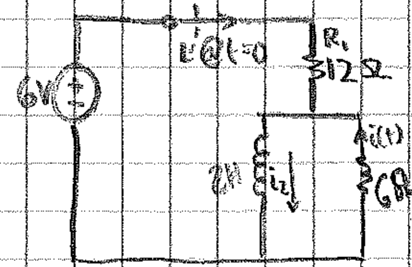
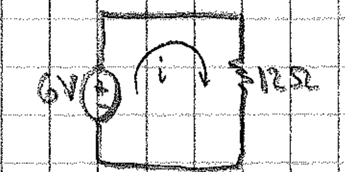
HW #7; Engr M20 Jared Fowler

1) Find $i(t)$ for $t>0$ and sketch the waveform in the following:



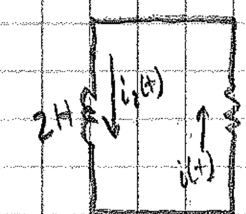
Note: Assume that @ $t<0$ the ckt is at stable state ⇒ inductor looks like a short.



$i = 6/12 = \underline{500\,mA}$ (I.C.)

Note: @ $t(0^+)$, the current will be 500 mA, b/c current through inductor cannot change instantaneously.

After switch open, we have:



Note that $i_L(t) = i(t)$ for $t>0$.

KVL: $L\frac{di}{dt} + iR = 0 \Rightarrow \underline{\frac{di}{dt} + \frac{iR}{L} = 0}$ (Eq1)

(Eq2) $i = i_n + i_f$ & $i_f = K_1$ b/c 0 is constant $\Rightarrow i_f \Rightarrow \cancel{\frac{di}{dt}}^{\,0} + K_1\frac{R}{L} = 0 \Rightarrow \underline{K_1 = 0}$

, Now for $i_n \Rightarrow \frac{di_n}{dt} + \frac{R}{L}i_n = 0$

Assumed solution of $i_n = K_2e^{st}$

Substitute into (Eq1) $\rightarrow \frac{dK_2e^{st}}{dt} + \frac{R}{L}K_2e^{st} = 0$

$s\cancel{K_2e^{st}} + \frac{R}{L}\cancel{K_2e^{st}} = 0$

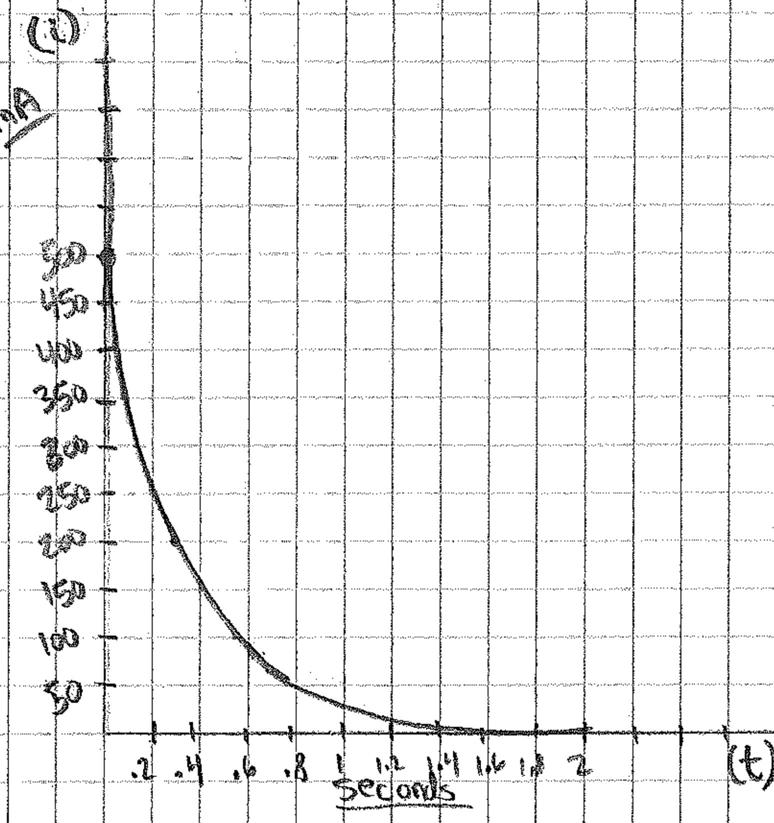
$s = -\frac{R}{L} = \frac{6}{2} = \underline{-3}$

Substitute into (Eq2)

$i(t) = K_2e^{-3t} + 0$

$i(0) = 500\,mA = K_2\cancel{e^{0}}^{\,1} \rightarrow \underline{K_2 = 0.5}$

$\therefore \boxed{i(t) = 0.5e^{-3t}}$



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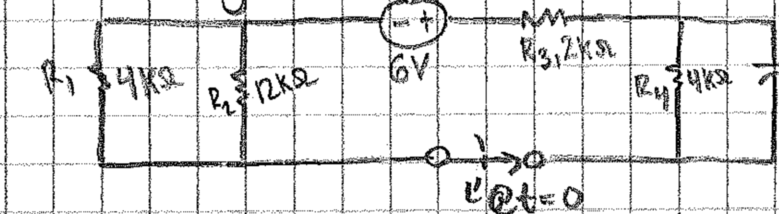
2) Find $V_c(t)$ for $t>0$ in the ckt below and plot the response including time interval prior to the switch opening.



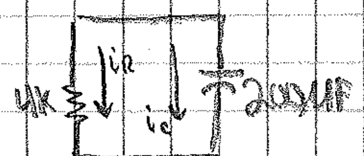
Note: Assume that @ $t<0$ the ckt is at stable state → capacitor looks like open

Note: @ $t<0$, $V_{R_4} = V_c$

KVL: $-6 + 2ki + 4ki + 3ki = 0$

$i = \frac{6}{9k} = \frac{2}{3}$ mA

$\therefore V_c = (4k)(\frac{2}{3}\text{mA}) = \underline{\frac{8}{3}V}$ (I.C.)



(Eq 1)

KCL: $i_c + i_R = 0 \rightarrow \left(\cancel{C}^{.0002}\frac{dV}{dt} + \frac{V_c}{4k} = 0\right) \cdot 10^4 = \frac{2dV}{dt} + \frac{10}{4}V_c = 0 \Rightarrow \frac{dV_c}{dt} + \frac{5}{4}V_c = 0$

(Eq 2)

$V_c(t) = V_n + V_f$ & $K_2 = V_f$ b/c 0 is constant $\Rightarrow V_f = \frac{d\cancel{K_2}^{0}}{dt} + \frac{5}{4}K_2 = 0 \Rightarrow \underline{K_2 = 0}$

Assumed solution, $V_n = K_1 e^{st}$

Substitute into (Eq 1)

$\frac{d}{dt}K_1 e^{st} + \frac{5}{4}K_1 e^{st} = 0$

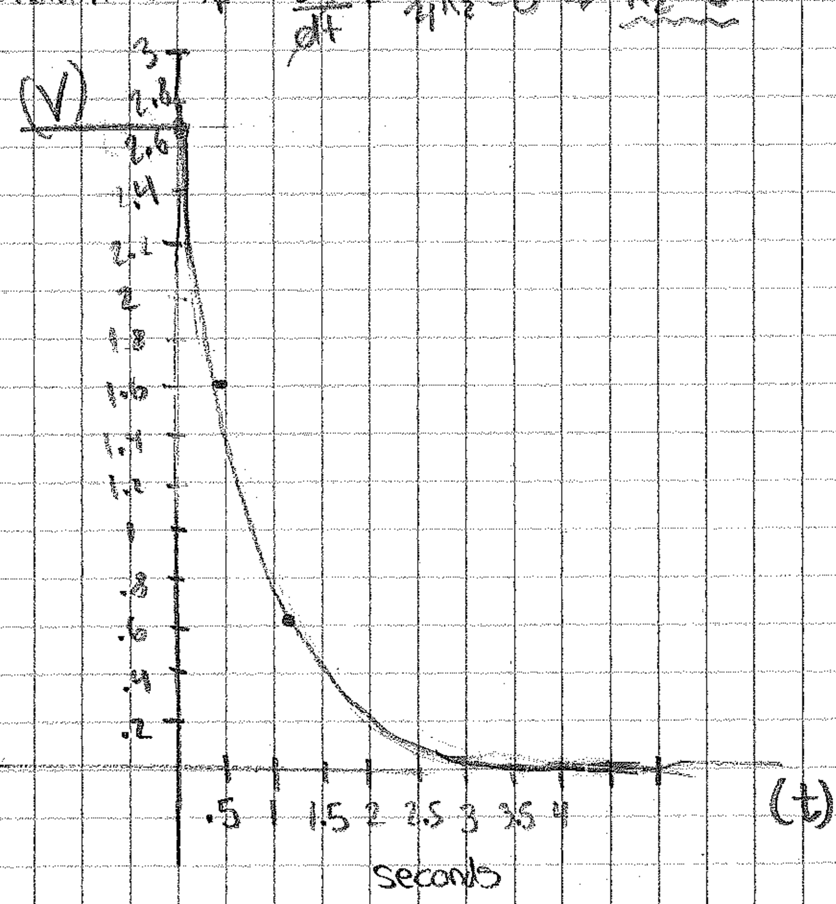
$sK_1 e^{st} + \frac{5}{4}K_1 e^{st} = 0 \Rightarrow \underline{s = -\frac{5}{4}}$

Back into (Eq 2)

$V_c(t) = K_1 e^{-\frac{5}{4}t}$

$V_c(0) = \frac{8}{3} = K_1 e^{0} \quad \therefore \underline{K = \frac{8}{3}}$

$\therefore \boxed{V_c(t) = \frac{8}{3}e^{-\frac{5}{4}t}}$



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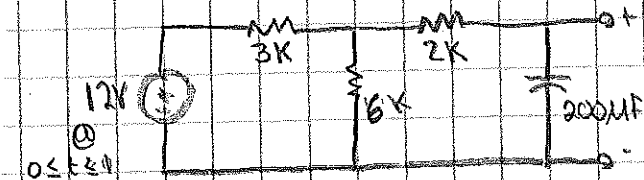
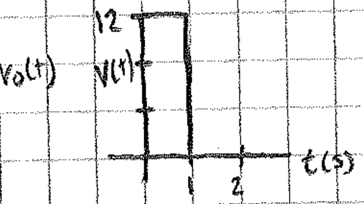
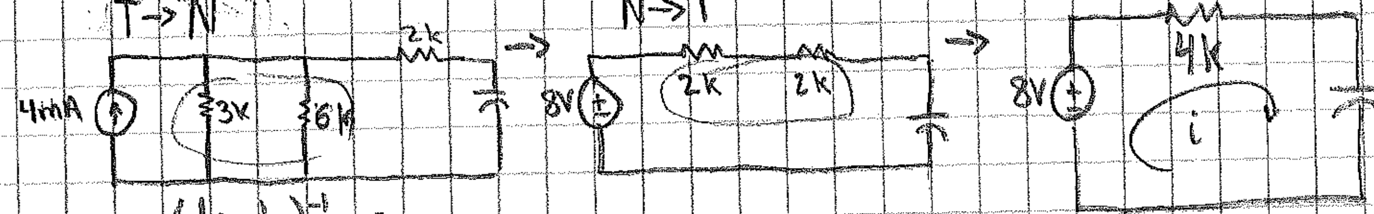
3) Determine the equation for the voltage for $V_o(t)$ for $t>0$ for the ckt below when subjected to the input pulse shown below.





T -> N   N -> T



$\left(\frac{1}{3k}+\frac{1}{6k}\right)^{-1} = 2k$

KCL: $i_R = i_c \Rightarrow \dfrac{8-V_c}{4k} = C\dfrac{dv}{dt} \Rightarrow \dfrac{dv}{dt} + \dfrac{V_c}{(4k)C} = \dfrac{8}{4k(C)} \Rightarrow \underbrace{\dfrac{dv}{dt} + 1.25V_c = 10}$ (Eq. 1)

Note: @ $t<0$, the voltage is $\underbrace{0}$ (I.C.)

for $0 \le t \le 1$, $V(t) = V_n + V_f$

$V_f = K_2$ b/c 10 is constant $\underset{(Eq1)}{\Rightarrow}$ $\dfrac{dK_2}{dt} + 1.25K_2 = 10 \Rightarrow \underline{K_2 = 8}$

$V_n = K_1e^{st} \underset{(Eq1)}{\Rightarrow} \dfrac{d}{dt}K_1e^{st} + 1.25K_1e^{st} = 0 \Rightarrow sK_1e^{st} + 1.25K_1e^{st} = 0 \Rightarrow \underline{S = -1.25}$

$\rightarrow V(t) = K_1e^{-1.25t} + 8$
$V(0) = 0 = K_1 + 8 \rightarrow \underline{K_1 = -8}$

$\therefore \boxed{@\ 0 \le t \le 1,\ V_c(t) = 8 - 8e^{-1.25t}}$

Note: $V(1) = 8 - 8e^{-1.25} = \underline{5.707}$ (I.C.)

for a sourceless circuit, $V(t) = Ke^{-1.25t}$ @ $t>1$

$V(1) = 5.707 = Ke^{-1.25} \Rightarrow \underline{K = 5.707e^{1.25}}$

$\therefore \boxed{@\ t>1,\ V(t) = 5.707e^{-1.25(t-1)}}$

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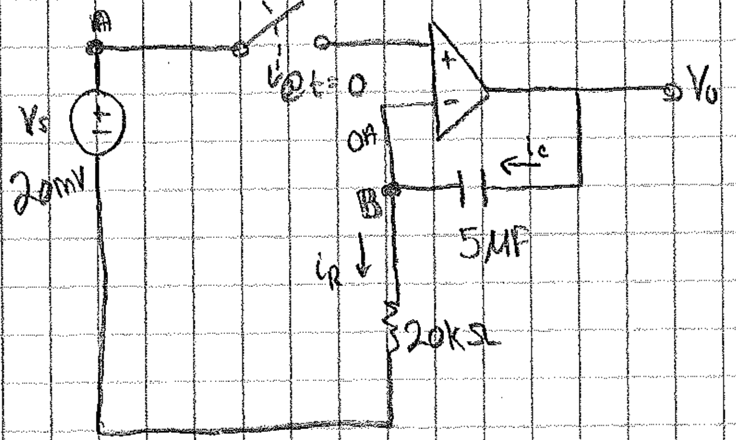
4) Find $V_o(t)$ for $t>0$ when $V_s = 20mV$ in the following CKT:



Note: @ t(0), $V_c = 0 \rightarrow V_o(0) = 20mV$ (I.C.)

KCL @ (-) input

$i_c + OA = i_R$

$C\frac{dV_c}{dt} = \frac{V_B - 0}{20K}$ Note: $V_A = V_B = 20mV$

$C\frac{dV_c}{dt} = \frac{20mV}{20K} \Rightarrow \frac{dV_c}{dt} = \frac{1mV}{1K \cdot 5\times10^{-6}} \Rightarrow \frac{dV_c}{dt} = .2$

$\int dV_o = \int 0.2\,dt$

$V_o(t) = 0.2t + C$ & $V(0) = 20mV = 0.2(0) + C \Rightarrow C = .02$

$\therefore \boxed{V_o(t) = 0.2t + 0.02 \quad @ \; t>0}$

Note: Eventually voltage will become saturated based upon amp-amp voltage supply inputs