Homework 1

Chapter 15

P15.5 $x = (4.00\text{ m})\cos(3.00\pi t + \pi)$; compare this with $x = A\cos(\omega t + \phi)$ to find

(a) $\omega = 2\pi f = 3.00\pi$ or $\boxed{f = 1.50\text{ Hz}}$

(b) $T = \dfrac{1}{f} = \boxed{0.667\text{ s}}$

(c) $A = \boxed{4.00\text{ m}}$

(d) $\phi = \boxed{\pi\text{ rad}}$

(e) $x(t = 0.250\text{ s}) = (4.00\text{ m})\cos(1.75\pi) = \boxed{2.83\text{ m}}$

P15.18 $m = 1.00$ kg, $k = 25.0$ N/m, and $A = 3.00$ cm. At $t = 0$, $x = -3.00$ cm.

(a) $$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{25.0\text{ N/m}}{1.00\text{ kg}}} = 5.00\text{ rad/s}$$

so that, $$T = \frac{2\pi}{\omega} = \frac{2\pi}{5.00} = \boxed{1.26\text{ s}}$$

(b) $$v_{\max} = A\omega = \left(3.00\times10^{-2}\text{ m}\right)(5.00\text{ rad/s}) = \boxed{0.150\text{ m/s}}$$

$$a_{\max} = A\omega^2 = \left(3.00\times10^{-2}\text{ m}\right)(5.00\text{ rad/s})^2 = \boxed{0.750\text{ m/s}^2}$$

(c) Because $x = -3.00$ cm and $v = 0$ at $t = 0$, the required solution is $x = -A\cos\omega t$, or

$$\boxed{x = 3.00\cos(5.00t + \pi)}$$

Then, $$v = \frac{dx}{dt} = \boxed{-15.0\sin(5.00t + \pi)}$$

and $$a = \frac{dv}{dt} = \boxed{-75.0\cos(5.00t + \pi)}$$

where x is in cm, v is in cm/s, and a is in cm/s^2.

Note: an equally valid solution with $\phi = 0$ is

$$x(t) = -(0.03\text{m})\cos[(5\text{ s}^{-1})t]$$

$$v(t) = \frac{dx}{dt} = \left(0.15\frac{\text{m}}{\text{s}}\right)\sin[(5\text{ s}^{-1})t]$$

$$a(t) = \frac{dv}{dt} = \left(0.75\frac{\text{m}}{\text{s}}\right)\cos[(5\text{ s}^{-1})t]$$

P15.29 (a) Energy is conserved by an isolated simple harmonic oscillator:

$$E=\frac{1}{2}kA^2=\frac{1}{2}mv^2+\frac{1}{2}kx^2\rightarrow\frac{1}{2}mv^2=\frac{1}{2}kA^2-\frac{1}{2}kx^2$$
$$\rightarrow\frac{1}{2}mv^2=\frac{1}{2}k\left(A^2-x^2\right)$$

When $x=A/3$,

$$\frac{1}{2}mv^2=\frac{1}{2}k\left(A^2-x^2\right)=\frac{1}{2}k\left[A^2-\left(\frac{A}{3}\right)^2\right]=\frac{1}{2}kA^2\left[1-\frac{1}{9}\right]$$
$$\frac{1}{2}mv^2=\frac{1}{2}kA^2\frac{8}{9}=\boxed{\frac{8}{9}E}$$

(b) When $x=A/3$,

$$\frac{1}{2}kx^2=\frac{1}{2}k\left(\frac{A}{3}\right)^2=\frac{1}{9}\left(\frac{1}{2}kA^2\right)=\boxed{\frac{1}{9}E}$$

(c) $$\frac{1}{2}kA^2=\frac{1}{2}mv^2+\frac{1}{2}kx^2=\frac{1}{2}\left(\frac{1}{2}kx^2\right)+\frac{1}{2}kx^2$$

$$\frac{1}{2}kA^2=\frac{3}{4}kx^2\rightarrow x=\boxed{\pm\sqrt{\frac{2}{3}}A}$$

(d) $\boxed{\text{No.}}$ The maximum potential energy of the system is equal to the total energy of the system: kinetic plus potential energy. Because the total energy must remain constant, the kinetic energy can never be greater than the maximum potential energy.

P15.31 (a) $F = k|x| = (83.8\ \text{N/m})(5.46\times10^{-2}\ \text{m}) = \boxed{4.58\ \text{N}}$

(b) $E = U_s = \frac{1}{2}kx^2 = \frac{1}{2}(83.8\ \text{N/m})(5.46\times10^{-2}\ \text{m})^2 = \boxed{0.125\ \text{J}}$

(c) While the block was held stationary at $x = 5.46$ cm, $\sum F_x = -F_s + F = 0$, or the spring force was equal in magnitude and oppositely directed to the applied force. When the applied force is suddenly removed, there is a net force $F_s = 4.58$ N directed toward the equilibrium position acting on the block. This gives the block an acceleration having magnitude

$$|a| = \frac{F_s}{m} = \frac{4.58\ \text{N}}{0.250\ \text{kg}} = \boxed{18.3\ \text{m/s}^2}$$

(d) At the equilibrium position, $PE_s = 0$, so the block has kinetic energy $K = E = 0.125$ J and speed

$$v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2(0.125\ \text{J})}{0.250\ \text{kg}}} = \boxed{1.00\ \text{m/s}}$$

(e) $\boxed{\text{Smaller.}}$ Friction would transform some kinetic energy into internal energy.

(f) $\boxed{\text{The coefficient of kinetic friction between the block and surface.}}$

(g) The block will come to a stop after sliding through distance $d = x = 0.054\ 6$ m.

$$\Delta E_{\text{mech}} = \Delta K + \Delta U = -f_k d$$

$$0 + \left(0 - \frac{1}{2}kx^2\right) = -f_k d = -\mu_k mgd \rightarrow \mu_k = \frac{kx^2}{2mgd} = \frac{kx^2}{2mgx} = \frac{kx}{2mg}$$

$$\rightarrow \mu_k = \frac{(83.8\ \text{N/m})(0.054\ 6\ \text{m})}{2(0.250\ \text{kg})(9.80\ \text{m/s}^2)} = \boxed{0.934}$$

P15.35 The period of a pendulum is the time for one complete oscillation and is given by $T = 2\pi\sqrt{\ell/g}$, where ℓ is the length of the pendulum.

(a) $T = \left(\dfrac{3.00 \text{ min}}{120 \text{ oscillations}}\right)\left(\dfrac{60 \text{ s}}{1 \text{ min}}\right) = \boxed{1.50 \text{ s}}$

(b) The length of the pendulum is

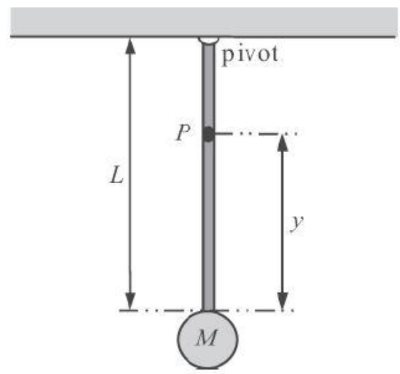
$$\ell = g\left(\frac{T^2}{4\pi^2}\right) = \left(9.80 \text{ m/s}^2\right)\left(\frac{(1.50 \text{ s})^2}{4\pi^2}\right) = \boxed{0.559 \text{ m}}$$

P15.59 Let F represent the tension in the rod.

(a) At the pivot,

$$F = Mg + Mg = \boxed{2Mg}$$

(b) A fraction of the rod's weight $Mg\left(\dfrac{y}{L}\right)$ as well as the weight of the ball pulls down on point P. Thus, the tension in the rod at point P is



ANS. FIG. P15.59

$$F = Mg\left(\frac{y}{L}\right) + Mg = \boxed{Mg\left(1 + \frac{y}{L}\right)}$$

(c) Relative to the pivot, $I = I_{\text{rod}} + I_{\text{ball}} = \dfrac{1}{3}ML^2 + ML^2 = \dfrac{4}{3}ML^2$.

For the physical pendulum, $T = 2\pi\sqrt{\dfrac{I}{mgd}}$, where $m = 2M$ and d is the distance from the pivot to the center of mass of the rod and ball combination. Therefore,

$$d = \frac{M(L/2) + ML}{M + M} = \frac{3L}{4}$$

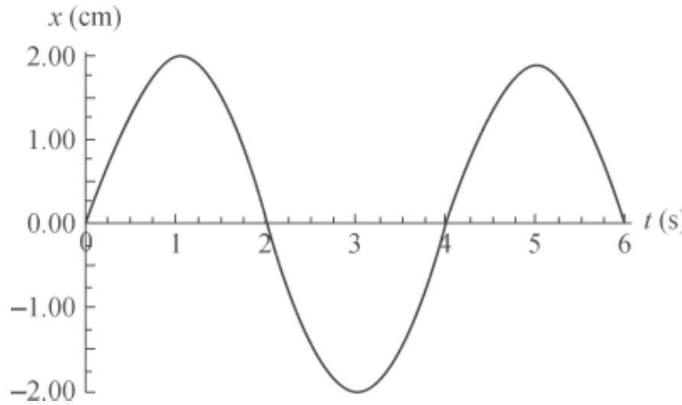
and $T = 2\pi\sqrt{\dfrac{(4/3)ML^2}{(2M)g(3L/4)}} = \boxed{\dfrac{4\pi}{3}\sqrt{\dfrac{2L}{g}}}$.

(d) For $L = 2.00$ m, $T = \dfrac{4\pi}{3}\sqrt{\dfrac{2(2.00 \text{ m})}{9.80 \text{ m/s}^2}} = \boxed{2.68 \text{ s}}$.

P15.64 (a) The amplitude is the magnitude of the maximum displacement from equilibrium (at $x = 0$). Thus, $\boxed{A = 2.00\text{ cm}}$.

(b) The period is the time for one full cycle of the motion. Therefore, $\boxed{T = 4.00\text{ s}}$.

(c) The angular frequency is $\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{4.00\text{ s}} = \boxed{\dfrac{\pi}{2}\text{ rad/s}}$.



ANS. FIG. P15.64

(d) The maximum speed is

$$v_{\max} = \omega A = \left(\frac{\pi}{2}\text{ rad/s}\right)(2.00\text{ cm}) = \boxed{\pi\text{ cm/s}}$$

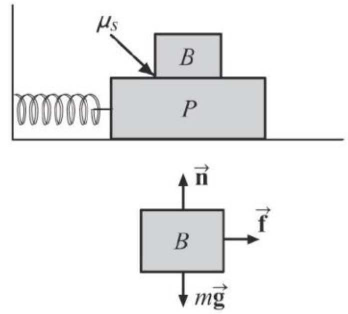
(e) The maximum acceleration is

$$a_{\max} = \omega^2 A = \left(\frac{\pi}{2}\text{ rad/s}\right)^2(2.00\text{ cm}) = \boxed{4.93\text{ cm/s}^2}$$

(f) The general equation for position as a function of time for an object undergoing simple harmonic motion with $x = 0$ when $t = 0$ and x increasing positively is $x = A\sin\omega t$. For this oscillator, this becomes

$$\boxed{x = 2.00\sin\left(\frac{\pi}{2}t\right)\text{, where } x \text{ is in centimeters and } t \text{ in seconds.}}$$

P15.65 The maximum acceleration of the oscillating system is $a_{\max} = A\omega^2 = 4\pi^2 Af^2$. The friction force exerted between the two blocks must be capable of accelerating Block B at this rate. Thus, if Block B is about to slip,



ANS. FIG. P15.65

$$f = f_{\max} = \mu_s n = \mu_s mg$$
$$= m\left(4\pi^2 Af^2\right)$$

which gives a maximum amplitude of oscillation of

$$A = \frac{\mu_s g}{4\pi^2 f^2} = \frac{(0.600)(980 \text{ cm/s}^2)}{4\pi^2(1.50 \text{ s}^{-1})^2} = \boxed{6.62 \text{ cm}}$$

P15.66 Refer to ANS. FIG. P15.65. The maximum acceleration of the oscillating system is $a_{\max} = A\omega^2 = 4\pi^2 Af^2$. The friction force exerted between the two blocks must be capable of accelerating Block B at this rate. Thus, if Block B is about to slip,

$$f = f_{\max} = \mu_s n = \mu_s mg = m\left(4\pi^2 Af^2\right)$$

which gives a maximum amplitude of oscillation of

$$A = \boxed{\frac{\mu_s g}{4\pi^2 f^2}}$$

P15.70 Please refer to ANS. FIG. P15.69. At equilibrium, we have

$$\sum \tau = 0 - mg\left(\frac{L}{2}\right) + kx_0 L$$

where x_0 is the equilibrium compression.

After displacement by a small angle (we assume cos $\theta \approx 1$),

$$\sum \tau = -mg\left(\frac{L}{2}\right) + kxL = -mg\left(\frac{L}{2}\right) + k\left(x_0 - L\theta\right)L = -k\theta L^2$$

But,

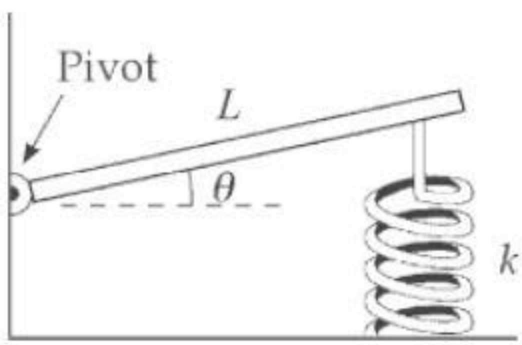
$$\sum \tau = I\alpha = \frac{1}{3}mL^2\frac{d^2\theta}{dt^2}$$

Comparing this result to the general form for simple harmonic motion in which the angular acceleration is opposite in direction and proportional to the displacement,

$$\frac{d^2\theta}{dt^2} = -\omega^2\theta$$

we find that

$$\omega^2 = \frac{3k}{m} \rightarrow \boxed{\omega = \sqrt{\frac{3k}{m}}}$$



ANS. FIG. P15.69

P15.75 (a) $T=\dfrac{2\pi}{\omega}=2\pi\sqrt{\dfrac{L}{g}}=2\pi\sqrt{\dfrac{2.23\text{ m}}{9.80\text{ m/s}^2}}=\boxed{3.00\text{ s}}$

(b) $E=\dfrac{1}{2}mv^2=\dfrac{1}{2}(6.74\text{ kg})(2.06\text{ m/s})^2=\boxed{14.3\text{ J}}$

(c) For a system of an isolated pendulum-Earth, mechanical energy is conserved. Relate the pendulum bob at the lowest point to the highest point:

$$\Delta K+\Delta U_g=0$$

$$\left(0-\frac{1}{2}mv^2\right)+(mgh-0)=0$$

$$mgh=\frac{1}{2}mv^2$$

$$h=\frac{v^2}{2g}=\frac{(2.06\text{ m/s})^2}{2(9.80\text{ m/s}^2)}=0.217\text{ m}$$

and

$$h=L-L\cos\theta=L(1-\cos\theta)$$

$$\cos\theta=1-\frac{h}{L}=1-\frac{0.217\text{ m}}{2.23\text{ m}}$$

$$\boxed{\theta=25.5^\circ}$$