The Traveler's Problem

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In this work we present the Traveler's Problem (TP), a computational task whose extensions and variations are often encountered by travelers around the world. The task is concerned with creating a valid travel schedule, using airplanes as a means of transportation and in accordance with certain constraints specified by the traveler.

Each instance of TP consists of:

1. A set of airports. Airports represent locations a traveler can begin their commute in, visit as a desired destination, or connect in on the way to their destination.

2. A set of flights. Each flight includes arrival and departure airports, as well as a price and a date.

3. The starting point, which is a single airport chosen from the set of airports

4. A set of destinations the traveler wants to visit, which is a subset of the set of airports.

A solution to any instance of TP is a sequence s of valid flights, $f_1, f_2, ..., f_k$. We say that a sequence contains valid flights if the flights in the sequence have the following properties:

1. The departure airport of the first flight in s, f_1, is equal to the arrival airport of the last flight in s, f_k is equal to the starting point.

2. For any two consecutive flights in s: f_i, f_{i+1}, we have that the arrival airport of f_i is equal to the departure airport of f_{i+1}

3. The flights in s have dates, which occur in increasing, temporal order (flights earlier in the list happen earlier in time).

4. All destinations the traveler wants to visit are included in at least one flight from s.

An optimal solution is a solution, which minimises the total sum of flight's prices.

Any solution (including any optimal solution) may include airports a traveler has not specified on their list of destinations. Such airports are called connections.

We note that the specification of TP as presented above may not reflect requirements of many travelers. Indeed, travelers may have additional preferences (soft contstraints) and requirements (hard constraints) with regards to their travel. For example, travelers may require to spend a minimum number of days in a given destination, or trade off an additional day at a destination for increase in total travel price. Travelers may require to spend a given date at a given destination, for example due to an event occuring on that date in this destination. Additionaly, connections may be penalised by some users, perhaps with penalty proportional to the waiting time. The traveler may have a certain price threshold (budget), such that if the total travel cost exceeds that threshold the traveler will discard a solution as invalid.

It is therefore suggested that any attempt at an investigation of TP assumes as an additional non-functional requirement, that any proposed model to solve TP is flexible and can be easily extended by adding, removing and modifying soft and hard constraints.

We present an example instance of TP and its most optimal solution.

Example. Adam is on holiday from 1st of August until and including 14th of August. He wants to go to Milan Malpensa, Berlin Tegel, Amsterdam and Paris Orly. His starting point is Glasgow International. Adam can visit the following airports during his trip:

- Glasgow International (G)
- Berlin Tegel (B)
- Milan Malpensa (M)
- Amsterdam (A)
- Paris Orly (P)
- Frankfurt am Main (F)
- London Heathrow (L)

All available flights that arrive to and depart from these airports are listed on Table1.

Solution. A valid solution of the TP instance in the example above is the sequence of flights s, where the each flight is represented by its flight number, specified in the first column of Table 1:

$$s = \{GA1, AP4, PM6, MF9, FB11, BG13\}$$

The total sum of the prices of the flights in s is £699.

Flight No	**Departs**	**Arrives**	**Date**	**Price (£)**
GA1	G	A	1 August	74
GF1	G	F	1 August	86
GL3	G	L	3 August	25
AP4	A	P	4 August	58
LG14	L	G	14 August	24
PL12	P	L	12 August	45
PM6	P	M	6 August	71
MF9	M	F	9 August	39
FB11	F	B	11 August	122
FM8	F	M	8 August	234
BG13	B	G	13 August	335
BL13	B	L	13 August	102

Table 1: List of flights with departure and arrival airports, flight date and price

Optimal Solution. The optimal solution of the example TP instance is the sequence of flightd s', where the each flight is represented by its flight number, specified in the first column of Table 1:

$$s' = \{GA1, AP4, PM6, MF9, FB11, BL13, LG14\}$$

The total sum of the prices of the flights in s' is £483.

s' is the optimal solution, as the total cost of the flights is minimised. In both s and s', airport F is connection. s' used a second connection, that is L.

In this work, we present the traveler's problem (TP). An instance of the problem serves as a worked example, for which two possible solutions are given, one of which is optimal. We have not found any extensive study on TP and its variations in the existing literature. An investigation of this problem could be of both theoretical and practical interest.