

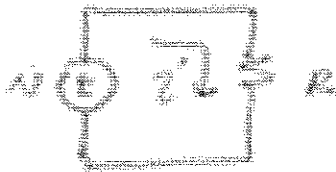
HW set 1

Soln 1, 2, 4

SP2018

Soln 1.

$$P(t) = 20 \text{ Gt} + \text{mW} \quad V = 10 \text{ Gt}$$



$$i(t) = \frac{P(t)}{V(t)} = \frac{(20 \text{ Gt} + \text{mW}) 10^{-3} \text{ W}}{10 \text{ Gt} + \text{V}}$$

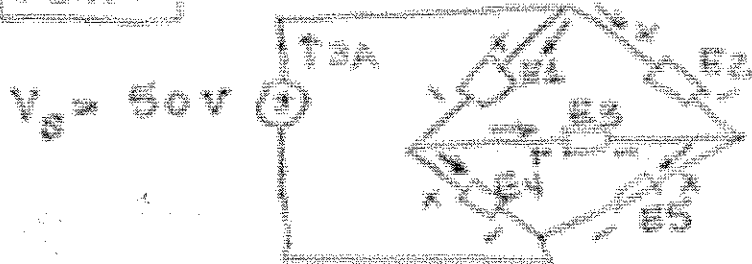
$$i(t) = 2 \text{ Gt} + \text{mA}$$

$$i(t) = 2 \text{ Gt} \text{ mA}$$

by ohm's Law: $V = iR$ $R = \frac{V}{i} = \frac{10 \text{ Gt} + \text{V}}{2 \text{ Gt} + \text{mA}}$

$$R = 5 (10^3) \Omega = 5 \text{ k}\Omega$$

Soln 2.



The $V_0 = 50$ is Applying Power $= 100 \text{ W}$

All 5 elements are absorbing Power $=$

$$= 10 + 10 + (10V_0 + 30 + 20) = (30 + V_0) \text{ W}$$

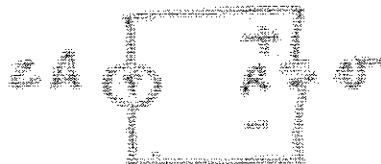
By Conservation of Energy (Power):

$$100 \text{ W} = (30 + V_0) \text{ W} \rightarrow V_0 = 20 \text{ V}$$

Soln 3.

$$1: V_R = IR \quad iR > 10 \text{ V}$$

$$R > 5 \Omega$$



$$2: P = i^2 R, \quad i^2 R < 25 \text{ W}$$

$$4R < 25 \quad R < \frac{25}{4} < 6.25 \Omega$$

$$5 \Omega < R < 6.25 \Omega$$

HW Set 1

Soln 5

SP 2018

Soln 5.

$$\text{Consumption per day} = \left(120W \frac{4h}{\text{day}} + 60W \frac{8h}{\text{day}} \right) \\ = 960 \frac{Wh}{\text{day}} = 0.96 \frac{kWh}{\text{day}}$$

$$\text{Energy Consumption per year (365 days)} : \\ (365) \text{ days} \left(0.96 \frac{kWh}{\text{day}} \right) = 350 kWh$$

$$\text{Yearly cost} = (350 kWh) \left(\frac{\$0.12}{kWh} \right) = \$42.05$$

Now you can perform Energy Consumption analysis and cost @ home, include all other appliances and add them.

Soln3.

For $0 < t < 6s$, assuming $q(0) = 0$,

$$q(t) = \int_0^t i dt + q(0) = \int_0^t 3t dt + 0 = 1.5t^2$$

$$\text{At } t=6, q(6) = 1.5(6)^2 = 54$$

For $6 < t < 10s$,

$$q(t) = \int_6^t i dt + q(6) = \int_6^t 18 dt + 54 = 18t - 54$$

$$\text{At } t=10, q(10) = 180 - 54 = 126$$

For $10 < t < 15s$,

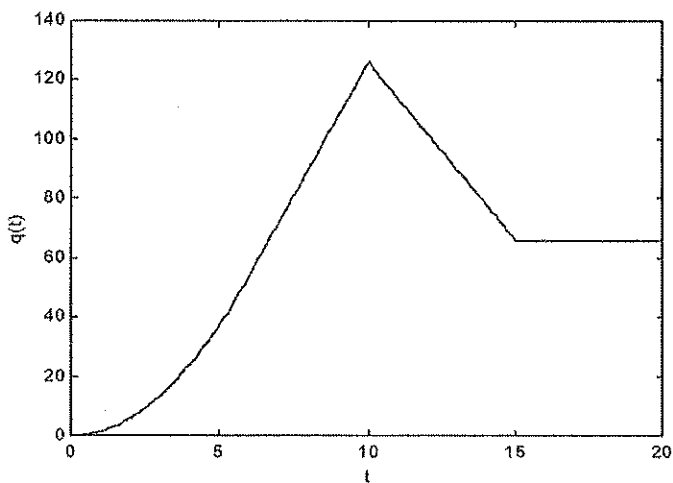
$$q(t) = \int_{10}^t i dt + q(10) = \int_{10}^t (-12) dt + 126 = -12t + 246$$

$$\text{At } t=15, q(15) = -12 \times 15 + 246 = 66$$

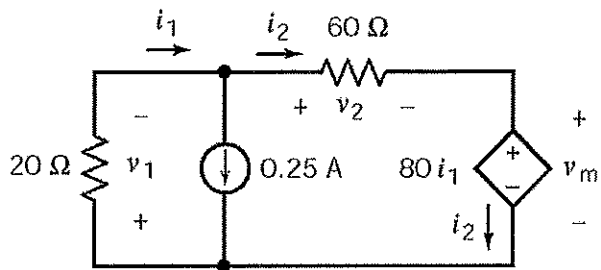
For $15 < t < 20s$,

$$\text{Thus, } q(t) = \int_{15}^t 0 dt + q(15) = 66$$

$$q(t) = \begin{cases} 1.5t^2 \text{ C, } 0 < t < 6s \\ 18t - 54 \text{ C, } 6 < t < 10s \\ -12t + 246 \text{ C, } 10 < t < 15s \\ 66 \text{ C, } 15 < t < 20s \end{cases}$$



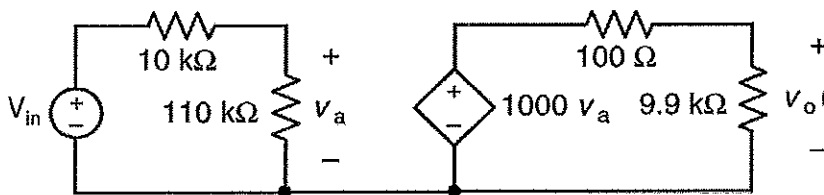
The plot of the charge is shown across.

SOLN1.

$$i_1 = i_2 + 0.25 \quad v_1 = 20i_1 \quad v_2 = 60i_2 = 60(i_1 - 0.25) = 60i_1 - 15$$

$$v_2 + 80i_1 + v_1 = 0 \Rightarrow (60i_1 - 15) + 80i_1 + 20i_1 = 0 \Rightarrow i_1 = \frac{15}{160} = 0.09375 \text{ A}$$

$$v_m = 80i_1 = 80(0.09375) = 7.5 \text{ V}$$

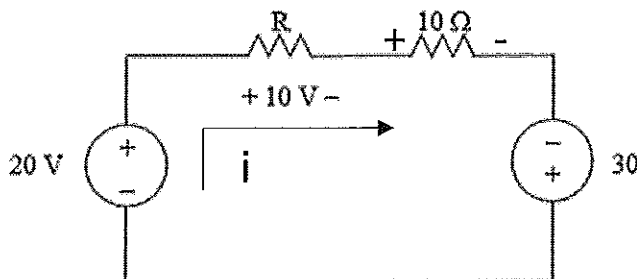
SOLN2.

Use Voltage DIV for the first loop to find V_a :

$$V_a = \frac{110k}{(10 + 110)k} 10\text{mV} = 9.2\text{mV}$$

Use Voltage DIV for the second loop to find V_o :

$$V_o = \frac{9900}{(100 + 9900)} (1000 \times 9.2\text{mV}) = 9.11\text{V}$$

SOLN3.

KVL around the loop:

$$-20 + 10 + 10i - 30 = 0$$

$$i = (40\text{V}/10\Omega) = 4\text{A}$$

Now: by Ohm's law

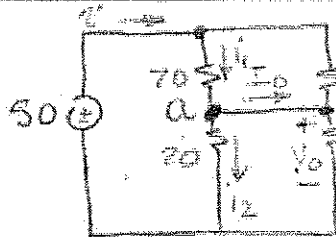
$$10\text{V} = iR = (4\text{A}) R$$

$$R = (10\text{V}/4\text{A}) = 2.5\Omega$$

HW Set 2

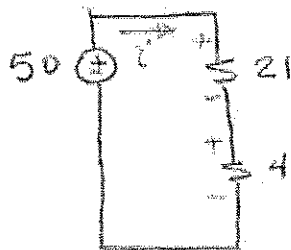
Soln 4.

SP 2018



$$70 \parallel 30 \quad R_{P1} = \frac{(30)(70)}{100} = 21\Omega$$

$$20 \parallel 5 \quad R_{P2} = \frac{(5)(20)}{25} = 4\Omega$$



$$i = \frac{50}{25} = 2A$$

$$V_{4\Omega} = V_{20\Omega} = V_0 = V_{5\Omega} = (2A)(4\Omega) = 8V$$

$$V_{21\Omega} = V_{70\Omega} = V_{30\Omega} = (2A)(21\Omega) = 42V$$

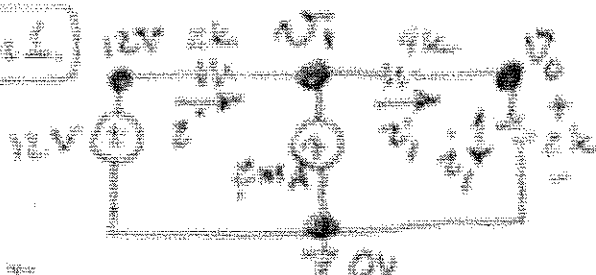
BY Ohm's Law:

$$i_1 = \frac{V_{21\Omega}}{70\Omega} = \frac{42}{70} = 0.6A$$

Also

$$i_2 = \frac{8V}{20\Omega} = 0.4A$$

Soln 1



Nodal analysis a

 V_1 -node - KCL

$$2 + 2mA = 0$$

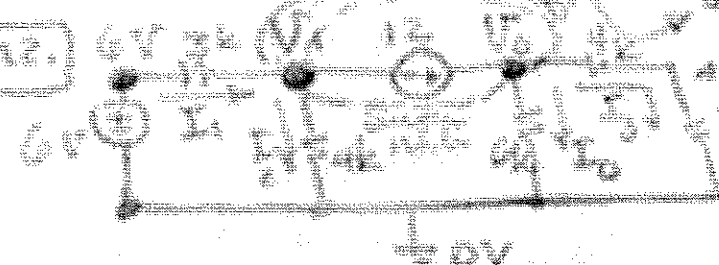
$$\frac{12 - V_1}{2k} + 2mA = \frac{V_1 - V_0}{4k} \quad \text{but } V_1 - V_0 = \frac{V_0}{2k}$$

$$0k \Rightarrow \frac{V_0}{2k} = \frac{V_0}{6k}$$

$$\text{Solving: } \frac{12 - V_1}{2k} + 2mA = \frac{V_1}{6k} \rightarrow V_1 = 12V$$

$$\text{KVL char/loop: } V_0 = (2k)I_1 = \left(\frac{V_1}{6k}\right)(6k) = 12V$$

Soln 2

4k + 2k
ans in series
= 6k

$$V_1 - V_0 = -12 \rightarrow V_1 = V_0 - 12 \quad \text{Eq 1}$$

$$\text{KCL @ Super node } I_1 = I_2 + I_3 + I_4$$

$$\frac{6 - V_1}{3k} = \frac{V_1}{4k} + \frac{V_0}{6k} + \frac{V_0}{6k} \quad \text{Eq 2}$$

$$\text{Sub Eq 1 in Eq 2:}$$

$$\frac{6 - (V_0 - 12)}{3k} = \frac{V_0 - 12}{4k} + \frac{2V_0}{6k} \rightarrow V_0 = 9.80V$$

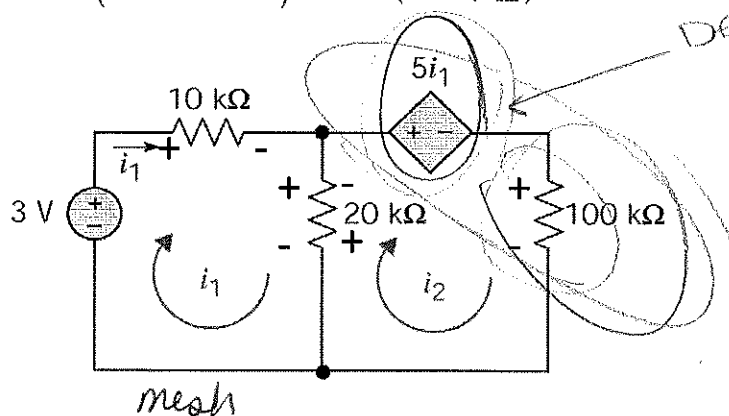
Apply KVL to left mesh: $-3 + 10 \times 10^3 i_1 + 20 \times 10^3 (i_1 - i_2) = 0 \Rightarrow 30 \times 10^3 i_1 - 20 \times 10^3 i_2 = 3$ (1)

Apply KVL to right mesh: $5 \times 10^3 i_1 + 100 \times 10^3 i_2 + 20 \times 10^3 (i_2 - i_1) = 0 \Rightarrow i_1 = 8i_2$ (2)

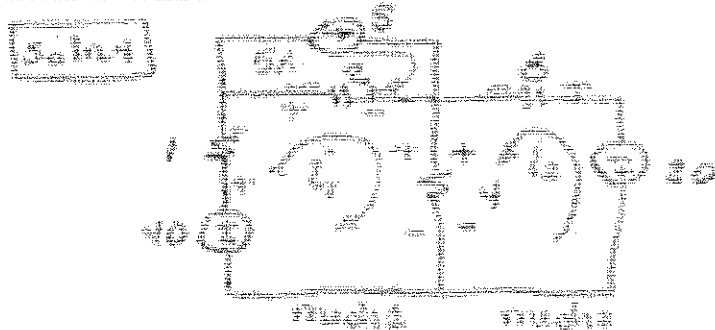
Solving (1) & (2) simultaneously $\Rightarrow i_1 = \frac{6}{55} \text{ mA}, i_2 = \frac{3}{220} \text{ mA}$

Power delivered to cathode $= (5i_1)(i_2) + 100(i_2)^2$
 $= 5\left(\frac{6}{55}\right)\left(\frac{3}{220}\right) + 100\left(\frac{3}{220}\right)^2 = 0.026 \text{ mW}$

\therefore Energy in 24 hr. $= (2.6 \times 10^{-5} \text{ W})(24 \text{ hr})(3600 \text{ s/hr}) = 2.25 \text{ J}$



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Mesh 1: KVL $-10 + 10i_1 + 20(i_1 - i_2) = 0$
 $\Rightarrow 7i_1 - 2i_2 = 10$

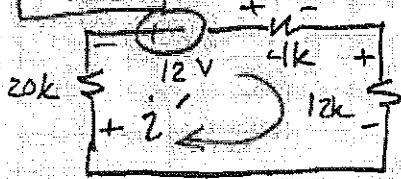
Mesh 2: KVL $+20 + 0i_2 + 100(i_2 - i_1) = 0$
 $\Rightarrow -10i_1 + 120i_2 = -20$

Solve EQ 1 & EQ 2 simultaneously:

$i_1 = 10 \text{ A}, i_2 = -5 \text{ A}$

$V_o = 10(i_2 - i_1) = 10(-5 - 10) = -150 \text{ V}$

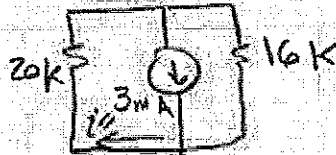
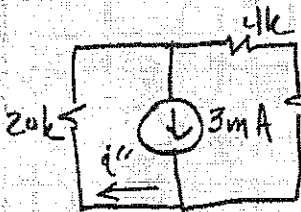
Soln 1. eliminate 3mA, 9mA sources:



$$KVL: +20ki' + 12 + 4ki' + 12ki' = 0$$

$$i' = -\frac{12}{36} = -\frac{1}{3} \text{ mA}$$

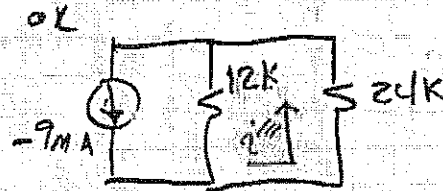
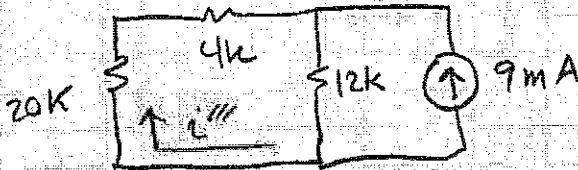
eliminate 9mA & 12V sources:



using C.D.:

$$i'' = 3\text{mA} \frac{16\text{K}}{36\text{K}} = \frac{4}{3} \text{ mA}$$

eliminate 3mA & 12V sources:

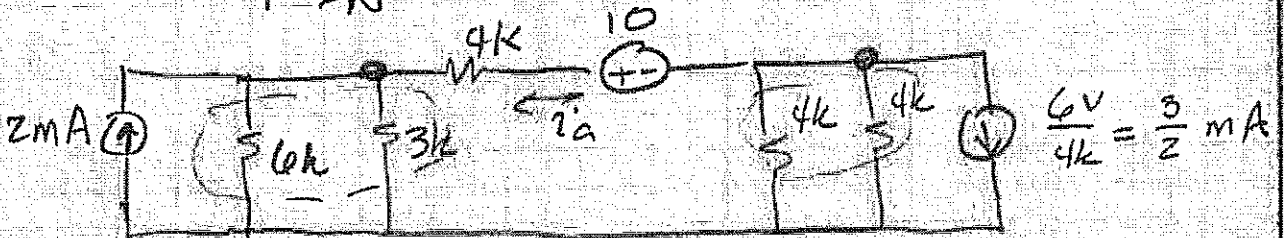
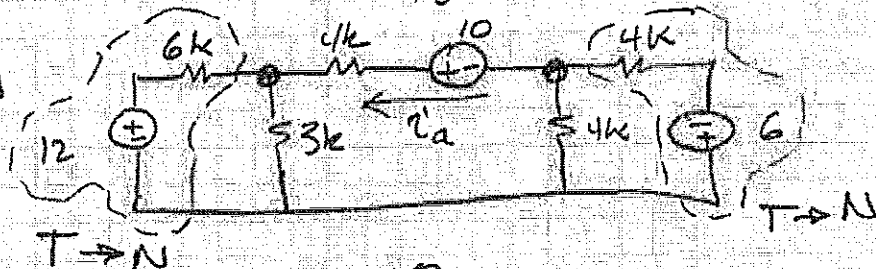


by C.D.:

$$i''' = (-9\text{mA}) \frac{12\text{K}}{36\text{K}} = -3\text{mA}$$

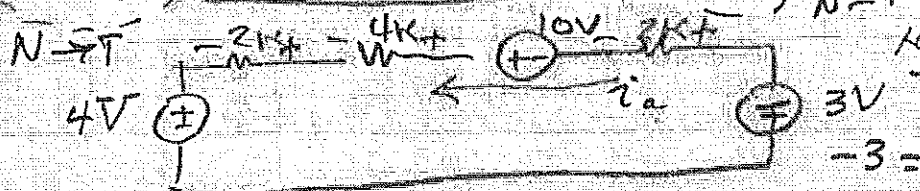
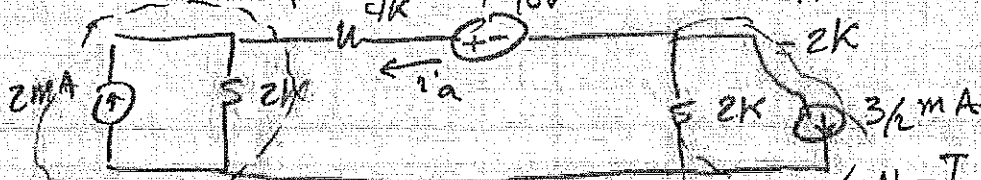
$$i = i' + i'' + i''' = -\frac{1}{3} \text{ mA} + \frac{4}{3} \text{ mA} - 3\text{mA} = \boxed{-2\text{mA}}$$

Soln 2.



$$6\text{k} \parallel 3\text{k} = \frac{(6)(3)}{9} = 2\text{k}$$

$$4\text{k} \parallel 4\text{k}$$

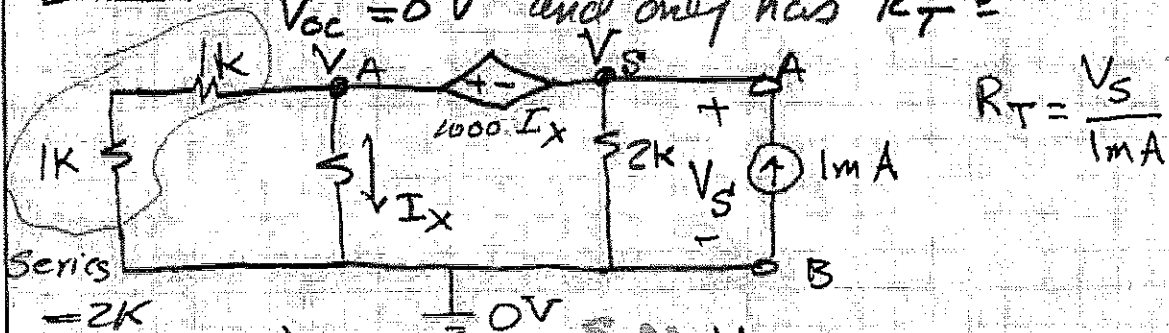


$$KVL: -4 - (2\text{k} + 4\text{k} + 2\text{k})i_a + 10 - 3 = 0$$

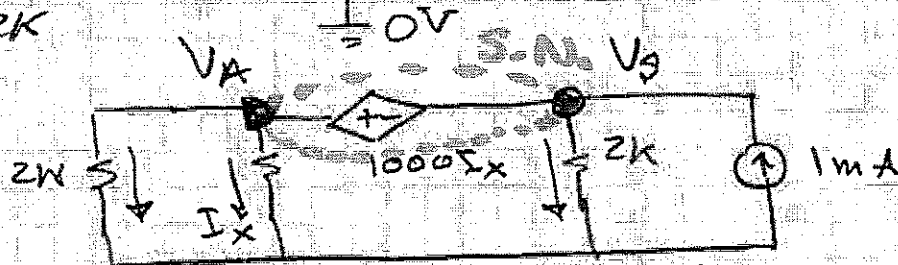
$$-3 = -8\text{k}i_a, i_a = \boxed{0.375 \text{ mA}}$$

Soln 3

The ckt contains only dep sources so $V_{oc} = 0V$ and only has $R_T =$



$$R_T = \frac{V_s}{1mA}$$



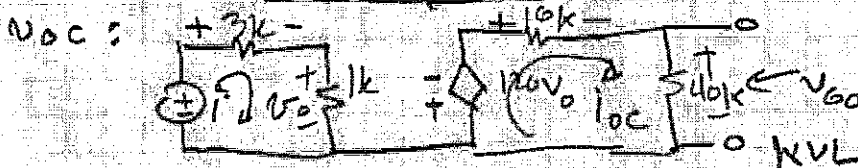
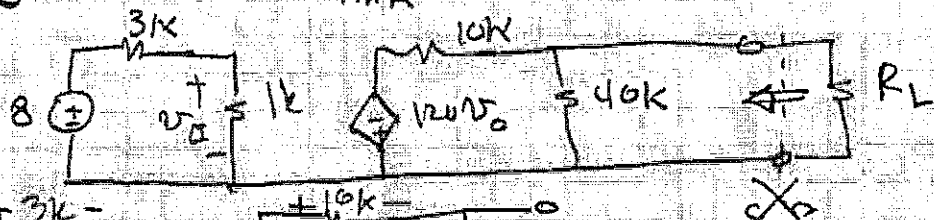
KCL @ super node $\frac{V_A}{2K} + \frac{V_A}{2K} + \frac{V_s}{2K} = 1mA$ EQ 1

Also: $\frac{V_A}{I_x} = 2K$ $V_A = 2kI_x$ Also $V_A - V_s = 1000I_x$

simplify: to get $V_A = 2V_s \Rightarrow$ EQ 1 $\frac{2V_s}{2K} = 1mA$

$V_s = \frac{2}{5}$ so $R_T = \frac{2/5}{1mA} = 0.4 \times 10^3 = \boxed{400\Omega}$

Soln 4

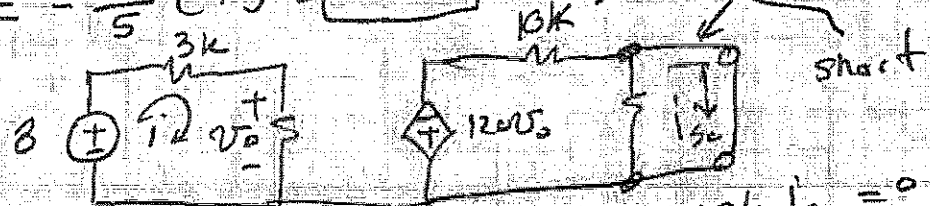


$-3k + 3ki + ik = 0$, $i = 2mA$, $+120V_o + 10k i_{oc} + 40k i_{oc} = 0$

$V_o = 2V$

$V_{oc} = -\frac{24}{5}(40) = \boxed{-192V}$

Find i_{sc}

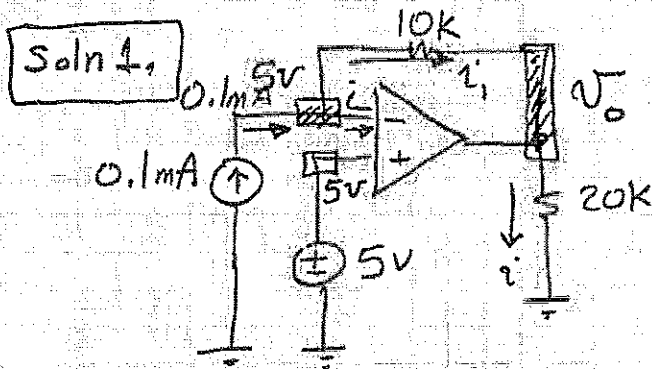


$V_o = 2V$ $+240 + 10k i_{sc} = 0$ $i_{sc} = \boxed{-24mA}$

$R_T = \frac{V_{oc}}{i_{sc}} = \frac{-192}{-24mA} = \boxed{8k\Omega}$

$i_{A1} = \frac{-192}{8k} = -12mA$

$P_{8k} = (-12mA)^2 \cdot 8k = \boxed{1.152W}$



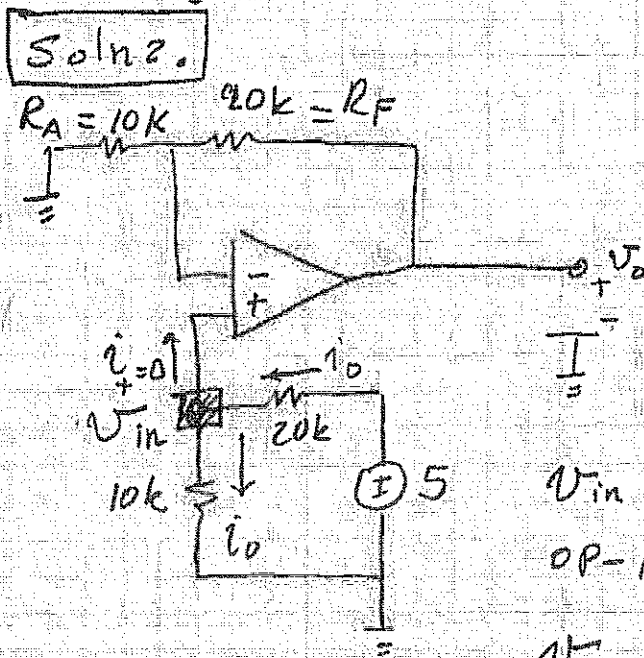
$i_- = 0$ because of principle of open

$V_+ = V_- = 5V$ " " " of short

KCL @ (-) input node of Amp: $0.1m = i_1 + i_2$
 $0.1mA = \frac{5 - V_o}{10k} \rightarrow V_o = 4$ As $i_2 = \frac{V_o - 0}{20k}$

$$i_2 = \frac{4V}{20k\Omega} = 2 \times 10^{-4} = \boxed{0.2mA}$$

know voltage @ (-) input is 5V so the voltage across the 0.1mA source is $\boxed{5V}$.



$$i_o = \frac{5V}{(20k + 10k)\Omega} = \boxed{0.1667mA}$$

V_{in} is found by V.D.:

$$V_{in} = 5 \frac{10k}{10k + 20k} = \frac{5}{3}V$$

V_{in} is input to the non-inv OP-Amp with Equation =

$$V_o = \left(1 + \frac{R_F}{R_A}\right) V_{in} = \left(1 + \frac{20}{10}\right) \frac{5}{3}$$

$$\boxed{V_o = 5V}$$

Soln3

by V.D.:

$$V_A = 5 \frac{10k}{30k} = \frac{5}{3}V$$

write KCL @ (-) input

$$A_2:$$

$$i_o = i_i$$

$$\frac{V_o - 0}{100k} = \frac{0 - V_B}{40k}$$

Also:

$$\text{KCL @ } V_A \text{ (-) input of } A_1: i_x = i_y$$

$$\frac{V_B - V_A}{10k} = \frac{V_A - 1}{20k}, \quad \frac{V_B - 5/3}{10k} = \frac{5/3 - 1}{20k} \Rightarrow V_B = 2V$$

$$\text{so } \frac{V_o - 0}{100k} = \frac{0 - 2}{40k} \Rightarrow \boxed{V_o = -5V}$$

Soln4.

by inspection (KCL):

$$i_s = i_i$$

KCL @ V_o node:

$$i_2 + i_1 = i_o$$

$$i_2 + i_s = i_o$$

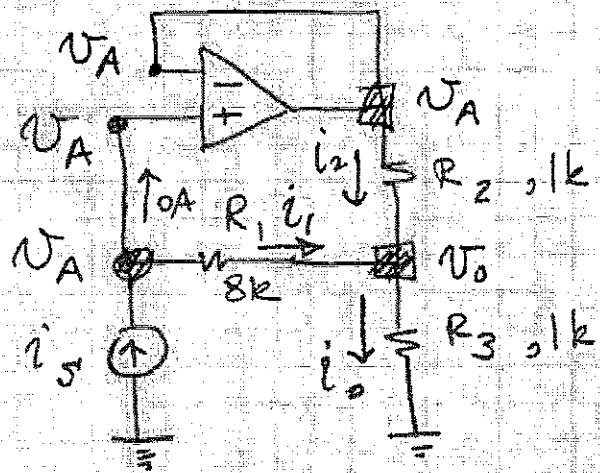
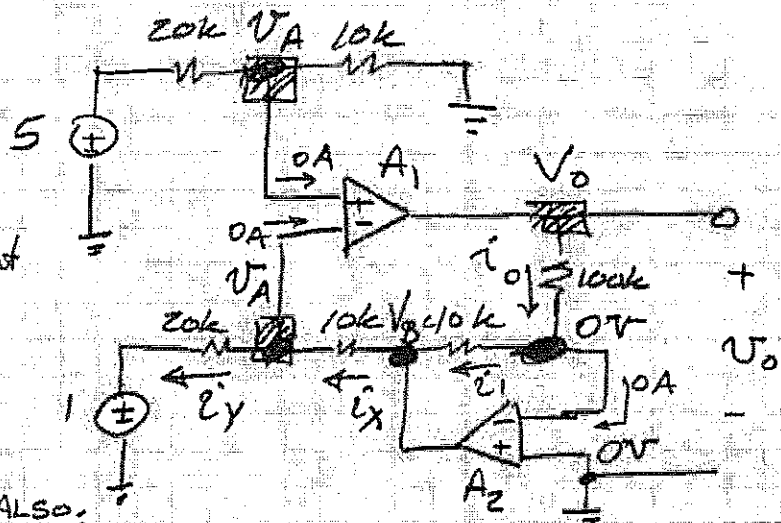
$$i_s = i_o - i_2$$

notice:

$$V_{R_2} = V_{R_1}, \quad i_s 8k = i_2 1k \Rightarrow i_2 = 8i_s$$

$$i_s = i_o - 8i_s \Rightarrow 9i_s = i_o \Rightarrow$$

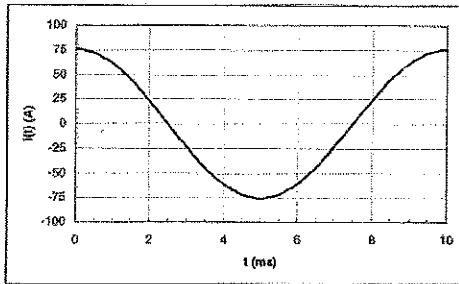
$$\frac{i_o}{i_s} = \boxed{9}$$



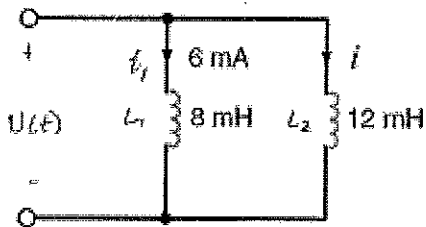
Solution 1.

$$i = C dv/dt \quad v = 12 \sin \omega t \quad \omega = \frac{2\pi}{T} \quad T = 10 \text{ ms} \Rightarrow \omega = 200\pi \text{ rad/s}$$

$$i = 75.4 \cos \omega t \text{ mA} \quad \omega = 200\pi \text{ rad/s}$$



Solution 2.



$$\left. \begin{aligned} \text{for } L_1 \quad i_1 &= \frac{1}{L_1} \int v(t) dt \\ \text{for } L_2 \quad i &= \frac{1}{L_2} \int v(t) dt \end{aligned} \right\} \quad \frac{i}{L_1} = \frac{L_2}{L_1}$$

$$i = L_1 \left(\frac{L_2}{L_1} \right)$$

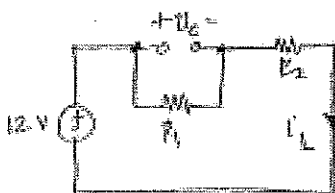
$$i = 4 \text{ mA}$$

Solution 3.

Since voltage source is constant, v_L and i_L are constant

$$i_L = C dv_L/dt = 0 \quad \text{if } v_L = L di_L/dt = 0$$

New circuit



$$i_L = \frac{12}{R_1 + R_2} = 40 \text{ mA}$$

$$v_L = \frac{12 R_1}{R_1 + R_2} = 8 \text{ V}$$

$$\omega_C = \frac{1}{2} C v_C^2 = \omega_L = \frac{1}{2} L i_L^2 \Rightarrow C = L \left(\frac{i_L}{v_C} \right)^2$$

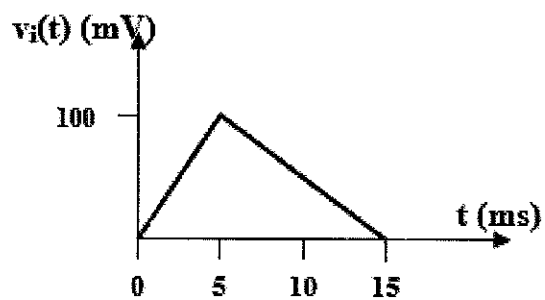
$$C = 2.5 \mu\text{F}$$

Solution 4.

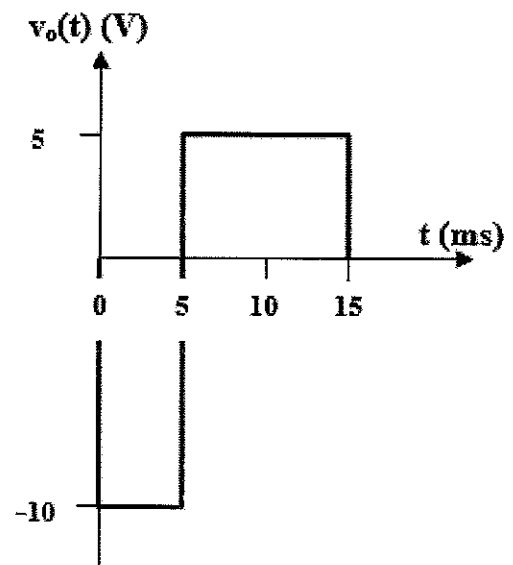
$$v_o = -RC \frac{dv_i}{dt}, \quad RC = 50 \times 10^3 \times 10 \times 10^{-6} = 0.5$$

$$v_o = -0.5 \frac{dv_i}{dt} = \begin{cases} -10, & 0 < t < 5 \\ 5, & 5 < t < 15 \end{cases}$$

The input is sketched in Fig. (a), while the output is sketched in Fig. (b).

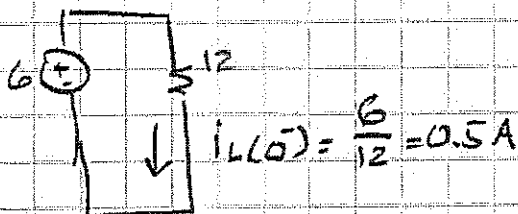
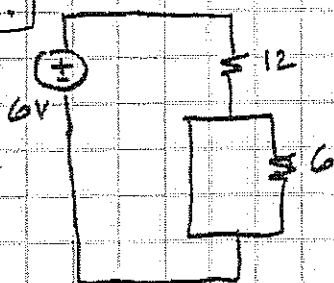


(a)

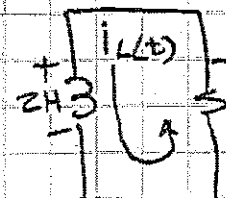


(b)

Soln 1.

For $t=0^-$,For $t=0^+$ or $t>0$:

KVL:



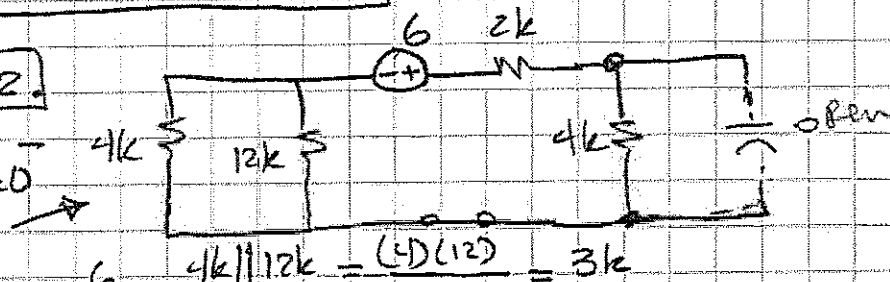
$$2 \frac{di_L}{dt} + 6i_L = 0 \Rightarrow$$

$$\frac{di_L}{dt} + 3i_L = 0, i_L = Ke^{st}$$

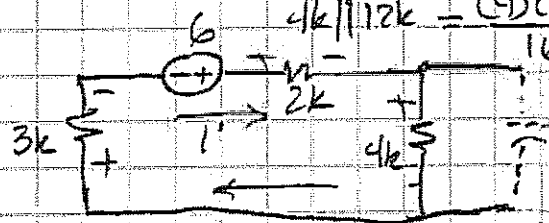
$$Kse^{st} + 3Ke^{st} = 0, s = -3, i_L(t) = Ke^{-3t}, i_L(0) = 0.5 A$$

$$i_L(t) = 0.5 e^{-3t} A$$

Soln 2.

For $t=0^-$ 

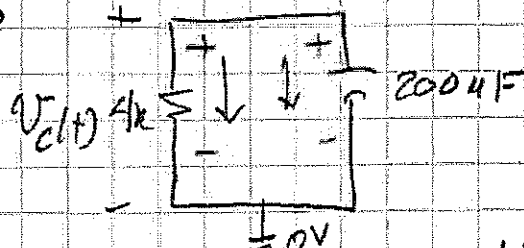
$$V_C(0^-) = V_{R_4}$$



$$+3ki - 6 + 2ki + 4ki = 0, i = \frac{6}{9} mA$$

$$i(0) = \frac{2}{3} mA, V_{R_4} = \frac{8}{3} V$$

$$So V_C(0^-) = V_C(0) = \frac{8}{3} V$$

For $t>0$ 

$$i_{4k} + i_C = 0$$

$$\frac{V_C - 0}{4k} + 200 \frac{dV_C}{dt} = 0$$

$$V_C + 200 \cdot 10^{-6} \cdot 4 \cdot 10^3 \frac{dV_C}{dt} = 0$$

$$\frac{dV_C}{dt} + 1.25 V_C = 0 \Rightarrow V_C(t) = Ke^{-1.25t}$$

$$V_C + 0.8 \frac{dV_C}{dt} = 0$$

$$V_C(0) = \frac{8}{3} \Rightarrow V_C(0) = \frac{8}{3} = Ke^0 \Rightarrow K = \frac{8}{3} = 2.67$$

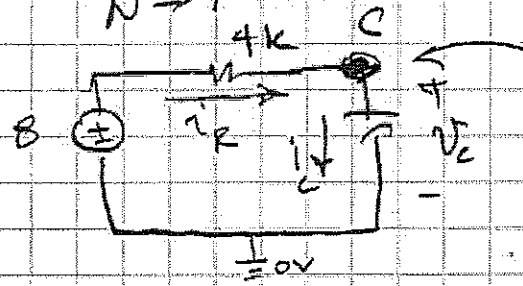
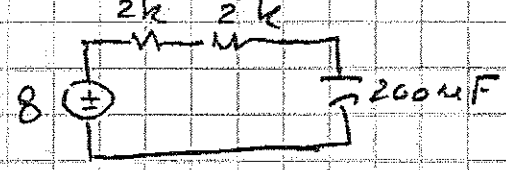
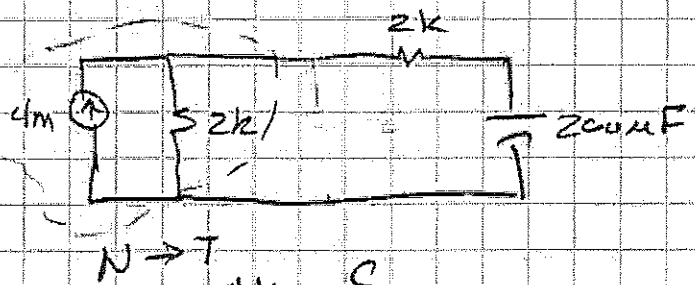
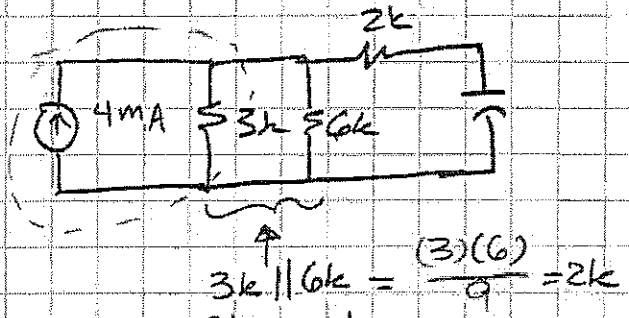
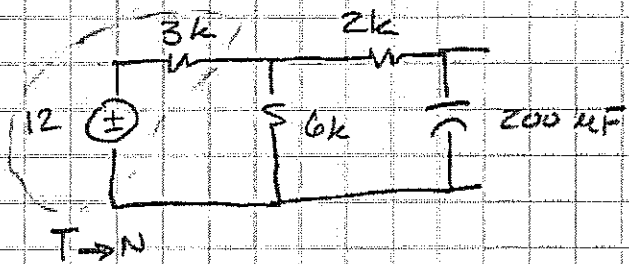
$$V_C(t) = 2.67 e^{-1.25t}$$

Soln 3

For $t=0$ or $t < 0$ $V(t) = 0$, then

$V_c(0) = V_0 = 0$

For $0 \leq t \leq 1$ $V(t) = 12V$



KCL @ C-node:
 $0 = i_R = i_C$
 $\frac{8 - V_c}{4k} = 200 \times 10^{-6} \frac{dV_c}{dt}$
 $8 - V_c = 0.8 \frac{dV_c}{dt}$

$\frac{dV_c}{dt} + \frac{V_c}{0.8} = \frac{8}{0.8} = 10$

$\frac{dV_c}{dt} + 1.25 V_c = 10$

try $V_{cf} = K_1 \rightarrow$ Sub $0 + 1.25 K_1 = 10, K_1 = 8$

$\frac{dV_c}{dt} + 1.25 V_c = 0 \Rightarrow V_{cn} = K_2 e^{-1.25t}$

total soln $V_c(t) = K_1 + K_2 e^{-1.25t} = 8 + K_2 e^{-1.25t}$

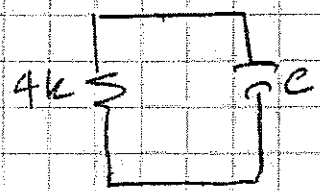
$V_c(0) = V_0 = 0 = 8 + K_2 e^0 \rightarrow K_2 = -8$

$V_c(t) = 8 - 8e^{-1.25t}$ for $0 \leq t \leq 1$

For $t > 1 \rightarrow$ the 12-V source goes to zero:

Also $t = 1^- \Rightarrow V(1) = 8 - 8e^{-1.25}$

$V(1) = 8 - 8(0.29) = 5.71V$



no source in the ckt
 so: the new soln:

$V_c(t) = K_3 e^{-1.25t}$ $V(1) = 5.71 = K_3 e^{-1.25(1)}$

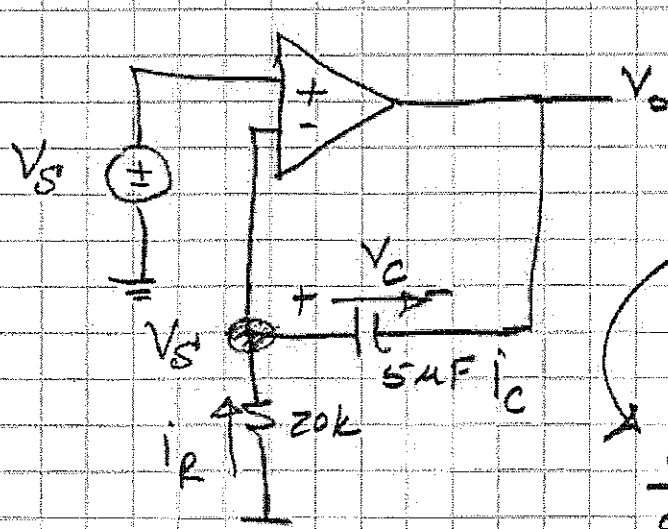
$K_3 = 5.71e$

$V_c(t) = 5.71 e^{-1.25(t-1)}$ for $t > 1$

For $t < 0$ Switch is open $V_c(0) = 0$, $V_s = 0$
 So $V_o(0) = 0$

For $t > 0$

KCL @ (-) input of OP-Amp



$$i_R = i_C$$

$$\frac{0 - V_s}{20k} = C \frac{dV_c}{dt}$$

$$\text{ALSO: } V_s - V_o = V_c$$

$$\frac{dV_c}{dt} = -\frac{V_s}{RC}, \quad dV_c = -\frac{V_s}{RC} dt$$

$$dV_c = -\frac{V_s}{RC} dt + V_c(0)$$

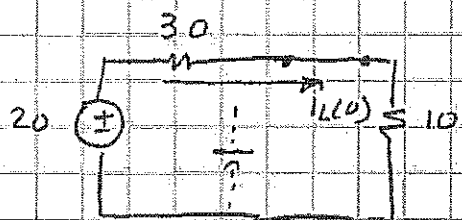
$$RC = 20 \cdot 10^3 \cdot 5 \cdot 10^{-6} = 0.1s$$

$$V_c = -\frac{V_s}{RC} t = -\frac{20}{0.1} t \text{ mV} \quad \text{since } V_s = 20 \text{ mV}$$

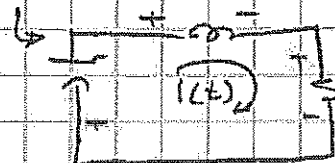
$$V_o = V_s - V_c = 20 \text{ mV} - \left(-\frac{20t}{0.1}\right) \text{ mV} = \boxed{20(1 + 10t) \text{ mV}}$$

Soln 1.

This is RLC series ckt.

for $t=0$ or $t < 0$:

$$V_C(0) = 0, \quad i_L(0) = \frac{20V}{40} = 0.5A$$

For $t > 0$ 

$$+4 \frac{di}{dt} + 10i + \frac{1}{0.25} \int i dt + V(0) = 0$$

$$4 \frac{d^2 i}{dt^2} + 10 \frac{di}{dt} + 4i = 0 \rightarrow \frac{d^2 i}{dt^2} + 2.5 \frac{di}{dt} + i = 0$$

$$s^2 + 2.5s + 1 = 0 \quad s = \frac{-2.5 \pm \sqrt{6.25 - 4}}{2} \quad \begin{aligned} & \sqrt{\frac{-2.5 + 1.5}{2}} = -0.5 \\ & \sqrt{\frac{-2.5 - 1.5}{2}} = -2 \end{aligned}$$

$$i(t) = k_1 e^{-2t} + k_2 e^{-.5t}$$

$$v(t) = \int (k_1 e^{-2t} + k_2 e^{-.5t}) \Rightarrow \frac{k_1}{-2} e^{-2t} + \frac{k_2}{-.5} e^{-.5t}$$

$$\Rightarrow v(t) = A e^{-2t} + B e^{-.5t}$$

$$v(0) = 0 = A + B = 0 \quad \boxed{A = -B} \quad \text{EQ1}$$

$$i_0 = i_L(t) = 0.5 \bigg|_{t=0} = C \frac{dv}{dt} \bigg|_{t=0} = 0.25 (-2A e^{-2t} - 0.5B e^{-.5t}) \bigg|_{t=0} = 0.5$$

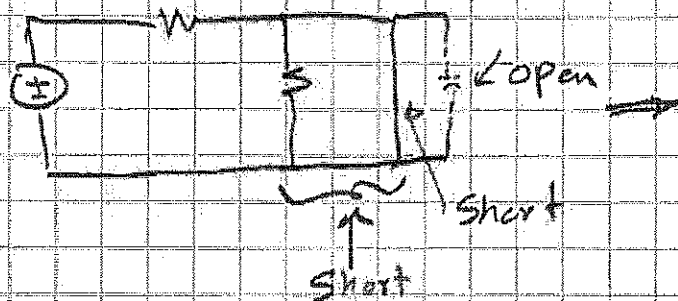
$$4 \times 0.25 (-2A e^{-2t} - 0.5B e^{-.5t}) = 0.5 \times 4 \quad \text{at } t=0$$

$$\boxed{-2A - 0.5B = 2} \quad \text{EQ2} \quad \text{Solving EQ1 \& EQ2: } A = -1.33 \text{ and } B = 1.33$$

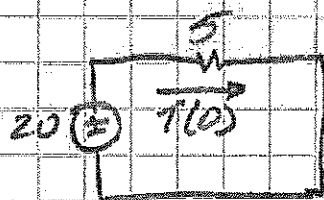
$$\boxed{v(t) = -1.33 e^{-2t} + 1.33 e^{-.5t}}$$

Soln 2. this is RLC parallel ckt:

For $t < 0$



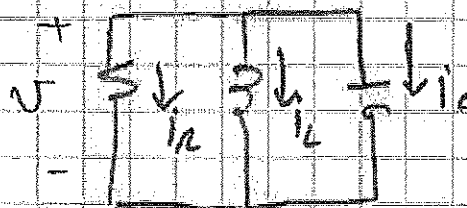
$$V_C(0) = 0$$



$$i(0) = 20/5 = 4A$$

So I.C : $V_C(0) = 0$
 $i(0) = 0$

For $t > 0$



$$i_R + i_L + i_C = 0$$

$$\frac{V}{R} + \frac{1}{L} \int V dt + i_C(0) +$$

$$C \frac{dV}{dt} = 0 \quad \text{EQ 1}$$

$\frac{d}{dt}$ of EQ 1:

$$C \frac{d^2 V}{dt^2} + \frac{1}{R} \frac{dV}{dt} + \frac{V}{L} = 0 \quad C=1F, R=5\Omega, L=0.25H$$

$$\Rightarrow S^2 + S + 4 = 0$$

$$S_1, S_2 = \frac{-1 \pm \sqrt{1-16}}{2} = \frac{-1 \pm \sqrt{-15}}{2} = \frac{-1 + j\sqrt{15}}{2}, \frac{-1 - j\sqrt{15}}{2}$$

$$S_1 = -0.5 + j1.94$$

$$S_2 = -0.5 - j1.94$$

$$V(t) = e^{-0.5t} (K_1 \cos 1.94t + K_2 \sin 1.94t)$$

$$V(0) = 0 = K_1 (\cos 0) + K_2 \sin(0) \Rightarrow K_1 = 0$$

$$i_C|_{t=0} = -i_R|_{t=0} - i_L|_{t=0} = -\frac{V(0)}{R} = -4$$

$$\frac{dV}{dt}|_{t=0} \Rightarrow C=1F, \frac{dV}{dt}|_{t=0} = -4$$

$$\frac{dV}{dt} = e^{-0.5t} (0.5)(K_1 \cos 1.94t + K_2 \sin 1.94t) + e^{-0.5t} (-1.94K_1 \sin 1.94t + 1.94K_2 \cos 1.94t)$$

$$\frac{dV}{dt}|_{t=0} = -0.5K_1 + 1.94K_2 \Rightarrow -0.5K_1 + 1.94K_2 = -4$$

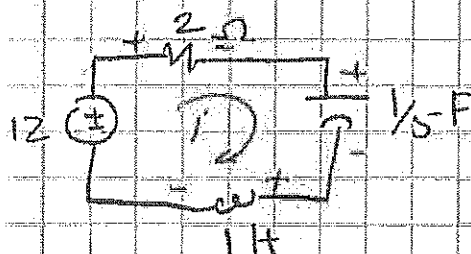
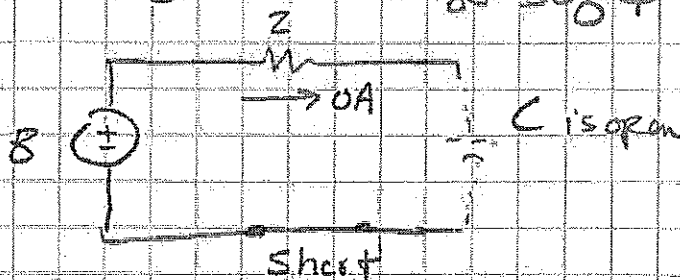
$$K_2 = -2.1$$

$$V(t) = -2.1 e^{-0.5t} \sin 1.94t$$

Soln3 for $t < 0$ I.C. =

$$V_{2\Omega} = 0, \text{ so } V_C(0) = 8V$$

$$\text{and } i_L(0) = 0 \text{ for } t > 0$$



$$-12 + 2i + V_C + V_C(0) + \frac{di}{dt} = 0$$

notice $i = C \frac{dV_C}{dt} \rightarrow \frac{di}{dt} = C \frac{d^2 V_C}{dt^2}$

$$-12 + 2C \frac{dV_C}{dt} + V_C + C \frac{d^2 V_C}{dt^2} = 0$$

$$-12 + \frac{2}{5} \frac{dV_C}{dt} + V_C + \frac{1}{5} \frac{d^2 V_C}{dt^2} = 0$$

$$\frac{d^2 V_C}{dt^2} + 2 \frac{dV_C}{dt} + 5V_C = 60 \rightarrow V_C(t) = V_{ch} + V_{cf}$$

$$V_{cf} = A, \quad 0 + 0 + 5A = 60 \quad \boxed{A = 12} \quad \text{so } V_{cf} = 12$$

$$s^2 + 2s + 5 = 0 \rightarrow s_1 = -1 + j2, \quad s_2 = -1 - j2$$

$$V_{ch} = e^{-t} (K_1 \cos 2t + K_2 \sin 2t)$$

$$V_C(t) = e^{-t} (K_1 \cos 2t + K_2 \sin 2t) + 12$$

$$V_C(0) = 8 = e^0 (K_1 + 0) + 12 \quad 8 = K_1 + 12, \quad K_1 = -4$$

from I.C.

$$i_C(0) = i_L(0) = 0 \quad i(0) = C \frac{dV_C(0)}{dt} = 0$$

$$\frac{dV_C(0)}{dt} = 0 \quad \frac{dV_C}{dt} = -(K_1 \cos 2t + K_2 \sin 2t) e^{-t}$$

$$+ e^{-t} (-K_1 2 \sin 2t + 2K_2 \cos 2t), \quad \frac{dV_C(0)}{dt} = -K_1 + 2K_2 = 0$$

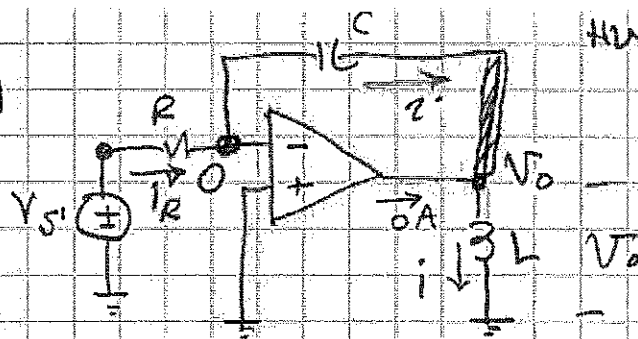
$$-(-4) + 2K_2 = 0, \quad K_2 = -2$$

$$\boxed{V_C(t) = -e^{-t} (4 \cos 2t + 2 \sin 2t) + 12}$$

Soln.

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KCL @ (-) input of Amp = $i_R = i$

$$\frac{V_s - 0}{R} = C \frac{d(0 - V_o)}{dt} \Rightarrow \frac{V_s}{R} = -C \frac{dV_o}{dt}$$

V_o is across the inductor L ,

$$V_o = L \frac{di}{dt} \Rightarrow$$

$$\frac{V_s}{R} = -C \frac{d}{dt} \left[\frac{dV_o}{dt} \right] L = -C \frac{d^2 V_o}{dt^2} L$$

$$\boxed{\frac{d^2 V_o}{dt^2} = -\frac{V_s}{RCL}}$$

Solution 1.

- (a) $4 \sin(\omega t - 30^\circ) = 4 \cos(\omega t - 30^\circ - 90^\circ) = \underline{4 \cos(\omega t - 120^\circ)}$
 (b) $-2 \sin(6t) = \underline{2 \cos(6t + 90^\circ)}$
 (c) $-10 \sin(\omega t + 20^\circ) = 10 \cos(\omega t + 20^\circ + 90^\circ) = \underline{10 \cos(\omega t + 110^\circ)}$

Solution2.

- (a) $v(t) = 10 \cos(4t - 60^\circ)$
 $i(t) = 4 \sin(4t + 50^\circ) = 4 \cos(4t + 50^\circ - 90^\circ) = 4 \cos(4t - 40^\circ)$

Thus, $i(t)$ leads $v(t)$ by 20° .

- (b) $v_1(t) = 4 \cos(377t + 10^\circ)$
 $v_2(t) = -20 \cos(377t) = 20 \cos(377t + 180^\circ)$

Thus, $v_2(t)$ leads $v_1(t)$ by 170° .

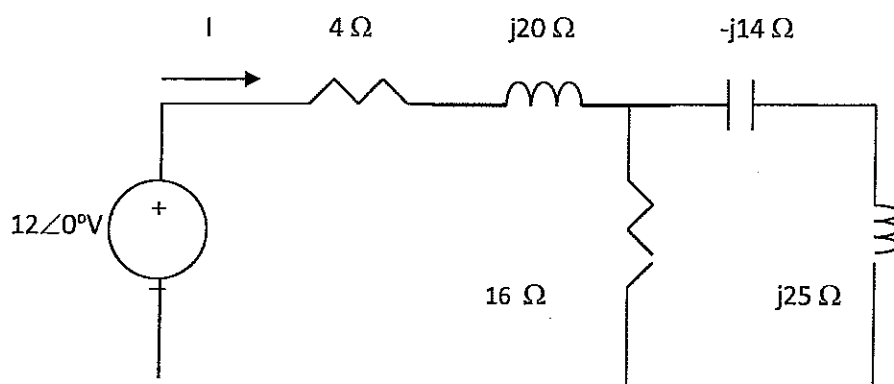
- (c) $x(t) = 13 \cos(2t) + 5 \sin(2t) = 13 \cos(2t) + 5 \cos(2t - 90^\circ)$
 $\mathbf{X} = 13\angle 0^\circ + 5\angle -90^\circ = 13 - j5 = 13.928\angle -21.04^\circ$

$$x(t) = 13.928 \cos(2t - 21.04^\circ)$$

$$y(t) = 15 \cos(2t - 11.8^\circ)$$

$$\text{phase difference} = -11.8^\circ + 21.04^\circ = 9.24^\circ$$

Thus, $y(t)$ leads $x(t)$ by 9.24° .

Solution3.

Solution 3 continued

$$Z_{eq} = 4 + j20 + 10 // (-j14 + j25) = \underline{9.135 + j27.47 \Omega}$$

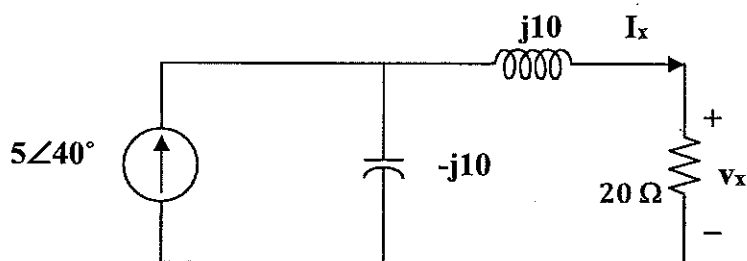
$$I = \frac{V}{Z_{eq}} = \frac{12}{9.135 + j27.47} = 0.4145 \angle -71.605^\circ$$

$$i(t) = \underline{0.4145 \cos(10t - 71.605^\circ) \text{ A}} = \underline{414.5 \cos(10t - 71.6^\circ) \text{ mA}}$$

Solution 4.

Since $\omega = 100$, the inductor = $j100 \times 0.1 = j10 \Omega$ and the capacitor = $1/(j100 \times 10^{-3})$

= $-j10 \Omega$.



Using the current dividing rule:

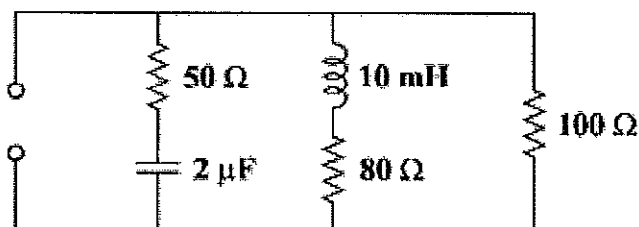
$$I_x = \frac{-j10}{-j10 + 20 + j10} 5 \angle 40^\circ = -j2.5 \angle 40^\circ = 2.5 \angle -50^\circ$$

$$V_x = 20 I_x = 50 \angle -50^\circ$$

$$v_x = \underline{50 \cos(100t - 50^\circ) \text{ V}}$$

Solution 5.

$$\omega = 2\pi f = 2(3.14)2000 = 12560$$



$$Z_1 = 50 + \frac{1}{j\omega C} = 50 + \frac{-j}{(2\pi)(2 \times 10^3)(2 \times 10^{-6})}$$

Solution 5 continued

$$Z_1 = 50 - j39.79$$

$$Z_2 = 80 + j\omega L = 80 + j(2\pi)(2 \times 10^3)(10 \times 10^{-3})$$

$$Z_2 = 80 + j125.66$$

$$Z_3 = 100$$

$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}$$

$$\frac{1}{Z} = \frac{1}{100} + \frac{1}{50 - j39.79} + \frac{1}{80 + j125.66}$$

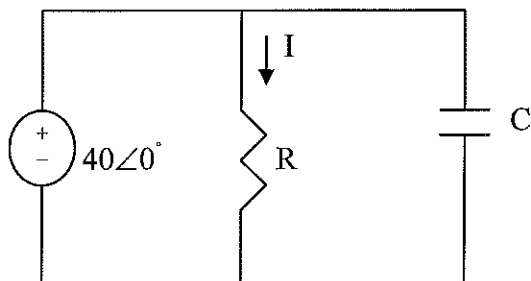
$$\frac{1}{Z} = 10^{-3} (10 + 12.24 + j9.745 + 3.605 - j5.663)$$

$$= (25.85 + j4.082) \times 10^{-3}$$

$$= 26.17 \times 10^{-3} \angle 8.97^\circ$$

$$Z = \underline{\underline{38.21 \angle -8.97^\circ \Omega}}$$

Solution1.



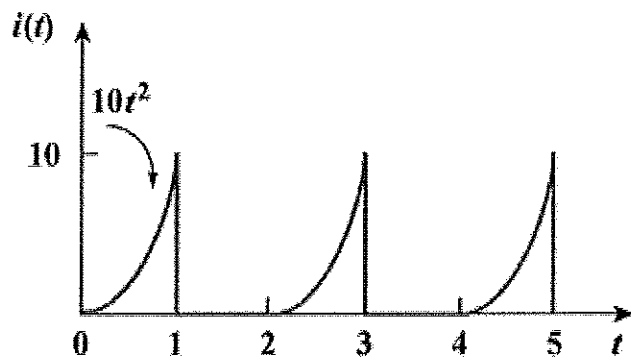
$$90 \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j90 \times 10^{-6} \times 2 \times 10^3} = -j5.5556$$

$$I = 40/60 = 0.6667 \text{ A} \text{ or } I_{\text{rms}} = 0.6667/1.4142 = 0.4714 \text{ A}$$

The average power delivered to the load is the same as the average power absorbed by the resistor which is

$$P_{\text{avg}} = |I_{\text{rms}}|^2 60 = \underline{\underline{13.333 \text{ W}}}.$$

Solution2.

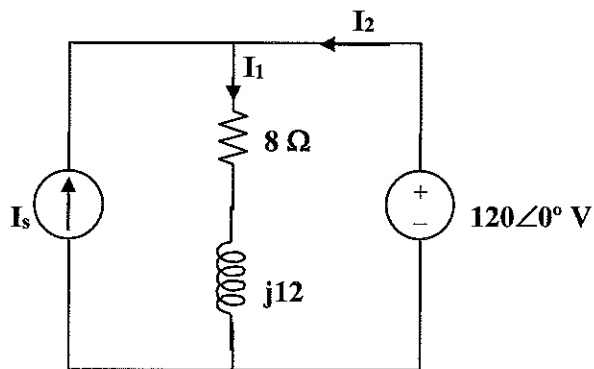


$$I_{\text{rms}}^2 = \frac{1}{2} \left[\int_0^1 (10t^2)^2 dt + \int_1^2 0 dt \right]$$

$$I_{\text{rms}}^2 = 50 \int_0^1 t^4 dt = 50 \cdot \frac{t^5}{5} \Big|_0^1 = 10$$

$$I_{\text{rms}} = \underline{\underline{3.162 \text{ A}}}$$

Solution3.



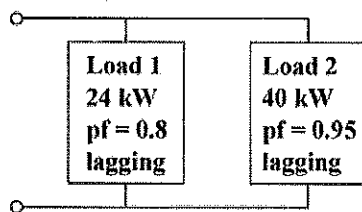
$$I_s + I_2 = I_1 \text{ or } I_s = I_1 - I_2$$

$$I_1 = \frac{120}{8 + j12} = 4.615 - j6.923$$

$$\text{But, } S = VI_2^* \longrightarrow I_2^* = \frac{S}{V} = \frac{2500 - j400}{120} = 20.83 - j3.333$$

$$\text{or } I_2 = 20.83 + j3.333$$

Solution4.



$$(a) \quad \theta_1 = \cos^{-1}(0.8) = 36.87^\circ$$

$$S_1 = \frac{P_1}{\cos\theta_1} = \frac{24}{0.8} = 30 \text{ kVA} \quad Q_1 = S_1 \sin\theta_1 = (30)(0.6) = 18 \text{ kVAR}$$

$$S_1 = 24 + j18 \text{ kVA} \quad \theta_2 = \cos^{-1}(0.95) = 18.19^\circ$$

$$S_2 = \frac{P_2}{\cos\theta_2} = \frac{40}{0.95} = 42.105 \text{ kVA} \quad Q_2 = S_2 \sin\theta_2 = 13.144 \text{ kVAR}$$

$$S_2 = 40 + j13.144 \text{ kVA} \quad S = S_1 + S_2 = 64 + j31.144 \text{ kVA}$$

$$\theta = \tan^{-1}\left(\frac{31.144}{64}\right) = 25.95^\circ \quad \text{pf} = \cos \theta = \underline{0.8992}$$

$$(b) \quad \theta_2 = 25.95^\circ, \quad \theta_1 = 0^\circ$$

$$Q_c = P[\tan \theta_2 - \tan \theta_1] = 64[\tan(25.95^\circ) - 0] = 31.144 \text{ kVAR}$$

$$C = \frac{Q_c}{\omega V_{\text{rms}}^2} = \frac{31,144}{(2\pi)(60)(120)^2} = \underline{5.74 \text{ mF}}$$

