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Date: January 24, 2018
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Lab 1: Voltage and Current Division

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Objective

Understand, and put into practice, voltage and current division concepts as well as the principles associated with the Wheatstone Bridge method for measuring resistance.

Theory

Note: Theories, concepts, and proofs heavily quoted from "Fundamentals of Electric Circuits" 5th edition.

Ohm's Law

The voltage *v* across a resistor is directly proportional to the current *i* flowing through the resistor. The constant of proportionality is defined as the resistance, *R*. Therefore:

$$v = iR$$

Kirchhoff's Current Law

The algebraic sum of currents entering a node (or a closed boundary) is zero. In other words, the sum of currents entering a node is equal to the sum of currents leaving a node.

$$\text{Assume a set of currents } i_k(t), k = 1, 2, \ldots, \text{flow into a node}$$

$$i_T(t) = i_1(t) + i_2(t) + i_3(t) + \ \ldots \quad \text{Algebraic sum of currents}$$

$$q_T(t) = \ q_1(t) + q_2(t) + q_3(t) + \cdots \quad \text{Integrate both sides.}$$

$$\text{Note: } q_k(t) = \int i_k(t)dt \ \text{ and } q_T(t) = \int i_T(t)dt$$

$$\boldsymbol{q_T(t) = 0 \rightarrow i_T(t) = 0} \quad \text{Law of conservation of electric charge}$$

Kirchhoff's Voltage Law

The algebraic sum of all voltages around a closed path (or loop) is zero. In other words, the sum of voltage drops is equal to the sum of voltage rises in a closed path. This law is based off, and proven by, the conservation of energy.

Voltage Division

Voltage can be "divided" by placing resistors in series. The equivalent resistance of any number of resistors connected in series is the sum of the individual resistances.

$$R_{eq} = R_1 + R_2 + R_3 + \cdots + R_N$$

Based upon KCL, the current running through each of these resistors is equal. Applying Ohm's law, the resistance is directly proportional to the voltage across each resistor, hence, "dividing" the voltage.

Current Division

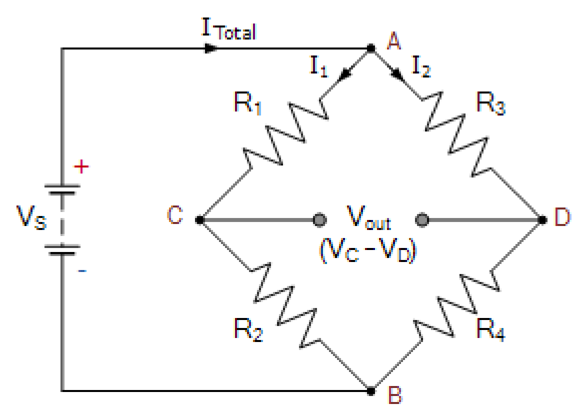
Current can be "divided" by placing resistors in parallel. The equivalent resistance of any number of resistors in parallel is the sum of the individual conductances.

$$R_{eq} = \left(\left(\frac{1}{R_1}\right) + \left(\frac{1}{R_2}\right) + \left(\frac{1}{R_3}\right) + \cdots + \left(\frac{1}{R_N}\right)\right)^{-1}$$

Based upon KVL, the voltage across each resistor is the same. Applying Ohm's law, each resistor's resistance is in inverse proportion to the current running through it, hence, "dividing" the current.

Wheatstone Bridge

Used to analyze two series strings in parallel. For the purpose of this experiment, by making the current through each series the same, the resistance of R_4 can be determined by adjusting the resistance of R_2 until V_{out} reads zero. (see Calculation 1.2)

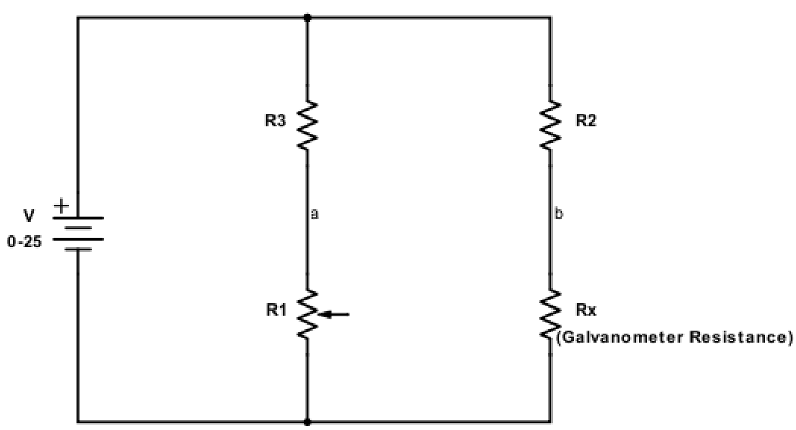


Procedure

Part 1:

A bridge circuit was created to determine the value of an unknown resistor, R_x. Specification for this circuit can be seen below (see Figure 1.1).

Figure 1.1



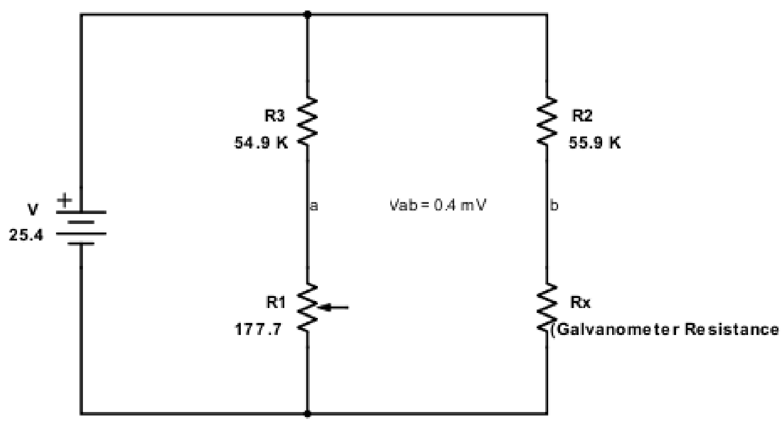
The current through R_x is limited to 0.5mA when the power supply is set at 25V. By limiting the current flowing through R_1 to also be 0.5mA, the value of R_x can be found by adjusting the variable resistance of R_1 until the voltage of V_{ab} = 0V.

Based upon KCL, the current flowing through R_3 and R_2 must also be limited to 0.5mA. Using Ohm's law, appropriate values for these two resistors were found to be greater than or equal to 50KΩ (see Calculation 1.1).

The expression for resistance R_x was derived in terms of R_1, R_2, R_3 (see Calculation 1.2).

The circuit (see Figure 1.2) was built and V_{ab} was monitored while R_1 was adjusted. Once V_{ab} was approximately zero, the resistance of R_1 was measured to be 177.7Ω. Using the expression derived in Calculation 1.2, the value of R_x was calculated to be **180.9Ω** (see Calculation 1.3).

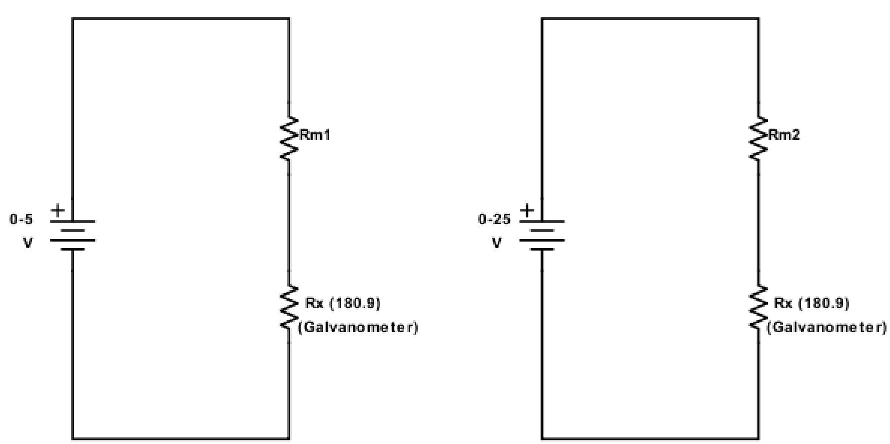
Figure 1.2



Part 2:

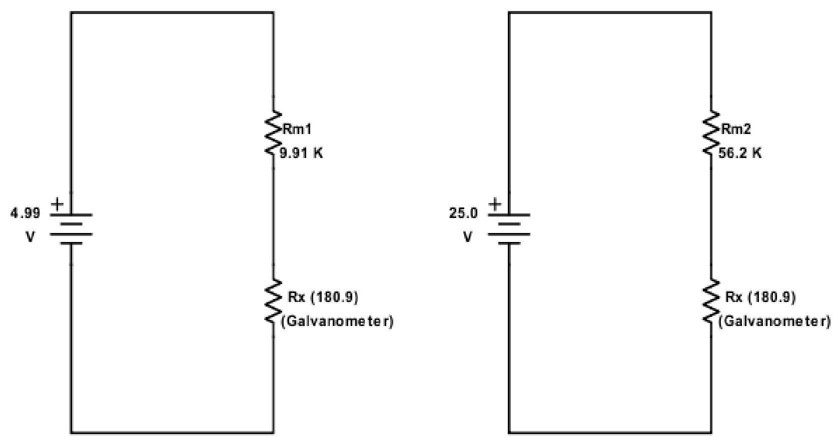
Two voltmeters were created, one having a 0-5 volt range and the other a 0-25 volt range, using the galvanometer from part 1 (see Figure 2.1).

Figure 2.1



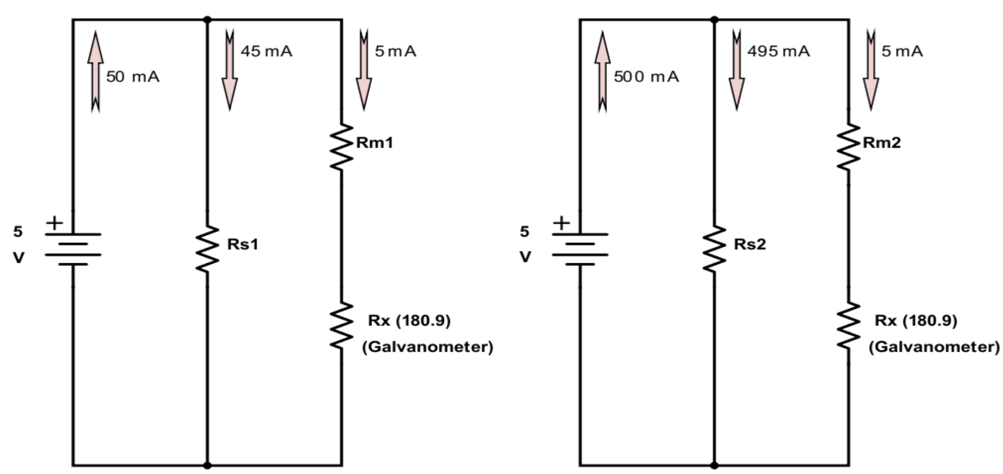
Given the current restriction of 0.5mA that can flow through the galvanometer, KVL and Ohm's laws were used to find appropriate minimum resistance values R_{m1} and R_{m2} which are **9819Ω** and **49819Ω** respectively (see Calculation 2.1). The circuits were then built as seen below (see Figure 2.2).

Figure 2.2



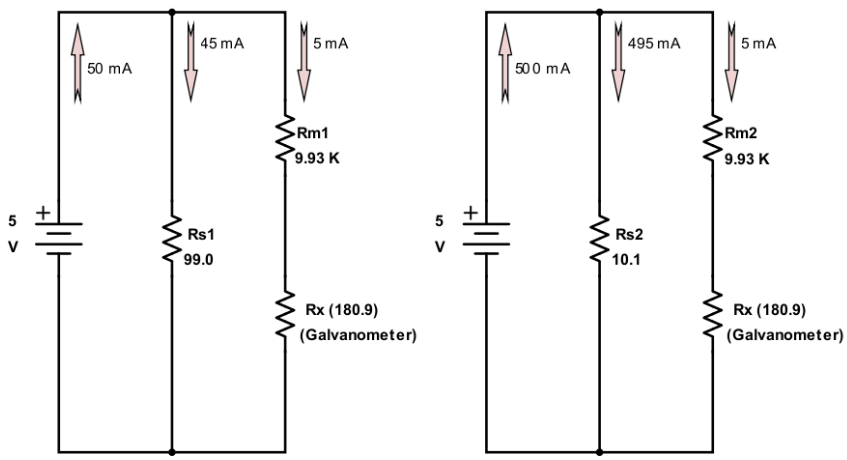
Two ammeters were created, one having a 0-50 mA range the other a 0-500 mA range, using the galvanometer from part 1 (see Figure 2.3).

Figure 2.3



Given the current restriction of the galvanometer, KCL and Ohm's law were used to determine the maximum resistance values for R_{s1} and R_{s2} which are **101.01Ω** and **10.101Ω** respectively (see Calculation 2.2). Because R_{m1} and R_{m2} continue to form a closed loop in their respective circuits, KVL applies the same as it did in Calculation 2.1. The circuits were then built as seen below (see Figure 2.4).

Figure 2.4



Part 3:

Given the circuit information below (see Figure 3.1a), the theoretical values for R_{L1}, R_{L2}, R_1, R_2, and R_S were found (see Figure 3.1b and Calculation 3.1). Note that the maximum power output is *assumed* to be 1.5 watts.

Figure 3.1a

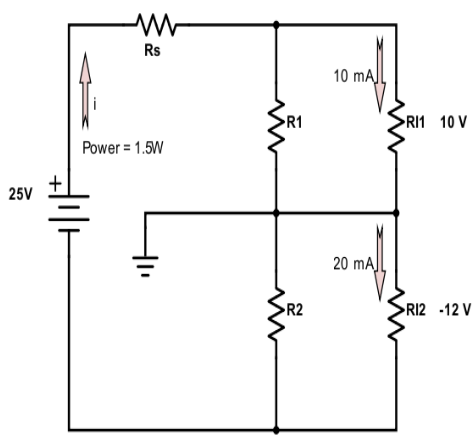
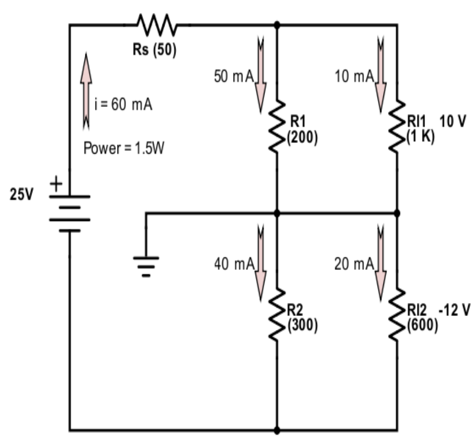


Figure 3.1b



The circuit was built and voltages were measured across R_{L1} and R_{L2} (see Figure 3.2a). These resistors were then removed and voltages were measured across R_1 and R_2 (see Figure 3.2b). The theoretical and real values were found to vary slightly, perhaps largely contributed to variance between the theoretical and actual resistor values and the assumption made on the power output. The Voltage Regulation (V.R.) was found to be **12%** across R_1 and **15%** across R_2 (see Calculation 3.2), which are both within the requirement of V.R. < 20%.

Figure 3.2a

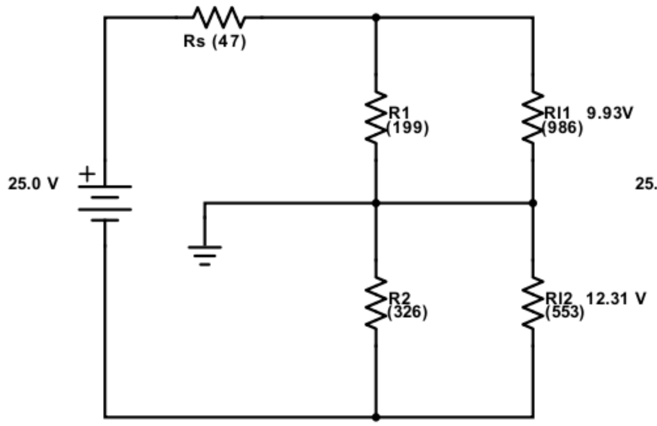
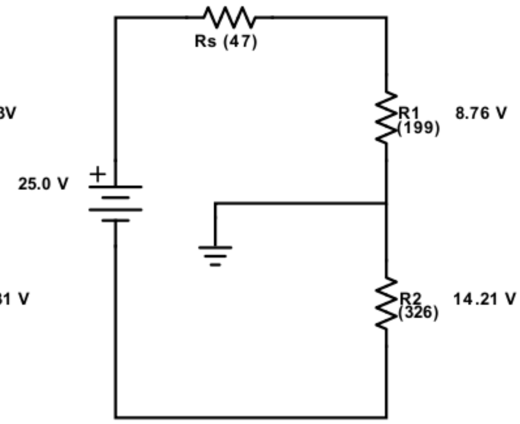


Figure 3.2b



Data & Calculations

Note:

For convenience, variables V (voltage), R (resistance), and I (current) will be subscripted based upon subscriptions in their respective diagrams. For example, the current across resistor R_3 will be represented as i_3, and the voltage across R_3 will be represented as V_3.

Calculation 1.1

$$V = iR \text{ and } i_3 < 0.5\ mA \text{ and } i_2 < 0.5\ mA$$

$$\textit{Note that the maximum voltage across } R_3 \textit{ is } 25V. \textit{ While it may never reach}$$

$$\textit{this value, setting } V_3 \textit{ to } 25V \textit{ will set } R_3 \textit{and } R_2 \textit{ to higher, safer values.}$$

$$Let\ V_3 = 25V \rightarrow 25 = 0.5mA(R_3)$$

$$R_3 \geq \frac{25}{0.0005} = \mathbf{50Kohms}$$

$$Repeat\ for\ R_2\ to\ also\ get\ \boldsymbol{R_2} \geq \mathbf{50Kohms}$$

Calculation 1.2

$$25V = V_3 + V_1 \text{ and } 25V = V_2 + V_x \quad \text{Using KVL (Kirchhoff's Voltage Law)}$$

$$25V = V_3 + V_1 \text{ and } 25V = V_2 + V_x$$

$$V_3 + V_1 = V_2 + V_x \quad \text{Use Ohm's Law}$$

$$i_3R_3 + i_1R_1 = i_2R_2 + i_xR_x \quad \textit{Note that } i_3 = i_1 \textit{ and } i_2 = i_x$$

$$i_3(R_3 + R_1) = i_2(R_2 + R_x)$$

$$R_x = \frac{i_3(R_3 + R_1)}{i_2} - R_2 \quad \text{Use Ohm's Law}$$

$$R_x = \frac{\left(\frac{V_3}{R_3}\right)(R_3 + R_1)}{\left(\frac{V_2}{R_2}\right)} - R_2 \quad \text{Note that } V_2 = V_3$$

$$R_x = \frac{(R_2R_3 + R_2R_1)}{R_3} - \frac{R_2R_1}{R_3}$$

$$\boldsymbol{R_x = \frac{R_2R_1}{R_3}} \quad \textbf{Formula 1.2.1}$$

Calculation 1.3

$$R_x = \frac{R_2R_1}{R_3} = \frac{(55.9K)(177.7)}{54.9K} = \mathbf{180.9\ ohms} \quad \text{Using formula 1.2.1}$$

Calculation 2.1

$V = V_x + V_m$ Using KVL (Kirchhoff's Voltage Law)

$V = R_x i_x + R_m i_m$ Use Ohm's Law. Note that $i_m = i_x$

$\mathbf{R_m} = \frac{(\mathbf{V} - (\mathbf{R_x i_x}))}{\mathbf{i_m}}$ **Formula 2.1.1**

$R_m = \frac{(5 - (180.9)(0.5mA))}{0.5mA} = \mathbf{9819\ ohms}$ Apply formula 2.1.1, with V = 5 Volts

$R_m = \frac{(25 - (180.9)(0.5mA))}{0.5mA} = \mathbf{49819\ ohms}$ Apply formula 2.1.1, with V = 25 Volts

Calculation 2.2

$Note: V_s = 5V, i_m = 0.5mA, i_x = 0.5mA$

$V_s = i_s R_s$ Use Ohm's Law.

$R_s = \frac{V_s}{i_s}$ **Formula 2.2.1, where $\mathbf{i_s} = (\boldsymbol{i} - \boldsymbol{i_x})$**

$R_{s1} = \frac{V_{s1}}{i_{s1}} = \frac{5}{50mA - .5mA} = \mathbf{101.01\ ohms}$ Apply formula 2.2.1, with i = 50 mA

$R_{s2} = \frac{V_{s2}}{i_{s2}} = \frac{5}{500mA - .5mA} = \mathbf{10.101\ ohms}$ Apply formula 2.2.1, with i = 500 mA

$Note\ that\ R_{m1} = R_{m2}.\ KVL\ applies\ to\ the\ same\ loop\ as\ it\ did\ in\ Calculation\ 2.1.\ Therefore, R_{m1} = \mathbf{9819\ ohms}$

Calculation 3.1

$P = Vi$ Definition of Power

$1.5 = 25i$

$i = 60\ mA$

$Note: i_1 = 50mA\ and\ i_2 = 40mA$ $Via\ KCL\ (Kirchhoff's\ Current\ Law)$

$25V = V_s + V_{l1} + V_{l2}$ Using KVL (Kirchhoff's Voltage Law)

$V_s = 25 - 10 - 12 = 3V$

$V_s = R_s i_s \rightarrow \boldsymbol{R_s} = \frac{V_s}{i_s} = \frac{3}{60mA} = \mathbf{50\ ohms}$ Use Ohm's Law

$\boldsymbol{R_{l1}} = \frac{V_{l1}}{i_{l1}} = \frac{10}{10mA} = \mathbf{1K\ ohm}$

$\boldsymbol{R_{l2}} = \frac{V_{l2}}{i_{l2}} = \frac{12}{20mA} = \mathbf{600\ ohm}$

$\boldsymbol{R_1} = \frac{V_1}{i_1} = \frac{10}{50mA} = \mathbf{200\ ohm}$

$$\boldsymbol{R_2} = \frac{V_2}{i_2} = \frac{12}{40mA} = \mathbf{300}\ \boldsymbol{ohm}$$

Calculation 3.2

Note: Definition of Voltage Regulation (V.R.) as follows:

$$VR = \left(\frac{(V_{oc} - V_l)}{V_l}\right) * 100\%$$

$$\boldsymbol{V}.\boldsymbol{R}._{\mathbf{1}} = \left(\frac{(V_{0c1} - V_{l1})}{V_{l1}}\right) * 100\% = = \left(\frac{(8.76 - 9.93)}{9.93}\right) * 100\% = \mathbf{12}\%$$

$$\boldsymbol{V}.\boldsymbol{R}._{\mathbf{2}} = \left(\frac{(V_{0c2} - V_{l2})}{V_{l2}}\right) * 100\% = = \left(\frac{(14.21 - 12.31)}{12.31}\right) * 100\% = \mathbf{15}\%$$

Discussion of Results

Part 1:

The experiment went as expected. The resistance of the galvanometer was found by using a Wheatstone bridge circuit. There was a bit of a snag with the galvanometer and potentiometer units. The first galvanometer unit had a very strange and volatile dial reading, and the first two potentiometer units were unable to reduce the read voltage to a near-zero value. After a bit of frustration, both the galvanometer and potentiometer units were replaced with working units.

Part 2:

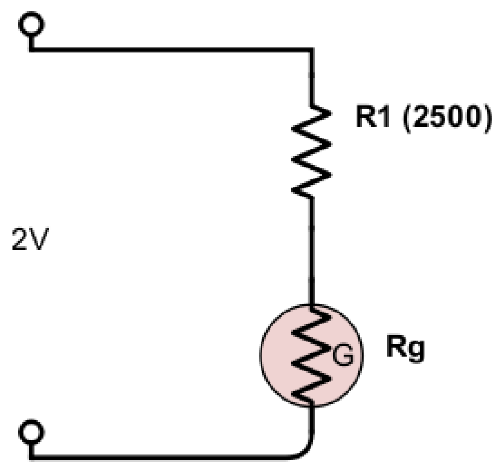
The experiments went as expected. The galvanometer voltmeter and ammeter were successfully created, and by using the selected resistors the galvanometer's dial was successfully limited to its 0.5mA range.

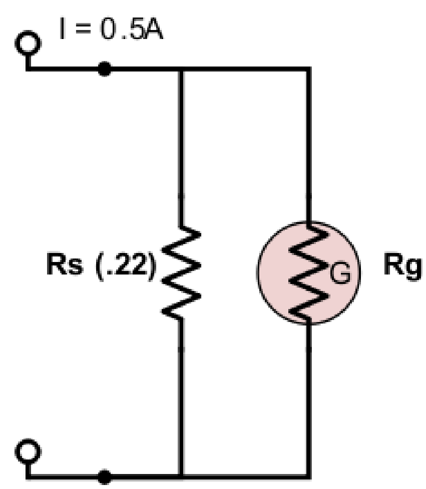
Part 3:

The measured voltage across R_1, R_{l1} was 9.93V, and across R_2, R_{l2} was 12.31V, which are comparable to the theoretical values of 10V and 12V respectively. While the calculated Voltage Regulation values, 12% and 15%, were within the acceptable thresholds of < 20%, they were far from ideal. It's interesting that the voltage increased across R_2 and decreased across R_1 when the load resistors were removed. Using Ohm's law, the current through Figure 3.2a is found to be 60mA, however, the current through Figure 3.2b is found to be only 44mA.

Appendix

Q: A particular galvanometer serves as a 2-V full scale voltmeter when a 2500 ohm is used as a multiplier resistor and it serves a 0.5 A ammeter when a 0.22 ohm shunt resistor is used. Determine the internal resistance of the galvanometer and the current required to produce full scale deflection.





$V_s = V_g \rightarrow .22i_s = R_g i_g$	KVL and Ohm's Law	1.1
$I = i_s + i_g \rightarrow 0.5A = i_s + i_g \rightarrow i_s = .5 - i_g$	KCL	1.2
$2V = V_1 + V_g \rightarrow 2 = 2500i_g + R_g i_g \rightarrow R_g i_g = 2 - 2500i_g$	KVL	1.3
$.22i_s = 2 - 2500i_g$	*Combine* 1.1 *and* 1.3	
$.22(.5 - i_g) = 2 - 2500i_g$	Insert 1.2 for i_s	
$\boldsymbol{i_g = 0.756\ mA}$	Solve for Current i_g	
$(0.756\ mA)R_g = 2 - 2500(0.756\ mA)$	Substitue found i_g *value into* 1.3	
$\boldsymbol{R_g = 145.5\ ohms}$	Solve for Resistance R_g	

To produce full scale deflection, the galvanometer should have an internal resistance of 145.5 ohms and there should be a current of 0.756 mA.