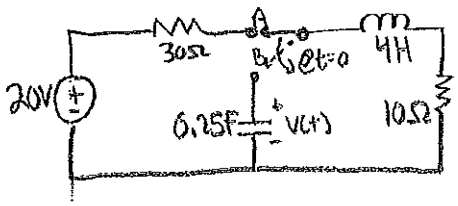
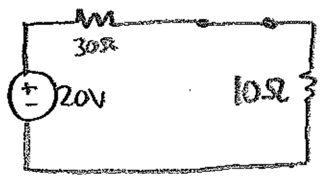
Engr M20, HW#8 Jared Fowler

1) The switch in figure below moves from position A to B @ $t=0$. Find $V(t)$ for $t>0$.



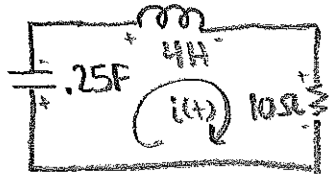
Note: @$t(0^-)$



$\rightarrow$ KVL: $-20+30i+10i=0 \rightarrow \underline{i(0^-)=500\text{mA}}$

Note: $V_R(0^-)=V_C(0^+) \rightarrow \underline{V_c(0^+)=5V}$ (I.C.)

Note: @$t(0^+)$



$\rightarrow$ KVL: $L\frac{di}{dt}+10i+V_c=0$ Note: $i=C\frac{dv}{dt}$, $\frac{di}{dt}=C\frac{d^2v}{dt^2}$

$\rightarrow LC\frac{d^2v}{dt^2}+10C\frac{dv}{dt}+V_c=0$

$\rightarrow \underline{\frac{d^2v}{dt}+\frac{5}{2}\frac{dv}{dt}+V=0}$ (EQ1)

$X=X_n+X_f$, $X_f=0$ because 0 is constant

$X_n \rightarrow Ke^{st} \rightarrow \frac{d^2(Ke^{st})}{dt^2}+\frac{5}{2}\frac{d(Ke^{st})}{dt}+Ke^{st}=0$

$s^2\cancel{Ke^{st}}+\frac{5}{2}s\cancel{Ke^{st}}+\cancel{Ke^{st}}=0 \rightarrow \underline{s^2+\frac{5}{2}s+1=0}$

Quadratic Formula: $s_1, s_2 = \frac{-\frac{5}{2}\pm\sqrt{\frac{25}{4}-4}}{2}$ $\begin{cases} -\frac{5}{4}+\frac{3}{4}=\underline{-\frac{1}{2}} \\ -\frac{5}{4}-\frac{3}{4}=\underline{-2} \end{cases}$ $\rightarrow \omega_0=1, Z=\frac{5}{4}$ "over-damped"

$V(t)=K_1e^{-\frac{1}{2}t}+K_2e^{-2t}$

$V(0)=5=K_1e^0+K_2e^0 \rightarrow \underline{K_1=5-K_2}$

$i(0)=0.5=C\left.\frac{dv}{dt}\right|_{t=0}=.25(-\frac{1}{2}K_1e^0+(-2)K_2e^0) \rightarrow \underline{2=-\frac{1}{2}K_1-2K_2}$

$\rightarrow 2=-\frac{1}{2}(5-K_2)-2K_2$

$\frac{4}{2}+\frac{5}{2}=-2K_2+\frac{1}{2}K_2 \rightarrow K_2=-3, K_1=8$ $\therefore$ $\boxed{V(t)=8e^{-\frac{1}{2}t}-3e^{-2t} \text{ @ } t>0}$

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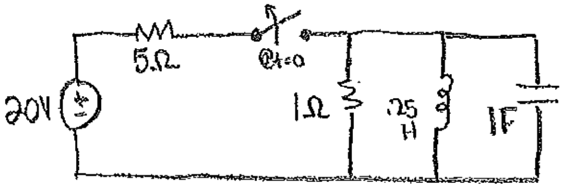
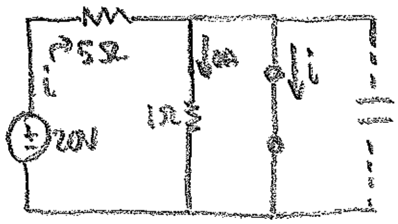
2) Find the voltage across the capacitor as a function of time for $t>0$ for the circuit in Fig below. Assume steady state exists @ $t=0$.



Note: @ $t(0^-)$



$\to i_L = 20/5 = \underline{4A = i(0^-)}$ (C acts like short @ start)

(I.C.)

$\underline{V_C(0^-) = V_L(0^-) = 0V}$

Note: @ $t(0^+)$

$\to$ KCL: $i_L + i_R + i_C = 0$

$\to \frac{d}{dt}\left[\frac{1}{L}\int v\,dt + i(0) + \frac{V}{R} + C\frac{dv}{dt} = 0\right]$

$\to \cancel{C}\frac{d^2v}{dt^2} + \frac{1}{R}\frac{dv}{dt} + \frac{1}{L}v = 0$

$\to \underline{\frac{d^2v}{dt^2} + \frac{dv}{dt} + 4v = 0}$ (Eq1)

$x = x_n + x_f$, $x_f = 0$

$x_n \to Ke^{st} \to s^2 + s + 4 = 0$

$\to \omega_0 = 2$, $1 = 2\zeta\omega_0 \to \zeta = 1/4$

$\to$ "under-damped"

QF: $s_1, s_2 = \frac{-1 \pm \sqrt{1-16}}{2}$ $\to$ $-\frac{1}{2} + j\frac{\sqrt{15}}{2}$, $-\frac{1}{2} - j\frac{\sqrt{15}}{2}$

$\to V(t) = e^{-\frac{1}{2}t}\left(k_1 \sin\frac{\sqrt{15}}{2}t + k_2 \cos\frac{\sqrt{15}}{2}t\right)$

$V(0) = 0 = e^0(k_1(0) + k_2(1)) \to \underline{k_2 = 0}$

$i(0) = 4 = C\frac{dv}{dt}\Big|_{t=0} = (1)\left[e^{-\frac{1}{2}t}k_1 \sin\frac{\sqrt{15}}{2}t\right]\frac{d}{dt}$

$= -\frac{1}{2}e^0 k_1 \cancel{\sin(0)} + (e^0)\left(\frac{\sqrt{15}}{2}\cos(0)k_1\right)$

$4 = \frac{\sqrt{15}}{2}k_1 \to k_1 = 8/\sqrt{15}$

$\therefore \boxed{V(t) = -e^{-\frac{1}{2}t}\left(\frac{8}{\sqrt{15}}\right)\sin\frac{\sqrt{15}}{2}t \quad @\, t>0}$

Note: Initial current through C will run bottom $\to$ top, ∴, equation is negated.

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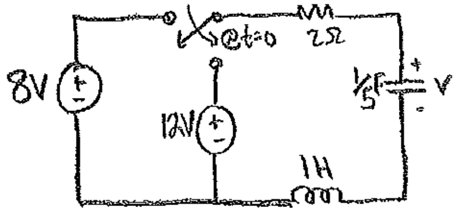
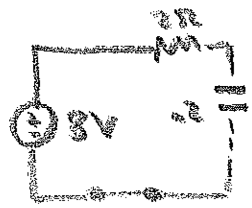
3) Find V for $t > 0$.



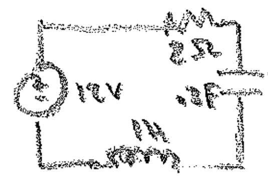
Note: @ $t(0^-)$



$\rightarrow \underline{i(0^-) = 0}$

$\rightarrow \underline{V_c(0^-) = 8V}$ b/c $V_R = \cancel{i}^{\,0} R = 0V.$

Note: @ $t(0^+)$



$\rightarrow$ KVL: $-12 + 2i + V + L\dfrac{di}{dt} = 0$ Note: $i = C\dfrac{dv}{dt}$, $\dfrac{di}{dt} = C\dfrac{d^2v}{dt^2}$

$$\rightarrow LC\frac{d^2v}{dt^2} + 2C\frac{dv}{dt} + V = 12$$

$$\rightarrow \underline{\frac{d^2v}{dt^2} + 2\frac{dv}{dt} + 5V = 60} \quad (EQ1)$$

$x = x_n + x_f$

$x_f = K \rightarrow 5K = 60 \rightarrow \underline{K = 12} = x_f$

$x_n \rightarrow Ke^{st} \rightarrow s^2 + 2s + 5 = 0$

Note: $\omega_0 = \sqrt{5}$, $\zeta = 2/\sqrt{5} < 1$

$\therefore$ "under-damped"

Q.E: $s_1, s_2 = \dfrac{-2 \pm \sqrt{4 - 20}}{2}$ $\begin{cases} -1 + j2 \\ -1 - j2 \end{cases}$

$\rightarrow V(t) = e^{-t}(k_1 \sin 2t + k_2 \cos 2t) + 12$

$V(0) = 8 = e^0(k_1(0) + k_2(1)) + 12 \rightarrow \underline{k_2 = -4}$

$i(0) = \cancel{0} = \cancel{C}\dfrac{dv}{dt}\Big|_{t=0} = \left[e^{-t}(k_1 \sin 2t - 4\cos 2t)\right]\dfrac{d}{dt}$

$0 = -\cancel{e^{-t}}(k_1 \cancel{\sin 2t} - 4\cos 2t) + \cancel{e^{-t}}(2k_1 \cos 2t + 8 \sin 2t)$

$0 = -1(-4) + 2k_1 \rightarrow \underline{k_1 = -2}$

$$\therefore \boxed{V(t) = e^{-t}(-2\sin 2t - 4\cos 2t) + 12}$$

@ $t > 0$

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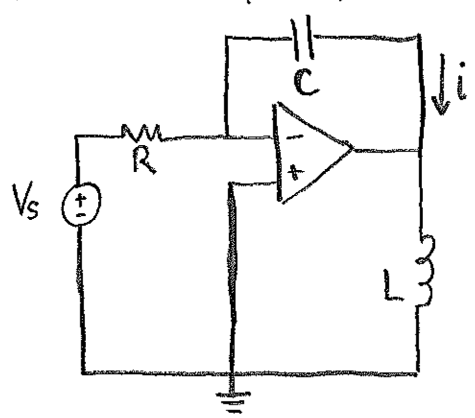
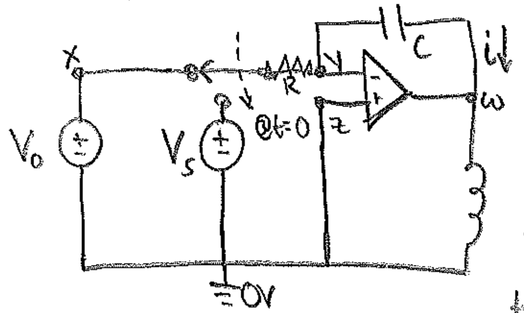
4) For the op-Amp Circuit below, find the differential equation for $i(t)$.



<u>Note:</u> The purpose of an op-amp is to measure the voltage between inputs and output the delta voltage multiplied by a multiplier. V_s is changing... let's see it as this:



NOW, at stable state there is no current across the capacitor, i.e., $i(0^-) = 0$ because capacitor acts like open in DC circuit in Stable State. Also @ $t(0)$, the voltage is $V(0^-) = V_0$

Note also that ground, 0V, is greater than op-amp output which will be negative, based upon inverting output. Current is drawn into the op amp more than delivered out. In this case, the inductor has little to <u>NO</u> effect upon the function $i(t)$ through capacitor.

KCL @ $t(0)$, $i_R + 0 = i_c$

$$\frac{V_s - V_y}{R} = C\frac{dV}{dt} \longrightarrow \frac{V_s}{R} = C\frac{dV}{dt} + \frac{V_y}{R} \longrightarrow \frac{dV}{dt} + \frac{V_y}{RC} = \frac{V_s}{RC}$$

Note: with L acting as short, KVL tells us that $V_y = V_c$ b/c $-V_s + \underbrace{V_R}_{V_s - V_c} + V_c = 0$

$$\frac{dV}{dt} + \frac{V}{RC} = \frac{V_s}{RC}$$

$X_f = K = V_s$, $X_n \to Ke^{st} \longrightarrow S = -1/RC$

$$V(t) = Ke^{-1/RC\,t} + V_s$$
$$V(0) = V_0 \to K = (V_0 - V_s)$$
$$\longrightarrow V(t) = (V_0 - V_s)e^{-1/RC\,t} + V_s$$

$$i(t) = C\frac{dV}{dt} = C\left[(V_0 - V_s)e^{-1/RC\,t} + V_s\right]\frac{d}{dt}$$

$$i(t) = \frac{-C}{RC}(V_0 - V_s)\,e^{-1/RC\,t}$$

$$\boxed{i(t) = \frac{1}{R}(V_s - V_0)\,e^{-1/RC\,t}}$$