Numerička matematika

1. Lagrangeov oblik IP

$x_0 = -1, \; y_0 = 1$
$x_1 = 1, \; y_1 = -1$
$x_2 = 2, \; y_2 = 13$
$x_3 = 3, \; y_3 = 69$
$x_4 = 4, \; y_4 = 221$

$$p_0(x) = \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)}$$

$$= \frac{(x-1)(x-2)(x-3)(x-4)}{(-1-1)(-1-2)(-1-3)(-1-4)}$$

$$= \frac{(x^2-3x+2)(x^2-7x+12)}{(+2)(-3)(-4)(-5)} = \frac{x^4-7x^3+12x^2-3x^3+21x^2-36x+2x^2-14x+24}{120}$$

$$= \frac{x^4-10x^3+35x^2-50x+24}{120}$$

$$p_1(x) = \frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)}$$

$$= \frac{(x+1)(x-2)(x-3)(x-4)}{(1+1)(1-2)(1-3)(1-4)}$$

$$= \frac{(x^2-x-2)(x^2-7x+12)}{(2)(-1)(-2)(-3)} = \frac{x^4-7x^3+12x^2-x^3+7x^2-12x-2x^2+14x-24}{-12}$$

$$= \frac{x^4-8x^3+17x^2+2x-24}{-12}$$

$$p_2(x) = \frac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)}$$

$$= \frac{(x+1)(x-1)(x-3)(x-4)}{(2+1)(2-1)(2-3)(2-4)}$$

$$= \frac{(x^2-1)\cdot(x^2-7x+12)}{3\cdot 1\cdot(-1)\cdot(-2)} = \frac{x^4-7x^3+12x^2-x^2+7x-12}{6}$$

$$= \frac{x^4-7x^3+11x^2+7x-12}{6}$$

$$p_3(x) = \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)} = \frac{(x+1)(x-1)(x-2)(x-4)}{(3+1)(3-1)(3-2)(3-4)}$$

$$= \frac{(x^2-1)(x^2-6x+8)}{4\cdot 2\cdot 1\cdot(-1)} = \frac{x^4-6x^3+8x^2-x^2+6x-8}{-8}$$

$$= \frac{x^4-6x^3+7x^2+6x-8}{-8}$$

$$P_4(x) = \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_0)(x_4-x_1)(x_4-x_2)(x_4-x_3)}$$

$$P_4(x) = \frac{(x+1)(x-1)(x-2)(x-3)}{(4+1)(4-1)(4-2)(4-3)}$$

$$P_4(x) = \frac{(x^2-1)(x^2-5x+6)}{5\cdot 3\cdot 2\cdot 1} = \frac{x^4-5x^3+6x^2-x^2+5x-6}{30}$$

$$P_4(x) = \frac{x^4-5x^3+5x^2+5x-6}{30}$$

$$P_4(x) = 1\cdot\frac{1}{120}(x^4-10x^3+35x^2-50x+24) - 1\cdot\frac{1}{12}\cdot(x^4-8x^3+17x^2+2x-24)$$
$$+13\cdot\frac{1}{6}(x^4-7x^3+14x^2+7x-12) + 69\cdot\frac{1}{-8}(x^4-6x^3+7x^2+6x-8)$$
$$+281\cdot\frac{1}{30}\cdot(x^4-5x^3+5x^2+5x-6)$$

$$P_4(x) = x^4 - x^3 + 2x^2 - 3$$

11. Newtonov obuk IP

$x_0 = -1 \quad y_0 = 1$
$x_1 = 1 \quad y_1 = -1$
$x_2 = 2 \quad y_2 = 13$
$x_3 = 3 \quad y_3 = 69$
$x_4 = 4 \quad y_4 = 221$

x_k	$f[-1]$	$f[-1,1]$	$f[-1,1,2]$	$f[-1,1,2,3]$	$f[-1,1,2,3,4]$
0	-1	1			
			-1		
1	1	-1		5	
			14		4
2	2	13		21	
			56		9
3	3	69		48	
			152		
4	4	221			

$$P_4(x) = 1 + (-1)(x+1) + 5(x+1)(x-1) + 4(x+1)(x-1)(x-2) + 1(x+1)(x-1)(x-2)(x-3)$$

$$= 1 - x - 1 + 5x^2 - 5 + 4(x^3 - 1)(x-2) + 1(x^2-1)(x^2-5x+6)$$

$$= -5 + 5x^2 - x + 4(x^3 - 2x^2 - x + 2) + (x^4 - 5x^3 + 6x^2 - x^2 + 5x - 6)$$

$$= -5 + 5x^2 - x + 4x^3 - 8x^2 - 4x + 8 + x^4 - 5x^3 + 5x^2 + 5x - 6$$

$$= x^4 - x^3 + 2x^2 - 3$$