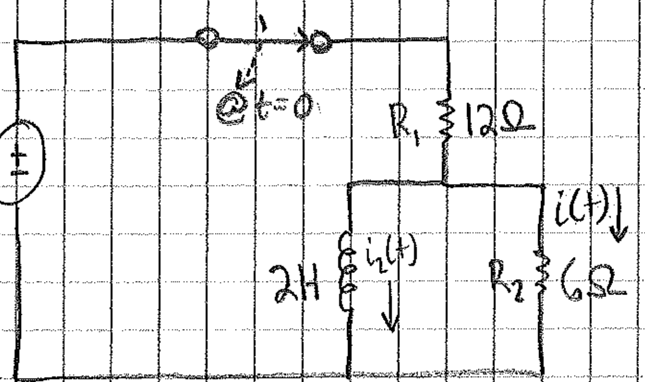
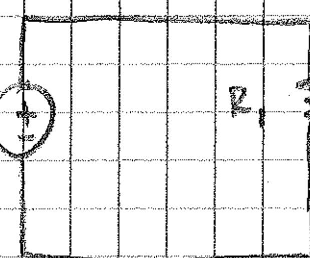
ENGR M20 HW # 7 Jared Fowler

1) Find $i(t)$ for $t > 0$ and sketch the waveform in the following:



<u>Note</u>: We'll assume that the circuit at $t<0$ is at a steady state. Inductors in a steady state D.C. circuit look like <u>a short</u>.

In stable state, we have the following circuit:

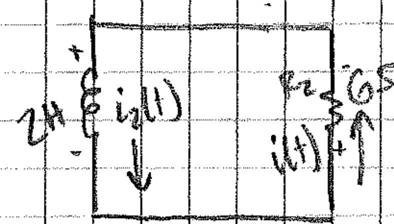


$\therefore i_0 = \frac{6}{12} = \underline{500\,mA}$

(I.C.)

b/c inductor CANNOT change instantaneously, @ $t=0$, $\underline{i_0 = i(0) = 500\,mA}$

<u>Note</u>: At $t>0$, the inductor will continue a current through the following circuit:



Note that $i(t) = i_2(t)$ for $t>0$.

Using KVL, $L\frac{di}{dt} + iR = 0$

$$\frac{di}{dt} + \frac{iR}{L} = 0 \quad \text{(Eq.1)}$$

(Eq2) $i = i_n + i_f$ & $i_f = K_1$ b/c 0 is const.

Substitute back into (Eq1)

$$\frac{d\cancel{K_1}^{\,0}}{\cancel{dt}} + \left(\frac{-K_1(\cancel{6})}{\cancel{2}}\right) = \cancel{0}$$

$\underline{K_1 = 0}$

Now for i_n (Natural)

$$\frac{di_n}{dt} + \frac{R}{L} i_n = 0$$

Assume the solution:

$$i_n = K_2 e^{st}$$

Substitute in...

$$\frac{d}{dt}(K_2 e^{st}) + \left(\frac{\cancel{R}^{\,6}}{\cancel{L}_{\,2}} K_2 e^{st}\right) = 0$$

$$\cancel{K_2} s \cancel{e^{st}} + (3\cancel{K_2}\cancel{e^{st}}) = 0$$

$$\underline{s = -3}$$

Sub into (Eq2)

$$i(t) = K_2 e^{-3t} + 0$$

$$i(0) = 500\,mA = K_2 \cancel{e^{0}}^{\,1}$$

$$\therefore K_2 = 500\,mA$$

$$\therefore \boxed{i(t) = .5e^{-3t}}$$

continue

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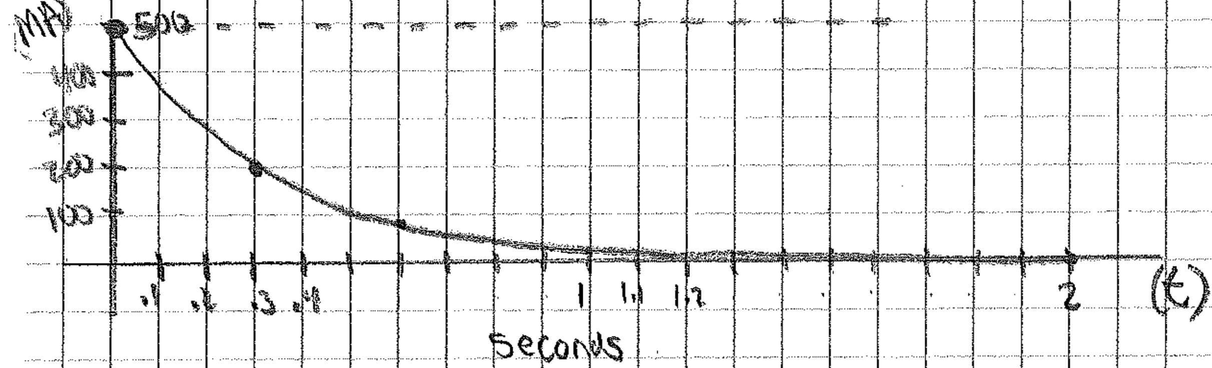
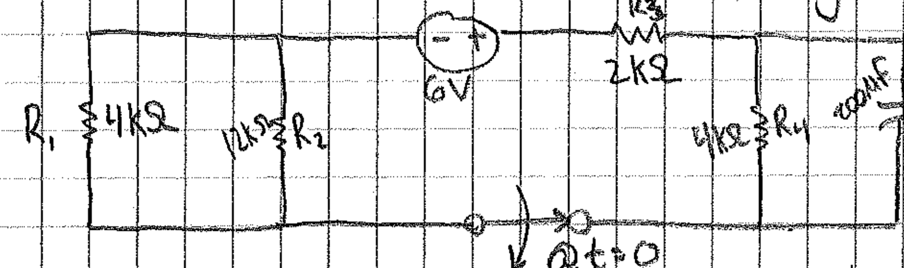
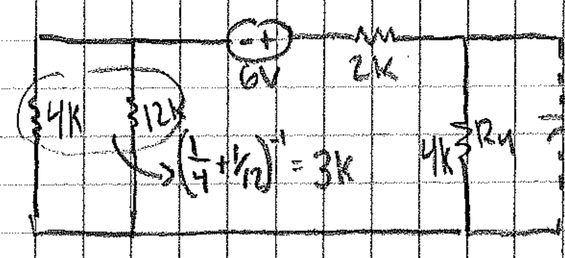


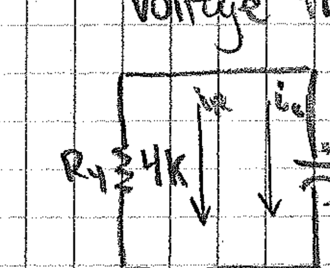
2) Find $V_c(t)$ for $t>0$ in the CKT below and plot the response including time interval prior to the switch opening.



Note: We'll assume that Circuit is steady @ $t<0$. At that phase, C acts like an open circuit.
@ $t<0$, we have the following:



Note: @ $t \geq 0$ the capacitor will supply voltage through this circuit:



KCL: $i_c + i_R = 0$

$$\left(\cancel{C}\frac{dv_c}{dt} + \frac{V_c}{4k} = 0\right) \cdot 10^4$$

$$2\frac{dv}{dt} + \frac{10}{4}V_c = 0$$

(Eq 1) $$\frac{dV_c}{dt} + \frac{5}{4}V_c = 0$$

Note: $V_{R_4} = V_c$ @ $t<0$, and at $t=0$, $t(0)$ b/c voltage across capacitor cannot change instantaneously

Using KVL: $-6 + 2ki + 4ki + 3ki = 0$

$$i = \frac{6}{9k} = 2/3 \text{ mA}$$

$\therefore V_{R_4} = V_{oc} = (4k)(2/3 \text{ mA})$

$= \underline{8/3 \text{ V}}$ (I.C.)

Assume solution in form: $V_c(t) = Ke^{st}$, substitute into (Eq 1)

$$\frac{d}{dt}Ke^{st} + \frac{5}{4}Ke^{st} = 0$$

$$sKe^{st} + \frac{5}{4}Ke^{st} = 0$$

$$\underline{s = -5/4}$$

continue

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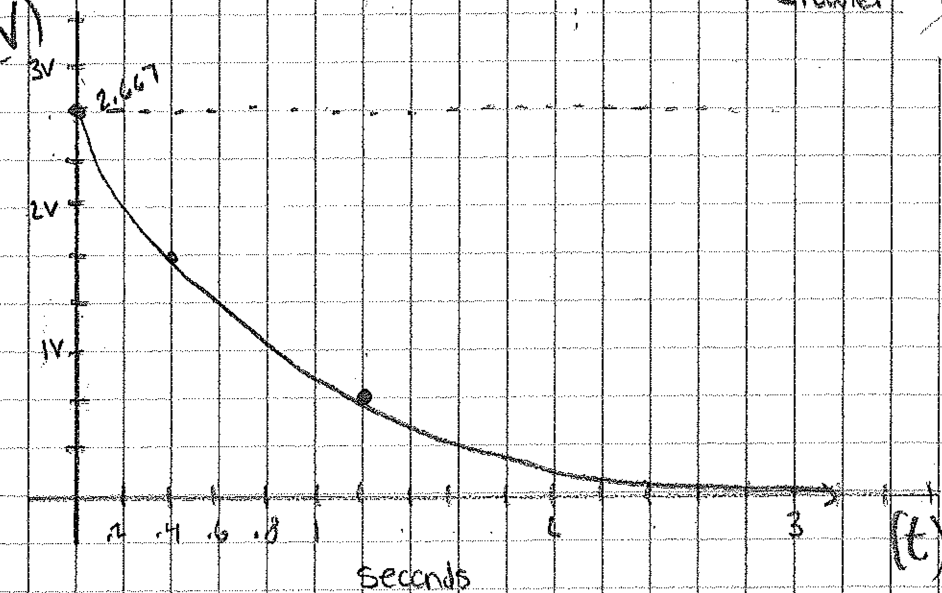
$V_c(t) = Ke^{-5/4\, t}$

@ $t = 0$, $V_c(0) = Ke^{0}$

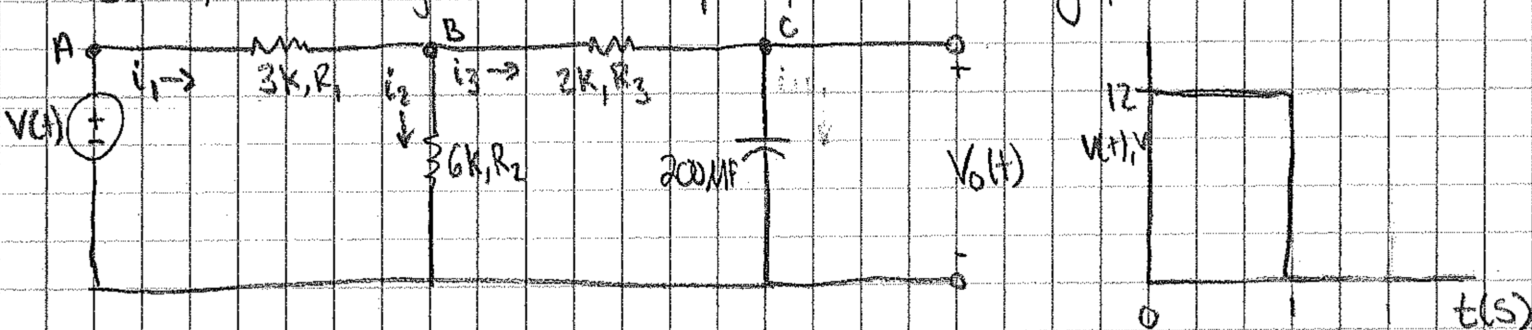
$V_c(0) = K$

∴ from our I.C., $K = 8/3\, V$

∴ $\boxed{V_c(t) = \frac{8}{3} e^{-\frac{5t}{4}}}$



(3) Determine the equation for the voltage for $V_o(t)$ for $t > 0$ for the CKT below, when subjected to the input pulse shown in graph.

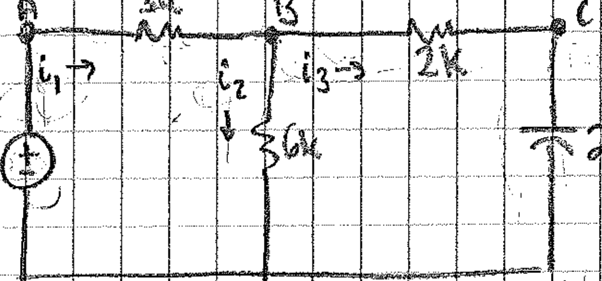


<u>Note</u>: We need to solve for 3 equations; $t<0$, $0<t<1$, $t>1$.

@ $t<0$, the voltage is 0, $V_0(0^-) = V_0(0) = 0$

∴ $\underline{@\ t<0,\ V_0(t) = 0}$

For $0 \le t \le 1$; use KCL:



$i_1 = i_2 + i_3$

$\frac{\cancel{V_A}^{12} - V_B}{3K} = \frac{V_B - 0}{6K} + C\frac{dV_c}{dt}$

$\frac{12}{3K} - \frac{\cancel{V_B}^{2V_B}}{\cancel{3K}_{6K}} = \frac{V_B}{6K} + \frac{C\,dV_c}{dt}$

$\overset{200\mu F}{C}\frac{dV_c}{dt} + \frac{3V_B}{6K} = \frac{12}{3K}$

KCL @ C

$i_3 = i_3$

$\frac{V_B - V_c}{2K} = \frac{C\,dV_c}{dt}$

$\frac{V_B}{2K} - \frac{V_c}{2K} = \frac{C\,dV_c}{dt}$

$V_B = \left(\frac{C\,dV_c}{dt} + \frac{V_c}{2K}\right)2K$

$V_B = (2K)\frac{C\,dV_c}{dt} + V_c$

↓ continue

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$$C\frac{dV_c}{dt} + \frac{3V_B}{6k} = \frac{12}{3k}, \quad V_B = (2k)C\frac{dV_c}{dt} + V_c$$

$$C\frac{dV_c}{dt} + \frac{1}{2k}\left(2k\,C\frac{dV_c}{dt} + V_c\right) = \frac{12}{3k}$$

$$C\frac{dV_c}{dt} + C\frac{dV_c}{dt} + \frac{V_c}{2k} = 12/3k$$

$$\frac{dV_c}{dt} + \frac{V_c}{(2k)(2C)} = \frac{12}{3k(2C)} \Rightarrow \underbrace{\frac{dV_c}{dt} + \frac{4}{5}V_c = 10}_{} \text{ (Eq. A)}$$

(Eq A)

$$\Rightarrow V_f = k_2 \Rightarrow \cancel{\frac{d k_2}{dt}}^{0} + \frac{4}{5}k_2 = 10$$

$$\underline{K_2 = 12.5}$$

Also, $\frac{dV_n}{dt} + \frac{4}{5}V_n = 0$, assumed sol. $V_n = K_1 e^{st}$

$\Rightarrow S = -4/5$

$\Rightarrow V = V_n + V_f = K_1 e^{-\frac{4}{5}t} + 12.5$

$V(0) = 0 \Rightarrow -12.5 = K_1 e^{0}$

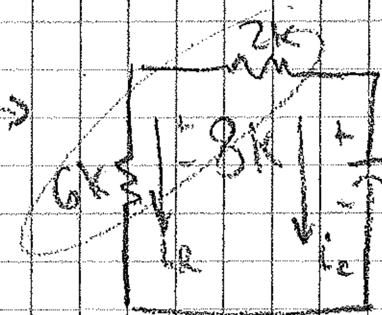
$\therefore \underline{K_1 = -12.5}$

$$\therefore \boxed{V_c(t) = 12.5 - 12.5e^{-\frac{4}{5}t} \quad @\ 0 \le t \le 1}$$

Note: @ $t = 1$, $V_c(1) = 12.5 - 12.5e^{-4/5}$

$= \underline{6.88\,\text{V}}$ (I.C.)

Finally, @ $t > 1$ we have the following CKT:



for $t > 1$, sourceless circuit.

$\rightarrow V(t) = Ke^{-\frac{4}{5}t}$

Find K →

$6.88 = V(1) = Ke^{-\frac{4}{5}(1)}$

$K = 6.88e^{4/5}$

$V(t) = 6.88e^{4/5} \cdot e^{-\frac{4}{5}t}$

$$\therefore \boxed{V(t) = 6.88e^{-\frac{4}{5}(t-1)} \quad \text{For } t > 1}$$

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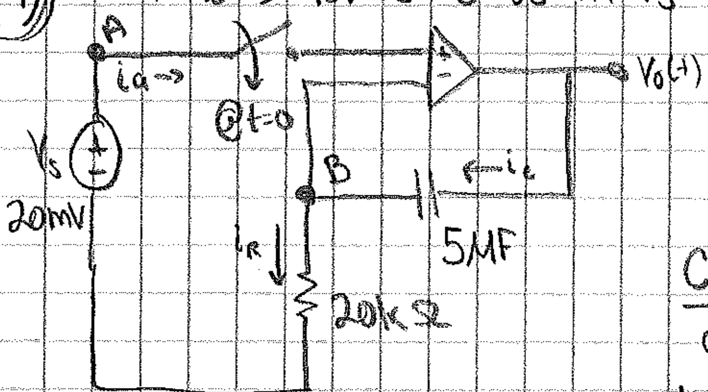
$\cancel{V_c(t) = Ke^{-1.6t} + 5.51}$

$\cancel{V_c(1) = Ke^{-1.6} \ldots}$

$\cancel{\therefore K = 5.51e^{1.6}}$

$\cancel{\therefore \boxed{@\ t>1,\ V_c(t) = 5.51e^{-1.6(t-1)}}}$

(4) Find $V_0(t)$ for $t > 0$ when $V_s = 20mV$ in the following circuit.



KCL @(−)

$i_c = i_R$

Note:

$V_B = V_A = 20mV$

$$C\frac{dV_0}{dt} = \frac{V_B - 0}{20K}$$

$$\frac{dV_0}{dt} - \frac{V_B}{.1} = 0 \Rightarrow \int \frac{dV_0}{dt} = \int .2\,dt$$

Note: @ t(0), t=0,

there is 0V across the capacitor. This means that @ t(0), that $V_0 = 20mV$. (I.C.)

$V_0(t) = .2t + C$

& $V_0(0) = 20mV = .2(0) + C$

$C = 20mV$

$$\therefore \boxed{V_0(t) = 0.2t + 0.02}$$