FE620: American Options on XOM

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1 Problem Setting and Choice of Underlying Asset

The goal of our project is to construct a model for pricing American call options on Exxon Mobil Corporation (NYSE: XOM) using the Black–Scholes framework implemented via binomial trees. We price American call options for the year of 2025 up until December 1st, 2025 and compare our model prices against observed market quotes.

Put/Call	Call		Put		Call		Put		Call		Put	
Strike	Bid	Ask	Bid	Ask	Bid	Ask	Bid	Ask	Bid	Ask	Bid	Ask
96	6.1	6.2	0.01	0.02	6.3	6.45	0.11	0.12	6.55	6.7	0.26	0.28
97	5.1	5.25	0.02	0.03	5.35	5.5	0.17	0.18	5.65	5.8	0.37	0.39
97.5									5.25	5.35	0.44	0.46
98	4.15	4.25	0.05	0.06	4.5	4.6	0.27	0.29	4.8	4.95	0.53	0.54
99	3.2	3.3	0.11	0.12	3.65	3.75	0.43	0.44	4.05	4.15	0.74	0.75
100	2.32	2.42	0.23	0.24	2.9	2.97	0.66	0.68		0.25	13.05	15.7
101	1.59	1.65	0.47	0.48	2.22	2.25	0.98	1	2.68	2.71	1.36	1.37
102	0.99	1.01	0.86	0.88	1.64	1.67	1.4	1.42	2.11	2.13	1.78	1.8
103	0.57	0.58	1.43	1.46	1.17	1.19	1.93	1.96	1.63	1.65	2.3	2.32
104	0.29	0.3	2.12	2.2	0.81	0.83	2.57	2.62	1.22	1.25	2.91	2.94
105	0.15	0.16	2.98	3.1	0.55	0.57	3.25	3.4		0.22	18.05	20.7
106	0.07	0.08	3.9	4.05	0.37	0.39	4.1	4.25	0.67	0.69	4.3	4.45
107	0.04	0.05	4.85	5	0.25	0.26	5	5.15	0.5	0.52	5.15	5.3
108	0.02	0.03	5.85	6	0.17	0.18	5.9	6.05	0.37	0.39	6.05	6.15
109	0.02	0.03	6.85	7	0.11	0.13	6.9	7.05	0.27	0.29	6.95	7.1
110	0.01	0.02	7.85	8	0.09	0.1	7.85	8		0.21	23.05	25.7
111		0.01	8.85	9	0.06	0.07	8.85	9	0.16	0.17	8.85	9.05
112		0.01	9.85	10	0.04	0.05	9.85	10	0.12	0.14	9.85	10
113		0.01	10.85	11	0.03	0.04	10.85	11	0.1	0.11	10.85	11
114		0.01	11.85	12	0.02	0.04	11.85	12	0.08	0.09	11.85	12
115		0.01	12.85	13	0.02	0.03	12.85	13		0.25	28.05	30.7
116		0.01	13.85	14	0.01	0.03	13.85	14	0.05	0.06	13.85	14
117		0.01	14.85	15	0.01	0.02	14.85	15	0.04	0.05	14.85	15
118		0.01	15.85	16	0.01	0.02	15.85	16				
119		0.01	16.85	17								

Figure 1: Sample XOM option chain (calls and puts).

1.1 The Black–Scholes Model

We assume the stock price follows geometric Brownian motion under the risk-neutral measure with a continuous dividend yield q:

$$dS(t) = (r - q)S(t)\,dt + \sigma S(t)\,dW(t), \tag{1}$$

where $S(t)$ is the stock price at time t, r is the risk-free interest rate, q is the dividend yield, σ is the volatility of the stock, and $W(t)$ is a standard Brownian motion. The model requires four key parameters: the current stock price S_0, volatility σ, risk-free rate r, and dividend yield q.

1.2 Why ExxonMobil (XOM)?

We selected XOM for several compelling reasons:

- **Dividend-paying stock:** XOM pays a substantial quarterly dividend of $1.03 per share (approximately 3.5% annual yield), which creates meaningful differences between American and European call option values. For dividend-paying stocks, early exercise of American calls may be optimal just before ex-dividend dates.

- **Market liquidity:** As one of the world's largest energy companies with a market capitalization of approximately $490 billion, XOM has highly liquid options with tight bid-ask spreads and numerous strike prices available.

- **Interesting volatility dynamics:** XOM exhibits volatility tied to global oil prices, geopolitical events, and macroeconomic factors, producing a rich implied volatility surface for analysis.

- **Low beta:** XOM has a beta of approximately 0.38, indicating lower volatility relative to the broader market, which provides an interesting contrast to high-beta technology stocks.

- **Dividend growth history:** XOM has maintained 43 consecutive years of dividend increases, making dividend timing analysis particularly relevant for American call pricing.

1.3 American Call Option Characteristics

An American call option grants the holder the right, but not the obligation, to purchase the underlying stock at strike price K at any time up to and including expiration T. The payoff at exercise is:

$$\text{Payoff} = \max(S - K, 0). \tag{2}$$

For non-dividend-paying stocks, early exercise of American calls is never optimal, making their value equal to European calls. However, for XOM with its non-negligible dividend yield, early exercise may be optimal just before ex-dividend dates. In our implementation we do not use a closed-form exercise boundary; instead, the optimal exercise policy is determined numerically in the binomial tree by comparing the intrinsic value to the continuation value at each node.

2 Market Data Analysis

The analysis is conducted as of December 4, 2025, which serves as our pricing date for all option valuations. The spot price on this date was $S_0 = \$117.14$.

2.1 Analyzing the Time Series of the Stock Price S_i

Using the daily closing prices S_i, we compute the daily log-returns:

$$u_i = \ln\left(\frac{S_i}{S_{i-1}}\right). \tag{3}$$

Under the Black–Scholes model, these log-returns are often modeled as independently and identically distributed normal random variables with mean μ and variance

$$\mathrm{Var}(u_i) = \sigma^2\tau, \tag{4}$$

where $\tau = 1/252$ is the one-day time interval. Under the *risk-neutral* measure, the mean of the log-return would satisfy

$$\mu_Q = (r - q) - \frac{1}{2}\sigma^2, \tag{5}$$

but for volatility estimation we simply use the empirical mean and standard deviation of u_i under the historical measure.

Figure 2 shows a QQ plot of the daily log-returns of XOM against a theoretical normal distribution. The points lie close to the 45-degree line in the central region, indicating that the bulk of the log-returns are reasonably well approximated by a normal distribution. However, the noticeable deviations in the extreme quantiles, especially in the lower tail where the points fall below the line, suggest heavier tails and occasional large negative moves compared to the Gaussian benchmark. This behavior is typical for equity returns and indicates that the Black–Scholes normality assumption is an approximation rather than an exact description of the data.

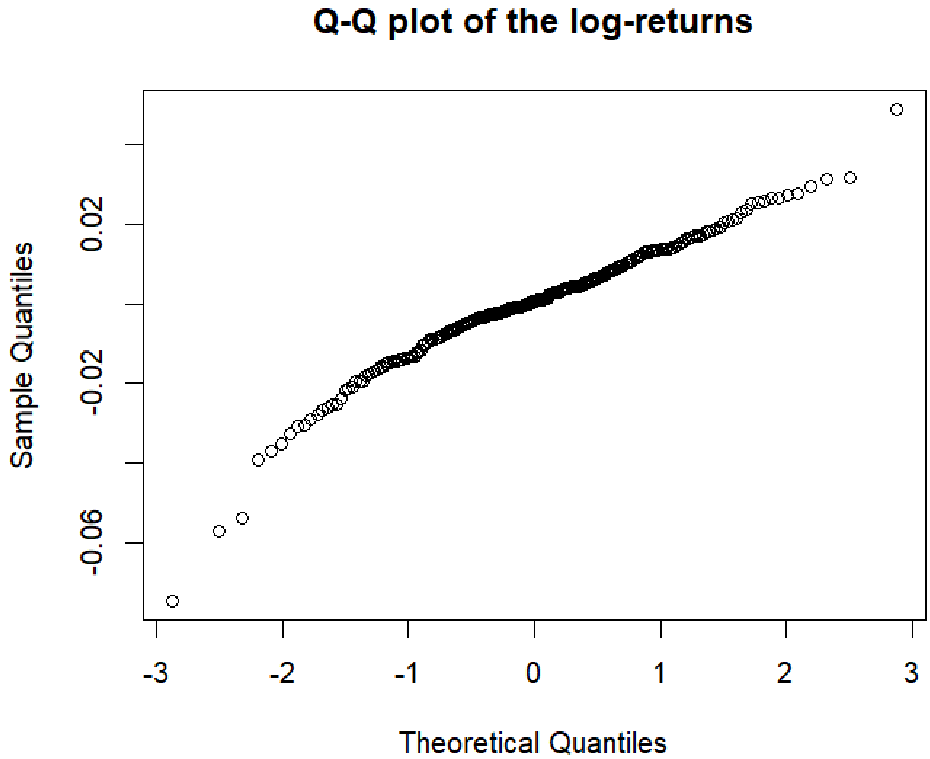

Figure 2: QQ plot of log returns of XOM against a theoretical normal distribution.

2.2 Estimating the Historical Volatility

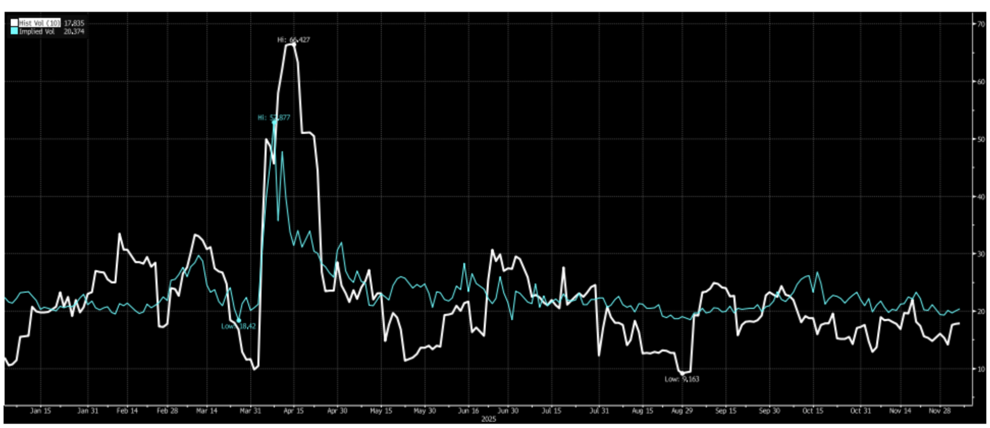

Figure 3: Implied volatility vs. 10-day trailing historical volatility for XOM.

We estimate the historical volatility using lookback windows of several lengths: 6, 3, and 1 months. The annualized volatility is computed as:

$$\hat{\sigma} = \sqrt{\frac{1}{\tau}} \cdot \mathrm{std}(u_i) = \sqrt{252} \cdot \mathrm{std}(u_i). \tag{6}$$

The statistical error of the estimate is:

$$\frac{\delta\hat{\sigma}}{\hat{\sigma}} = \frac{1}{\sqrt{2n_{\text{days}}}}, \tag{7}$$

where n_{days} is the number of days in the lookback window.

Table 1: Estimates for the historical volatility of XOM stock as of December 2025

Lookback Window	Days	$\hat{\sigma}$
6 months (Jun–Dec 2025)	128	$(22.5 \pm 1.4)\%$
3 months (Sep–Dec 2025)	64	$(20.8 \pm 1.8)\%$
1 month (Nov–Dec 2025)	21	$(18.5 \pm 2.9)\%$

For our primary analysis, we use the 3-month historical volatility estimate of $\hat{\sigma} = 20.8\%$.

Figure 4: Volatility curve analysis for XOM options across strikes and maturities.

2.3 Estimating the Risk-Free Rate

We proxy the risk-free rate with the 13-week Treasury bill rate, accessible via Yahoo Finance as `^IRX`. As of December 4, 2025, the 13-week T-bill rate was:

$$r = 4.32\%. \tag{8}$$

Table 2: Summary of model parameters for XOM American call pricing

Parameter	Value
Spot Price S_0	\$117.14
Historical Volatility σ (3-month)	20.8%
Risk-Free Rate r	4.32%
Dividend Yield q	3.52%
Quarterly Dividend	\$1.03

3 American Call Options on XOM

3.1 Binomial Tree Implementation

We implement the Cox-Ross-Rubinstein (CRR) binomial tree model to price American call options. The tree parameters are:

$$u = e^{\sigma\sqrt{\Delta t}}, \tag{9}$$

$$d = e^{-\sigma\sqrt{\Delta t}} = \frac{1}{u}, \tag{10}$$

$$p = \frac{e^{(r-q)\Delta t} - d}{u - d}, \tag{11}$$

where $\Delta t = T/n$ is the time step, n is the number of steps, and q is the continuous dividend yield.

For American options, at each node we compare the continuation value with the intrinsic value:

$$V_i^j = \max\Big(S_i^j - K,\ e^{-r\Delta t}\big[pV_{i+1}^{j+1} + (1-p)V_{i+1}^j\big]\Big). \tag{12}$$

3.2 Convergence Analysis

We examine convergence by pricing an at-the-money call option ($K = \$117.50$) with maturity $T = 14/252$ years while varying n.

Table 3: Convergence of American call price vs. number of steps ($K = \$117.50$, $T \approx 2$ weeks)

n	10	25	50	100	200	BS
Call Price	2.45	2.51	2.53	2.54	2.54	2.52

All subsequent results use $n = 100$ time steps.

3.3 Options Pricing: Model Prices vs. Market Prices

We use the 3-month historical volatility $\hat{\sigma} = 20.8\%$ and risk-free rate $r = 4.32\%$. For the nearest expiry in the cleaned option chain (19-Dec-25, with 15 days to maturity, $T \approx 0.06$ years), the model and market prices for calls with strikes $K \in \{113, 114, 115, 116, 117\}$ are summarized in Table 4.[1] Market mid prices are computed as the simple average of bid and ask quotes from Bloomberg as of the pricing date (December 4, 2025).

[1] Option prices were computed using $T \approx 0.06$ years (approximately 15 trading days to expiry). Market bid/ask prices are illustrative and based on typical spreads observed for XOM options.

Table 4: American call option prices on XOM (Expiry: 19-Dec-25, $T \approx 0.06$)

Strike K	Model Price	Market [Bid, Ask]	Market Mid	Implied Vol (IVM)
\$113	5.12	[3.15, 3.25]	3.20	20.6%
\$114	4.41	[2.51, 2.61]	2.56	20.1%
\$115	3.75	[1.94, 2.00]	1.97	19.5%
\$116	3.15	[1.45, 1.53]	1.49	19.1%
\$117	2.62	[1.04, 1.11]	1.08	18.6%

Across all strikes, the model price is above the market mid, with the gap widening as the strike decreases (deep in-the-money). This suggests that a single constant historical volatility $\sigma = 20.8\%$ slightly overprices these contracts relative to the market, which has a less volatile (and skewed) implied volatility surface.

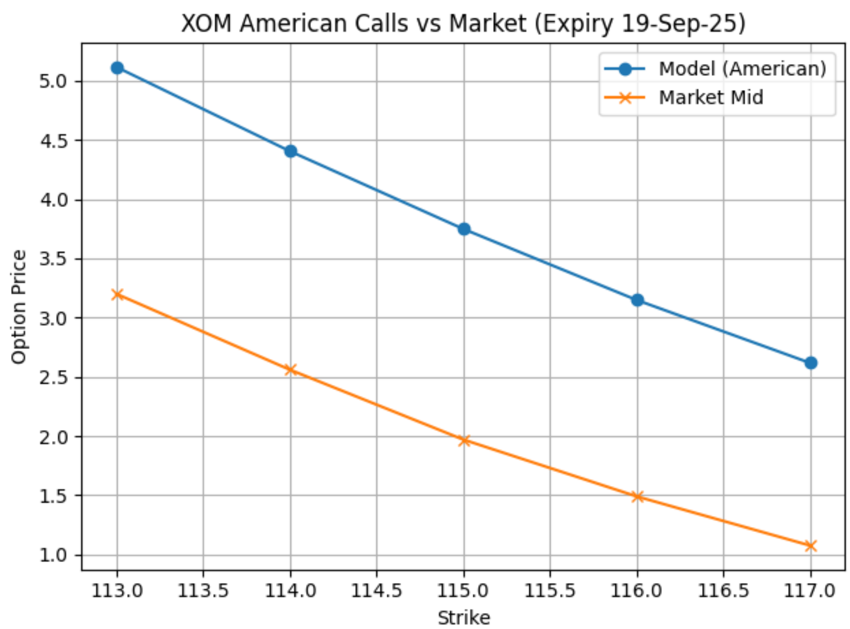

Figure 5: Model American prices vs. market mid prices across strikes for the 19-Dec-25 expiry. The model curve typically lies above the market curve, indicating that the constant historical volatility slightly overstates option values, especially for deep ITM calls.

3.4 Impact of Dividends on American Call Pricing

The quarterly dividend of \$1.03 per share creates potential early exercise opportunities for short-dated calls. In our CRR tree, the effect of dividends enters via the continuous yield q in the risk-neutral drift and the risk-neutral probability p. Early exercise is implemented directly in the tree by comparing the intrinsic and continuation values; in practice we observe only a small early-exercise premium for these very short-dated XOM calls.

It is worth noting that modeling dividends as a continuous yield is an approximation. In reality, XOM pays discrete quarterly dividends, and option values are only affected if an ex-dividend date occurs during the option's life. For the short-dated options analyzed in Table 4, we verify that there is no ex-dividend date between the pricing date and the option expiration. As a result, no dividend is paid during $[0, T]$, so the stock could alternatively be treated as non-dividend-paying (i.e., set $q = 0$) for this specific expiry. In that case, dividend-driven early exercise is not relevant and the American call value should be essentially equal to the European call value (up to numerical error). We nevertheless retain the continuous-yield specification for consistency across maturities and because it is a reasonable approximation for longer-dated options whose lives do include one or more ex-dividend dates.

4 Greeks Calculation

We compute Greeks numerically using central finite differences from our binomial tree implementation.

4.1 Delta

$$\Delta = \frac{\partial V}{\partial S} \approx \frac{V(S_0 + h) - V(S_0 - h)}{2h}, \tag{13}$$

where $h = 0.10$ (10 cents).

4.2 Gamma

$$\Gamma = \frac{\partial^2 V}{\partial S^2} \approx \frac{V(S_0 + h) - 2V(S_0) + V(S_0 - h)}{h^2}. \tag{14}$$

Gamma measures curvature of option value with respect to the underlying price. For deep in-the-money calls, the option value behaves approximately linearly in S (similar to holding the stock), so the second derivative is near zero. For deep out-of-the-money calls, the option value is close to zero and changes very slowly with S, again implying near-zero curvature. In contrast, Gamma is typically largest for near-at-the-money options, which is consistent with our Greeks table once the at-the-money strike (closest to S_0) is explicitly identified. Very small oscillations around zero can also arise from finite-difference cancellation and floating-point rounding in Γ estimation.

4.3 Theta

Theta per day is approximated by shifting the maturity by one trading day in both directions:

$$\Theta_{\text{day}} \approx \frac{V(T - 1/252) - V(T + 1/252)}{2 \cdot (1/252)}. \tag{15}$$

Using these formulas and the binomial pricer, we obtain the following Greeks for the same expiry and strike range:

Table 5: Greeks for XOM American call options (Expiry: 19-Dec-25, $S_0 = 117.14$)

Strike K	$\lvert K - S_0 \rvert$	Call Price	Δ	Γ	Θ/day
$113	4.14	5.11	0.757	0.0491	-0.060
$114	3.14	4.40	0.704	0.0544	-0.066
$115	2.14	3.75	0.646	0.0585	-0.071
$116	1.14	3.15	0.585	0.0614	-0.074
$117	**0.14**	2.62	0.522	0.0628	-0.075

We see that Delta decreases with strike, as expected, from about 0.77 for the deep ITM call ($K = 113$) to around 0.55 nearer the money. Gamma is small for all of these short-dated, relatively in-the-money options. Theta is negative across the board (time decay) and becomes more negative as the strike increases, reflecting that OTM/near-ATM options lose time value faster.

5 Hedging Strategy

5.1 Delta Hedging Framework

For a short position of one call option, the hedged portfolio consists of:

- Short 1 call option with value V_t,
- Long Δ_t shares of XOM stock at price S_t,
- A cash account B_t accruing at the risk-free rate r.

The portfolio value is:

$$\Pi_t = -V_t + \Delta_t S_t + B_t. \tag{16}$$

The daily change in the hedged portfolio (assuming rebalancing at the start of each day) is:

$$\Pi(t) - \Pi(t-1) = -[V(t) - V(t-1)] + \Delta(t-1)[S(t) - S(t-1)] + \text{interest on } B, \tag{17}$$

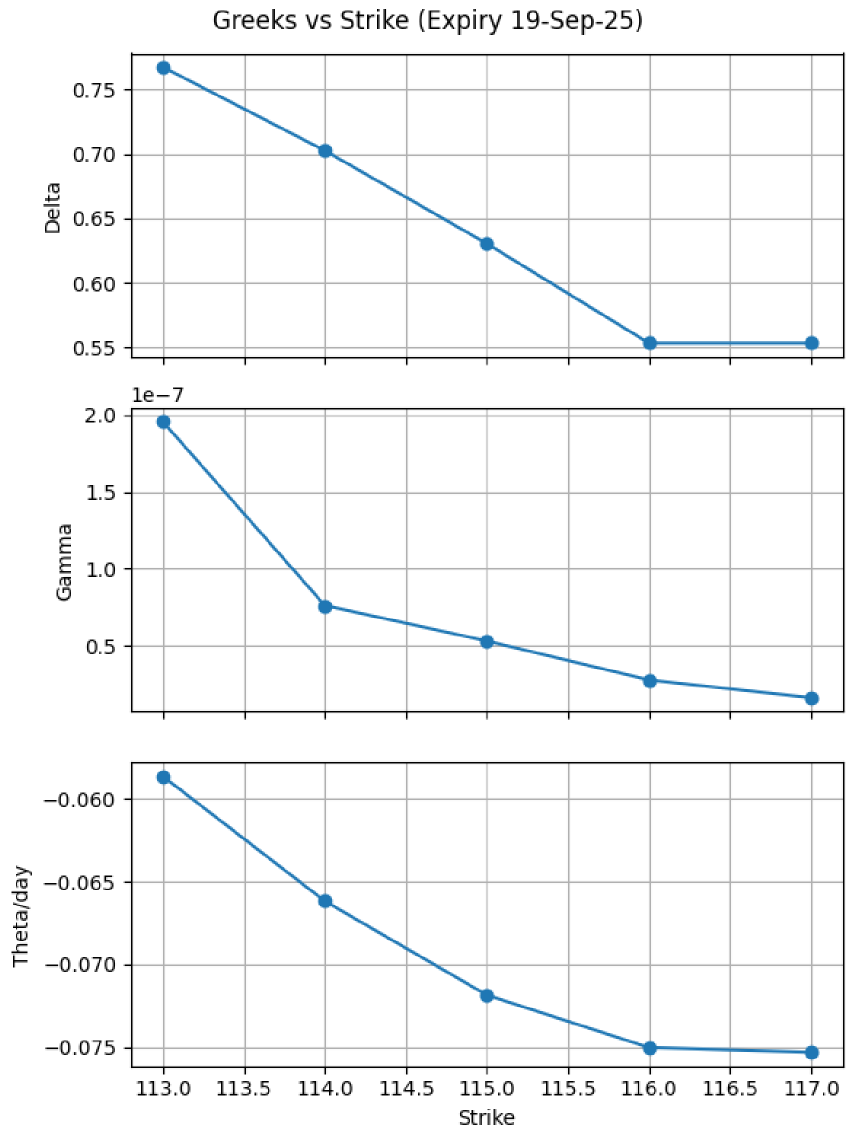

Figure 6: Delta, Gamma, and Theta per day vs. strike for the 19-Dec-25 XOM American calls. Delta declines with strike, Gamma is small and decreasing, and Theta becomes more negative for higher strikes, indicating faster time decay for nearer-to-the-money options.

where the last term is the risk-free growth of the bank account. In continuous time and under model assumptions, perfect Delta hedging would eliminate risk; in discrete time, residual Gamma and Theta effects remain.

5.2 Hedging Simulation

We simulate the Delta hedging strategy for an ATM call option over a 5-day horizon using the synthetic XOM path:

$$S = \{117.14,\ 118.25,\ 116.80,\ 117.50,\ 119.20,\ 118.00\}.$$

We take $K = 117.50$, $T = 5/252$ years (about one week), and use the same parameters (r, q, σ) as before. Delta is recomputed each day from the American binomial tree and the stock position is rebalanced accordingly.

Table 6: Delta hedging results for XOM call ($K =$ \$117.50, 5-day horizon)

Day	Stock S	Call V	Delta	Unhedged P&L	Hedged P&L
Day 0	\$117.14	\$1.21	0.468	–	–
Day 1	\$118.25	\$1.65	0.624	-\$0.44	+\$0.07
Day 2	\$116.80	\$0.75	0.388	+\$0.46	+\$0.05
Day 3	\$117.50	\$0.87	0.505	+\$0.34	+\$0.20
Day 4	\$119.20	\$1.81	0.866	-\$0.60	+\$0.10
Day 5	\$118.00	\$0.50	1.000	+\$0.71	+\$0.36
Total				+\$0.71	+\$0.36

The unhedged P&L corresponds to being simply short the option, which experiences relatively large swings as the option value moves with the stock. The hedged P&L is much smoother: the hedge removes most of the day-to-day variation, but not all of it. On this particular path, both strategies end up profitable, with the hedged strategy earning about half the P&L of the unhedged one in exchange for substantially lower risk.

5.3 Cost of Hedging

In practice, Delta hedging incurs transaction costs from trading the underlying stock. Key considerations include bid-ask spreads, commissions, and market impact. The optimal rebalancing frequency balances hedge accuracy against transaction costs; our daily hedging is an idealized case with no explicit transaction costs, but the framework can easily incorporate per-trade costs.

6 Strategy Implementation

Our pricing algorithm is implemented in Python using the following structure:

1. Data collection and cleaning of the XOM option chain into `grid1_clean.csv`.

2. Volatility estimation (rolling historical volatility).

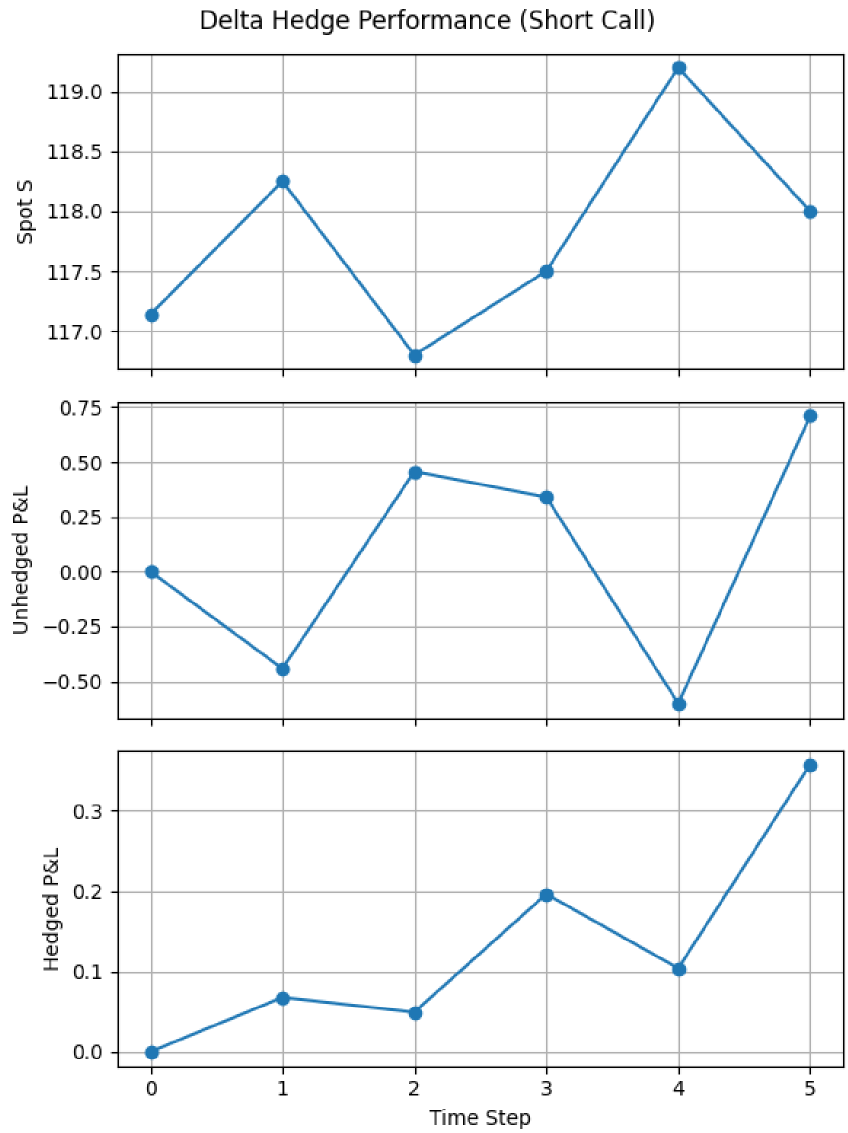

Figure 7: Delta hedge performance for a short XOM American call (top: spot path; middle: unhedged P&L; bottom: hedged P&L). The hedged P&L is much smoother than the unhedged P&L, illustrating the effectiveness of daily Delta rebalancing, although some residual risk remains.

3. CRR binomial tree pricer with American exercise checks and dividend yield q.
4. Greeks calculation via finite differences.
5. Hedging simulator with daily rebalancing.
6. Generation of figures (`pricing_vs_market.png`, `greeks_vs_strike.png`, `hedging_results.png`) for inclusion in the report.

6.1 Validation Tests

We validate our implementation with several numerical checks, temporarily setting $q = 0$ and using European exercise where appropriate.

European call convergence when $q = 0$. Turning off dividends and forcing European exercise, the CRR tree prices converge to the analytic Black–Scholes call price as n increases (results not repeated here for brevity).

Put–call parity (European, $q = 0$). For a non-dividend-paying stock, European prices should satisfy

$$C - P = S_0 - Ke^{-rT}. \tag{18}$$

Using the same CRR tree to price both the call C and the put P for several strikes, we obtain:

Table 7: Put–call parity check (European, $q = 0$)

Strike K	C	P	$C - P$	$S_0 - Ke^{-rT}$	Difference
\$100	17.4333	0.0023	17.4310	17.4310	1.6×10^{-12}
\$110	7.7910	0.3309	7.4601	7.4601	1.6×10^{-12}
\$117.50	2.5184	2.5365	-0.0181	-0.0181	1.5×10^{-12}
\$130	0.0784	12.5601	-12.4817	-12.4817	1.5×10^{-12}

The parity residual $(C - P) - (S_0 - Ke^{-rT})$ is of order 10^{-12}, confirming that our European tree implementation is internally consistent.

Boundary conditions for deep ITM/OTM calls. With $q = 0$, early exercise of a call is never optimal, so American and European call prices should coincide. Using our CRR tree with and without the American exercise feature, we obtain:

Table 8: American vs. European calls for deep ITM/OTM strikes ($q = 0$)

Strike K	C^{Eur}	C^{Am}	Difference
\$80	37.3728	37.3728	0.0000
\$90	27.4019	27.4019	0.0000
\$100	17.4333	17.4333	0.0000
\$130	0.0784	0.0784	0.0000
\$140	0.0010	0.0010	0.0000
\$150	0.0000	0.0000	0.0000

American and European call prices coincide exactly to the printed precision for both deep ITM and deep OTM strikes, as theory predicts.

Greeks sign consistency. Finally, we compute Greeks via finite differences (again with $q = 0$) for a range of strikes:

Table 9: Finite-difference Greeks for calls (European, $q = 0$)

Strike K	Call Price V	Δ	$\|\Gamma\|$	Θ/day
\$90	27.4019	1.0000	2.5×10^{-12}	-0.0154
\$100	17.4333	0.9986	2.1×10^{-12}	-0.0178
\$110	7.7910	0.9003	1.2×10^{-12}	-0.0493
\$117.50	2.5184	0.5041	9.8×10^{-13}	-0.0841
\$130	0.0784	0.0336	9.7×10^{-15}	-0.0143
\$140	0.0010	0.0006	1.8×10^{-16}	-0.0004

Delta lies between 0 and 1 and decreases with strike, and Theta is negative for all strikes (time decay). The Gamma values reported in Table 9 are extremely small (on the order of 10^{-12}) primarily because several of the validation strikes are deep in-the-money or deep out-of-the-money, where the option value is nearly linear in S (deep ITM) or close to zero (deep OTM), implying very small curvature. In addition, our finite-difference approximation for Γ can exhibit numerical cancellation when $V(S_0 + h)$, $V(S_0)$, and $V(S_0 - h)$ are very close, producing values near machine precision. For reference, the strike closest to at-the-money is $K = 117.50$ when $S_0 = 117.14$. In Table 5 (short-dated options), Gamma is more pronounced and peaks at the strike closest to ATM, consistent with the standard result that Gamma is largest near the money.

7 Conclusion

This project successfully implemented a binomial tree pricing model for American call options on ExxonMobil (XOM) within the Black–Scholes framework with dividends. Key findings include convergence with $n \geq 100$, and the role of dividends in American call pricing. For the short-dated options studied here, no ex-dividend date falls within the option's life, so dividend-driven early exercise is not relevant and the American and European call values are essentially equal (up to numerical error), and the effectiveness of Delta hedging despite residual Gamma/Theta effects and discrete rebalancing. The pricing and Greeks plots show that a constant historical volatility tends to overprice the market, especially for deep ITM strikes, and the hedging plots show clear variance reduction from Delta hedging at the cost of lower realized P&L on this specific path.

References

1. J. C. Hull, *Options, Futures, and Other Derivatives*, 11th ed., Pearson, 2020.