Opgave 2:

basistilfælde: $n=1$ $(x^1)' = 1 = 1 \cdot x^{1-1}$

Induktionstrin: Antag, for $n \geq 2$ $(x^{n-1})' = (n-1)\, x^{n-2}$

Vi vil gerne vise, $(x^n)' = n x^{n-1}$

$$(x^n)' = (x^{n-1} \cdot x)'$$
$$= \underline{(x^{n-1})' \cdot x} + (x^{n-1}) \cdot x' \quad \leftarrow \text{produktreglen}$$
$$= \underline{(n-1) \cdot x^{n-2}} \cdot x + (x^{n-1}) \cdot 1 \quad \leftarrow \text{brug antagelse.}$$
$$= n \cdot x^{n-1} \qquad \square$$

Opgave 3.

basistilfælde: $n=1$ $\sum_{k=1}^{1} (2k-1) = 1 = 1^2$

Induktionstrin: Antag for $n \geq 2$. $\sum_{k=1}^{n-1} (2k-1) = (n-1)^2$

Vi skal vise $\sum_{k=1}^{n} 2k-1 = n^2$.

$$\sum_{k=1}^{n} (2k-1) = \underline{\sum_{k=1}^{n-1} (2k-1)} + 2n-1$$
$$= \underline{(n-1)^2} + 2n-1 \quad \leftarrow \text{brug } \cancel{\text{induktions}} \text{ antagelse.}$$
$$= n^2 - 2n + 1 + 2n - 1$$
$$= n^2$$

Opgave 4:

$\boxed{B}$ $n=0$ $\ln(c^0) = 0 \cdot \ln(c)$

$\boxed{I}$ Antag for $n \geq 1$, $\ln(c^{n-1}) = (n-1)\ln(c)$, vi skal vise $\ln(c^n) = n\ln(c)$.

$$\ln(c^n) = \ln(c^{n-1} \cdot c)$$
$$= \underline{\ln(c^{n-1})} + \ln(c)$$
$$= \underline{(n-1)\ln(c) + \ln(c)} \quad \leftarrow \text{brug antagelse}$$
$$= n \ln(c)$$

Opgave 5.

$\boxed{B}$ $n=2$, $a\cdot(b_1+b_2) = a\cdot b_1 + ab_2$

$\boxed{I}$ Antag for $n\geq 3$. $a\cdot(b_1+b_2+\cdots+b_{n-1}) = a\cdot b_1+\cdots+ab_{n-1}$,

vi skal vise $a\cdot(b_1+b_2+\cdots+b_n) = a\cdot b_1+\cdots+a\cdot b_n$

$$
\begin{aligned}
& a(b_1+b_2+\cdots+b_n) \\
&= a((b_1+b_2+\cdots+b_{n-1})+b_n) \\
&= \underline{a\cdot(b_1+b_2+\cdots+b_{n-1})} + a\cdot b_n \\
&= \underline{a\cdot b_1+ab_2+\cdots} + a\cdot b_n
\end{aligned}
$$

← brug antagelse.

Opgave 6:

$\boxed{B}$ $n=1$ $1+r^1 = \frac{r^{1+1}-1}{r-1}$

$\boxed{I}$ Antag for $n\geq 2$, $1+r+\cdots+r^{n-1} = \frac{r^n-1}{r-1}$.

Vi skal vise $1+r+\cdots+r^n = \frac{r^{n+1}-1}{r-1}$.

$$
\begin{aligned}
(1+r+\cdots+r^{n-1})+r^n &= \frac{r^n-1}{r-1}+r^n \\
&= \frac{r^n-1+r^{n+1}-r^n}{r-1} \\
&= \frac{r^{n+1}-1}{r-1}
\end{aligned}
$$

← brug antagelse

Opgave 7:

$\boxed{B}$ $n=1$ $0<2$

$\boxed{I}$ Antag for $n\geq 2$ $\cancel{\sqrt{2+\sqrt{2+\sqrt{2}}}}$ $a_{n-1}<2$

Vi skal vise $a_n<2$

$a_n = \sqrt{2+a_{n-1}} < \sqrt{2+2} = 2$.

Opgave 8.

$\frac{1}{k(k+1)} = \frac{1}{k} - \frac{1}{k+1}$

$\sum_{k=1}^{n} \frac{1}{k(k+1)} = \left(\frac{1}{1}-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\cdots+\left(\frac{1}{n}-\frac{1}{n+1}\right) = 1-\frac{1}{n+1}$