Lab partners: Jared Fowler Team #:

Chikheang Soeng Day of Lab: Thursday

Date: March 7, 2019

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Spherical Concave Mirrors & Thin Lenses

Purpose:

To measure the focal length of converging mirrors and lenses by a variety of techniques.

Required Equipment and Supplies:

A spherical concave mirror, a positive lens of known index of refraction, optical bench or table and accessories, low-power He-Ne laser, mm ruler, small white-light source, optical mounts, rods, rod connectors, pinch clamps, plastic head pin, beam splitter & mounted plane mirror (or a second He-Ne), metric mm ruler, spherometer and flat glass plate, vernier calipers.

Caution:

Even though the lasers used in this lab are low power they can cause retinal damage if the beam enters your eye. Never look directly into the laser beam and be cautious not to allow reflections of the beam from shiny objects to enter your eye.

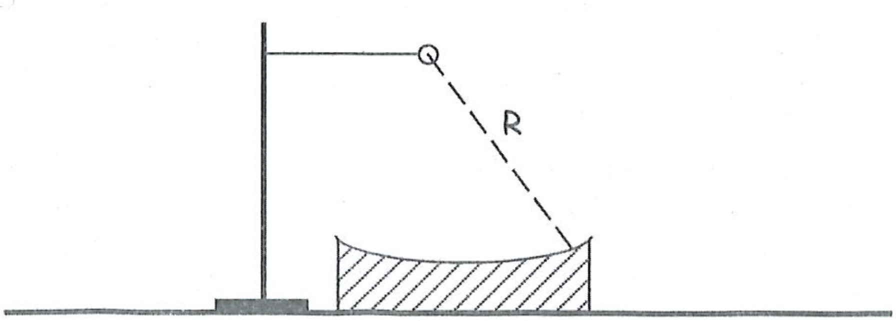
Introduction:

In this experiment we will study the focusing of light using both converging mirrors and lenses. A number of techniques for finding the focal length of mirrors and lenses will be introduced. While performing the experiment you should think about which technique is probably the most accurate (why?) and which is (are) the most convenient for practical use in the lab.

Procedure:

Part A: The focal length of a concave mirror (Important: use the same mirror during this entire portion of the experiment!)

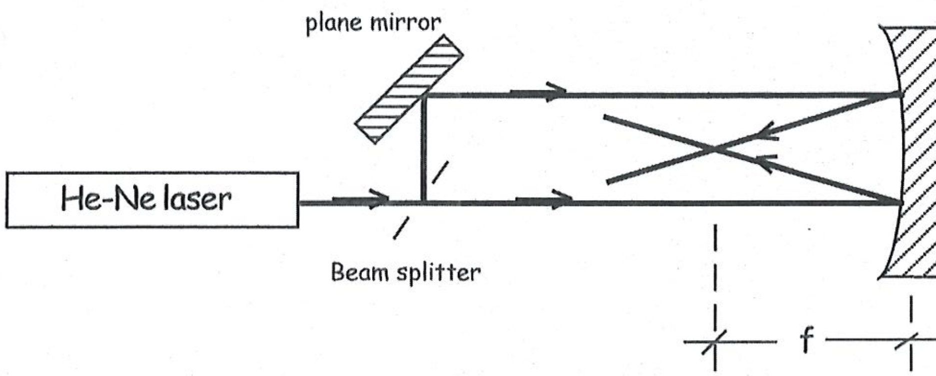
1. Place a concave mirror face up on a table. Attach a small, round marker (a plastic pin head works fine) to a stand so that the marker is suspended directly over the mirror as shown in the figure below.



Adjust the height of the marker until an image of the marker and the actual marker appear the same size when viewed from directly above the mirror. Using a mm ruler, measure the distance from the center of the marker to the surface of the mirror. Be careful not to scratch the mirror's surface! The distance you have measured is the radius of curvature of the mirror, R/2 is the focal length of the mirror. Record your value (+/- estimated uncertaint) in the box below.

$f_{method\ 1}$ = 6 +/- 4 cm

2. Now we will measure the focal length of the same mirror in a different way. Arrange two He-Ne laser beams (using a beam splitter or a second laser) so that they direct parallel beams onto the mirror's surface as shown below.



It is very important that you adjust the two beams striking the mirror so that they are very parallel to each other. Measuring the beam's separation with a mm ruler at several points along their path will help you with this adjustment. Carefully measure the distance from the point at which the beams cross to the mirror surface. (A small piece of paper taped onto a non-shiny rod or pencil will act as a good target to help you identify the crossing point.) Record this value in the box below.

$f_{method\ 2}$ = 19.5 +/- 0.5 cm

3. Place a lighted object box at a distance about 1.5 to 3 times the focal length away from the mirror. Move a frosted glass plate in front of the mirror until the image of the object box is clearly seen. Measure s and s' and use the formula,

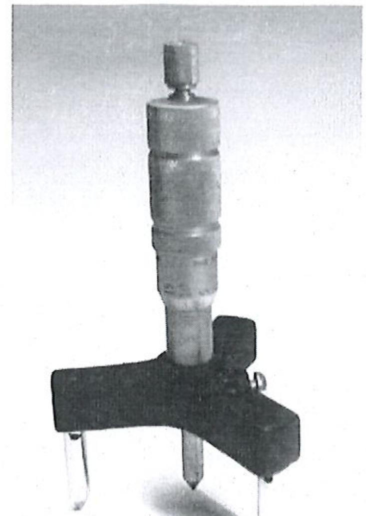
$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

s = object distance (cm) 65   s' = image distance (cm) 32

to find the focal length of the mirror. Record this value in the box below.

$f_{method\ 3}$ = 21 +/- 2 cm

4. For the last technique we will use an instrument called a spherometer to accurately measure the radius of curvature of the concave mirror. The spherometer is comprised of three fixed legs 120o apart and a central, movable leg with micrometer thread. Initially the spherometer should be placed on a known flat piece of glass to make sure that it is properly zeroed (check with the professor if you are unsure how to do this). Carefully place the spherometer onto the mirror surface (avoid scratching the mirror) and using the small clutch knob advance the movable leg until it just touches the surface. Read the value (in mm) from the micrometer barrel and record it in the space below.

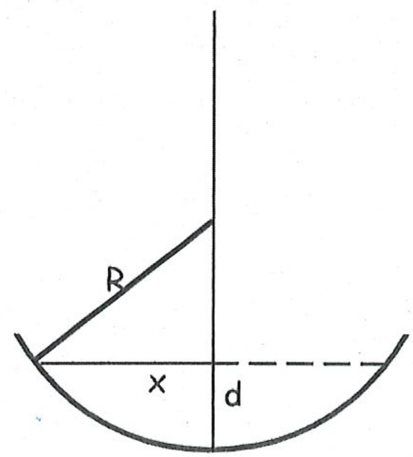


d = 0.55 mm $\delta d = 0.05$

Now use the formula,

$$R = (x^2 + d^2)/2d = 2f$$

where x is the distance from the movable leg to a fix leg on the spherometer (either 33.0 mm or $\boxed{22.5\text{ mm}}$ in our adjustable model) and d is the distance the micrometer is advanced below the plane of the fixed legs when it is in contact with the mirror surface.



The radius and then thefocal length (0.500 R) of the concave spherical mirror can be found. Enter your result in the box below.

$f_{method\ 4}$ = 23 +/- 2 cm

Summarize your result in the table below. Give the % difference assuming that the spherometer technique is the "accepted" value.

Method	Result	% difference
focal length by parallax method	f_1 = 6 cm	74 %
focal length by laser ray tracing	f_2 = 19.5 cm	16 %
focal length by image, object distance	f_3 = 21 cm	9 %
focal length by spherometer	f_4 = 23 cm	

Things to think about: Which method is the most accurate; which is the most convenient? Can any of these methods be used to measure the focal length of a convex (diverging) mirror?

Discussion Part A

In the box below discuss the relative merits (or demerits) of the four techniques for finding the focal length of the mirror. ***Things to think about: Which method is the most accurate; which is the most convenient? Were the differences between the results obtained by each method larger or smaller than the estimated uncertainty? Can any of these methods be used to measure the focal length of a convex (diverging) mirror?***

Technique 4 - And General Note

The "accepted" value for the focal length (23cm) is assumed to be the value measured by the spherometer. While the instrument can be very precise, a 10% uncertainty was determined after comparing several readings and error propogation. Because the margin of error for the "accepted" value is large, it's no suprise that the percent discrepancies for most other techniques are greater than their respective margins of error. This is a quick and convenient technique which can be used for concave & convex mirrors alike.

Technique 1

A "quick and dirty" method, easy to execute but yielded terrible results. The focal length was found to be 6cm with a 67% uncertainty. The percent difference from the theoretical value is 74% which is not within the error margin. This method could also be used for convex mirrors, the only difference being that the image would be upright and virtual.

Technique 2

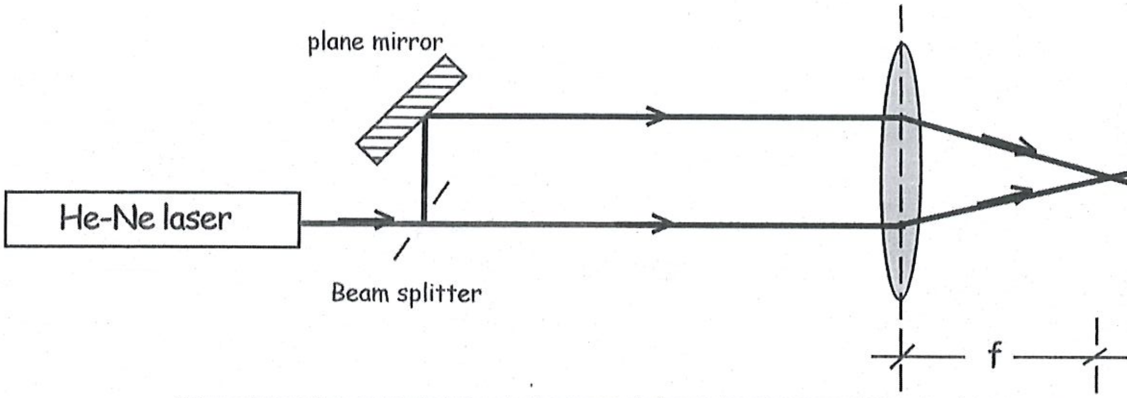
This has the potential to be very accurate, however, it requires the most special equipment and careful setup. Great care should be taken to ensure that the two beams are parallel with each other and with the principal axis. The focal length was found to be 19.5cm with a 3% uncertainty. The percent difference from the theoretical value is 16% which is not within the error margin. This method relies upon the convergence of laser beams and would, therefore, not be suitable for convex mirrors which would diverge the laser beams.

Technique 3

An accurate method, not as precise as technique 2 but requires less equipment. The focal length was found to be 21cm with a 10% uncertainty. The percent difference from the theoretical value is 9% which is within the error margin! This method relies upon the creation of a real image and would, therefore, not be suitable for convex mirrors which only produce virtual images.

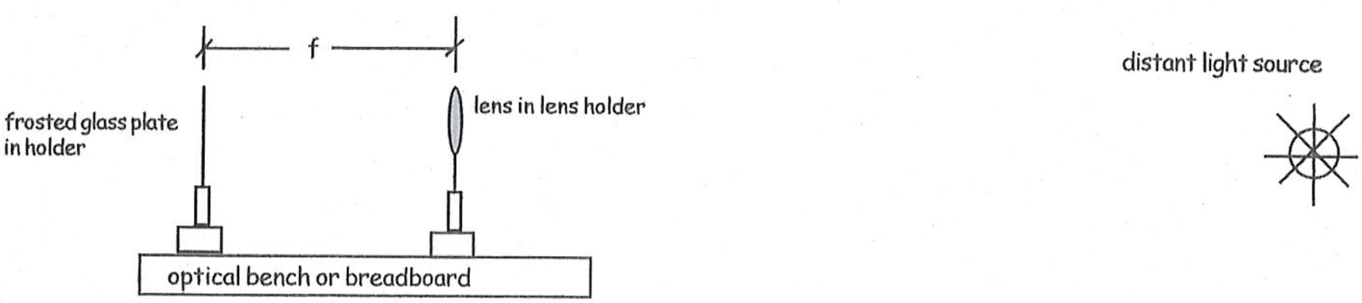
Part B: The focal length of a positive lens (Important: use the same converging lens during this entire portion of the experiment!)

1. We will measure the focal length of a positive lens in a manner similar to the second method we used with the concave mirror. Set up the apparatus as shown in the figure below. For safety, the lens should be mounted in a lens holder and attached to an optical bench. Just like you did previously, align the two laser beams so that their rays are parallel then measure the distance from the center of the lens to the point at which the rays cross. This is the focal length of the lens. Record this value in the box below.



$f_{method\ 1}$ = 26 +/- 0.5 cm

2. Your professor will set up a small, bright light source on one side of the lab. To improve accuracy, set up your apparatus as far away from the light sources as possible. As shown in the figure below, mount a frosted glass plate on the optical bench on the far side of the lens, away from the source.



By the time the rays from the source strike your lens they are almost parallel therefore the image distance is almost the same as the focal length of the lens. Slide the frosted glass plate until a sharp image of the light source is observed. Measure the distance from the center of the lens to the glass plate. This is the focal length (to the thin lens approximation). Enter the result below.

$f_{method\ 2}$ = 21 +/- 0.5 cm

3. In a manner very similar to what you used for the spherical mirror, place the lighted object box at a distance about 1.5 to 3 times the focal length away from the lens. Move a frosted glass plate on the opposite side of the lens until the image of the object box is clearly seen. Measure s and s' (to the center of the lens) and again use the formula,

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

s = object distance (cm) 33   s' = image distance (cm) 46

to find the focal length of the lens. Enter the results in the box below.

$f_{method\ 3}$ = 19 +/- 1 cm

4. ~~For the last technique we will use again use a spherometer to accurately measure the radius of curvature of your lens (see page three of this lab for details). Measure both radii of curvature of the lens with the spherometer and enter in the spaces below:~~

~~R_1 = ________ cm R_2 = ________ cm~~

~~Ask your professor for the index of refraction of the glass used in your lens.~~

~~n_{glass} = ________~~

~~Now use the thin lens lens maker's equation to calculate the focal length and enter your result in the box below.~~

$$\frac{1}{f} = (n_r - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

~~$f_{method\ 4}$ = ________ +/- ________ cm~~

As we did for the concave mirror, summarize you results in a table similar to the one below. Assume the spherometer measurement gives the "accepted" value for focal length.

focal length by laser ray tracing	f_1 = 26 cm	~~0.5~~	%
focal length by distant object	f_2 = 21 cm	~~0.5~~	%
focal length by image, object distance	f_3 = 19 cm	~~1~~	%
~~focal length by spherometer~~	~~f_4 = ____ cm~~		

Discussion Part B

In the box below discuss the relative merits (or demerits) of the four techniques for finding the focal length of the lens. As in Part A, here are a few things to think about: Which method is the most accurate; which is the most convenient? Were the differences between the results obtained by each method larger or smaller than the estimated uncertainty? Can any of these methods be used to measure the focal length of a diverging (negative) lens?

General Note:
Due to the small size of the lens, no focal length "accepted" value could be found withe the spherometer. As such, there are no percent discrepencies discussed for this portion of the experiment.

Technique 1
Like technique 2 for the concave mirror, this similar method has the potential for very accurate results. A fair ammount of effort is needed to properly setup the lasers, ensuring that they are parallel to each other and with the principal axis of the lens. The focal length was found to be 26 cm with a 2% uncertainty. This method relies upon the convergence of laser beams and would, therefore, not be suitable for diverginging lenses which would diverge the beams.

Technique 2
A relatively easy experiment to setup with minimal equipment and decent results. This method was overall the most convenient of those done for the converging lens. The light rays reaching the lens were assumed "close" to parallel, but not precisely. Perhaps the experiment could be more accurate if the sun were used as the distant light source instead. The focal length was found to be 21cm with a 3% uncertainty. This method relies upon the convergence of parrallel light rays and would, therefore, not be suitable for diverging lenses which would diverge the rays.

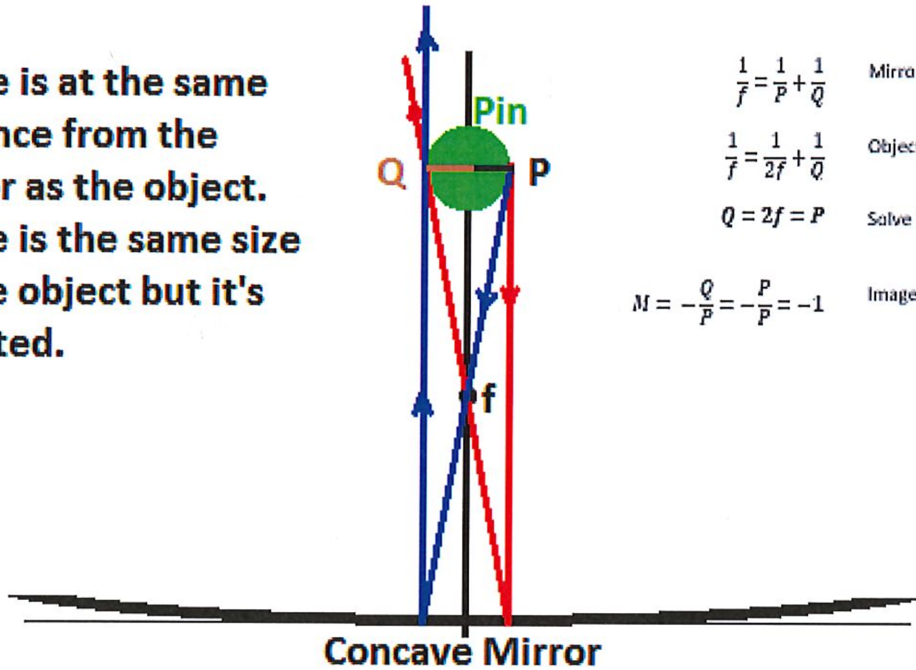
Technique 3
An accurate method, not as precise as technique 1 but requires less special equipment. The focal length was found to be 19cm with a 6% uncertainty. This method relies upon the creation of a real image and would, therefore, not be suitable for diverging lenses which would produce a virtual image on the same side of the lens as the object.

The End! :-)

Ray Trace - 1A

Stressed-Physicist
Eyeball

Image is at the same distance from the mirror as the object. Image is the same size as the object but it's inverted.



$\frac{1}{f}=\frac{1}{P}+\frac{1}{Q}$	Mirror Equation
$\frac{1}{f}=\frac{1}{2f}+\frac{1}{Q}$	Object placed at center of curvature, or 2f.
$Q=2f=P$	Solve for Image location.
$M=-\frac{Q}{P}=-\frac{P}{P}=-1$	Image is same size but inverted.

Jared Fowler

Spherometer Equation – Error Propagation

$$R = 2f = \frac{x^2 + d^2}{2d}$$ Spherometer Equation

$$\frac{\partial R}{\partial x} = \mathbf{2x}$$ Take partial derivative for x

Quotient Rule:

Take partial derivative for d

$$\left(\frac{f}{g}\right)' = \frac{(f'g - g'f)}{g^2}$$ Use Quotient Rule

$$f = (x^2 + d^2) \rightarrow f' = 2d$$
$$g = d \rightarrow g' = 1$$

$$\frac{\partial R}{\partial d} = \frac{1}{2}\left(\frac{2dd - 1(x^2 + d^2)}{d^2}\right) = \frac{2d^2 - d^2 - x^2}{2d^2} = \frac{\mathbf{d^2 - x^2}}{\mathbf{2d^2}}$$

$$\delta R = \left\{\left[\delta d\left(\frac{\partial R}{\partial d}\right)\right]^2 + \left[\delta x\left(\frac{\partial R}{\partial x}\right)\right]^2\right\}^{\frac{1}{2}}$$ Definition of absolute error

$$\frac{\delta R}{R} = \left\{\left[\frac{\delta d\left(\frac{\partial R}{\partial d}\right)}{R}\right]^2 + \left[\frac{\delta x\left(\frac{\partial R}{\partial x}\right)}{R}\right]^2\right\}^{\frac{1}{2}}$$ Definition of relative error

$$\frac{\delta R}{R} = \left\{\left[\frac{\delta d\left(\frac{d^2 - x^2}{2d^2}\right)}{\frac{x^2 + d^2}{2d}}\right]^2 + \left[\frac{\delta x(2x)}{\frac{x^2 + d^2}{2d}}\right]^2\right\}^{\frac{1}{2}}$$ Substitute

$$\frac{\boldsymbol{\delta R}}{\boldsymbol{R}} = \left\{\left[\frac{\boldsymbol{\delta d(d^2 - x^2)}}{\boldsymbol{d(x^2 + d^2)}}\right]^2 + \left[\frac{\boldsymbol{\delta x(4dx)}}{\boldsymbol{x^2 + d^2}}\right]^2\right\}^{\frac{1}{2}}$$

Simplify
Absolute error for d is taken as the max delta for several measurements. (0.05mm)
Treating absolute error of x as 0.

$$\frac{\delta R}{R} = \left\{\left[\frac{(0.05mm)((0.55mm)^2 - (22.5mm)^2)}{(0.55mm)((22.5mm)^2 + (0.55mm)^2)}\right]^2\right\}^{\frac{1}{2}} = 0.091 \rightarrow \mathbf{10\%}$$ Solve

Conclusion

The experiments, with the exception of A1, yielded results which were "within the ballpark", or general scope of agreement, with each other. The focal length of the concave mirror was found to be between 19 to 25 centimeters, and the focal length of the converging lens was found to be between 18 to 26 centimeters.

Type	#	$Focal\ Length\ (cm)$	$\%\ Uncertainty$	$\%\ Difference$	Within Error Margin
Concave Mirror	A1	6	67%	74%	NO
	A2	19.5	3%	16%	NO
	A3	21	10%	9%	YES
	A4	23	10%	X	X
Converging Lens	B1	26	2%	X	X
	B2	21	3%	X	X
	B3	19	6%	X	X
	B4	X	X	X	X

The experiments with the greatest potential for accuracy also required the greatest attention to setup and precision. Referring to A2 and B1, great care was needed to ensure that the laser beams were parallel with each other as well as with the principal axis of the mirror/lens. Any deviation was a source of error. Other sources of error included misperception of when images came into best focus (methods A3, B2, B3) and misperception while comparing object and image size (method A1). These errors were accounted for by taking the delta of the maximum and minimum lengths at which the perceptual changes were barely noticeable.

As discussed in more detail in "Discussion Part A", the "accepted" value for the focal length of the concave mirror, obtained in A4, had a 10% uncertainty. As such, the percent differences should only be regarded as percent difference with respect to technique A4 and not to an exact value. The experiment could be improved by having an exact accepted value provided for both the mirror and lens.

Technique B2 could be improved by using the Sun as the distant light source instead of a light bulb in order to ensure that the instance rays are parallel with the lens. (This would essentially be the same concept as a focused beam from a magnifying glass used by kids to burn leaves and ants.)

Jared Fowler

Spherical Concave Mirrors and Lenses – Pre-lab Work

Prove the focal length by spherometer formula: $R = \frac{x^2+d^2}{2d} = 2f$

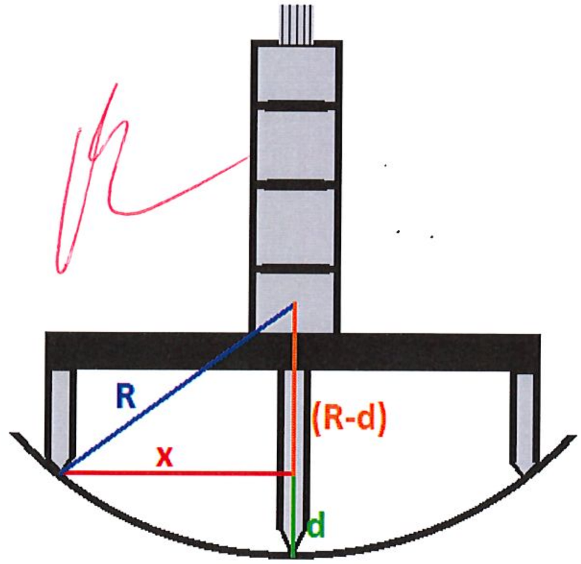
Use Pythagorean Theorem

$$R^2 = x^2 + (R-d)^2$$

$$R^2 = x^2 + R^2 - 2dR + d^2$$

$$2dR = x^2 + d^2$$

$$\boldsymbol{R = \frac{x^2 + d^2}{2d}}$$



Consider a ray of light (red), parallel to the principle axis, incident on a spherical mirror. Based upon the law of reflection $A = A = B = B'$. By the law of corresponding angles angle PCD is equal to B and $E = 2B$.

$$\frac{h}{R} = sinB$$

$$\frac{h}{f} = tanE = tan2B$$

If the aperture of the mirror is small, then

$$sin\theta \approx tan\theta \approx \theta$$

$$\frac{h}{R} = \frac{h}{2F} \rightarrow \boldsymbol{R = 2F}$$