98/100

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total deductions -6
bonus +4

Lab 1: Voltage and Current Division

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Objective

Understand, and put into practice, voltage and current division concepts as well as the principles associated with the Wheatstone Bridge method for measuring resistance.

Theory

Note: Theories, concepts, and proofs heavily quoted from "Fundamentals of Electric Circuits" 5th edition.

Ohm's Law

The voltage *v* across a resistor is directly proportional to the current *i* flowing through the resistor. The constant of proportionality is defined as the resistance, *R*. Therefore:

$$v = iR$$

Kirchhoff's Current Law

The algebraic sum of currents entering a node (or a closed boundary) is zero. In other words, the sum of currents entering a node is equal to the sum of currents leaving a node.

Assume a set of currents $i_k(t), k = 1, 2, \ldots,$ flow into a node

$$i_T(t) = i_1(t) + i_2(t) + i_3(t) + \ldots \quad \textit{Algebraic sum of currents}$$

$$q_T(t) = q_1(t) + q_2(t) + q_3(t) + \cdots \quad \text{Integrate both sides.}$$

$$\text{Note: } q_k(t) = \int i_k(t)dt \text{ and } q_T(t) = \int i_T(t)dt$$

$$q_T(t) = 0 \rightarrow i_T(t) = 0 \quad \textit{Law of conservation of electric charge}$$

Kirchhoff's Voltage Law

The algebraic sum of all voltages around a closed path (or loop) is zero. In other words, the sum of voltage drops is equal to the sum of voltage rises in a closed path. This law is based off, and proven by, the conservation of energy.

Voltage Division

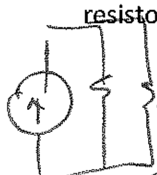
Voltage can be "divided" by placing resistors in series. The equivalent resistance of any number of resistors connected in series is the sum of the individual resistances.

$$R_{eq} = R_1 + R_2 + R_3 + \cdots + R_N$$

Based upon KCL, the current running through each of these resistors is equal. Applying Ohm's law, the resistance is directly proportional to the voltage across each resistor, hence, "dividing" the voltage.

Current Division

Current can be "divided" by placing resistors in parallel. The equivalent resistance of any number of resistors in parallel is the sum of the individual conductances.



show $i_2 = i \frac{R_1}{R_1+R_2}$

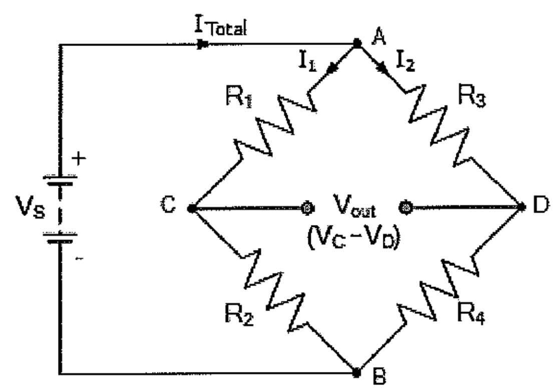
$$R_{eq} = \left(\left(\frac{1}{R_1}\right) + \left(\frac{1}{R_2}\right) + \left(\frac{1}{R_3}\right) + \cdots + \left(\frac{1}{R_N}\right)\right)^{-1}$$

-3

Based upon KVL, the voltage across each resistor is the same. Applying Ohm's law, each resistor's resistance is in inverse proportion to the current running through it, hence, "dividing" the current.

Wheatstone Bridge

Used to analyze two series strings in parallel. For the purpose of this experiment, by making the current through each series the same, the resistance of R_4 can be determined by adjusting the resistance of R_2 until V_{out} reads zero. (see Calculation 1.2)



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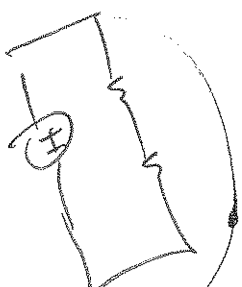

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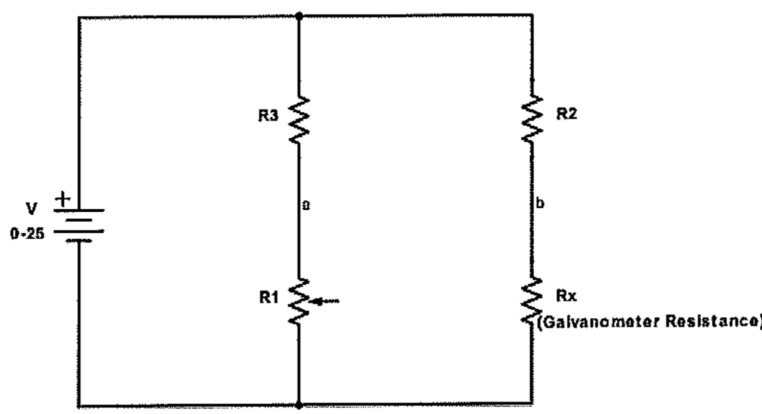
Procedure

Part 1:

A bridge circuit was created to determine the value of an unknown resistor, R_x. (see Figure 1.1)

Figure 1.1



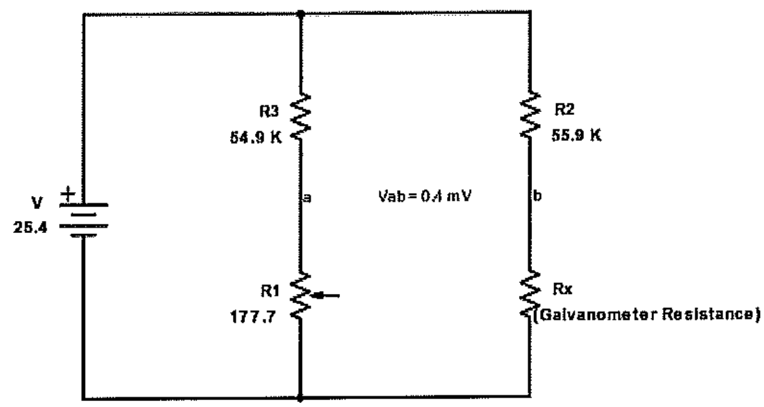
The current through R_x is limited to 0.5mA when the power supply is set at 25V. By limiting the current flowing through R_1 to also be 0.5mA, the value of R_x can be found by adjusting the variable resistance of R_1 until the voltage of V_{ab} = 0V.

Based upon KCL, the current flowing through R_3 and R_2 must also be limited to 0.5mA. Using Ohm's law, appropriate values for these two resistors were found to be greater than or equal to **50KΩ** (see Calculation 1.1).

The expression for resistance R_x was derived in terms of R_1, R_2, R_3 (see Calculation 1.2).

The circuit (see Figure 1.2) was built and V_{ab} was monitored while R_1 was adjusted. Once V_{ab} was approximately zero, the resistance of R_1 was measured to be **177.7Ω**. Using the expression derived in Calculation 1.2, the value of R_x was calculated to be **180.9Ω** (see Calculation 1.3).

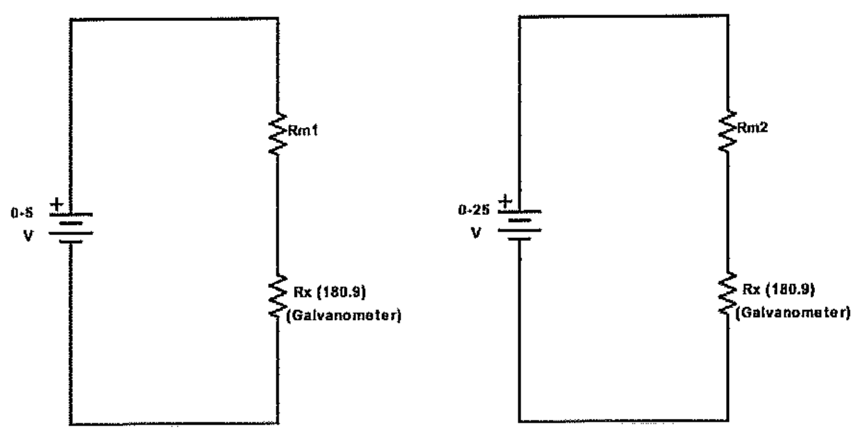
Figure 1.2



Part 2:

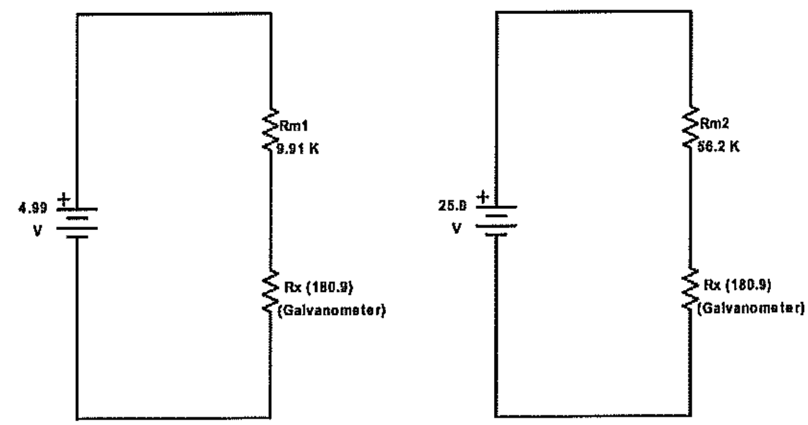
Two voltmeters were created, one having a 0-5 volt range and the other a 0-25 volt range, using the galvanometer from part 1 (see Figure 2.1).

Figure 2.1



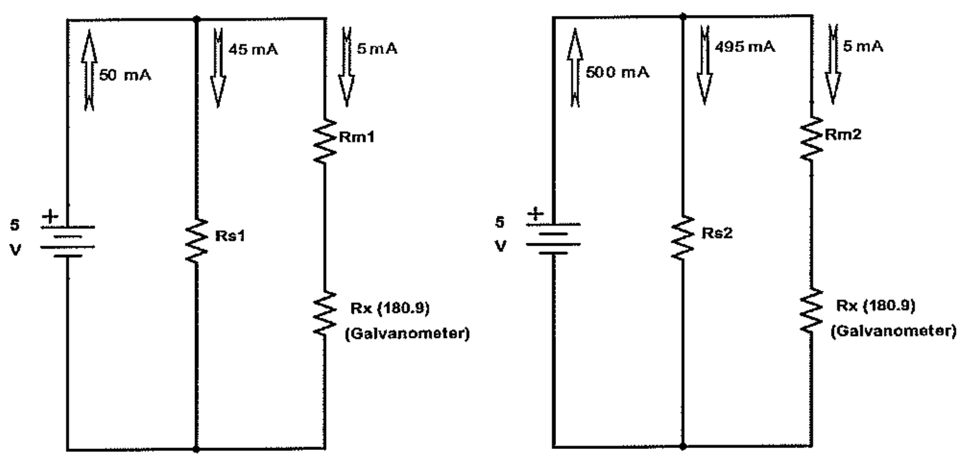
Given the current restriction of 0.5mA that can flow through the galvanometer, KVL and Ohm's laws were used to find appropriate minimum resistance values R_{m1} and R_{m2} which are **9819Ω** and **49819Ω** respectively (see Calculation 2.1). The circuits were then built as seen below (see Figure 2.2).

Figure 2.2



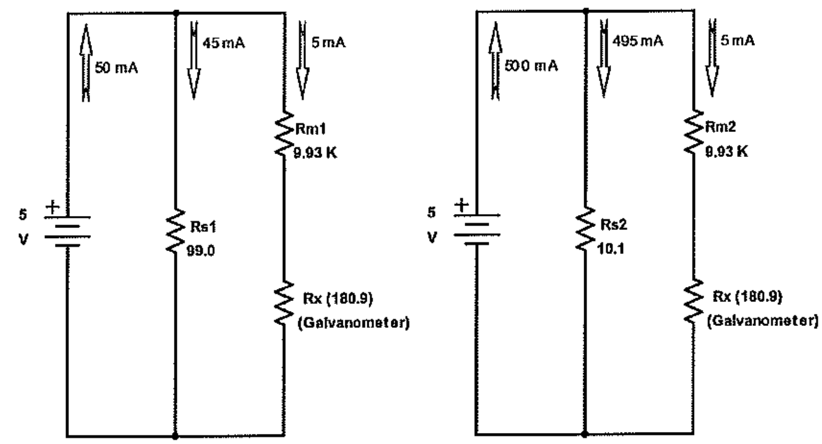
Two ammeters were created, one having a 0-50 mA range the other a 0-500 mA range, using the galvanometer from part 1 (see Figure 2.3).

Figure 2.3



Given the current restriction of the galvanometer, KCL and Ohm's law were used to determine the maximum resistance values for R_{s1} and R_{s2} which are **101.01Ω** and **10.101Ω** respectively (see Calculation 2.2). Because R_{m1} and R_{m2} continue to form a closed loop in their respective circuits, KVL applies the same as it did in Calculation 2.1. The circuits were then built as seen below (see Figure 2.4).

Figure 2.4



Part 3:

Given the circuit information below (see Figure 3.1a), the theoretical values for R_{L1}, R_{L2}, R_1, R_2, and R_S were found (see Figure 3.1b and Calculation 3.1). Note that the maximum power output is *assumed* to be 1.5 watts.

Figure 3.1a

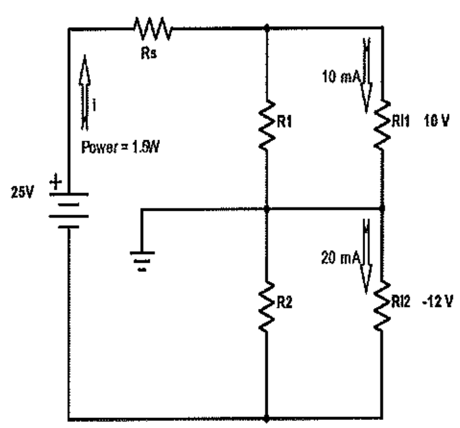
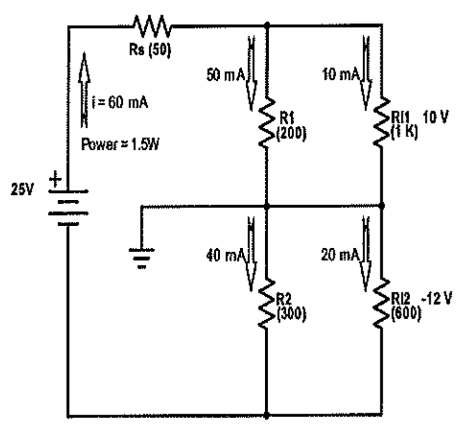


Figure 3.1b



The circuit was built and voltages were measured across R_{L1} and R_{L2} (see Figure 3.2a). These "load" resistors were then removed and voltages were measured across R_1 and R_2 (see Figure 3.2b). The

Voltage Regulation (V.R.) was found to be **12%** across R_1 and **15%** across R_2 (see Calculation 3.2), which are both within the requirement of V.R. < 20%.

Figure 3.2a

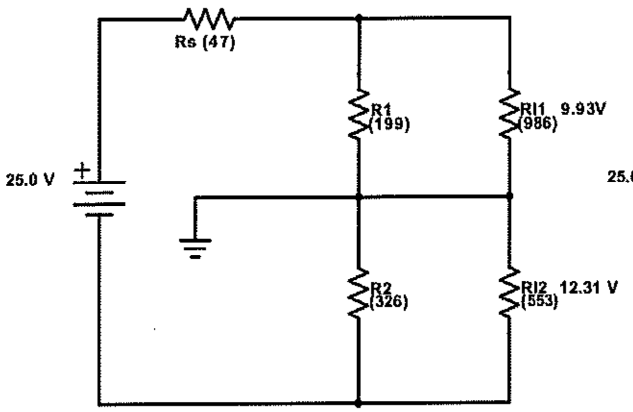
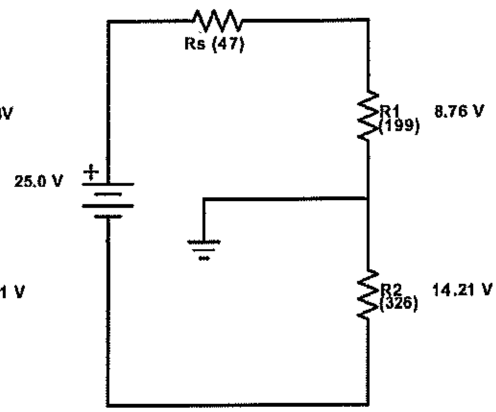


Figure 3.2b



Data & Calculations

Note:

For convenience, variables V (voltage), R (resistance), and I (current) will be subscripted based upon subscriptions in their respective diagrams. For example, the current across resistor R_3 will be represented as i_3, and the voltage across R_3 will be represented as V_3.

Calculation 1.1

$$V = iR \text{ and } i_3 < 0.5\ mA \text{ and } i_2 < 0.5\ mA$$

Note that the maximum voltage across R_3 is 25V. While it may never reach this value, setting V_3 to 25V will set R_3 and R_2 to higher, safer values.

$$\text{Let } V_3 = 25V \to 25 = 0.5mA(R_3)$$

$$R_3 \geq \frac{25}{0.0005} = \mathbf{50Kohms}$$

Repeat for R_2 to also get $R_2 \geq \mathbf{50Kohms}$

Calculation 1.2

$$25V = V_3 + V_1 \text{ and } 25V = V_2 + V_x \quad \text{Using KVL (Kirchhoff's Voltage Law)}$$

$$25V = V_3 + V_1 \text{ and } 25V = V_2 + V_x$$

$$V_3 + V_1 = V_2 + V_x \quad \text{Use Ohm's Law}$$

$$i_3R_3 + i_1R_1 = i_2R_2 + i_xR_x \quad \text{Note that } i_3 = i_1 \text{ and } i_2 = i_x$$

$$i_3(R_3 + R_1) = i_2(R_2 + R_x)$$

$$R_x = \frac{i_3(R_3 + R_1)}{i_2} - R_2 \quad \text{Use Ohm's Law}$$

$$R_x = \frac{\left(\frac{V_3}{R_3}\right)(R_3 + R_1)}{\left(\frac{V_2}{R_2}\right)} - R_2 \quad \text{Note that } V_2 = V_3$$

$$R_x = \frac{(R_2R_3 + R_2R_1)}{R_3} - \frac{R_2R_1}{R_3}$$

$$R_x = \frac{R_2R_1}{R_3} \quad \textbf{Formula 1.2.1}$$

this should be in ^

Calculation 1.3

$$R_x = \frac{R_2R_1}{R_3} = \frac{(55.9K)(177.7)}{54.9K} = \mathbf{180.9\ ohms} \quad \text{Using formula 1.2.1}$$

Calculation 2.1

$$V = V_x + V_m$$ Using KVL (Kirchhoff's Voltage Law)

$$V = R_x i_x + R_m i_m$$ Use Ohm's Law. Note that $i_m = i_x$

$$\mathbf{R_m = \frac{(V - (R_x i_x))}{i_m}}$$ **Formula 2.1.1**

$$R_m = \frac{(5 - (180.9)(0.5mA))}{0.5mA} = \mathbf{9819\ ohms}$$ Apply formula 2.1.1, with V = 5 Volts

$$R_m = \frac{(25 - (180.9)(0.5mA))}{0.5mA} = \mathbf{49819\ ohms}$$ Apply formula 2.1.1, with V = 25 Volts

Calculation 2.2

$$Note: V_s = 5V, i_m = 0.5mA, i_x = 0.5mA$$

$$V_s = i_s R_s$$ Use Ohm's Law.

$$\mathbf{R_s = \frac{V_s}{i_s}}$$ **Formula 2.2.1, where $i_s = (i - i_x)$**

$$R_{s1} = \frac{V_{s1}}{i_{s1}} = \frac{5}{50mA - .5mA} = \mathbf{101.01\ ohms}$$ Apply formula 2.2.1, with i = 50 mA

$$R_{s2} = \frac{V_{s2}}{i_{s2}} = \frac{5}{500mA - .5mA} = \mathbf{10.101\ ohms}$$ Apply formula 2.2.1, with i = 500 mA

*Note that $R_{m1} = R_{m2}$. KVL applies to the same loop as it did in Calculation 2.1. Therefore, $R_{m1} =$ **9819 ohms***

Calculation 3.1

$$P = Vi$$ Definition of Power

$$1.5 = 25i$$

$$i = 60\ mA$$

$$Note: i_1 = 50mA\ and\ i_2 = 40mA$$ *Via KCL (Kirchhoff's Current Law)*

$$25V = V_s + V_{l1} + V_{l2}$$ Using KVL (Kirchhoff's Voltage Law)

$$V_s = 25 - 10 - 12 = 3V$$

$$V_s = R_s i_s \rightarrow R_s = \frac{V_s}{i_s} = \frac{3}{60mA} = \mathbf{50\ ohms}$$ Use Ohm's Law

$$R_{l1} = \frac{V_{l1}}{i_{l1}} = \frac{10}{10mA} = \mathbf{1K\ ohm}$$

$$R_{l2} = \frac{V_{l2}}{i_{l2}} = \frac{12}{20mA} = \mathbf{600\ ohm}$$

$$R_1 = \frac{V_1}{i_1} = \frac{10}{50mA} = \mathbf{200\ ohm}$$

$$R_2 = \frac{V_2}{i_2} = \frac{12}{40mA} = \mathbf{300\ ohm}$$

Calculation 3.2

Note: Definition of Voltage Regulation (V.R.) as follows:

$$VR = \left(\frac{(V_{oc} - V_l)}{V_l}\right) * 100\%$$

$$\boldsymbol{V.R._1} = \left(\frac{(V_{0c1} - V_{l1})}{V_{l1}}\right) * 100\% == \left(\frac{(8.76 - 9.93)}{9.93}\right) * 100\% = \mathbf{12\%}$$

$$\boldsymbol{V.R._2} = \left(\frac{(V_{0c2} - V_{l2})}{V_{l2}}\right) * 100\% == \left(\frac{(14.21 - 12.31)}{12.31}\right) * 100\% = \mathbf{15\%}$$

Discussion of Results

Part 1:

The experiment went as expected. The resistance of the galvanometer was found by using a Wheatstone bridge circuit. There was a bit of a snag with the galvanometer and potentiometer units. The first galvanometer unit had a very strange and volatile dial reading, and the first two potentiometer units were unable to reduce the read voltage to a near-zero value. After a bit of frustration, both the galvanometer and potentiometer units were replaced with working units.

Part 2:

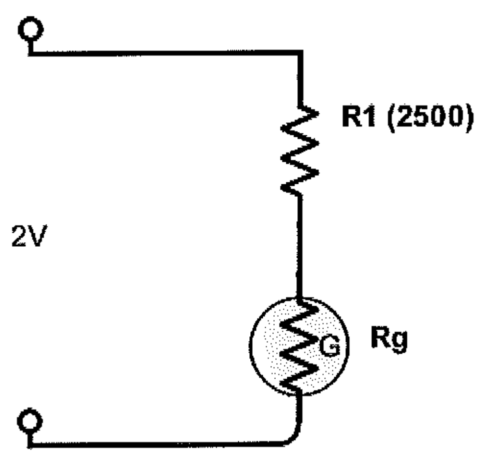
The experiments went as expected. The galvanometer voltmeter and ammeter were successfully created, and by using the selected resistors the galvanometer's dial was successfully limited to its 0.5mA range. This experiment put into practice both voltage division in the Voltmeter scenarios, and current division in the Ammeter scenarios.

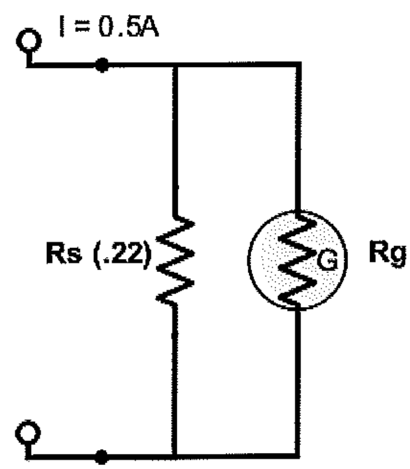
Part 3:

The measured voltage across R_1, R_{l1} was 9.93V, and across R_2, R_{l2} was 12.31V, which are very similar to the theoretical values of 10V and 12V respectively. While the calculated Voltage Regulation values, 12% and 15%, were within the acceptable threshold of 20%, they were far from ideal. It's interesting that the voltage increased across R_2 and decreased across R_1 when the load resistors were removed. Using Ohm's law, the current through Figure 3.2a is found to be 60mA, however, the current through Figure 3.2b is found to be only 44mA. This makes sense, as the DC Voltage source no longer needed to provide as much current to maintain 25 volts. Because the resistance is held at a constant, this implies that the Voltage and Current are proportional to one another, which is confirmed by Ohm's law.

Appendix

Q: A particular galvanometer serves as a 2-V full scale voltmeter when a 2500 ohm is used as a multiplier resistor and it serves a 0.5 A ammeter when a 0.22 ohm shunt resistor is used. Determine the internal resistance of the galvanometer and the current required to produce full scale deflection.





$V_s = V_g \rightarrow .22i_s = R_g i_g$	KVL and Ohm's Law	1.1
$I = i_s + i_g \rightarrow 0.5A = i_s + i_g \rightarrow i_s = .5 - i_g$	KCL	1.2
$2V = V_1 + V_g \rightarrow 2 = 2500i_g + R_g i_g \rightarrow R_g i_g = 2 - 2500i_g$	KVL	1.3
$.22i_s = 2 - 2500i_g$	*Combine* 1.1 *and* 1.3	
$.22(.5 - i_g) = 2 - 2500i_g$	Insert 1.2 for i_s	
$i_g = \mathbf{0.756\ mA}$	Solve for Current i_g	
$(0.756\ mA)R_g = 2 - 2500(0.756\ mA)$	Substitue found i_g *value into* 1.3	
$R_g = \mathbf{145.5\ ohms}$	Solve for Resistance R_g	

To produce full scale deflection, the galvanometer should have an internal resistance of **145.5 ohms** and there should be a current of **0.756 mA.**

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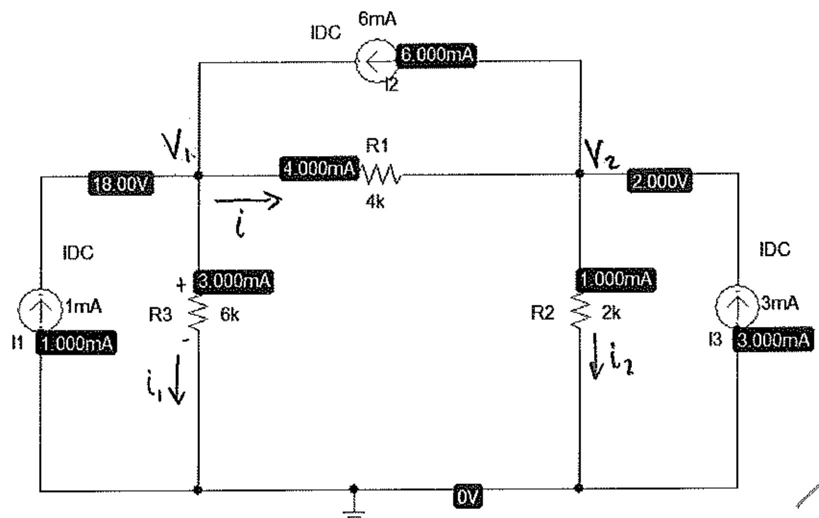
99/100

Lab 2 – PSPICE

P1.



KCL @ V_1:

$$1mA + 6mA = i + i_1$$

$$7mA = \frac{V_1 - V_2}{4K} + \frac{V_1 - 0}{6K}$$

$$7mA = \frac{V_1}{4K} - \frac{V_2}{4K} + \frac{V_1}{6K}$$

$$7mA = \frac{3V_1}{12K} - \frac{3V_2}{12K} + \frac{2V_1}{12K}$$

$$84 = 5V_1 - 3V_2 \quad (1)$$

KCL @ V_2:

$$i + 3mA = 6mA + i_2$$

$$3mA = i - i_2$$

$$3mA = \frac{V_1 - V_2}{4k} - \left(\frac{V_2 - 0}{2k}\right)$$

$$3mA = \frac{V_1}{4K} - \frac{V_2}{4K} - \frac{V_2}{2K}$$

$$3mA = \frac{V_1}{4K} - \frac{V_2}{4K} - \frac{2V_2}{4k}$$

$$12 = V_1 - 3V_2 \quad (2)$$

$$84 = 5(12 + 3V_2) - 3V_2$$

$$84 = 60 + 12V_2$$

$$12V_2 = 24$$

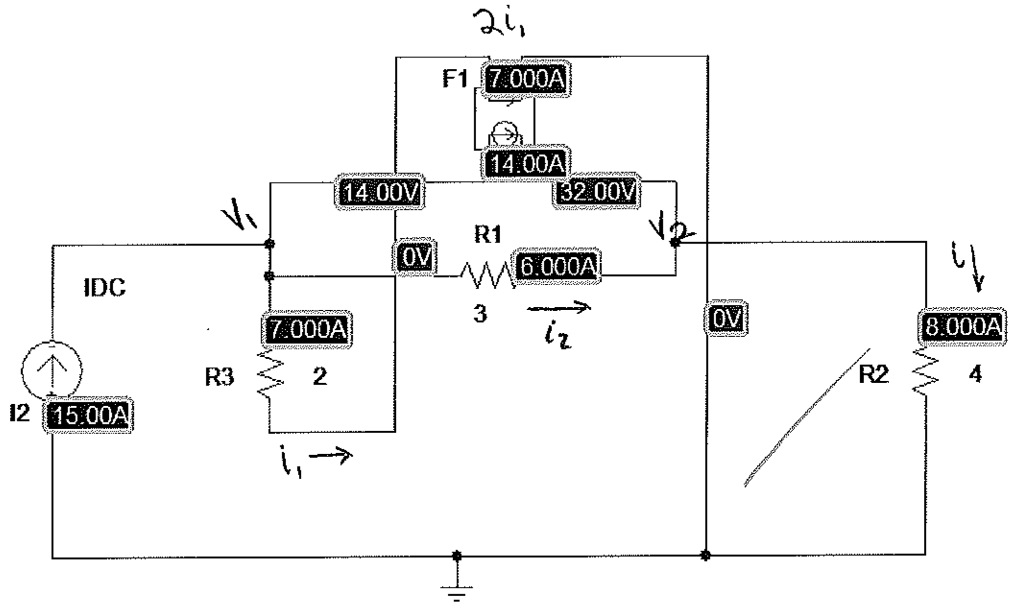
$$\boxed{V_2 = 2V}$$ ✓

$$12 = V_1 - 3(2)$$

$$\boxed{V_1 = 18V}$$ ✓

$$i = \frac{V_1 - V_2}{4K} = \frac{18-2}{4k} = \boxed{4mA}$$ ✓

P2.



KCL @ V_1:

$15A = i_1 + i_2 + 2i_1$

$15A = 3i_1 + i_2$

$15A = 3\left(\frac{V_1 - 0}{2}\right) + \left(\frac{V_1 - V_2}{3}\right)$

$15A = \frac{3V_1}{2} + \frac{V_1}{3} - \frac{V_2}{3}$

$15A = \frac{9V_1}{6} + \frac{2V_1}{6} - \frac{2V_2}{6}$

$90 = 11V_1 - 2V_2 \quad (1)$

KCL @ V_2:

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−1

$i = 2i_1 + i_2$

$\frac{V_2 - 0}{4} = 2\left(\frac{V_1 - 0}{2}\right) + \left(\frac{V_1 - V_2}{3}\right)$

$\frac{V_2}{4} = V_1 + \frac{V_1}{3} - \frac{V_2}{3}$

$3V_2 = 12V_1 + 4V_1 - 4V_2$

$7V_2 = 16V_1$

$V_2 = \frac{16}{7}V_1 \quad (2)$

$90 = 11V_1 - 2\left(\frac{16}{7}V_1\right)$

$90 = 11V_1 - \frac{32}{7}V_1$

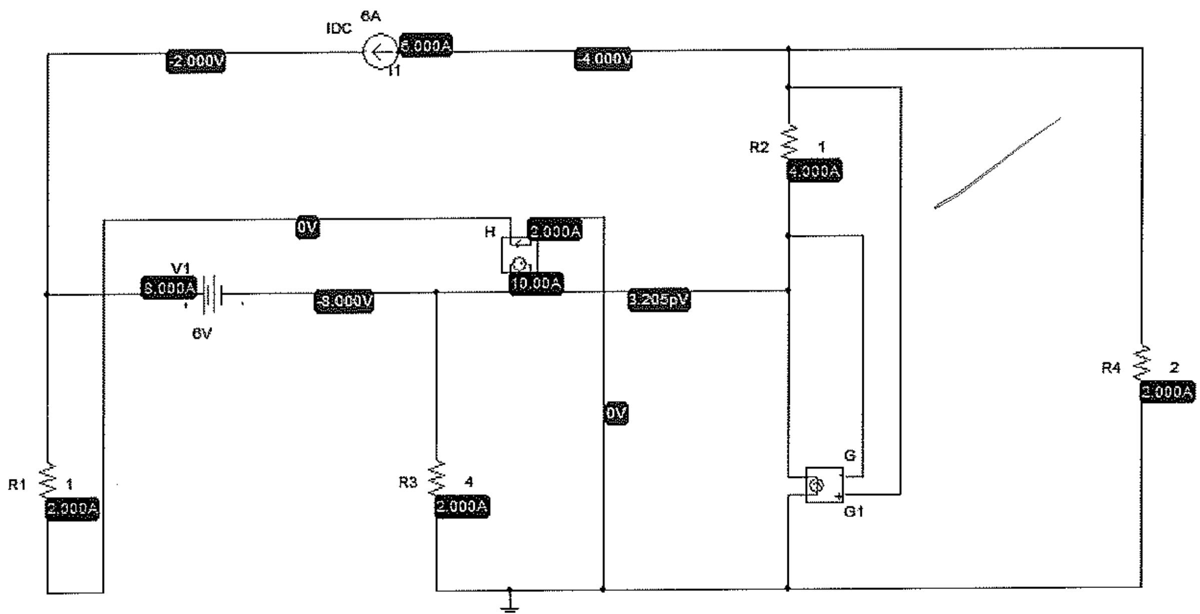
$90 = \frac{45}{7}V_1$

$V_1 = 90 \cdot \left(\frac{7}{45}\right) = \boxed{14V}$ ✓

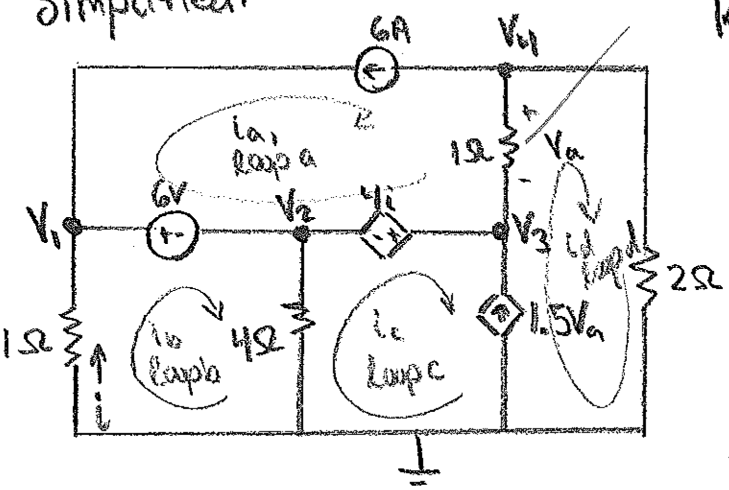
$V_2 = \frac{16}{7}(14) = \boxed{32V}$ ✓

$i = \frac{V_2}{4} = \frac{32}{4} = \boxed{8A}$ ✓

P3.



Simplified:



KVL @ loop b: (Note: $i_b = i$)

$1 i_b + 6 + 4(i_b - i_c) = 0$

$\underline{5i - 4i_c + 6 = 0}$ (1)

KVL @ loop c:

$4(i_c - i_b) - 4i = 0$

$4i_c - 4i - 4i = 0$

$4i_c - 8i = 0$

$\underline{i_c = 2i}$ (3)

KVL @ loop d:

$2i_d + 1(i_d - i_a) = 0$

$2i_d + i_d - i_a = 0$

$3i_d = \cancel{i_a}\ -6$

$\underline{i_d = -2}$ (4)

KVL @ loop a:

$1(i_a - i_d) + 4i - 6V = 0$

$i_a - i_d + 4i - 6 = 0$

Note: $i_a = -6$

$\therefore\ \underline{4i - i_d = 12}$ (2)

$\rightarrow i_d = -4$ ...

ask about this...

(1,3): $5i - 4(2i) + 6 = 0$

$5i - 8i + 6 = 0$

$-3i = -6$

$\underline{i = 2}$ (5)

& $\underline{i_c = 4}$ (6)

$\therefore V_1 = iR = -2(1) = \boxed{-2V}$

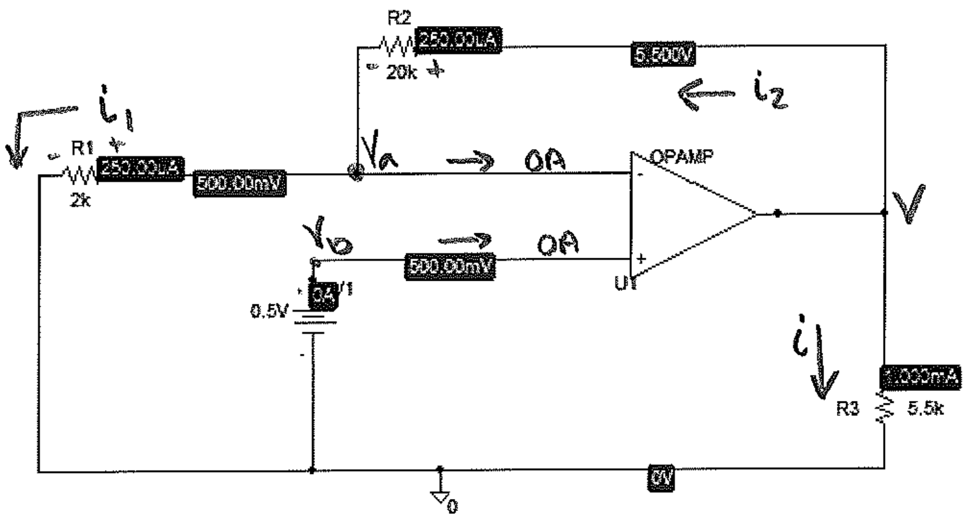
$V_2 = R(i_b - i_c) = 4(2-4) = \boxed{-8V}$

$V_a = 1(i_a - i_d) = -4V$

$V_3 = V_4 - V_a = -4V - (-4V) = \boxed{0V}$

$V_4 = R(i_d) = 2(-2) = \boxed{-4V}$

P4.



KCL @ (-) amp:

$i_2 = i_1 + 0A$

$$\frac{V-V_a}{20K} = \frac{V_a-0}{2K}$$

$$\frac{V}{20K} - \frac{V_a}{20K} = \frac{V_a}{2K}$$

$$\frac{V}{20K} - \frac{V_a}{20K} = \frac{10V_a}{20K}$$

$V = 11V_a$

Note: Principle of short

$\rightarrow V_a = V_b$

and $V_b = .5V$

$\rightarrow V_a = .5V$

$\therefore V = 11(.5)$

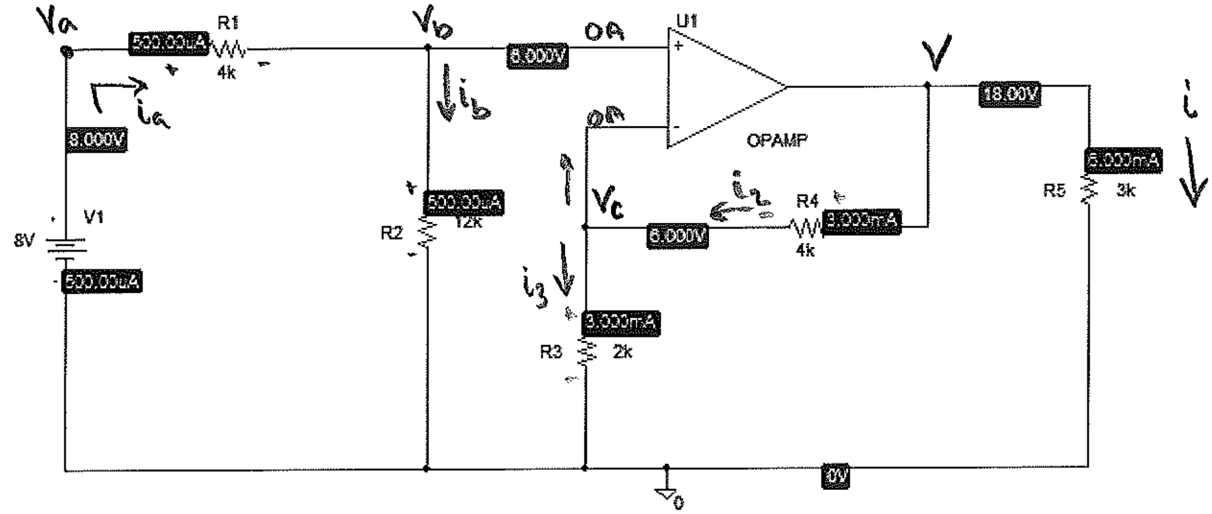
$\boxed{V = 5.5V}$

$V = iR$

$5.5 = i(5.5K)$

$\boxed{i = 1mA}$

P5.



KCL @ (+) amp:

$i_b + 0A = i_a$

$\frac{V_b - 0}{12K} = \frac{V_a - V_b}{4K}$

$\frac{V_b}{12K} = \frac{V_a}{4k} - \frac{V_b}{4K}$

$\frac{V_b}{12K} = \frac{3V_a}{12K} - \frac{3V_b}{12K}$

$V_b + 3V_b = 3V_a$

$4V_b = 3V_a$

Note: $V_a = 8V$

$V_b = 3(8)/4$

$V_b = 6V$

By principle of short

$V_b = V_c = 6V$

KCL @ V_c

$i_2 = 0A + i_3$

$\frac{V - V_c}{4K} = \frac{V_c - 0}{2K}$

$\frac{V}{4K} - \frac{V_c}{4K} = \frac{V_c}{2K}$

$V - V_c = 2V_c$

$V = 3V_c$

$V = 3(6)$

$\boxed{V = 18V}$ ✓

$V = iR$

$18 = i(3K)$

$i = 18/3K$

$\boxed{i = 6mA}$ ✓

Name: **Jared Fowler**

Date: March 10, 2018
Class: Engr M20/L – Moorpark College
Instructor: Hadi Darejeh

total deductions −10

bail 95/100

Lab 3: Introduction to Operational Amplifiers

Lab Partners: Roland Terezon
Daniel Alaya

Objective

Gain practical experience with OP-AMPS through building integrated circuits and using analytical tools, including the oscilloscope, to analyze.

Theory

Note: Theories, concepts, and proofs heavily quoted from "Fundamentals of Electric Circuits" 5th edition.

OP-AMP

An active circuit element designed to perform mathematical operations of addition, subtraction, multiplication, division, differentiation, and integration. The op-amp consists of a complex arrangement of resistors, transistors, capacitors, and diodes. For this lab, however, op-amps will be treated as a black-box with four inputs and one output, as seen in the figure 1.1. The (-) and (+) specify inverting and noninverting inputs, respectively. Positive and negative power supplies are connected to +Vss and -Vss, respectively. The op-amp senses the difference between the inverting and noninverting inputs, multiplies it by the gain A, and causes the resulting voltage to appear at the output.

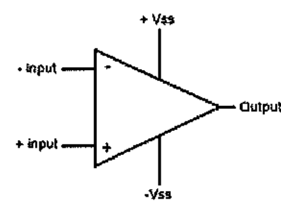


Figure 1.1

$$V_{out} = A\left(V_{in(+)} - V_{in(-)}\right)$$ **Theorem 1.1**

The output voltage is limited by the input power supply voltages, +Vss and -Vss. That is,

$$-Vss \leq V_{out} \leq +Vss$$ **Theorem 1.2 (See Appendix)**

To facilitate the understanding of op-amp circuits, ideal op-amps are generally assumed. Two important properties of the ideal op-amp are:

1. The currents into both the inverting and noninverting inputs are 0. (Principle of open.)
2. For feedback applications, the voltage across $V_{in(+)}$ and $V_{in(-)}$ is 0. (Principle of virtual short.)

D.C. Amplifier

The inverting amplifier reverses the polarity of the input signal while amplifying it. Both the input voltage and feedback are connected to the op-amps negative (-) input. As proven below, the output voltage is directly determined by the feedback resistor, R2, and the input resistor R3. A voltmeter connected across V4 and common ground is used to analyze the output voltage.

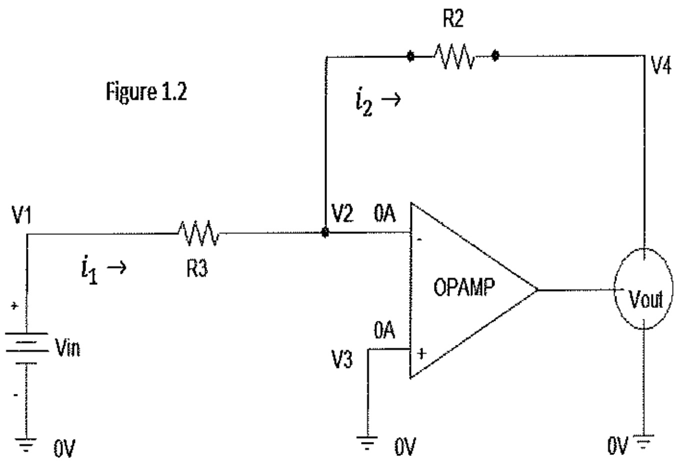


Figure 1.2

$Note: V_1 = V_{in}, V_4 = V_{out}$

$Note: V_3 = V_2 = 0V$ Principle of Virtual Short

$Note: Current\ @\ opamp's\ (\pm) = 0A$ Principle of Open

$i_1 + 0A = i_2$ $KCL\ @\ V_2$

$$\frac{V_1 - V_2}{R_3} = \frac{V_2 - V_4}{R_2}$$ Nodal Analysis

$$\frac{V_1 - 0}{R_3} = \frac{0 - V_4}{R_2}$$ Substitute value of V_2

$$V_4 = -\frac{R_2}{R_3} V_1$$ Solve for V_{out}

$$V_{out} = -\frac{R_2}{R_3} V_{in}$$ **Equation 1.1**

A.C. Amplifier

Same principle as the D.C. inverting amplifier except the input voltage is Alternating Current type. (A.C.) An oscilloscope connected across V4 and common ground is used to analyze the output voltage V_{pp} (Peak to Peak) and V_{rms} (DC-Equivalent Voltage).

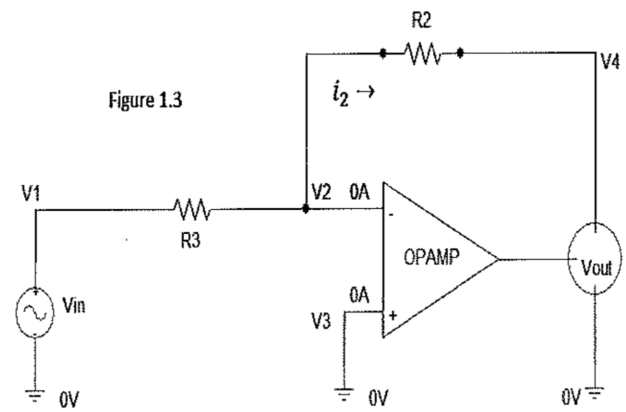


Figure 1.3

Unity Gain Inverter

Same principle as the D.C./A.C. inverting amplifier except that the resistances of R2 and R3 are equal, or ideally so. This op-amp configuration is generally used to transfer a voltage from a circuit with a high output impedance level, to a second circuit with a low impedance level. Because resistor values vary, for this lab Equation 1.1 will still be used to determine the amplification value.

Non-Inverting Op-Amp

The noninverting amplifier is an op amp circuit designed to provide a positive voltage gain. Unlike the inverting op-amp, the input voltage and feedback are connected to the positive (+) input of the op-amp. Though the gain is slightly different than the inverting amplifier, the value is still dependent upon the loopback resistor, R2, and the input resistor, R3. See proof below.

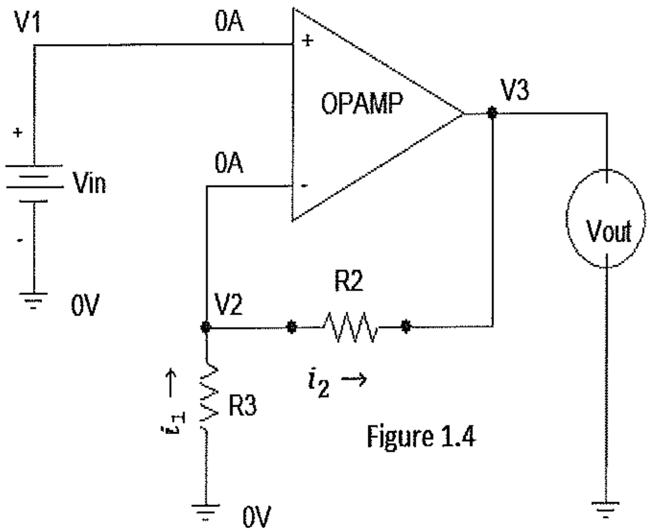


Figure 1.4

$$Note: V_1 = V_{in}, V_3 = V_{out}$$

$$Note: V_1 = V_2 = V_{in}$$ Principle of Virtual Short

$$Note: Current\ @\ opamp's\ (\pm) = 0A$$ Principle of Open

$$i_1 + 0A = i_2$$ $KCL\ @\ V_2$

$$\frac{0 - V_2}{R_3} = \frac{V_2 - V_3}{R_2}$$ Nodal Analysis

$$\frac{V_3}{R_2} = \frac{V_2}{R_2} + \frac{V_2}{R_3}$$

$$V_3 = V_2 + \frac{R_2 V_2}{R_3}$$

$$V_3 = V_2\left(1 + \frac{R_2}{R_3}\right)$$ Solve for V_{out}

$$V_{out} = V_{in}\left(1 + \frac{R_2}{R_3}\right)$$ **Equation 1.2**

Summing Op-Amp

The summing amplifier is an op amp circuit that combines several inputs and produces an output that is the weighted sum of the inputs. As seen in figure 1.5, several input voltages all connect to V2, the negative (-) input of the op-amp. As proven below, the output voltage is the combination of each input, which is amplified based upon the loopback resistor and the input's input resistor. The same idea can be applied in a noninverting architect, in which each voltage input would produce an output voltage based upon equation 1.2.

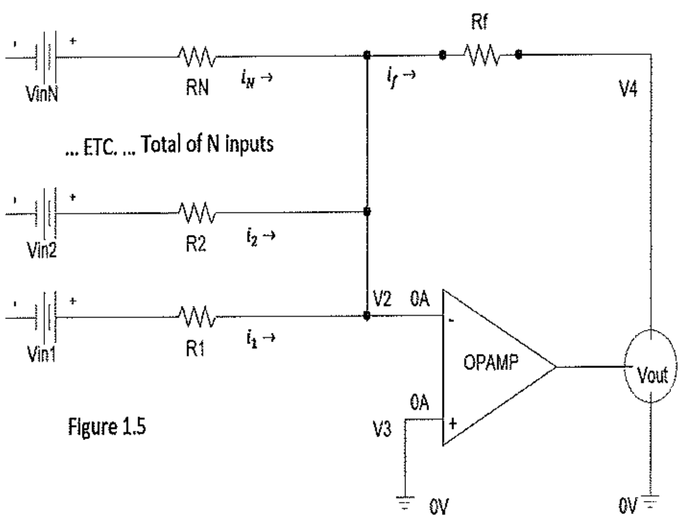


Figure 1.5

$$Note: V_4 = V_{out}$$

$$Note: V_2 = V_3 = 0V$$ Principle of Virtual Short

$$Note: Current\ @\ opamp's\ (\pm) = 0A$$ Principle of Open

$$i_1 + i_2 + \cdots + i_N + 0A = i_f$$ $KCL\ @\ V_2$

$$\frac{(V_{in1} - 0)}{R_1} + \frac{V_{in2} - 0}{R_2} + \cdots + \frac{V_{inN} - 0}{R_N} = \frac{0 - V_4}{R_f}$$ Nodal Analysis

$$\frac{V_4}{R_f} = \frac{-V_{in1}}{R_1} - \frac{V_{in2}}{R_2} - \cdots - \frac{V_{inN}}{R_N}$$ Solve for V_{out}

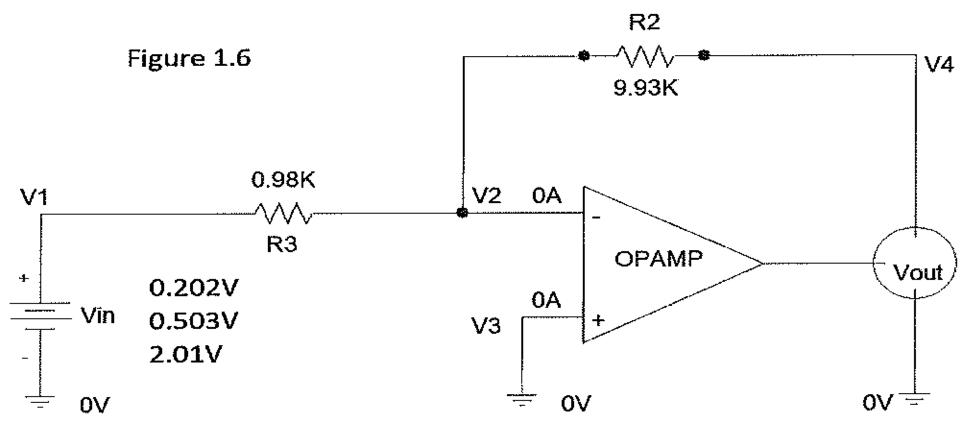
$$V_{out} = -V_{in1}\left(\frac{R_f}{R_1}\right) - V_{in2}\left(\frac{R_f}{R_2}\right) - \cdots - V_{inN}\left(\frac{R_f}{R_N}\right)$$ **Equation 1.3**

Procedure

Part 1:

A D.C. amplifier, as seen in figure 1.6, was built. The feedback resistor, R2, was measured at 9.93K-ohms, and the input resistor, R3, was measured at .98K-ohms. The output voltage was measured for three different input voltages. (0.202V, 0.503V, and 2.01V)

Figure 1.6

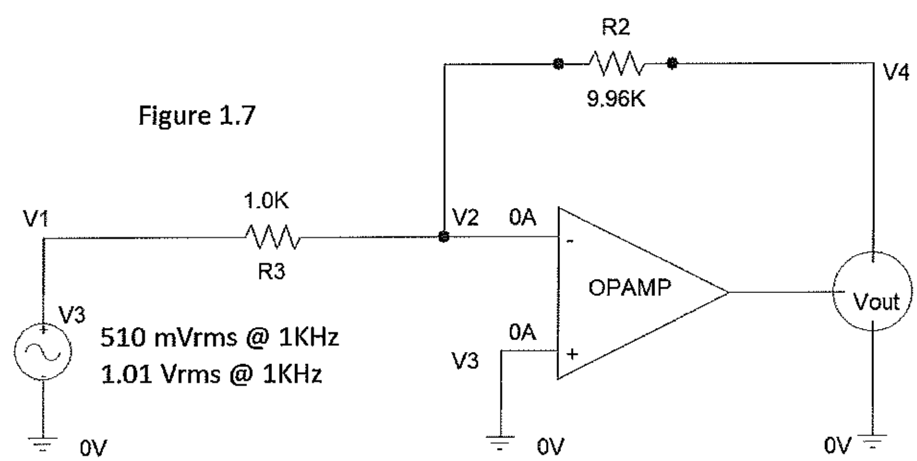


V_{in}	Feedback Resistor	Input Resistor	Amplification (Calculation 1.2)	Theoretical Vout	Measured Vout	% Error (Calculation 1.1)
0.202V	9.93K	0.98K	10.13	-2.05V	2.05V	0
0.503V	9.93K	0.98K	10.13	-5.10V	-5.07V	0.59
2.01	9.93K	0.98K	10.13	-20.37V	-7.92V	61.10

Part 2:

An A.C. amplifier, as seen in figure 1.7, was built. The feedback resistor, R2, was measured at 9.96K-ohms, and the input resistor, R3, was measured at 1.00K-ohms. The output voltage was measured for an A.C. input of 503mV_{RMS} @ 1KHz, and for 0.998V_{RMS} @ 1KHZ.

Figure 1.7

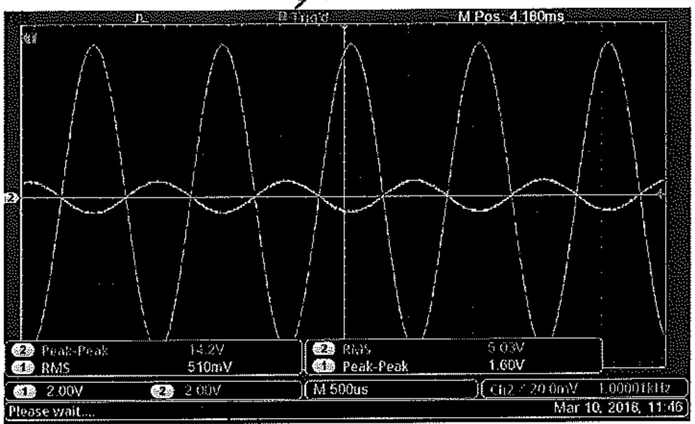


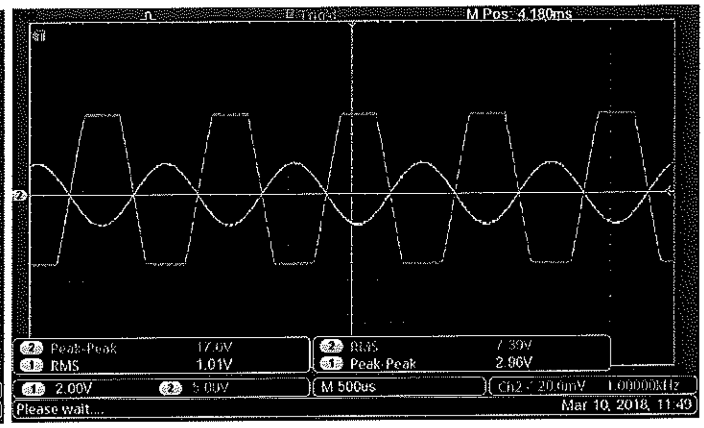
V_{in}	Feedback Resistor	Input Resistor	Amplification (Calculation 1.2)	Theoretical Vout	Measured Vout	% Error (Calculation 1.1)
510 mVrms	9.96K	1.0K	9.96	-5.08 Vrms	-5.03V	0.98
1.01 Vrms	9.96K	1.0K	9.96	-10.06 Vrms	-7.39V	26.5

Label ~~Label~~ o'scope Pics

CH1 ViN

CH2 Vout





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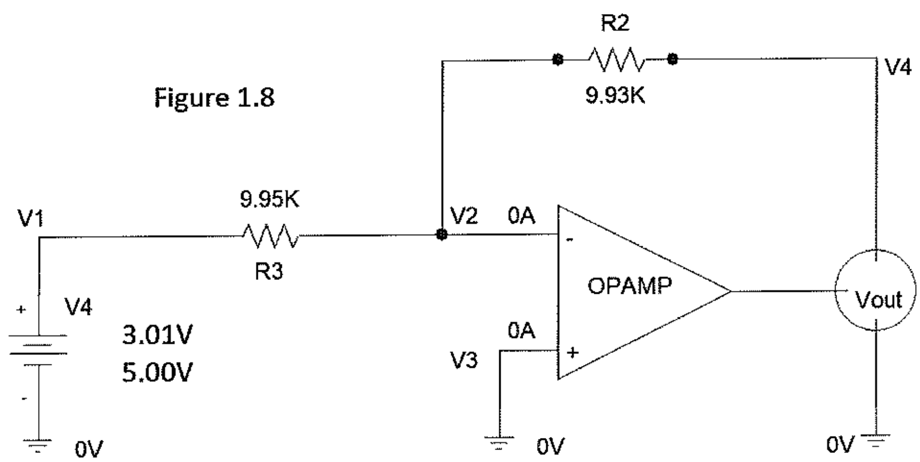
Part 3a:

A D.C. unity gain inverter, as seen in figure 1.8, was built. The feedback resistor, R2, was measured at 9.93K-ohms, and the input resistor, R3, was measured at 9.95K-ohms. The output voltage was measured for D.C. input voltages of 3.01V and 5.00V.

Figure 1.8



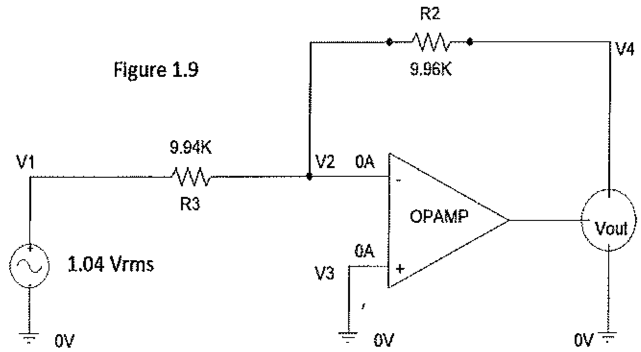
V_{in}	Feedback Resistor	Input Resistor	Amplification (Calculation 1.2)	Theoretical Vout	Measured Vout	% Error (Calculation 1.1)
3.01V	9.93K	9.95K	.998	-3.00V	3.01V	0.33
5.00V	9.93K	9.95K	.998	-4.99V	-4.99V	0

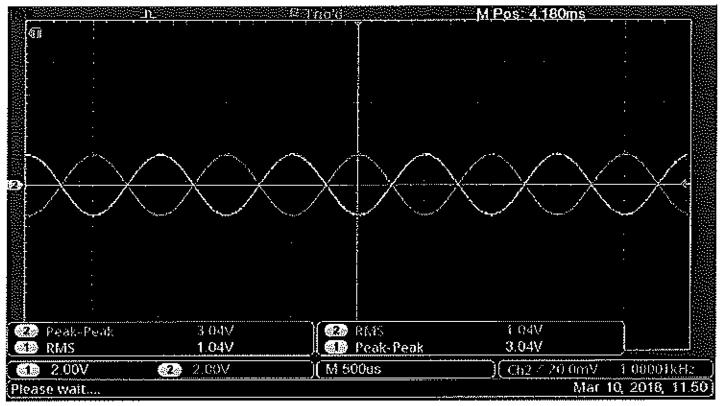
Part 3b:

An A.C. unity gain inverter, as seen in figure 1.9, was built. The feedback resistor, R2, was measured at 9.96K-ohms, and the input resistor, R3, was measured at 9.94K-ohms. The input voltage was set to 1.04 V_{RMS} @ 1KHz, and the output voltage was analyzed and measured with an oscilloscope.

V_{in}	Feedback Resistor	Input Resistor	Amplification (Calculation 1.2)	Theoretical Vout	Measured Vout	% Error (Calculation 1.1)
1.04 Vrms	9.96K	9.94K	1.002	-1.04V	-1.04V	0

Figure 1.9





Part 4a:

A D.C. noninverting amplifier, as seen in figure 1.10, was built. The feedback resistor, R2, was measured at 9.93K-ohms, and the input resistor, R3, was measured at 0.99K-ohms. The output voltage was measured for D.C. input voltages of 0.200V, 0.501V, and 2.01V.

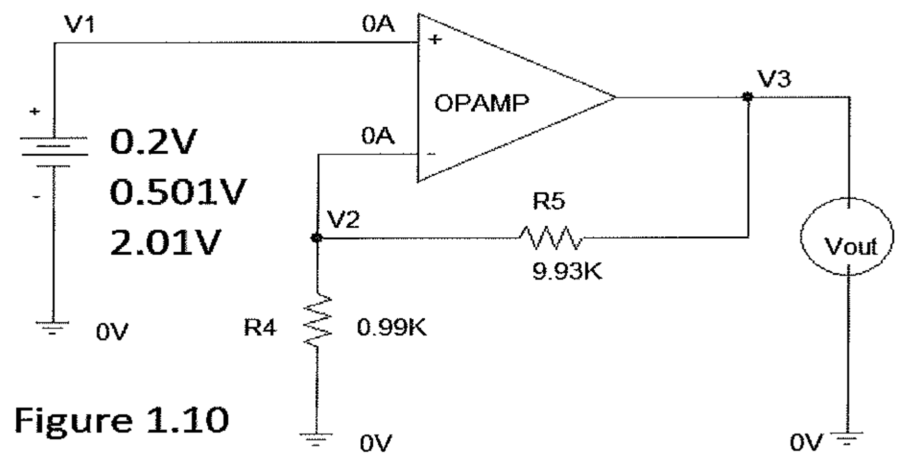


Figure 1.10

V_{in}	Loopback Resistor	Input Resistor	Amplification (Calculation 1.3)	Theoretical Vout	Measured Vout	% Error (Calculation 1.1)
0.200V	9.93K	0.99K	11.03	2.21V	2.20V	0.45
0.501V	9.93K	0.99K	11.03	5.53V	5.54V	0.18
2.01	9.93K	0.99K	11.03	22.17V	9.25V	~~58.23~~

no need to calculate %err. Saturation.

Part 4b:

An A.C. noninverting amplifier, as seen in figure 1.11, was built. The feedback resistor, R2, was measured at 9.96K-ohms, and the input resistor, R3, was measured at 1.0K-ohms. The output voltage was measured for A.C. input 509 V_{RMS}.

V_{in}	Loopback Resistor	Input Resistor	Amplification (Calculation 1.3)	Theoretical Vout	Measured Vout	% Error (Calculation 1.1)
509 mVrms	9.96K	1.0K	10.96	5.58V	5.54V	0.72

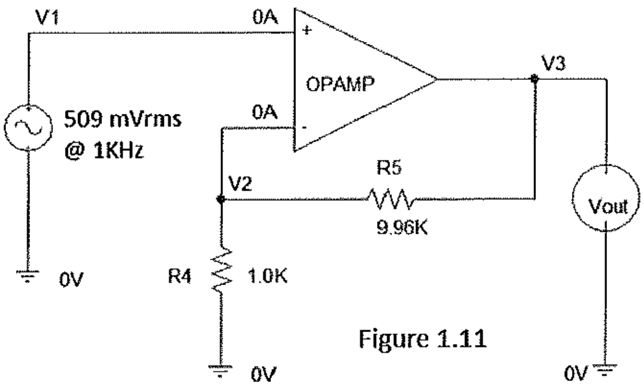
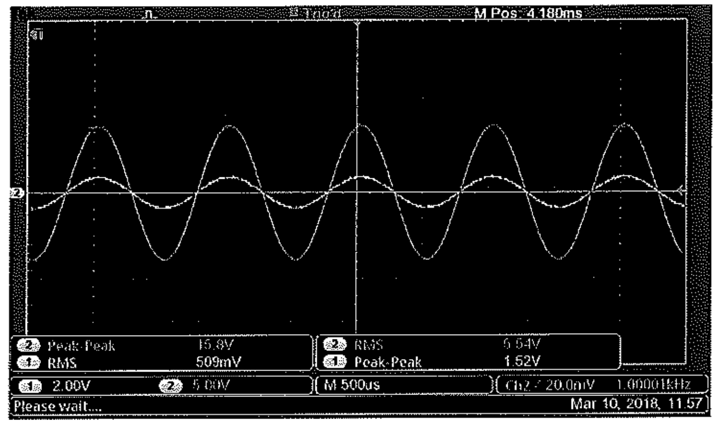


Figure 1.11



Part 5:

Three summing op-amp circuits were built. The first was a dual D.C. voltage input of 2.00V and 3.00V, with a feedback resistor value of 9.96K-ohm and input resistor values of 9.95K-ohm and 9.92k-ohm, as seen in figure 1.12. The 9.92K-ohm was then swapped with a 4.65K-ohm resistor and the input voltages were changed to 2.00V and 2.48V, as seen in figure 1.13. Output voltages were then measured with a voltmeter.

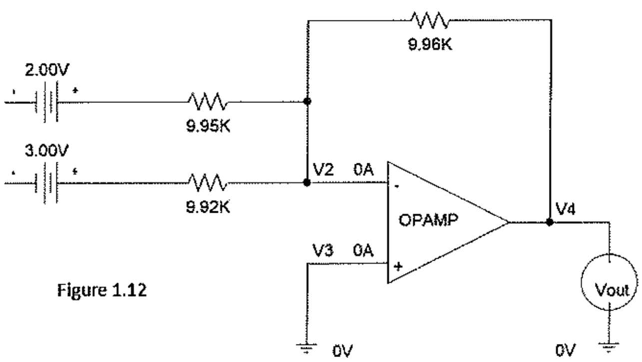


Figure 1.12

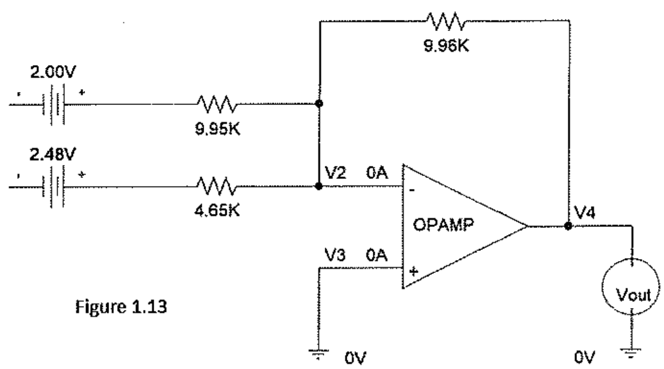


Figure 1.13

V_{in}	Loopback Resistor	Input Resistor	Amplification (Calculation 1.4)	Theoretical Vout	Measured Vout	% Error (Calculation 1.1)
2.00V	9.96K	9.95K	1.001	2.002V + 3.012V =	-4.97	0.88
3.00V		9.92K	1.004	-5.014		
2.00V	9.96K	9.95K	1.001	2.002V + 5.312V =	-7.27V	0.55
2.48V		4.65K	2.142	-7.31V		

The third circuit was built with one D.C. voltage source and one A.C. voltage source. Connected in series with the A.C. source is a 100 µF capacitor. The feedback resistor was measured at 9.96K-ohms, the D.C. input resistor was measured at 9.94K-ohm, and the A.C. input resistor was measured at 9.92K-ohm, as seen in figure 1.14. The resulting output voltage was then analyzed and measured with an oscilloscope.

V_{in}	Loopback Resistor	Input Resistor	Amplification (Calculation 1.4)	Theoretical Vout	Measured Vout	% Error (Calculation 1.1)
2.00V	9.96K	9.94K	1.002	-1.42V	-1.45	2.11
1.41 Vrms		9.92K	1.004			

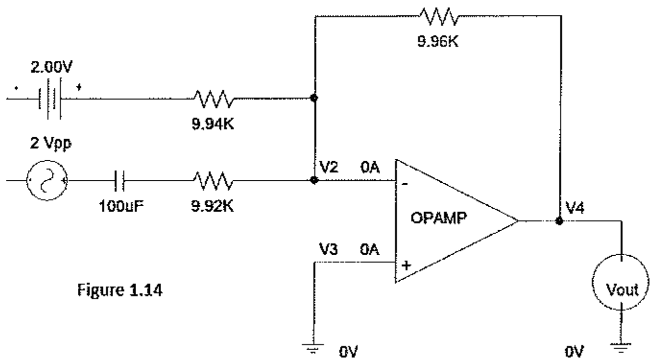
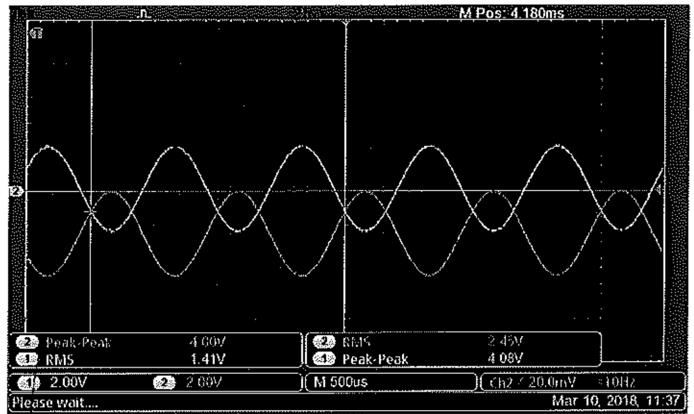


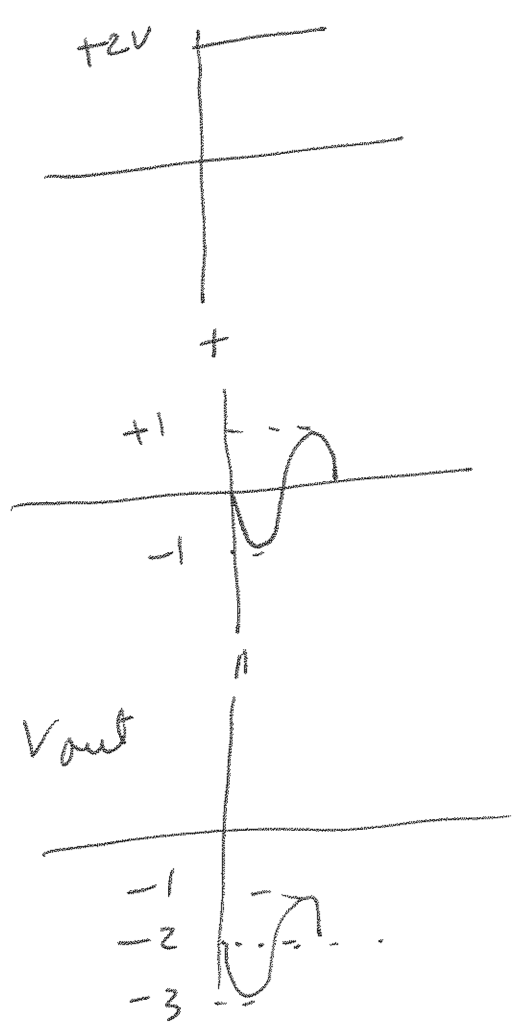
Figure 1.14



in theory :

$V_{iN_1} = 2V\ dc$

$V_{iN_2} = 2V_{pp}$



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Data & Calculations

Complete Data Table

V_{in}	Loopback Resistor	Input Resistor	Amplification	Theoretical Vout	Measured Vout	% Error
Part 1						
0.202V	9.93K	0.98K	10.13	-2.05V	2.05V	0
0.503V	9.93K	0.98K	10.13	-5.10V	-5.07V	0.59
2.01	9.93K	0.98K	10.13	-20.37V	-7.92V	~~61.10~~ SAT
Part 2						
510 mVrms	9.96K	1.0K	9.96	-5.08 Vrms	-5.03V	0.98
1.01 Vrms	9.96K	1.0K	9.96	-10.06 Vrms	-7.39V	26.5
Part 3						
3.01V	9.93K	9.95K	.998	-3.00V	3.01V	0.33
5.00V	9.93K	9.95K	.998	-4.99V	-4.99V	0
1.04 Vrms	9.96K	9.94K	1.002	-1.04V	-1.04V	0
Part 4						
0.200V	9.93K	0.99K	11.03	2.21V	2.20V	0.45
0.501V	9.93K	0.99K	11.03	5.53V	5.54V	0.18
2.01	9.93K	0.99K	11.03	22.17V	9.25V	~~58.23~~ SAT
509 mVrms	9.96K	1.0K	10.96	5.58V	5.54V	0.72
Part 5						
2.00V 3.00V	9.96K	9.95K 9.92K	1.001 1.004	2.002V + 3.012V = -5.014	-4.97	0.88
2.00V 2.48V	9.96K	9.95K 4.65K	1.001 2.142	2.002V + 5.312V = -7.31V	-7.27V	0.55
2.00V 1.41 Vrms	9.96K	9.94K 9.92K	1.002 1.004	-1.42V	-1.45	2.11

Calculation 1.1

$$\%Error \rightarrow \frac{|V_{theoretical} - V_{measured}|}{|V_{theoretical}|} x\ 100$$

Example: Part 1:

$$\frac{|5.10 - 5.07|}{5.10} * 100 = 0.59\%$$

Calculation 1.2

Use Equation 1.1

$$V_{out} = -\frac{R_2}{R_3} V_{in}$$

Example: Part 3

$$V_{out} = -\left(\frac{9.93}{9.95}\right)(3.01) = -3.00$$

Calculation 1.3

$$Use\ Equation\ 1.2$$

$$V_{out} = V_{in}\left(1 + \frac{R_2}{R_3}\right)$$

Example: Part 4a

$$V_{out} = 0.200V\left(1 + \frac{9.93K}{0.99K}\right) = 2.21$$

Calculation 1.4

$$Use\ Equation\ 1.3$$

$$V_{out} = -V_{in1}\left(\frac{R_f}{R_1}\right) - V_{in2}\left(\frac{R_f}{R_2}\right) - \cdots - V_{inN}\left(\frac{R_f}{R_N}\right)$$

Example: Part 5a

$$V_{out} = -2.00V\left(\frac{9.96K}{9.95K}\right) - 3.00V\left(\frac{9.96K}{9.92K}\right) = -5.01$$

Discussion of Results

Part 1:

Experiment went as expected. The differences in input voltages were successfully amplified and inverted. Output voltages for the 0.202V and .503V yielded very low percent errors. The 2V input yielded a 61.10%. This output was expected because the supply voltages were limited to -10V and +10V. See Theorem 1.2 and Op-Amp output voltage saturation in appendix. As verification, the supply voltages were temporarily bumped up to +/- 25V, which in turn allowed the output voltage reading to reach its predicted value.

Part 2:

Experiment went as expected. The observed sine wave was inverted and amplified based upon input voltage difference and amplification value. As in par 1 circuit 3, the second circuit resulted in a high percent error of 26.5%. Once again, this is due to Op-Amp output voltage saturation. Verification of this was done by increasing the supply voltages to +/- 25V which in turn allowed the output voltage reading to reach its predicted value and form a nice, non-clipped, sine wave.

Part 3:

Experiment went as expected. The difference in input voltages was successfully inverted while keeping the about same voltage value. Very low percent errors were yielded.

Part 4:

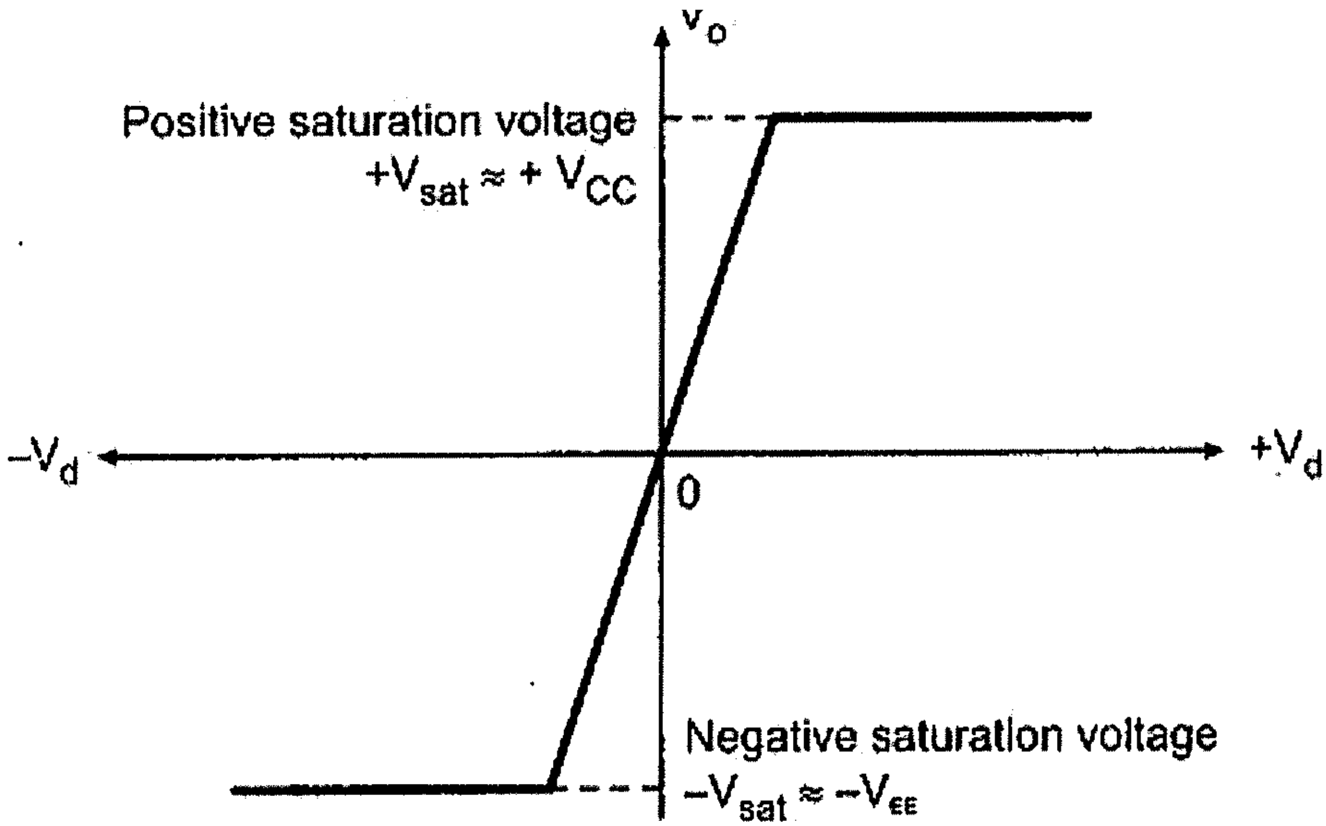
Experiment went as expected. The differences in input voltages were successfully amplified for both the D.C. and A.C. circuits. Once again, the effects of Op-Amp output voltage saturation was seen for the third D.C. circuit (2V input), resulting in a percent error of 58.23%.

Part 5:

Experiment went as expected. The differences in input voltages were successfully amplified, added together, and inverted for both the D.C. and A.C. circuits. Of special note was the behavior of the mixed D.C. and A.C. circuit. Analyzing the oscilloscope output, the sine wave has indeed been inverted. Instead of adding to the amplitude of the wave, however, the D.C. amplification vertically offsets the wave by -2V, which is the calculated amplified output voltage for the D.C. input voltage.

Appendix

Op-Amp Output Voltage Saturation



100/100 Excellent Lab

Name: **Jared Fowler**

Date: ~~April 8, 2018~~
Class: Engr M20/L – Moorpark College
Instructor: Hadi Darejeh

Lab 4: Transient Response

Lab Partners: Roland Terezon
Daniel Alaya

Objective

Analyze first-order circuits using standardized methods and PSPICE, and compare the theoretical results with those found in the lab experiment.

Theory

Note: Theories, concepts, and proofs heavily quoted from "Fundamentals of Electric Circuits" 5th edition.

First-Order Circuits

Contain only one energy storage element (capacitor or inductor), and that can, therefore, be described using only a first order differential equation.

Transient Response

Time required for a capacitor to fully charge is equivalent to about 5 times constants or 5T. This transient response time T, is measured in terms of **τ = R x C**, in seconds. **(EQ 0.1)**

Low-Pass Filter

A filter that passes signals with a frequency lower than a certain cutoff frequency and attenuates signals with frequencies higher than the cutoff frequency.

The low-pass filter circuit analyzed in this lab is shown in Figure 0.1. Note that resistor R1 is placed in series with, and before, the capacitor C1. The circuit is powered by a square wave of 10Vpp @ 200Hz.

The voltage across C1 is derived below. The derivation consists of two output equations, (EQ 1.3) and (EQ 1.4), which represent high peak and low peak voltages of the square wave respectively for one period, 0<t<5ms.

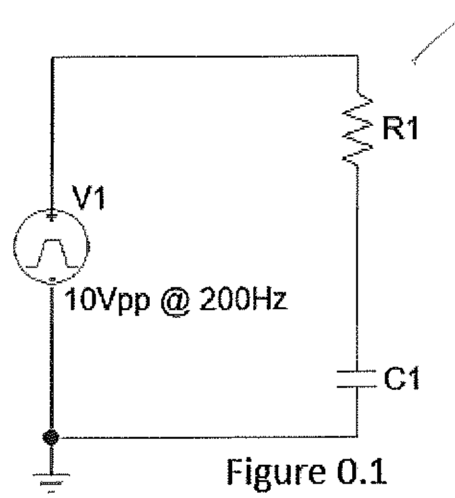


Figure 0.1

$$\textit{Given that } V_1 \textit{ is } 10Vpp \ @ \ 200Hz$$

$$\textit{Note}: @\, t < 0, t(0)\ V_c = 0V \qquad \text{I.C.}$$

$$i_r = i_c \qquad \text{K.C.L.}$$

$$\frac{(10 - V_c)}{R_1} = \frac{cdv}{dt}$$

$$\frac{cdv}{dt}+\frac{V_c}{R_1}=\frac{10}{R_1}$$

$$\frac{dv}{dt}+\frac{V_c}{R_1c}=\frac{10}{R_1c}$$ **(EQ 1.1)**

$$V_c(t)=V_n+V_f$$ **(EQ 1.2)**

$$V_f=k_2$$ Because right side is constant

$$\frac{dk_2}{dt}+\frac{k_2}{R_1c}=\frac{10}{R_1c}$$ Substitute into (EQ 1.1), and solve for k2

$$k_2=10$$

$$V_n=k_1e^{st}$$ Assumed solution

$$\frac{dk_1e^{st}}{dt}+\frac{k_1e^{st}}{R_1c}=0$$ Substitute into (EQ 1.1), and solve for s

$$sk_1e^{st}+\frac{k_1e^{st}}{R_1c}=0$$

$$s+\frac{1}{R_1c}=0$$

$$s=-\frac{1}{R_1c}$$

$$V_c(t)=k_1e^{-\frac{1}{R_1c}t}+10$$ Substitute into (EQ 1.2)

$$V(0)=0=k_1+10$$ Solve for k1 at t=0

$$k_1=-10$$

$$@\,0\le t\le\ 2.5ms,\quad V_c(t)=10-10e^{-\frac{1}{R_1c}t}$$ **(EQ 1.3)**

$$V_c(2.5ms)=V_2$$ I.C.

$$V_c(t)=k_1e^{-\frac{1}{R_1c}t}$$ Source-less CKT

$$V_c(2.5ms)=V_2=k_1e^{-\frac{1}{R_1c}(2.5)}$$ Solve for k1 at t=2.5ms

$$k_1=V_2e^{\frac{1}{R_1c}(2.5)}$$

$$@\,2.5ms\le t\le\ 5ms,\quad V_c(t)=\left(V_2e^{\frac{1}{R_1c}(2.5)}\right)e^{-\frac{1}{R_1c}t}$$ **(E.Q. 1.4)**

High-Pass Filter

A filter that passes signals with a frequency higher than a certain cutoff frequency and attenuates signals with frequencies lower than the cutoff frequency.

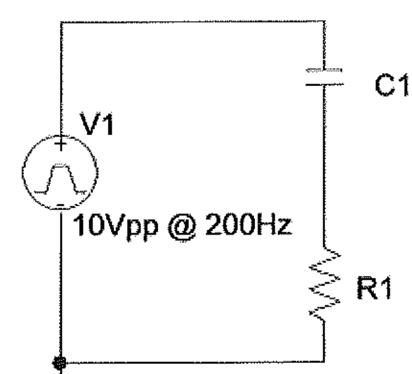


Figure 0.2

The high-pass filter circuit analyzed in this lab is shown in Figure 0.2. Note that resistor R1 is placed in series with, and after, the capacitor C1. The circuit is powered by a square wave of 10Vpp @ 200Hz.

The voltage across C1 is derived below. The derivation consists of two output equations, (EQ 2.2) and (EQ 2.3), which represent high peak and low peak voltages of the square wave respectively for one period, 0<t<5ms.

Given that V_1 is 10Vpp @ 200Hz

$Note{:}\ @\ t(0), V_r = 10V$	I.C. (Vc cannot change instantaneously.)
$Note{:}\ V_1 = V_c + V_R \rightarrow V_R = V_1 - V_c$	**(EQ 2.1)**
⋮	Follow procedure from Equation 1 to arrive at (EQ 1.3), then substitute in (EQ 2.1)
$V_R(t) = 10 - \left(10 - 10e^{-\frac{1}{R_1 c}t}\right)$	Simplify
$@\ 0 \le t \le 2.5ms, \quad V_R(t) = 10e^{-\frac{1}{R_1 c}t}$	**(EQ 2.2)**
⋮	Follow procedure from Equation 1 to arrive at (EQ 1.4), then substitute in (EQ 2.1)
$@\ 2.5ms \le t \le 5ms, \quad V_R(t) = 0 - \left(V_2 e^{\frac{1}{R_1 c}(2.5)}\right) e^{-\frac{1}{R_1 c}t}$	**(EQ 2.3)**

Big Charged Capacitor to Smaller Uncharged Capacitor

The circuit as shown in Figure 0.3 is analyzed in this lab. Capacitor C1 reaches steady state while in series with the voltage source V1 and resistor R2. To be in steady state means that C1 acts like an "open" in the circuit, and has been charged to match the voltage of V1. Capacitor C2 is completely discharged. The "switch" is then flipped, disabling the first circuit and enabling a circuit of C1, R1, ammeter, and C2 in series. C1 discharges which causes a current to flow through the circuit. The current flowing through the circuit in respect to time is derived below.

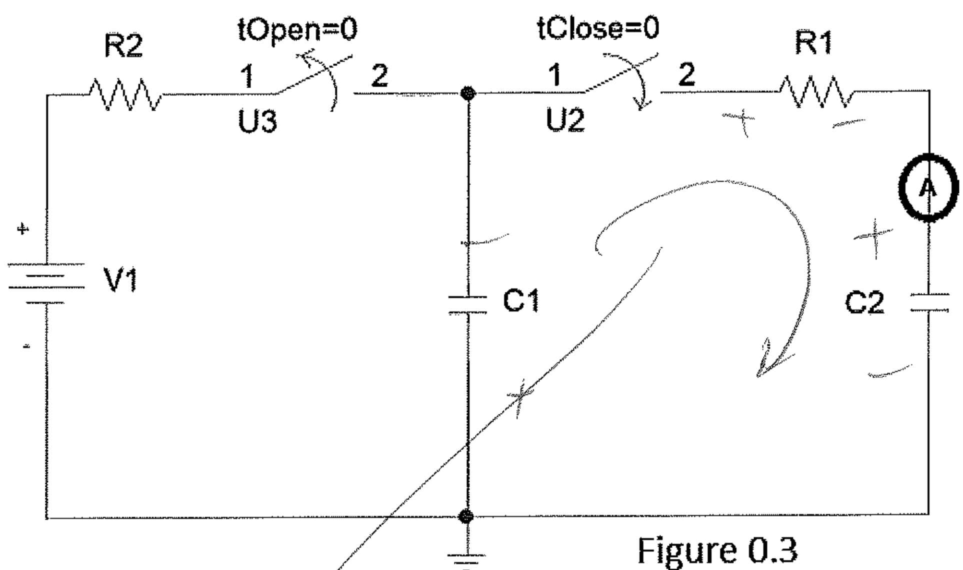


Figure 0.3

Note: At $t(0)$ the switches open and close to produce a circuit represented by $C_1, R_1,$ and C_2

$V_{c1} + V_{R1} + V_{c2} = 0$	K.V.L.
$\frac{d}{dt}\left[\frac{1}{c_1}\int idt + iR_1 + \frac{1}{c_2}\int idt = 0\right]$	Substitute voltages with capacitor-current relationship equations. Take the derivative of the entire equation.
$\frac{1}{c_1}i + \frac{R_1 di}{dt} + \frac{1}{c_2}i = 0$	
$\frac{R_1 di}{dt} + i\left(\frac{1}{c_2} + \frac{1}{c_1}\right) = 0$	

$$\frac{di}{dt} + i\left(\frac{c_1 + c_2}{R_1 c_2 c_1}\right) = 0$$ **(EQ 3.1)**

$$x(t) = x_n + x_f$$ **(EQ 3.2)**

$$x_f = k \rightarrow k = 0$$ Right side is constant and 0

$$x_n = ke^{st}$$

$$\frac{dke^{st}}{dt} + ke^{st}\left(\frac{c_1 + c_2}{R_1 c_2 c_1}\right) = 0$$ Substitute into EQ 3.1

$$s = -\left(\frac{c_1 + c_2}{R_1 c_1 c_2}\right)$$ Solve for s

$$i(t) = ke^{-\left(\frac{c_1+c_2}{R_1 c_1 c_2}\right)t} + 0$$ Substitute into EQ 3.2

$$Note: At\ t(0),\ c_1 = V_o\ and\ c_2 = 0V. \rightarrow i(0) = \frac{V_o}{R_1}\quad (I.C.)$$

$$i(0) = \frac{V_o}{R_1} = ke^0 \rightarrow k = \frac{V_o}{R_1}$$ Solve for k

$$i(t) = \frac{V_o}{R_1} e^{-\left(\frac{c_1+c_2}{R_1 c_1 c_2}\right)t}$$ **(EQ 3.3)**

Op-Amp Integrator

Operational amplifier circuit that performs the mathematical operation of Integration.

The integrator circuit analyzed in this lab is shown in Figure 0.4. Note that the feedback line is populated with capacitor C1. The resistor in parallel with C1 acts as a stabilizer. The circuit is powered by a square wave of 2.5Vpp @ 1KHz.

The Op-Amp output voltage in respect to time is derived below. The derivation consists of two output equations, (EQ 4.2) and (EQ 4.3), which represent high peak and low peak voltages of the square wave respectively for one period, 0<t<1ms.

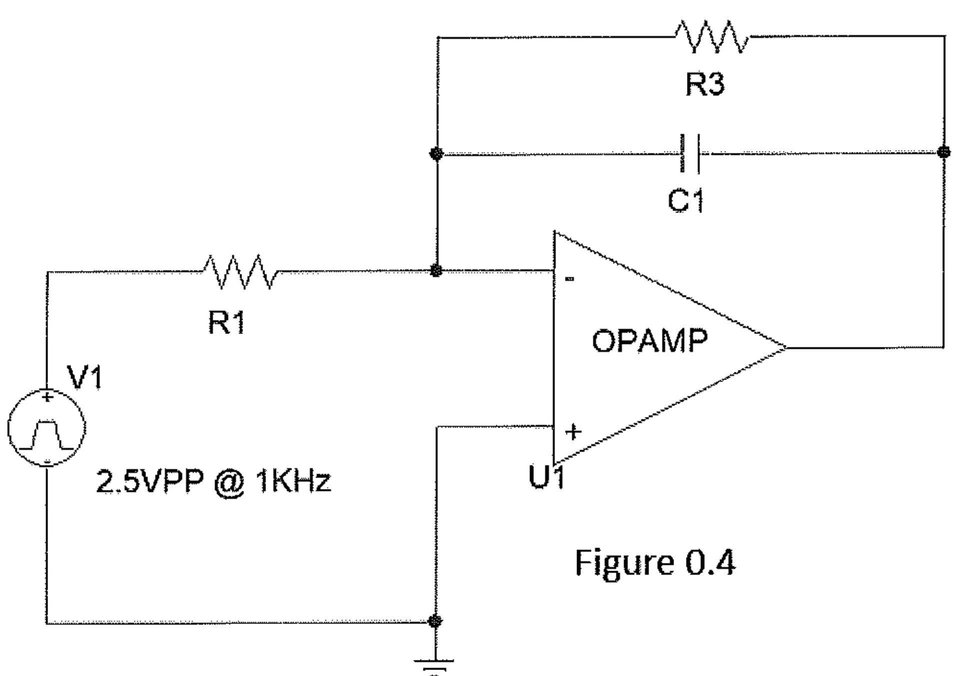


Figure 0.4

$$Given\ that\ V_1\ is\ 2.5Vpp\ @\ 1KHz$$

$$Note: R_3\ acts\ as\ a\ stablizer. It\ will\ not\ be\ included\ in\ this\ derivation.$$

$$Note: @\ t(0), V_c = 0$$ I.C. (Vc cannot change instantaneously)

$$Note: V_{OpAmp}^{-} = V_{OpAmp}^{+} = 0V \quad \&\& \quad i_{OpAmp}^{-} = i_{OpAmp}^{+} = 0A$$ Principles of Short and Open for OpAmps

$$i_r + 0 = i_c$$ K.C.L. @ OpAmp (-)

$$\frac{V_1 - 0}{R_1} = \frac{cdV_c}{dt}$$

$$\frac{cdV_c}{dt} = \frac{V_1}{R_1}$$

$$\frac{dV_c}{dt} = \frac{V_1}{R_1 c}$$ **(EQ 4.1)**

$$\int dV_c = \int \frac{V_1}{R_1 c} dt$$

$$V_c(t) = \frac{V_1}{R_1 c} t + C \ldots V_c(0) = 0 \to C = 0$$

$$@\ 0 \le t \le 500us, \quad V_c(t) = V_o(t) = \frac{-V_1}{R_1 c} t$$ **(EQ 4.2)** **Negate b/c inverse op-amp**

$$V_o(500us) = V_2$$ I.C.

$$\frac{dV_c}{dt} = \frac{V_1}{R_1 c}$$ **(EQ 4.1)**

$$\int dV_c = \int \frac{V_1}{R_1 c} dt$$

$$V_c(t) = \frac{V_1}{R_1 c} t + C \ldots V_c(500us) = V_2 = \frac{V_1}{R_1 c}(500us) + C$$

$$C = V_2 - \frac{V_1}{R_1 c}(500us)$$

$$@\ 500us \le t \le 1000us, \quad V_o(t) = \frac{-V_1}{R_1 c} t + \left(V_2 - \frac{V_1}{R_1 c}(500us)\right)$$ **(EQ 4.3)** **Negate b/c inverse op-amp**

Op-Amp Differentiator

Operational amplifier circuit performs the mathematical operation of Differentiation.

The differentiator circuit analyzed in this lab is shown in Figure 0.5. Note that the op-amp negative input line is populated with capacitor C1. The resistor in series with C1 acts as a stabilizer. The circuit is powered by a sawtooth wave of 2.5Vpp @ 1KHz.

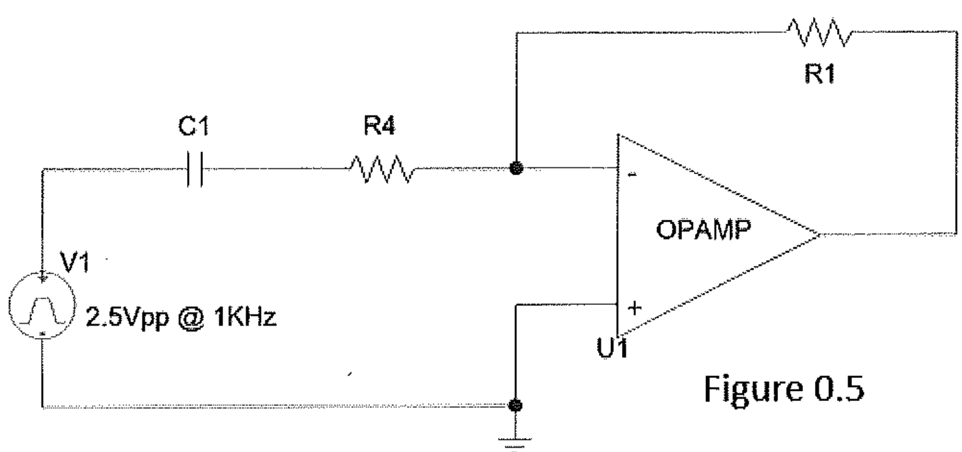


Figure 0.5

The Op-Amp output voltage in respect to time is derived below. The derivation consists of two output equations, (EQ 5.3) and (EQ 5.5), which represent high peak and low peak voltages of the sawtooth wave respectively for one period, 0<t<1ms.

$$\textit{Given that } V_1 \textit{ is } 2.5Vpp \ @ \ 1KHz$$

$$\textit{Note: } R_4 \textit{ acts as a stablizer. It will not be included in this derivation.}$$

$$\textit{Note: } @ \ \boldsymbol{t(0), V_c = 0}$$ I.C. (Vc cannot change instantaneously.)

$$\textit{Note: } \boldsymbol{V^-_{OpAmp} = V^+_{OpAmp} = 0V \quad \&\& \quad i^-_{OpAmp} = i^+_{OpAmp} = 0A}$$ Principles of Short and Open for OpAmps

$$\textit{Note: } \boldsymbol{V_1 - V_c = 0 \rightarrow V_1 = V_c}$$ **(EQ 5.1)**

$$\textit{Note: } m_{v1} = \frac{-1.25 - 1.25}{0.0005 - 0} = -5000, \ b_{v1} = 1.25 \rightarrow$$ **(EQ 5.2)**

$$\boldsymbol{V_1(t) = -5000t + 1.25}$$

$$i_r + i_c = 0$$ K.C.L. @ OpAmp (-)

$$\frac{0 - V_o}{R_1} + \frac{cdV_1}{dt} = 0$$

$$V_o = \frac{(cR_1)d(-5000t + 1.25)}{dt}$$ Take the derivative.

$$\boldsymbol{@ \ 0 \leq t \leq 500us, \quad V_o = -5000(cR_1)}$$ **(EQ 5.3)**

$$\textit{Note: } m_{v1b} = \frac{1.25 - -1.25}{0.001 - 0.0005} = 5000, \ b_{v1b} = -1.25 \rightarrow$$ **(EQ 5.4)**

$$\boldsymbol{V_1(t) = 5000t - 1.25}$$

$$i_r + i_c = 0$$ K.C.L. @ OpAmp (-)

$$\frac{0 - V_o}{R_1} + \frac{cdV_1}{dt} = 0$$

$$\frac{0 - V_o}{R_1} + \frac{cdV_1}{dt} = 0$$

$$V_o = \frac{(cR_1)d(5000 - 1.25)}{dt}$$ Take the derivative.

$$\boldsymbol{@ \ 500us \leq t \leq 1000us, \quad V_o(t) = 5000(cR_1)}$$ **(EQ 5.5)**

Procedure

Part 1:

The low-pass filter circuit, as seen in Figure 1.1, was built. The resistor was measured at 9.93K-ohms and the capacitor was of unknown value. An oscilloscope was used to analyze and measure the charge/discharge curve of the capacitor with respect to input voltage, as seen in Figure 1.2. The time to charge was found to be around 820us. Utilizing this data, the capacitor was calculated to have a capacitance of **16.5nF**. (Calculation 1.1) The real value was then measured by a separate device to be **14.9nF**, which is **10.7%** different than the calculated value. (Calculation 1.2) The waveforms were calculated to be $V_c(t) = 10 - 10e^{-6758.72t}$, @ 0<t<2.5ms, and $V_c(t) = (2.17E8)e^{-6758.72t}$ @2.5ms<t<5ms for the square wave's high and low voltage outputs respectively for one period, 0 < t < 5ms. (Calculation 1.3) The circuit was built and analyzed in PSPICE, as seen in Figure 1.3.

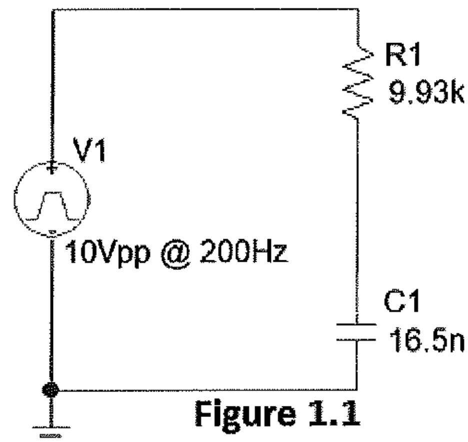


Figure 1.1

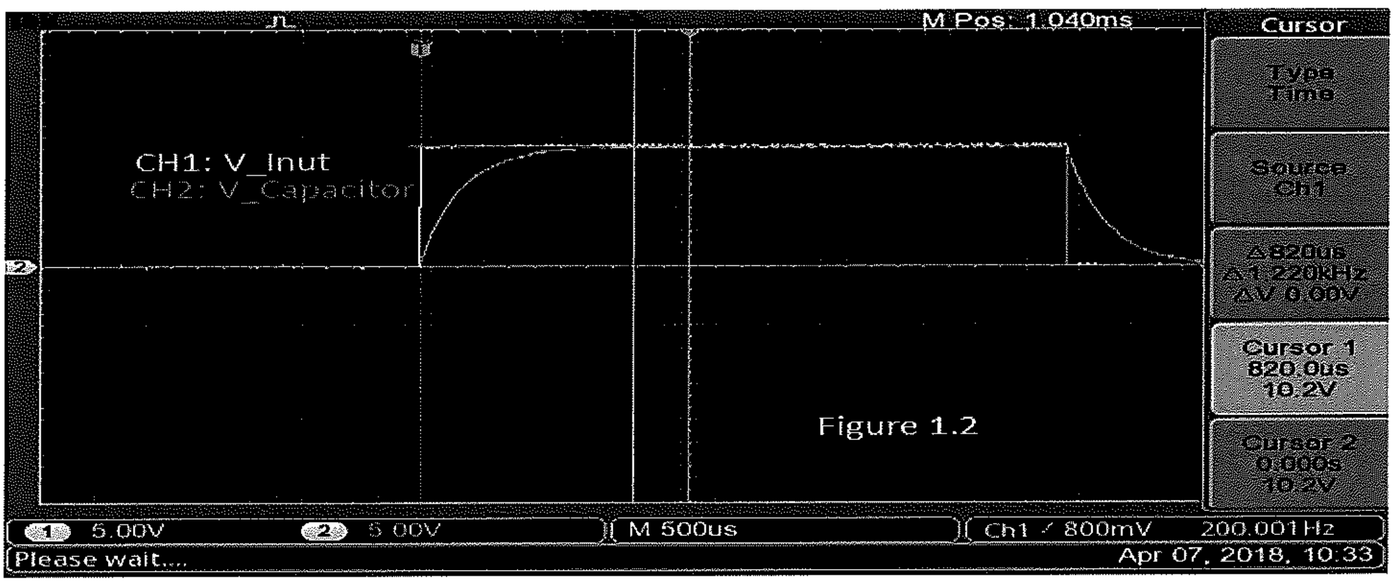


Figure 1.2

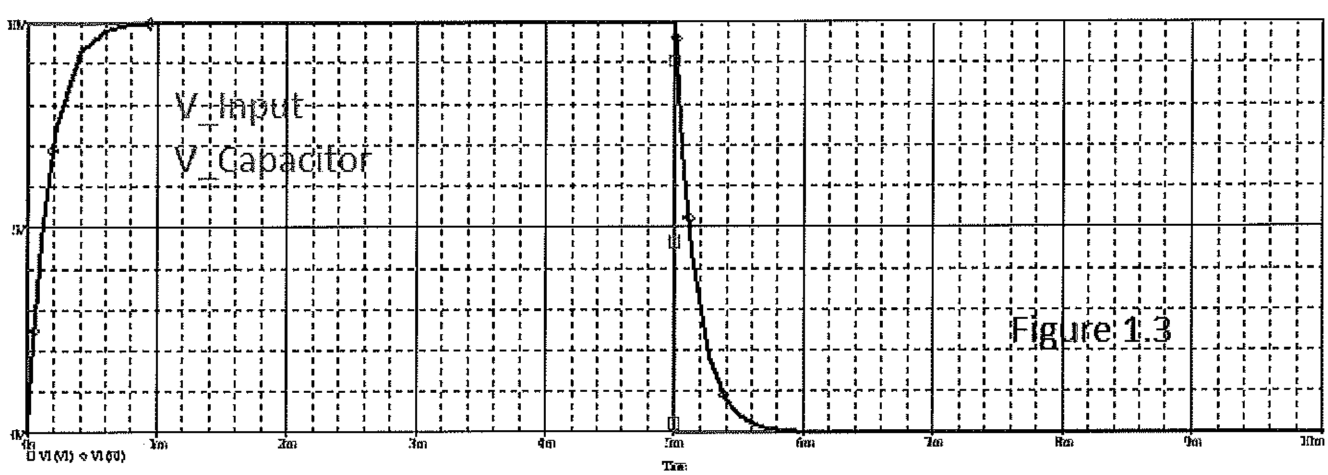


Figure 1.3

Part 2:

The high-pass filter circuit, as seen in Figure 2.1, was built, simply by switching the capacitor and resistor from part 1. An oscilloscope was used to analyze and measure the charge/discharge curve of the capacitor with respect to input voltage, as seen in Figure 2.2. The waveforms were calculated to be $V_c(t) = 10e^{-6758.72t}$ @ $0<t<2.5$ms, and $V_c(t) = -(2.17E8)e^{-6758.72t}$ @ $2.5\text{ms}<t<5$ms, for the square wave's high and low voltage outputs respectively for one period, $0 < t < 5$ms. (Calculation 2.1) The circuit was built and analyzed in PSPICE, as seen in Figure 2.3.

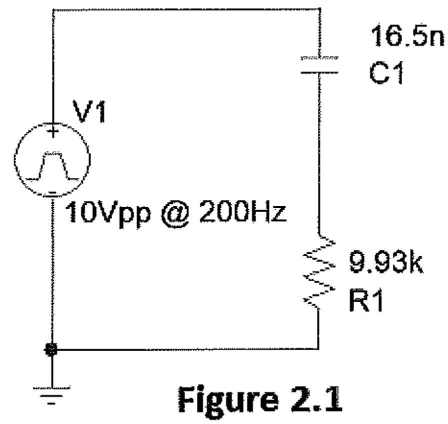


Figure 2.1

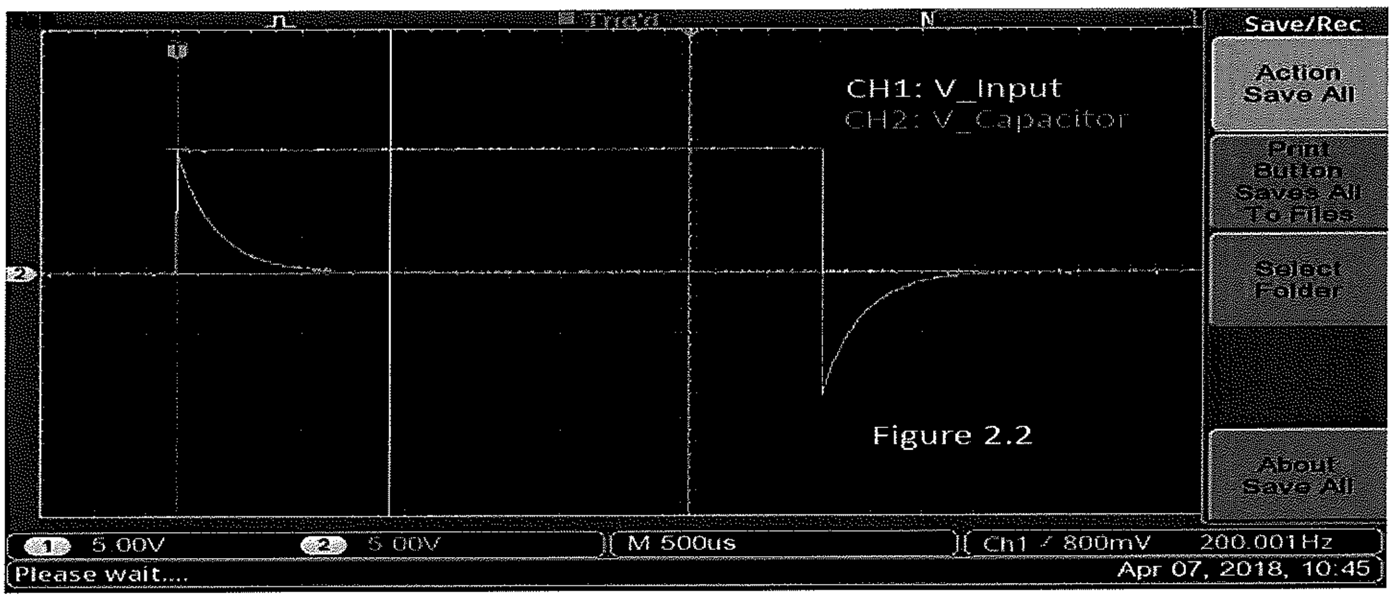


Figure 2.2

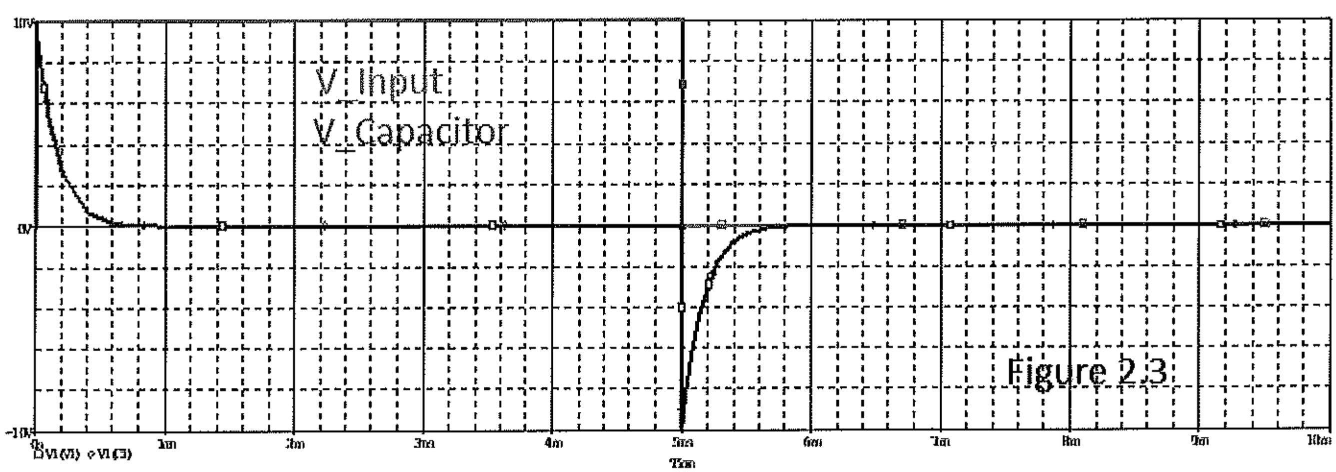


Figure 2.3

Part 3:

The circuit, as seen in Figure 3.1, was built. Resistor R1 was measured at 4.98M-Ohms. An ammeter (100-ohm) was connected in series with the circuit between R1 and C2. Capacitor C1 fully charged, and C2 fully discharged. The switch was pulled and the current flowing through the C1, R1, ammeter, C2 circuit was recorded at 1 second intervals. These data points were analyzed with Excel to produce the waveform and equation as show in Figure 3.2. The waveform was calculated to be $i(t) = (5 * 10^{-6})e^{-0.02t}$ (Calculation 3.1), which is very close to the waveform calculated by excel, $y = (5 * 10^{-6})e^{-0.01x}$. The circuit was built and analyzed in PSPICE, as seen in Figure 3.3.

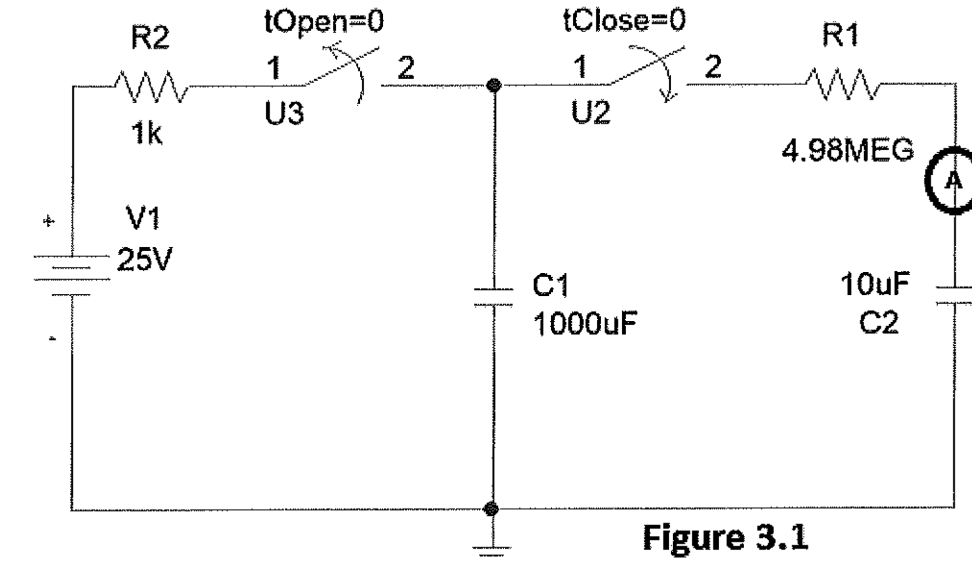


Figure 3.1

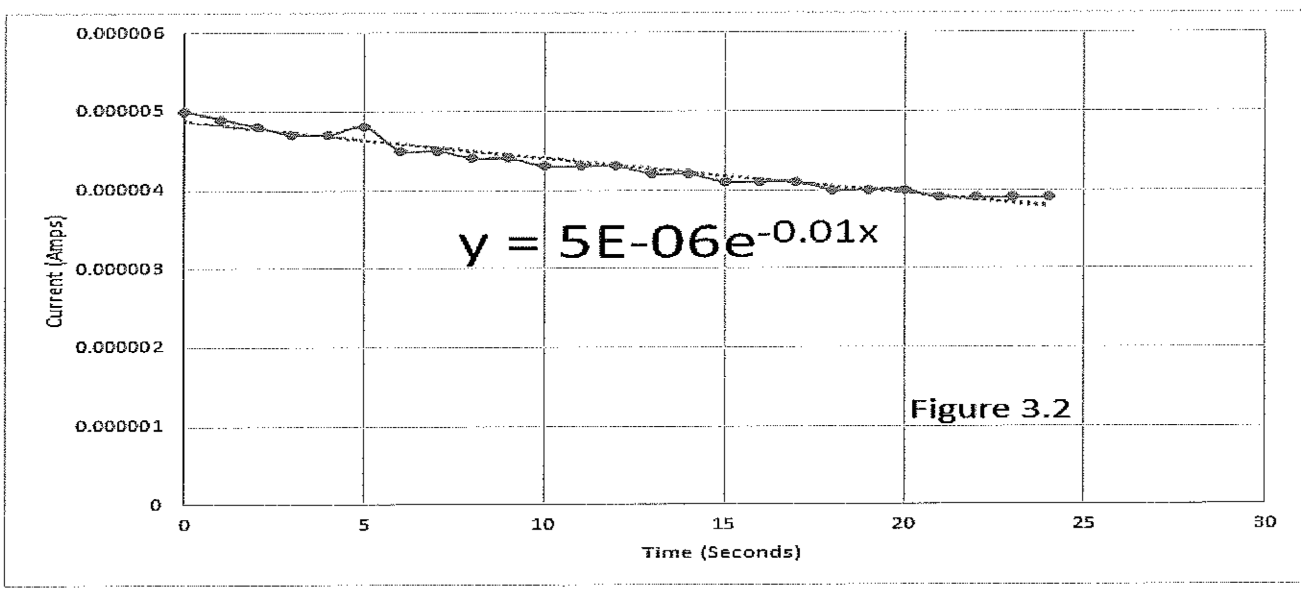


Figure 3.2

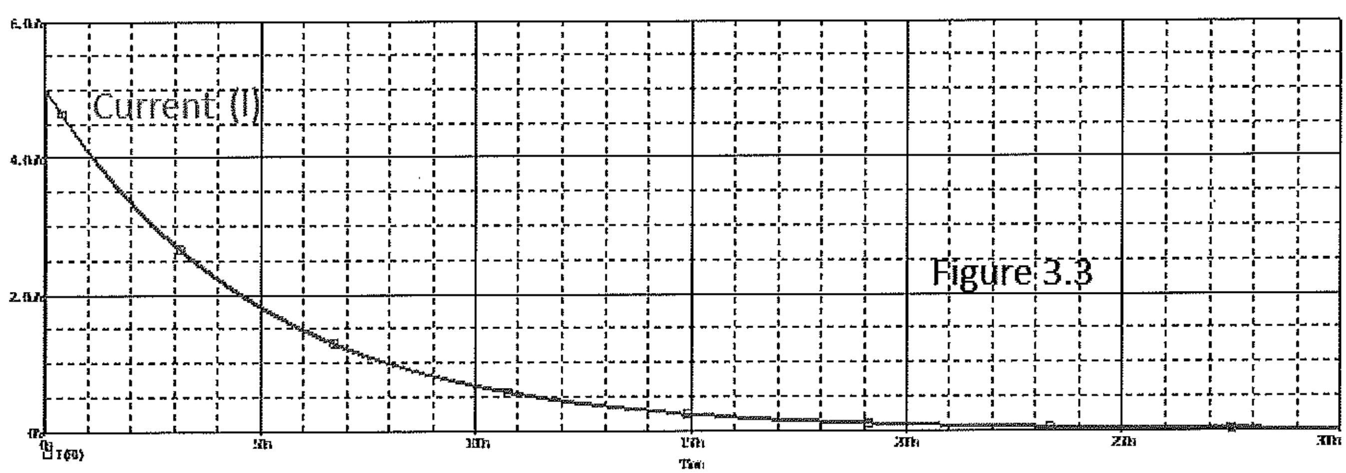


Figure 3.3

Part 4:

The op-amp integrator circuit, as seen in Figure 4.1, was built. R1 was measured at 9.93k-ohm. An oscilloscope was used to analyze and measure the op-amp output voltage in relation to the input voltage, as seen in Figure 4.2. The waveforms were calculated to be $V_o(t) = 1258.1t$ @ 0<t<500us, and $V_o(t) = -1258.1t + 1.25$ @ 500us<t<1000us for the square wave's low and high voltage outputs respectively for one period, 0 < t < 1ms. (Calculation 4.1) The circuit was built and analyzed in PSPICE, as seen in Figure 4.3.

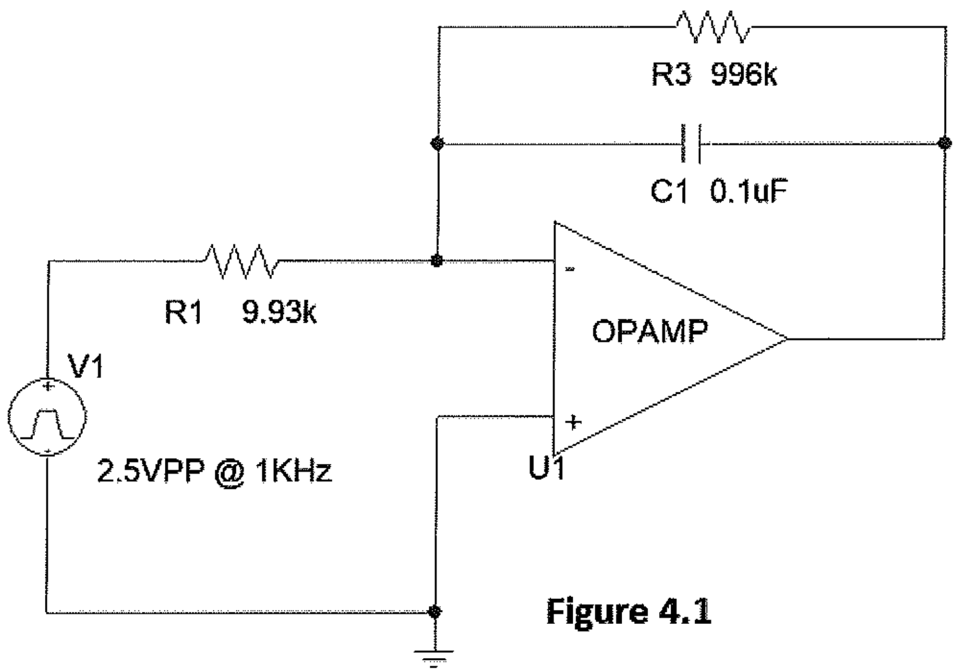


Figure 4.1

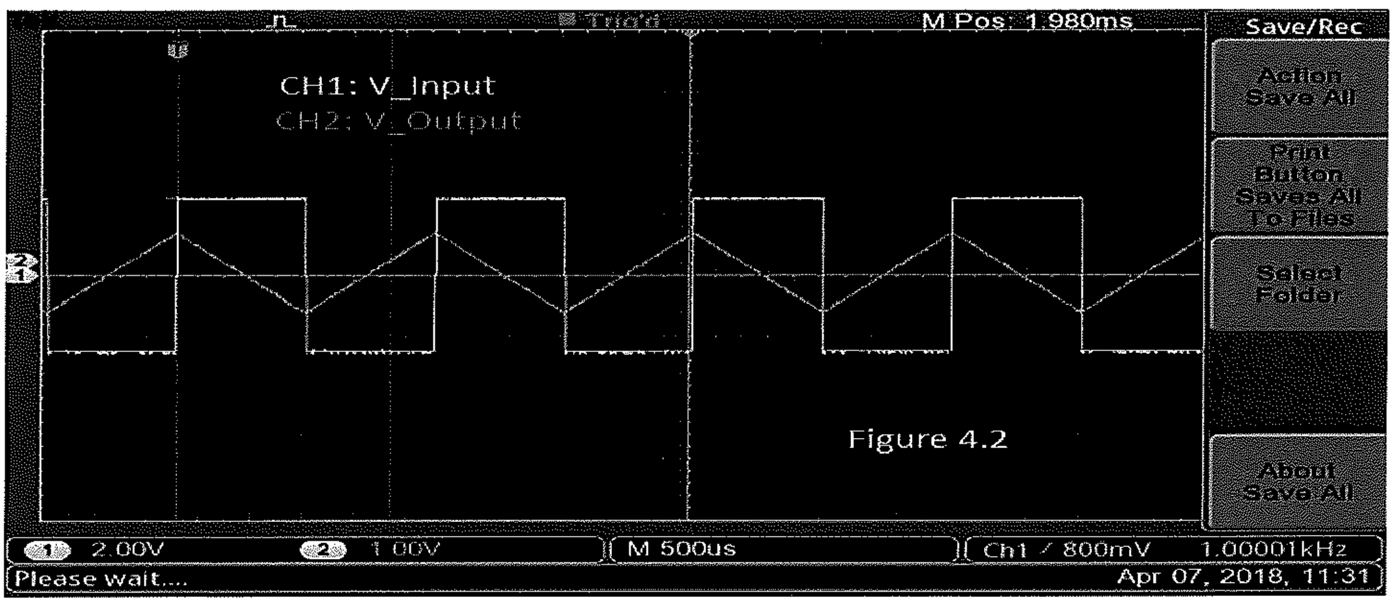


Figure 4.2

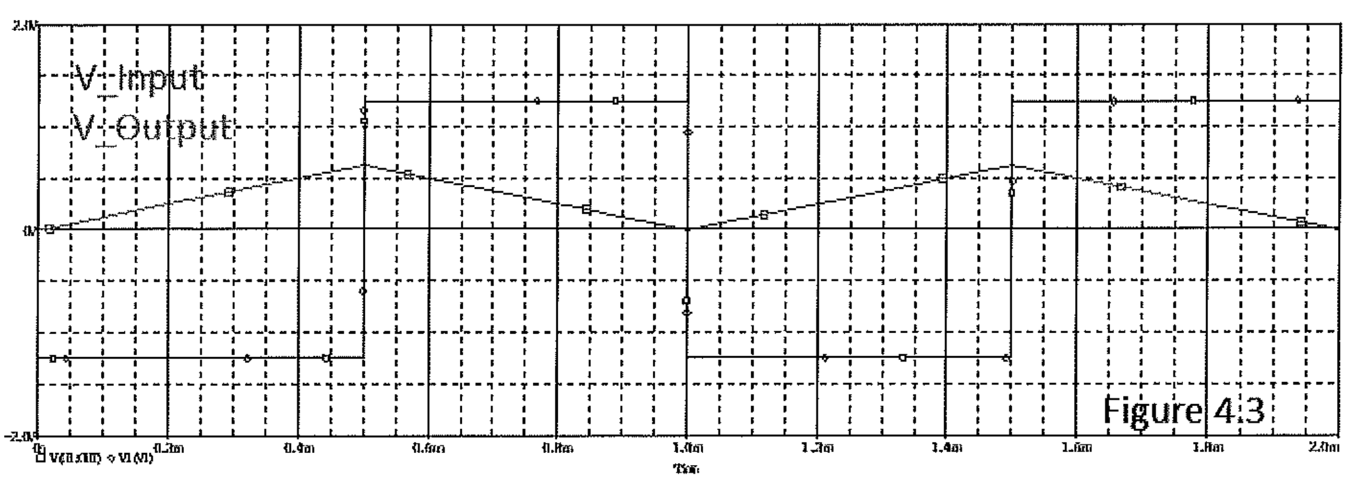


Figure 4.3

Part 5:

The op-amp differentiator circuit, as seen in Figure 5.1, was built. R1 was measured at 9.93k-ohm. An oscilloscope was used to analyze and measure the op-amp output voltage in relation to the input voltage, as seen in Figure 5.2. The waveforms were calculated to be $V_o(t) = -4.79$ @ $0<t<500us$ and $V_o(t) = 4.79$ @ $500us<t<1000us$ for the sawtooth wave's high and low voltage outputs respectively for one period, 0 < t < 1ms. (Calculation 5.1) The circuit was built and analyzed in PSPICE, as seen in Figure 5.3. The sawtooth waveform was created in PSPICE using a square wave input with a manipulated transition time to match ½ the period, thus producing a sawtooth wave.

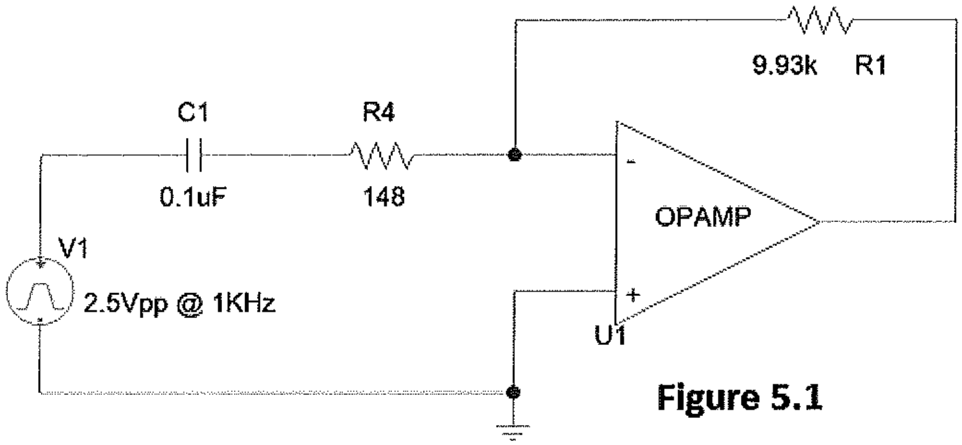


Figure 5.1

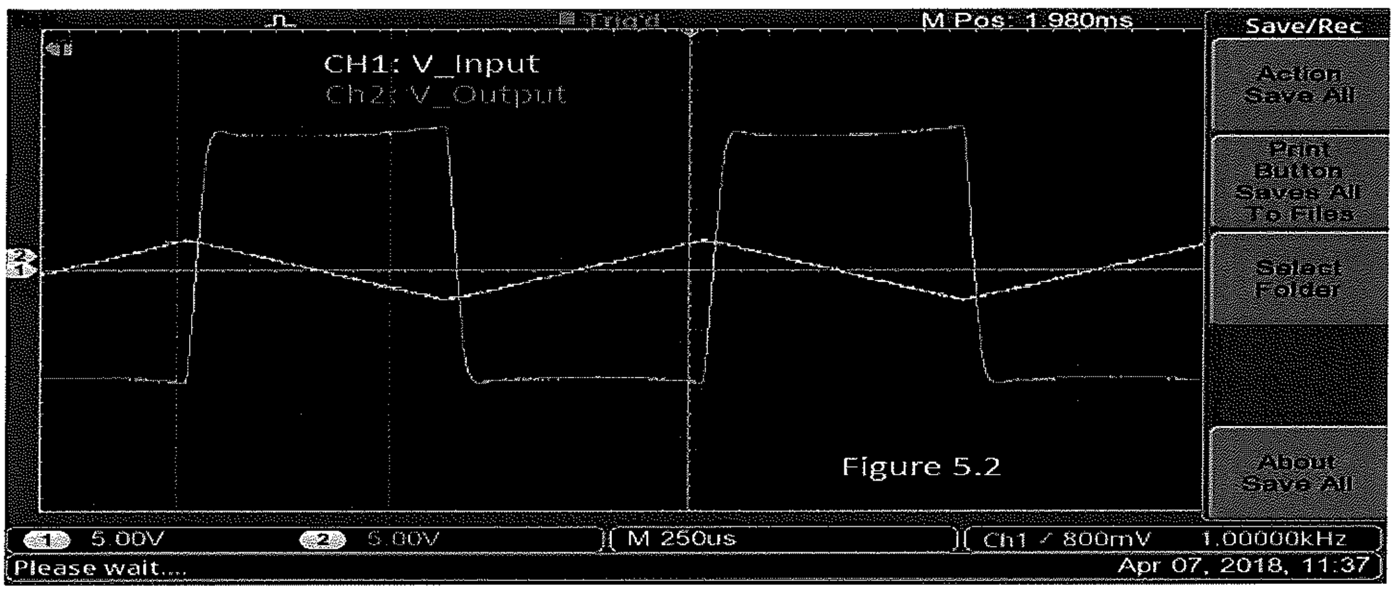


Figure 5.2

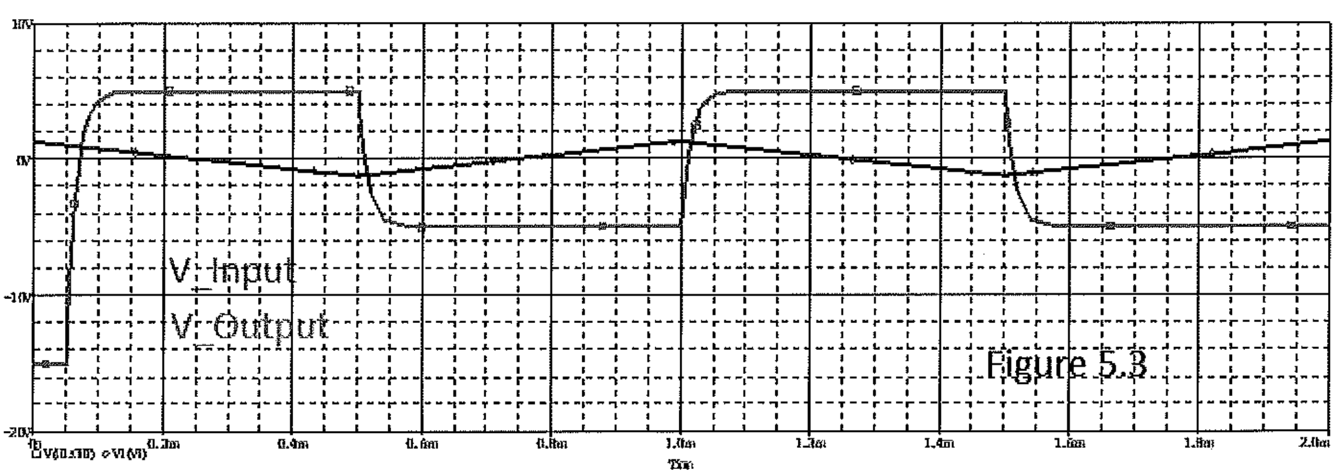


Figure 5.3

Data & Calculations

Calculation 1.1

$$\textit{Use Equation } 0.1$$

$$5t = RC$$

$$C = \frac{5t}{R} = \frac{5(820us)}{9.93k} = 16.5nF$$

Calculation 1.2

$$\%Error \to \frac{|x_{theoretical} - x_{measured}|}{|x_{theoretical}|} x\ 100$$

$$\frac{|14.9nF - 16.5nF|}{14.9nF} * 100 = 10.7\%$$

Calculation 1.3

$$V_c(t) = 10 - 10e^{e^{\frac{-1}{9.93k(14.9nF)}}}$$

Substitute measured values into (EQ 1.3)

$$@\ 0 \le t \le\ 2.5ms, \quad V_c(t) = 10 - 10e^{-6758.72t}$$

$$V_c(2.5ms) = 10 - 10e^{\frac{-1}{9.93k(14.9nF)}(2.5ms)}$$

I.C.

$$V_c(2.5ms) = V_2 = {\sim}10$$

$$V_c(t) = 10e^{\frac{1}{9.93k(14.9nF)}(2.5ms)}e^{-\frac{1}{9.93k(14.9nF)}t}$$

Substitute measured values and calculated V_2 into (EQ 1.4)

$$@\ 2.5ms \le t \le\ 5ms, \quad V_c(t) = (2.17E8)e^{-6758.72t}$$

Calculation 2.1

$$V_c(t) = 10e^{e^{\frac{-1}{9.93k(14.9nF)}}}$$

Substitute measured values into (EQ 2.2)

$$@\ 0 \le t \le\ 2.5ms, \quad V_c(t) = 10e^{-6758.72t}$$

$$V_R(2.5ms) = 10e^{\frac{-1}{9.93k(14.9nF)}(2.5ms)}$$

I.C.

$$V_R(2.5ms) = V_2 = {\sim} - 10$$

$$V_R(t) = \left(-10e^{\frac{1}{9.93k(14.9nF)}(2.5)}\right)e^{\frac{-1}{9.93k(14.9nF)}t}$$

Substitute measured values and calculated V_2 into (EQ 2.3)

$$@\ 2.5ms \le t \le\ 5ms, \quad V_c(t) = -(2.17E8)e^{-6758.72t}$$

Calculation 3.1

$$i(t) = \frac{25}{4980000}e^{-\left(\frac{0.001+0.00001}{(0.001*0.0001*4980000)}\right)t}$$

Substitute measured values into (EQ 3.3)

$$i(t) = (5 * 10^{-6})e^{-0.02t}$$

Simplify. Compare to $i(t) = (5 * 10^{-6})e^{-0.01t}$

Calculation 4.1

$$V_o(t) = \frac{--1.25}{993k(0.1uF)}t$$ Substitute measured values into (EQ 4.2)

$$@\, 0 \leq t \leq 500us, \quad V_o(t) = 1258.1t$$ Simplify.

$$V_o(500us) = 1258.1(0.0005) = 0.629$$ I.C.

$$V_o(t) = \frac{-1.25}{9930 * 00000001}t + \left(0.629 - \frac{-1.25}{9.93k(0.1uF)}(500us)\right)$$ Substitute measured values into (EQ 4.3)

$$@\, 500us \leq t \leq 1000us, \quad V_o(t) = -1258.1t + 1.25$$ Simplify.

Calculation 5.1

$$@\, 0 \leq t \leq 500us, \quad V_o(t) = -5000(9930 * 0.0000001)$$ Substitute measured values into (EQ 5.3)

$$@\, 0 \leq t \leq 500us, \quad V_o(t) = -4.79$$ Simplify.

$$@\, 500us \leq t \leq 1000us, \quad V_o(t) = 5000(9930 * 0.0000001)$$ Substitute measured values into (EQ 5.5)

$$@\, 0 \leq t \leq 500us, \quad V_o(t) = 4.79$$ Simplify.

Discussion of Results

Part 1:

Experiment went as expected. The estimated/calculated capacitor value (16.5uF) came close to the actual value (14.9nF). The 10.7% error was most likely a factor of human error when choosing the start and end time points on the oscilloscope. The charge and discharge waveforms of the capacitor were clearly seen. The derived equation and theoretical graph produced with PSPICE matched nicely with the experimental output captured by the oscilloscope.

Part 2:

Experiment went as expected. The change in voltage across the resistor, due to the charge and discharge waveforms of the capacitor, were clearly seen. The derived equation and theoretical graph produced with PSPICE matched nicely with the experimental output captured by the oscilloscope.

Part 3:

The final results of the experiment were as expected, however, the experiment itself did not go very smoothly. The current flowing through the circuit in response to the large capacitor's discharge was clearly seen. Contrary to the instruction to take data points every 20 seconds, data points were taken every 1 second for 30 seconds. While 20 second intervals would work great in ideal conditions, it was found that the ammeter was incapable of accurately measuring the current once it dropped below 3.5uA or so. As a result, this portion of the experiment was repeated after having taken data points every 20 seconds for 15 minutes and receiving poor results. Microsoft's Excel produced an accurate equation based upon the gathered data points which nicely corresponded with the derived equation and PSPICE graph.

Part 4:

Experiment went as expected. The op-amp output voltage was clearly seen to correspond to the integration of the input voltage wave. The derived equation and theoretical graph produced with PSPICE matched nicely with the experimental output captured by the oscilloscope.

Part 5:

Experiment went as expected. The op-amp output voltage was clearly seen to correspond to the derivative of the input voltage wave. The derived equation and theoretical graph produced with PSPICE matched nicely with the experimental output captured by the oscilloscope.

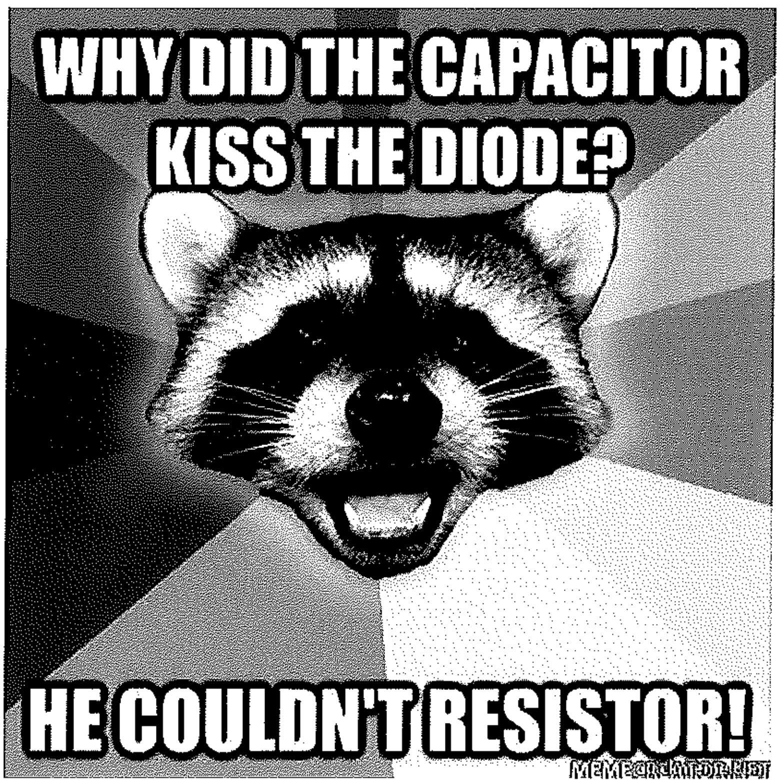
Appendix

A freshman goes into Radio Shack and asks for a capacitor.

"Will that be cash? Asks the clerk.

"Nah, charge it." Says the freshman.

Cool



Name: **Jared Fowler** Excellent lab 100/100

Date: May 1, 2018

Class: Engr M20/L – Moorpark College

Instructor: Hadi Darejeh

Lab 5: Second Order Circuits

total deductions (-3)

but

Lab Partners: Roland Terezon

Daniel Alaya

Objective

Analyze second-order circuits using standardized methods and PSPICE, and compare the theoretical results with those found in the lab experiment.

Theory

Note: Theories, concepts, and proofs heavily quoted from "Fundamentals of Electric Circuits" 5th edition & Wikipedia.

Second-Order Circuits

A circuit which is characterized by a second-order differential equation. It consists of resistors and the equivalent of two energy storage elements.

Capacitor, C

Device used to store an electric charge, consisting of one or more pairs of conductors separated by an insulator. The voltage across a capacitor in respect to time: $V_c(t) = \frac{1}{C}\int i_C dt + V_C(0)$, and the current in respect to time: $i_C(t) = C\frac{dv}{dt}$.

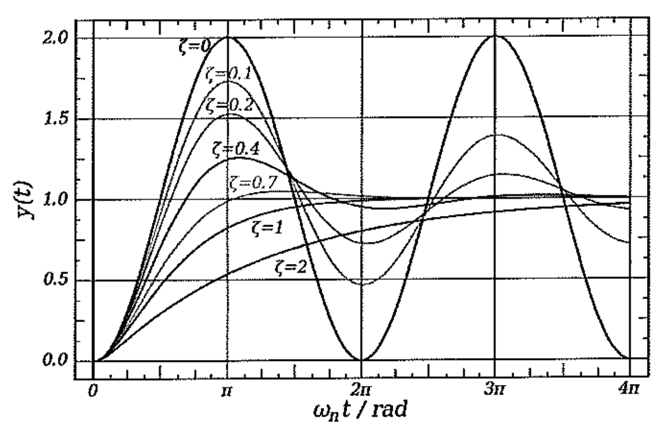
Inductor, L

Device that stores energy in a magnetic field when electric current flows through it. An inductor typically consists of an insulated wire wound into a coil around a core. The voltage across an inductor in respect to time: $V_L(t) = L\frac{di}{dt}$, and the current in respect to time: $i_L(t) = \frac{1}{L}\int v_L dt + i_L(0)$.

Damping Ratio

Dimensionless measure describing how oscillations in a system decay after a disturbance. The value is denoted by ζ (zeta, z), which can vary from undamped (z=0), underdamped (z<1), critically damped (z=1), and overdamped (z>1).

Damping is caused by the resistance in the circuit. It determines whether or not the circuit will resonate naturally (that is, without a driving source). Circuits which will resonate in this way are described as underdamped and those that will not are overdamped.



$$Damped\ Frequency:$$

$$\omega = \frac{2\pi}{T_{Period}}$$ **EQ 0.0.1**

$$Damping\ Ratios:$$

$$\xi = \frac{R}{2\omega L}$$ RCL in Series **EQ 0.0.2**

$$\xi = \frac{1}{2\omega RC}$$ CL in Parallel **EQ 0.0.3**

$$Differential\ Equations - Expected\ results\ based\ off\ Daming\ Ratio$$

$$if\ \xi = 1 \rightarrow x(t) = k_1 e^{s_1 t} + k_2 t e^{s_2 t}$$ **EQ 0.1** "Critical Damping"

$$if\ \xi > 1 \rightarrow x(t) = k_1 e^{s_1 t} + k_2 e^{s_2 t}$$ **EQ 0.2** "Over-Damped'

$$if\ \xi < 1 \rightarrow x(t) = e^{at}(k_1 \cos(\omega t) + k_2 \sin(\omega t))$$ **EQ 0.3** "Under-Damped"

$$if\ \xi = 0 \rightarrow x(t) = k_1 \cos(\omega t) + k_2 \sin(\omega t)$$ **EQ 0.4** "Un-Damped"

RCL in Series

The circuit shown in Figure 0.1 is analyzed in this lab. In this circuit, a capacitor, inductor, and resistor are in series. A square wave pulse is used for the input voltage. The pulse's high and low times are large enough to allow a complete charge and discharge of the system over the course of one period. The voltage across the resistor is derived below. Note that the waveform will depend upon the damping ratio as seen in EQ 1.3.

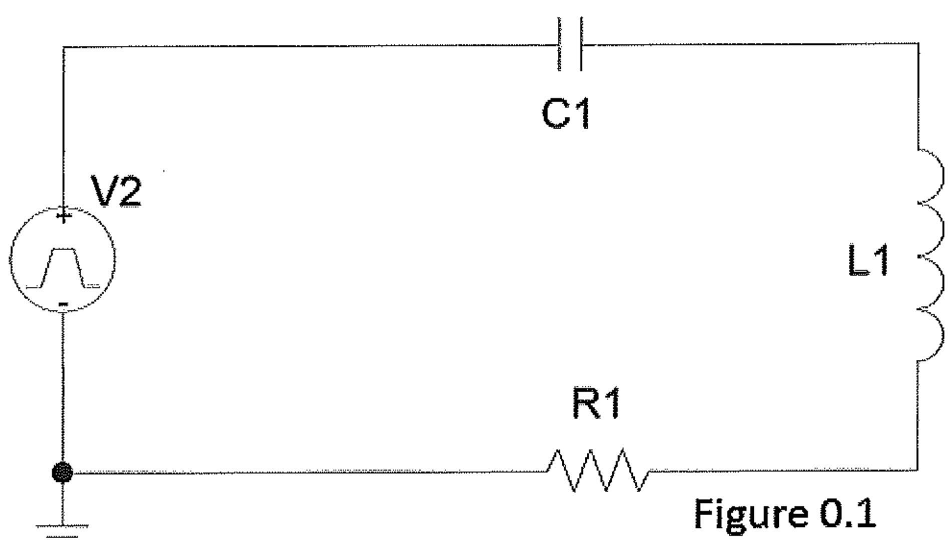


Figure 0.1

$$Solve\ for\ voltage\ across\ R_1, that\ is, V_R(t)$$

$$V_s = V_c + V_L + V_R$$

KVL

Note: $V_R = iR \rightarrow i = \frac{V_R}{R}$

$$\left[V_s = \frac{1}{c}\int idt\ V_C(0) + \frac{Ldi}{dt} + V_R\right]\frac{d}{dt}$$

Substitute and take derivative

$$\frac{dV_R}{dt} + \frac{1}{C}i + \frac{Ld^2i}{dt^2} = 0$$

$$\frac{L}{R}\left(\frac{d^2V_R}{dt^2}\right) + \frac{dV_R}{dt} + \frac{V_R}{RC} = 0$$

Substitute current for V_R

$$\boldsymbol{\frac{d^2V_R}{dt^2} + \frac{R}{L}\left(\frac{dV_R}{dt}\right) + \frac{1}{LC}(V_R) = 0}$$

EQ 1.1

$$x = x_f + x_n$$

$$x_f = 0$$

Because right side of '=' is constant 0.

$$x_n = ke^{st}$$

Differential Equations. (DE)

$$\frac{d^2ke^{st}}{dt^2} + \frac{Rdke^{st}}{Ldt} + \frac{1}{CL}ke^{st} = 0$$

$$\boldsymbol{S^2 + \frac{R}{L}s + \frac{1}{CL} = 0}$$

EQ 1.2

$$Note: \boldsymbol{\omega^2 = \frac{1}{CL}}\ and\ 2\omega\xi = \frac{R}{L} \rightarrow \xi = \boldsymbol{\frac{R\sqrt{C}}{2\sqrt{L}}}$$

EQ 1.3

$$\boldsymbol{s_1, s_2 = -\frac{R}{2L} \pm \frac{\sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{CL}}}{2}}$$

EQ 1.4
Quadratic Equation.

$$From\ here, solve\ for\ k_1\ and\ k_2\ based\ upon\ expected\ output\ from\ damping\ factor\ \xi.$$
$$See\ EQ\ 0.1 - EQ\ 0.4$$

$$Solve\ for\ k_1\ by\ solving\ at\ V_R(0).$$
$$Solve\ for\ k_2\ by\ solving\ for\ i, and\ then\ using\ the\ initial\ voltage\ across\ Inductor\ which\ is\ V_s$$

RCL in Parallel

The circuit shown in Figure 0.2 is analyzed in this lab. In this circuit, a capacitor and inductor in parallel. A square wave pulse is used for the input voltage. The pulse's high and low times are large enough to allow a complete charge and discharge of the system over the course of one period. The voltage across the capacitor is derived below. Note that the waveform will depend upon the damping ratio as seen in EQ 2.4.

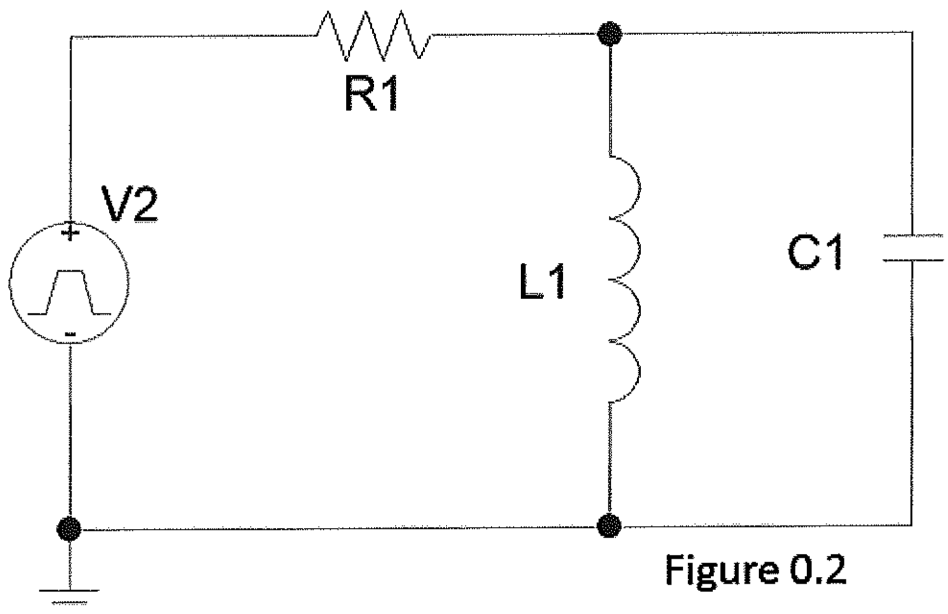


Figure 0.2

$$i_R = i_L + i_C$$ KCL

$$\frac{V - V_C}{R} = \frac{1}{L}\int V_C dt + i(0) + \frac{CdV_C}{dt}$$

$$\frac{d}{dt}[EQ\ 2.1]$$ Take Derivative.

$$\boldsymbol{\frac{d^2V_C}{dt^2} + \frac{dV_C}{dtRC} + \frac{1}{LC}V_C = 0}$$ **EQ 2.1**

$$x = x_f + x_n$$

$$x_f = 0$$ Because right side of '=' is constant 0.

$$x_n = ke^{st}$$ Differential Equations. (DE)

$$\frac{d^2ke^{st}}{dt^2} + \frac{dke^{st}}{RCdt} + \frac{1}{LC}ke^{st} = 0$$

$$\boldsymbol{S^2 + \frac{1}{RC}s + \frac{1}{LC} = 0}$$ **EQ 2.2**

$$Note: \boldsymbol{\omega^2 = \frac{1}{LC}}\ and\ 2\omega\xi = \frac{1}{RC} \rightarrow \xi = \boldsymbol{\frac{\sqrt{LC}}{2RC}}$$ **EQ 2.3**

$$\boldsymbol{s_1, s_2 = -\frac{1}{2RC} \pm \frac{\sqrt{\left(\frac{1}{RC}\right)^2 - \frac{4}{LC}}}{2}}$$ **EQ 2.4**
Quadratic Equation.

From here, solve for k_1 and k_2 based upon expected output from damping factor ξ.

$$See\ EQ\ 0.1 - EQ\ 0.4$$

$$Solve\ for\ k_1\ by\ solving\ at\ V(0).$$
$$Solve\ for\ k_2\ by\ taking\ the\ derivaative\ of\ the\ entire\ equation, convert\ to\ a\ function\ of\ current\ by\ using\ the\ relation{:}\ i(t) = \frac{CdV(t)}{dt}, and\ solving\ at\ i(0)$$

Internal Resistance of Power Supply (Square Wave Generator)

The square wave generator used in Figures 0.1 and 0.2 has an internal resistance. This resistance can be found by sampling the voltage across the generator when no load is present, and then again when a load is present. It's easiest to picture the internal resistance as a resistor in series with the voltage source, which Figure 0.3 illustrates. This is mere voltage division, where V_0 is the voltage with no load, and V_L is the voltage across the added load, R_L. The internal resistance, R_i, is derived below.

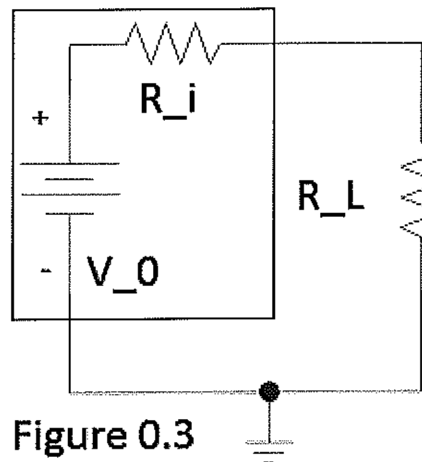


Figure 0.3

$$V_L = V_0\left(\frac{R_L}{R_L + R_i}\right)$$ Voltage Division

$$\left(\frac{V_0}{V_L}\right)^{-1} = \left(\frac{R_L}{R_L + R_i}\right)^{-1}$$

$$R_L\left(\frac{V_0}{V_L}\right) = R_L + R_i$$

$$R_i = R_L\left(\frac{V_0}{V_L} - 1\right)$$ **EQ 3.1**

Procedure

Part 1:

The internal resistance of the square wave generator was determined. The circuit, as seen in Figure 1.1, was constructed. The voltage across the voltage source was first measured without any load resistor, R_L. Five additional voltage readings were taken with the load resistor connected, each time the load resistor being changed to a different value. The results can be seen in the table below.

The internal resistance was found experimentally to be 50.25 Ohms. The square wave generator voltage source claims to have an internal resistance of 50 Ohms. The experimental and actual internal resistances differ by 0.5%.

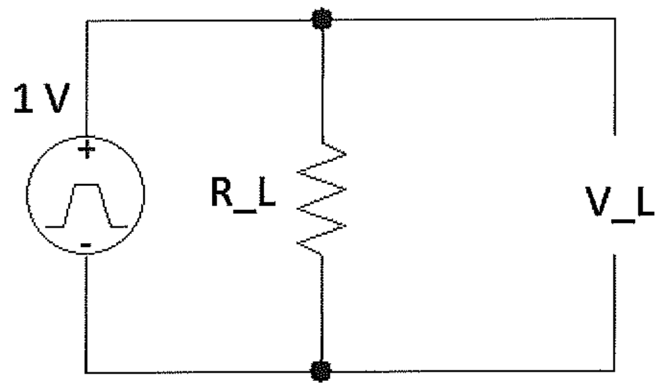


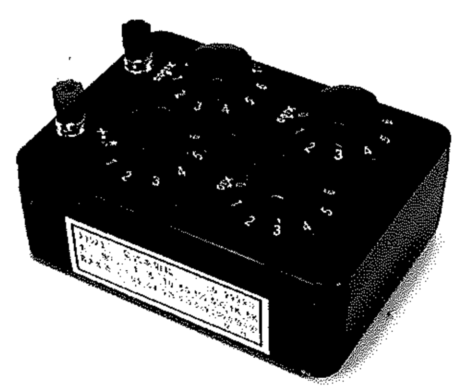
Figure 1.1

V_{in}	Resistance (R)	V_{out}	Internal Resistance (R_i) – See (Calculation 1.1)
1 V_{RMS}	OPEN	2.01	-
1 V_{RMS}	300	1.72	50.58
1 V_{RMS}	200	1.60	51.25
1 V_{RMS}	100	1.34	50.00
1 V_{RMS}	50	1.00	50.50
1 V_{RMS}	25	0.68	48.90

Average: 50.25-Ohm
(Theoretical Value: 50 Ohms) -> % Error: 0.5% - See (Calculation 1.2)

Part 2:

The internal resistance of the inductor was found using an ohmmeter. The ohmmeter determines the resistance by sending a small current through the measured object and determining the voltage drop. While the inductor will initially read at a higher resistance due to its anti-current-change behavior, the inductor will eventually act like a short in the circuit, at which time the internal resistance can be measured. This internal resistance is the resistance of the material itself which makes up the inductor. Per this lab, an inductor decade box was used which means that the internal resistance changes based upon the box setting. At the 70mH setting, the internal resistance was measured to be around 0.5-Ohm. Because the internal resistance is such a low value, it will not be taken into consideration in the calculations.



Part 3a:

The circuit shown in Figure 3.1 was built. An oscilloscope was used to analyze both the input voltage from the square wave generator, and the voltage across the 6.8K-ohm resistor.

PSpice was used to acquire a theoretical output curve as seen in Figure 3.1.1. The experimental output curve measured with the oscilloscope is shown in Figure 3.1.2. Both curves accurately identify the curve as being overdamped. The theoretical equation was found to be: $V_R(t) = 1.64e^{-18066t} - 1.64e^{79078}$ (Calculation 3.1)

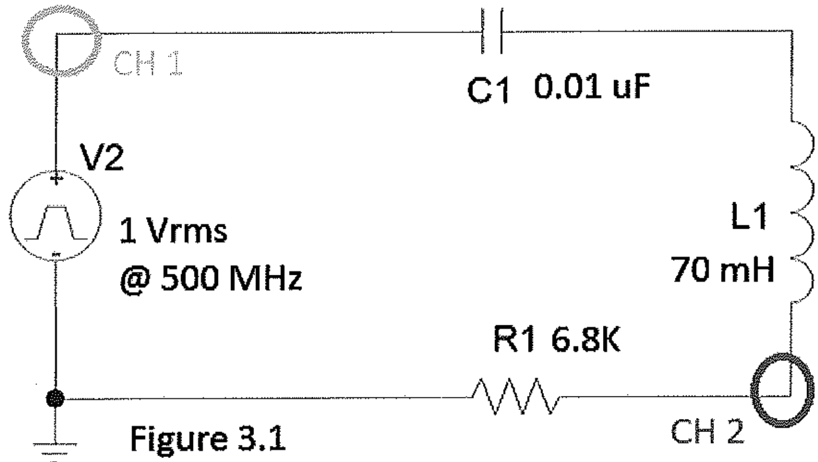


Figure 3.1

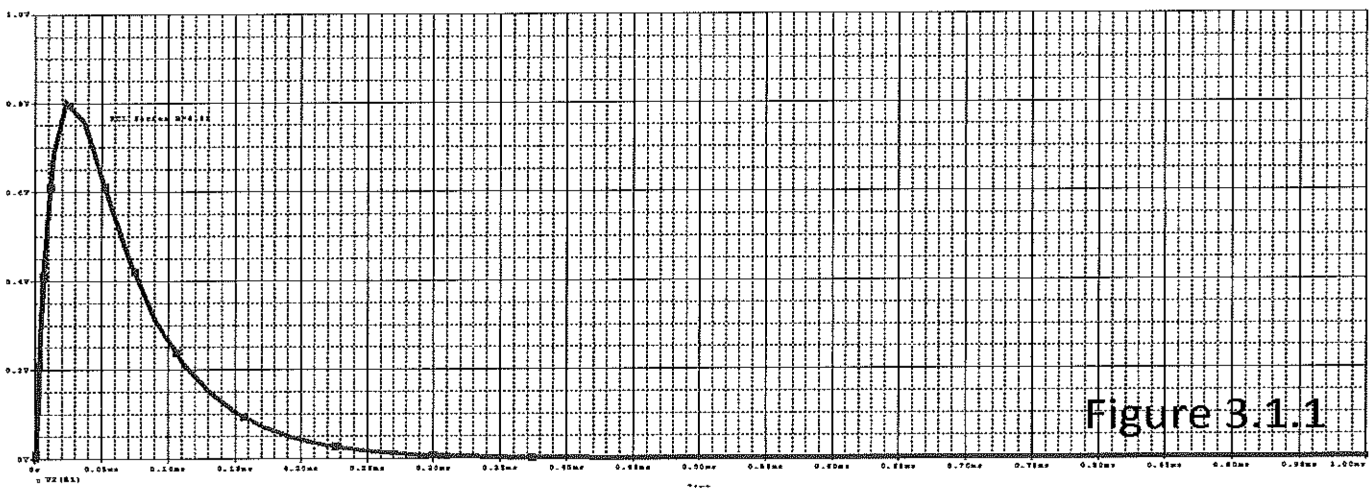


Figure 3.1.1

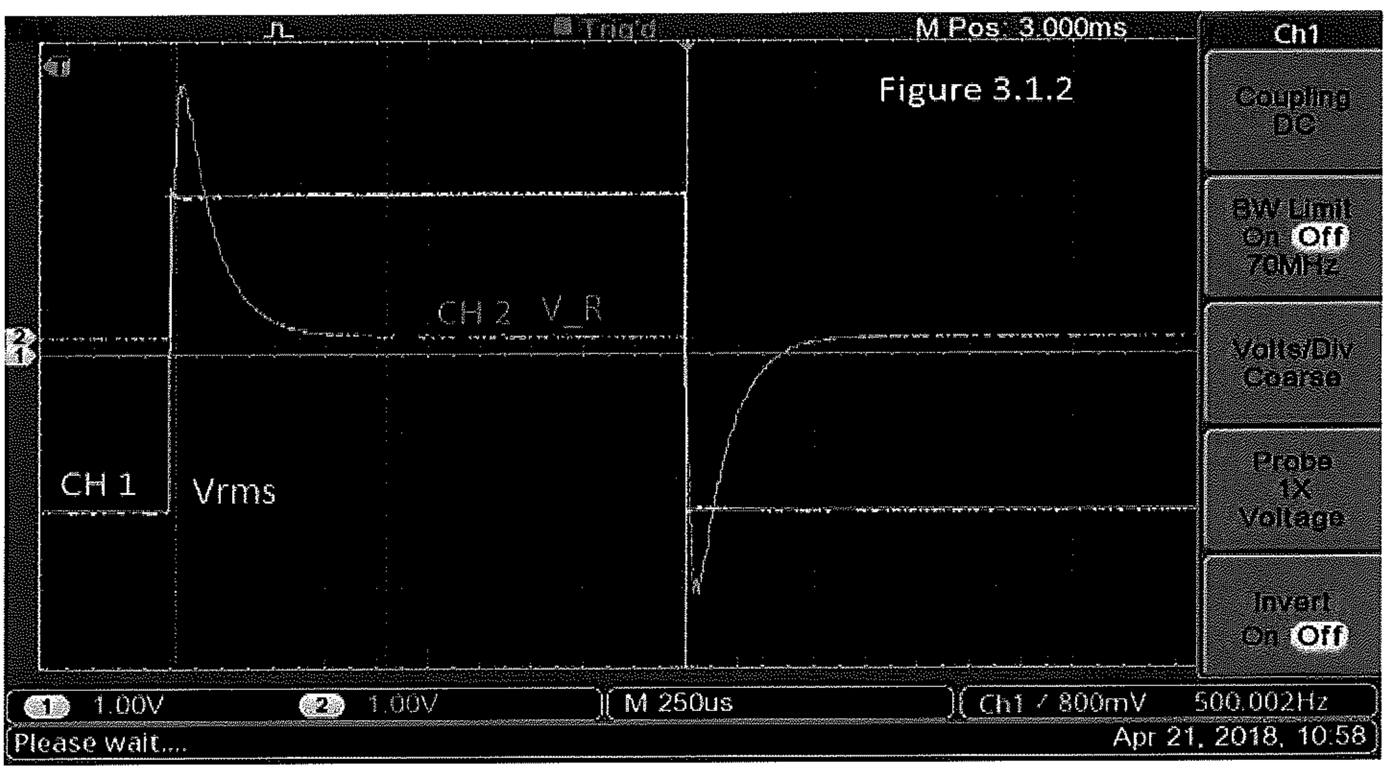


Figure 3.1.2

Part 3b:

The circuit shown in Figure 3.2 was built. An oscilloscope was used to analyze both the input voltage from the square wave generator, and the voltage across the 560-ohm resistor.

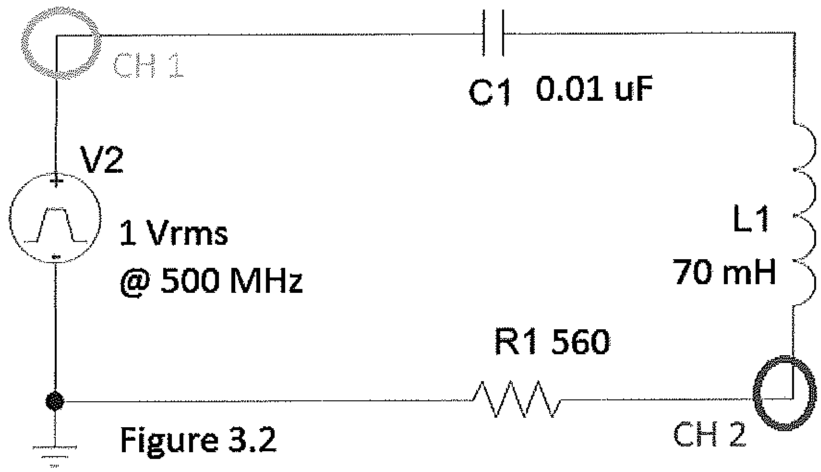


Figure 3.2

PSpice was used to acquire a theoretical output curve as seen in Figure 3.2.1. The experimental output curve measured with the oscilloscope is shown in Figure 3.2.2. Both curves accurately identify the curve as being underdamped. The theoretical equation was found to be: $V_R(t) = e^{-4000t}(0.213sin(37584t))$ (Calculation 3.2) From the oscilloscope reading, the oscillating period was observed to be about 175us. The damping frequency and ratio were estimated to be 35903 rads/sec and 0.111 respectively. (Calculation 3.2.1)

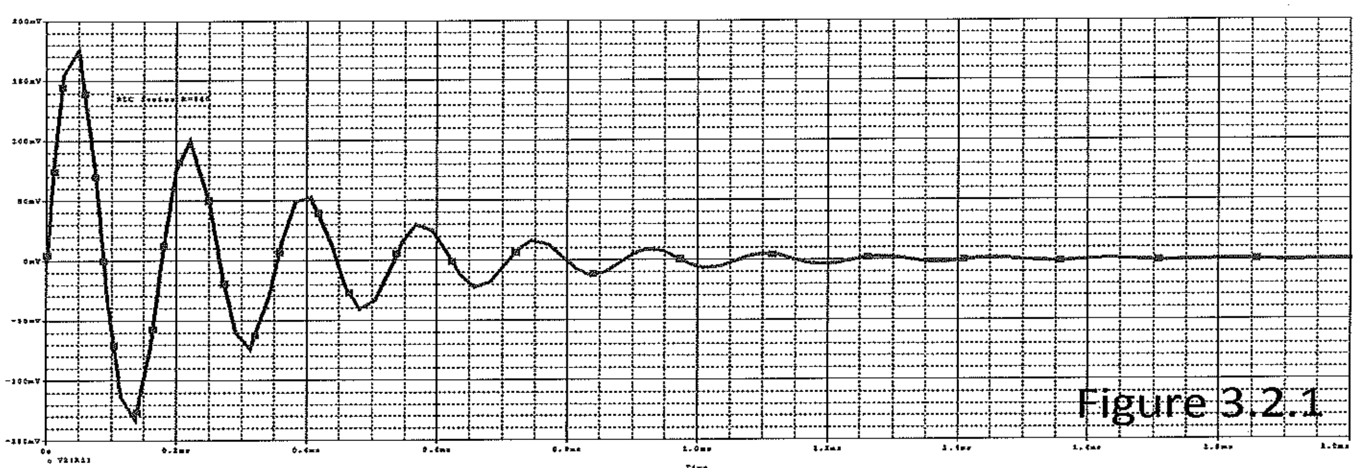


Figure 3.2.1

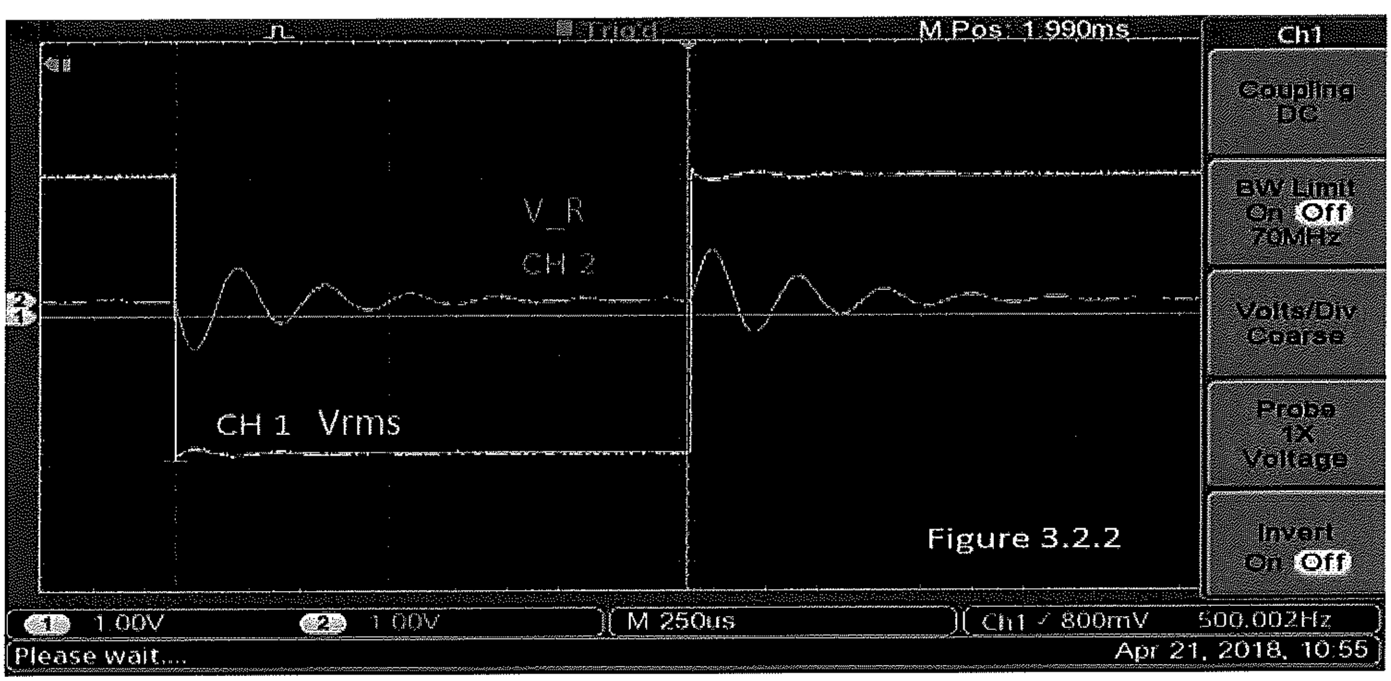


Figure 3.2.2

Part 3c:

The circuit shown in Figure 3.3 was built. An oscilloscope was used to analyze both the input voltage from the square wave generator, and the voltage across the capacitor.

PSpice was used to acquire a theoretical output curve as seen in Figure 3.3.1. The experimental output curve measured with the oscilloscope is shown in Figure 3.3.2. Both curves accurately identify the curve as being underdamped. The theoretical equation was found to be: $V_c(t) = e^{-7353t}(0.397sin(37074t))$ (Calculation 3.3) From the oscilloscope reading, the oscillating period was observed to be about 175us. The damping frequency and ratio were estimated to be 35903 rads/sec and 0.205 respectively. (Calculation 3.3.1)

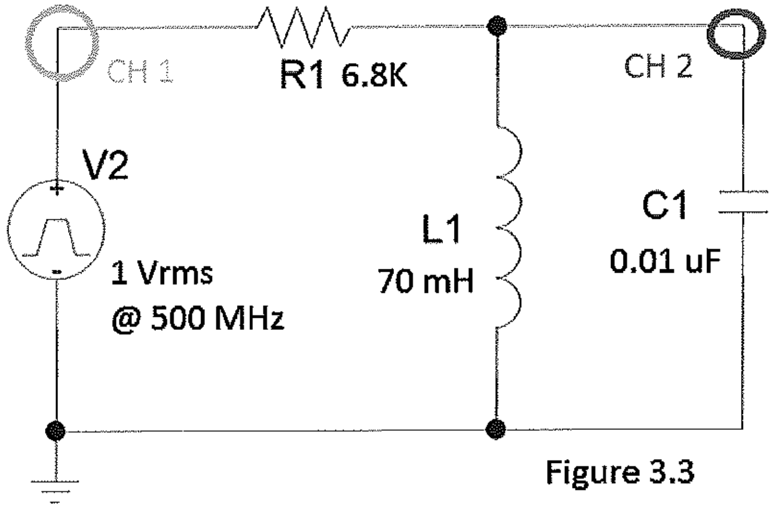


Figure 3.3

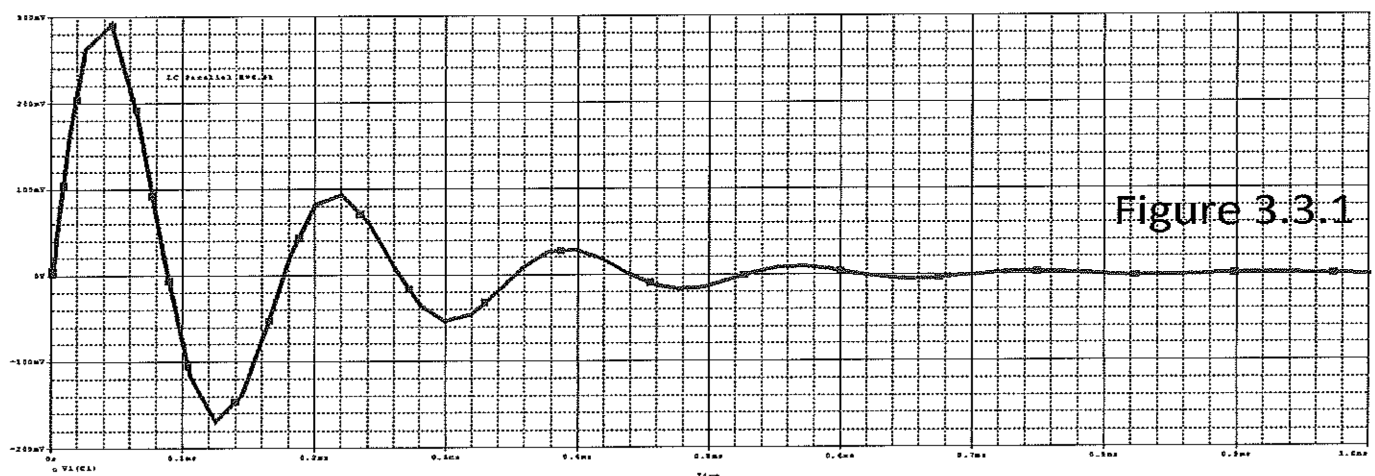


Figure 3.3.1

Label plots

CH1 Vin

CH2 Vout

(-3)

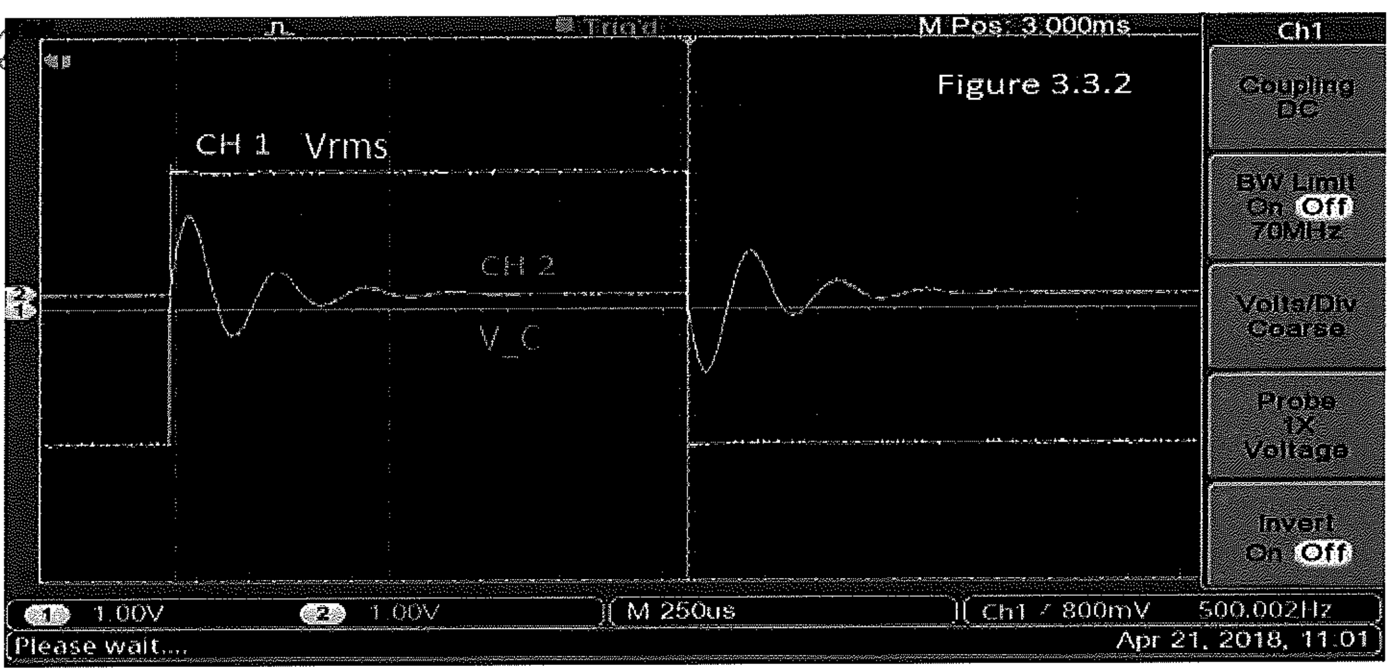


Figure 3.3.2

Part 3d:

The circuit shown in Figure 3.4 was built. An oscilloscope was used to analyze both the input voltage from the square wave generator, and the voltage across the capacitor.

PSpice was used to acquire a theoretical output curve as seen in Figure 3.4.1. The experimental output curve measured with the oscilloscope is shown in Figure 3.4.2. Both curves accurately identify the curve as being overdamped. The theoretical equation was found to be: $V_C(t) = 0.091e^{-8337t} - 0.091e^{-170159t}$ (Calculation 3.4)

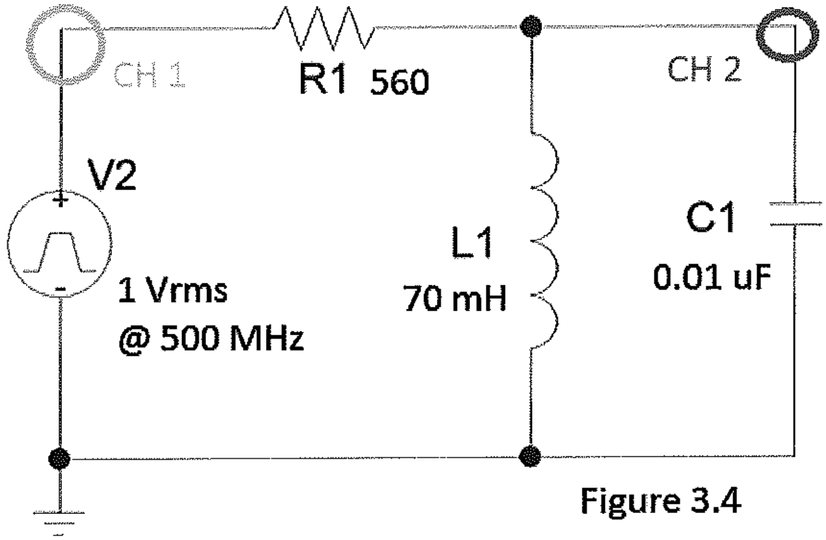


Figure 3.4

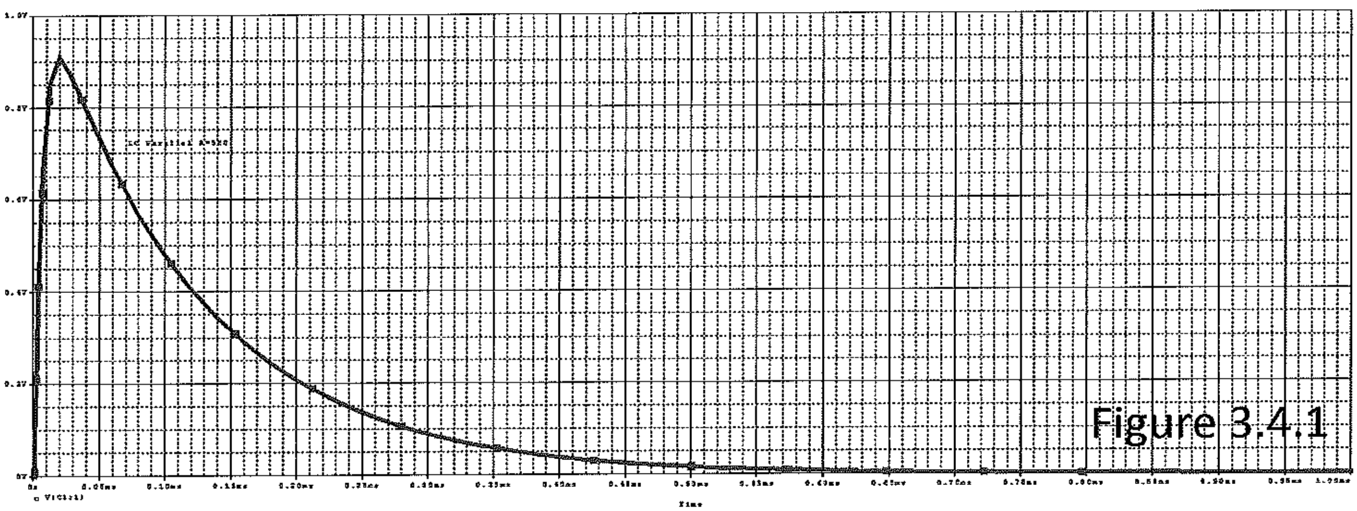


Figure 3.4.1

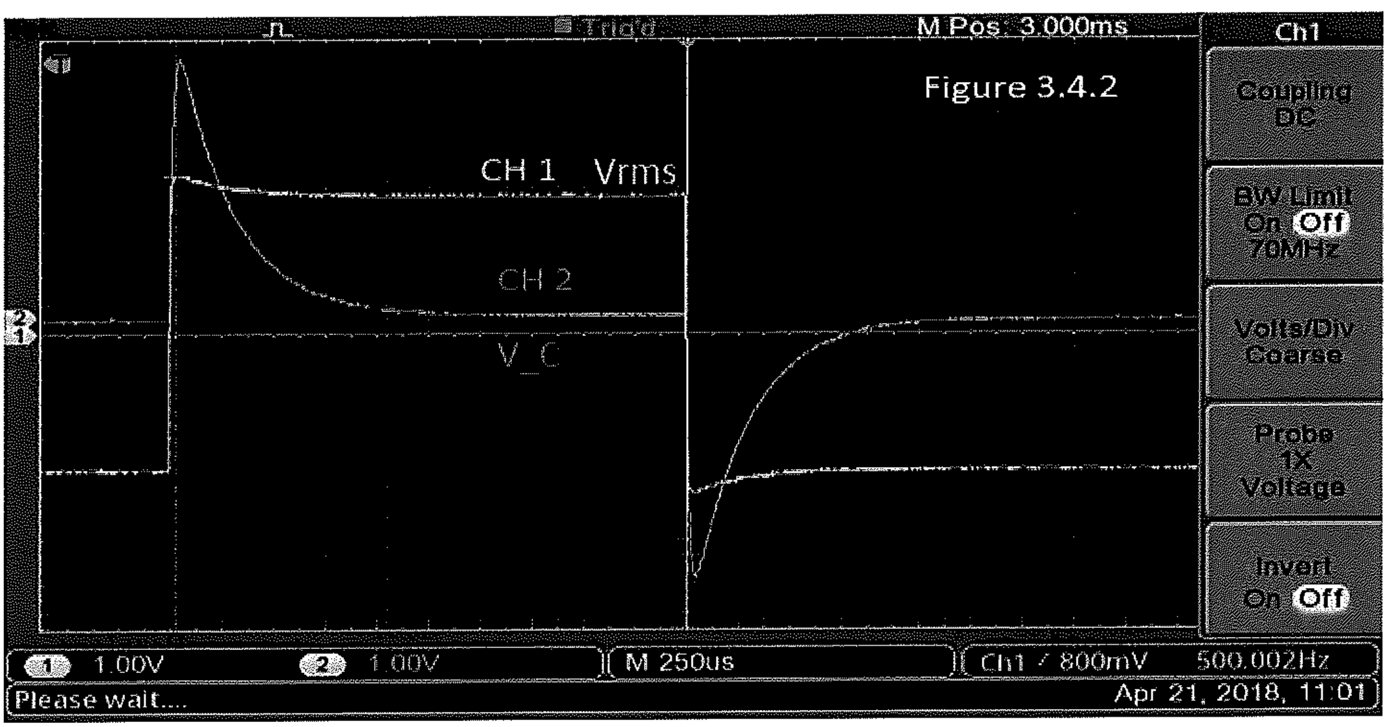


Figure 3.4.2

Data & Calculations

Calculation 1.1

$$R_i = R_L\left(\frac{V_0}{V_L} - 1\right)$$ Use EQ 3.1

$$R_i = R_L\left(\frac{2.01}{V_L} - 1\right)$$ Plug in known values.

$$R_i = 300\left(\frac{2.01}{1.72} - 1\right)$$ Solve for each scenario...
Example: Load resistor is 300 Ohms

$$R_i = 50.58$$

Calculation 1.2

$$\% \; Error = \frac{|A_{theoretical} - A_{experimental}|}{A_{theoretical}} x100$$ Definition of % error.

$$\% \; Error = \frac{|50.0 - 5.25|}{50.0} x100$$ Plug in actual resistance and averaged experimental resistance.

$$\% \; Error = 0.5\%$$ Solve

Calculation 3.1

$$\frac{d^2V_R}{dt^2} + \frac{R}{L}\left(\frac{dV_R}{dt}\right) + \frac{1}{LC}(V_R) = 0$$ Use EQ 1.1

$$\frac{d^2V_R}{dt^2} + 97143\left(\frac{dV_R}{dt}\right) + 1428570000(V_R) = 0$$ Rewrite with actual values for R, L, C

$$\xi = \frac{R\sqrt{C}}{2\sqrt{L}} = 1.285 \rightarrow Overdamped$$ Use EQ 1.3

$$s_1, s_2 = -\frac{R}{2L} \pm \frac{\sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{CL}}}{2}$$ Use EQ 1.4

$$s_1, s_2 = -48572 \pm 30506 = -18066, -79078$$

$$if\; \xi > 1 \rightarrow x(t) = k_1e^{s_1t} + k_2e^{s_2t}$$ Use EQ 0.2

$$V_R(t) = k_1 e^{-18066t} + k_2 e^{-79078t}$$
$$V_R(0) = 0 = k_1 + k_2 \rightarrow k_1 = -k_2$$

$$Note: i = \frac{V}{R}, V_L = \frac{Ldi}{dt}$$

$$i_R(t) = \frac{1}{6800}\left(k_1 e^{-18066t} - k_1 e^{-79078t}\right)$$

Solve for initial voltage across Inductor

$$V_L(t) = 0.00001(-18066k_1 e^{-18066t} + 79078k_2 e^{-79078t})$$

$$V_L(0) = 1V = .00001(-18066k_1 + 79078k_1)$$
$$\rightarrow k_1 = 1.64, k_2 = -1.64$$

$$\boldsymbol{V_R(t) = 1.64e^{-18066t} - 1.64e^{79078}}$$

Calculation 3.2

$$\frac{d^2V_R}{dt^2} + \frac{R}{L}\left(\frac{dV_R}{dt}\right) + \frac{1}{LC}(V_R) = 0$$

Use EQ 1.1

$$\frac{d^2V_R}{dt^2} + 8000\left(\frac{dV_R}{dt}\right) + 1428570000(V_R) = 0$$

Rewrite with actual values for R, L, C

$$\xi = \frac{R\sqrt{C}}{2\sqrt{L}} = 0.106 \rightarrow Underdamped$$

Use EQ 1.3

$$s_1, s_2 = -\frac{R}{2L} \pm \frac{\sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{CL}}}{2}$$

Use EQ 1.4

$$s_1, s_2 = -4000 \pm j37584$$

$$if\ \xi < 1 \rightarrow x(t) = e^{at}(k_1\cos(\omega t) + k_2\sin(\omega t))$$

Use EQ 0.3

$$V_R(t) = e^{-4000t}(k_1\cos(37584t) + k_2\sin(37584t))$$
$$V_R(0) = 0 = k_1$$

$$Note: i = \frac{V}{R}, V_L = \frac{Ldi}{dt}$$

$$i_R(t) = \frac{e^{-4000t}}{560}(k_2\sin(37584t))$$

Solve for initial voltage across Inductor

$$V_L(t) = 0.000125(-4000e^{-4000t}(k_2\sin(37584t)) + e^{-4000t}(k_2 * 37584\cos(37584t))$$

$$V_L(0) = 1V = .000125(37584k_2)$$
$$\rightarrow k_2 = 0.213$$

$$V_R(t) = e^{-4000t}(0.213\sin(37584t))$$

Calculation 3.2.1

$$\omega = \frac{2\pi}{T_{Period}}$$

Use EQ 0.0.1
"Experimental"

$$\omega = \frac{2\pi}{175us} = 35903 \rightarrow \xi = 0.111$$

$$Note: \omega^2 = \frac{1}{CL} \ and \ 2\omega\xi = \frac{R}{L} \rightarrow \xi = \frac{R\sqrt{C}}{2\sqrt{L}}$$

Use EQ 1.3
"Theoretical"

$$\omega = \frac{1}{CL} = 37796 \rightarrow \xi = 0.106$$

$$\%\ Diff\ \omega = \frac{|35903 - 37796|}{37796} x100 = 5\%$$

$$\%\ Diff\ \xi = \frac{|0.111 - 0.106|}{0.106} x100 = 4.7\%$$

Calculation 3.3

$$\frac{d^2V_C}{dt^2} + \frac{dV_C}{dtRC} + \frac{1}{LC}V_C = 0$$

Use EQ 2.1

$$\frac{d^2V_C}{dt^2} + \frac{14706dV_C}{dt} + 1428570000V_C = 0$$

Rewrite with actual values for R, L, C

$$\xi = \frac{\sqrt{LC}}{2RC} = 0.195 \rightarrow Underdamped$$

Use EQ 2.3

$$s_1, s_2 = -\frac{1}{2RC} \pm \frac{\sqrt{\left(\frac{1}{RC}\right)^2 - \frac{4}{LC}}}{2}$$

Use EQ 2.4

$$s_1, s_2 = -7353 \ \pm j37074$$

$$if\ \xi < 1 \rightarrow x(t) = e^{at}(k_1\cos(\omega t) + k_2\sin(\omega t))$$

Use EQ 0.3

$$V_C(t) = e^{-7353t}(k_1\cos(37074t) + k_2\sin(37074t))$$
$$V_C(0) = 0 = \ k_1$$

$$Note: i_C = \frac{CdV}{dt}$$

$$i_C(t) = .01uF(-7353e^{-7353t}(k_2\sin(37074t)) + e^{-7353t}(k_2 * 37074\cos(37074t))$$

$$i_C(0) = i_R - i_L = \frac{1V}{6800} - 0 = 0.147mA$$
$$0.147mA = i_C(0) = .01uF(k_2 * 37074)$$
$$\rightarrow k_2 = 0.397$$

$$\boldsymbol{V_C(t) = e^{-7353t}(0.397sin(37074t))}$$

Calculation 3.3.1

$$\omega = \frac{2\pi}{T_{Period}}$$

Use EQ 0.0.1
"Experimental"

$$\omega = \frac{2\pi}{175us} = 35903 \rightarrow \xi = 0.205$$

$$Note: \omega^2 = \frac{1}{LC} \text{ and } 2\omega\xi = \frac{1}{RC} \rightarrow \xi = \frac{\sqrt{LC}}{2RC}$$
$$\omega = \frac{1}{LC} = 37796 \rightarrow \xi = 0.195$$

Use EQ 2.3
"Theoretical"

$$\% \, Diff \; \omega = \frac{|35903 - 37796|}{37796} x100 = 5\%$$

$$\% \, Diff \; \xi = \frac{|0.205 - 0.195|}{0.195} x100 = 5.13\%$$

Calculation 3.4

$$\frac{d^2V_C}{dt^2} + \frac{dV_C}{dtRC} + \frac{1}{LC}V_C = 0$$

Use EQ 2.1

$$\frac{d^2V_C}{dt^2} + \frac{178571dV_C}{dt} + 1428570000V_C = 0$$

Rewrite with actual values for R, L, C

$$\xi = \frac{\sqrt{LC}}{2RC} = 2.36 \rightarrow Overdamped$$

Use EQ 2.3

$$s_1, s_2 = -\frac{1}{2RC} \pm \frac{\sqrt{\left(\frac{1}{RC}\right)^2 - \frac{4}{LC}}}{2}$$

Use EQ 2.4

$$s_1, s_2 = -89286 \pm 80891 = -8337, -170159$$

$$if \; \xi > 1 \rightarrow x(t) = k_1 e^{s_1 t} + k_2 e^{s_2 t}$$

Use EQ 0.2

$$V_C(t) = k_1 e^{-8337t} + k_2 e^{-170159t}$$
$$V_C(0) = 0 = k_1 + k_2 \rightarrow k_1 = -k_2$$

$$Note: i_C = \frac{CdV}{dt}$$

$$i_C(t) = (0.01uF)(-8337k_1e^{-8337t} + 170159k_1e^{-170159t})$$

$$i_C(0) = i_R - i_L = \frac{1V}{6800} - 0 = 0.147mA$$
$$0.147mA = i_C(0) = .01uF(k_1 * 161882)$$
$$\rightarrow k_1 = 0.091$$

$$\boldsymbol{V_C(t) = 0.091e^{-8337t} - 0.091e^{-170159t}}$$

Discussion of Results

Part 1:

Experiment went as expected. The experimentally found internal resistance of the square wave generator was found to be 50.25-Ohms, which is 0.5% different than the actual manufacture claimed value of 50-Ohms.

Part 2:

Experiment went as expected. The internal resistance of an inductor was practically negligible at less than 1-Ohm, which was expected. Ideal theoretical inductors have an internal resistance of 0-Ohms.

Part 3:

Experiment went as expected. The RCL circuit output exhibited an overdamped waveform with the 6.8K-Ohm resistor, and an underdamped waveform with a 560-Ohm resistor, while the parallel RCL circuit output exhibited an underdamped waveform with the 6.8K-Ohm resistor, and an overdamped waveform with a 560-Ohm resistor. In each case, the theoretical and experimental results matched-up nicely. Based upon the underdamped waveforms, the damping factor and frequencies were calculated and compared to theoretical values. As seen in Calculation 3.2.1, there was only a 5% difference between the theoretical and experimental values for the RCL circuit, and as seen in Calculation 3.3.1, there was only a 5% difference between the theoretical and experimental values for the parallel RCL circuit.

Appendix