Jared Fowler

The Speed of Sound in Thin Metal Rods

(A Formal Lab)

Purpose:

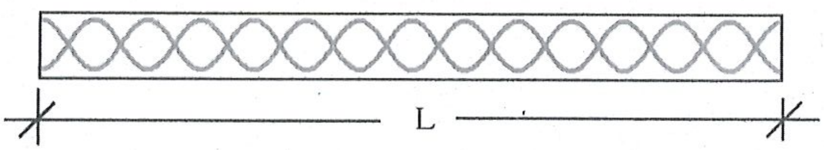
To measure the speed of sound waves traveling longitudinally (axially) down thin metal rods.

Apparatus:

Metal rods (steel, aluminum, brass, copper) with attached piezoelectric transducers (PZT), foam rubber blocks to support the rods, function generator, oscilloscope, frequency counter (optional), cables and connectors, meter stick or metric steel tape, disposable foam ear plugs, "official" MC formal lab data collection form.

Theory:

Longitudinal acoustics waves can form "standing wave" patterns in a thin metal rod in much the same manner as standing waves form in an open ended air-filled pipe. As shown in the figure below the waves reflect from each end of the thin rod with a displacement antinode (or pressure node, the proof of this will be left to a more advanced course).



Realize that the sketch above is not a "picture" of the wave; the acoustic waves we will initially measure are longitudinal (or "compressional"), not transverse waves. Rather think of the sketch as a graph of the time varying position of metal atoms as they vibrate longitudinally in the rod. Now for a standing wave to form, the rod length must be a multiple N (the harmonic number) times the half wavelength so that there will be a displacement antinode formed symmetrically at either end of the rod.

$$L = N\left(\frac{\lambda_1}{2}\right) = \left(\frac{v}{2f_1}\right)$$

Likewise, the next highest harmonic frequency that forms a standing wave must contain one additional half wavelength,

$$L = (N+1)\left(\frac{\lambda_2}{2}\right) = (N+1)\left(\frac{v}{2f_2}\right)$$

Now we solve each equation for frequency and then take the difference,

$$f_1 = \frac{Nv}{2L} \quad and \quad f_2 = \frac{(N+1)v}{2L}$$

$$\Delta f = f_2 - f_1 = \frac{v}{2L}(N+1-N) = \frac{v}{2L}$$

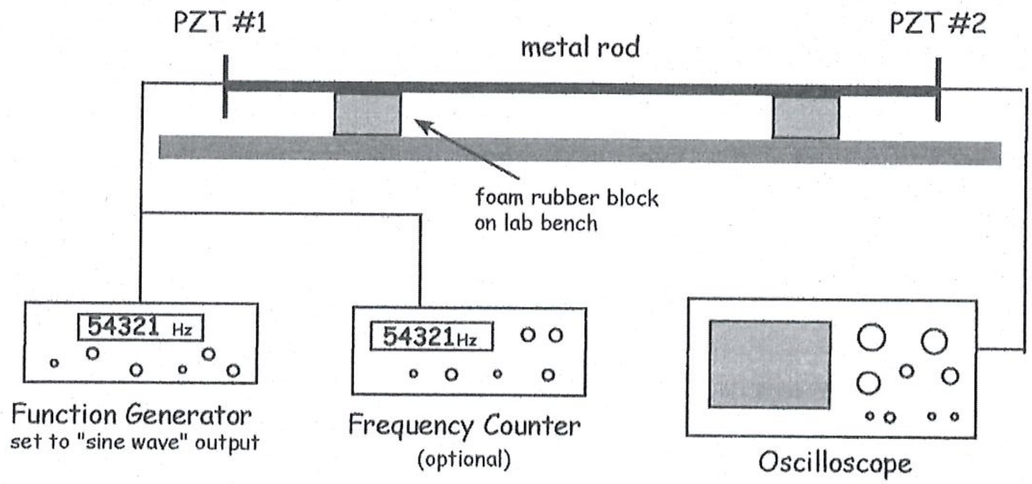
Or we can finally write,

$$\boxed{v = 2L(\Delta f)}$$

Therefore if we measure the frequency spacing between adjacent "longitudinal modes" of the acoustic wave in the metal rod and then multiply by twice the length of the rod, we will have indirectly measured the speed of the longitudinal (axial) acoustic wave.

Procedure:

1. Set up the apparatus as shown in the figure below.



2. Adjust the function generator (start around 50 kHz) until the signal produced by PZT #1 forms a resonant standing wave in the metal rod. Resonance is detected by adjusting the frequency until a maximum signal amplitude from PZT # 2 is displayed on the oscilloscope. Hints: More accurate determination of the resonant frequency can be detected by selecting a sensitive vertical scale on the oscilloscope and adjusting the vertical position of the trace until just the top of the waveform is displayed. Next to the frequency of each resonance record the approximate amplitude (in volts) of the resonance as displayed on the oscilloscope. Generally you will only want to consider the largest resonances which should correspond to the longitudinal modes of the rod. The smaller resonances can confuse your calculations. They could be caused by a variety of factors including: transverse modes of the rod, harmonic distortion in the signal generator and PZTs, and resonance modes of the circular metal PZT plates. Remember also that the longitudinal resonances should be evenly spaced in frequency. Repeat the above procedure until at least 10 different resonant frequencies are found.

3. Repeat the entire procedure above for a second rod made of a different material.

4. Choose either of the rods you previously used and repeat the procudure, but start at a much lower frequency of about 3 kHz.

5. Make sure to record the length and type of metal of each rod as well as the model and serial number (if available) of each major piece of equipment used in the experiment. Have your data collection sheet signed and dated by the professor!

Computations and Analysis:

1. Calculate an average frequency spacing for the resonances in each rod. Calculate the standard deviation and eliminate any value from the calculation of an average value that lies more than two standard deviations away from the mean.

2. Using the average value of Δf for each rod, calculate the speed of sound waves in the particular metal. Keep your low frequency data and result separate.

3. Display your final results in a table similar to that shown below (but you only need do two rods plus one again at low frequency). <u>Clearly indicate which results were obtained from high frequency and which one was from low frequency resonances.</u>

material	$v_{expt(}$(m/s)	$v_{accepted}$(m/s)	% difference	
~~Aluminum~~		~~5000~~		
Brass (.7Cu/.3Zn)	3550 m/s ± 1%	3480	2%	← 50kHz +
	3580 m/s ± 1%		3%	← 3kHz +
~~Copper~~		~~3750~~		
~~Steel (1% carbon)~~		~~5180~~		
~~Steel (mild)~~		~~5200~~		

(Accepted values are from the CRC Handbook of Chemistry & Physics)

4. Estimate the experimental uncertainty in each result and include it in your tabulated results.

5. In you discussion of errors, note whether the actual difference between the experimental and accepted values is larger or smaller than the estimated experimental uncertainty. Which should typically be bigger? Both the steel and brass rods are alloys. Is the actual alloy ratio of our rods precisely that assumed in the CRC values? You may wish to refer to the CRC handbook and other reference sources and see how much of a difference the alloy ratio changes the speed of sound. Could this help explain any experimental "error" that you noted in your results? Did your result for v derived from the low frequency resonances agree with the high frequency result? Should it? Discuss.

NOTES:

Rod Material: Brass
Rod Length: (0.597 +/- 0.002) meters
Frequency Uncertainty: +/- 10Hz. This means that for Δf, the uncertainty is +/- 20Hz.
Theoretical Velocity: 3480 m/s

N	Frequency 'f' (Hz)	Change In Frequeny $\Delta f = (f_N - f_{N-1})$
1	50910	-
2	53895	2985
3	56870	2975
4	59838	2968
5	62827	2989
6	65795	2968
7	68765	2970
8	71725	2960
9	74686	2961
10	77645	2959
	Δf Average	2971
	Δf Standard Deviation σ	10
	Δf Average such that: $\Delta f < (f_{average} + 2\sigma)$ & $\Delta f > (f_{average} - 2\sigma)$	2971
1	3000	-
2	6000	3000
3	9000	3000
4	12000	3000
5	15000	3000
6	18015	3015
7	21005	2990
8	24000	2995
9	27000	3000
10	30000	3000
	Δf Average	3000
	Δf Standard Deviation	6
	Δf Average such that: $\Delta f < (f_{average} + 2\sigma)$ & $\Delta f > (f_{average} - 2\sigma)$	2998

Calculation 1.1 – Speed of Sound in Brass Rod (High Frequency)

$$v = 2L(\Delta f)$$

$$v = 2(0.597m)(2971Hz)$$

$$v = 3547 \to \mathbf{3550\ m/s}$$

Calculation 1.2 – Speed of Sound in Brass Rod (Low Frequency)

$$v = 2L(\Delta f)$$

$$v = 2(0.597m)(2998Hz)$$

$$v = \mathbf{3580\ m/s}$$

Calculation 1.3 – Percent Discrepancies

$$\%\ Discrepancy = \left|\frac{V_{Theoretical} - V_{Experimental}}{V_{Theoretical}}\right| * 100$$

High Frequency:

$$\%\ Discrepancy = \left|\frac{3480 - 3550}{3480}\right| * 100 = \mathbf{2\%}$$

Low Frequency:

$$\%\ Discrepancy = \left|\frac{3480 - 3580}{3480}\right| * 100 = \mathbf{3\%}$$

Calculation 1.4 - Error Propagation

$$V = 2L(\Delta f)$$

Starting Equation

$$\frac{\partial V}{\partial L} = 2(\Delta f), \qquad \frac{\partial V}{\partial(\Delta f)} = 2L$$

Take partial derivatives in respect for length and delta frequency

$$\delta V = \left\{\left[\delta L\left(\frac{\partial V}{\partial L}\right)\right]^2 + \left[\delta(\Delta f)\left(\frac{\partial V}{\partial(\Delta f)}\right)\right]^2\right\}^{\frac{1}{2}}$$

Definition for absolute error.

$$\frac{\delta V}{V} = \left\{\left[\frac{\delta L\left(\frac{\partial V}{\partial L}\right)}{V}\right]^2 + \left[\frac{\delta(\Delta f)\left(\frac{\partial V}{\partial(\Delta f)}\right)}{V}\right]^2\right\}^{\frac{1}{2}}$$

Definition for relative error.

$$\frac{\delta V}{V} = \left\{\left[\frac{\delta L(2(\Delta f))}{2L(\Delta f)}\right]^2 + \left[\frac{\delta(\Delta f)(2L)}{2L(\Delta f)}\right]^2\right\}^{\frac{1}{2}} = \left\{\left[\frac{\delta L}{L}\right]^2 + \left[\frac{\delta(\Delta f)}{(\Delta f)}\right]^2\right\}^{\frac{1}{2}}$$

Substitute in partial derivatives and V. Simplify. Note: $\delta L = \pm 0.002m$, $\delta(\Delta f) = \pm\, 20Hz$.

$$\frac{\delta V}{V} = \left\{\left[\frac{0.002m}{0.597m}\right]^2 + \left[\frac{20}{2959}\right]^2\right\}^{\frac{1}{2}} = 0.007 \to 1\%\ error$$

Use the smallest calculated delta frequency to estimate largest possible error.

Conclusion

The experimental speed of sound through a brass rod was $3550\frac{m}{s} \pm 1\%$ for higher frequencies starting at 50KHz, and $3580\frac{m}{s} \pm 1\%$ for lower frequencies starting at 3KHz. The percent discrepancies of these results were 2% and 3% respectively. Unfortunately, these percent differences are not within the margin of error. Perhaps as a general rule, experiments will typically have a greater margin of error with respect to the percent discrepancies. This experiment, however, seems to be an exception to that rule. Despite the sources of error, including the length measurement of the rod and human observational error while finding the correct frequencies, the measured values were still exceptionally accurate and consistent. The minimal variation of output mathematically contributed to a very low margin of error. It's likely that the given theoretical value given for the speed of sound through brass wasn't accurate for the brass rod used in the experiment. Consider the fact that brass is an alloy of copper and zinc. According to the *CRC Handbook of Chemistry and Physics 85th Edition*, the speed of sound through copper is 3750 m/s, and 3850 m/s through zinc. Brass rated at a copper-zinc ratio of 7:3 has a speed of sound measured at 3480 m/s. Without venturing into the complexities of how the copper-zinc ratios could affect the speed of sound, it's a reasonable guess that the speed would vary based upon the associated changes of the material's bulk modulus and density factors which share the following relationship with wave velocity: $v = \left(\frac{B}{p}\right)^{\frac{1}{2}}$. The experiment could be improved by providing the exact copper-zinc ratio and a method to compute a more accurate value for the theoretical speed of sound through the brass.

The experimental values for both low and high frequency starting points yielded similar results as expected, however, the higher frequency data points were less consistent than the lower frequency data points. This could have been the result of observer uncertainty, other physical anomalies, or **perhaps** a need for greater frequency precision as the frequency to wavelength ratio grew. (Note: $v = \lambda f$)

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Speed of Sound in Metal Rods

Data Sheet

15

DATA Point	Rod #1 (Brass) Note: 5V_{pp}		Rod #2		Rod #1 (Brass) Note: 0.5V_{pp}	
	Frequency (Hz)	Amplitude (V)	Frequency (Hz)	Amplitude (V)	Frequency (Hz)	Amplitude (V)
1	50,910	0.060			3,000	0.030
2	53,895	0.038			6,000	0.080
3	56,870	0.016			9,000	0.150
4	59,838	0.012			12,000	0.060
5	62,827	0.014			15,000	0.040
6	65,795	0.016			18,015	0.120
7	68,765	0.022			21,005	0.100
8	71,725	0.034			24,000	0.010
9	74,686	0.040			27,000	0.010
10	77,645	0.032			30,000	0.010

Second Rod Portion

- Eliminated by Professor.

Length of Rod: 59.7 cm ± 0.2 cm

Uncertainties: f = ± 10 Hz, A = ± 5 mV

Note: This implies that Δf's uncertainty is ±20 Hz.