1.

$$\begin{bmatrix} 1.19 & 2.11 & -100 & 1 \\ 14.2 & -0.112 & 12.2 & -1 \\ 0 & 100 & -99.9 & 1 \\ 15.3 & 0.11 & -13.1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1.12 \\ 3.44 \\ 2.15 \\ 4.16 \end{bmatrix}$$

① ⟷ ④

$$\begin{bmatrix} 15.3 & 0.11 & -13.1 & -1 \\ 14.2 & -0.112 & 12.2 & -1 \\ 0 & 100 & -99.9 & 1 \\ 1.19 & 2.11 & -100 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4.16 \\ 3.44 \\ 2.15 \\ 1.12 \end{bmatrix}$$

$\times\left(-\frac{14.2}{15.3}\right)$ (row 1 → row 2)

$\times\left(-\frac{1.19}{15.3}\right)$ (row 1 → row 4)

$$\begin{bmatrix} 15.3 & 0.11 & -13.1 & -1 \\ 0 & -0.214 & 24.36 & -0.0719 \\ 0 & 100 & -99.9 & 1 \\ 0 & 2.1 & -98.98 & 1.078 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4.16 \\ -0.4209 \\ 2.15 \\ 0.79644 \end{bmatrix}$$

② ⟷ ③

$$\begin{bmatrix} 15.3 & 0.11 & -13.1 & -1 \\ 0 & 100 & -99.9 & 1 \\ 0 & -0.214 & 24.36 & -0.0719 \\ 0 & 2.1 & -98.98 & 1.078 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4.16 \\ 2.15 \\ -0.4209 \\ 0.79644 \end{bmatrix}$$

$\times\left(+\frac{0.214}{100}\right)$ (row 2 → row 3)

$\times\left(-\frac{2.1}{100}\right)$ (row 2 → row 4)

③ ⟷ ④

$$\Rightarrow \begin{bmatrix} 1513 & 0.11 & -13.1 & -1 \\ 0 & 100 & -99.9 & 1 \\ 0 & 0 & -96.88 & 1.007 \\ 0 & 0 & 0 & 0.1194 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4.16 \\ 2.15 \\ 0.7513 \\ -0.1229 \end{bmatrix}$$

$x_4 = -1.1822$

$x_3 = -0.10207$

$x_2 = 0.101264$

$x_1 = 0.176$

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$$A=\begin{bmatrix} 4 & 1 & -1 & 0 \\ 1 & 3 & -1 & 0 \\ -1 & -1 & 6 & 2 \\ 0 & 0 & 2 & 5 \end{bmatrix}, \quad A \cdot A^{-1} = I$$

$$\left[\begin{array}{cccc|cccc} 4 & 1 & -1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 3 & -1 & 0 & 0 & 1 & 0 & 0 \\ -1 & -1 & 6 & 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 5 & 0 & 0 & 0 & 1 \end{array}\right]$$

$\Rightarrow$ ① ÷ 4 $\Rightarrow$ $\left[\begin{array}{cccc|cccc} 1 & \frac{1}{4} & -\frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & 0 \end{array}\right]$ — ①

② − ① $\Rightarrow$ $\left[\begin{array}{cccc|cccc} 0 & \frac{11}{4} & -\frac{3}{4} & 0 & -\frac{1}{4} & 1 & 0 & 0 \end{array}\right]$

③ − ① × (−1) $\Rightarrow$ $\left[\begin{array}{cccc|cccc} 0 & -\frac{3}{4} & \frac{23}{4} & 2 & \frac{1}{4} & 0 & 1 & 0 \end{array}\right]$

✓ ② ÷ $\frac{11}{4}$ $\Rightarrow$ $\left[\begin{array}{cccc|cccc} 0 & 1 & -\frac{3}{11} & 0 & -\frac{1}{11} & \frac{4}{11} & 0 & 0 \end{array}\right]$

③ + ② × $\frac{3}{4}$ $\Rightarrow$ $\left[\begin{array}{cccc|cccc} 0 & 0 & \frac{23}{4}-\frac{3}{11}\times\frac{3}{4} & 2 & \frac{8}{44} & \frac{11}{3} & 1 & 0 \end{array}\right]$

① − ② × $\frac{1}{4}$ $\Rightarrow$ $\left[\begin{array}{cccc|cccc} 1 & 0 & \frac{-2}{11} & 0 & \frac{3}{11} & -\frac{1}{11} & 0 & 0 \end{array}\right]$

✓ ③ ÷ $\frac{61}{11}$ $\Rightarrow$ $\left[\begin{array}{cccc|cccc} 0 & 0 & 1 & \frac{22}{61} & \frac{1}{33} & \frac{121}{183} & \frac{11}{61} & 0 \end{array}\right]$

④ − ③ × 2 $\Rightarrow$ $\left[\begin{array}{cccc|cccc} 0 & 0 & 0 & 5-\frac{44}{61} & -\frac{2}{33} & -\frac{242}{183} & -\frac{22}{61} & 1 \end{array}\right]$

√ ④ ÷ $\frac{261}{61}$ ⇒ [0 0 0 1 | -0.0153 -0.309 -0.0844 $\frac{61}{261}$]

√ ① + ③ × 0.1818

[1 0 0 0.0656 | 0.2787 -0.082 0.0328 0]

√ ② + ③ × 0.2727

[0 1 0 0.0983 | -0.082 0.3777 0.0492 0]

① − 0.0656 ④ , ② − 0.0983 ④ , ③ − 0.3606 R4

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 0.2797 & -0.0805 & 0.0383 & -0.0153 \\ 0 & 1 & 0 & 0 & -0.0805 & 0.3793 & 0.0575 & -0.023 \\ 0 & 0 & 1 & 0 & 0.0383 & 0.0575 & 0.2107 & -0.0843 \\ 0 & 0 & 0 & 1 & -0.0153 & -0.023 & -0.0843 & 0.2337 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 0.2797 & -0.0805 & 0.0383 & -0.0153 \\ -0.0805 & 0.3793 & 0.0575 & -0.023 \\ 0.0383 & 0.0575 & 0.2107 & -0.0843 \\ -0.0153 & -0.023 & 0.0843 & 0.2337 \end{bmatrix}$$

3、

$$A = L \cdot U$$

$$l_1 = a_{11} = 3 \qquad l_{ii} = a_{ii} - l_{i+1,i} \cdot u_{i,i+1}$$

$$u_1 = \frac{a_{12}}{l_1} = \frac{-1}{3} \qquad u_i = \frac{c_i}{l_i}$$

$$l_{11} = 3$$

$$l_{22} = 3 - (-1)(-\tfrac{1}{3}) = 2.667$$

$$l_{33} = 3 - (-1)(-0.375) = 2.625$$

$$l_{44} = 3 - (-1)(-0.381) = 2.619$$

$$u_{12} = \frac{-1}{3}, \quad u_{34} = \frac{-1}{2.625}$$

$$u_{23} = \frac{-1}{2.667}$$

$LY = b$

$$\begin{bmatrix} 3 & 0 & 0 & 0 \\ -1 & 2.667 & 0 & 0 \\ 0 & -1 & 2.625 & 0 \\ 0 & 0 & -1 & 2.619 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 4 \\ 1 \end{bmatrix}$$

$$L \cdot y = \begin{bmatrix} 2 \\ 3 \\ 4 \\ 1 \end{bmatrix} \Rightarrow \quad y_1 = \frac{2}{3}$$

$$y_2 = \left(3 + \frac{2}{3}\right)/2.667 = 1.375$$

$$y_3 = (4 + 1.375)/2.625 = 2.048$$

$$y_4 = (1 + 2.048)/2.619 = 1.164$$

$U \cdot X = y \Rightarrow$

$$\begin{bmatrix} 1 & -0.333 & 0 & 0 \\ 0 & 1 & -0.375 & 0 \\ 0 & 0 & 1 & -0.381 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0.667 \\ 1.375 \\ 2.048 \\ 1.164 \end{bmatrix}$$

$$x_4 = y_4 = 1.164$$

$$x_3 = 2.048 + 0.381 \times 1.164 = 2.491$$

$$x_2 = 1.375 + 0.375 \times 2.491 = 2.309$$

$$x_1 = 0.669 + 0.333 \times 2.309 = 1.436$$

$$X = \begin{bmatrix} 1.436 \\ 2.309 \\ 2.491 \\ 1.436 \end{bmatrix}$$