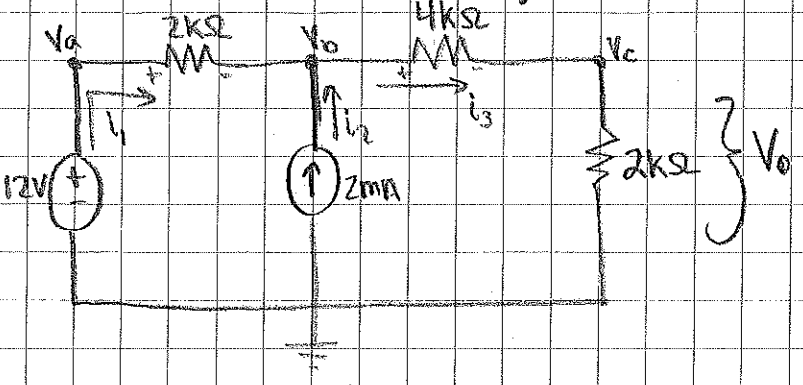
Engr. M20 - HW#3

Jared Fowler

1) Find V_o across the 2k resistor, using nodal analysis.



KCL @ V_b → $i_3 = i_1 + 2mA$

$$\frac{V_b - V_c}{4k} = \frac{V_a - V_b}{2k} + 2mA$$

$$\frac{V_b}{4k} - \frac{V_c}{4k} = \frac{V_a}{2k} - \frac{V_b}{2k} + 2mA$$

$$V_b - V_c = 2V_a - 2V_b + 8$$

$$2V_a - 3V_b + V_c = -8 \quad (1)$$

KCL @ V_c → $i_3 = i_3$

$$\frac{V_b - V_c}{4k} = \frac{V_c - 0}{2k}$$

$$\frac{V_b}{4k} - \frac{V_c}{4k} = \frac{V_c}{2k}$$

$$V_b - V_c = 2V_c$$

$$V_b = 3V_c \quad (2)$$

Note: $V_a = 12V$ (3)

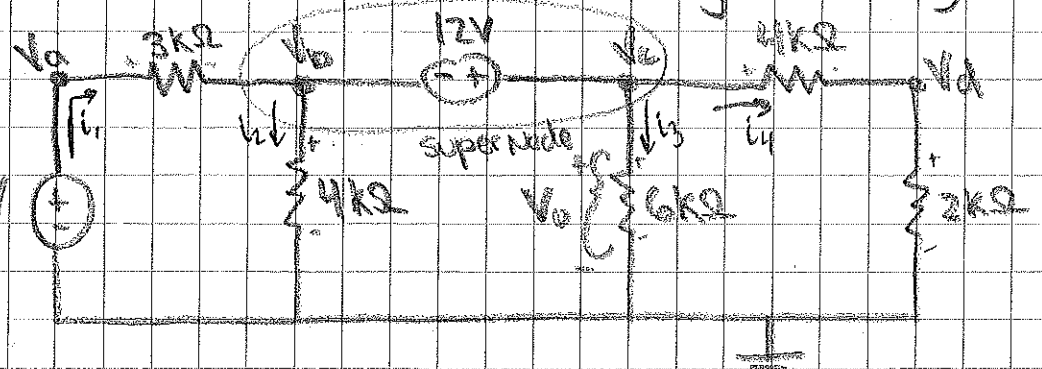
(1) → $2(12) - 9V_c + V_c = -8$

$$8V_c = 32$$

$$V_c = 4V$$

& $V_c = V_o$ ∴ $V_o = \boxed{4V}$

2) Find V_o across the 6k resistor, using nodal analysis.



Note: $V_a = 6V$ (1) & $V_c - V_b = 12V$ (2)

KCL @ Super Node: $i_1 = i_2 + i_3 + i_4$

$$\frac{V_a - V_b}{3k} = \frac{V_b - 0}{4k} + \frac{V_c - 0}{6k} + \frac{V_c - V_d}{4k}$$

$$\frac{V_a}{3k} - \frac{V_b}{3k} = \frac{V_b}{4k} + \frac{V_c}{6k} + \frac{V_c}{4k} - \frac{V_d}{4k}$$

$$4V_a - 4V_b = 3V_b + 2V_c + 3V_c - 3V_d$$

$$4(6) - 7V_b = 5V_c - 3V_d$$

$$24 = 7V_b + 5V_c - 3V_d \quad (3)$$

KCL @ V_d: $i_4 = i_5$

$$\frac{V_c - V_d}{4k} = \frac{V_d - 0}{2k}$$

$$\frac{V_c}{4k} - \frac{V_d}{4k} = \frac{V_d}{2k}$$

$$V_c - V_d = 2V_d$$

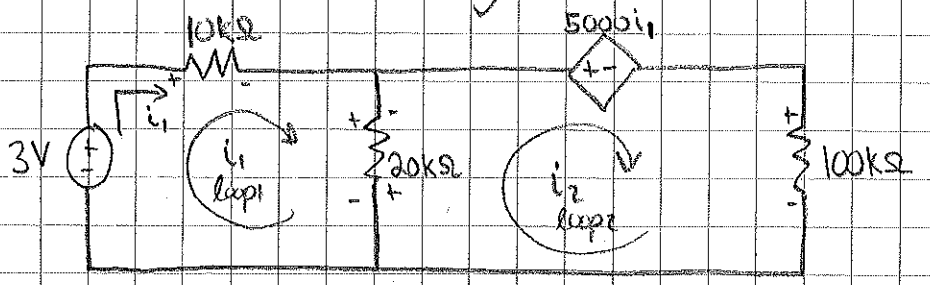
$$V_c = 3V_d \quad (4)$$

$\rightarrow$ $24 = 7(V_c - 12) + 5V_c - 3\left(\frac{V_c}{3}\right)$

$24 = 7V_c - 84 + 5V_c - V_c$

$108 = 11V_c \rightarrow V_c = 9.82$ & $V_c = V_o = \boxed{9.82V}$

3) Find the energy delivered to the cathode during a 24-hour period. (The cathode is represented by the dependent voltage source and the 100 kΩ resistor. Use Mesh analysis to solve this problem.



KVL @ loop 1: $-3V + 10k i_1 + 20k(i_1 - i_2) = 0$

$-3V + 30k i_1 - 20k i_2 = 0$ (1)

KVL @ loop 2: $5000 i_1 + 100k i_2 + 20k(i_2 - i_1) = 0$

$120k i_2 - 15k i_1 = 0$

$8 i_2 = i_1$ (2)

$\rightarrow -3V + 30k(8 i_2) - 20k i_2 = 0$

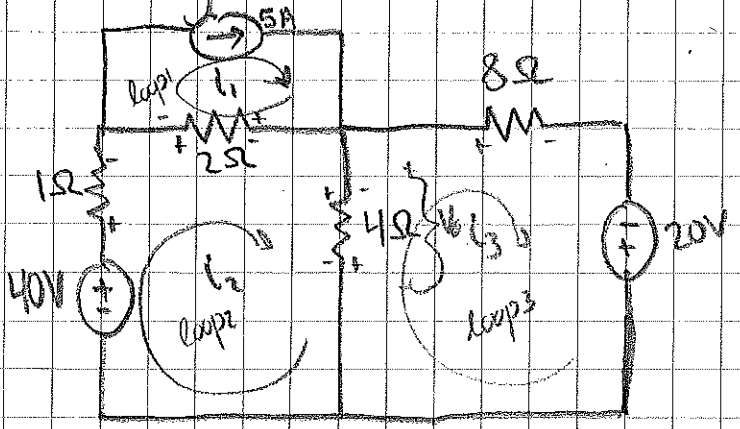
$220k i_2 = 3V$

$i_2 = 0.0136\ mA$ (3)

$P = Vi = i^2R = (100k)(.0000136)^2 = 0.018\ mW\text{-}S$ (4)

$$\int_0^{(24)(60)(60)} 0.018\,dt = \left[0.018t + c\right]_0^{86400} = \boxed{1.6\,W}$$

4) Apply mesh analysis to find V_o across 4Ω in the circuit below:



KVL @ loop 2: $-40V + i_2 + 2(i_2 - 5) + 4(i_2 - i_3) = 0$

$-40 + i_2 + 2i_2 - 10 + 4i_2 - 4i_3 = 0$

$50 = -4i_3 + 7i_2$ (1)

KVL @ loop 3: $-20 + 4(i_3 - i_2) + 8i_3 = 0$

$-20 + 4i_3 - 4i_2 + 8i_3 = 0$

$20 = -4i_2 + 12i_3$

$5 = -i_2 + 3i_3$ (2)

$\rightarrow$ $50 = 7(3i_3 - 5) - 4i_3$

$50 = 21i_3 - 35 - 4i_3$

$85 = 17i_3$

$i_3 = 5A$ (3)

$\therefore V_o = Ri = 4(5) = \boxed{20V}$