Lab 4: The Logistic Map

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Abstract

In this lab, we expose some simulations with the logistic equation. Verhulst invented this equation in 1838 [1] and since them several researchers have used it to understand the basic aspects of the exponential growth with a limit [2].

1 The Logistic Equation

Researchers have applied the logistic to many areas. For instance, in population dynamics, the describe the population growth; in sociology the spreading of a rumor and in Economics, the relationship between commodity, quantity and price [2]. In this equation, the ratio of existing population to the maximum possible population at time two is equal to a constant times the ratio of existing population to the maximum possible population at time one, which in turns is multiplied by the 1 minus the same ratio at times one. The letter r designates that constant.

$$x_{n+1} = rx_n(1 - x_n) \tag{1}$$

2 When r increases...

After many iterations of the previous equation, something strange happens. When r exceeds 3.8, regular bifurcations occur until the system goes to chaos. Figure 1 showed the consequences of having r = 2.8 and figure 2 shows the consequences of having r = 3.8. Interestingly, after r passes a particular value, bifurcations start happening that makes and at after the r 3.8 the system becomes chaotic.

3 Vectorized logistic map

It is possible to vectorize the logistic map. Here I will show the code for the system under different rs

import numpy as np import matplotlib.pyplot as plt n = 10 r = np.linspace(2.5, 4.0, n) x = 1e-5 * np.ones(n) iterations = 30 x = logistic(r, x) X = np.zeros((iterations,n)) for i in range(1,iterations): x = logistic(r, x) X[i,:] = x plt.plot(X[:,[1]]) plt.plot(X[:,3],'.') for i in range(n): plt.plot(X[:,i],'.') plt.title('r = ' + str(r[i])) plt.show()

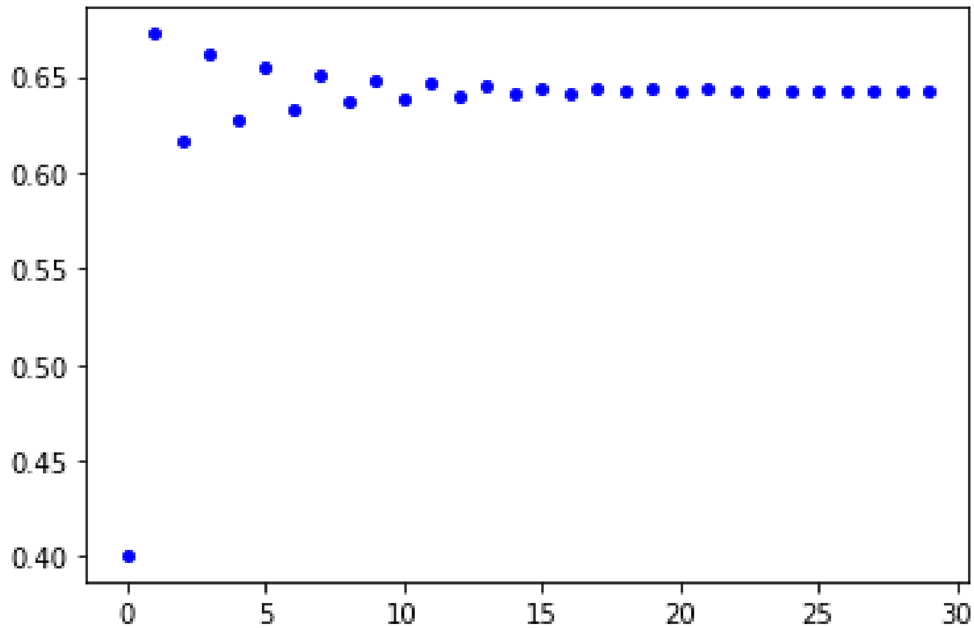


Figure 1: Behavior dependent on r, r = 2.8.

References

[1] R. May, ”Simple Mathematical Models With Very Complicated Dynamics,” Nature **26 (5560)**,457 (1993).

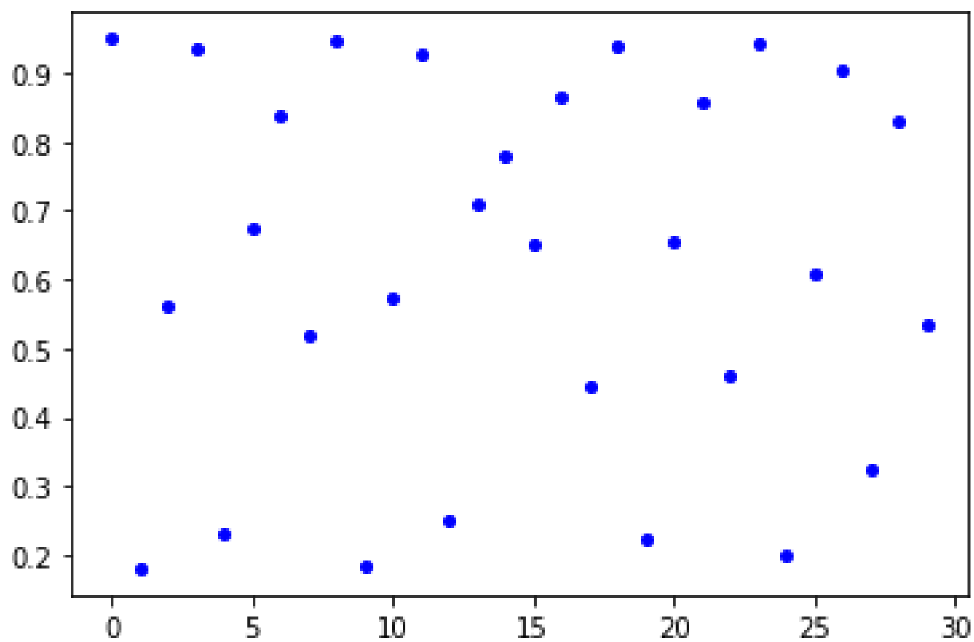
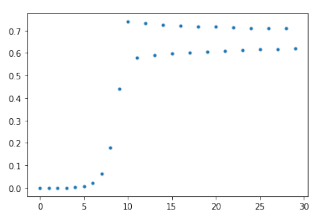
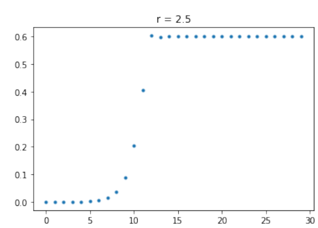


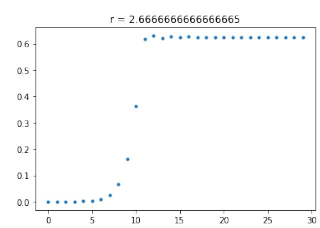
Figure 2: Behavior dependent on r, r = 3.8.



(a)



(b)



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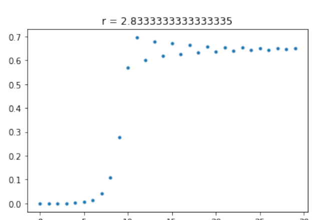

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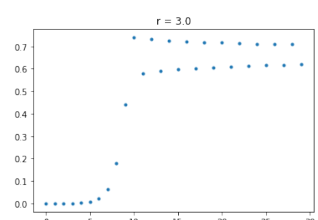

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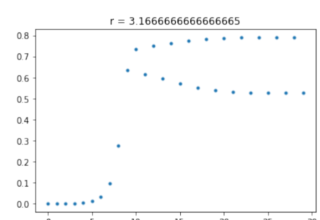

(f)

Figure 3: Vectorized logistic map