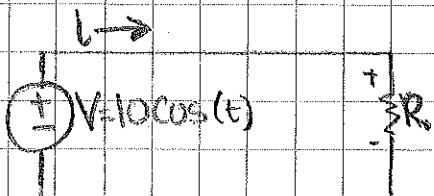


- 1) A resistor absorbs an instantaneous power of $20\cos^2(t)$ mW when connected across $V = 10\cos(t)$ V, voltage source. Find i and R .



$$P = Vi$$

$$20\cos^2(t) \text{ mW} = 10\cos(t)V \cdot i$$

$$i = \frac{0.02\cos^2(t)}{10\cos(t)}$$

$$i = 0.002\cos(t) \text{ A} = \boxed{2\cos(t) \text{ mA}}$$

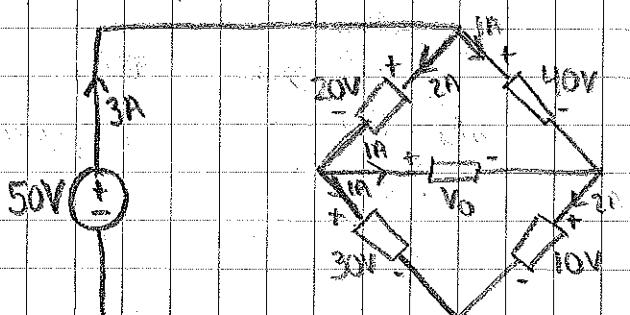
$$V = iR$$

$$10\cos(t)V = 0.002\cos(t)R$$

$$R = \frac{10\cos(t)}{0.002\cos(t)}$$

$$R = 5000 \Omega = \boxed{5K\Omega}$$

- 2) Find V_o for the circuit below:



Use KVL: $\sum_{m=1}^M V_m = 0$ (for any closed loop)

$$\text{... } 50 \dots -50V + 20V + V_o + 10V = 0$$

$$V_o = \boxed{20V}$$

3) If the current flowing through an element is given by:

$$i(t) = \begin{cases} 3t \text{ A}, & 0 \leq t \leq 6 \text{ s} \\ 18 \text{ A}, & 6 < t \leq 10 \text{ s} \\ -12 \text{ A}, & 10 < t \leq 15 \text{ s} \\ 0, & t > 15 \text{ s} \end{cases}$$

Given: $q(0) = 0$

Plot the charge stored in the element over $0 \leq t \leq 20 \text{ s}$.

Note:

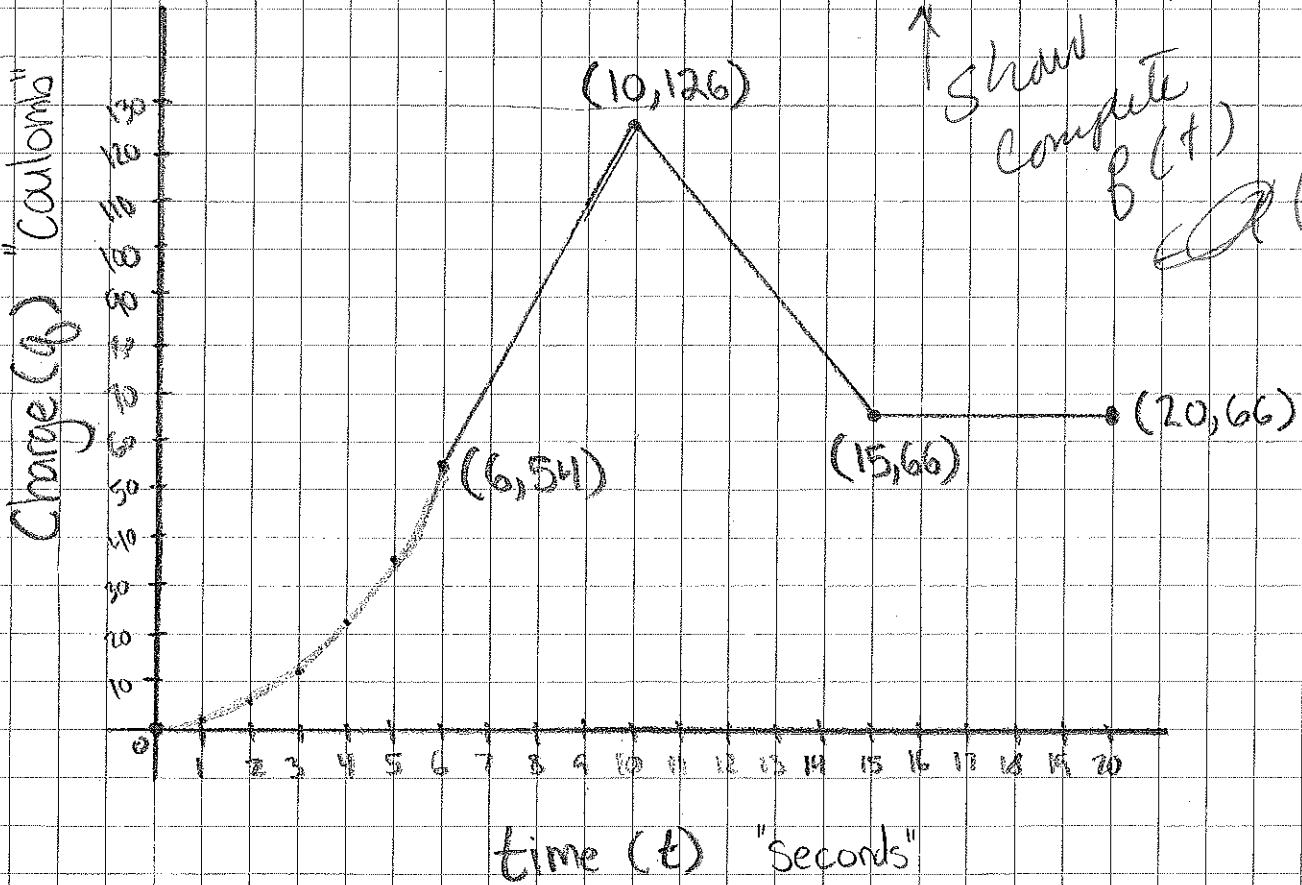
$$q(t) = \int i dt \rightarrow$$

$$Q = \int_{t_1}^{t_2} i dt$$

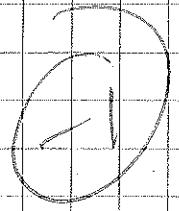
$$\int_0^6 3t dt = \frac{3}{2}t^2 \Big|_0^6 \rightarrow @ t=6, q=54$$

$$\int_6^{10} 18 dt = 18t \Big|_6^{10} \rightarrow @ t=10, q=54+72=126$$

$$\int_{10}^{15} -12 dt = -12t \Big|_{10}^{15} \rightarrow @ t=15, q=126-60=66$$



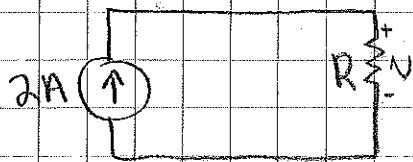
Show
Compute
 $f(t)$



- 4) Specify the resistance R in Figure below so that both of the following conditions are satisfied:

1. $V > 10V$

2. The power absorbed by the resistor is less than $25W$.



Use the resistor table to just choose resistor(s) nominal values that satisfy the design.

Note:

$$V = iR$$

$$V = 2R, V > 10 \rightarrow R > 5\Omega$$

$$P = VI = I^2R, P < 25W$$

$$25 < I^2R \rightarrow R < 6.25\Omega$$

... so, we can use one of the following: $5.1\Omega, 5.6\Omega, 6.2\Omega$

- 5) In a household, a 120-W PC is run for 4 hours/day, while a 60-W bulb burns for 8 hours/day. If the utility company charges \$0.12/kWh, calculate how much the household pays per year on the PC and the bulb.

Note:

$$\star 1 \text{ year} = 365 \text{ days} = 8760 \text{ hours} = 525600 \text{ min} = 31536000 \text{ s}$$

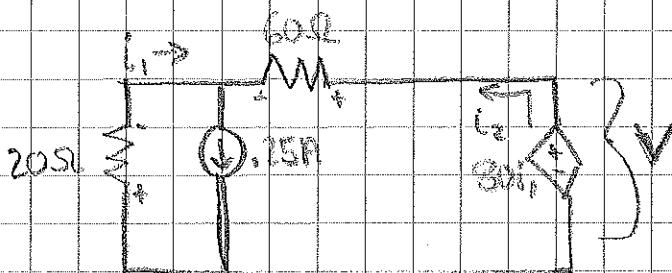
$$120W \cdot (60 \cdot 60 \cdot 4 \cdot 365) \text{ s} = 630720000 \text{ W-s}$$

$$60W \cdot (60 \cdot 60 \cdot 8 \cdot 365) \text{ s} = 630720000 \text{ W-s}$$

$$\frac{\$0.12}{\text{kW-h}} \cdot 2(630720 \text{ kW-s}) \cdot \frac{1 \text{ kW-h}}{(60 \cdot 60) \text{ kW-s}} = \$42.05$$

... and on leap year: \$42.16

D) Determine the value of the voltage that is measured by the meter in the circuit below:



Note: KCL $\rightarrow .25A = i_1 + i_2 \quad (1)$

KVL $\rightarrow -80i_1 + 60i_2 - 20i_3 = 0$

$$60i_2 = 100i_1$$

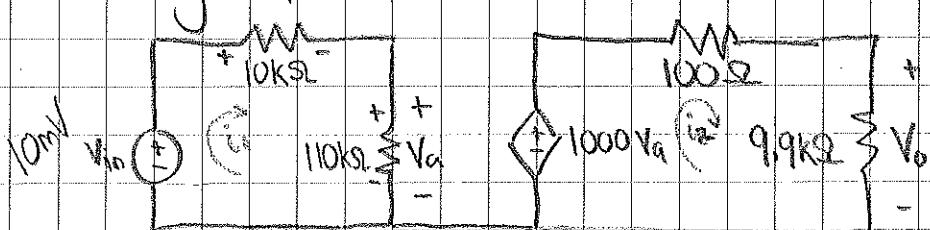
$$i_1 = \frac{3}{5}i_2 \quad (2)$$

$$.25 = \frac{3}{5}i_2 + i_2 \quad (1, 2)$$

$$.25 = \frac{8}{5}i_2$$

$$i_2 = .156 \rightarrow i_1 = .09375 \rightarrow V = \boxed{7.5V}$$

2) The voltage input of the circuit is 10mV. Determine $V_o(t)$.



Note: KVL $\rightarrow 10mV = 10k_i_1 + 110k_i_1$

$$10mV = 120k_i_1$$

$$\therefore i_1 = .0000000833A \quad (1)$$

$$\therefore V_a = i_1(110k) = 9.167mV \quad (2)$$

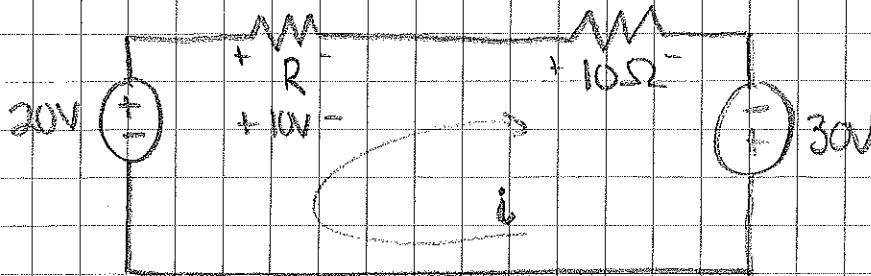
KVL $\rightarrow 1000V_a + 9.9k_i_3 = 100i_2 + 9.9k_i_3$

$$9.17 = 10k_i_2$$

$$\therefore i_2 = 0.917mA \quad (3)$$

$$V_o = (i_2)(9.9k) = (0.917)(9.9) = \boxed{9.075V}$$

3) Find R in the circuit below:

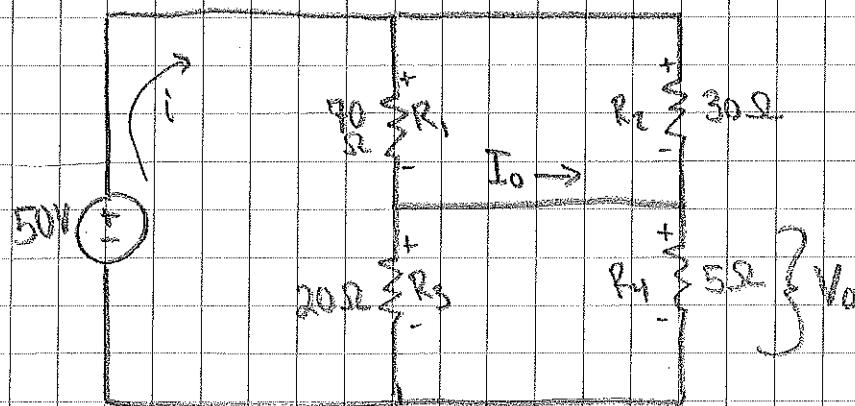


Note: KVL $\rightarrow -20V + 10V + 10i - 30V = 0$
 $10i = 40V$
 $i = 4A$ (1)

Ohm's $\rightarrow R = \frac{V_R}{I}$ (2)

$\therefore R = \frac{V_R}{I} = \frac{10}{4} = 2.5\Omega$

4) Calculate V_0 and I_0 in the circuit below:



Note: $V_{R_1, R_2} \rightarrow R_{12} = \left(\frac{1}{70} + \frac{1}{30} \right)^{-1} = 21\Omega$ (1)

$V_{R_3, R_4} \rightarrow R_{34} = \left(\frac{1}{20} + \frac{1}{5} \right)^{-1} = 4\Omega$ (2)

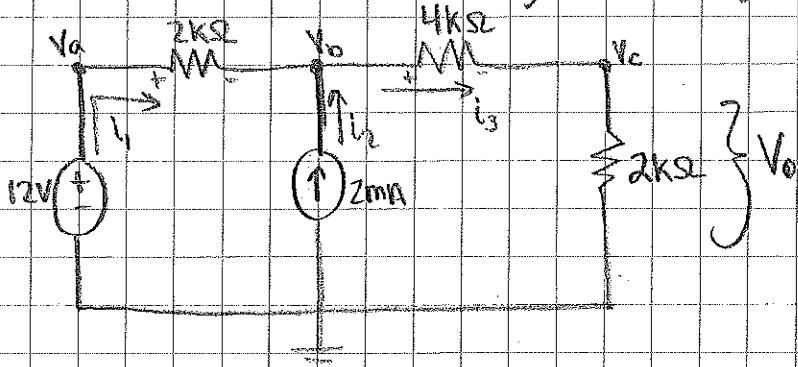
KVL $\rightarrow 50V = 21i + 4I_0 \rightarrow i = \frac{2A}{7} \rightarrow V_0 = V_{R_3} = (4)(2) = 8V$ (3)

b/c $V_0 = 8V$, this means that $i_3 = \frac{8}{20} = \frac{2}{5}$, (4)

and Using KCL $50 = V_{R_1} + 8 \rightarrow V_{R_1} = 42V \rightarrow i_1 = \frac{42}{70} = \frac{3}{5}$ (5)

KCL $\rightarrow i_1 = I_0 + i_3 \rightarrow \frac{3}{5} = I_0 + \frac{2}{5} \rightarrow I_0 = 0.2A$

1) Find V_o across the $2k\Omega$ resistor, using nodal analysis. 19
20



$$\text{KCL at } V_b \rightarrow i_2 = i_1 + 2\text{mA}$$

$$\frac{V_b - V_c}{4k} = \frac{V_a - V_b}{2k} + 2\text{mA}$$

$$\frac{V_b}{4k} - \frac{V_c}{4k} = \frac{V_a}{2k} - \frac{V_b}{2k} + 2\text{mA}$$

$$\text{KCL at } V_c \rightarrow i_3 = i_2$$

$$\frac{V_b - V_c}{4k} = \frac{V_c - 0}{2k}$$

$$\frac{V_b}{4k} - \frac{V_c}{4k} = \frac{V_c}{2k}$$

$$V_b - V_c = 2V_a - 2V_b + 8$$

$$V_b - V_c = 2V_c$$

$$2V_a - 3V_b + V_c = -8 \quad (1)$$

$$V_b = 3V_c \quad (2)$$

$$\text{Note: } V_a = 12V \quad (3)$$

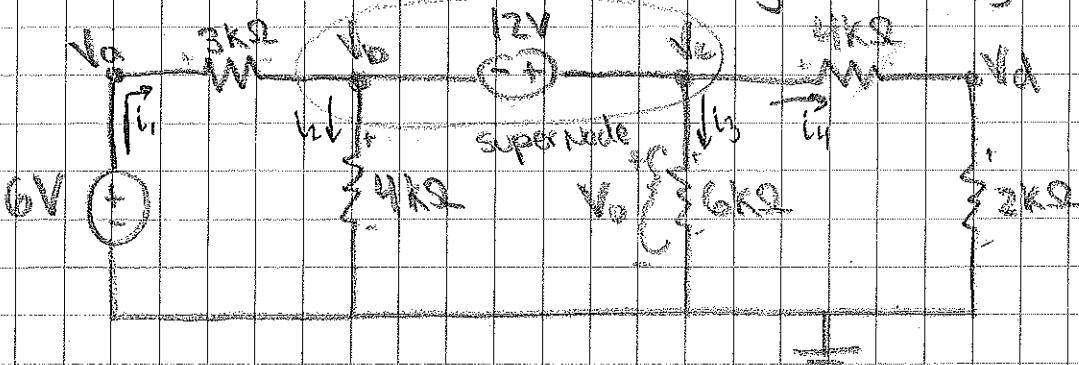
$$(1) \rightarrow 2(3) - 9V_c + V_c = -8$$

$$8V_c = 32$$

$$V_c = 4V$$

$$V_c = V_o \therefore V_o = \boxed{4V}$$

2) Find V_o across the $6k\Omega$ resistor, using nodal analysis.



$$\text{Note: } V_a = 6V \quad (1) \quad \sum V_b - V_d = 12V \quad (2)$$

KCL @ Super Node: $i_1 = i_2 + i_3 + i_4$

$$\frac{V_a - V_b}{3k} - \frac{V_b - 0}{4k} + \frac{V_c - 0}{5k} + \frac{V_d - V_d}{4k}$$

$$\frac{V_a - V_b}{3k} - \frac{V_b}{4k} = \frac{V_b}{4k} + \frac{V_c}{5k} + \frac{V_d - V_d}{4k}$$

$$4V_a - 4V_b = 3V_b + 2V_c + 3V_d - 3V_d$$

$$4V_a - 7V_b = 5V_c - 3V_d$$

$$24 = 7V_b + 5V_c - 3V_d \quad (3)$$

KCL @ V_d : $i_3 = i_4$

$$\frac{V_c - V_d}{4k} = \frac{V_d - 0}{2k}$$

$$\frac{V_c}{4k} - \frac{V_d}{4k} = \frac{V_d}{2k}$$

$$V_c - V_d = 2V_d$$

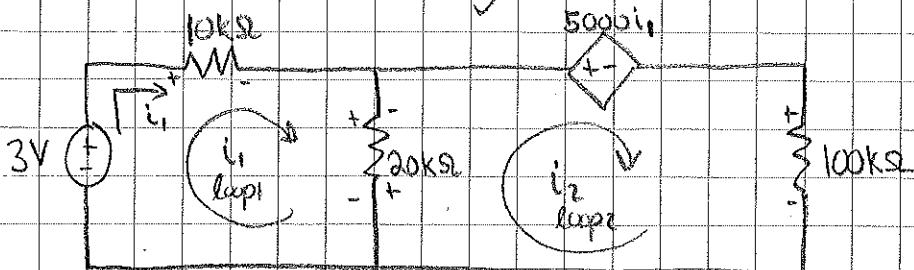
$$V_c = 3V_d \quad (4)$$

$$\rightarrow 24 = 7(V_c - 12) + 5V_c - 3\left(\frac{V_c}{3}\right)$$

$$24 = 7V_c - 84 + 5V_c - V_c$$

$$108 = 11V_c \Rightarrow V_c = 9.82 \quad \therefore V_d = V_b = \boxed{9.82V}$$

- 3) Find the energy delivered to the cathode during a 24-hour period.
 (The cathode is represented by the dependent voltage source and the 100 k Ω resistor. Use Mesh analysis to solve this problem.)



$$\text{KVL @ Loop 1: } -3V + 10ki_1 + 20k(i_1 - i_2) = 0 \\ -3V + 30ki_1 - 20ki_2 = 0 \quad (1)$$

$$\text{KVL @ Loop 2: } 5000i_1 + 100ki_2 + 20k(i_2 - i_1) = 0 \\ 120ki_2 - 15ki_1 = 0 \\ 8i_2 = i_1 \quad (2)$$

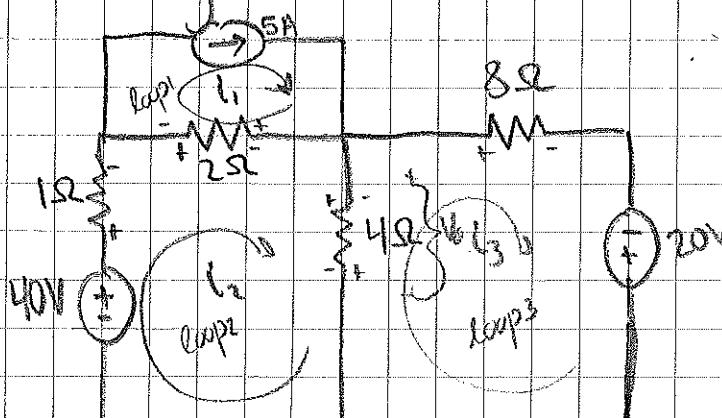
$$\rightarrow -3V + 30(8i_2) - 20ki_2 = 0 \\ 220ki_2 = 3V \\ i_2 = 0.0136 \text{ mA} \quad (3)$$

$$P = Vi = i^2 R = (100k)(0.000136)^2 = 0.018 \text{ mW-S} \quad (4)$$

$$\int_{0}^{(24 \cdot 60 \cdot 60)} 0.000136 dt = [0.000136t + C]_{0}^{86400} = 11.6 \text{ W}$$

Note: $i_1 = 0.000136A \rightarrow 36(V) = 3.6 \text{ W} \rightarrow 28.8 \text{ J}$

- 4) Apply mesh analysis to find V_o across 4Ω in the circuit below:



Traoder

$$\text{KVL @ loop 2: } -40V + i_2 + 2(i_2 - 5) + 4(i_2 - i_3) = 0$$

$$-40 + i_2 + 2i_2 - 10 + 4i_2 - 4i_3 = 0$$
$$50 = 7i_2 - 4i_3 \quad (1)$$

$$\text{KVL @ loop 3: } -20 + 4(i_3 - i_2) + 8i_3 = 0$$

$$-20 + 4i_3 - 4i_2 + 8i_3 = 0$$
$$20 = 4i_2 + 12i_3$$
$$5 = i_2 + 3i_3 \quad (2)$$

$$\begin{aligned} \rightarrow 50 &= 7(3i_3 - 5) - 4i_3 \\ 50 &= 21i_3 - 35 - 4i_3 \\ 85 &= 17i_3 \\ i_3 &= 5A \quad (3) \end{aligned}$$

$$\therefore V_o = Ri = 4(5) = \boxed{20V}$$

a.a

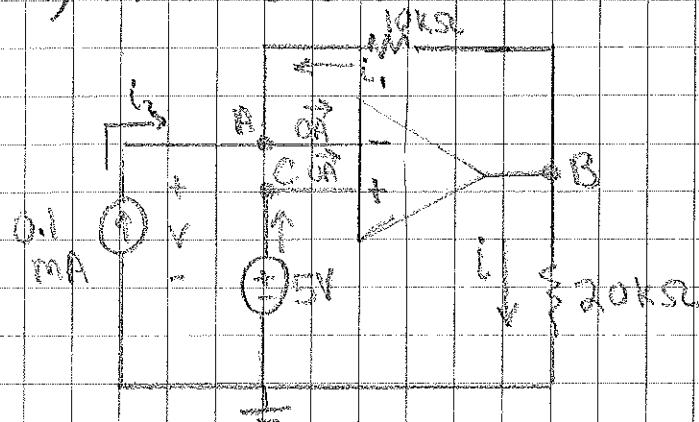
bⁿ

$(3^k)^2$
 3^{2k}

$3 \cdot 3^2 \cdot 3^3 \cdot 3^4$

20

10

1) Find V and i in the circuit below

KCL @ (-) opmp:

$$i_1 + i_2 = 0A$$

Note: Principle of Shunt $\rightarrow V_B = V_C$ Note: $V = 5V \Rightarrow V_B = 5V$ (Eq 2)

$$\frac{V_B - V_A}{10k} + 0mA = 0A$$

$$\frac{V_B - V_A}{10k} + 0.1mA = 0$$

$$V_B - V_A + 1V = 0 \text{ (Eq 1)}$$

$$(Eq 1, Eq 2) \Rightarrow V_B - 5 + 1 = 0$$

$$V_B = 4V \text{ (Eq 3)}$$

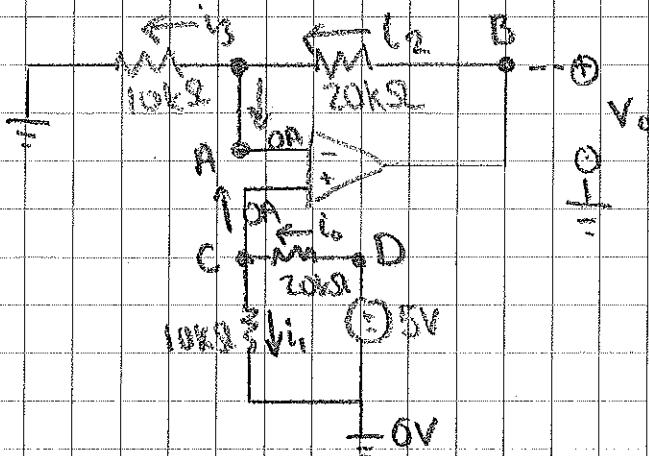
Note: $V = iR \Rightarrow 4 = i(20k)$

$$i = 200 \mu A$$

Note: $V = V_A \Rightarrow V = 5V$

2) Find V_o and i_o in the circuit below:

JFader



KCL @ C:

$$i_0 = i_1 + 0A$$

$$\frac{V_B - V_C}{20k} = \frac{V_C - 0}{10k}$$

$$\frac{V_B}{2k} - \frac{V_C}{20k} = \frac{2V}{20k}$$

Note: $V_B = 5V$

$$5 - V_C = 2V_C$$

$$3V_C = 5$$

$$V_C = 5/3V \quad (\text{Eq 1})$$

Note: Principle of Short $\rightarrow V_A = V_C \Rightarrow V_A = 5/3V$

KCL @ A:

$$i_2 = i_3 + 0A$$

$$\frac{V_B - V_A}{20k} = \frac{V_A - 0}{10k}$$

$$\frac{V_B}{2k} - \frac{V_A}{20k} = \frac{2V_A}{20k}$$

$$V_B = 3V_A = 3(5/3)$$

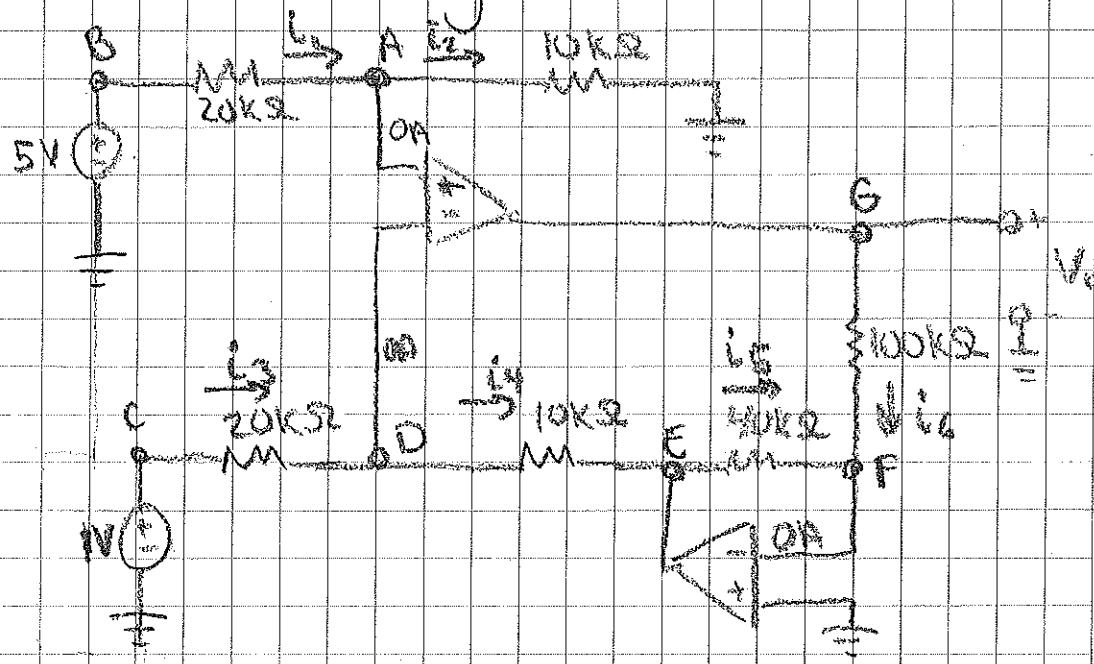
$$\therefore V_B = 5V \quad (\text{Eq 3})$$

Note: $V_B = V_o \rightarrow V_o = 5V$

Note: $V_{10\Omega} = (V_B - V_C) = i_1(20k)$

$$\frac{5 - 5/3}{20k} = 10 \Rightarrow i_1 = 166.7mA$$

3) Find V_o in the following Circuits



Note: $V_B = 5V$; $V_o = ?V$; $V_F = 0V$ (Principle of short)

KCL @ n:

$$i_1 = i_2 + 0A$$

$$\frac{V_B - V_A}{20k} = \frac{V_B - 0}{10k}$$

$$\frac{V_B}{20k} + \frac{V_B}{10k} = \frac{2V_B}{20k}$$

$$+V_B = 3V_B \Rightarrow V_A = 5V_3 = 1.667V \text{ (eq1)}$$

$$\therefore V_D = 1.667V \text{ (Principle of short)}$$

$$V_F = 2V \text{ (eq2)}$$

KCL @ D:

$$i_3 = i_4 + 0A$$

$$\frac{V_B - V_D}{20k} = \frac{V_D - V_E}{10k}$$

$$\frac{V_B}{20k} - \frac{V_D}{20k} = \frac{2V_B}{20k} - \frac{2V_D}{20k}$$

$$3V_B/20k = 1 + 2V_D/20k$$

$$V_D = 2V$$

KCL @ F:

$$\frac{5V}{20k} - \frac{5V}{20k} + \frac{2V}{10k} - \frac{2V}{20k} = 0$$

$$i_5 + i_6 = 0A$$

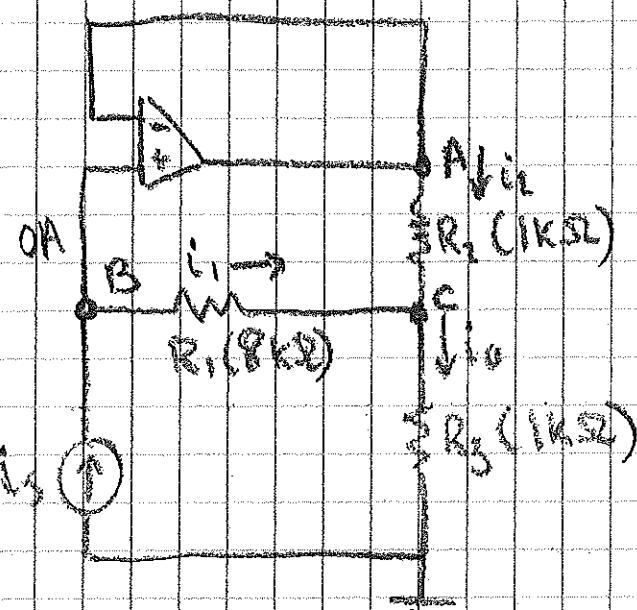
$$\frac{V_C - V_E}{20k} + \frac{V_E - V_F}{10k} = 0$$

$$10 = -2V_E$$

$$\therefore V_E = -5V$$

$$\text{Note: } V_E = V_o \Rightarrow V_o = -5V$$

- 4) A non-inverting current amplifier is portrayed in CKT below. Calculate the gain A_{vA} . Take $R_1 = 8k\Omega$, and $R_2 = R_3 = 1k\Omega$



Note: $V_A = V_B$ by principle of short.

$$KCL @ B$$

$$i_5 = i_1 + i_2$$

$$i_5 = \frac{V_B - V_C}{8k}$$

$$i_5 = \frac{V_B - V_C}{8k} \quad (1)$$

$$KCL @ C$$

$$i_1 + i_2 = i_0$$

$$\frac{V_B - V_C}{8k} + \frac{V_A - V_C}{1k} = \frac{V_C - 0}{1k}$$

$$\frac{V_B - V_C}{8k} + \frac{V_A}{8k} + \frac{8V_A - 8V_C}{8k} = \frac{8V_C}{8k}$$

$$V_B - V_C + 8V_A - 8V_C = 8V_C$$

$$\frac{V_B}{8k} + 3V_A = 17V_C$$

$$3V_A = 17V_C$$

$$V_B = V_A = \frac{17}{3}V_C \quad (2)$$

$$(1,2) \Rightarrow i_5 = \frac{17V_C}{8k}$$

$$i_5 = \frac{V_C}{8k}$$

$$i_5 = \frac{V_C}{8k} \quad (3)$$

$$\therefore i_0 = V_C/1k$$

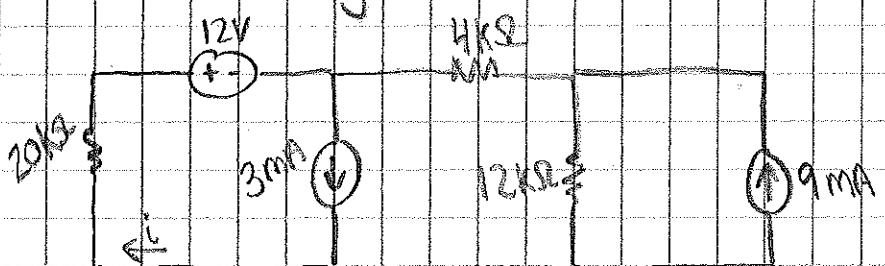
$$\therefore V_C = i_0 k \quad (4)$$

$$(3,4) \Rightarrow i_5 = \frac{i_0 k}{8k}$$

$$\frac{i_5}{i_0} = \frac{1}{8}$$

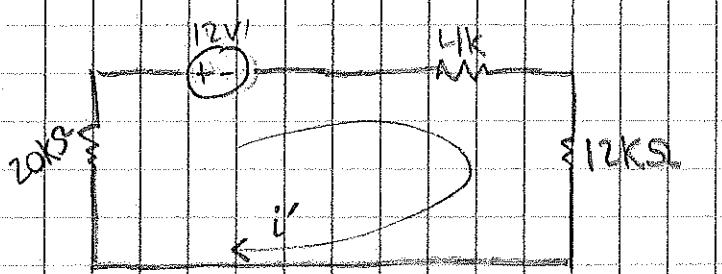
HW #3

Jared Fowler

1) Find i using superposition.3 Sources

12V, 3mA, 9mA

Keep 12V, Kill Others



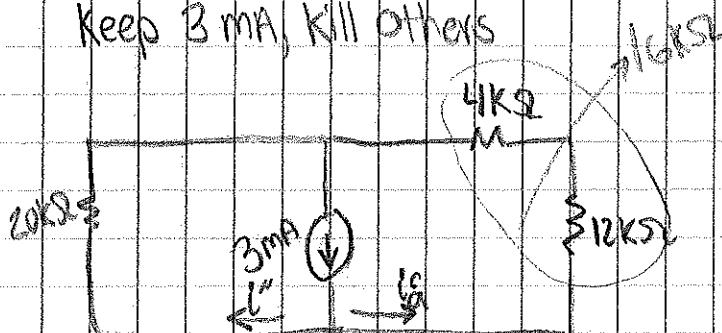
Using KVL:

$$20k(i') + 12V + 4k(i') + 12k(i') = 0$$

$$i' = -12 / (20k + 4k + 12k)$$

$$i' = -\frac{1}{3} \text{ mA} \quad (1)$$

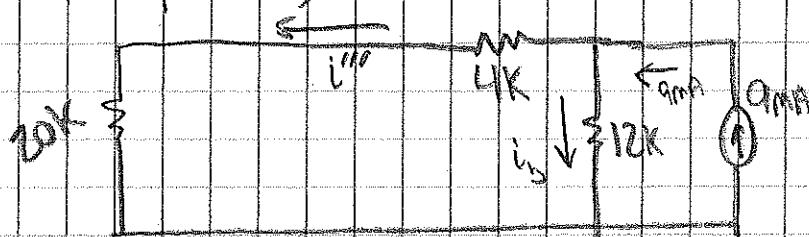
Keep 3 mA, Kill Others



Use Current Division:

$$i'' = 3 \text{ mA} \left(\frac{16k}{36k} \right) = 3 \text{ mA} \left(\frac{4}{9} \right) = \frac{1}{3} \text{ mA} \quad (2)$$

Keep 9 mA, Kill Others



Use Current Division

$$i''' = 9 \text{ mA} \left(\frac{12k}{36k} \right) = 9 \text{ mA} \left(\frac{1}{3} \right) = 3 \text{ mA} \quad (3)$$

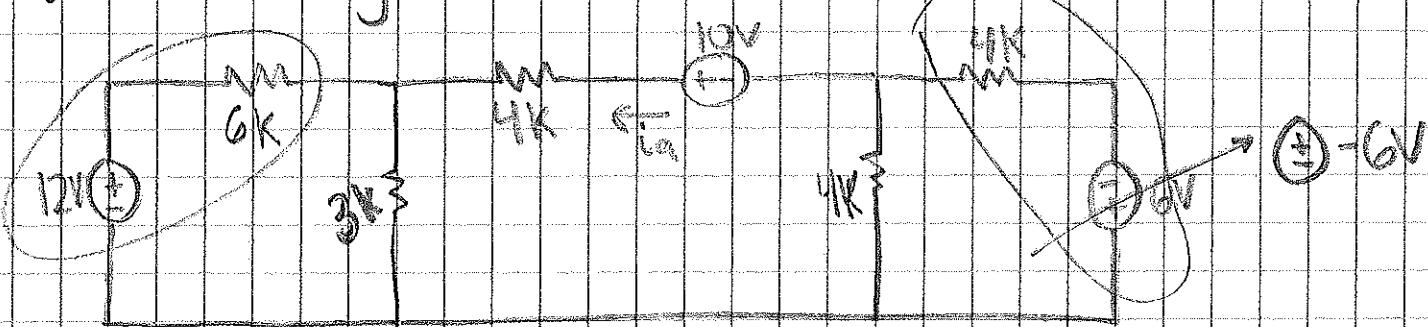
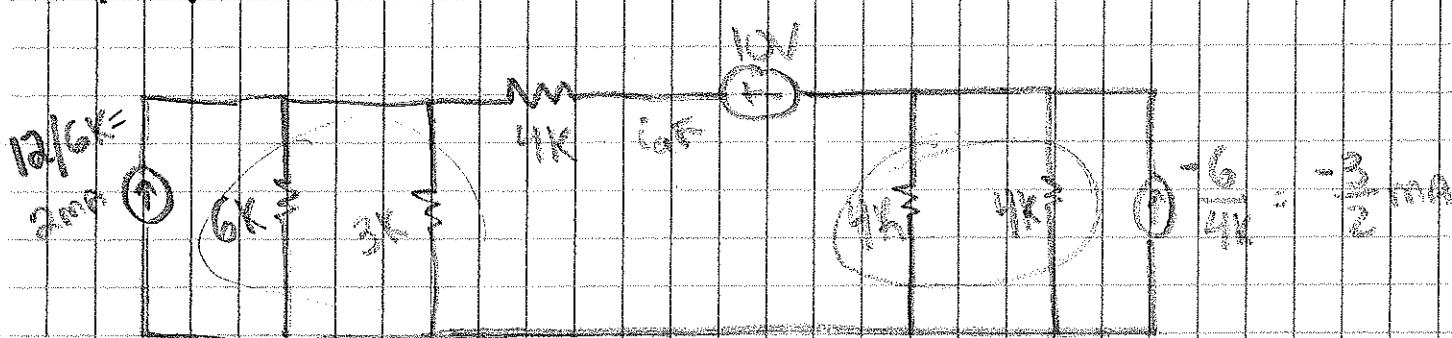
$$\text{So: } i = i'' + i''' - i'$$

$$= -\frac{1}{3} \text{ mA} + \frac{1}{3} \text{ mA} - 3 \text{ mA}$$

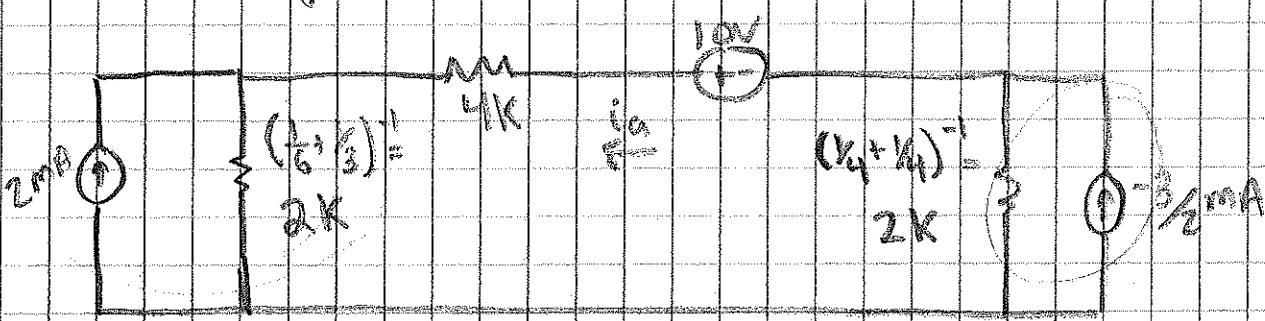
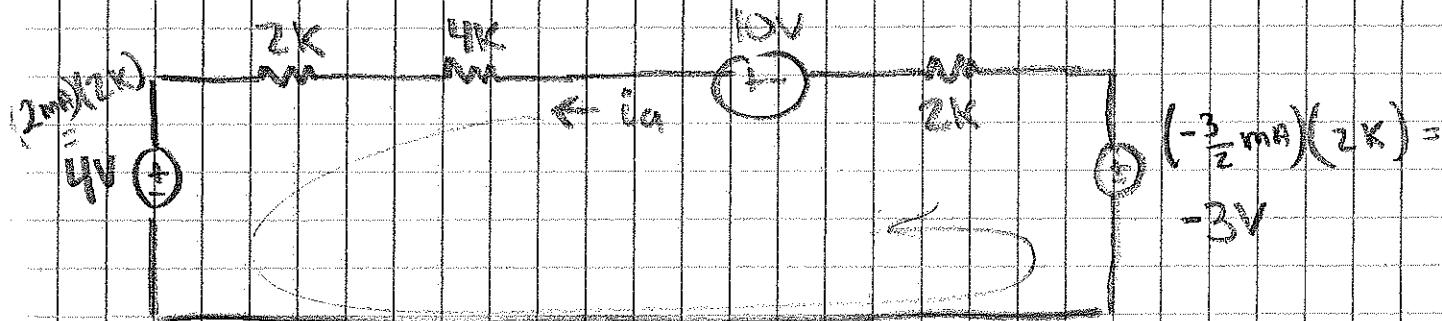
$$i = -2 \text{ mA}$$

JFaster

2) Find i_a using source transformation.

 $T \rightarrow N$ 

use equivalence rule for parallel resistor

 $N \rightarrow T$ 

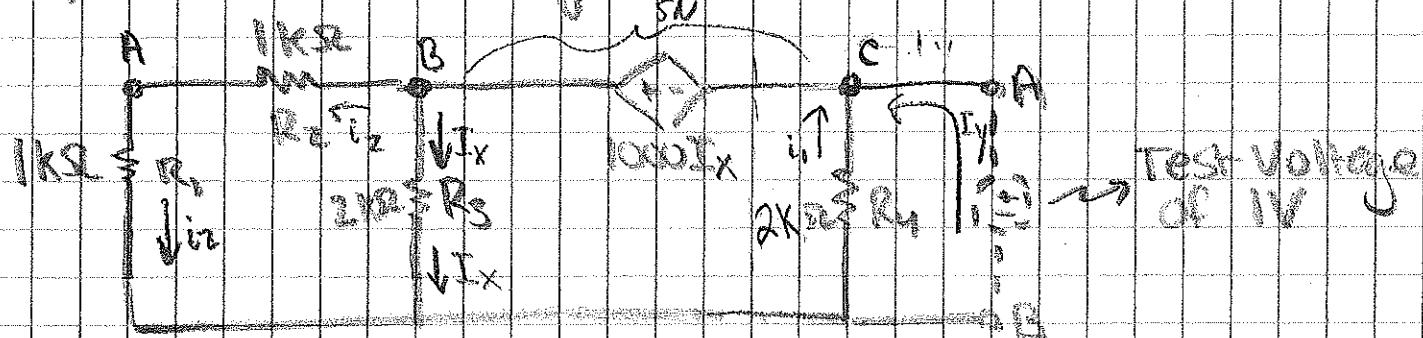
use KVL:

$$(4k)(i_a) + (2k)(i_w) + 4V - (-3V) + (2k)(i_d) - 10V = 0$$

$$(8k)(i_a) - 3V = 0 \Rightarrow i_a = \frac{3}{8} \text{ mA} = 375 \text{ mA}$$

I Foster

3) Find the Thevenen equivalent between A-B terminal.



Note: In a V_T dependent circuit, there is no voltage output in the Thevenen equivalent. What we are trying to find is the equivalent resistance of the entire circuit, R_T , such that it is the same regardless of voltage.

For example: if $(V_m/I_m) = R_T \rightarrow (V_n/I_n) = R_T$

So, want to find:

$$R_T = \frac{V_{test}}{I_{test}} = 400\Omega$$

$$\text{Let } V_{test} = V_o = 1V$$

$$\text{Note: } i_1 = i_2 \Rightarrow \frac{V_m - V_B}{2k} = \frac{V_B - V_A}{1k} \Rightarrow 2V_B = V_B + V_A \quad (1)$$

KCL @ Super Node:

$$I_y + i_1 = I_x + i_2$$

$$I_y + \frac{0 - V_c}{2k} = \frac{V_B - 0}{2k} + \frac{V_B - V_A}{1k}$$

$$I_y - \frac{V_c}{2k} = \frac{V_B}{2k} + \frac{V_B - V_A}{1k}$$

$$(2k)I_y - V_c = V_B + 2V_B - 2V_A \quad (3)$$

$$(1, 2, 3) \Rightarrow (2k)I_y - (V) = (6/0) - 2(V) \\ (2k)I_y = 5V \Rightarrow I_y = 5A$$

$$\text{Note: } V_B - V_c = 1000I_x \Leftrightarrow I_x = \frac{V_B}{2k}$$

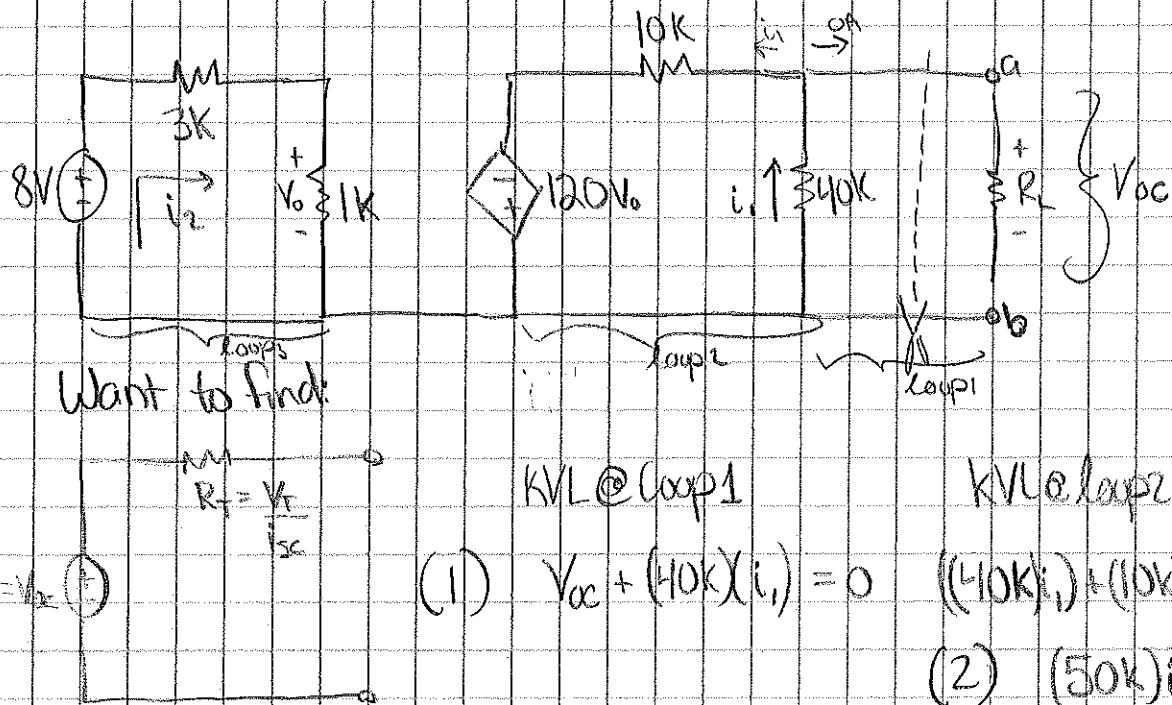
$$V_B - V_c = \frac{1k}{2k} V_B$$

$$\frac{1}{2}V_B = V_c \quad (2) \rightarrow V_c = \frac{1}{2}V_B$$

$$\frac{1}{2}V_c = V_A \dots (4) \\ \frac{1}{2}V_B = 2V$$

$$\therefore I_y = \frac{5}{2k} A \Leftrightarrow R_T = \frac{1}{\frac{5}{2k}} = 400\Omega$$

4) For the circuit below, what resistor connected across terminals a-b will absorb maximum power from the circuit? What is that power?



KVL @ Loop 3

$$-8V + 3k(i_2) + 1k(i_2) = 0$$

$$-8V + 4k(i_2) = 0$$

$$\text{where } V_o = (1k)i_2 \quad (1)$$

$$4k i_2 = 8$$

$$i_2 = 2 \text{ mA} \quad (3)$$

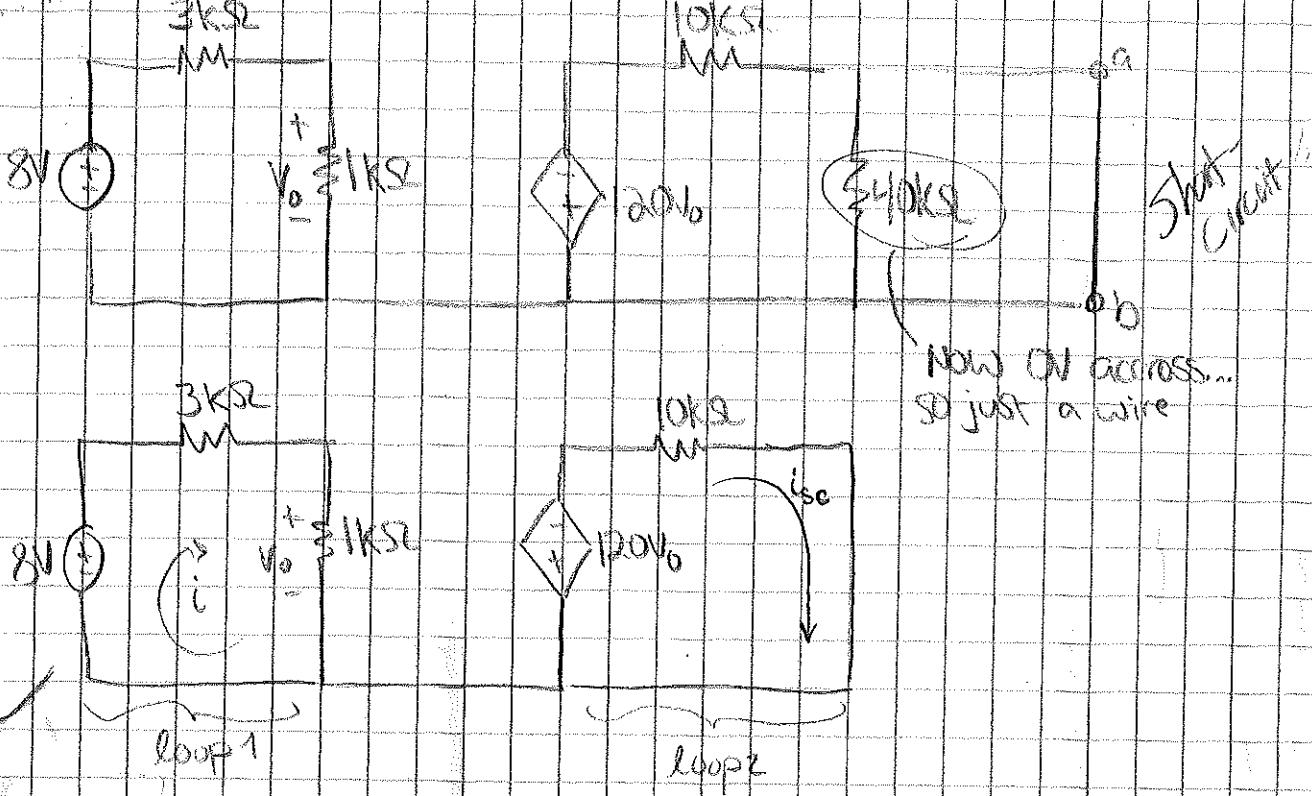
$$V_o = 2V \quad (-)$$

$$(2, 4) \rightarrow (50k)i_1 - 120(2) = 0$$

$$i_1 = 2.4 \text{ mA} \quad (5)$$

$$(5, 1) \rightarrow V_{oc} + 10k(2.4 \text{ mA}) = 0 \rightarrow V_{oc} = -19.2V$$

CONTINUE



KVL @ loop 1:

$$-8V + 3kI + 1kI = 0 ; V_o = 1kI$$

$$8V = +4kI$$

$$I = +2\text{mA}$$

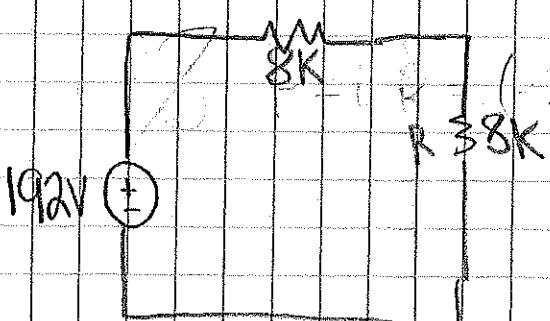
$$; V_o = +2V$$

$$120V_o + 10kV_{sc} = 0$$

$$120(+2) + 10kV_{sc} = 0$$

$$V_{sc} = -2.4\text{mA}$$

$$R_T = \frac{V_T}{i_{sc}} = \frac{-192V}{-2.4\text{mA}} = 8k\Omega$$

Max power absorbed when $R = R_T$

$$V = iR$$

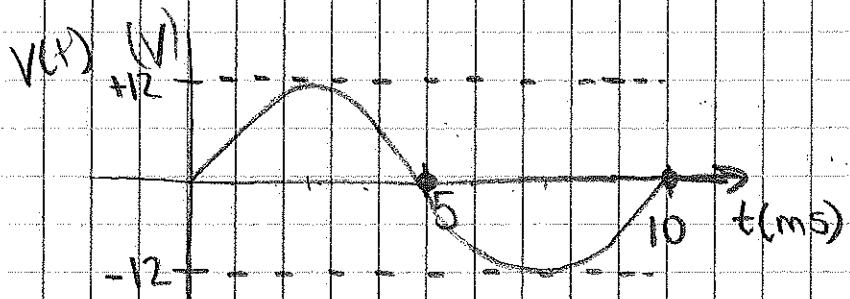
$$192 = i(16k)$$

$$i = 192/16k = \frac{12}{8k} = 1.5\text{mA}$$

Note: $P_{max} = \frac{V^2_{th}}{4R_{th}}$

$$P = \frac{192^2}{4(8k)} = 1.152\text{W}$$

- 1) The Voltage across a 10 mF capacitor is shown below. Determine and plot the waveform for the capacitor current.



Note: $i = C \frac{dv}{dt}$; Amplitude: 12; Period: 5ms

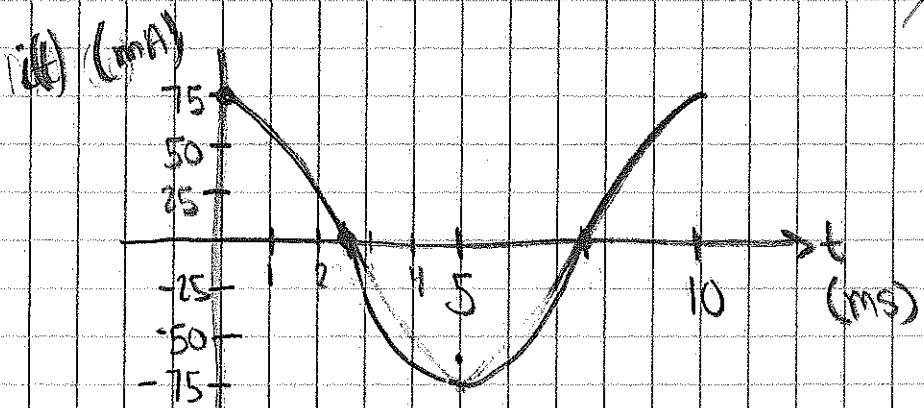
The Voltage plot takes on this form:

$$V(t) = 12 \sin(200\pi t) \quad \text{b/c} \quad n = 2\pi/0.01 = 200 \text{ rad/s}$$

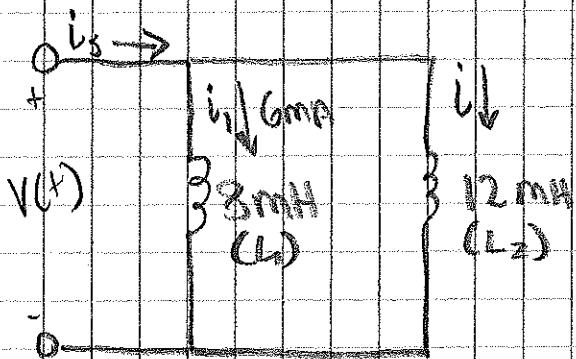
$$i(t) = (10\text{ mF}) / (12 \sin(200\pi t) \cdot \frac{d}{dt})$$

$$i(t) = (10 \times 10^{-3} \text{ F})(12)(200\pi)(\cos(200\pi t))$$

$$i(t) = 0.0754 \cos(200\pi t)$$



2) Two inductors are connected in parallel as shown in the CKT below. Find i .



$$V = \frac{dV}{dt}$$

$$\frac{1}{L_1} + \frac{1}{L_2}$$

$$\left(\frac{1}{6} + \frac{1}{12}\right)$$

$$L = \frac{L_1 L_2}{L_1 + L_2}$$

$$6\text{mH} \parallel 12\text{mH}$$

Note that the voltage across both inductors is equal.

$$i_1 = \frac{L_2}{L_1 + L_2} i_s$$

$$i = \frac{L_1}{L_1 + L_2} i_s$$

$$6\text{mH} = \frac{12\text{mH}}{20\text{mH}} i_s$$

$$i = \frac{3\text{mH}}{20\text{mH}} (10\text{mA})$$

$$i_s = 10\text{mA}$$

$$i = \frac{3}{5} (10\text{mA})$$

$$(20\text{mH}) \parallel \frac{dV}{dt} = 0$$

$$i_s = \frac{5}{12} i_s$$

$$i = 4\text{mA}$$

$$i_s = \frac{20}{20+6} i_s$$

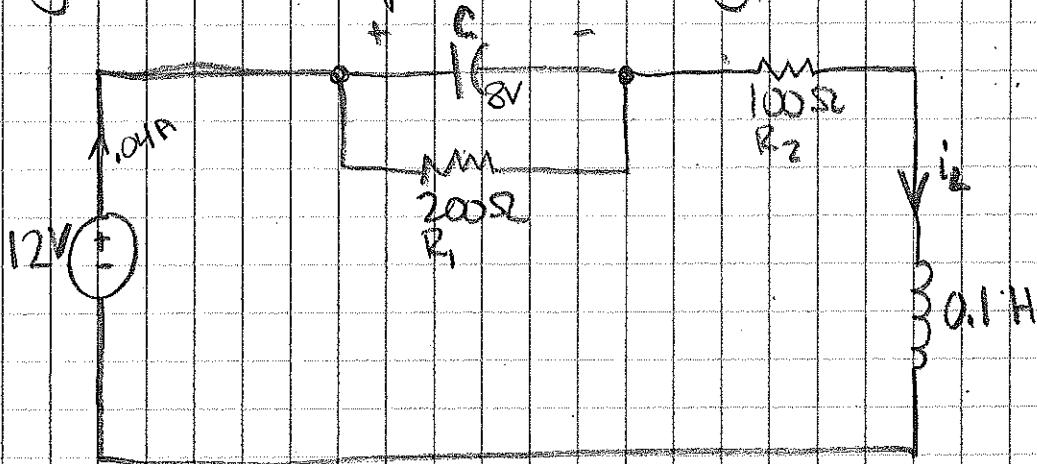
$$20\text{mH} \parallel i_s$$

$$(6\text{mH}) \parallel 8\text{mH}$$

$$i_s = \frac{18}{28} i_s$$

$$2 \parallel 5$$

- 3) Find the Value of C if the energy stored in the capacitor in the Figure below is equal to stored energy in the inductor.



Note: energy stored (w) = $\frac{1}{2}CV_c^2$ $\therefore w = \frac{1}{2}Li_L^2$

~~$\frac{1}{2}CV_c^2 = \frac{1}{2}Li_L^2$~~

~~$CV_c^2 = Li_L^2$~~

Note: $V_c = V_{R_2}$, and in DC Circuit an inductor has 0V, and a capacitor acts like an open circuit...

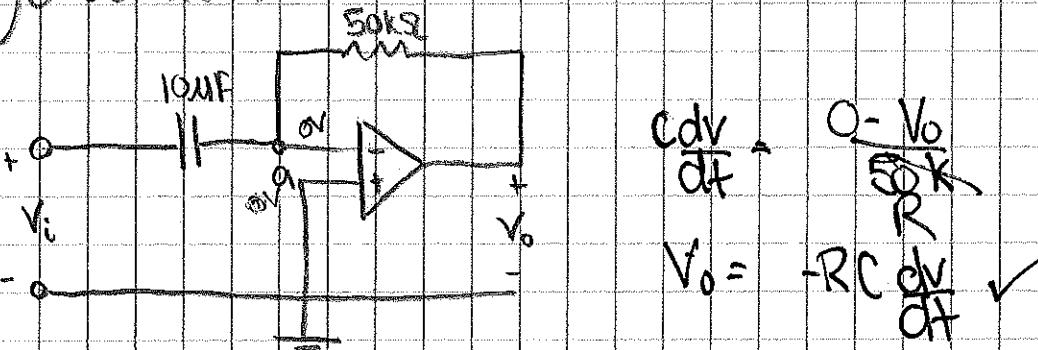
~~$C(200i)^2 = Li_L^2$~~

~~$C(40000)i^2 = li_L^2$~~

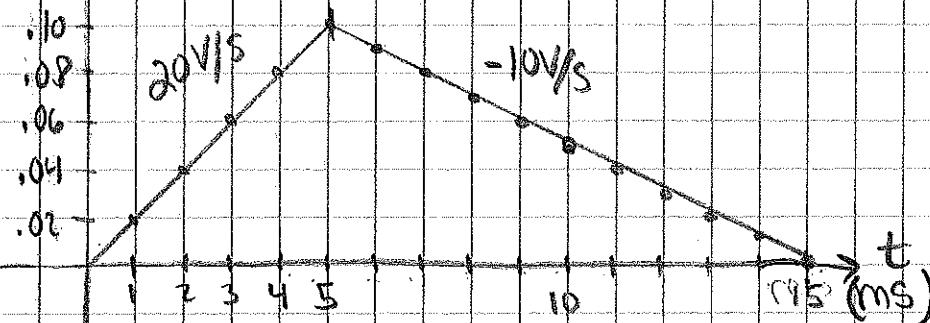
~~$C = \frac{1}{40000} = \frac{1}{40000}$~~

~~$C = 3 \text{ MF}$~~

4) A voltage waveform has the following characteristics: a positive slope of 20 V/s for 5 ms followed by a negative slope of -10 V/s for 10 ms . If the waveform is applied to a differentiator with $R = 50 \text{ k}\Omega$, $C = 10 \mu\text{F}$, sketch the output voltage waveform.



$$V_o = -RC \frac{dv}{dt}$$



Note: $V_o = -RC \frac{dv_i}{dt} \Rightarrow$ f. assume: $t < 0 : V_o = 0 \text{ V}$

$$0 \leq t \leq 5 \text{ ms} \quad V_o = (50 \times 10 \times 10^{-6})(20 \text{ V/s})$$

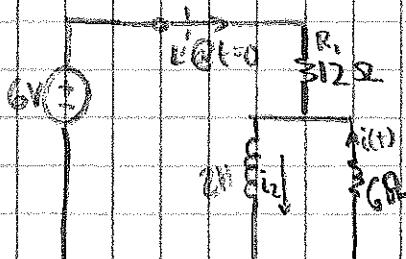
$$V_o = 10 \text{ V}$$

$$5 \text{ ms} < t \leq 15 \text{ ms} \quad V_o = (50 \times 10 \times 10^{-6})(-10 \text{ V/s})$$

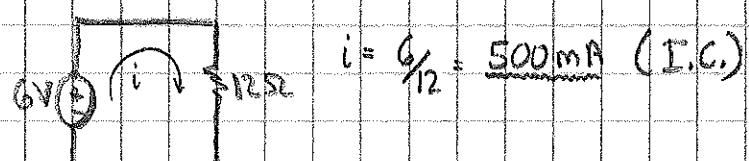
$$V_o = 5 \text{ V}$$



1) Find $i(t)$ for $t > 0$ and sketch the waveform in the following:



Note: Assume that at $t < 0$ the ckt is at stable state \rightarrow inductor looks like a short.



Note: At $t(0^+)$, the current will be 500mA, b/c current through inductor cannot change instantaneously.

After switch open, we have:



Note that $i_L(t) = i(t)$ for $t > 0$.

$$\text{KVL: } L \frac{di}{dt} + iR = 0 \Rightarrow \frac{di}{dt} + \frac{iR}{L} = 0 \quad (\text{Eqg})$$

$$(\text{Eqf}) \quad i = i_n + i_F \quad \text{if } R = K_1 \text{ b/c } 0 \text{ is constant} \Rightarrow i_F \Rightarrow \frac{di}{dt} + \frac{K_1 R}{L} = 0 \Rightarrow K_1 = 0$$

$$\text{Now for } i_n \Rightarrow \frac{di_n}{dt} + \frac{R}{L} i_n = 0$$

$$\text{Assumed solution of } i_n = K_2 e^{st}$$

$$\text{Substitute into (Eqf)} \rightarrow \frac{dK_2 e^{st}}{dt} + \frac{R}{L} K_2 e^{st} = 0$$

$$s K_2 e^{st} + \frac{R}{L} K_2 e^{st} = 0$$

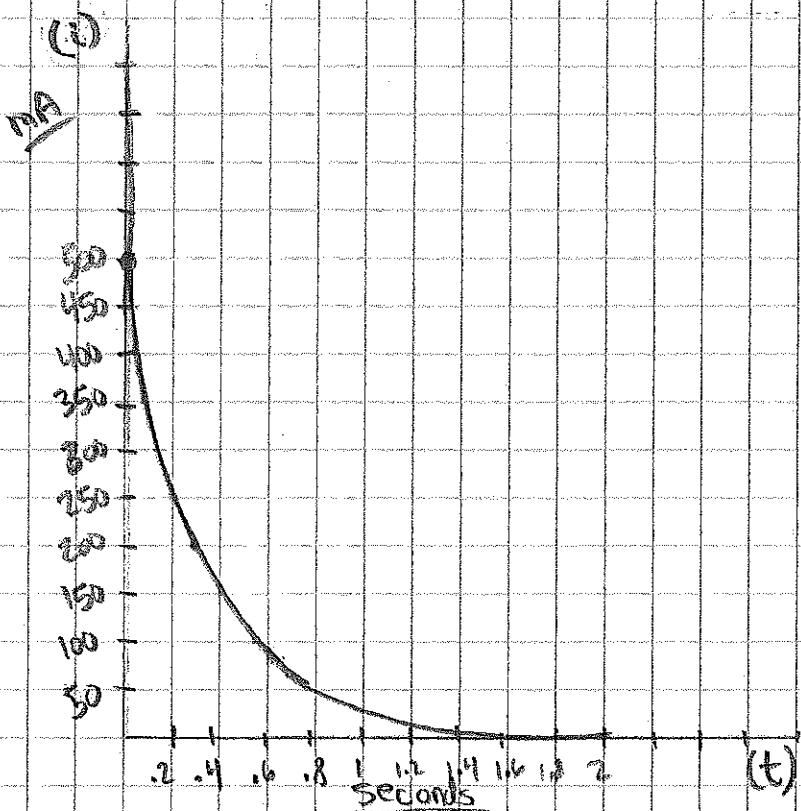
$$s = -\frac{R}{L} = \frac{6}{2} = -3$$

$$\text{Substitute into (Eqf)}$$

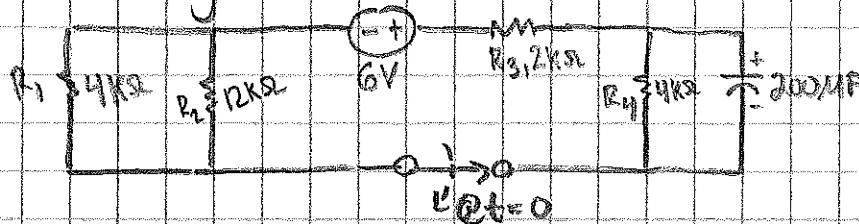
$$i(t) = K_2 e^{-3t} + 0$$

$$i(0) = 500 \text{ mA} = K_2 e^{0} \Rightarrow K_2 = 0.5$$

$$\therefore i(t) = 0.5 e^{-3t}$$



2) Find $V_c(t)$ for $t \geq 0$ in the ckt below and plot the response including time interval prior to the switch opening.



Note: @ $t < 0$, $V_{R_4} = V_c$

$$\text{KVL: } -6 + 2ki + 4ki + 3ki = 0$$

$$i = \frac{6}{9k} = \frac{2}{3} \text{ mA}$$

$$\therefore V_c = 4k \left(\frac{2}{3} \text{ mA} \right) = \frac{8}{3} \text{ V (DC)}$$



Note: Assume that @ $t > 0$ the ckt is at stable state
→ capacitor looks like open



$$\text{KCL: } i_C + i_R = 0 \rightarrow \left(\frac{d^3 V_c}{dt^3} + \frac{V_c}{4k} = 0 \right) \cdot 10^4 = \frac{2dV_c}{dt} + \frac{10k}{4} = 0 \Rightarrow \frac{dV_c}{dt} + \frac{5k}{4} V_c = 0$$

(Eq1)

$$V_c(t) = V_0 e^{kt} \quad \& \quad k_2 = V_0 \text{ b/c } 0 \text{ is constant} \Rightarrow V_0 = \frac{dV_c}{dt} \Big|_{t=0} \cdot \frac{5k}{4} K_2 = 0 \Rightarrow K_2 = 0$$

Assumed solution, $V_n = K_1 e^{st}$

Substitute into (Eq1)

$$\frac{d}{dt} K_1 e^{st} + \frac{5k}{4} K_1 e^{st} = 0$$

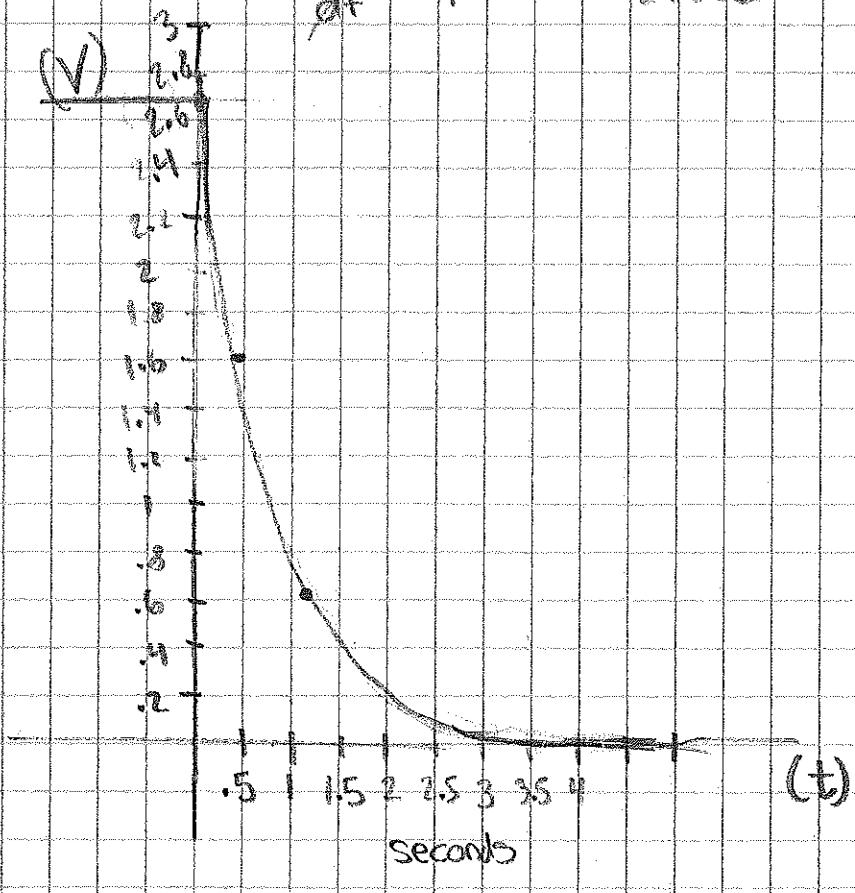
$$s K_1 e^{st} + \frac{5k}{4} K_1 e^{st} = 0 \Rightarrow s = -\frac{5k}{4}$$

Back into (Eq1)

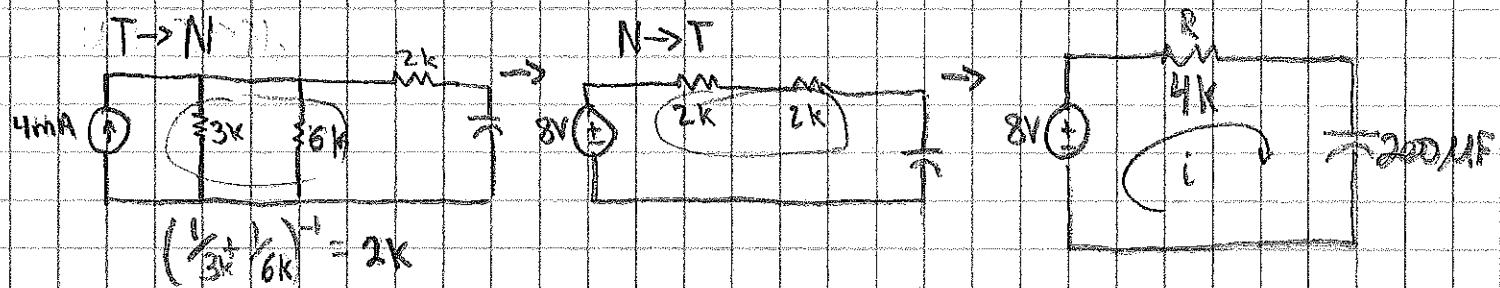
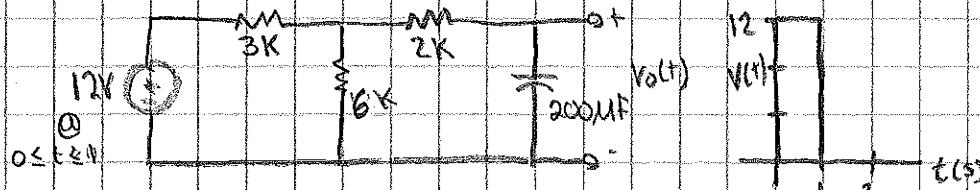
$$V_c(t) = K_1 e^{-\frac{5k}{4} t}$$

$$V_c(0) = \frac{8}{3} = K_1 e^{0} \quad \therefore K_1 = \frac{8}{3}$$

$$\therefore V_c(t) = \frac{8}{3} e^{-\frac{5k}{4} t}$$



3) Determine the equation for the voltage for $V_o(t)$ for $t > 0$ for the circuit below when subjected to the input pulse shown below.



$$\text{KCL: } i_R = i_c \Rightarrow \frac{8 - V_c}{4k} = \frac{CdV}{dt} \Rightarrow \frac{dV + V_c}{dt} + \frac{8}{4kC} = \frac{8}{4kC} \Rightarrow \frac{dV + 1.25V_c}{dt} = 10 \quad (\text{Eq. 1})$$

Note: @ $t < 0$, the voltage is 0 (I.C.)

for $0 < t < 1$, $V(t) = V_n + V_f$

$$V_f = K_2 \text{ b/c 10 is constant} \Rightarrow \frac{dK_2}{dt} + 1.25K_2 = 10 \Rightarrow K_2 = 8$$

$$V_n = K_1 e^{st} \Rightarrow \frac{d}{dt} K_1 e^{st} + 1.25K_1 e^{st} = 0 \Rightarrow sK_1 e^{st} + 1.25K_1 e^{st} = 0 \Rightarrow s = -1.25$$

$$\rightarrow V(t) = K_1 e^{-1.25t} + 8 \\ V(0) = 0 = K_1 + 8 \Rightarrow K_1 = -8 \quad \therefore \text{at } t > 1, V(t) = 3 - 8e^{-1.25t}$$

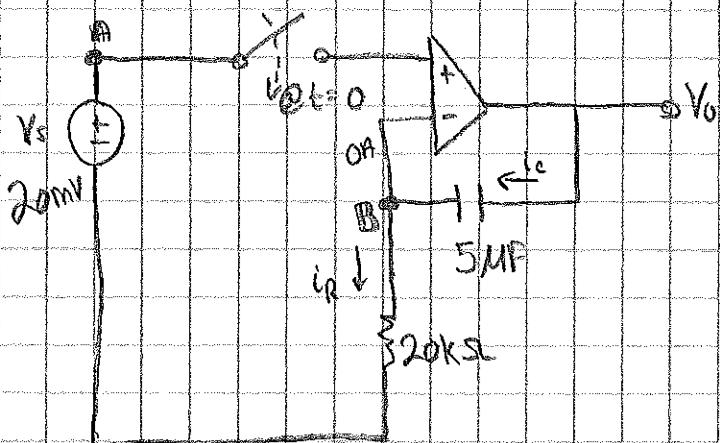
$$\text{Note: } V(1) = 3 - 8e^{-1.25} = 5.707 \text{ (I.C.)}$$

for a sourceless circuit, $V(t) = K e^{-1.25t}$ @ $t > 1$

$$V(1) = 5.707 = K e^{-1.25} \Rightarrow K = 5.707 e^{1.25}$$

$$\therefore \text{at } t > 1, V(t) = 5.707 e^{-1.25(t-1)}$$

4) Find $V_o(t)$ for $t > 0$ when $V_s = 20mV$ in the following CKT:



Note: At $t(0)$, $V_c = 0 \rightarrow V_o(0) = 20mV$ (I.C.)

KCL at (-) input

$$i_c + i_A = i_R$$

$$\frac{CdV_c}{dt} = \frac{V_B - 0}{20k}$$

Note: $V_B = V_s = 20mV$

$$CdV_c = \frac{20mV}{20k} \Rightarrow \frac{dV_c}{dt} = \frac{1mV}{1k \cdot 5 \times 10^3} \Rightarrow \frac{dV_c}{dt} = .2$$

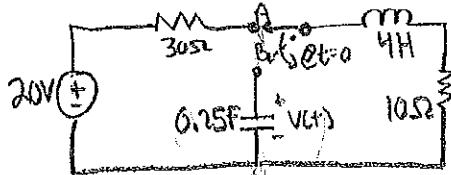
$$\int dV_o = \int 0.2 dt$$

$$V_o(t) = 0.2t + C \quad ; \quad V(0) = 20mV = 0.2(0) + C \Rightarrow C = 0.02$$

$$\therefore V_o(t) = 0.2t + 0.02 \quad @ t > 0$$

Note: Eventually voltage will become saturated based upon op-amp voltage supply inputs

- I) The switch in figure below moves from position A to B @ $t=0$.
 Find $V(t)$ for $t > 0$.



Note: $\text{At } t(0)$

$\rightarrow \text{KVL: } -20 + 30i + 10i = 0 \rightarrow i(0) = 500 \text{ mA}$

Note: $V_L(0) = V_C(0) \rightarrow V_C(0) = 5V$ (I.C.)

Note: $\text{At } t(0^+)$

$\rightarrow \text{KVL: } L \frac{di}{dt} + 10i + V_C = 0$ Note: $i = \frac{dV}{dt}$, $\frac{di}{dt} = \frac{d^2V}{dt^2}$

$\rightarrow L \frac{d^2V}{dt^2} + 10 \frac{dV}{dt} + V_C = 0$

$\rightarrow \frac{d^2V}{dt^2} + \frac{5}{2} \frac{dV}{dt} + V = 0$ (Eqn)

$$X = X_h + X_p, \quad X_p = 0 \text{ because } 0 \text{ is constant}$$

$$X_h \rightarrow ke^{st} \rightarrow \frac{d^2(ke^{st})}{dt^2} + \frac{5}{2} \frac{d(ke^{st})}{dt} + ke^{st} = 0$$

$$s^2 ke^{st} + \frac{5}{2}s k e^{st} + k e^{st} = 0 \rightarrow s^2 + \frac{5}{2}s + 1 = 0$$

Quadratic formula: $s_1, s_2 = \frac{-\frac{5}{2} \pm \sqrt{\frac{25}{4} - 4}}{2} \quad \begin{cases} -\frac{5}{4} + \frac{3}{4} = -\frac{1}{2} \\ -\frac{5}{4} - \frac{3}{4} = -2 \end{cases} \rightarrow \omega_0 = 1, Z = \frac{5}{4}$

"Over-damped"

$$V(t) = K_1 e^{-\frac{1}{2}t} + K_2 e^{-2t}$$

$$V(0) = 5 = K_1 e^0 + K_2 e^0 \rightarrow K_1 = 5 - K_2$$

$$i(0) = 0.5 = \left. \frac{dV}{dt} \right|_{t=0} = .25(-\frac{1}{2}K_1 e^0 + (-2)K_2 e^0) \rightarrow 2 = -\frac{1}{2}K_1 - 2K_2$$

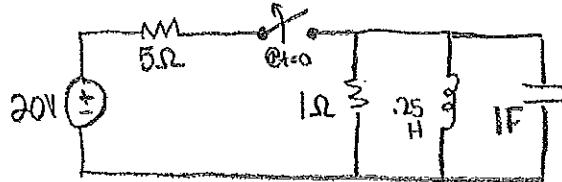
$$\rightarrow 2 = -\frac{1}{2}(5 - K_2) - 2K_2$$

$$\frac{1}{2} + \frac{5}{2} = -2K_2 + \frac{1}{2}K_2 \rightarrow K_2 = -3, K_1 = 8$$

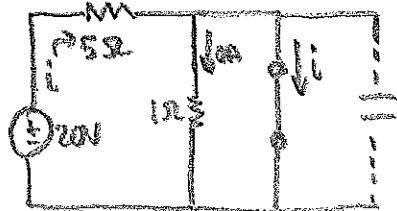
©

$$V(t) = 8e^{-\frac{1}{2}t} - 3e^{-2t} \quad @ t > 0$$

- 2) Find the Voltage across the capacitor as a function of time for $t > 0$ for the circuit in Fig below. Assume Steady State exists at $t = 0$.



Note:
at $t=0$



$$\rightarrow i_L = 20/5 = 4A = I(0)$$

$$V_C(0) = V_L(0) = 0V$$

(Counts like sum of stat)

(I.C.)

Note:
at $t=0^+$



$$\rightarrow KCL: i_L + i_R + i_C = 0$$

$$\rightarrow \frac{d}{dt} \left[\frac{1}{L} (V(t) - V(0)) + \frac{V}{R} + C \frac{dV}{dt} \right] = 0$$

$$X = X_R + X_L, \quad X_L = 0$$

$$\rightarrow \frac{d^2V}{dt^2} + \frac{1}{R} \frac{dV}{dt} + \frac{1}{L} V = 0$$

$$X_L \rightarrow KE^2 \rightarrow S^2 + S + 4 = 0$$

$$\rightarrow \frac{dV}{dt^2} + \frac{1}{R} \frac{dV}{dt} + 4V = 0 \quad (\text{eq1})$$

$$\rightarrow \omega_0 = 2, \quad T = 2\pi\omega_0 \rightarrow 2 = X_L$$

\rightarrow "under-damped"

$$\text{QF: } S_1, S_2 = \frac{-1 \pm \sqrt{1+16}}{2} = \frac{-1 \pm \sqrt{17}}{2}$$

$$= -\frac{1}{2} \pm \frac{\sqrt{15}}{2}$$

$$\rightarrow V(t) = e^{-\frac{1}{2}t} (k_1 \sin \frac{\sqrt{15}}{2} t + k_2 \cos \frac{\sqrt{15}}{2} t)$$

$$V(0) = 0 = e^0 (k_1(0) + k_2(1)) \rightarrow k_2 = 0$$

$$i(0) = V = \left. \frac{dV}{dt} \right|_{t=0} = 0 \left[e^{-\frac{1}{2}t} k_1 \sin \frac{\sqrt{15}}{2} t + k_2 \cos \frac{\sqrt{15}}{2} t \right] \Big|_{t=0}$$

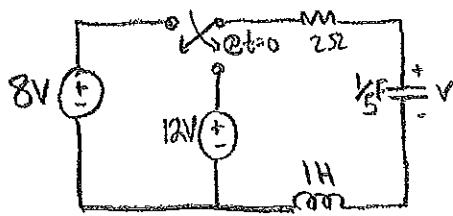
$$= -k_1 e^0 \left[k_1 \sin(0) + (0) \left(\frac{\sqrt{15}}{2} \cos(0) \right) k_1 \right]$$

$$0 = \frac{\sqrt{15}}{2} k_1 \rightarrow k_1 = \frac{8}{\sqrt{15}}$$

Note: initial current through C will run bottom \rightarrow top, ...
Equation is negated.

$$\therefore V(t) = -e^{-\frac{1}{2}t} \left(\frac{8}{\sqrt{15}} \right) \sin \frac{\sqrt{15}}{2} t \quad \text{at } t > 0$$

3) Find V for $t > 0$.



Note: $\text{At } t=0$ $\rightarrow i(0) = 0$
 $\rightarrow V_L(0) = 8V$ b/c $V_R = j/\omega = 0V$.

Note: $\text{At } t=0$ $\rightarrow \text{KVL: } -12 + 2i + V + L\frac{di}{dt} = 0$ Note: $i = C\frac{dv}{dt}$, $L\frac{di}{dt} = \frac{d^2v}{dt^2}$
 $\rightarrow LC\frac{d^2v}{dt^2} + 2Cd\frac{dv}{dt} + V = 12$ $\frac{d^2v}{dt^2}, \frac{Cd\frac{dv}{dt}}{dt}$
 $\rightarrow \frac{d^2v}{dt^2} + 2\frac{dv}{dt} + 5v = 60 \quad (\text{eq1})$

$$X_F = K \rightarrow 5K = 60 \rightarrow K = 12 = X_F$$

$$X_H = K\omega^2 \rightarrow S^2 + 25 + 5 = 0$$

Note: $\omega_0 = \sqrt{5}$, $\zeta = 2/\sqrt{5} < 1$
 \therefore "under-damped"

$$\text{QE: } S_{1,2} = \frac{-2 \pm \sqrt{4 - 20}}{2} = \frac{-2 \pm j\sqrt{16}}{2} = -1 \pm j4$$

$$\rightarrow V(t) = e^{-t}(k_1 \sin 2t + k_2 \cos 2t) + 12$$

$$V(0) = 3 = e^0(k_1(0) + k_2(1)) + 12 \rightarrow k_2 = -9$$

$$i(t) = \frac{dV}{dt} = \frac{d}{dt}[e^{-t}(k_1 \sin 2t + k_2 \cos 2t)] = e^{-t}(-k_1 \sin 2t - 2k_1 \cos 2t + k_2 \sin 2t - 2k_2 \cos 2t)$$

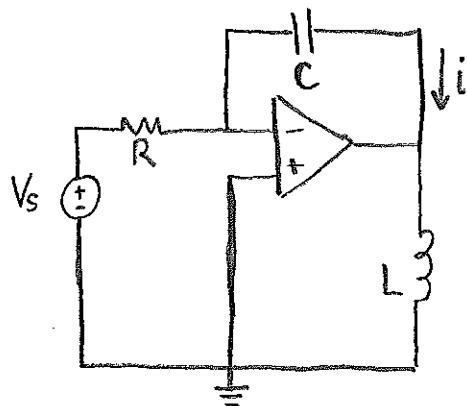
$$0 = -e^{-t}(k_1 \sin 2t - 4k_1 \cos 2t) + e^{-t}(2k_2 \cos 2t + 8k_2 \sin 2t)$$

$$0 = -4(k_1) + 2k_2 \Rightarrow k_2 = 2k_1$$

$$\therefore V(t) = e^{-t}(-2 \sin 2t - 4 \cos 2t) + 12$$

$e^{-t} > 0$

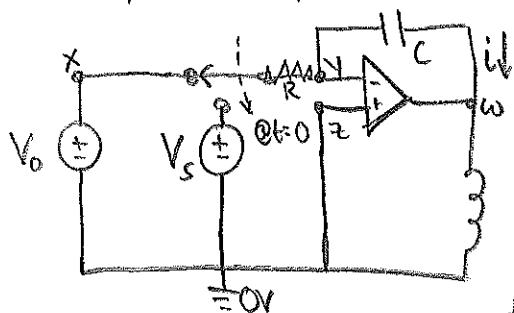
4) For the op-amp circuit below, find the differential equation for $i(t)$.



see below

V7.(-2)

Note: The purpose of an op-amp is to measure the voltage between inputs and output the delta voltage multiplied by a multiplier. V_s is changing... let's see it as this:



Now, at stable state there is no current across the capacitor, i.e., $i(0^+) = 0$ because capacitor acts like open in DC circuit in Stable State. Also at $t(0)$, the voltage is $V(0^+) = V_o$

Note also that ground, OV, is greater than op-amp output which will be negative, based upon inverting output. Current is drawn into the op-amp more than delivered out. In this case, the inductor has little to NO effect upon the function $i(t)$ through capacitor.

$$\text{KCL at } t(0), \quad i_R + 0 = i_C$$

$$\frac{V_s - V_l}{R} = \frac{CdV}{dt} \rightarrow \frac{V_s}{R} = \frac{CdV}{dt} + \frac{V_l}{R} \rightarrow \frac{dV}{dt} + \frac{V_l}{RC} = \frac{V_s}{RC}$$

Note: with L acting as short, KVL tells us that $V_l = V_c$ b/c $-V_s + \frac{V_s}{R} + V_c = 0$

$$\frac{dV}{dt} + \frac{V}{RC} = \frac{V_s}{RC} \quad \text{if } X_f = K = V_s, \quad X_n \rightarrow Ke^{st} \rightarrow S = -\frac{1}{RC}$$

$$V(t) = \frac{CdV}{dt} = C \left[(V_o - V_s) e^{-\frac{t}{RC}} + V_s \right] \frac{d}{dt}$$

$$V(t) = Ke^{-\frac{t}{RC}} + V_s$$

$$V(0) = V_o \rightarrow K = (V_o - V_s)$$

$$\rightarrow V(t) = (V_o - V_s) e^{-\frac{t}{RC}} + V_s$$

$$i(t) = \frac{-C}{RC} (V_o - V_s) e^{-\frac{t}{RC}}$$

$$i(t) = \frac{1}{R} (V_s - V_o) e^{-\frac{t}{RC}}$$

1) Express the following functions in cosine form:

a) $4\sin(\omega t - 30^\circ) \rightarrow 4\cos(\omega t - 120^\circ)$

b) $-2\sin(6t) \rightarrow -2\cos(6t - 90^\circ)$

c) $-10\sin(\omega t + 20^\circ) \rightarrow -10\cos(\omega t - 70^\circ)$

2) For the following pairs of sinusoids, determine which are leads and by how much.

a) $v(t) = 10\cos(4t - 60^\circ)$ and $i(t) = 4\sin(4t + 50^\circ)$

$$i(t) = 4\cos(4t - 40^\circ) \quad \therefore [i(t) \text{ leads } v(t) \text{ by } 20^\circ]$$

b) $V_1(t) = 4\cos(377t + 10^\circ)$ and $V_2(t) = -20\cos(377t)$

$$V_2(t) = 20\cos(377t + 180^\circ)$$

$$\therefore [V_2(t) \text{ leads } V_1(t) \text{ by } 170^\circ]$$

c) $x(t) = 13\cos 2t + 5\sin 2t$ and $y(t) = 15\cos(2t - 11.8^\circ)$

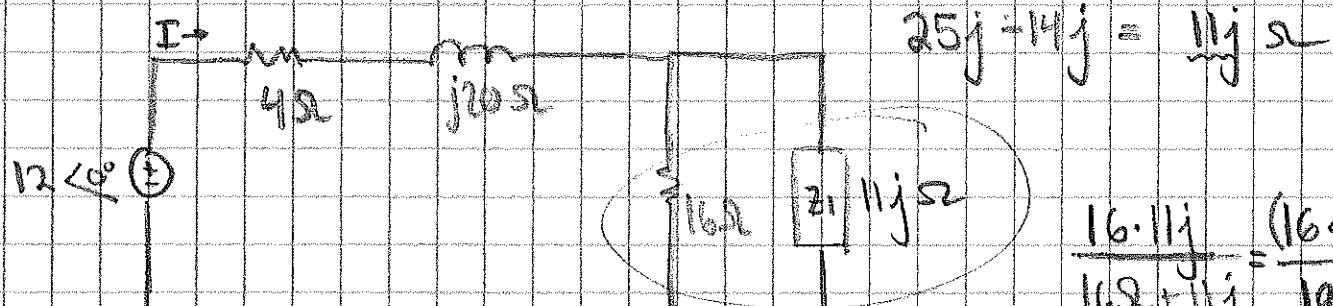
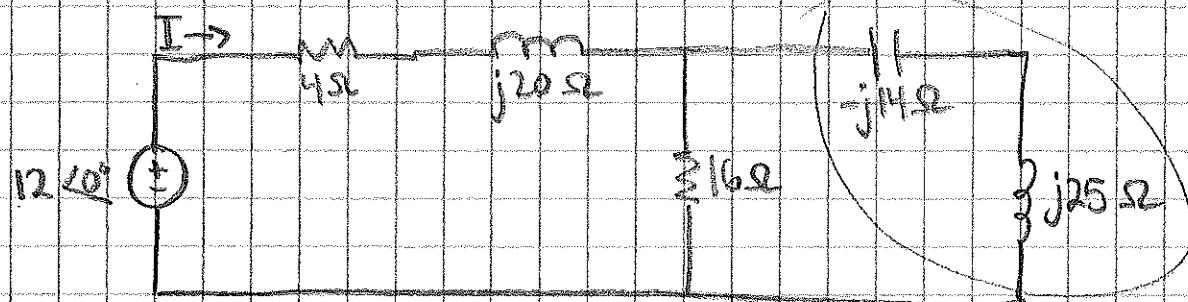
Note: $A\cos \omega t + B\sin \omega t = C\cos(\omega t - \theta)$, $C = \sqrt{A^2 + B^2}$
 $\theta = \tan^{-1} \frac{B}{A}$

$$\rightarrow x(t) = \sqrt{13^2 + 5^2} \cos(2t - 21.01^\circ)$$

$$\rightarrow x(t) = 13.93 \cos(2t - 21.01^\circ)$$

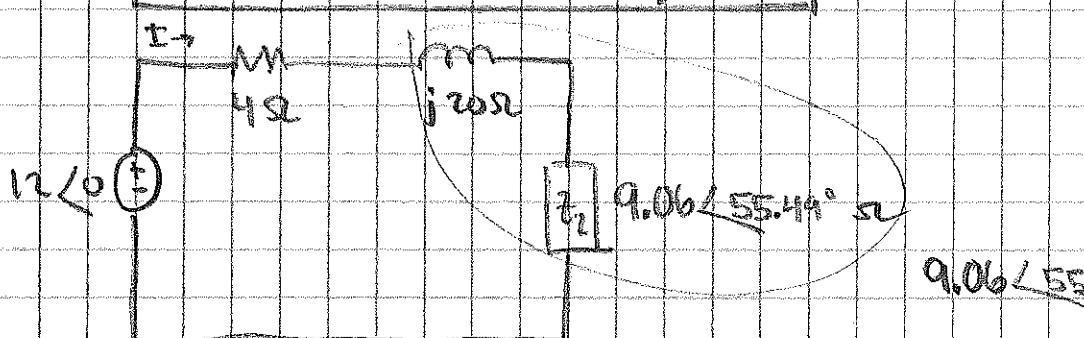
$$[y(t) \text{ lead } x(t) \text{ by } 9.24^\circ]$$

3) For the circuit shown, find Z_{eq} and use that to find current I .
 Let $\omega = 10 \text{ rad/s}$.



$$\frac{16 \cdot 11i}{16i + 11i} = \frac{(16\angle 0^\circ)(11\angle 90^\circ)}{16i + 11i} = 19.42 \angle 34.51^\circ$$

$$9.06 \angle 55.49^\circ \Omega$$



$$9.06 \angle 55.49^\circ = 5.13 + 7.46j \Omega$$

$$R = 9.06 \sin(55.49^\circ) \rightarrow R = 9.06 \times 0.819 \rightarrow R = 7.46 \Omega$$

$$5.13 + 7.46j + 20j \\ = 5.13 + 27.46j \Omega$$

$$\dots \text{Now add the } 4\Omega \rightarrow Z_{eq} = 9.13 + 27.46j \Omega$$

$$I = \frac{V}{R} = \frac{12\angle 0^\circ}{28.94 \angle 71.61^\circ} = .41 \angle -71.61^\circ$$

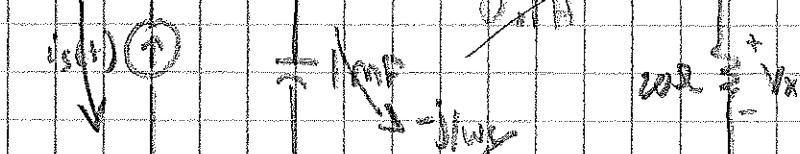
$$\dots \text{or } Z_{eq} = 28.94 \angle 71.61^\circ \Omega$$

$$I = .41 \angle -71.61^\circ A \rightarrow i(t) = 0.41 \cos(10t - 71.61^\circ) A$$

... Not sure what you're looking for... giving us ω ...

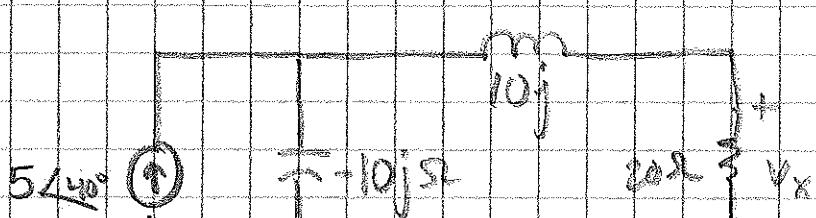
4) Determine V_x in the circuit shown $i_3(t) = 5\cos(100t + 140^\circ)$.

$$j\omega L \rightarrow j\omega L = 100 \cdot 6j = 10j \Omega$$

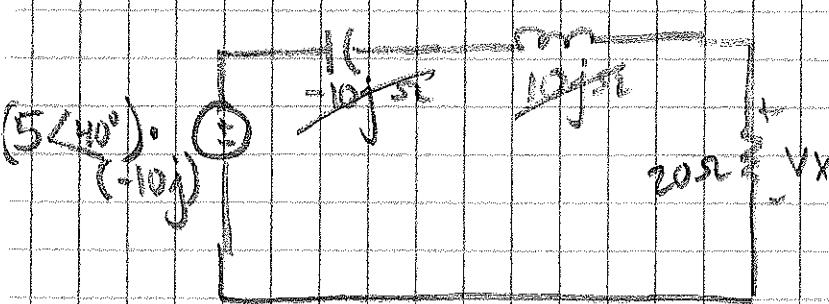


$$5\angle 40^\circ$$

$$\frac{-j}{100 \cdot (0.001)} = -10j \Omega$$



N → T



$$\rightarrow V_x = (5\angle 40^\circ)(-10j) \rightarrow 10\angle -90^\circ$$

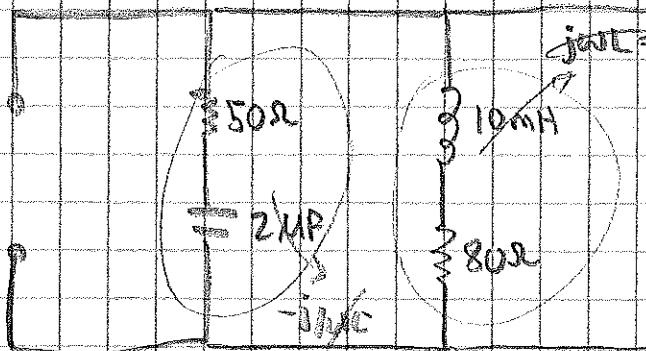
$$= (5\angle 40^\circ)(10\angle -90^\circ)$$

$$V_x = 50\angle -50^\circ \text{ V}$$

$$\text{or } V_x = 32.14 - 32.3j \text{ V}$$

$$\text{or } V_x = 50\cos(100t - 50^\circ) \text{ V}$$

- 5) The network below is part of a schematic assembly in an industrial electronic sensing device. What is the total impedance of the circuit at 2 kHz?



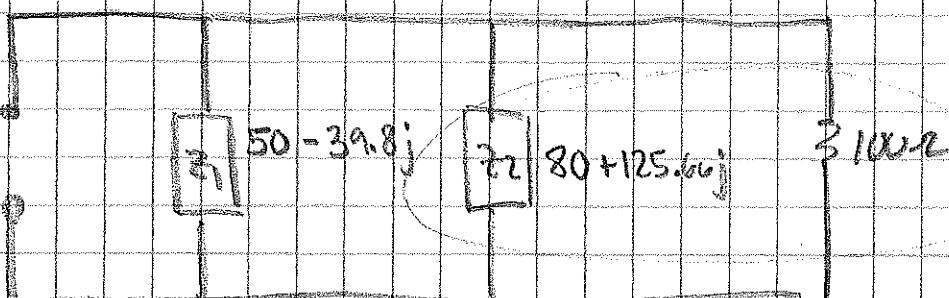
$$j\omega L = 125.66 \text{ V} \quad \text{Note: } \omega = 2\pi f, \quad f = \frac{1}{T}$$

3100Ω

$2000\text{Hz} = f$

$$\rightarrow \omega = 2\pi(2\text{K}) =$$

$$12566.4 \frac{\text{rad}}{\text{s}}$$



$$\frac{(80 + 125.66j)(100)}{100 + 80 + 125.66j} = \frac{(148.97 \angle 1.52^\circ)(100 \angle 0^\circ)}{180 + 125.66j}$$

$$= 14897 \angle 57.52^\circ$$

$$214.5 \angle 34.90^\circ$$

$$= 67.9 \angle 22.6^\circ$$

Now the final...

$$\frac{(67.9 \angle 22.6^\circ)(50 - 39.8j)}{(50 - 39.8j) + 67.9 \angle 22.6^\circ} = \frac{(67.9 \angle 22.6^\circ)(63.91 \angle -38.52^\circ)}{(50 - 39.8j) + (62.69 + 26.1j)} = \frac{4339.5 \angle -15.92^\circ}{112.69 - 13.7j}$$

$$Z = 4339.5 \angle -15.92^\circ$$

$$= 113.52 \angle -6.90^\circ = 38.23 \angle -9.02^\circ \Omega \text{ or } 37.8 - 6j \Omega$$