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Team #:

Day of Lab: Thursday @ 1pm

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Systems of Thin Lenses

Purposes:

1. To demonstrate the validity of the thin lenses in contact formula.
2. To measure the focal length of a negative (diverging) lens using a combination of lenses.
3. To construct a Galilean beam expander/collimator and measure its lateral magnification.
4. To observe the operation of the beam expander as a refracting telescope.

Required Equipment and Supplies:

Two thin lenses of unknown focal length, positive lens with a focal length of approximately 15 cm, negative lens of know index of refraction but unknown focal length, optical mounts, rods, rod connectors, metric mm tape measure and ruler, spherometer with flat glass plate, lighted object box, frosted glass plate and holder.

Introduction:

In this lab we will study combinations of thin lenses, all of which are assumed to be described by the thin lens formula,

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f} \qquad \text{Equation 1}$$

where p = object distance, q = image distance, and f = the focal length.

Although this equation is an approximation, it generally works well if the thickness of the lens (measured vertex to vertex) is much smaller (say 1% or less) than the focal length. As discussed in lecture, for more accurate calculations, the position of the principle planes must be found and the focal length adjusted for the lens thickness. Alternately, the matrix method may be applied for a thick lens. But in reality, most modern optical design is done using ray tracing software which is accurate to at least the fifth power in the expansion of the sine and the cosine functions.

Procedure Part A: Addition of focal length formula for thin lenses in contact

1. In lecture we discussed the addition of thin lens formulas:

$$\frac{1}{f_1}+\frac{1}{f_2}+\ldots=\frac{1}{f_{equivalent}} \qquad \text{Equation 2}$$

$$\text{or} \quad P_1+P_2+\ldots=P_{equivalent} \qquad \text{Equation 3}$$

where f is measured in cm but the power of the lens P is usually measured in diopters (reciprocal meters).

+20
+30

Using one of the techniques used in the Thin Lens and Mirror lab, measure the focal length of the two positive lenses supplied by the instructor. Record both focal lengths in the spaces below, including an estimated uncertainty.

f_1 = 20 +/- 1 cm f_2 = 30 +/- 1 cm

Using Equation 2, calculate the equivalent focal length of the two lenses above when placed in contact. Enter that value in the space below.

$f_{equivalent}$ = 12 +/- 1 cm

2. Now carefully tape both thin lenses together with masking tape around the circumference (make sure they're in contact) and place the two lenses in a single lens holder. Again, using one of the three techniques you learned in the Thin Lens and Mirror lab, measure the equivalent focal length of the combination of thin lenses.

Calculations Part A:

In the box below show your calculation for the equivalent focal length of the two thin lenses taped together. Calculate an uncertainty in $f_{equivalent}$, and show all your calculation and results in the box below. Also calculate a percent difference

$$\frac{1}{f_1}+\frac{1}{f_2}=\frac{1}{f_{equivalent}}=\left(\left(\frac{1}{20cm}\right)+\left(\frac{1}{30cm}\right)\right)^{-1}=12cm$$

$$f_{equiv}=\left(\left(\frac{1}{f_1}\right)+\left(\frac{1}{f_2}\right)\right)^{-1}$$

Use chain rule to obtain partial derviatives.

$$\frac{df(u)}{dx}=\frac{df}{du}*\frac{du}{dx}$$

$$f(u)^{-1}, u=\left(\frac{1}{f_1}+\frac{1}{f_2}\right)\rightarrow\frac{d}{du}(u^{-1})\frac{\partial f_{equiv}}{\partial f}\left(\frac{1}{f_1}+\frac{1}{f_2}\right)$$

$$\frac{\partial f_{equiv}}{\partial f_1}=\frac{f_2^2}{(f_2+f_1)^2},\ \frac{\partial f_{equiv}}{\partial f_2}=\frac{f_1^2}{(f_2+f_1)^2}$$

$$\frac{\partial f_{equiv}}{f_{equiv}}=\left\{\left[\frac{\left(\frac{\partial f_{equiv}}{\partial f_1}\right)\delta f_1}{f_{equiv}}\right]^2+\left[\frac{\left(\frac{\partial f_{equiv}}{\partial f_2}\right)\delta f_2}{f_{equiv}}\right]^2\right\}^{\frac{1}{2}}$$

$$\frac{\partial f_{equiv}}{f_{equiv}}=\left\{\left[\frac{\frac{f_2^2}{(f_2+f_1)^2}\delta f_1}{\left(\left(\frac{1}{f_1}\right)+\left(\frac{1}{f_2}\right)\right)^{-1}}\right]^2+\left[\frac{\frac{f_1^2}{(f_2+f_1)^2}\delta f_2}{\left(\left(\frac{1}{f_1}\right)+\left(\frac{1}{f_2}\right)\right)^{-1}}\right]^2\right\}^{\frac{1}{2}}$$

$$\frac{\partial f_{equiv}}{f_{equiv}}=\left\{\left[\frac{\frac{30cm^2}{(30cm+20cm)^2}(1cm)}{\left(\left(\frac{1}{20cm}\right)+\left(\frac{1}{30cm}\right)\right)^{-1}}\right]^2+\left[\frac{\frac{20cm^2}{(30cm+20cm)^2}(1cm)}{\left(\left(\frac{1}{20cm}\right)+\left(\frac{1}{30cm}\right)\right)^{-1}}\right]^2\right\}^{\frac{1}{2}}$$

$$=0.033\rightarrow 4\%\rightarrow 0.5cm$$

$$\%\,difference=\frac{|Value_{theoretical}-Value_{experimental}|}{Value_{theoretical}}*100$$

$$\%\,difference=\frac{|12-12|}{12}*100=0\%$$

$f_{equivalent}$ = 12 +/- 0.5 (4%) cm % difference = 0 %

Discussion Part A:

Taking into account the sum of the uncertainties that you estimated for your two values of $f_{equivalent}$, did you demonstrate the validity of Equation 2 ? For example, was your percent difference smaller than the combined uncertainties (also expressed as a percentage)? Would using a different method to measure the focal lengths have improved the precision of your results? Discuss in the box below.

The combined lenses created an equivalent focal length of 12cm. Using error propogation, the uncertainty was calculated to be 0.5cm, or 4%. The theoretical value for the new focal length was 12cm. As such, the percent difference between the theoretical and experimental values is 0%, which is within the margin of error of 4%. These results completely comply with, and strongly support, equation 2.

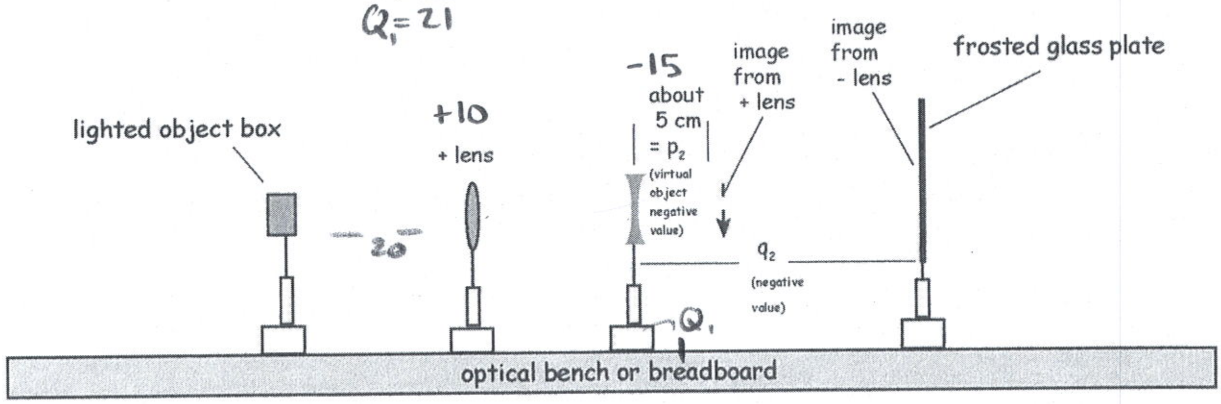
While tedious to setup, the parallel-laser-beam-to-converging-point-on-a-back-surface technique is a very accurate method. The most difficult portion of this experiment is the needed precision in setting the beams parallel with each other and the principal axis. Other techniques, though easier to setup and execute, would most likely not yield as accurate results.

Procedure Part B: Measuring the focal length of a diverging (negative) lens

1. Measuring the focal length of a diverging lens or mirror is not as easy as it is for a converging element because real images of real objects cannot directly be formed, therefore the measurement must be made indirectly. One common technique that we use here is to form a real image using a converging lens and then use that image as a virtual object for the diverging lens.

2. Set up the apparatus as shown below. Some modifications in the exact position of the components may be needed depending on the particular focal lengths of the lenses you use (see the professor for assistance).

P_1 20
Q_1 = 21



3. With the negative lens removed from the system, use the frosted glass plate to find the position of the real image formed by the positive lens. Note the position of the image and place the unknown negative lens about 5 cm to the left of the first image position. The distance from the first image to the center of the negative lens is = p_2 the virtual object distance (a negative quantity). $\frac{1}{F} = \frac{1}{P} + \frac{1}{Q}$ $\frac{1}{-15} = \frac{1}{-5} + \frac{1}{Q}$

4. Slide the frosted plate to the right until a new focused image is obtained. Measure the new image distance, this is q_2 (a positive quantity) and use the thin lens equation to find the focal length of the diverging lens (a negative quantity). Record your values for p_2 and q_2 in the spaces provided below. If no focused real image can be seen on the frosted screen, see your professor.

p_2 = - 5 cm	q_2 = + 7.5 cm

5. If the diameter of the lens is large enough, use the spherometer and the known index of refraction (ask your professor or lab tech for the value) to find the focal length. Record your values of R_1 and R_2 in the spaces below.

R_1 = ________ +/- ________ cm	R_2 = ________ +/- ________ cm
n = ________	

Calculations Part B:

Using your values for p_2 and q_2, use Equation 1 to calculate the focal length of the negative lens. If available, use the spherometer data and the known value of n_r to also calculate the focal length. Show your calculations in the box below (including uncertainty) and enter your results in the spaces provided.

$$\frac{1}{P_2}+\frac{1}{Q_2}=\frac{1}{f}=\left(\left(\frac{1}{-5cm}\right)+\left(\frac{1}{7.5cm}\right)\right)^{-1}=-15cm$$

$$f=\left(\left(\frac{1}{P_2}\right)+\left(\frac{1}{Q_2}\right)\right)^{-1}$$

Use chain rule to obtain partial derviatives.

$$\frac{df(u)}{dx}=\frac{df}{du}*\frac{du}{dx}$$

$$f(u)^{-1}, u=\left(\frac{1}{P_2}+\frac{1}{Q_2}\right)\rightarrow\frac{d}{du}(u^{-1})\frac{\partial f}{[\partial P\mid\partial Q]}\left(\frac{1}{P_2}+\frac{1}{Q}\right.$$

$$\frac{\partial f}{\partial P_2}=\frac{Q_2^2}{(P_2+Q_2)^2},\ \frac{\partial f}{\partial Q_2}=\frac{P_2^2}{(P_2+Q_2)^2}$$

$$\frac{\partial f}{f}=\left\{\left[\frac{\left(\frac{\partial f}{\partial P_2}\right)\delta P_2}{f}\right]^2+\left[\frac{\left(\frac{\partial f}{\partial Q_2}\right)\delta Q_2}{f}\right]^2\right\}^{\frac{1}{2}}$$

$$\frac{\partial f}{f}=\left\{\left[\frac{\frac{Q_2^2}{(P_2+Q_2)^2}\delta P_2}{\left(\left(\frac{1}{P_2}\right)+\left(\frac{1}{Q_2}\right)\right)^{-1}}\right]^2+\left[\frac{\frac{P_2^2}{(P_2+Q_2)^2}\delta Q_2}{\left(\left(\frac{1}{P_2}\right)+\left(\frac{1}{Q_2}\right)\right)^{-1}}\right]^2\right\}^{\frac{1}{2}}$$

$$\frac{\partial f}{f}=\left\{\left[\frac{\frac{7.5cm^2}{(-5cm+7.5cm)^2}(0.5cm)}{\left(\left(\frac{1}{-5cm}\right)+\left(\frac{1}{7.5cm}\right)\right)^{-1}}\right]^2+\left[\frac{\frac{5cm^2}{(-5cm+7.5cm)^2}(0.5cm)}{\left(\left(\frac{1}{-5cm}\right)+\left(\frac{1}{7.5cm}\right)\right)^{-1}}\right]^2\right\}^{\frac{1}{2}}$$

$$=0.33\rightarrow 33\%\rightarrow 5cm$$

Note: This high uncertainty is not suprising, given that the distance from object P_2 to image Q_2 has a net value of 2.5cm, and absolute error was taken as 0.5cm which is 20% of 2.5cm

$$\%\,difference=\frac{|Value_{theoretical}-Value_{experimental}|}{Value_{theoretical}}*100$$

$$\%\,difference=\frac{|15-15|}{15}*100=0\%$$

$f_{\text{thin lens formula}}$ = - 15 +/- 5 cm (33%) $f_{\text{spherometer}}$ = ~~_____ +/- _____~~ cm

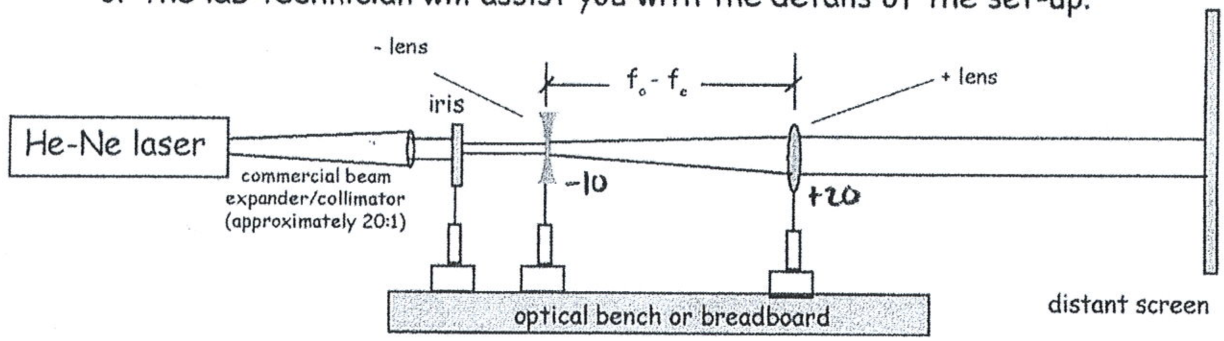
% difference = 0 %

Part C: Construction and Measurement of a Galilean Beam Expander/Collimator

Beam expander/collimators are often used in conjunction with lasers to increase beam size while at the same time decreasing beam divergence. The Galilean expander constructed here is simply a Galilean astronomical telescope used in reverse. As discuss in lecture, with a well-engineered expander the lateral magnification is simply the negative ratio of the positive lens' focal length divided by the focal length of the negative lens. In the ideal case, the output beam divergence of a Gaussian laser beam will be reduced (relative to the input beam divergence) by that same ratio.

Procedure Part C:

1. Assemble the beam expander as shown in the figure below. Your professor or the lab technician will assist you with the details of the set-up.



2. Adjust the spacing between the lenses until the output beam is collimated (check its diameter with a metric ruler close to the second lens and again on a distant wall or screen), Be careful not to reflect the laser beam into your eye!

3. Carefully measure the beam diameter D_1 with a metric ruler as it exits the iris (or use a more accurate method if available - ask the professor). Also measure the collimated beam diameter D_2 on the distant screen. Use the ratio of beam sizes to obtain the experimental value for the lateral magnification of the beam expander/collimator. Enter those values in the spaces below.

D_1 = 0.86 +/- 0.02 cm	D_2 = 1.68 +/- 0.02 cm
experimental	$M_L = D_2/D_1$ = 1.95 +/- 2%

4. Use the formula $M_L = -(f_o/f_e)$ to obtain the theoretical value for the beam expander's lateral magnification. Enter that value in the space below along with a percent difference between the theoretical value and experimental value calculated above.

theoretical	$M_L = f_o/f_e$ = 2 +/- 0%
	% difference = 3 %

Were your two values for M_L equal within the limits of the experimental uncertainty? Discuss briefly in the box below.

$$M_L = \frac{D_2}{D_1} = \frac{1.68cm}{0.86cm} = 1.95$$

$$M_L = \frac{D_2}{D_1}$$

$$\frac{\partial M}{\partial D_1} = -\frac{D_2}{D_1^2}, \frac{\partial M}{\partial D_2} = \frac{1}{D_1}$$

$$\frac{\partial M}{M} = \left\{\left[\frac{\left(\frac{\partial M}{\partial D_1}\right)\delta D_1}{M}\right]^2 + \left[\frac{\left(\frac{\partial M}{\partial D_2}\right)\delta D_2}{M}\right]^2\right\}^{\frac{1}{2}}$$

$$\frac{\partial M}{M} = \left\{\left[\frac{\left(-\frac{D_2}{D_1^2}\right)\delta D_1}{\frac{D_2}{D_1}}\right]^2 + \left[\frac{\left(\frac{1}{D_1}\right)\delta D_2}{\frac{D_2}{D_1}}\right]^2\right\}^{\frac{1}{2}}$$

$$\frac{\partial M}{M} = \left\{\left[\frac{\delta D_1}{D_1}\right]^2 + \left[\frac{\delta D_2}{D_2}\right]^2\right\}^{\frac{1}{2}} = \left\{\left[\frac{0.02cm}{0.86cm}\right]^2 + \left[\frac{0.02cm}{1.68}\right]^2\right\}^{\frac{1}{2}}$$

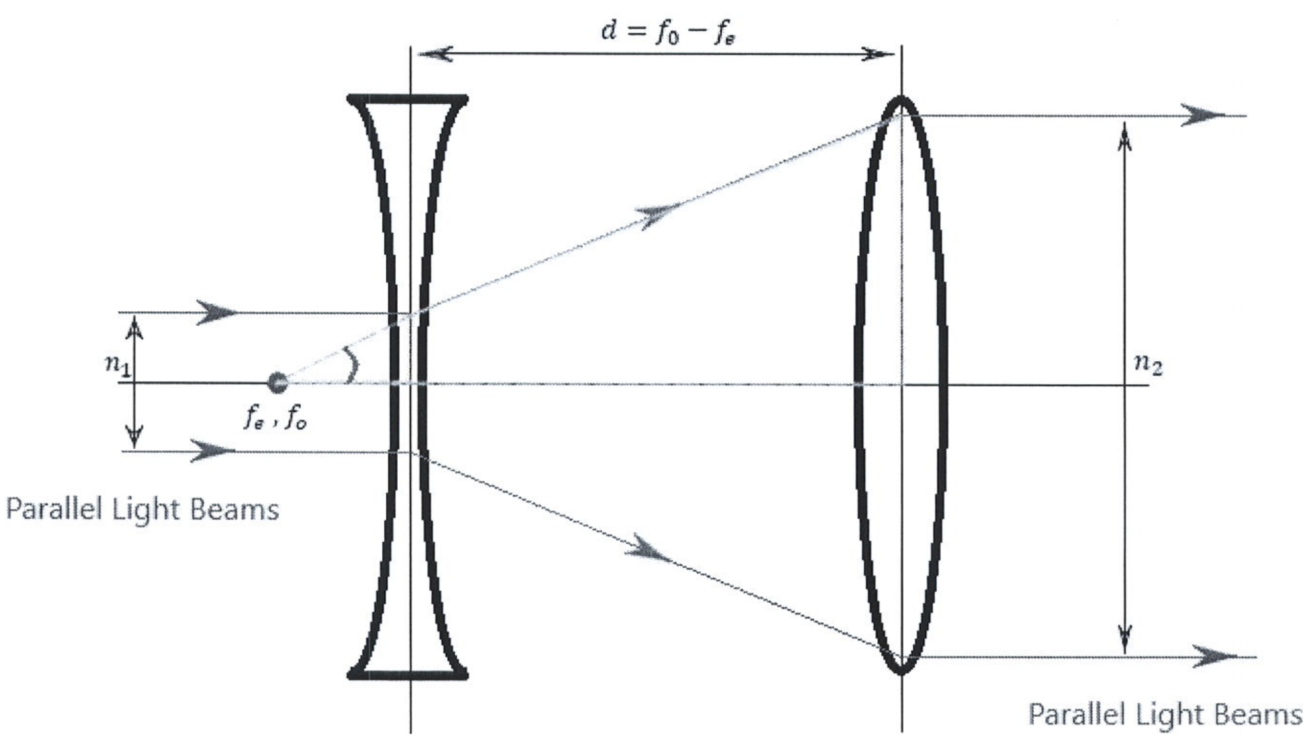
$$= 0.03cm \rightarrow 2\%$$

$$\% \, difference = \frac{|Value_{theoretical} - Value_{experimental}|}{Value_{theoretical}} * 100$$

$$\% \, difference = \frac{|2 - 1.95|}{2} * 100 = 3\%$$

The percent descrepancy is 3%, which is not within the margin of error of 2%. Regardless, relative to the small measurements the experiment yielded excellent results.

Jared Fowler



$$\frac{1}{f_e} = \frac{1}{P_1} + \frac{1}{Q_1}$$	Thin lens formula
$$\frac{1}{f_e} = \frac{1}{\infty} + \frac{1}{Q_1}$$	Incoming parallel light beams indicate that the object is at infinite distance.
$$Q_1 = f_e$$	Solve. Note that the focal length of a diverging lens is negative.
$$\rightarrow P_2 = \lvert d\rvert + \lvert f_e\rvert$$	
$$\frac{1}{f_o} = \frac{1}{P_2} + \frac{1}{Q_2}$$	Thin lens formula
$$\frac{1}{f_o} = \frac{1}{d + \lvert f_e\rvert} + \frac{1}{\infty}$$	Outgoing parallel light beams indicate that the image is at infinite distance.
$$f_o = d + \lvert f_e\rvert \rightarrow \boldsymbol{d} = \boldsymbol{f_o} - \lvert \boldsymbol{f_e}\rvert$$	Solve.
$$tan\theta = \frac{n_1}{2f_e} \; and \; tan\theta = \frac{n_2}{2f_o}$$	(see Diagram – purple and green triangles)
$$\frac{n_1}{2f_e} = \frac{n_2}{2f_o} \rightarrow \frac{n_1}{f_e} = \frac{n_2}{f_o} \rightarrow \frac{n_2}{n_1} = \frac{f_o}{f_e}$$	
$$Note: \; Magnificaiton \; 'M' = \frac{h_i}{h_o} = \frac{n_2}{n_1} \rightarrow \; \boldsymbol{M} = \frac{\boldsymbol{f_o}}{\boldsymbol{f_e}}$$	Magnification is defined as image height over object height. Solve.

Conclusion

Experiment A:

The equivalent focal length of two combined lenses, one with focal length 20cm and the other with focal length 30cm, was found to be 12cm with a 4% uncertainty. The percent difference from the theoretical value of 12cm is 0%, which is within the margin of error. The experiment required great precision to be taken, ensuring that the two laser beams were parallel with each other and with the principal axis of the lenses. Because of the small diameter of the lenses, it was difficult to position the lasers close enough together without having interference from the surrounding apparatus. The experiment could be improved by using a beam splitter with a single laser.

Experiment B:

The focal length of the diverging lens was found to be -15cm with a 33% uncertainty. The uncertainty was obtained by error propagation and is surprisingly high at first glance. With closer observation, however, the high value makes sense when comparing the ratio of absolute error (0.5cm) to the distance between the virtual object and final image (2.5cm). The percent difference from the theoretical value of -15cm is 0%, which is within the margin of error. The experiment could be improved by using different lenses and/or repositioning the apparatus such that the distance between the virtual object and final image is increased. This could be achieved by simply moving the diverging lens closer to the converging lens, making P_2 -10cm instead of -5cm.

Experiment C:

The magnification of the beam expander was experimentally found to be 1.95 with a 2% uncertainty. The percent difference from the theoretical value of 2 is 3%, which is not within the margin of error. While diameters D_1 and D_2 were measured with a very accurate tool, it was difficult to properly measure the final expanded beam. Also, with the small magnitude of magnification, the ratio of absolute error to measured value is high. The experiment could be improved by using an iris with a larger diameter, or by using different lenses and/or repositioning the apparatus to produce a larger final beam, thus reducing the error to measured value ratio.