Homework 2

Chapter 16

P16.9 (a) We note that $\sin\theta = -\sin(-\theta) = \sin(-\theta+\pi)$, so the given wave function can be written as

$$y(x,t) = (0.350)\sin(-10\pi t + 3\pi x + \pi - \pi/4)$$

Comparing, $10\pi t - 3\pi x + \pi/4 = kx - \omega t + \phi$. For constant phase, x must increase as t increases, so the wave travels in the positive x direction. Comparing the specific form to the general form, we find that

$$v = \frac{\omega}{k} = \frac{10\pi}{3\pi} = 3.33 \text{ m/s}.$$

Therefore, the velocity is $\boxed{\left(3.33\hat{\mathbf{i}}\right) \text{ m/s}}$.

(b) Substituting $t = 0$ and $x = 0.100$ m, we have

$$y(0.100\ 0) = (0.350 \text{ m}) \sin\left(-0.300\pi + \frac{\pi}{4}\right) = -0.054\ 8 \text{ m}$$

$$= \boxed{-5.48 \text{ cm}}$$

(c) $k = \dfrac{2\pi}{\lambda} = 3\pi$: $\lambda = \boxed{0.667 \text{ m}}$ $\quad \omega = 2\pi f = 10\pi$: $f = \boxed{5.00 \text{ Hz}}$

(d) $v_y = \dfrac{\partial y}{\partial t} = (0.350)(10\pi)\cos\left(10\pi t - 3\pi x + \dfrac{\pi}{4}\right)$

$$v_{y,\max} = (10\pi)(0.350) = \boxed{11.0 \text{ m/s}}$$

P16.16 (a) At $x = 2.00$ m, $y = \boxed{0.100\sin(1.00 - 20.0t)}$. Because this disturbance varies sinusoidally in time, it describes simple harmonic motion.

(b) At $x = 2.00$ m, compare $y = 0.100\sin(1.00 - 20.0t)$ to $A\cos(\omega t + \phi)$:

$$\begin{aligned} y &= 0.100\sin(1.00 - 20.0t) = -0.100\sin(20.0t - 1.00) \\ &= 0.100\cos(20.0t - 1.00 + \pi) \\ &= 0.100\cos(20.0t + 2.14) \end{aligned}$$

so $\omega = 20.0$ rad/s and $f = \dfrac{\omega}{2\pi} = \boxed{3.18\text{ Hz}}$

P16.24 (a) For the first equation,

$$f = \frac{1}{T} \rightarrow T = \frac{1}{f} \rightarrow [T] = \frac{1}{[f]} = \frac{1}{\mathrm{T}^{-1}} = \mathrm{T}$$

$\boxed{\text{units are seconds}}$

$$v = \sqrt{\frac{T}{\mu}} \rightarrow T = \mu v^2 \rightarrow [T] = [\mu v^2] = \frac{\mathrm{M}}{\mathrm{L}}\left(\frac{\mathrm{L}}{\mathrm{T}}\right)^2 = \frac{\mathrm{ML}}{\mathrm{T}^2}$$

$\boxed{\text{units are newtons}}$

(b) $\boxed{\text{The first } T \text{ is period of time; the second is force of tension.}}$

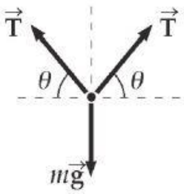
P16.25 The down and back distance is 4.00 m + 4.00 m = 8.00 m.

The speed is then $v = \dfrac{d_{\text{total}}}{t} = \dfrac{4(8.00\text{ m})}{0.800\text{ s}} = 40.0\text{ m/s} = \sqrt{\dfrac{T}{\mu}}$.

Now, $\mu = \dfrac{0.200\text{ kg}}{4.00\text{ m}} = 5.00 \times 10^{-2}\text{ kg/m}$.

So $T = \mu v^2 = (5.00 \times 10^{-2}\text{ kg/m})(40.0\text{ m/s})^2 = \boxed{80.0\text{ N}}$.

P16.30 From the free-body diagram $mg = 2T \sin\theta$



ANS. FIG. P16.30

$$T = \frac{mg}{2\sin\theta}$$

The angle θ is found from

$$\cos\theta = \frac{3L/8}{L/2} = \frac{3}{4}$$

$$\therefore \theta = 41.4°$$

(a) $$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{mg}{2\mu\sin\theta}}$$

$$= \sqrt{\frac{mg}{2\mu\sin 41.4°}} = \left(\sqrt{\frac{9.80\ \text{m/s}^2}{2\left(8.00\times10^{-3}\ \text{kg/m}\right)\sin 41.4°}}\right)\sqrt{m}$$

or $\boxed{v = (30.4\ )\sqrt{m}, \text{ where } v \text{ is in meters per second and } m \text{ is in kilograms.}}$

(b) $v = 60.0 = 30.4\sqrt{m}$ and $\boxed{m = 3.89\ \text{kg}}$

P16.36 The frequency and angular frequency of the wave are

$$f = \frac{v}{\lambda} = \frac{30.0\ \text{m/s}}{0.500\ \text{s}} = 60.0\ \text{Hz} \quad \text{and} \quad \omega = 2\pi f = 120\pi\ \text{rad/s}$$

The power that is required is then

$$P = \frac{1}{2}\mu\omega^2 A^2 v$$

$$= \frac{1}{2}\left(\frac{0.180\ \text{kg}}{3.60\ \text{m}}\right)(120\pi\ \text{rad/s})^2(0.100\ \text{m})^2(30.0\ \text{m/s})$$

$$= \boxed{1.07\ \text{kW}}$$

P16.39 Comparing

$$y = 0.350\sin\left(10\pi t - 3\pi x + \frac{\pi}{4}\right)$$

with

$$y = A\sin(kx - \omega t + \phi) = A\sin(\omega t - kx - \phi + \pi)$$

we have

$$k = 3\pi\ \mathrm{m}^{-1},\ \omega = 10\pi\ \mathrm{s}^{-1},\ \text{and } A = 0.350\ \mathrm{m}$$

Then,

$$v = f\lambda = (2\pi f)\left(\frac{\lambda}{2\pi}\right) = \frac{\omega}{k} = \frac{10\pi\ \mathrm{s}^{-1}}{3\pi\ \mathrm{m}^{-1}} = 3.33\ \mathrm{m/s}$$

(a) The rate of energy transport is

$$\begin{aligned} P &= \frac{1}{2}\mu\omega^2 A^2 v \\ &= \frac{1}{2}(75\times10^{-3}\ \mathrm{kg/m})(10\pi\ \mathrm{s}^{-1})^2(0.350\ \mathrm{m})^2(3.33\ \mathrm{m/s}) \\ &= \boxed{15.1\ \mathrm{W}} \end{aligned}$$

(b) Recall that $vT = \lambda$. The energy per cycle is

$$\begin{aligned} E_\lambda &= P\,T = \frac{1}{2}\mu\omega^2 A^2 \lambda \\ &= \frac{1}{2}(75.0\times10^{-3}\ \mathrm{kg/m})(10\pi\ \mathrm{s}^{-1})^2(0.350\ \mathrm{m})^2\left(\frac{2\pi}{3\pi\ \mathrm{m}^{-1}}\right) \\ &= \boxed{3.02\ \mathrm{J}} \end{aligned}$$

P16.43 The linear wave equation is $\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$.

If $\quad y = e^{b(x-vt)}$

Then $\quad \frac{\partial y}{\partial t} = -bve^{b(x-vt)} \quad$ and $\quad \frac{\partial y}{\partial x} = be^{b(x-vt)}$

$$\frac{\partial^2 y}{\partial t^2} = b^2v^2e^{b(x-vt)} \quad \text{and} \quad \frac{\partial^2 y}{\partial x^2} = b^2e^{b(x-vt)}$$

Therefore, $\frac{\partial^2 y}{\partial t^2} = v^2\frac{\partial^2 y}{\partial x^2}$, demonstrating that $e^{b(x-vt)}$ is a solution.

P16.51 (a) The wave function becomes

$$0.175\text{ m} = (0.350\text{ m})\sin\left[(99.6\text{ rad/s})t\right]$$

or $\quad \sin\left[(99.6\text{ rad/s})t\right] = 0.500$

The smallest two angles for which the sine function is 0.500 are 30.0° and 150°, i.e., 0.523 6 rad and 2.618 rad.

$$(99.6\text{ rad/s})t_1 = 0.523\,6\text{ rad, thus } t_1 = 5.26\text{ ms}$$

$$(99.6\text{ rad/s})t_2 = 2.618\text{ rad, thus } t_2 = 26.3\text{ ms}$$

$$\Delta t \equiv t_2 - t_1 = 26.3\text{ ms} - 5.26\text{ ms} = \boxed{21.0\text{ ms}}$$

(b) Distance traveled by the wave

$$= \left(\frac{\omega}{k}\right)\Delta t = \left(\frac{99.6\text{ rad/s}}{1.25\text{ rad/m}}\right)(21.0\times10^{-3}\text{ s}) = \boxed{1.68\text{ m}}$$

(c) To find the tension in the string, we first compute the wave speed

$$v = \lambda f = \frac{\omega}{k} = \frac{50.0\ \text{s}^{-1}}{0.800\ \text{m}^{-1}} = 62.5\ \text{m/s}$$

then,

$$v = \sqrt{\frac{T}{\mu}} \text{ gives } T = \mu v^2 = \left(\frac{12.0 \times 10^{-3}\ \text{kg}}{1.00\ \text{m}}\right)(62.5\ \text{m/s})^2 = \boxed{46.9\ \text{N}}$$

The maximum transverse force is very small compared to the tension, more than a thousand times smaller.

P16.61 (a) $P(x) = \frac{1}{2}\mu\omega^2 A^2 v = \frac{1}{2}\mu\omega^2 A_0^2 e^{-2bx}\left(\frac{\omega}{k}\right) = \boxed{\frac{\mu\omega^3}{2k} A_0^2 e^{-2bx}}$

(b) $P(0) = \boxed{\frac{\mu\omega^3}{2k} A_0^2}$

(c) $\frac{P(x)}{P(0)} = \boxed{e^{-2bx}}$

NOTE: problem 16-60 included for reference for problem 16-64

P16.60 Imagine a short transverse pulse traveling from the bottom to the top of the rope. When the pulse is at position x above the lower end of the rope, the wave speed of the pulse is given by $v=\sqrt{\frac{T}{\mu}}$, where $T=\mu x g$ is the tension required to support the weight of the rope below position x.

Therefore, $v=\sqrt{gx}$.

But $v=\frac{dx}{dt}$, so that $dt=\frac{dx}{\sqrt{gx}}$

and $t=\int_0^L \frac{dx}{\sqrt{gx}}=\frac{1}{\sqrt{g}}\left.\frac{\sqrt{x}}{\frac{1}{2}}\right|_0^L \approx \boxed{2\sqrt{\frac{L}{g}}}$

P16.64 Refer to Problem 60. At distance x from the bottom, the tension is $T = \left(\frac{mxg}{L}\right) + Mg$, so the wave speed is:

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{TL}{m}} = \sqrt{xg + \left(\frac{MgL}{m}\right)} = \frac{dx}{dt} \rightarrow dt = \frac{dx}{\sqrt{xg + \left(\frac{MgL}{m}\right)}}$$

(a) Then

$$t = \int_0^t dt = \int_0^L \left[xg + \left(\frac{MgL}{m}\right)\right]^{-1/2} dx$$

gives

$$t = \frac{1}{g} \frac{\left[xg + \left(MgL/m\right)\right]^{1/2}}{\frac{1}{2}}\Bigg|_{x=0}^{x=L}$$

$$t = \frac{2}{g}\left[\left(Lg + \frac{MgL}{m}\right)^{1/2} - \left(\frac{MgL}{m}\right)^{1/2}\right]$$

$$\boxed{t = 2\sqrt{\frac{L}{mg}}\left(\sqrt{M+m} - \sqrt{M}\right)}$$

(b) When $M = 0$,

$$t = 2\sqrt{\frac{L}{g}}\left(\frac{\sqrt{m} - 0}{\sqrt{m}}\right) = \boxed{2\sqrt{\frac{L}{g}}}$$

(c) As $m \to 0$ we expand

$$\sqrt{M+m} = \sqrt{M}\left(1+\frac{m}{M}\right)^{1/2} = \sqrt{M}\left(1+\frac{1}{2}\frac{m}{M}-\frac{1}{8}\frac{m^2}{M^2}+\cdots\right)$$

to obtain
$$t = 2\sqrt{\frac{L}{mg}}\left(\sqrt{M}+\frac{1}{2}\left(\frac{m}{\sqrt{M}}\right)-\frac{1}{8}\left(m^2/M^{3/2}\right)+\cdots-\sqrt{M}\right)$$

$$t \approx 2\sqrt{\frac{L}{g}}\left(\frac{1}{2}\sqrt{\frac{m}{M}}\right) = \boxed{\sqrt{\frac{mL}{Mg}}}$$

where we neglect terms $\frac{1}{8}\left(\frac{m^2}{M^{3/2}}\right)$ and higher because terms with m^2 and higher powers are very small.