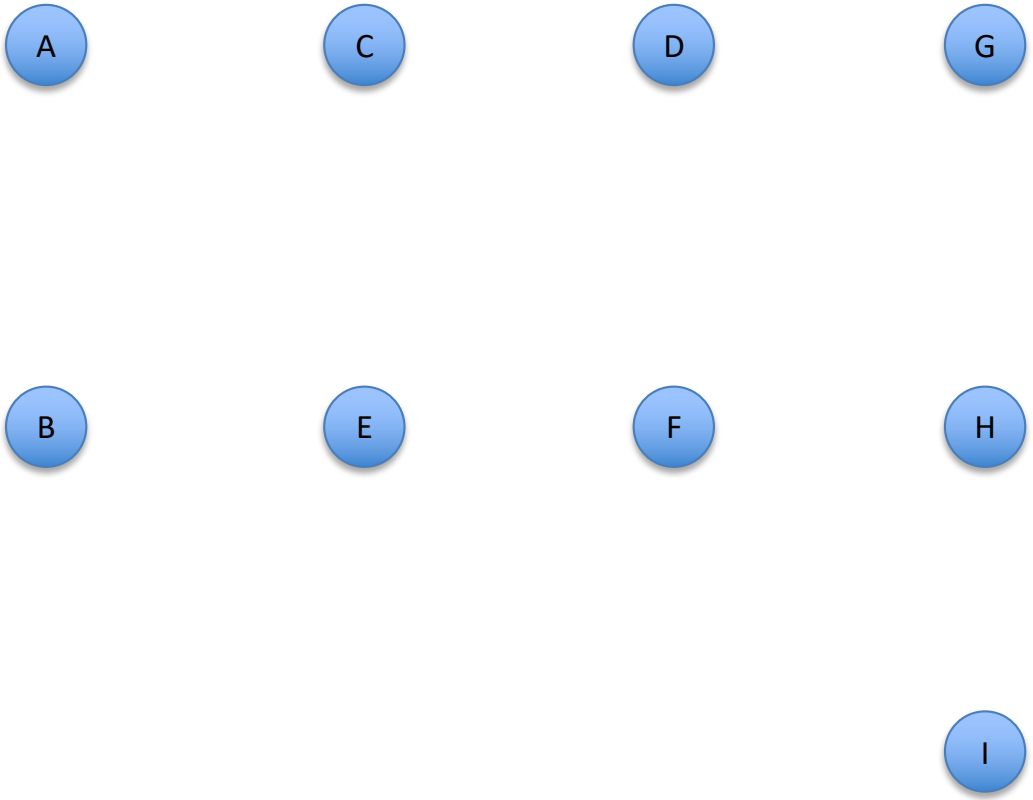
1.- Consider the project given by the following table of prerequisites:

Activity	A	B	C	D	E	F	G	H	I
Needed time (days)	3	6	8	7	5	11	3	3	2
Prerequisites	–	–	A, B	C, E	B	E	D	F, G	B

a) Compute the minimum number of days needed to complete the project.
b) Obtain the critical path and its weight explaining its meaning.
c) Compute the maximum delay allowed for task E without affecting the duration of the entire project.

Exercices

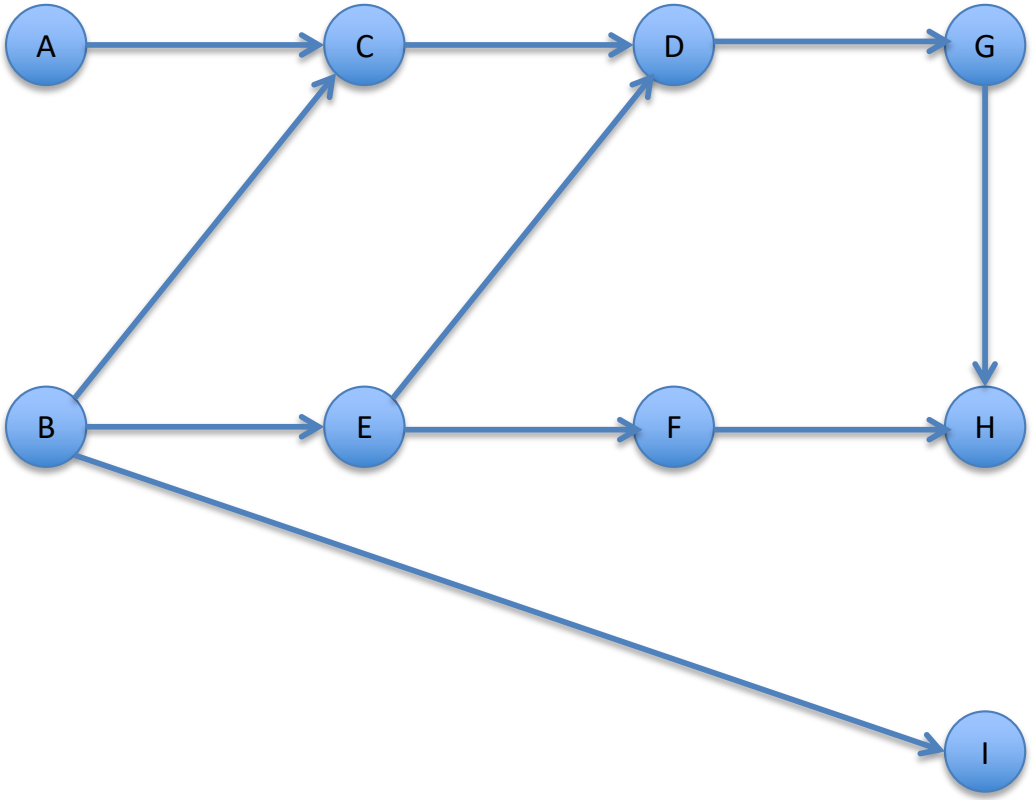
First: Represent each activity using a vertex



Activity	A	B	C	D	E	F	G	H	I
Needed time (days)	3	6	8	7	5	11	3	3	2
Prerequisites	–	–	A, B	C, E	B	E	D	F, G	B

Exercices

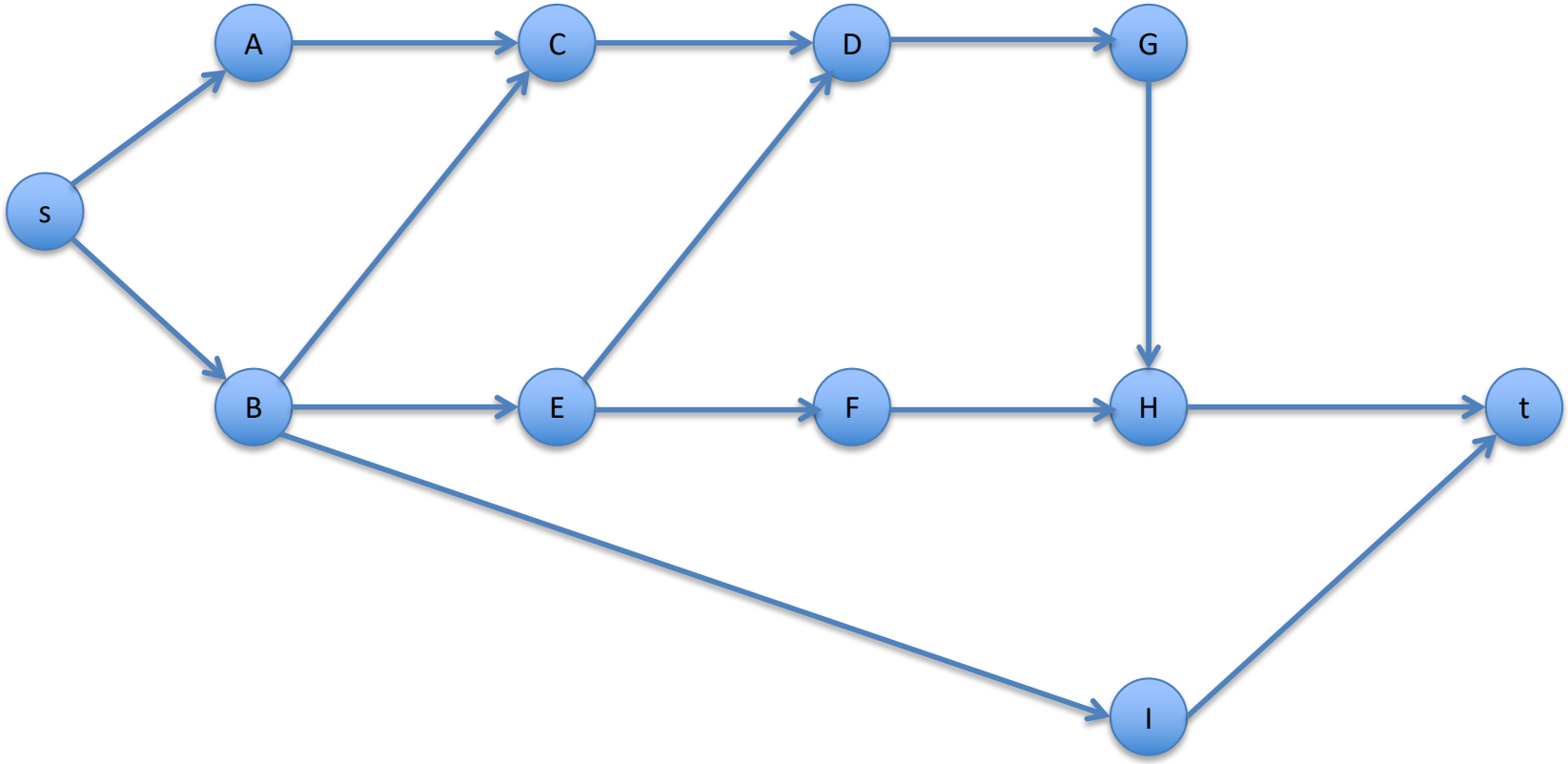
Second: Draw the arcs from the prerequisits.



Activity	A	B	C	D	E	F	G	H	I
Needed time (days)	3	6	8	7	5	11	3	3	2
Prerequisites	–	–	A, B	C, E	B	E	D	F, G	B

Exercices

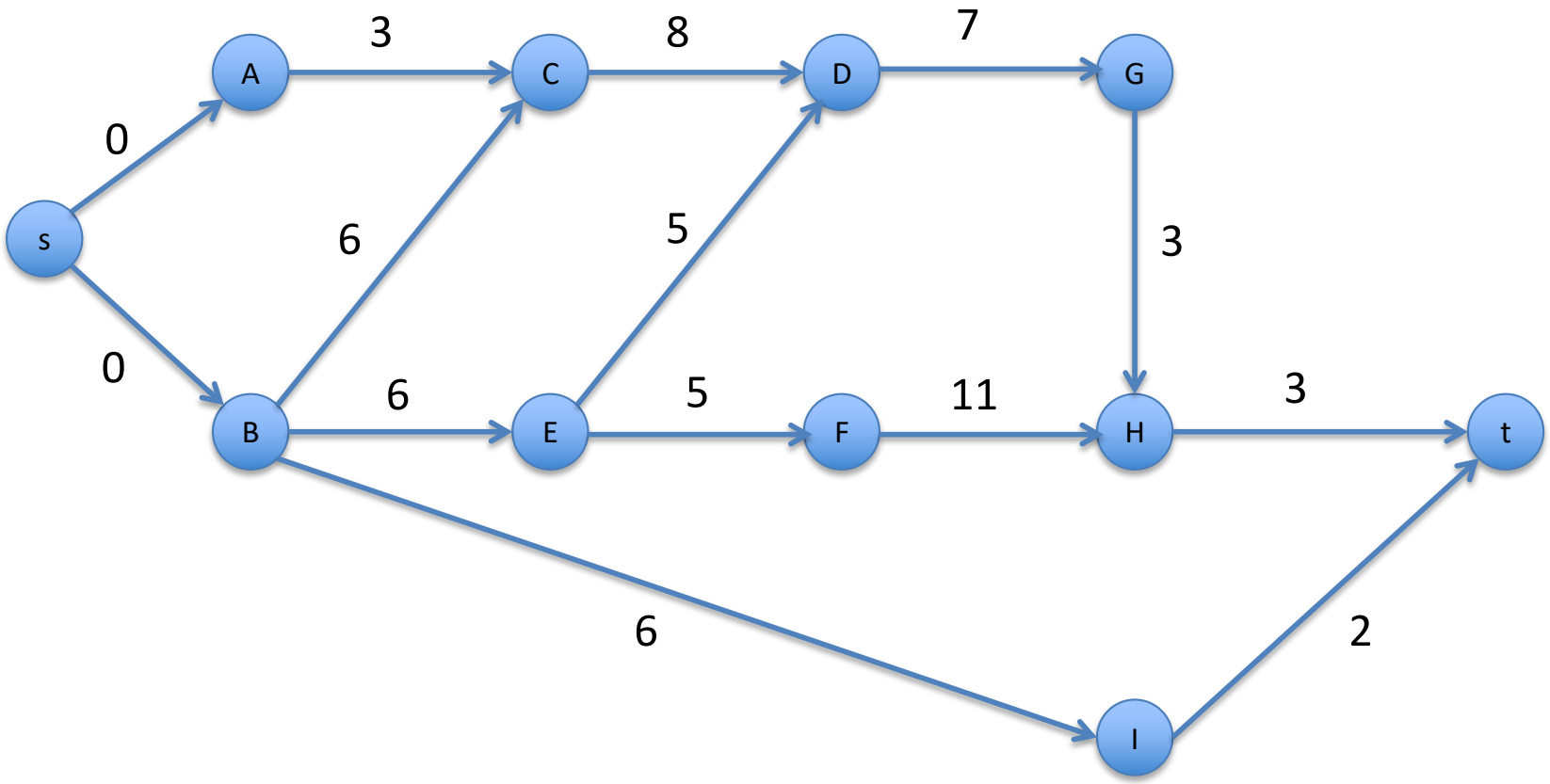
Third: Add fictitious vertices



A	B	C	D	E	F	G	H	I
3	6	8	7	5	11	3	3	2
–	–	A, B	C, E	B	E	D	F, G	B

Exercices

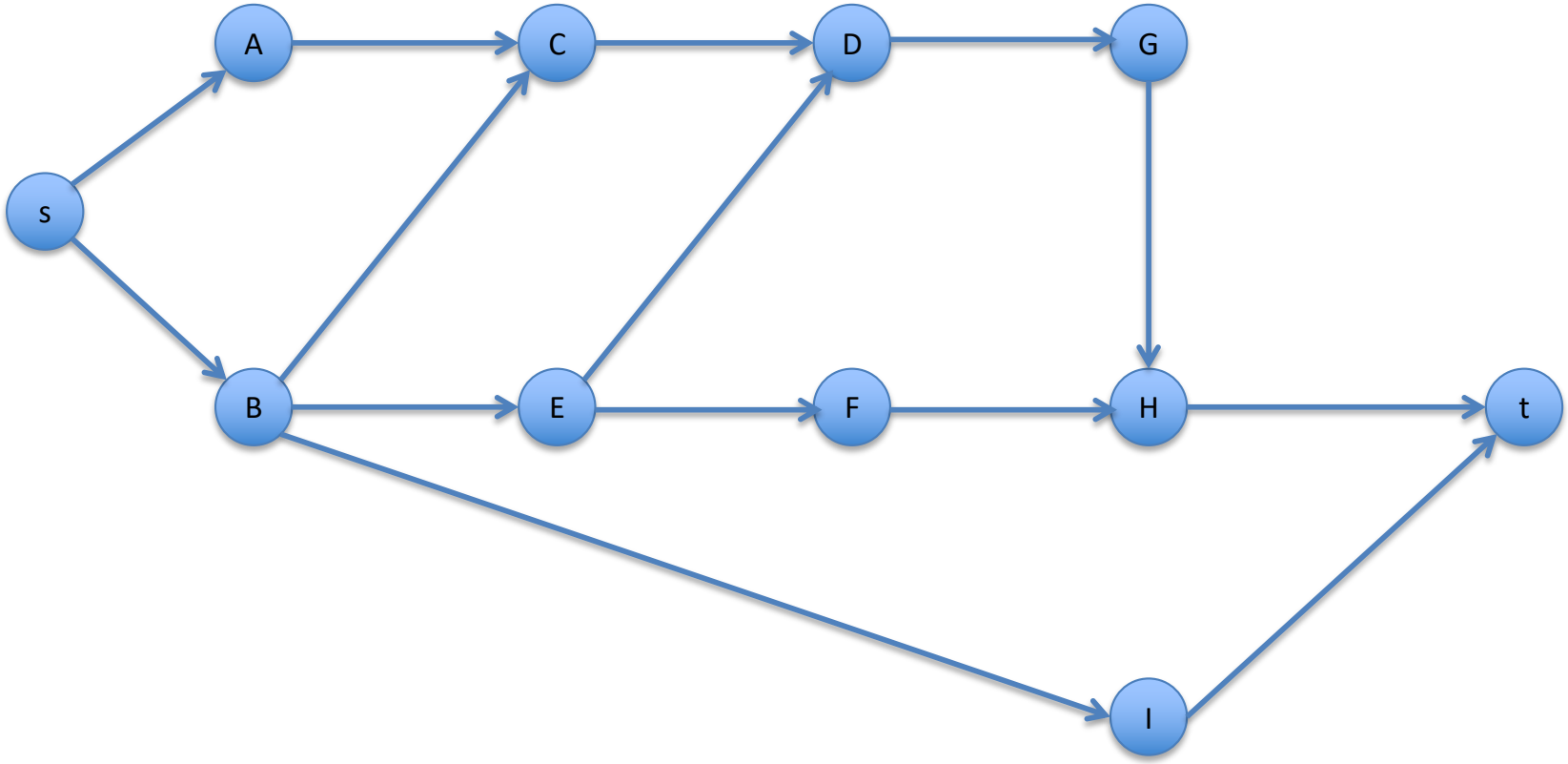
Forth: Add weights



Activity	A	B	C	D	E	F	G	H	I
Needed time (days)	3	6	8	7	5	11	3	3	2
Prerequisites	–	–	A, B	C, E	B	E	D	F, G	B

Exercices

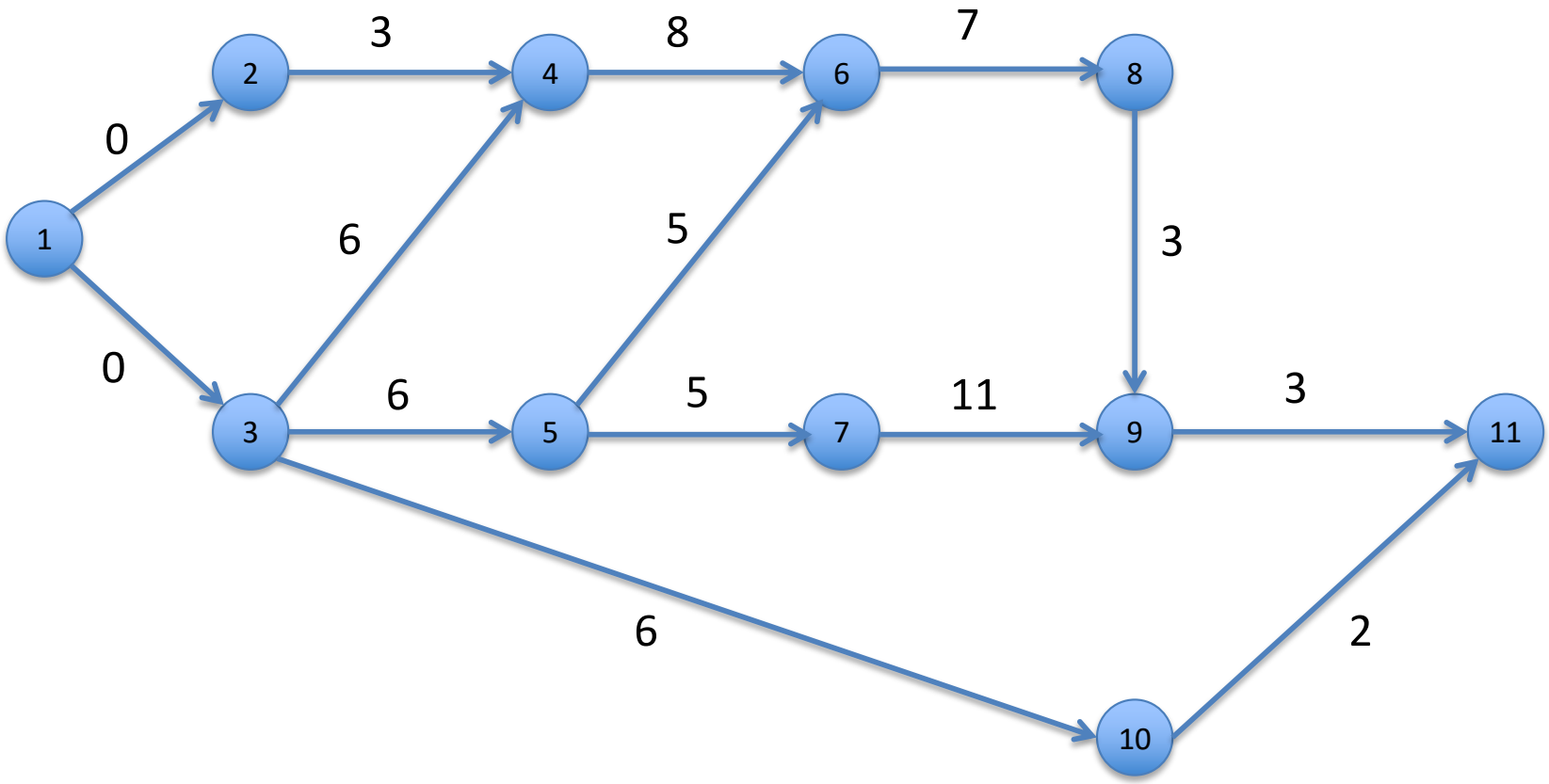
5: Apply the numbering algorithm



Vertex:	s	A	B	C	D	E	F	G	H	I	t
Number:	1	2	3	4	6	5	7	8	9	10	11

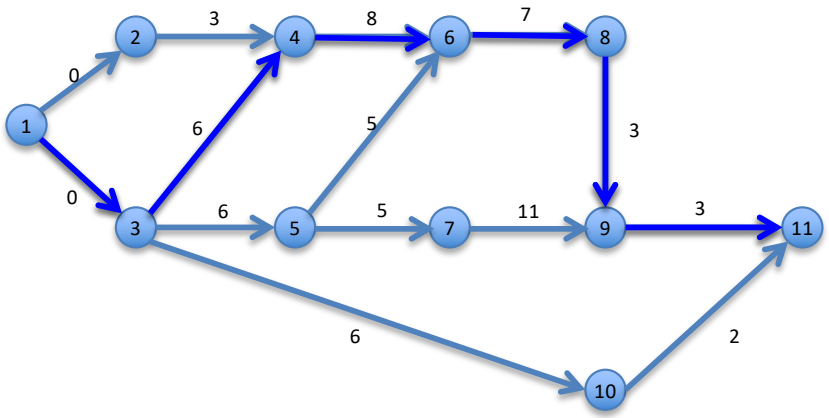
Exercices

New numbering



Exercices

Bellman



$u_1=0,$

$u_2=w_{12}=0$

$u_3=w_{13}=0$

$u_4=\max\{u_2+w_{24}\,,\ u_3+w_{34}\}=\max\{0+3,\ 0+6\}=6$

$u_5=\max\{u_3+w_{35}\}=\max\{0+6\}=6$

$u_6=\max\{u_4+w_{46}\,,\ u_5+w_{56}\}=\max\{6+8,\ 6+5\}=14$

$u_7=\max\{u_5+w_{57}\}=\max\{6+5\}=11$

$u_8=\max\{u_6+w_{68}\}=\max\{14+7\}=21$

$u_9=\max\{u_7+w_{79}\,,\ u_8+w_{89}\}=\max\{11+11,\ 21+3\}=24$

$u_{10}=\max\{u_3+w_{3,10}\}=\max\{0+6\}=6$

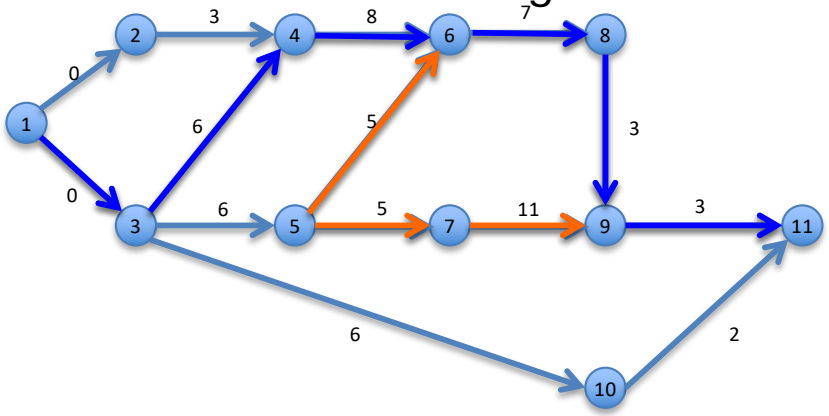
$u_{11}=\max\{u_9+w_{9,11}\,,\ u_{10}+w_{10,11}\}=\max\{24+3,\ 6+2\}=27$

Critical path

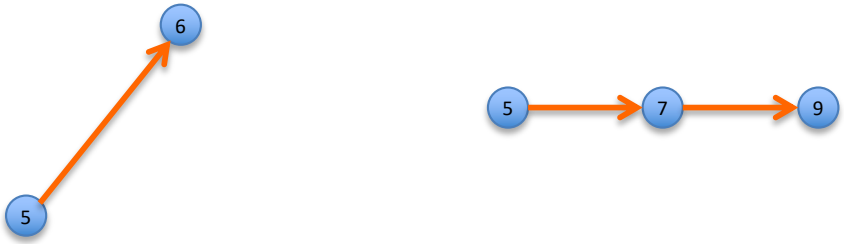
Critical path (new numbering)	1	3	4	6	8	9	11
Original critical path	s	B	C	D	G	H	t

Exercices

Compute the maximum delay allowed for task a7 without affecting the duration of the entire project.



Consider the different paths that link the activity E (5 in the new numbering) with the critical path.



Suppose activity E is delayed x days. The two previous paths must satisfy:

$$\begin{cases} u_5 + 5 + x \le u_6 \\ u_5 + 5 + 11 + x \le u_9 \end{cases} \quad \begin{cases} 11 + x \le 14 \\ 22 + x \le 24 \end{cases} \quad \begin{cases} x \le 3 \\ x \le 2 \end{cases} \rightarrow \boxed{x \le 2}$$

Maximum delay allowed for activity E: 2 days