Lab partners: Jared Fowler | Team #:

Chikheang Soeng | Day of Lab: Thursday

Date: Feb.7, 2019

Measuring the Index of Refraction of Glass and Water

Purpose:

To measure the index of refraction of glass and water.

Required Equipment and Supplies:

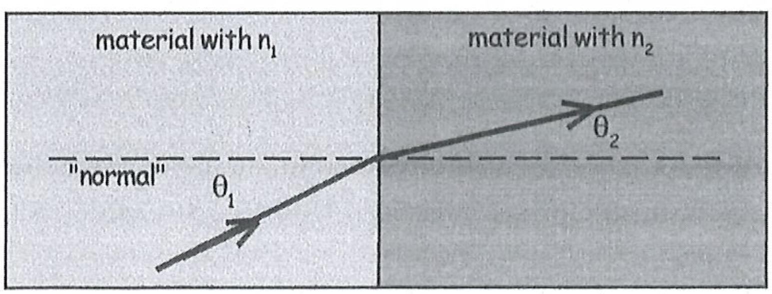
corkboard, pins, copier paper, metric ruler, protractor, glass cube, plastic refraction tank, deionized water supply, equilateral glass prism, He-Ne laser with mount

Discussion:

In lecture we have studied Snell's law, the law of refraction,

$$n_1 \sin \theta_1 = n_2 \sin \theta_2 \qquad (1)$$

where n_1 and n_2 are the indexes of refraction of materials on either side of a boundary and the angle of incidence θ_1 and the angle of refraction θ_2 are measured with respect to the normal to the surface as shown in the figure below.



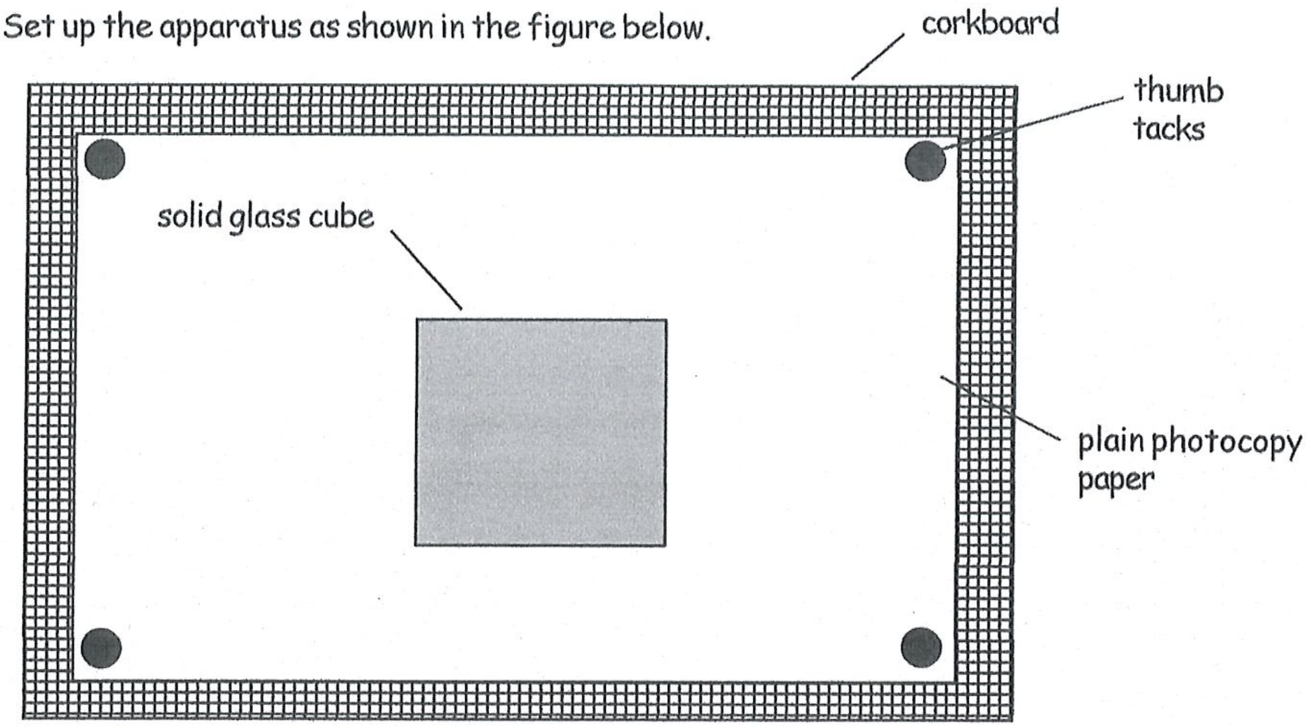
The index of refraction is a dimensionless number which tells us how much light will slow down below its speed in a vacuum ($c = 3.00 \times 10^8$ m/s) as it propagates through material. The speed of light v in a material of index of refraction n is given by the simple relationship,

$$v = \frac{c}{n} \qquad (2)$$

As discussed in lecture, it is this slowing down of the wavefront which causes the change in direction of the wave that we refer to as refraction.

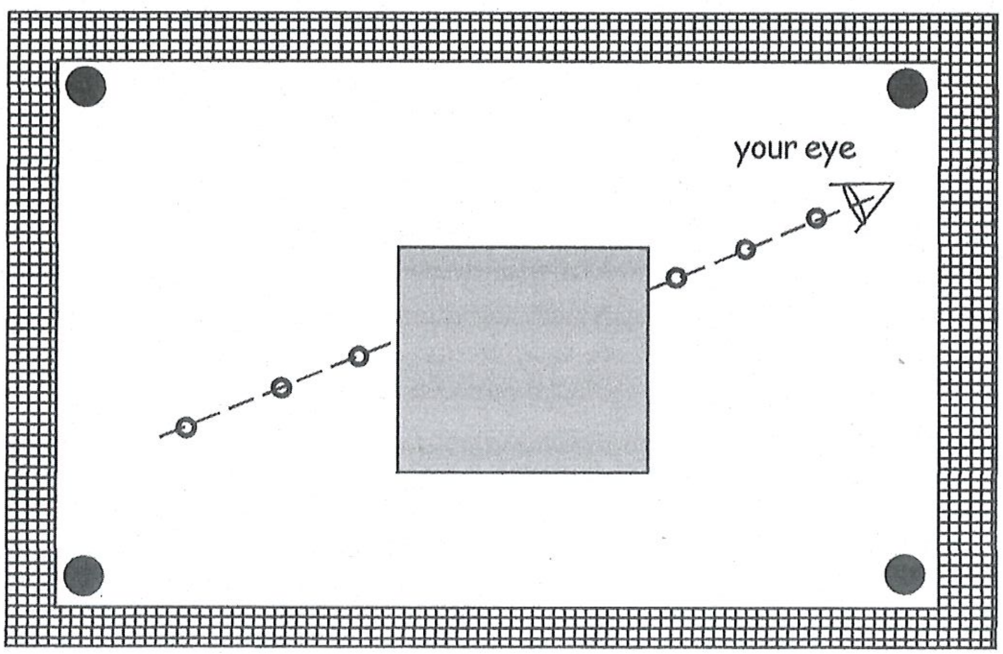
Procedure Part 1 - Measuring the Index of Refraction of a Glass Cube.

1. Set up the apparatus as shown in the figure below.



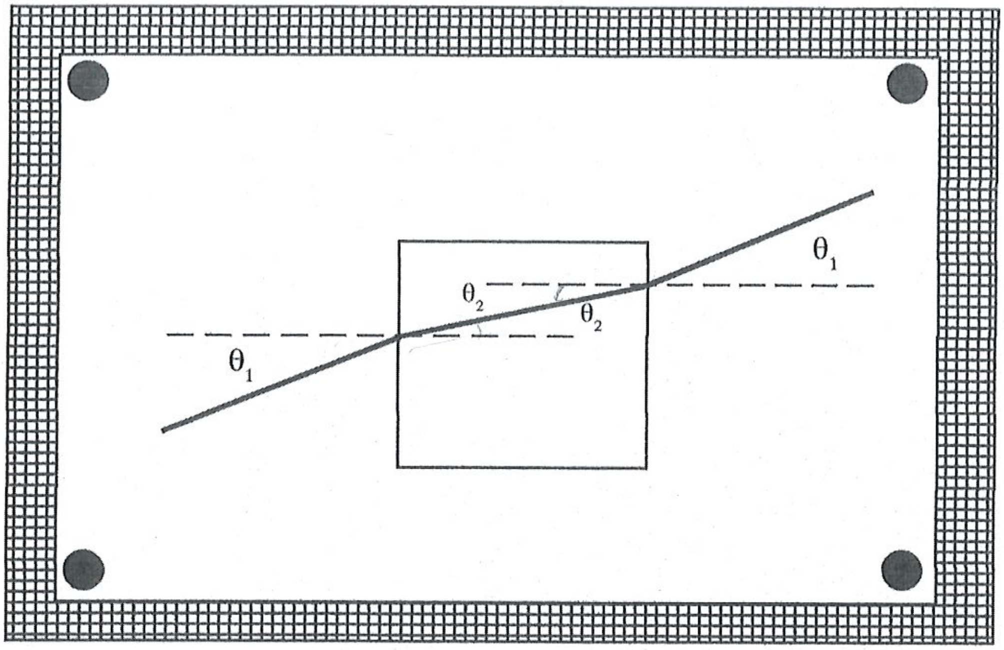
top view

2. Before you do anything else, trace a fine pencil line around the edge of the cube - just in case you bump it during the experiment!



top view

3. Looking through one side of the cube, carefully place three pins in a line on either side of the cube so that they appear to your eye to form a single line. Your results will come out better if the angle of the line you define relative to the normal is not too small (say at least 30^o).



top view

4. Don't remove the cube yet, but when you do you will be able to draw a ray trace as shown above. If all goes well your two angles inside the glass should be equal and your two angles outside the glass should be equal. Now use your six pins to define two more rays through the glass cube using the same technique..

5. If you remembered to trace around the cube (did you?) you can now remove the cube and connect the pin marks as shown in the sample above. Using a protractor, measure the angles for all three cases. Try to measure to an accuracy of +/- 0.5°. *(For the A+ students: A more precise alternative is to use a mm ruler and measure the angles by the inverse tangent.)* Enter your results in the table below.

Plastic

Trial #	θ_1	θ_1	$\theta_{1average}$	θ_2	θ_2	$\theta_{2\ average}$	n_2
1	28.9°	30.7°	29.8°	20.1°	21.8°	21.0°	1.39
2	44.3°	42.2°	43.3°	32.0°	33.3°	32.7°	1.27
3	59.8°	59.7°	59.8°	34.7°	37.1°	35.9°	1.47

6. To fill in the last column on the right you will have to use Snell's law and assume that the index of refraction of air is 1.00. Show **one sample** of your calculation in the box on the next page.

Sample Snell's Law Calculation

Done for Trial 1

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \rightarrow n_2 = \frac{n_1 \sin\theta_1}{\sin\theta_2}$$

$$n_2 = \frac{(1)\sin(29.8^\circ)}{\sin(21.0^\circ)} = 1.39$$

Our average value of n_2 for ~~glass~~ Plastic = 1.38

6. Believe it or not, when the College purchased the glass cubes the manufacturer didn't tell us the "accepted" value for the index, so you have no way of calculating a % error. So like in real research we can only calculate a % difference relative to some other "research group". Get the results from one other lab team and show your calculations for a % difference in the box below.

We compared our results to Team # ________

Another Team: 1.46
Internet: 1.49

% difference (other team) 0.055 → 6%

% difference (Internet) 0.073 → 8%

Summing up Part #1:

In the box below, briefly state what you think were the major sources of error in this experiment. Are there any ways that the potential error could be reduced?

See Discussion of Results

Procedure Part 2 - Measuring the Index of Refraction of Water

1. This part should be easy! Just replace the glass cube with a plastic tank filled with deionized water and do it all again! A blank data table is shown below. The only difference is that we do know the index of deionized water (1.33) so you can calculate a % error.

Trial #	θ_1	θ_1	$\theta_{1average}$	θ_2	θ_2	$\theta_{2\ average}$	n_2
1	27.5°	27.4°	27.5°	20.6°	20.8°	20.7°	1.31
2	46.1°	43.5°	44.8°	27.6°	34.5°	31.1°	1.36
3	31.9°	29.0°	30.5°	22.8°	23.6°	23.2°	1.29

Sample Snell's Law Calculation

Done for Trial 1

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \rightarrow n_2 = \frac{n_1 \sin\theta_1}{\sin\theta_2}$$

$$n_2 = \frac{(1)\sin(27.5°)}{\sin(20.7°)}$$

$$n_2 = 1.31$$

Our average value of n_2 for water = 1.32

% error = 0.008 → 1%

Staple your ray traces to your completed report.

Jared Fowler

Error Propagation – Snell's Law

$$n_1 sin\theta_1 = n_2 sin\theta_2$$ Snell's Law

$$n_2 = \frac{sin\theta_1}{sin\theta_2}$$ Solve for n_2 and assume n_1 is 1.0

$$\frac{\partial n_2}{\partial \theta_1} = \frac{cos\theta_1}{sin\theta_2}, \quad \frac{\partial n_2}{\partial \theta_2} = -\frac{sin\theta_1 cos\theta_2}{\sin^2\theta_2}$$ Take partial derivatives for θ_1 and θ_2

$$\delta n_2 = \left\{\left[\delta\theta_1\left(\frac{\partial n_2}{\partial \theta_1}\right)\right]^2 + \left[\delta\theta_2\left(\frac{\partial n_2}{\partial \theta_2}\right)\right]^2\right\}^{\frac{1}{2}}$$ Definition for absolute error.

$$\frac{\delta n_2}{n_2} = \left\{\left[\frac{\delta\theta_1\left(\frac{\partial n_2}{\partial \theta_1}\right)}{n_2}\right]^2 + \left[\frac{\delta\theta_2\left(\frac{\partial n_2}{\partial \theta_2}\right)}{n_2}\right]^2\right\}^{\frac{1}{2}}$$ Definition for relative error.

$$\frac{\delta n_2}{n_2} = \left\{\left[\frac{\delta\theta_1\left(\frac{cos\theta_1}{sin\theta_2}\right)}{\frac{sin\theta_1}{sin\theta_2}}\right]^2 + \left[\frac{\delta\theta_2\left(-\frac{sin\theta_1 cos\theta_2}{\sin^2\theta_2}\right)}{\frac{sin\theta_1}{sin\theta_2}}\right]^2\right\}^{\frac{1}{2}}$$ Substitute

$$\frac{\delta n_2}{n_2} = \{[\delta\theta_1 cot\theta_1]^2 + [\delta\theta_2 cot\theta_2]^2\}^{\frac{1}{2}}$$ Simplify

NOTE 1.1

For each experiment, use the smallest measured angle data set to estimate maximum error. Take the absolute error for both θ_1 and θ_2 to be the maximum angle difference between each congruent angle set, that is, θ_1 to θ_1 and θ_2 to θ_2 for each trial. For example, the maximum difference of θ_1 to θ_1 for plastic was on trial 2, where $\boldsymbol{\delta\theta_1 = 44.3 - 42.2 = 2.1}$

Plastic: $\boldsymbol{\delta\theta_1 = 2.1°,\ \ \delta\theta_2 = 2.4°}$
Water: $\boldsymbol{\delta\theta_1 = 2.9°,\ \ \delta\theta_2 = 6.9°}$

Plastic:

$$\frac{\delta n_2}{n_2} = \left\{\left[\left(\frac{2.1\pi}{180}\right)\cot(29.8°)\right]^2 + \left[\left(\frac{2.4\pi}{180}\right)\cot(21.0°)\right]^2\right\}^{\frac{1}{2}} = 0.127 \rightarrow \mathbf{13\%}\ \boldsymbol{error}$$

Water:

$$\frac{\delta n_2}{n_2} = \left\{\left[\left(\frac{2.9\pi}{180}\right)\cot(27.5°)\right]^2 + \left[\left(\frac{6.9\pi}{180}\right)\cot(20.7°)\right]^2\right\}^{\frac{1}{2}} = 0.333 \rightarrow \mathbf{34\%}\ \boldsymbol{error}$$

Percent Discrepancies

$$\%\ Discrepancy = \left|\frac{Value_{Theoretical} - Value_{Experimental}}{Value_{Theoretical}}\right| * 100$$ Definition of percent discrepancy.

$$\%\ Discrepancy = \left|\frac{1.49 - 1.38}{1.49}\right| * 100 = \mathbf{8\%}$$ Plastic

$$\%\ Discrepancy = \left|\frac{1.33 - 1.32}{1.33}\right| * 100 = \mathbf{1\%}$$ Water

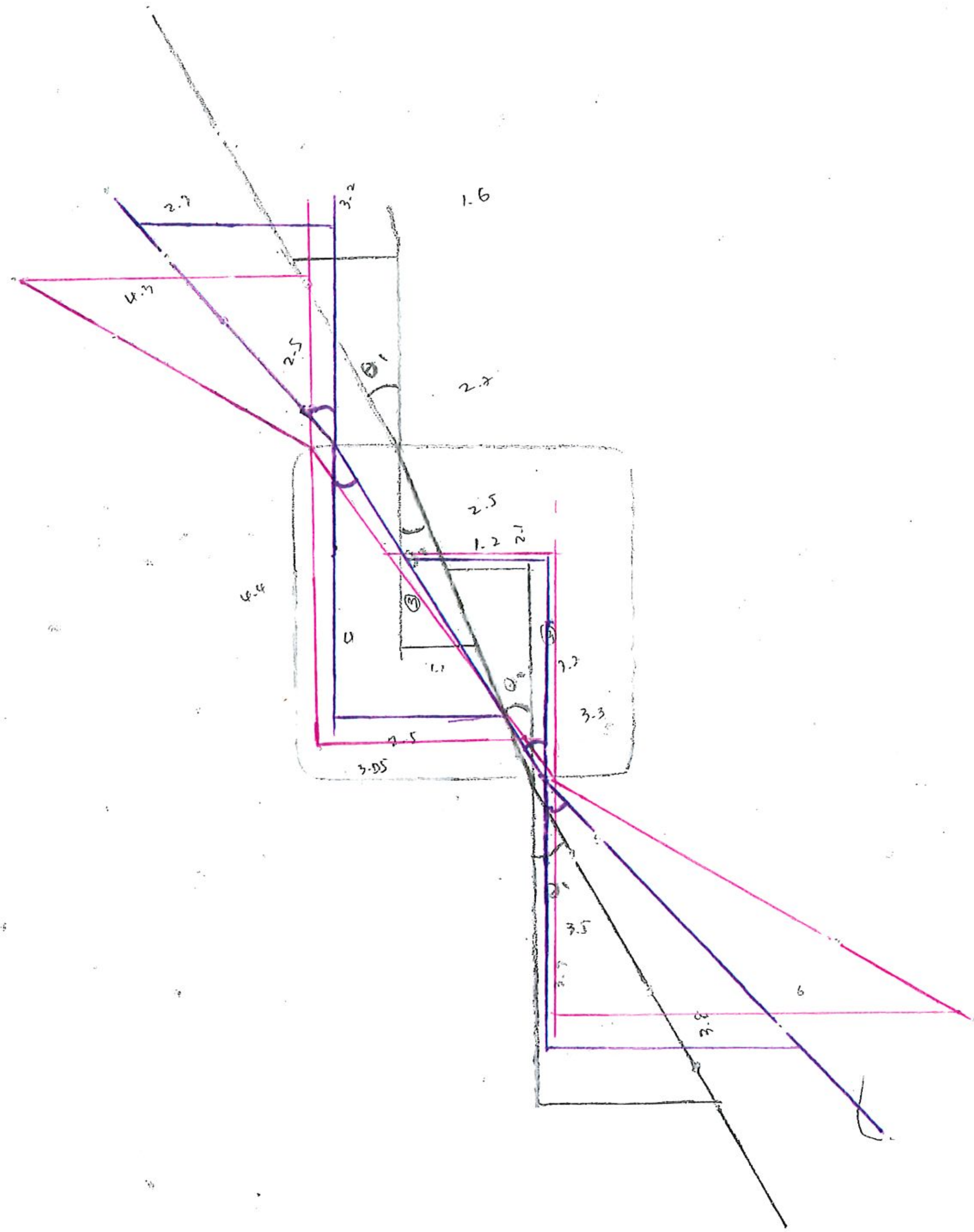
Jared Fowler

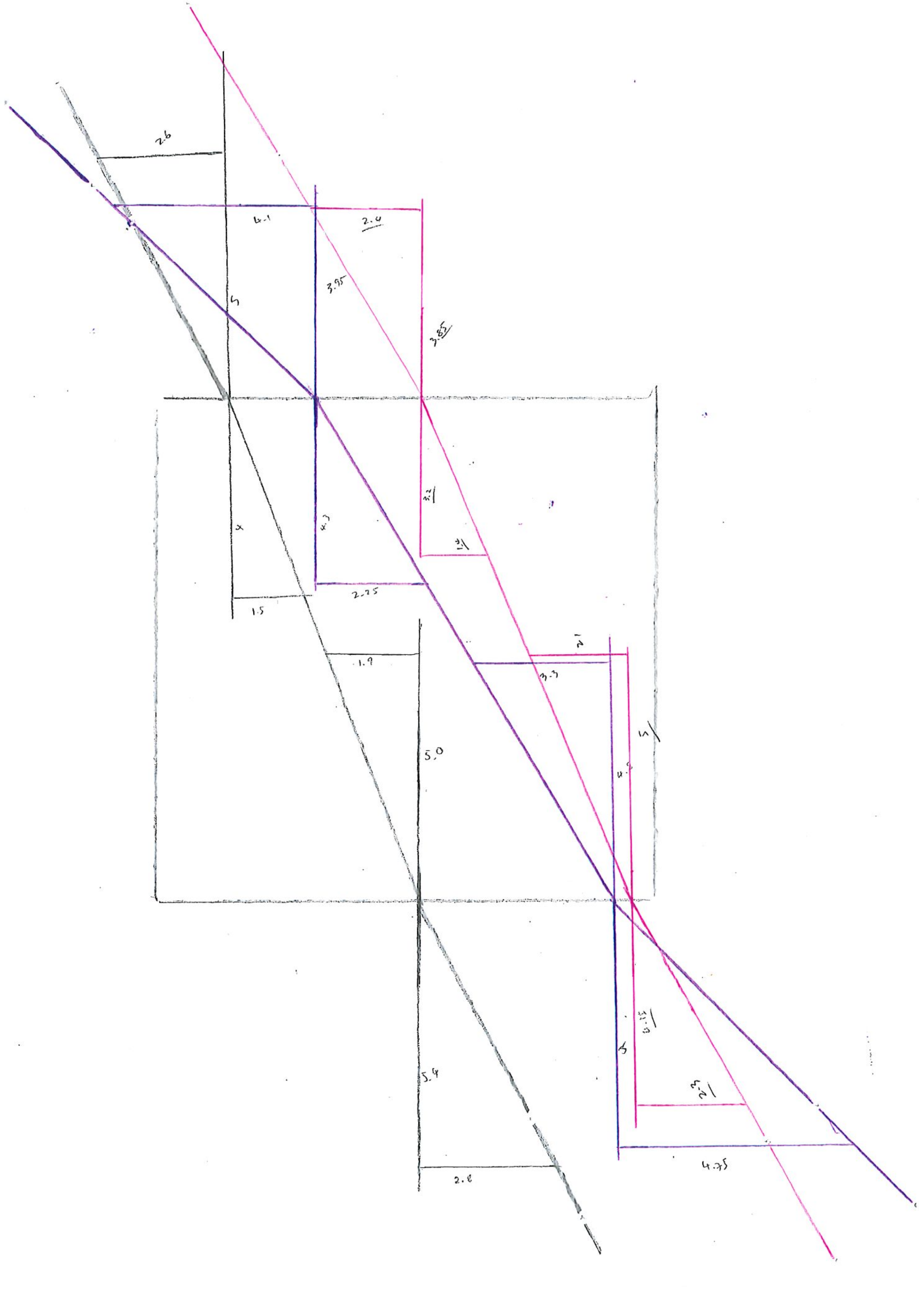
Discussion of Results / Conclusion

The experiments yielded favorable results consistent with the theoretical indices of refraction of plastic (1.49) and water (1.33). As no theoretical index of refraction for the plastic cube was given, a value from the National Physical Laboratory (www.npl.co.uk) was used. The index of refraction of plastic was experimentally found to be $\mathbf{1.38 \pm 13\%}$, and water $\mathbf{1.32 \pm 34\%}$. The percent discrepancies of these results are **8%** and **1%** respectively, which are both well-within the allotted margins of error.

The margins of error were found using an error propagation technique. The absolute errors for each angle, θ_1 and θ_2, were analyzed separately for each experiment, and determined to be the maximum differences between measured congruent angles for all three trials (see **Note 1.1**). This generous margin of error helps account for the major sources of error, primarily that of human observation. Placing pins on both sides of a cube while looking through the cube such that the pins lined-up with human visual perception was undoubtedly the most erroneous portion of the experiment. The water cube proved to be even more erroneous due to the larger distance between pens on each side of the cube, thus increasing the observational fuzziness related to human nearsighted versus farsighted focus. Interestingly, the overall discrepancy was greater for the plastic than it was for the water, even though the water had a greater margin of error. It's worth noting that the theoretical value for plastic may be incorrect as the exact type of plastic, which is unknown, can significantly change the index of refraction.

The experiment could have been significantly improved by using a laser to penetrate through the cube in place of human observation of ambient light passing through the cube. This would lead to a much more accurate ray trace, and in turn, very precise angles.


2.7
3.2
1.6
4.7
2.5
2.7
2.5
1.2
4.4
4
1.1
3.2
3.3
2.5
3.05
3.5
6
3.8


2.6
4.1
2.4
3.95
5
3.85
x
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1.5
2.25
1.9
3.3
5.0
5.4
2.8
4.75