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Visibility graph network analysis of gold price time series

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HIGHLIGHTS

- Gold price time series have been analyzed by visibility network method.
- The logarithmic gold price time series appears a multifractal Brownian series.
- The return series of log-gold price appears a multifractal Gaussian noise series.
- The visibility graphs of price series and return series are both small world networks.
- The price series is a hierarchy structure in agreement with Elliot's Wave Theory.

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ABSTRACT

Mapping time series into a visibility graph network, the characteristics of the gold price time series and return temporal series, and the mechanism underlying the gold price fluctuation have been explored from the perspective of complex network theory. The network degree distribution characters, which change from power law to exponent law when the series was shuffled from original sequence, and the average path length characters, which change from $L \sim \ln N$ into $\ln L \sim \ln N$ as the sequence was shuffled, demonstrate that price series and return series are both long-rang dependent fractal series. The relations of Hurst exponent to the power-law exponent of degree distribution demonstrate that the logarithmic price series is a fractal Brownian series and the logarithmic return series is a fractal Gaussian series. Power-law exponents of degree distribution in a time window changing with window moving demonstrates that a logarithmic gold price series is a multifractal series. The Power-law average clustering coefficient demonstrates that the gold price visibility graph is a hierarchy network. The hierarchy character, in light of the correspondence of graph to price fluctuation, means that gold price fluctuation is a hierarchy structure, which appears to be in agreement with Elliot's experiential Wave Theory on stock price fluctuation, and the local-rule growth theory of a hierarchy network means that the hierarchy structure of gold price fluctuation originates from persistent, short term factors, such as short term speculation.

1. Introduction

The fluctuation of asset price is the center of concerns in financial markets. It attracts speculators, who consider it an opportunity to engage in arbitrage, and disturbs investors, who believe it a risk to be managed. Temporal properties and mathematical description of financial asset prices are the key factors considered in modern quantitative finance, based on which financial models have been proposed to determine derivative product prices and investment portfolios, and to manage risk. Since it was suggested first in 1900 by French mathematician Bachelier [1], that asset prices might be described

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by what today is known as a random walk [2], random walk has been the main model adopted in financial theories, such as the Black–Scholes formula, the most popular method for option pricing [3]. A random walk is a random process where the increments are uncorrelated. It is believed in Fama's theory that, in an efficient market, asset price should respond directly and sufficiently to relevant information, which should arrive stochastically, and that daily changes of asset price are random, independent and not correlated [4]. As data accumulate, real market data provide a sample for the tests of properties of the financial temporal series. One test is the statistical distribution test, most of which have shown that asset price fluctuations are not normal distributions, and some corrected models such as the Levy process have been proposed [5]. Another test is the independence test. In traditional analysis, this test is demonstrated by autocorrelation, or frequency spectrum of asset price time series. Most of these researches have shown that the autocorrelation function of a finance asset price series has a short characteristic time scale, as short as a few trading minutes and finance price change has practically no memory [6]. Although these traditional analyses have provided some cues about the characteristics of finance asset price fluctuations, they still do not reveal the complex essence of finance asset price fluctuation.

In the past decades, the possible nonlinear structures of random-like financial asset price temporal series, chaos and fractality, have been examined [7,8]. From the view of nonlinear physics, researches have explored the possible determinate mechanism underlying the complex financial asset price series, which looks stochastic. These researches are extremely important, because chaos and fractal mean that price change, which looks random from traditional statistics, is not random, and not only financial models based on random walk, but major financial models based on martingale postulation, which just demands independent price change, are not available. The earliest nonlinear physical research of financial temporal series might be started by Mandelbrot's and Mantegna and Stanley's works [8,9], but the essentially identical work about the nonlinear fractal, the self-affine characteristic, was started earlier by Hurst in 1950 [10] as a statistic method called rescaled analysis (R/S). Today, the scale exponent based on Hurst's method, Hurst's exponent is used as an index to indicate whether a temporal series analyzed is temporally independent according to whether Hurst's exponent is $1/2$ (see appendices and reference in Ref. [11] for temporal series characteristics and their relation to the frequency spectrum, autocorrelation and Hurst exponent).

Thought Hurst's exponent is extensively used, a variety of Hurst exponent computing methods have been developed to estimate the temporal dependence, and many researches about financial price have reported the long-range dependence, a different result from the traditional autocorrelation analysis, Hurst exponent computation is still a problem [12,13]. According to Serinaldi [13], the many computed Hurst exponent is not, but index related to Hurst. This makes the financial temporal characteristic based on Hurst unreliable. Another problem about the Hurst index is the limited data length effect in Hurst estimation. In essence, the Hurst exponent of some random like processes such as Brownian motion is an infinite limit. When finite data is used to estimate, long-range relations are partially broken into finite series and local dynamics corresponding to a particular temporal window are overestimated. As data length increases, the estimated Hurst converges to its Hurst index so slowly, that the Hurst of real financial data inevitably deviates from its real value. According to Michel Couillard and Matt Davison, the finite Brownian motion data sets will always give a value of the Hurst exponent larger than $1/2$ and without an appropriate statistical test such a value can mistakenly be interpreted as evidence of long term memory [14]. In a more precise statistical significance test for the Hurst exponent, some financial time series are not affirmed the previously claimed long-term memory, and Brownian motion cannot be rejected as a model for price dynamics [14].

Multifractality is another promising index grasping complex characteristic of temporal series, which has been more and more adopted to analyze financial series [15–17], but which attitude of analyzed sequence the multifractality reflects is not distinct, except that the temporal correlation, the kurtosis and tail heaviness of the probability density distribution of the analyzed sequence are the factors that affect multifractalities [18,19].

Recently, graph network methods have been developed to analyze time series. In these methods, a time series is mapped into a graph network, and the characteristics of the time series are believed to be inherited in the mapped network which can be analyzed from a complex network perspective. Among these, the visibility graph network method has been demonstrated to be able to show the fractal characteristic of a time series by power law degree distribution characteristic and small world path length characteristic of the corresponding network [20]. The Hurst index of a fractal Brownian series has been further demonstrated linearly related to the power-law exponent of graph degree distribution [21]. This visibility graph method, as an efficient approach without the drawbacks of the Hurst index estimation, has been successfully used to characterize human strive intervals [21], occurrence of hurricanes in the United States [22], foreign exchange rates [23], energy dissipation rates in three-dimensional fully developed turbulence [24], human heartbeat dynamics [25,26], electroencephalogram series [27], daily streamflow series [28], and stock indices [29,30], but has still not been used to analyze commodities.

Gold is a rare precious metal, which is used in industry, and has always been a widely accepted international currency. As financial markets disturb, and currencies flourish, gold has been demonstrating more and more financial characteristics, as an investing and hedging tool. However, its price temporal characteristic has been rarely analyzed except for the recent multifractal test based on the Detrended Rescale Analysis of Hurst index estimation [31] and wavelet transform analysis [19]. In this paper, a gold price series is mapped into a visibility graph and analyzed from the view of the network. This work demonstrated that the gold price series and its return series are fractal, and more other characteristics of gold price temporal series have also been revealed from its network characteristics, including average length of network, clustering coefficient, and the relation of network degree distribution to Hurst index. Also, the mechanism underlying the temporal characteristics of gold price is analyzed through the network growth theory.

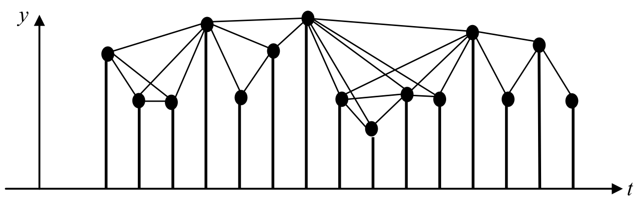

Fig. 1. Example of visibility graph network.

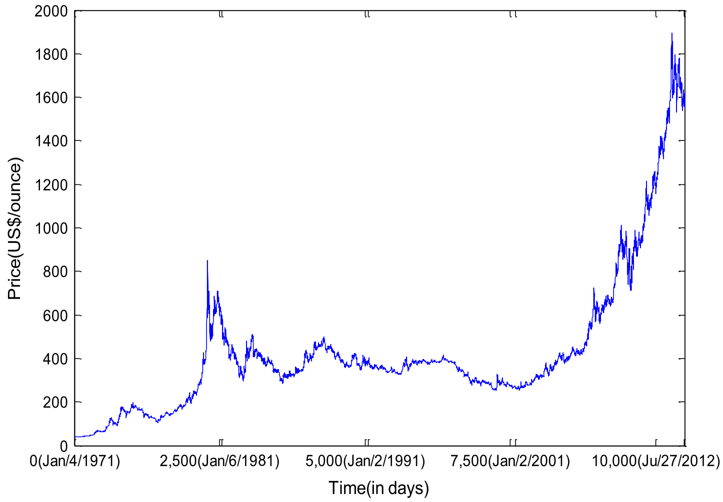

Fig. 2. Gold price series from 1971 to 2012. The ticks in the time axis are labeled in days every 2500 days and the first trading day in every 10 years (except the last day of the data, Ju/27/2012) respectively, in order to relate the series number with a historic date. From the labels, one can see that there are about 2500 trading days in a period of ten years.

2. Visibility graph method

The visibility graph method was first proposed by L. Lacasa et al. in 2008 [20] as a new way to characterize time series from associated network topology. It provided reliable correlation detection and Hurst index estimation through mapped graph network characteristics [20,21,30]. For a time series $[y_t : t = t_1, t_2, \ldots, t_N]$, a series of bars with heights proportional to their values are in Fig. 1. Any two data, (y_p, t_p) and (y_q, t_q) are visible when there is no other bar (y_i, t_i) between them blocking the connecting line of their tops. That is:

$$y_i < y_p + (y_q - y_p)\frac{t_i - t_p}{t_q - t_p}. \tag{1}$$

Connecting any two data which are visible forms a visibility graph network, in which every data is a node. This graph network conserves and inherits characteristics of the time series. Ordered (periodic) series convert into regular graphs, random series convert into exponential random graphs and fractal series into scale-free graphs [20]. For fractal Brownian series, the Hurst exponent can be reliably estimated by the power-law exponent of degree distribution of the graph [21]. Also, the time series temporal characters and the underlying mechanism can be interpreted from the view of graph theory.

3. Data analyzed

The analyzed data are gold fixings from the London Bullion Market Association, which was obtained from the London Bullion Market Association's web site, http://www.lbma.org.uk/pages/?page_id=53&title=gold_fixings. In a usual trading day, the London Bullion Market Association gives two gold fixings, an AM fixing and PM fixing. We have adopted the PM fixing as the daily gold price in a usual trading day. In some particular days, there was only an AM fixing or PM fixing, which is adopted as that day's gold price. The gold prices were in US dollars per ounce. Before 1971, the Bretton Woods system pegged the United States dollar to gold at a rate of US$35 per troy ounce, and the gold price in US dollars had not varied in that period. So we adopt the price after the year 1971. Fig. 2 is the analyzed gold price time series over about 41.5 years from the first trading day of 1971 to July 27, 2012.

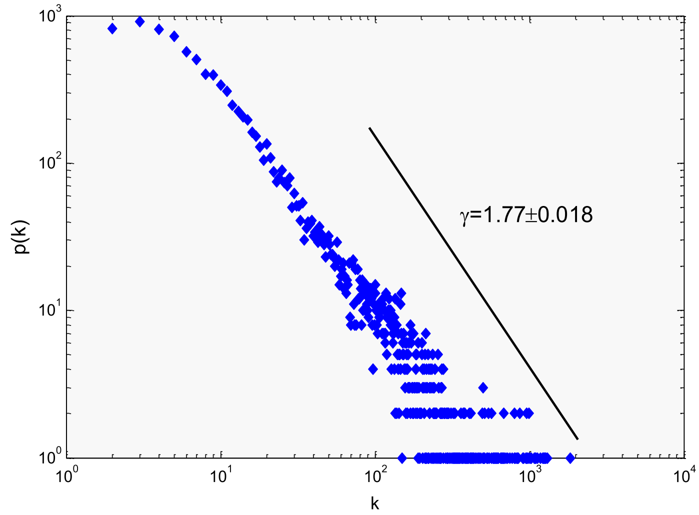

Fig. 3. Degree distribution of gold price visibility graph.

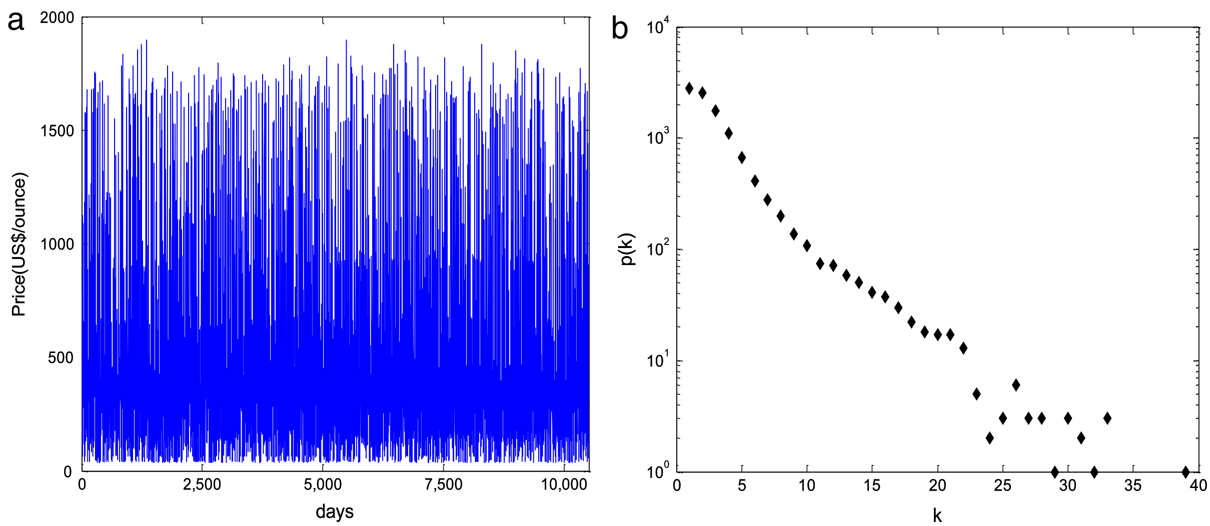

Fig. 4. Shuffled gold price series (a) and degree distribution of its visibility graph (b).

4. Results

4.1. Long-range correlation

According to L. Lacasa et al. [20], the visibility graph network of a fractal time series will have a power law degree distribution. To test the self-affine character of the gold price series, the degree distribution of gold price series was computed. Fig. 3 is the degree distribution of gold price series. It manifests a power law degree distribution. To compare with this result, the degree distribution of shuffled gold price series, which has lost its original temporal correlation, is also computed. Fig. 4 demonstrates that the shuffled gold prices series is not of power law, but of exponent law, a random series' graph network character [20]. This reveals that the gold price series is a fractal time series with long-range correlation.

We have also computed the degree distribution of gold price return series (gold price differences in consecutive days). Fig. 5(a), (b) are the gold price return series and its degree distribution. Fig. 5(c), (d) are the shuffled return series and their degree distributions. From these computations, we can see that the return series of gold price series is also a self-affine series with long-range correlation.

4.2. Multifractal Brownian series

According to L. Lacasa [20] and Ni et al. [30], a fractal Brownian series would have a Hurst exponent H linearly related to the exponent of power-law degree distribution γ, $\gamma = 3 - 2H$ and their increment series, a Gaussian series has an

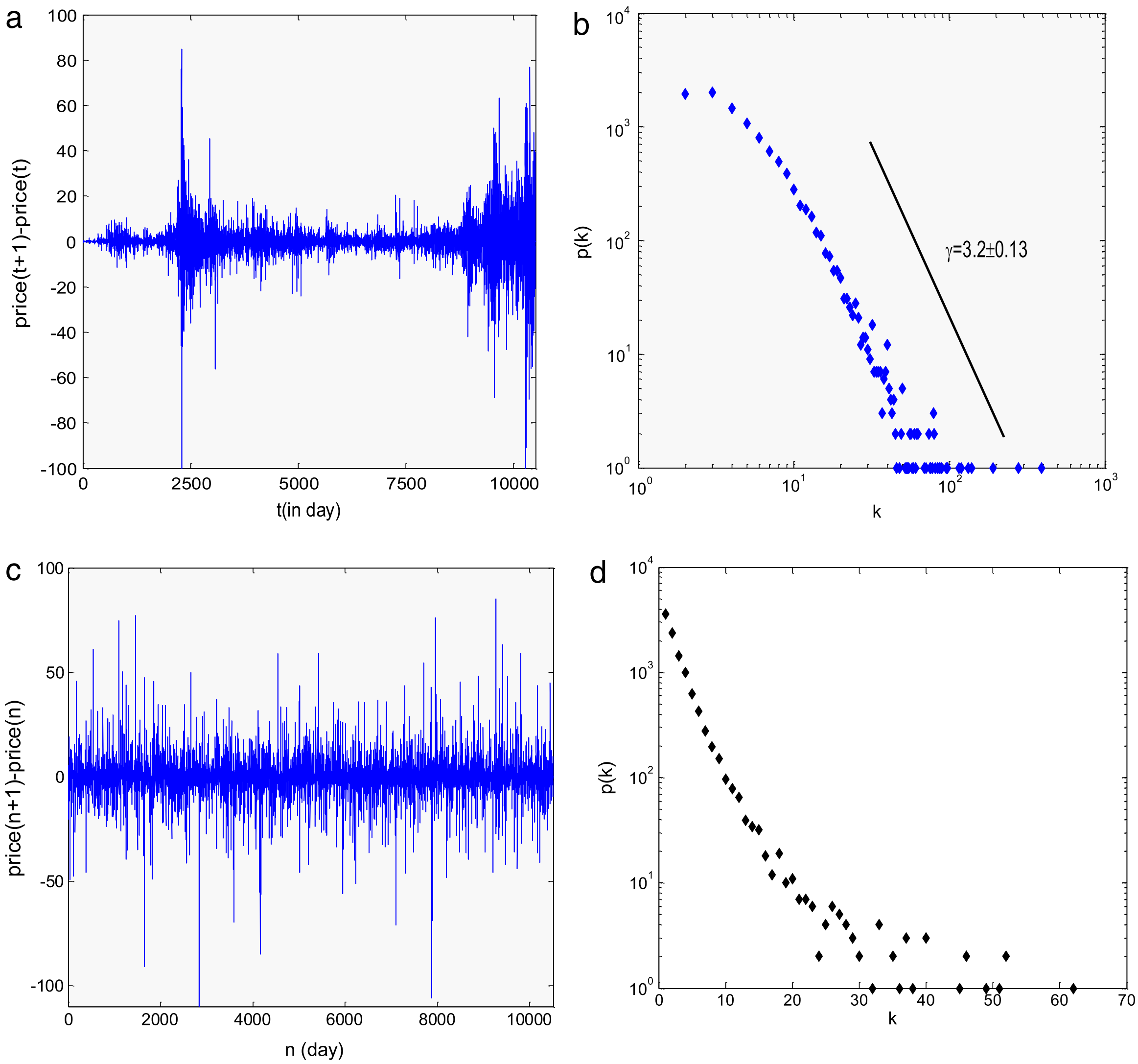

Fig. 5. Gold price return series and its degree distribution a, b: original gold price return series and its degree distribution of visibility network. c, d: Shuffled return series and its degree distribution of visibility network.

exponent of power-law degree distribution $\gamma = 5 - 2H$. To test whether the gold price series is a fractal Brownian series, we have computed degree distributions of the gold price series, price return series (price$(t+1)$ − price(t)), Logarithmic price series ($\log_{10}$ price(t)) and logarithmic return series ($\log_{10}$ price$(t+1) - \log_{10}$ price(t)), and also their Hurst exponents. Due to the tricky of the least squares fit estimation of the power-law degree distribution exponent, the maximum-likelihood estimation [32,33] was used to calculate the power-law degree distribution exponents:

$$\gamma = 1 + n\left[\sum_{i=1}^{n} \log \frac{k_i}{k_{\min}}\right]^{-1}, \tag{2}$$

where $k_{\min}$ is the smallest value of k (degree) from which the power law behavior holds (typically, $k_{\min} = 10$), k_i ($k_i \geq k_{\min}$), $i = 1, \dots, n$, are the values taken into account and n is the number of the values.

The Hurst exponents in fractal Brownian series test were computed by wavelet transform (WT) method [34,35]. The WT of temporal series $\{y_i | i = 1, \dots\cdots N\}$ is calculated as $W(s, a) = \frac{1}{a}\sum_{i=1}^{N} y_i \cdot g(\frac{i-s}{a})$, where $g(\cdot)$, the wavelet, is a 3 order derivative of Gaussian function, and a is the scale. Determining the maximum positions of WT $\{s_1, s_2, \dots\cdots, s_M\}$, for a fractal series, in the long scale limit the partition function obeys a power law,

$$Z(a, q) = \sum_{s=s_1}^{s_M} |W(s, a)|^q \propto a^{\tau(q)}.$$

Table 1
Degree distribution power-law exponent γ and the theoretical prediction.

	Hurst index	Power-law exponent γ	Theoretical prediction γ
Price series	$H = 0.5047$	1.77 ± 0.018	fBm 1.99
Price return		3.20 ± 0.13	fGn 3.99
Logarithm series	$H = 0.5396$	1.88 ± 0.03	fBm 1.92
Logarithm return series		3.89 ± 0.22	fGn 3.92

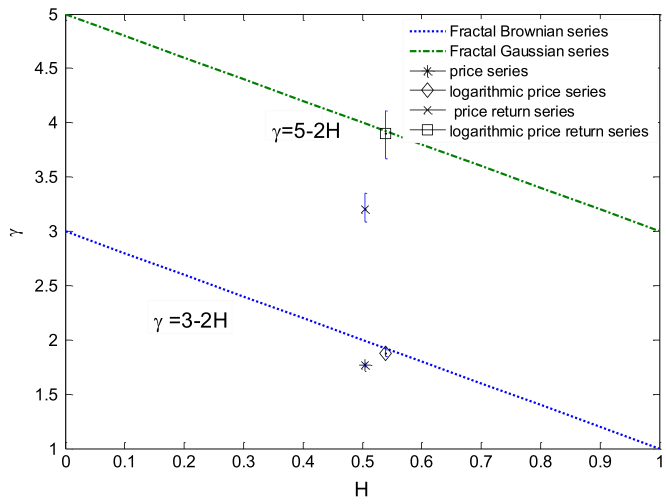

Fig. 6. Hurst exponents and the degree power-law exponents. The logarithmic series has a Hurst exponent and degree power-law exponent relation of fractal Brownian motion, and the logarithmic return series has a Gaussian noise relation.

For a multifractal series, $\tau(q)$ is a nonlinear function of q, and $h(q) = \frac{d\tau}{dq}$ is the function of q, which means that different subsets of the series have different local Hurst exponents $h(q)$, which quantifies the local singularity behavior of the series related with the local scaling.

If $\tau(q)$ is a straight line, $h(q)$ is a constant independent of q, different subsets of the series have common local Hurst exponents, and the series is a monofractal series.

In this calculation, $h(2)(q = 2)$ is the Hurst exponent of the series, which can be determined from the equation

$$qh(q) = \tau(q) + 1.$$

That is, the Hurst exponent of the series is

$$H = h(2) = \frac{1}{2}\left[\tau(2) + 1\right].$$

Table 1 is the Hurst exponents, the degree distribution power-law exponents and the theoretical prediction power-law exponents based on theoretical relations of Hurst exponent to degree power-law exponent for fractal Brownian motion and fractal Gaussian noise. In the table, it is shown that the price series and return series have power-law exponents more than 20% different from the theoretical predictions, and the logarithmic price series and logarithmic return series have power-law exponents with differences not more than 3% from the theoretical predictions, and within the range of calculation error. So, in relation to a Hurst exponent with degree power-law exponent, the logarithmic series accords with a fractal Brownian motion and the logarithmic return series accords with a fractal Gaussian noise. Fig. 6 illustrates this result.

To test whether the degree distribution changes with time, or whether a different subset series has a different degree distribution, we have calculated degree distributions in a moving window with 2500 data points (about ten years). This calculation demonstrated that as the calculation window moves, the power-law exponent of degree distribution changes. Fig. 8 demonstrates this change. The average of the moving degree power-law exponents, 2.38, is not equal to that of the whole data, 1.88, indicated in Fig. 7(c). This result can be understood, because the total connections in 2500 data windows, which did not include the connections of these data to the others outside the windows, are not the same in the whole data. This fact and that the differences of power-law exponents in different data windows are much larger than estimation errors of the power-law exponents mean that the changes of power-law exponents in moving data windows are not a sideproduct of the finite data point effect, but the time dependent character of the series.

As a fractal Brownian motion, the Hurst exponent which has a linear relation to its degree power-law exponent [21], time dependent power-law exponent of degree distribution means that there are different Hurst exponents in different local subsets. A time-varying Hurst exponent can be interpreted in terms of the multifractal properties of asset prices, which implies that different moments of a series are associated with different scaling laws [36]. The wavelet transform analysis

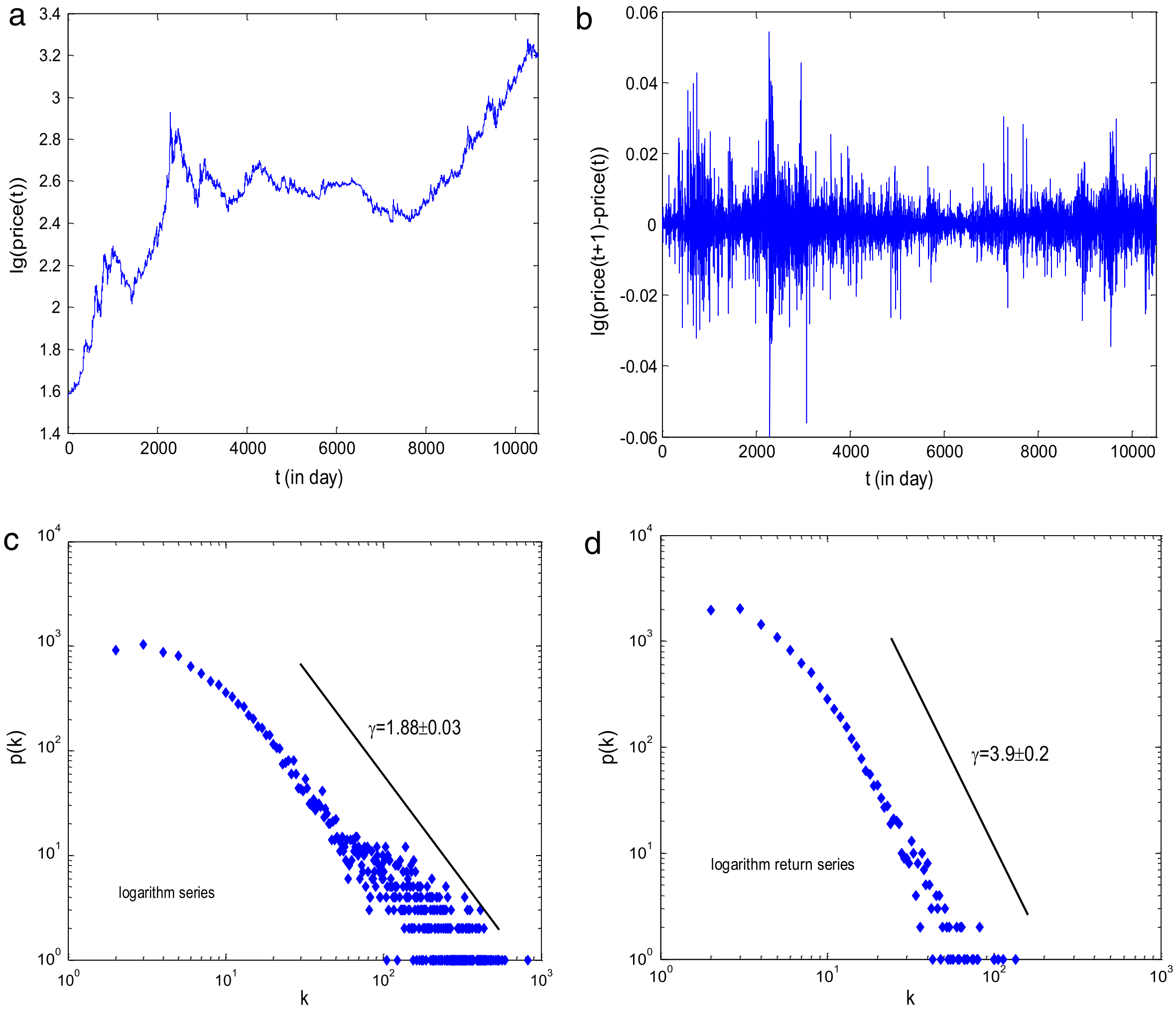

Fig. 7. Logarithmic price series, logarithmic price return series and their degree distributions. a, b: logarithmic gold price series and its degree distribution. c, d: logarithmic price return series and its degree distribution. The logarithmic series has a Hurst exponent H and degree power-law exponent γ relation of fractal Brownian movement and the logarithmic return series has a Gaussian noise relation. This is demonstrated in Fig. 6 and Table 1.

of the series confirmed this. Fig. 9 is $\tau(q)$ of different q calculated by wavelet transform. The $\tau(q)$ is not a straight line. It means that the series is a multifractal series. This result is also in accordance with the multifractal detrended fluctuation analysis [31] and Wavelet transform analysis [19].

According to X-H Ni's research [30], the linear relation of Hurst exponent with degree power-law exponent is not affected by multifractal character. So the logarithmic series is a multifractal Brownian motion and its return series is a Multifractal Gaussian noise.

4.3. Small world network

To test time series characteristics further, we have investigated the variance of average minimum path lengths of the mapping visibility network with data increasing. Fig. 10(a) and (b) are the average minimum path lengths of visibility networks for price series and the shuffled series. It is demonstrated that the average minimum path length of the shuffled price series network is scale-invariant, which manifests power-law growth with the data number N. However, the visibility network of the original price series manifests a small world characteristic.

Fig. 10(c) is the average minimum path length of the network for price return series. It demonstrates that the network for price return series is a small world network. The network for shuffled return series is scale-invariant.

The difference between original temporal series and the shuffled series in character of average path length demonstrates that the original data series is correlated. From this graph, this phenomena can be explained in terms of hub repulsion. In graphs for the original series, hubs, corresponding the highest values, are not hidden from each other, but in graphs for the shuffled series, hubs are hidden from each other [37]. From the point of view of network growth, the small world network

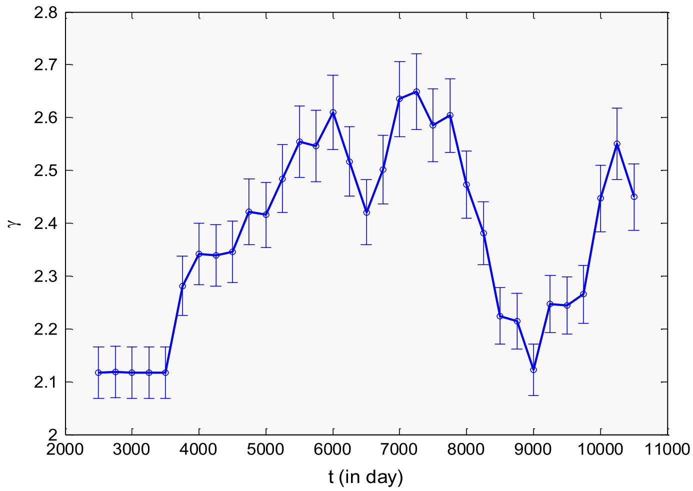

Fig. 8. Degree distribution power-law exponents of logarithmic price series change with time. The calculation window is 2500 data points (about ten years).

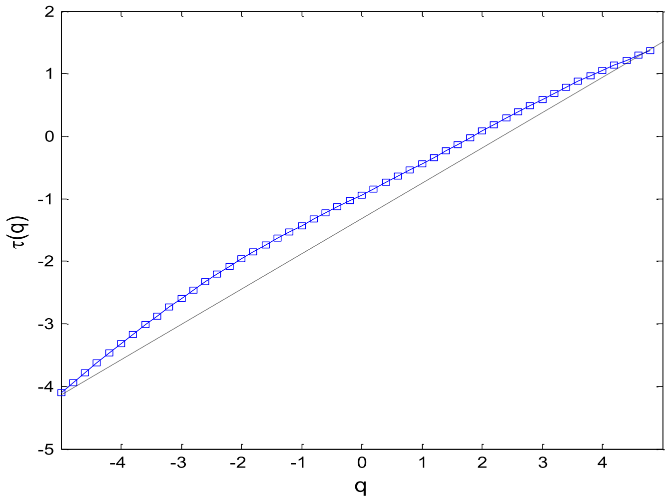

Fig. 9. Renyi exponent $\tau(q)$ for logarithmic gold price series obtained from wavelet transform. A monofractal series will demonstrate a linear $\tau(q)$ function, otherwise the series is a multifractal series.

means that in the processing of network growth, newly added nodes have some preferential attachment instead of random attachment [38]. This means that the gold price and gold price return does not change in a random manner, but they are attracted, limited, or affected by previous prices or return values. From Fig. 10, we can see that the mean path length of the price return series graph is larger than that of the price series graph. This may imply that the return graph, and the corresponding price return series, are less correlated than the price graph and the price series.

4.4. Hierarchy network

In a network, nodes may cluster in some groups. In a group, nodes are connected densely, and nodes in different groups are connected loosely. Different groups may cluster into a larger group, and a group may contain small sub-groups. This structure is a hierarchy structure. In a hierarchy network, nodes in a local sub-group usually have low degree, and different local groups are connected by high degree nodes in these groups. The clustering coefficient of a node is defined by the ratio of the real edge number of its connected nodes to the possible edge number. In a hierarchy network, high degree nodes, which are usually at the top level of the hierarchy, usually have low clustering coefficients, and low degree nodes, which are usually at the low level of the hierarchy, have high clustering coefficients, and the average clustering coefficient decreases with degree. Typically, the average clustering coefficient–degree relation in a hierarchy network is power law [38,39], $C(k) \sim k^{-\alpha}$.

Fig. 11 is the average clustering coefficient variation with degree. It demonstrates that relations of clustering coefficient to degree for price series graph, price return series graph, price logarithmic series graph and logarithmic return graph are all power law and that networks for them are all hierarchy network.

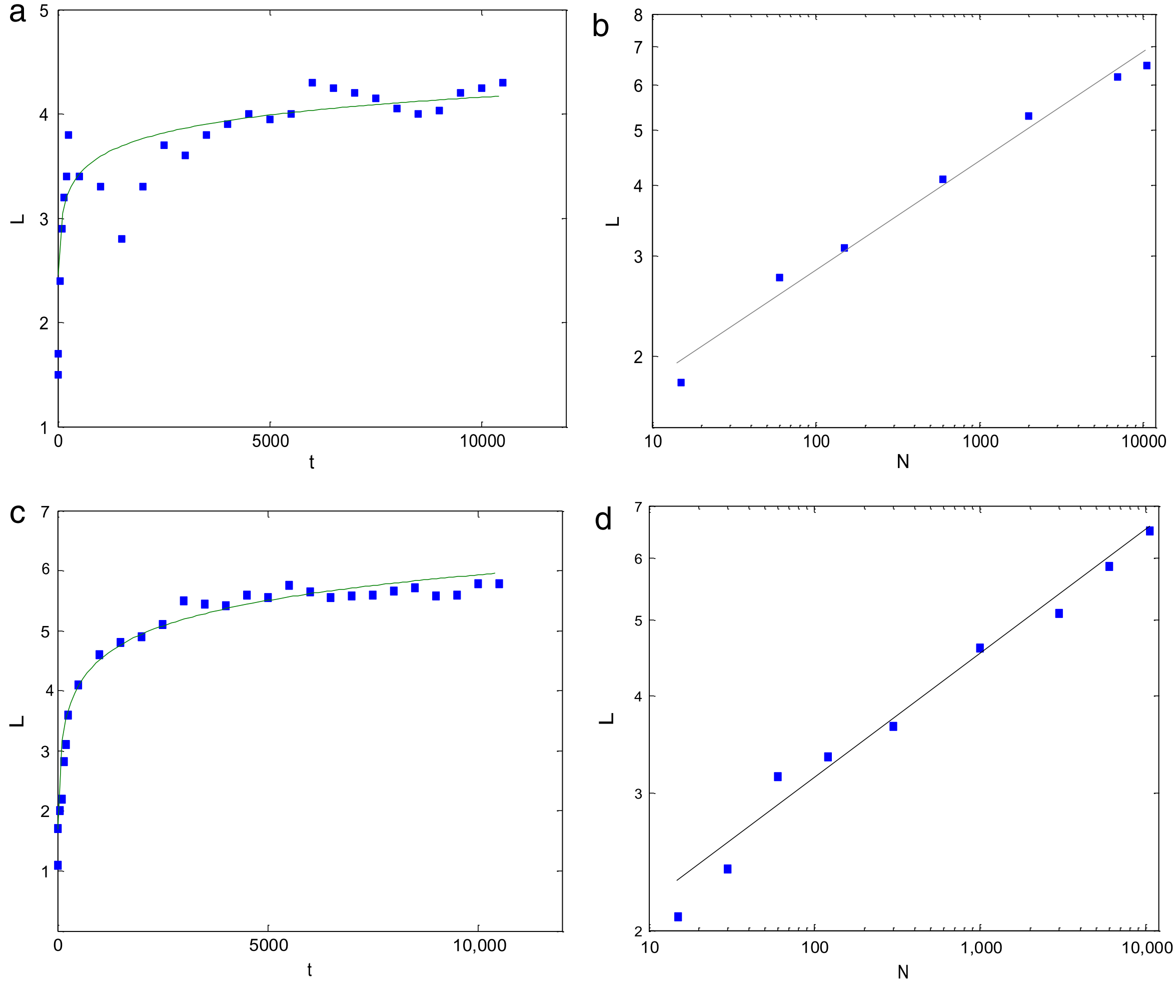

Fig. 10. Average minimum path length. a, b: mean path lengths for original and shuffled price series, $L = 1.89 + 0.25 \ln N$, $L = 1.32N^{0.18}$; c, d: mean path lengths for original and shuffled price return series $L = 0.28 + 0.61 \ln N$, $L = 1.57N^{0.15}$. Scatter points are calculated values and solid lines are fit lines.

After deeply observing the character of nodes clustering in groups with high clustering coefficients, we can grasp the characteristic of temporal series behind the hierarchy characteristic of the graph network. In a price–time graph, points in a local concave curve would correspond to nodes densely connected in the visibility graph with high clustering coefficient because they are highly visible to each other. Points at the left terminal and right terminal of a concave are local maximum which are more visible and have higher degree than others in the concave, because they link this concave to another one. This means that points in a local concave of a price–time graph correspond to nodes of a sub-group in a hierarchy visibility graph. A hierarchy visibility graph means that the corresponding price–time graph is composed of a series of concaves which connect and construct a larger concave. If we consider a concave as a wave in a temporal series, hierarchy character means that a temporal series, such as gold price, changes in the form of a wave, and waves, which are composed of sub-waves, construct a higher grade wave. This is most similar to the wave theory of stock price by R.N. Elliott, who proposed the Wave Theory about stock price changing in nature from his experience as the base of the investing technique [40]. In the Wave Theory, stock price fluctuations are considered as waves of 15 different levels from Submicro, MiniScule to Supermillennium. Waves of a lower level, which have a shorter time range, compose waves of a higher level, which have a longer time range. Though hierarchy construction of Elliott's Wave Theory is well known, and can be expected from the concept of log-periodicity originated from the criticality theory of phase transition [41–44], it is just an experiential judgment, and has never been demonstrated in a scientific way. The visibility network analysis of this work seems a statistical support to this experiential judgment and the concept of log-periodicity.

Now that the hierarchy of waves can be demonstrated by the visibility network, the mechanism underlying the hierarchy of waves can also been interpreted by network theory. From the view of network growth, the small world characteristic and hierarchy characteristic originate from the network growth local rule, the evolution rules of network growth involving a vertex and its neighbors. The local rule that results in preferential attachment will also result in hierarchy [38,39,45,46]. This means that the hierarchy of gold price series is original from persistent short term factors, such as short term speculation.

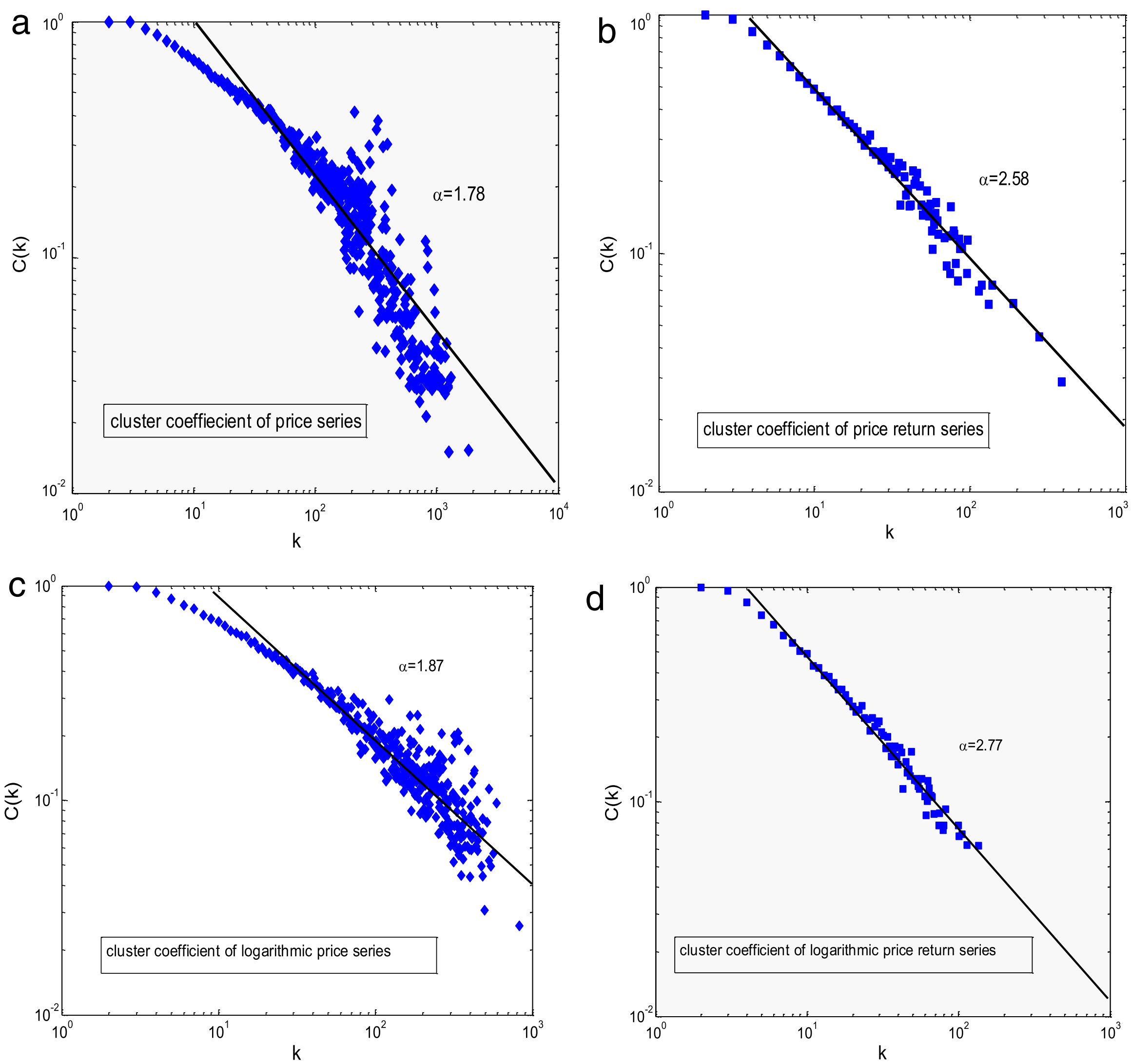

Fig. 11. Clustering coefficients of visibility graph a: price visibility graph, b: price return visibility graph, c: logarithmic price visibility graph, d: logarithmic price return visibility graph.

5. Summary and conclusion

We have investigated the gold price evolution from a network perspective by the visibility network method. This graph analysis has shown that the gold price series and gold price return series are long-term correlated, fractal series with a power-law degree distribution of visibility graph network. The relation of the Hurst exponent with degree distribution power-law exponent for logarithm price series agrees with that of fractal Brownian motion $\gamma = 3 - 2H$, and for logarithmic price return series agrees with that of fractal Gaussian noise, $\gamma = 5 - 2H$. The fact that the degree distribution power-law exponent in a time window changes with the moving of window demonstrates that a logarithmic gold price series is a series of time-variant Hurst, that is, a multifractal series. The clustering coefficient analysis demonstrated that visibility networks for gold price series and price return series are hierarchy networks. As points in a local concave price–time graph are densely connected and with high clustering coefficients, they correspond to the sub-group in the hierarchy network which are connected with each other and form a larger group. This hierarchy characteristic of a gold price visibility network means that gold price fluctuations are in a form of waves, which are in agreement with Elliott's Wave Theory about stock price change, that the stock price changes in a series of waves with hierarchy structure. The mean path length analysis demonstrates that the price graph and price return graph are small world networks. According to complex network theory, the small world characteristic and hierarchy characteristic are all original from the network growing local rule. This result means that the gold price local fluctuations originate from short term persistent factors. We cannot determine what the factors are, but suggest that short term speculation is one of these factors.

This work is significant not only to financial practice in that it provides the mathematic model of gold price temporal series for financial investment analysis, and, for the first time, demonstrates from a network perspective that gold price fluctuation, in agreement with the experiential Wave Theory in finance investment, is a hierarchical structure of waves, but also to theoretical research because the revealed hierarchy characteristic of the visibility network of gold price temporal series combined with the growth theory of the hierarchical network suggests the direction of research on the mechanism underlying gold price fluctuation.

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