Homework # 1

Ch 15 # 5, 18, 29, 31, 35, 59, 70

15-5) $x(t) = (4m)\cos[(3\pi s^{-1})t + \pi]$

Determine

basic eqn. $x(t) = A\cos(\omega t + \phi)$

a) frequency

$\omega = 2\pi f \Rightarrow f = \frac{\omega}{2\pi} = \frac{3\pi s^{-1}}{2\pi} = \boxed{\frac{3}{2}Hz = 1.5Hz = f}$

b.) period

$T = \frac{1}{f} = \frac{1}{3/2 Hz} = \boxed{\frac{2}{3}s = 0.67s = T}$

c.) amplitude

$\boxed{A = 4m}$

d.) phase constant

$\boxed{\phi = \pi}$

e.) $x(t = \frac{1}{4}s)$

$x(t = \frac{1}{4}s) = (4m)\cos[(3\pi s^{-1})(\frac{1}{4}s) + \pi)$

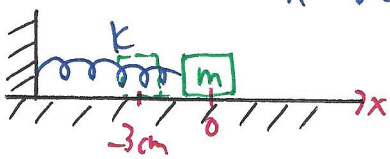
$= (4m)\cos[\frac{3}{4}\pi + \pi)$

$= (4m)\cos(\frac{7}{4}\pi)$

$\boxed{x(t = \frac{1}{4}s) = 2.83m}$

5-18) $m = 1\,kg$ glider $m = 1\,kg$, $k = 25\,\frac{N}{m}$

At $t=0$, released from rest at $x = -3\,cm$



a.) Find period

$$\omega = \sqrt{\frac{k}{m}} = 2\pi f = \frac{2\pi}{T}$$

$$\Rightarrow T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{1\,kg}{25\,\frac{N}{m}}} = \boxed{\frac{2}{5}\pi\,s \approx 1.26\,s = T}$$

b.) Find the maximum speed and acceleration

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{25\,\frac{N}{m}}{1\,kg}} = 5\,s^{-1}$$

$$x(t) = A\cos(\omega t + \phi)$$

$$V(t) = \frac{dx}{dt} = -\underbrace{\omega A}_{V_{max}}\sin(\omega t + \phi)$$

$$a(t) = \frac{dv}{dt} = -\underbrace{\omega^2 A}_{a_{max}}\cos(\omega t + \phi)$$

$$V_{max} = \omega A = (5\,s^{-1})(3\,cm) = \boxed{15\,\frac{cm}{s} = 0.15\,m/s = V_{max}}$$

$$a_{max} = \omega^2 A = \omega V_{max} = (5\,s^{-1})(15\,\frac{cm}{s}) = \boxed{75\,cm/s^2 = 0.75\,m/s^2 = a_{max}}$$

c.) Find $x(t)$, $V(t)$, $a(t)$

At $t=0$, $x(0) = -3\,cm = A\cos\phi$

$V(0) = 0 = -\omega A\sin\phi \Rightarrow \phi = 0$

$\Rightarrow x(0) = -3\,cm = A$

$$\boxed{\begin{array}{lcl} x(t) = -(0.03\,m)\cos[(5\,s^{-1})t] & & x(t) = (0.03\,m)\cos[(5\,s^{-1})t + \pi] \\ V(t) = \frac{dx}{dt} = +(0.15\,m/s)\sin[(5\,s^{-1})t] & \text{OR} & V(t) = \frac{dx}{dt} = -(0.15\,m/s)\sin[(5\,s^{-1})t + \pi] \\ a(t) = \frac{dv}{dt} = +(0.75\,m/s^2)\cos[(5\,s^{-1})t] & & a(t) = \frac{dv}{dt} = -(0.75\,m/s^2)\cos[(5\,s^{-1})t + \pi] \end{array}}$$

5-29) SHO w/ amplitude A and total energy E

a.) b.) Determine K and U when $x = \frac{1}{3}A$

$$U = \frac{1}{2}kx^2 = \frac{1}{2}k\left(\frac{1}{3}A\right)^2 = \frac{1}{9}\left(\frac{1}{2}kA^2\right)$$

$$E = \frac{1}{2}kA^2 = K + U$$

$$\Rightarrow \boxed{U = \frac{1}{9}E}$$

$$\Rightarrow K = E - U = E - \frac{1}{9}E = \boxed{\frac{8}{9}E = K}$$

c.) For what x does $K = \frac{1}{2}U$?

$$K = \frac{1}{2}U$$

$$E - U = \frac{1}{2}U$$

$$E = \frac{3}{2}U$$

$$\cancel{\frac{1}{2}}kA^2 = \frac{3}{2}\left(\cancel{\frac{1}{2}}kx^2\right)$$

$$\Rightarrow x^2 = \frac{2}{3}A^2$$

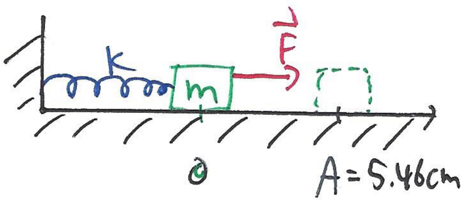
$$\boxed{x = \pm\sqrt{\frac{2}{3}}A}$$

d.) Are there any x where $K > U_{max}$? Explain.

No! Energy is conserved ⇒ energy is transformed from K to U and vice versa but $E = K + U$ is constant.

$K_{max} = U_{max} = E$

5-31)



$m = \frac{1}{4}$ kg
$k = 83.8 \frac{N}{m}$

a.) Find $|\vec{F}|$

$F = -kx \Rightarrow |\vec{F}| = kA = (83.8 \frac{N}{m})(0.0546m) = \boxed{4.58N = |\vec{F}|}$

b.) What is total energy stored?

$E = K + U = \frac{1}{2}kA^2 = \frac{1}{2}(83.8 \frac{N}{m})(0.0546m)^2 = \boxed{0.125J = E}$

c.) Find $|\vec{a}|$ just after applied force is removed

$\Sigma\vec{F} = m\vec{a} \Rightarrow a = \frac{F}{m} = \frac{kA}{m} = \frac{4.58N}{\frac{1}{4}kg} = \boxed{18.3 m/s^2 = a}$

d.) Find v when block reaches x=0.

$E = K + U = \frac{1}{2}mv^2 + \cancel{\frac{1}{2}kx^2}^{\,0}$

$\Rightarrow v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2(0.125J)}{\frac{1}{4}kg}} = \boxed{1.00 m/s = v}$

OR $\cancel{K_i}^{\,0} + U_i + \cancel{W_{nc}}^{\,0} = K_f + \cancel{U_f}^{\,0}$

i = m at max A and v=0
f = m at x=0

$\frac{1}{2}kA^2 = \frac{1}{2}mv^2$

$\Rightarrow v = \sqrt{\frac{k}{m}}A = \sqrt{\frac{2E}{m}}$ ✓

e.) If $\mu \neq 0$, would answer to d) be larger or smaller?

$\Rightarrow$ $\boxed{\text{v smaller because some energy converted to heat, sound, etc.}}$

f.) What other info required to find answer to d)? $\boxed{\Rightarrow \text{require } \mu_k}$

g.) What is largest μ_k that would allow m to reach x=0?

$\cancel{K_i}^{\,0} + U_i + W_{nc} = \cancel{K_f}^{\,0} + \cancel{U_f}^{\,0}$

initial - m at A
final - m at x=0 with v=0 to just barely get there

$\frac{1}{2}kA^2 - \mu_k mgA = 0$

$\mu_k^{max} = \frac{kA}{2mg} = \frac{(83.8 \frac{N}{m})(0.0546m)}{2(\frac{1}{4}kg)(9.8 m/s^2)}$

$\boxed{\mu_k^{max} = 0.934}$

Recall!
$W = \int \vec{F} \cdot d\vec{r}$

15-35) Simple pendulum makes 120 complete oscillations in 3 min $(g = 9.8\,m/s^2)$

Find a) period of pendulum and b) its length.

a.) T = time to complete one cycle/oscillation

$$= \frac{\text{total time}}{\text{number of oscillations}}$$

$$= \frac{3\text{ min}}{120} \cdot \frac{60\text{ s}}{\text{min}}$$

$$\boxed{T = \frac{3}{2}\text{ s}}$$

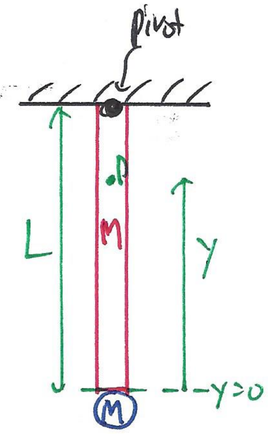
b.) $\omega = \sqrt{\frac{g}{L}} = 2\pi f = \frac{2\pi}{T}$

$$\Rightarrow \frac{g}{L} = \left(\frac{2\pi}{T}\right)^2$$

$$L = \left(\frac{T}{2\pi}\right)^2 g = \left(\frac{\frac{3}{2}\text{ s}}{2\pi}\right)^2 (9.8\,m/s^2)$$

$$\boxed{L = 0.559\text{ m}}$$

15-59)



Determine T in rod at

a) pivot (top)

$\sum \vec{F} = m\vec{a}$

$T - 2Mg = 0$

$\Rightarrow T = 2Mg$

b.) point P

$\sum \vec{F} = m\vec{a}$

$T - Mg - \frac{y}{L}Mg = 0$

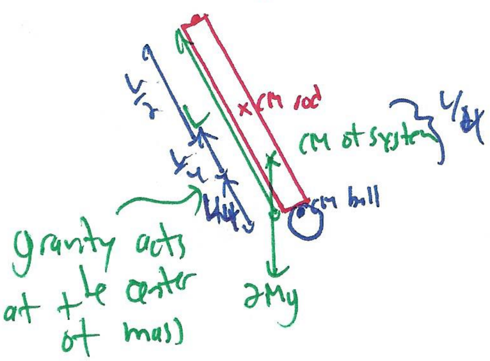
Mg $\quad m_{rod}g = \frac{y}{L}Mg$

$\boxed{T = \left(1 + \frac{y}{L}\right)Mg}$

c.) Find period T

physical pendulum $\Rightarrow$ $\omega = \sqrt{\frac{mgd}{I}}$

$m = 2M$



$y_{cm} = \frac{\sum y_i m_i}{\sum m_i} = \frac{0(M) + \frac{L}{2}(M)}{M + M} = \frac{L}{4}$ ✓

d = distance from pivot + CM $= \frac{3}{4}L$

$I = \sum m r_i^2 = I_{rod} + I_{ball} = \frac{1}{3}ML^2 + ML^2 = \frac{4}{3}ML^2$

$\omega = \sqrt{\frac{mgd}{I}} = 2\pi f = \frac{2\pi}{T}$

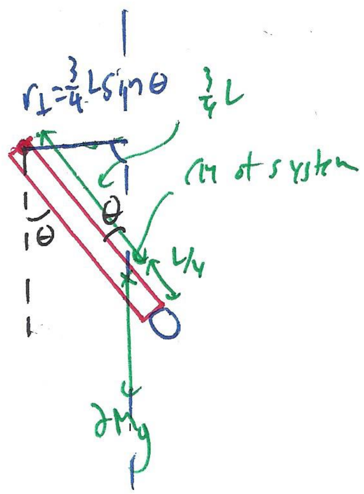
$\Rightarrow T = 2\pi\sqrt{\frac{I}{mgd}} = 2\pi\sqrt{\frac{\frac{4}{3}ML^2}{(2M)g\left(\frac{3}{4}L\right)}} = 2\pi\sqrt{\frac{8}{9}\frac{L}{g}} = \boxed{\frac{4\pi}{3}\sqrt{\frac{2L}{g}} = T}$

d) Find T is L = 2m

$T = \frac{4\pi}{3}\sqrt{\frac{2L}{g}} = \frac{4\pi}{3}\sqrt{\frac{2(2m)}{(9.8\,m/s^2)}} = \boxed{2.68s = T}$

15-59) (cont.)

c.) OR



worked out I before

$$\sum \tau = I\alpha$$

$$-2Mg\frac{3}{4}L\sin\theta = \frac{4}{3}ML^2 \frac{d^2\theta}{dt^2}$$

$$\Rightarrow \quad \frac{d^2\theta}{dt^2} = -\frac{9}{8}\frac{g}{L}\sin\theta$$

For small angles $\sin\theta \approx \theta$

$$\Rightarrow \quad \frac{d^2\theta}{dt^2} = -\frac{9}{8}\frac{g}{L}\theta$$

$$\Rightarrow \quad \omega = \sqrt{\frac{9}{8}\frac{g}{L}} = \frac{3}{2}\sqrt{\frac{g}{2L}} = \frac{2\pi}{T}$$

$$\Rightarrow T = \frac{4\pi}{3}\sqrt{\frac{2L}{g}}$$ ✓

15–**64.** An object attached to a spring vibrates with simple harmonic motion as described by Figure P15.64. For this motion, find (a) the amplitude, (b) the period, (c) the

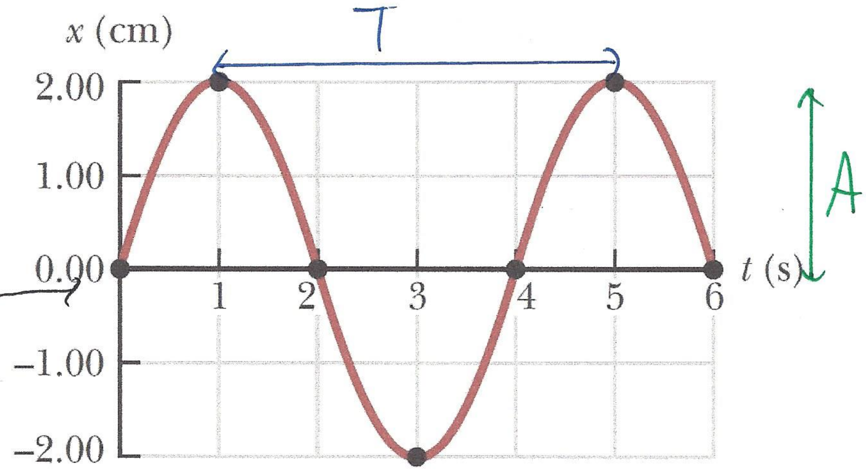


Figure P15.64

a.) $\boxed{A = 2.0\,cm = 2\times10^{-2}\,m}$

b.) $\boxed{T = 4\,s}$

c.) $\omega = 2\pi f = \frac{2\pi}{T} = \frac{2\pi}{4s} = \boxed{\frac{\pi}{2}\frac{rad}{s} = 1.57\frac{rad}{s} = \omega}$

d.) $x(t) = A\cos(\omega t + \phi)$

$V(t) = \frac{dx}{dt} = \underbrace{-\omega A}_{V_{max}}\sin(\omega t + \phi) \Rightarrow V_{max} = \omega A = \left(\frac{\pi}{2}\frac{rad}{s}\right)(2cm)$

$\boxed{V_{max} = \pi\frac{cm}{s} = 3.14\times10^{-2}\,m/s}$

e.) $a(t) = \frac{dv}{dt} = \underbrace{-\omega^2 A}_{a_{max}}\cos(\omega t + \phi) \Rightarrow a_{max} = \omega^2 A = V_{max}\omega = (3.14\times10^{-2}\,m/s)(1.57\frac{rad}{s})$

$\boxed{a_{max} = 4.93\times10^{-2}\,m/s^2}$

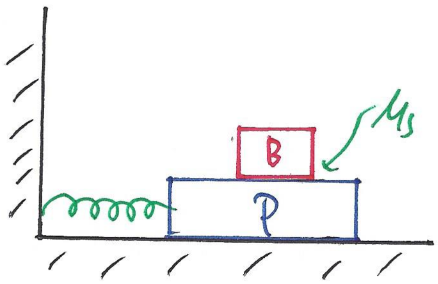
f.) $x(t) = A\cos(\omega t + \phi)$

$x(0) = 0 = A\cos\phi \Rightarrow \phi = -\frac{\pi}{2}$ $x = A\cos(\omega t - \frac{\pi}{2}) = A\sin\omega t$

looks like sine on figure!

$\boxed{x(t) = A\sin\omega t = (2\times10^{-2}\,m)\sin[(1.57\frac{rad}{s})t]}$

15-66)



Both P, B oscillate together with frequency f.
What is the maximum amplitude of oscillation if block B is not to slip?

For max A and B not slip

$$\Rightarrow \quad f_s = f_s^{max} = \mu_s n_B = \mu_s m_B g = m_B a_{max}$$

$$\Rightarrow \quad a_{max} = \mu_s g$$

P/B oscillate

$$x(t) = A\cos(\omega t + \phi)$$

$$v(t) = \frac{dx}{dt} = -\omega A \sin(\omega t + \phi)$$

$$a(t) = \frac{dv}{dt} = -\underbrace{\omega^2 A}_{a_{max}} \cos(\omega t + \phi)$$

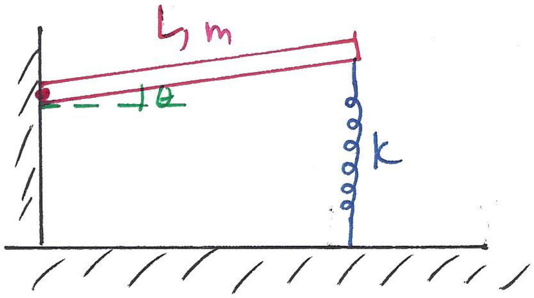
$$a_{max} = a_{max}$$

$$\omega^2 A = \mu_s g$$

$$A = \frac{\mu_s g}{\omega^2} \quad ; \quad \omega = 2\pi f$$

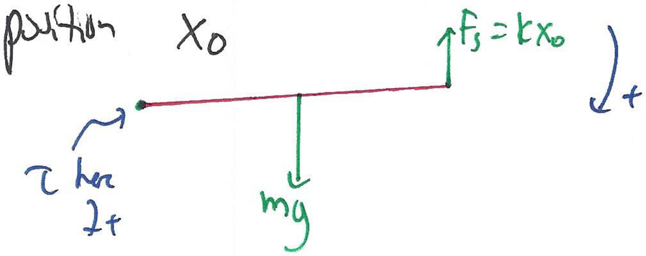
$$\Rightarrow \boxed{A = \frac{\mu_s g}{4\pi^2 f^2}}$$

15-70)



Find ω for small θ

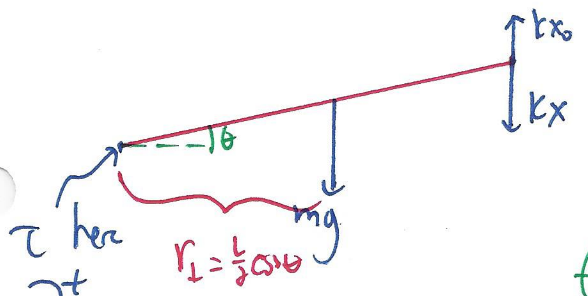
First find the equilibrium position x_0



$$\sum \tau = I\alpha$$

$$mg\frac{L}{2} - kx_0 L = 0$$

Now for small displacements around the horizontal



$$\sum \tau = I\alpha$$

$$mg\frac{L}{2}\cos\theta - kx_0 L\cos\theta + kxL\cos\theta = -\frac{1}{3}mL^2\frac{d^2\theta}{dt^2}$$

for small angle, $\cos\theta \approx 1$

$$\underbrace{mg\frac{L}{2} - kx_0 L}_{=0} + kxL = -\frac{1}{3}mL^2\frac{d^2\theta}{dt^2}$$

$x = L\theta$

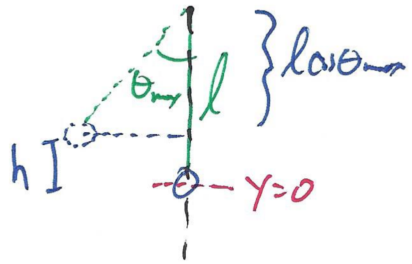
$$-kL^2\theta = \frac{1}{3}mL^2\frac{d^2\theta}{dt^2}$$

$$\frac{d^2\theta}{dt^2} = -3\frac{k}{m}\theta$$

compare $\Longleftrightarrow$ $\frac{d^2x}{dt^2} = -\omega^2 x$ Eqn of SHM

$$\Rightarrow \boxed{\omega = \sqrt{3\frac{k}{m}}}$$

15-75) Simple pendulum with $l = 2.23\,m$ and mass $m = 6.74\,kg$ has initial speed $2.06\,m/s$ at its equilibrium position.
Determine a) its period; b) total energy and c) max angular displacement



$$\Rightarrow l = l\cos\theta_{max} + h$$
$$\Rightarrow l\cos\theta_{max} = l - h$$
$$\cos\theta_{max} = 1 - \frac{h}{l}$$

a.) $\omega = \sqrt{\frac{g}{l}} = \frac{2\pi}{T}$

$$T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{2.23\,m}{9.8\,m/s^2}}$$

$$\boxed{T = 3.00\,s}$$

b.) $E = K + U$

$$E_i = K_i + \cancel{U_i}^{\,0} = \frac{1}{2}mv^2 = \frac{1}{2}(6.74\,kg)(2.06\,m/s)^2$$

$$\boxed{E_i = 14.30\,J}$$

c.) θ_{max}

from diagram above $\cos\theta_{max} = 1 - \frac{h}{l}$

Use conservation of energy to find h

$$K_i + \cancel{U_i}^{\,0} + \cancel{W_{nc}}^{\,0} = \cancel{K_f}^{\,0} + U_f \qquad v = 0 \text{ at max height}$$

$$\frac{1}{2}mv^2 = mgh$$
$$\Rightarrow h = \frac{v^2}{2g}$$

$$\Rightarrow \cos\theta_{max} = 1 - \frac{v^2}{2gl} = 1 - \frac{(2.06\,m/s)^2}{2(9.8\,m/s^2)(2.23\,m)} = 0.903$$

$$\Rightarrow \boxed{\theta_{max} = 25.46^\circ}$$