ENGR M20 HW#9 Jared Fowler

1) Express the following functions in Cosine form:

a) $4\sin(\omega t - 30^\circ) \rightarrow 4\cos(\omega t - 120^\circ)$

b) $-2\sin(6t) \rightarrow -2\cos(6t - 90^\circ)$

c) $-10\sin(\omega t + 20^\circ) \rightarrow -10\cos(\omega t - 70^\circ)$

2) For the following pairs of sinusoids, determine which one leads and by how much.

a) $v(t) = 10\cos(4t - 60^\circ)$ and $i(t) = 4\sin(4t + 50^\circ)$

$i(t) = 4\cos(4t - 40^\circ) \quad \therefore \boxed{i(t) \text{ leads } v(t) \text{ by } 20^\circ}$

b) $v_1(t) = 4\cos(377t + 10^\circ)$ and $v_2(t) = -20\cos(377t)$

$v_2(t) = 20\cos(377t + 180^\circ)$

$\therefore \boxed{v_2(t) \text{ leads } v_1(t) \text{ by } 170^\circ}$

c) $x(t) = 13\cos 2t + 5\sin 2t$ and $y(t) = 15\cos(2t - 11.8^\circ)$

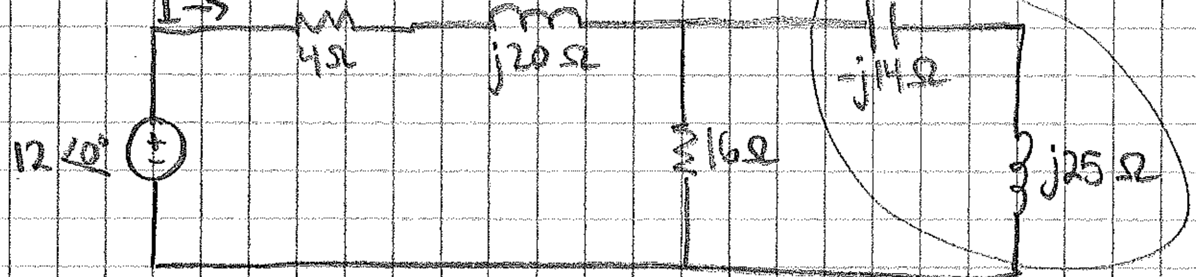
Note: $A\cos\omega t + B\sin\omega t = C\cos(\omega t - \theta)$, $C = \sqrt{A^2 + B^2}$, $\theta = \tan^{-1}\frac{B}{A}$

$\rightarrow x(t) = \sqrt{13^2 + 5^2}\cos(2t - 21.04^\circ)$

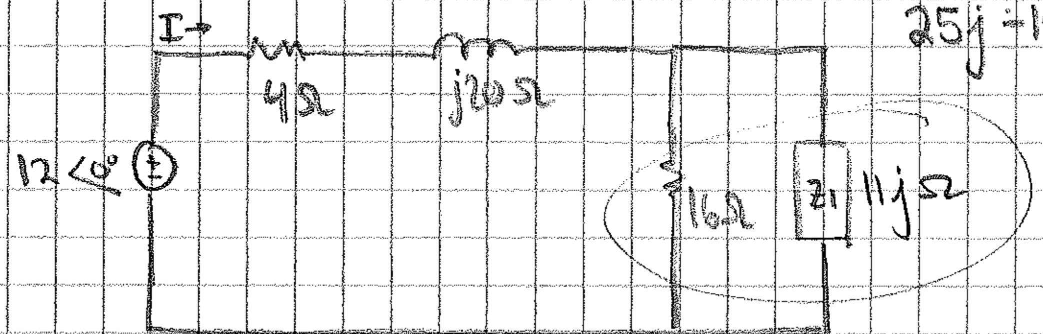
$\rightarrow x(t) = 13.93\cos(2t - 21.04^\circ)$

$\boxed{y(t) \text{ lead } x(t) \text{ by } 9.24^\circ}$

3) For the circuit shown, find Z_{eq} and use that to find current I. Let $\omega = 10$ rad/s.

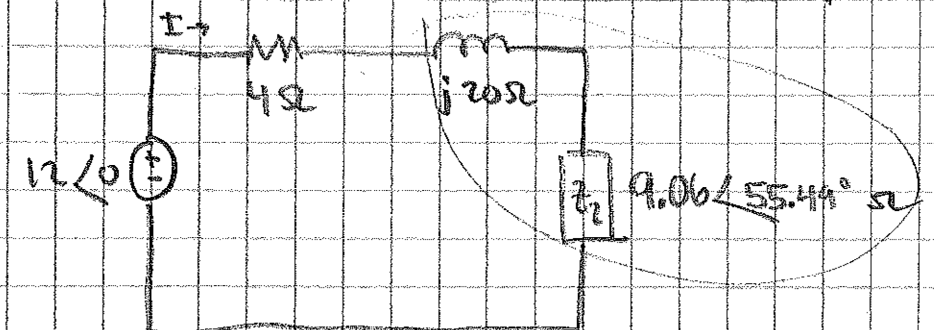


$25j - 14j = \underline{11j}\ \Omega$

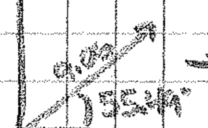


$$\frac{16 \cdot 11j}{16\Omega + 11j} = \frac{(16\angle 0°)(11\angle 90°)}{19.42\angle 34.51°}$$

$\underline{9.06\angle 55.49°\ \Omega}$



$9.06\angle 55.49° = \underline{5.13 + 7.46j\ \Omega}$



→ $B = 9.06\sin(55.49)$
$A = 9.06\cos(55.49)$

$5.13 + 7.46j + 20j$

$= \underline{5.13 + 27.46j}\ \Omega$

... Now add the 4Ω $\longrightarrow$ $\boxed{Z_{eq} = 9.13 + 27.46j}\ \Omega$

... or $\boxed{Z_{eq} = 28.94\angle 71.61°}\ \Omega$

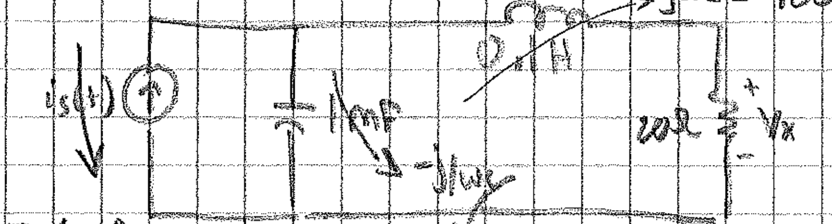
$$I = \frac{V}{R} = \frac{12\angle 0}{28.94\angle 71.61°} = .41\angle -71.61°$$

$\boxed{I = .41\angle -71.61°\ A}$ $\longrightarrow$ $\underline{i(t) = 0.41\cos(10t - 71.61°)\ A}$

... Not sure what your looking for... giving us ω...

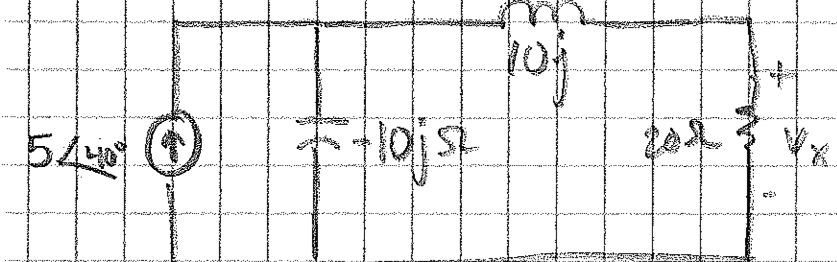
4) Determine V_x in the circuit shown. Let $i_s(t) = 5\cos(100t + 40°)$.

$j\omega L = 100\cdot(.1)j = \underline{10j\ \Omega}$

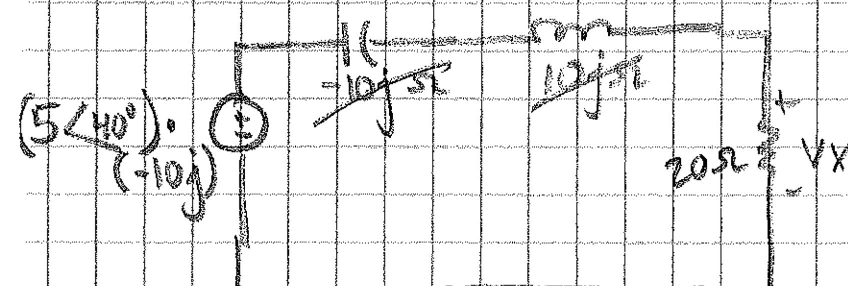


$\underline{5\angle 40°}$

$$\frac{-j}{100\cdot(.001)} = \underline{-10j\ \Omega}$$



N→T

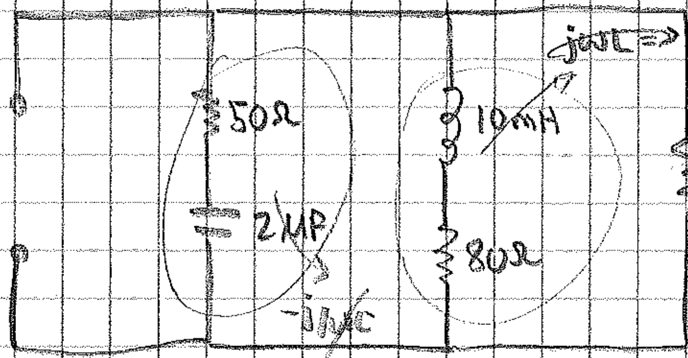


$\rightarrow V_x = (5\angle 40°)(\underbrace{-10j}_{10\angle -90°})$

$= (5\angle 40°)(10\angle -90°)$

$$\boxed{\begin{aligned} V_x &= 50\angle -50°\ \text{V} \\ &\text{or} \\ V_x &= 32.14 - 38.3j\ \text{V} \\ &\text{or} \\ V_x &= 50\cos(100t - 50°)\ \text{V} \end{aligned}}$$

5) The network below is part of a schematic describing an industrial electronic sensing device. What is the total impedance of the circuit @ 2 KHz?

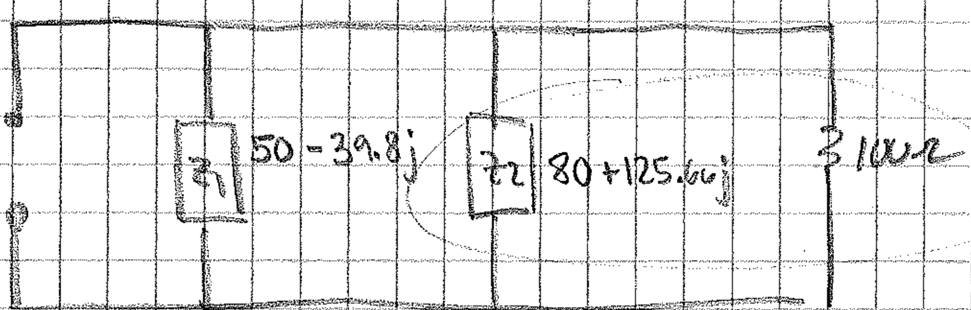


$j\omega L \Rightarrow 125.66\,j\ \Omega$ Note: $\omega = 2\pi f$, $f = \frac{1}{T}$

$2000\text{ Hz} = f$

$\rightarrow \omega = 2\pi(2k) = 12566.4\ \frac{rad}{s}$

$$\frac{-j}{(12566.4)(2\times10^{-6})} = -39.8j\ \Omega$$



$$\frac{(80+125.66j)(100)}{100+80+125.66j} = \frac{(148.97\angle 57.52°)(100\angle 0°)}{180+125.66j}$$

$$= \frac{14897\angle 57.52°}{2195\angle 34.92°} = 67.9\angle 22.6°$$

Now the final...

$$\frac{(67.9\angle 22.6)(50-39.8j)}{(50-39.8j)+67.9\angle 22.6} = \frac{(67.9\angle 22.6)(63.91\angle -38.52)}{(50-39.8j)+(62.69+26.1j)} = \frac{4339.5\angle -15.92}{112.69-13.7j}$$

$$Z = \frac{4339.5\angle -15.92}{113.52\angle -6.90} = \boxed{38.23\angle -9.02°\ \Omega \text{ or } 37.8-6j\ \Omega}$$