Exercise 1. (A concrete JNF computation) Consider the matrix

$$A = \begin{pmatrix} 4 & 1 & 0 \\ -11 & -3 & -1 \\ 5 & 2 & 2 \end{pmatrix}.$$

I will tell you right now that the only eigenvalue of A is 1.

(a) Given the fact that the only eigenvalue of A is 1, find a basis for the 1-eigenspace of A. (This is a kernel computation, which you know how to do.)

(b) Based on your computation from part (a), how many Jordan chains will you need to find in order to span $\mathbb{R}^3$?

(c) (Optional) Write down a basis for the degree 2 generalized eigenspace of A.

(d) Choose a generalized eigenvector for A of maximum degree. Write down its Jordan chain.

(e) Find a basis for $\mathbb{R}^3$ consisting of Jordan chains. Let B be the matrix with those vectors as its columns. Compute B^{-1}.

(f) Write down the matrix of A with respect to this basis by computing $B^{-1}AB$. Notice that this is the JNF.

(g) If you were writing an exercise for someone else to compute a JNF, how would you come up with a matrix that makes it so the problem isn't trivial, but also so that it isn't too hard? Asking for a friend...