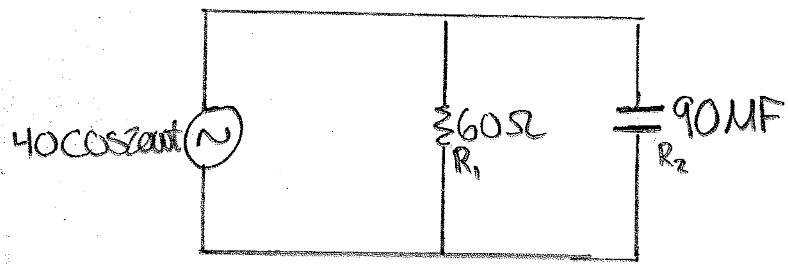
Jared Fowler

Engr M20 - HW#11

1) A load consists of a 60Ω resistor in parrallel with a 90 μF capacitor. If the load is connected to a voltage source $V_s(t) = 40\cos 2000t$, find the average power delivered to the load

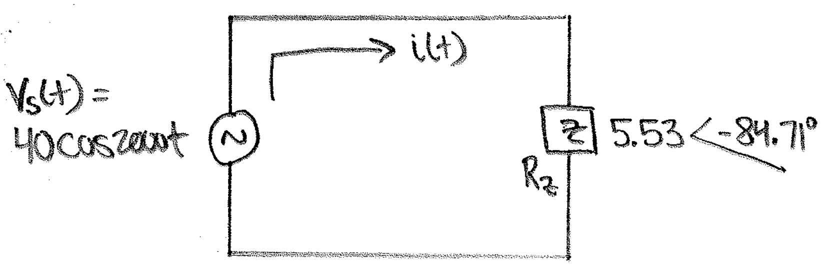


Note: $P_{AVG} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$

$V_m = 40$, $\theta_v = 0°$, $\omega = 2000$

Note: $90\mu F \to -j/\omega c \to -j/(90\mu F)(2000) = -5.556j$

$$R_z = \frac{R_1 R_2}{R_1 + R_2} = \frac{(60\angle 0°)(5.56\angle -90°)}{60 - 5.56j} = \frac{333.36\angle -90°}{60.257\angle -5.29} = 5.53\angle -84.71°$$



$$i(t) = \frac{V(t)}{R_z} = \frac{40\cos 2000t}{5.53\angle -84.71°} = \frac{40\angle 0°}{5.53\angle -84.71°} = 7.23\angle 84.71° = 7.23\cos(2000t + 84.71°)$$

$\to I_m = 7.23$, $\theta_i = 84.71°$

$$\therefore P_{AVG} = \frac{1}{2}(40)(7.23)\cos(0 - 84.71) = \boxed{13.33\ W}$$

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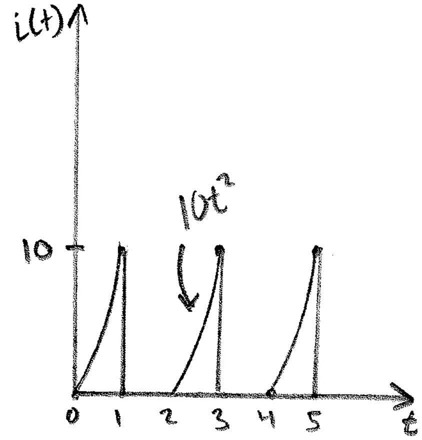
2) Obtain the rms value of the current waveform shown:



Note: $I_{RMS} = \left[\frac{1}{T}\int_0^T i^2(t)\,dt\right]^{\frac{1}{2}}$

Note: $0 < t < 1$, $i(t) = 10t^2$
$1 < t < 2$, $i(t) = 0$

$$I_{RMS} = \left[\frac{1}{2}\int_0^1 (10t^2)^2\,dt + \frac{1}{2}\int_1^2 (0)^2\,dt\right]^{1/2}$$

$$= \left[\frac{1}{2}\int_0^1 100t^4\,dt\right]^{1/2}$$

$$= \left[50\left(\frac{1}{5}t^5\right)\Big|_0^1\right]^{1/2}$$

$$I_{RMS} = [10]^{1/2} = \boxed{3.16\text{ A}}$$

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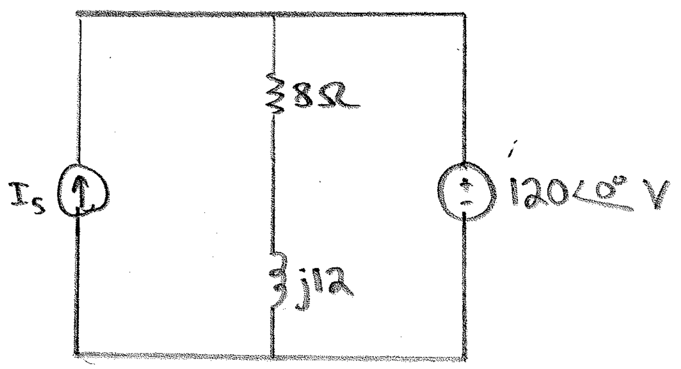
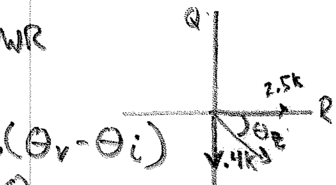
3) Determine I_s in the circuit shown if the voltage source supplies 2.5kW and 0.4 KVAR (leading)



Note: $S = \underbrace{2.5\text{kW}}_{\text{avg. PWR}} - 0.4j\ \text{KVAR}$

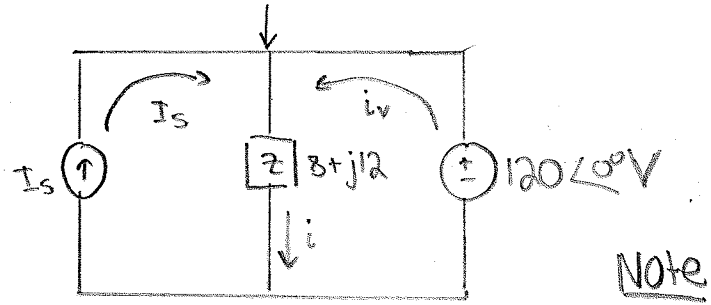


also, $p.f. = \cos(\theta_v - \theta_i)$

$\theta_z = \cancel{\theta_v}^{0°} - \theta_i$

$\theta_i = -\left(\tan^{-1}\left(\frac{.4k}{2.5k}\right)\right) = 9.09°$

$\rightarrow p.f. = \cos(-9.09°)$
$= 0.9874$



Note: KCL: $i = I_s + i_v$

Note: $P = V_{RMS} I_{VRMS}(p.f.) \rightarrow I_{VRMS} = \dfrac{P}{V_{RMS}(p.f.)}$

$V_m = 120 \rightarrow V_{RMS} = \dfrac{120}{\sqrt{2}} = 84.8528\text{ V}$

and $P = P_{AVG} = .2.5\text{kW}$

$\therefore I_{RMS} \rightsquigarrow i_{V_{RMS}} = (2.5K)/((84.8528)(0.9874)) = 29.8388\text{ A}$

Note: $V = IR \rightarrow I = \dfrac{V}{R} \rightarrow i = \dfrac{V}{R_z} = \dfrac{120\angle 0°}{8+j12} = \dfrac{120\angle 0°}{14.42\angle 56.31°} = 8.3205\angle -56.31°$

$i_{RMS} = i_m/\sqrt{2} = \dfrac{8.3205}{\sqrt{2}} = 5.88348\text{ A}$

$\therefore I_s = i - i_v = 5.88348 - 29.8388 = \boxed{-24\text{A}}$

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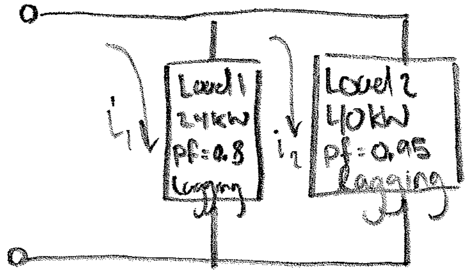
4) A 120-V_{RMS} 60 Hz source supplies two loads connected in parallel, as shown.

a) Find the power factor of the parallel combination.

b) Calculate the value of the capacitance connected in parallel that will raise the power factor to unity.



Note: $V_S = (120)(\sqrt{2}) \angle \theta_V^\circ$

Note: $\omega = (2\pi)(f) = (2\pi)(60)$

Note: Load 1: $S_{L_1} = 24kW + Qj$ Load 2: $S_{L_2} = 40kW + Qj$

$p.f. = .8$ $p.f. = .95$

$\rightarrow .8 = \cos(\theta_z)$ $\rightarrow .95 = \cos(\theta_z)$

$\rightarrow \theta_z = \cos^{-1}(.8)$ $\rightarrow \theta_z = \cos^{-1}(.95)$

$= 36.87^\circ$ $= 18.195^\circ$

$\rightarrow Q = 24k(\tan(+36.87)) = 18000\,VA$ $Q = 40k(\tan(18.195^\circ)) = 13148\,VA$

$\rightarrow S_{L_1} = 24kW + 18000j\,VA$ $\rightarrow S_{L_2} = 40kW + 13148j\,VA$

$\rightarrow S_{L_1} + S_{L_2} = 64\,kW + 31148j\,VAR$

$\therefore \tan^{-1}\left(\frac{31148}{64k}\right) = \theta_z = 25.952^\circ \rightarrow p.f. = \cos(25.952) = \boxed{0.90}$

Note: p.f. to unity $\Rightarrow Q = 0$ in $S = P + Qj$

... So, we need a capacitor which $Q = -31148\,VAR$

$\rightarrow S_c = -31148j\,VAR$, Note: $S = \frac{V_{RMS}^2}{Z^*} = -j\omega C V_{RMS}^2$

$\rightarrow \cancel{-}31148\cancel{j}\,VAR = \cancel{-j}((2\pi)(60))C(120^2)$

$\rightarrow C = \frac{31148}{(2\pi)(60)(120^2)} = \boxed{5.74mF}$