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Phys M020C
Thurs 1-3:50pm

20/20

ERROR PROPAGATION EXERCISES

Determine the calculated value using the given values in the given equations. Be sure to **include the units in your answer**. Using the error propagation method described above, calculate the percent error in the calculated value. For this exercise, **your percent error is to be given to two significant figures.**

Hand in this answer sheet. Work the problems neatly on scratch paper and staple your work to this sheet.

1. $A=xy$, $x = 3.0cm \pm 0.1cm$, $y = 4.0cm \pm 0.1cm$ — $12.0\ cm^2 \pm 4.2\%$

2. $f=x+y$, for x and y given in problem # 1 — $7.0\ cm \pm 2.1\%$

3. $f=x-y$, for x and y given in problem # 1 — $-1.0\ cm \pm 15\%$

4. $z=3x+2y$, for x and y given in problem # 1 — $17.0\ cm \pm 2.2\%$

5. $g = \frac{2h}{t^2}$ for $h = 2.00m \pm 3\%$, $t = 0.630s \pm 4\%$ — $10.1\ m/s^2 \pm 8.6\%$

6. $T = 2\pi\sqrt{\frac{M}{k}}$, $M = 2.5Kg \pm 6\%$, $k = \frac{100N}{m} \pm 2\%$ — $0.99\ s \pm 3.2\%$

7. $d = \left(\frac{5.00}{cm^2 g}\right)(ML^3)$, $M = 30.0g \pm 2\%$, $L = (20.3 \pm 0.2)cm$ — $(1.25 \times 10^6)\ cm \pm 3.6\%$

8. $z = x^2 + y^2$, $x = 3.0cm \pm 2\%$, $y = 4.0cm \pm 2\%$ — $25\ cm^2 \pm 3.0\%$

9. $z = \frac{5a^3 - (2cm)b^2}{C}$, $a = 2.0cm \pm 1\%$, $b = 3.0cm \pm 1\%$, $C = 11.0cm \pm 2\%$ — $2.0\ cm^2 \pm 6.1\%$

10. $h = d\sin\theta$, $d = 1.00m \pm 0.05m$, $\theta = 10^\circ \pm 1^\circ$ — $0.17\ m \pm 12\%$

Hint: Convert 1° to radians

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① $A = xy$, $x = 3.00\text{cm} \pm 0.1\text{cm}$, $y = 4.00\text{cm} \pm 0.1\text{cm}$

$\frac{\partial A}{\partial x} = y$, $\frac{\partial A}{\partial y} = x$, $A = (3.00)(4.00) = \boxed{12.0\ \text{cm}^2}$

$$\frac{\partial A}{A} = \left\{ \left[\frac{\left(\frac{\partial A}{\partial x}\right)\delta x}{A} \right]^2 + \left[\frac{\left(\frac{\partial A}{\partial y}\right)\delta y}{A} \right]^2 \right\}^{1/2}$$

$$= \left\{ \left[\frac{y\,\delta x}{xy} \right]^2 + \left[\frac{x\,\delta y}{xy} \right]^2 \right\}^{1/2}$$

$$= \left\{ \left[\frac{\delta x}{x} \right]^2 + \left[\frac{\delta y}{y} \right]^2 \right\}^{1/2} = \left\{ \left[\frac{0.1}{3} \right]^2 + \left[\frac{0.1}{4} \right]^2 \right\}^{1/2}$$

$= 0.0416 \rightarrow \boxed{4.2\%}$

② $f = x + y$, $x = 3.00 \pm 0.1\text{cm}$, $y = 4.00\text{cm} \pm 0.1\text{cm}$

$\frac{\partial f}{\partial x} = 1$, $\frac{\partial f}{\partial y} = 1$, $f = 3.00 + 4.00 = \boxed{7.00\ \text{cm}}$

$$\frac{\partial f}{f} = \left\{ \left[\frac{\left(\frac{\partial f}{\partial x}\right)\delta x}{f} \right]^2 + \left[\frac{\left(\frac{\partial f}{\partial y}\right)\delta y}{f} \right]^2 \right\}^{1/2}$$

$$= \left\{ \left[\frac{\delta x}{x+y} \right]^2 + \left[\frac{\delta y}{x+y} \right]^2 \right\}^{1/2}$$

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$$= \left\{ \left[\frac{0.1}{7.00} \right]^2 + \left[\frac{0.1}{7.00} \right]^2 \right\}^{1/2}$$

$$= 0.0202 \rightarrow \boxed{2.1\%}$$

③ $f = x - y$, $x = 3.00\,cm \pm 0.1\,cm$, $y = 4.00\,cm \pm 0.1\,cm$

$$\frac{\partial f}{\partial x} = 1, \quad \frac{\partial f}{\partial y} = -1, \quad f = 3.00 - 4.00 = \boxed{-1.00\,cm}$$

$$\frac{\delta f}{f} = \left\{ \left[\frac{\left(\frac{\partial f}{\partial x}\right)\delta x}{f} \right]^2 + \left[\frac{\left(\frac{\partial f}{\partial y}\right)\delta y}{f} \right]^2 \right\}^{1/2}$$

$$= \left\{ \left[\frac{\delta x}{x - y} \right]^2 + \left[\frac{-\delta y}{x - y} \right]^2 \right\}$$ (goes (+))

$$= \left\{ \left[\frac{0.1}{-1} \right]^2 + \left[\frac{0.1}{-1} \right]^2 \right\}^{1/2} = \left\{ 2(.1^2) \right\}^{1/2}$$

$$= 0.141 \rightarrow \boxed{15\%}$$

④ $z = 3x + 2y$, $x = 3.00\,cm \pm 0.1\,cm$, $y = 4.00\,cm \pm 0.1\,cm$

$$\frac{\partial z}{\partial x} = 3, \quad \frac{\partial z}{\partial y} = 2, \quad z = 3(3.00) + 2(4.00) = \boxed{17.0\,cm}$$

$$\frac{\delta z}{z} = \left\{ \left[\frac{\left(\frac{\partial z}{\partial x}\right)\delta x}{z} \right]^2 + \left[\frac{\left(\frac{\partial z}{\partial y}\right)\delta y}{z} \right]^2 \right\}^{1/2}$$

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$$= \left\{ \left[\frac{3\delta x}{3x+2y} \right]^2 + \left[\frac{2\delta y}{3x+2y} \right]^2 \right\}^{1/2}$$

$$= \left\{ \left[\frac{3(0.1)}{3(3)+2(4)} \right]^2 + \left[\frac{2(0.1)}{3(3)+2(4)} \right]^2 \right\}^{1/2}$$

$$= 0.0212 \rightarrow \boxed{2.2\%}$$

⑤ $g = 2h/t^2$, $h = 2.00\text{m} \pm 3\%$, $t = 0.630\text{s} \pm 4\%$

$$\frac{\partial g}{\partial h} = \frac{2}{t^2}, \quad \frac{\partial g}{\partial t} = \frac{-4h}{t^3}, \quad g = 2(2.00)/.630^2 = \boxed{10.1\ \text{m/s}^2}$$

$$\frac{\delta g}{g} = \left\{ \left[\frac{\left(\frac{\partial g}{\partial h}\right)\delta h}{g} \right]^2 + \left[\frac{\left(\frac{\partial g}{\partial t}\right)\delta t}{g} \right]^2 \right\}^{1/2}$$

$$= \left\{ \left[\frac{\left(\frac{2}{t^2}\right)(\delta h)}{\left(\frac{2h}{t^2}\right)} \right]^2 + \left[\frac{\left(\frac{-4h}{t^3}\right)(\delta t)}{\left(\frac{2h}{t^2}\right)} \right]^2 \right\}^{1/2}$$

$$= \left\{ \left[3\% \right]^2 + \left[2(4\%) \right]^2 \right\}^{1/2}$$

$$= 8.544 \rightarrow \boxed{8.6\%}$$

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⑥ $T = 2\pi\left(\frac{M}{K}\right)^{1/2}$, $M = 2.50\,kg \pm 6\%$, $K = \frac{100\,N}{m} \pm 2\%$

$\frac{\partial T}{\partial M} = \frac{\pi}{K^{1/2}M^{1/2}}$, $\frac{\partial T}{\partial K} = \frac{-\pi M^{1/2}}{K^{3/2}}$, $T = 2\pi\left(\frac{2.5\,kg}{100\,N/m}\right) = \boxed{0.99\left(\frac{kg\cdot m}{N}\right)^{1/2}}$

Note: this is seconds

$$\frac{\delta T}{T} = \left\{\left[\frac{\left(\frac{\partial T}{\partial M}\right)\delta M}{T}\right]^2 + \left[\frac{\left(\frac{\partial T}{\partial K}\right)\delta K}{T}\right]^2\right\}^{1/2}$$

$$= \left\{\left[\frac{\left(\frac{\pi}{K^{1/2}M^{1/2}}\right)\delta M}{\left(\frac{2\pi M^{1/2}}{K^{1/2}}\right)}\right]^2 + \left[\frac{\left(\frac{-\pi M^{1/2}}{K^{3/2}}\right)\delta K}{\left(\frac{2\pi M^{1/2}}{K^{1/2}}\right)}\right]^2\right\}^{1/2}$$

$$= \left\{\left(\frac{\delta M}{2M}\right)^2 + \left(\frac{\delta K}{K2}\right)^2\right\}^{1/2}$$

Note: $\delta M = (2.5)(.06) = .15$

$\delta K = (100)(.02) = 2$

$$= \left\{\left[\frac{6\%}{2}\right]^2 + \left[\frac{2\%}{2}\right]^2\right\}^{1/2}$$

$= 3.16 \rightarrow \boxed{3.2\%}$

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(7) $d = \left(\frac{5.00}{cm^3 g}\right)(ML^3)$, $M = 30.0g \pm 2\%$, $L = (20.3 \pm 0.2)cm$

$\frac{\partial d}{\partial M} = 5L^3$, $\frac{\partial d}{\partial L} = 15ML^2$, $d = (5.00)(30)(20.3^3) = \boxed{1.25 \times 10^6 cm}$

$$\frac{\partial d}{d} = \left\{\left[\frac{\left(\frac{\partial d}{\partial M}\right)\delta M}{d}\right]^2 + \left[\frac{\left(\frac{\partial d}{\partial L}\right)\delta L}{d}\right]^2\right\}^{1/2}$$

$$= \left\{\left[\frac{(5L^3)\,\delta M}{5ML^3}\right]^2 + \left[\frac{(15ML^2)\,\delta L}{5ML^3}\right]^2\right\}^{1/2}$$

$$= \left\{\left[\frac{\delta M}{M}\right]^2 + \left[\frac{3\,\delta L}{L}\right]^2\right\}^{1/2}$$

Note: $\delta L = .2 \rightarrow \frac{\delta L}{L} = 0.99\%$

$$= \left\{2^2 + ((3)(.99))^2\right\}^{\frac{1}{2}}$$

$= 3.60 \rightarrow \boxed{3.6\%}$

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(8) $z = x^2 + y^2$, $x = 3.00\text{cm} \pm 2\%$, $y = 4.00\text{cm} \pm 2\%$

$\frac{\partial z}{\partial x} = 2x$, $\frac{\partial z}{\partial y} = 2y$, $z = (3.00^2) + (4.00^2) = \boxed{25.0\text{ cm}^2}$

$$\frac{\delta z}{z} = \left\{ \left[\frac{\left(\frac{\partial z}{\partial x}\right)\delta x}{z} \right]^2 + \left[\frac{\left(\frac{\partial z}{\partial y}\right)\delta y}{z} \right]^2 \right\}^{1/2}$$

$$= \left\{ \left[\frac{2x\,\delta x}{x^2+y^2} \right]^2 + \left[\frac{2y\,\delta y}{x^2+y^2} \right]^2 \right\}^{1/2}$$

Note: $\delta x = (3)(.02) = .06$

$\delta y = (4)(.02) = .08$

$$= \left\{ \left[\frac{2(3)(.06)}{3^2+4^2} \right]^2 + \left[\frac{2(4)(.08)}{3^2+4^2} \right]^2 \right\}^{1/2}$$

$= 0.0293 \rightarrow \boxed{3.0\%}$

(9) $z = \frac{5a^3 - (2\text{cm})b^2}{c}$, $a = 2.00\text{cm} \pm 1\%$, $b = 3.00\text{cm} \pm 1\%$, $c = 11.0\text{cm} \pm 2\%$

$\frac{\partial z}{\partial a} = \frac{15a^2}{c}$, $\frac{\partial z}{\partial b} = \frac{-4b}{c}$, $\frac{\partial z}{\partial c} = \frac{-(5a^3 - 2b^2)}{c^2}$, $z = \frac{5(2^3) - 2(3^2)}{11} = \boxed{2.0\text{cm}^3}$

$$\frac{\delta z}{z} = \left\{ \left[\frac{\left(\frac{\partial z}{\partial a}\right)\delta a}{z} \right]^2 + \left[\frac{\left(\frac{\partial z}{\partial b}\right)\delta b}{z} \right]^2 + \left[\frac{\left(\frac{\partial z}{\partial c}\right)\delta c}{z} \right]^2 \right\}^{1/2}$$

$$\frac{\delta z}{z} = \left\{ \left[\frac{\left(\frac{15a^2}{c}\right)\delta a}{\frac{5a^3 - 2b^2}{c}} \right]^2 + \left[\frac{\left(\frac{-4b}{c}\right)\delta b}{\frac{5a^3 - 2b^2}{c}} \right]^2 + \left[\frac{\left(\frac{-(5a^3 - 2b^2)}{c^2}\right)\delta c}{\frac{5a^3 - 2b^2}{c}} \right]^2 \right\}^{1/2}$$

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$$= \left\{ \left[\frac{15a^2 \delta a}{5a^3 - 2b^2} \right]^2 + \left[\frac{4b\,\delta b}{5a^3 - 2b^2} \right]^2 + \left[\frac{\delta c}{c} \right]^2 \right\}^{1/2}$$

Note: $\delta a = (2)(.01) = 0.02$
$\delta b = (3)(.01) = 0.03$
$\delta c = (11)(.02) = 0.22$

$$= \left\{ \left[\frac{15(2^2)(.02)}{5(2^3) - 2(3^3)} \right]^2 + \left[\frac{4(3)(.03)}{5(2^3) - 2(3^3)} \right]^2 + \left(\frac{.22}{11} \right)^2 \right\}^{1/2}$$

$= 0.0603 \rightarrow \boxed{6.1\%}$

(10) $h = d\sin\theta$, $d = 1.00\text{m} \pm 0.05\text{m}$, $\theta = 10° \pm 1°$

$\frac{\partial h}{\partial d} = \sin\theta$, $\frac{\partial h}{\partial \theta} = d\cos\theta$, $h = (1)\sin(10°) = \boxed{0.17\text{m}}$

$$\frac{\partial h}{h} = \left\{ \left[\frac{\left(\frac{\partial h}{\partial d}\right)\delta d}{h} \right]^2 + \left[\frac{\left(\frac{\partial h}{\partial \theta}\right)\delta\theta}{h} \right]^2 \right\}^{1/2}$$

$$= \left\{ \left[\frac{(\sin\theta)\,\delta d}{d\sin\theta} \right]^2 + \left[\frac{d\cos\theta\,\delta\theta}{d\sin\theta} \right]^2 \right\}^{1/2}$$

$$= \left\{ \left[\frac{\delta d}{d} \right]^2 + \left[(\cot\theta)\delta\theta \right]^2 \right\}^{1/2}$$

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Note: Convert to radians $\rightarrow \frac{(1^\circ)\pi}{180} = \frac{\pi}{180}$ rads

$$= \left\{ \left(\frac{(.05)}{1} \right)^2 + \left(\cot(10^\circ)\left(\frac{\pi}{180} \right) \right)^2 \right\}^{1/2}$$

$$= 0.111 \rightarrow \boxed{12\%}$$