Uge 11

November 11, 2023

[ ]:
```python
from sympy import *
init_printing()
```

1 Opgave 5

1.1 a)

[ ]:
```python
A = Matrix([
    [2, 2],
    [-1, 4]
])
A
```

[ ]: $\begin{bmatrix} 2 & 2 \\ -1 & 4 \end{bmatrix}$

[ ]:
```python
v1 = Matrix([1 - I, 1])
v2 = Matrix([1 + I, 1])
v1, v2
```

[ ]: $\left(\begin{bmatrix} 1-i \\ 1 \end{bmatrix}, \begin{bmatrix} 1+i \\ 1 \end{bmatrix} \right)$

[ ]:
```python
beta_id_gamma = Matrix.hstack(v1, v2)
beta_id_gamma
```

[ ]: $\begin{bmatrix} 1-i & 1+i \\ 1 & 1 \end{bmatrix}$

$${}_\beta[\mathrm{id}_{\mathbb{C}^2}]_\gamma = \begin{bmatrix} 1-i & 1+i \\ 1 & 1 \end{bmatrix}$$

[ ]:
```python
gamma_id_beta = beta_id_gamma.inv()
gamma_id_beta
```

[ ]: $\begin{bmatrix} \frac{i}{2} & \frac{1}{2} - \frac{i}{2} \\ -\frac{i}{2} & \frac{1}{2} + \frac{i}{2} \end{bmatrix}$

1.2 b)

[ ]:
```
simplify(gamma_id_beta * A * beta_id_gamma)
```

[ ]:
$$\begin{bmatrix} 3+i & 0 \\ 0 & 3-i \end{bmatrix}$$

$$Av_1 = \lambda_1 v_1, Av_2 = \lambda_2 v_2$$

$$A \begin{bmatrix} | & | \\ v_1 & v_2 \\ | & | \end{bmatrix} = \begin{bmatrix} | & | \\ \lambda_1 v_1 & \lambda_2 v_2 \\ | & | \end{bmatrix} = \begin{bmatrix} | & | \\ v_1 & v_2 \\ | & | \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

$$\begin{bmatrix} | & | \\ v_1 & v_2 \\ | & | \end{bmatrix} = {}_\gamma[\mathrm{id}_{\mathbb{C}^2}]_\beta$$

$$A \ {}_\gamma[\mathrm{id}_{\mathbb{C}^2}]_\beta = {}_\gamma[\mathrm{id}_{\mathbb{C}^2}]_\beta \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

$${}_\beta[\mathrm{id}_{\mathbb{C}^2}]_\gamma \ A \ {}_\gamma[\mathrm{id}_{\mathbb{C}^2}]_\beta = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$