ALGEBRA

ARITHMETIC PROPERTIES

ASSOCIATIVE $a(bc) = (ab)c$

COMMUTATIVE $a + b = b + a$ and $ab = ba$

DISTRIBUTIVE $a(b + c) = ab + ac$

ARITHMETIC OPERATIONS EXAMPLES

$$ab + ac = a(b + c)$$

$$a\left(\frac{b}{c}\right) = \frac{ab}{c}$$

$$\frac{\left(\frac{a}{b}\right)}{c} = \frac{a}{bc}$$

$$\frac{a}{\left(\frac{b}{c}\right)} = \frac{ac}{b}$$

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$

$$\frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd}$$

$$\frac{a - b}{c - d} = \frac{b - a}{d - c}$$

$$\frac{a + b}{c} = \frac{a}{c} + \frac{b}{c}$$

$$\frac{ab + ac}{a} = b + c, a \neq 0$$

$$\frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)} = \frac{ad}{bc}$$

PROFESOR MUNDIAL

QUADRATIC EQUATION

For the equation

$ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

EXPONENT PROPERTIES

$$a^n a^m = a^{n+m}$$

$$(a^n)^m = a^{nm}$$

$$(ab)^n = a^n b^n$$

$$a^{-n} = \frac{1}{a^n}$$

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n = \frac{b^n}{a^n}$$

$$\frac{a^n}{a^m} = a^{n-m} = \frac{1}{a^{m-n}}$$

$$a^0 = 1, a \neq 0$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$\frac{1}{a^{-n}} = a^n$$

$$a^{\frac{n}{m}} = \left(a^{\frac{1}{m}}\right)^n = (a^n)^{\frac{1}{m}}$$

PROFESOR MUNDIAL

PROFESOR MUNDIAL

PROPERTIES OF INEQUALITIES

If $a < b$ then $a + c < b + c$ and $a - c < b - c$

If $a < b$ and $c > 0$ then $ac < bc$ and $a/c < b/c$

If $a < b$ and $c < 0$ then $ac > bc$ and $a/c > b/c$

PROPERTIES OF COMPLEX NUMBERS

$$i = \sqrt{-1}$$

$$i^2 = -1$$

$$\sqrt{-a} = i\sqrt{a}, \quad a \geq 0$$

$$(a + bi) + (c + di) = a + c + (b + d)i$$

$$(a + bi) - (c + di) = a - c + (b - d)i$$

$$(a + bi)(c + di) = ac - bd + (ad + bc)i$$

$$(a + bi)(a - bi) = a^2 + b^2$$

$$|a + bi| = \sqrt{a^2 + b^2}$$

$$\overline{(a + bi)} = a - bi$$

$$\overline{(a + bi)}(a + bi) = |a + bi|^2$$

$$\frac{1}{(a + bi)} = \frac{(a - bi)}{(a + bi)(a - bi)} = \frac{a - bi}{a^2 + b^2}$$

RADICAL PROPERTIES

$a, b \geq 0$ for even n

$$\sqrt[n]{a} = a^{\frac{1}{n}}$$

$$\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$$

$$\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$$

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

$\sqrt[n]{a^n} = a$, if n is odd

$\sqrt[n]{a^n} = |a|$, if n is even

PROFESOR MUNDIAL

LOGARITHM PROPERTIES

if $y = \log_b x$ then $b^y = x$

$\log_b b = 1$ and $\log_b 1 = 0$

$$\log_b b^x = x$$

$$b^{\log_b x} = x$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$\log_b(x^r) = r \log_b x$$

$$\log_b(xy) = \log_b x + \log_b y$$

$$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$$

PROFESOR MUNDIAL

COMMON FACTORING EXAMPLES

$$x^2 - a^2 = (x + a)(x - a)$$

$$x^2 + 2ax + a^2 = (x + a)^2$$

$$x^2 - 2ax + a^2 = (x - a)^2$$

$$x^2 + (a + b)x + ab = (x + a)(x + b)$$

$$x^3 + 3ax^2 + 3a^2x + a^3 = (x + a)^3$$

$$x^3 + a^3 = (x + a)(x^2 - ax + a^2)$$

$$x^3 - a^3 = (x - a)(x^2 + ax + a^2)$$

$$x^{2n} - a^{2n} = (x^n - a^n)(x^n + a^n)$$

ABSOLUTE VALUE

$$|a| = \begin{cases} a, & \text{if } a \geq 0 \\ -a, & \text{if } a < 0 \end{cases}$$

$$|a| = |-a|$$

$$|a| \geq 0$$

$$|ab| = |a||b|$$

$$\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$$

$$|a + b| \leq |a| + |b|$$

PROFESOR MUNDIAL

COMPLETING THE SQUARE

$$ax^2 + bx + c = a(\ldots)^2 + \text{constant}$$

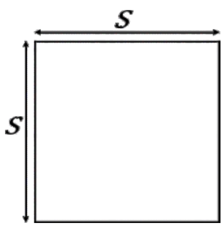
1. Divide by the coefficient a.
2. Move the constant to the other side.
3. Take half of the coefficient b/a, square it and add it to both sides.
4. Factor the left side of the equation.
5. Use the square root property.
6. Solve for x.

GEOMETRY

SQUARE

$P = 4s$

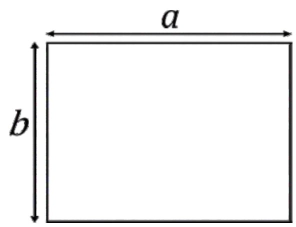
$A = s^2$



RECTANGLE

$P = 2a + 2b$

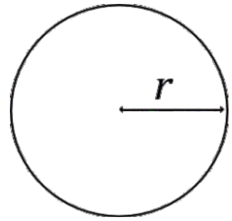
$A = ab$



CIRCLE

$P = 2\pi r$

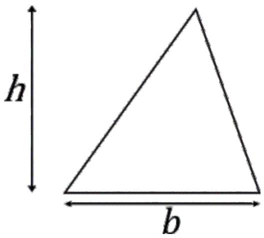
$A = \pi r^2$



TRIANGLE

$P = a + b + c$

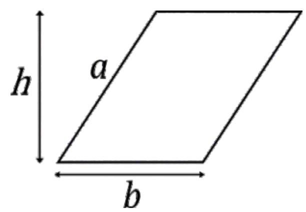
$A = \frac{1}{2} bh$



PARALLELOGRAM

$P = 2a + 2b$

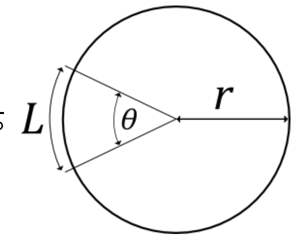
$A = bh$



CIRCULAR SECTOR

$L = \pi r \frac{\theta}{180°}$

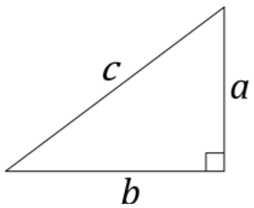
$A = \pi r^2 \frac{\theta}{360°}$



PYTHAGOREAN THEOREM

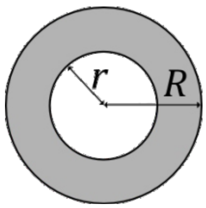
$a^2 + b^2 = c^2$

$c = \sqrt{a^2 + b^2}$



CIRCULAR RING

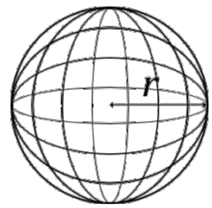
$A = \pi(R^2 - r^2)$



SPHERE

$S = 4\pi r^2$

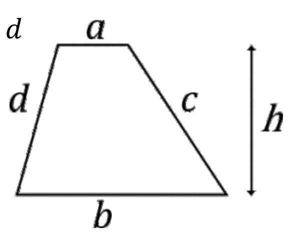
$V = \frac{4\pi r^3}{3}$



TRAPEZOID

$P = a + b + c + d$

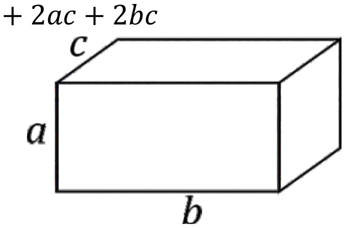
$A = h\frac{a + b}{2}$



RECTANGULAR BOX

$A = 2ab + 2ac + 2bc$

$V = abc$

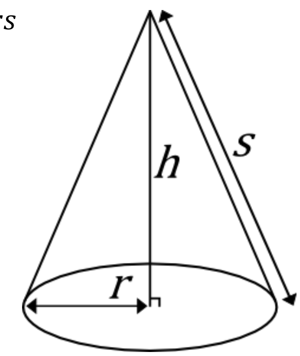


RIGHT CIRCULAR CONE

$A = \pi r^2 + \pi rs$

$s = \sqrt{r^2 + h^2}$

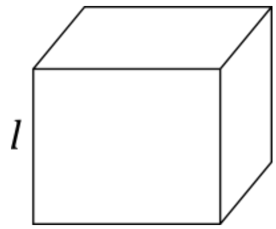
$V = \frac{1}{3}\pi r^2 h$



CUBE

$A = 6l^2$

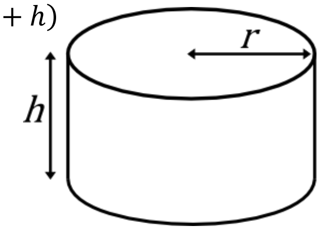
$V = l^3$



CYLINDER

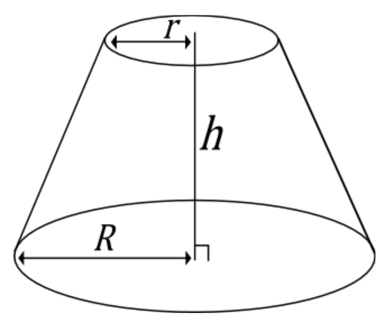
$A = 2\pi r(r + h)$

$V = \pi r^2 h$



FRUSTUM OF A CONE

$V = \frac{1}{3}\pi h(r^2 + rR + R^2)$



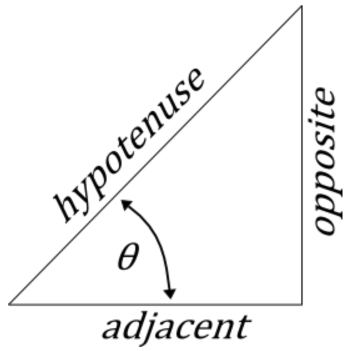
TRIGONOMETRY I

RIGHT TRIANGLE DEFINITION

$\sin\theta = \dfrac{opposite}{hypotenuse}$ $\quad \csc\theta = \dfrac{hypotenuse}{opposite}$

$\cos\theta = \dfrac{adjacent}{hypotenuse}$ $\quad \sec\theta = \dfrac{hypotenuse}{adjacent}$

$\tan\theta = \dfrac{opposite}{adjacent}$ $\quad \cot\theta = \dfrac{adjacent}{opposite}$



TRIG FUNCTIONS RANGE

$-1 \le \sin\theta \le 1$

$-1 \le \cos\theta \le 1$

$-\infty \le \tan\theta \le \infty$

$\csc\theta \ge 1$ and $\csc\theta \le -1$

$\sec\theta \ge 1$ and $\sec\theta \le -1$

$-\infty \le \cot\theta \le \infty$

UNIT CIRCLE DEFINITION

$\sin\theta = y$

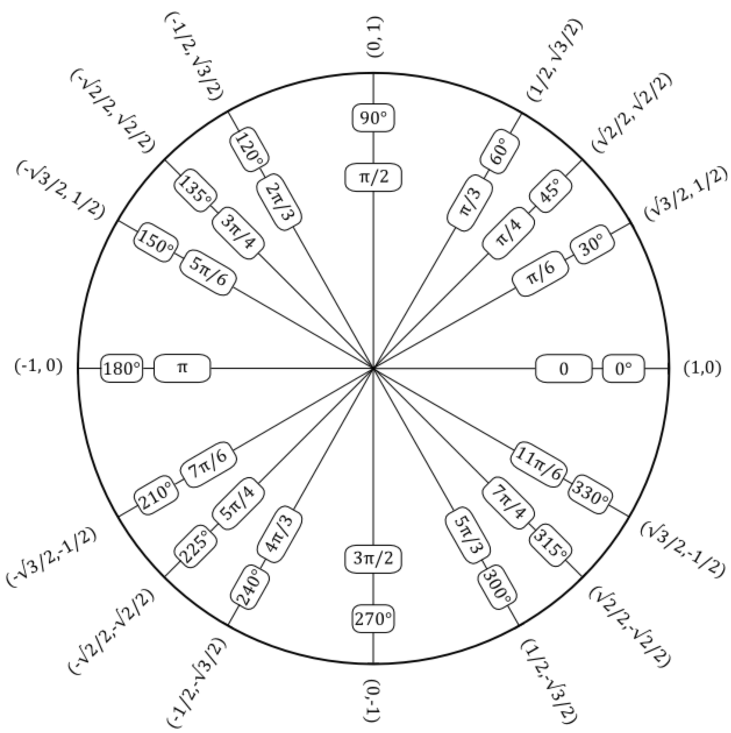
$\cos\theta = x$

$\tan\theta = \dfrac{y}{x}$

$\csc\theta = \dfrac{1}{y}$

$\sec\theta = \dfrac{1}{x}$

$\cot\theta = \dfrac{x}{y}$



TRIG FUNCTIONS DOMAIN

$\sin\theta$, θ can be any angle

$\cos\theta$, θ can be any angle

$\tan\theta$, $\theta \ne \left(n + \dfrac{1}{2}\right)\pi, \quad n = 0, \pm1, \pm2, \ldots$

$\csc\theta$, $\theta \ne n\pi, \quad n = 0, \pm1, \pm2, \ldots$

$\sec\theta$, $\theta \ne \left(n + \dfrac{1}{2}\right)\pi, \quad n = 0, \pm1, \pm2, \ldots$

$\cot\theta$, $\theta \ne n\pi, \quad n = 0, \pm1, \pm2, \ldots$

TRIG FUNCTIONS PERIOD

$\sin(\omega\theta) \rightarrow T = \dfrac{2\pi}{\omega}$

$\cos(\omega\theta) \rightarrow T = \dfrac{2\pi}{\omega}$

$\tan(\omega\theta) \rightarrow T = \dfrac{\pi}{\omega}$

$\csc(\omega\theta) \rightarrow T = \dfrac{2\pi}{\omega}$

$\sec(\omega\theta) \rightarrow T = \dfrac{2\pi}{\omega}$

$\cot(\omega\theta) \rightarrow T = \dfrac{\pi}{\omega}$

INVERSE TRIG FUNCTION NOTATION

$\sin^{-1} x \equiv \arcsin x \equiv \text{Asin}\, x$

$\cos^{-1} x \equiv \arccos x \equiv \text{Acos}\, x$

$\tan^{-1} x \equiv \arctan x \equiv \text{Atan}\, x$

INVERSE TRIG DOMAIN

$\sin^{-1} x : -1 \le x \le 1$

$\cos^{-1} x : -1 \le x \le 1$

$\tan^{-1} x : -\infty \le x \le \infty$

INVERSE TRIG FUNCTION RANGE

$-\dfrac{\pi}{2} \le \sin^{-1} x \le \dfrac{\pi}{2}$

$0 \le \cos^{-1} x \le \pi$

$-\dfrac{\pi}{2} \le \tan^{-1} x \le \dfrac{\pi}{2}$

TRIGONOMETRY II

TANGENT IDENTITIES

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

RECIPROCAL IDENTITIES

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sin\theta = \frac{1}{\csc\theta}$$

$$\sec\theta = \frac{1}{\cos\theta} \qquad \cos\theta = \frac{1}{\sec\theta}$$

$$\cot\theta = \frac{1}{\tan\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

PYTHAGOREAN IDENTITIES

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$\cot^2\theta + 1 = \csc^2\theta$$

PERIODIC IDENTITIES

$$\sin(\theta + 2\pi n) = \sin\theta$$

$$\cos(\theta + 2\pi n) = \cos\theta$$

$$\tan(\theta + \pi n) = \tan\theta$$

$$\csc(\theta + 2\pi n) = \csc\theta$$

$$\sec(\theta + 2\pi n) = \sec\theta$$

$$\cot(\theta + \pi n) = \cot\theta$$

EVEN/ODD IDENTITIES

$$\sin(-\theta) = -\sin\theta$$

$$\cos(-\theta) = \cos\theta$$

$$\tan(-\theta) = -\tan\theta$$

$$\csc(-\theta) = -\csc\theta$$

$$\sec(-\theta) = \sec\theta$$

$$\cot(-\theta) = -\cot\theta$$

DOUBLE ANGLE IDENTITIES

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$

$$= 2\cos^2\theta - 1$$

$$= 1 - 2\sin^2\theta$$

$$\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$$

HALF ANGLE IDENTITIES

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\tan\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

LAW OF COSINES

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

$$b^2 = a^2 + c^2 - 2ac\cos\beta$$

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

PRODUCT TO SUM IDENTITIES

$$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$$

$$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]$$

$$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$$

$$\cos\alpha\sin\beta = \frac{1}{2}[\sin(\alpha+\beta) - \sin(\alpha-\beta)]$$

SUM TO PRODUCT IDENTITIES

$$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\sin\alpha - \sin\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

LAW OF SINES

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$

LAW OF TANGENTS

$$\frac{a-b}{a+b} = \frac{\tan\left[\frac{1}{2}(\alpha-\beta)\right]}{\tan\left[\frac{1}{2}(\alpha+\beta)\right]}$$

$$\frac{b-c}{b+c} = \frac{\tan\left[\frac{1}{2}(\beta-\gamma)\right]}{\tan\left[\frac{1}{2}(\beta+\gamma)\right]}$$

$$\frac{a-c}{a+c} = \frac{\tan\left[\frac{1}{2}(\alpha-\gamma)\right]}{\tan\left[\frac{1}{2}(\alpha+\gamma)\right]}$$

SUM/DIFFERENCES IDENTITIES

$$\sin(\alpha\pm\beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha\pm\beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\tan(\alpha\pm\beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

MOLLWEIDES FORMULA

$$\frac{a+b}{c} = \frac{\cos\left[\frac{1}{2}(\alpha-\beta)\right]}{\sin\left(\frac{1}{2}\gamma\right)}$$

COFUNCTION IDENTITIES

$$\sin\left(\frac{\pi}{2}-\theta\right) = \cos\theta$$

$$\csc\left(\frac{\pi}{2}-\theta\right) = \sec\theta$$

$$\tan\left(\frac{\pi}{2}-\theta\right) = \cot\theta$$

$$\cos\left(\frac{\pi}{2}-\theta\right) = \sin\theta$$

$$\sec\left(\frac{\pi}{2}-\theta\right) = \csc\theta$$

$$\cot\left(\frac{\pi}{2}-\theta\right) = \tan\theta$$

CALCULUS I

DERIVATIVE DEFINITION

$$\frac{d}{dx}(f(x)) = f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

BASIC PROPERTIES

$$(cf(x))' = c(f'(x))$$

$$(f(x) \pm g(x))' = f'(x) \pm g'(x)$$

$$\frac{d}{dx}(c) = 0$$

MEAN VALUE THEOREM

If f *is* differentiable on the interval (a,b) and continuous at the end points there exists a c in (a,b) such that

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

PRODUCT RULE

$$(f(x)g(x))' = f(x)'g(x) + f(x)g(x)'$$

QUOTIENT RULE

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

POWER RULE

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

CHAIN RULE

$$\frac{d}{dx}\big(f(g(x))\big) = f'(g(x))g'(x)$$

COMMON DERIVATIVES

$$\frac{d}{dx}(x) = 1$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}(a^x) = a^x \ln(a)$$

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(\ln(x)) = \frac{1}{x}, x > 0$$

$$\frac{d}{dx}(\ln|x|) = \frac{1}{x}$$

$$\frac{d}{dx}(\log_a(x)) = \frac{1}{x\ln(a)}$$

PROFESOR MUNDIAL

PROFESOR MUNDIAL

CHAIN RULE AND OTHER EXAMPLES

$$\frac{d}{dx}([f(x)]^n) = n[f(x)]^{n-1}f'(x)$$

$$\frac{d}{dx}\left(e^{f(x)}\right) = f'(x)e^{f(x)}$$

$$\frac{d}{dx}(\ln[f(x)]) = \frac{f'(x)}{f(x)}$$

$$\frac{d}{dx}(\sin[f(x)]) = f'(x)\cos[f(x)]$$

$$\frac{d}{dx}(\cos[f(x)]) = -f'(x)\sin[f(x)]$$

$$\frac{d}{dx}(\tan[f(x)]) = f'(x)\sec^2[f(x)]$$

$$\frac{d}{dx}(\sec[f(x)]) = f'(x)\sec[f(x)]\tan[f(x)]$$

$$\frac{d}{dx}(\tan^{-1}[f(x)]) = \frac{f'(x)}{1+[f(x)]^2}$$

$$\frac{d}{dx}\left(f(x)^{g(x)}\right) = f(x)^{g(x)}\left(\frac{g(x)f'(x)}{f(x)} + \ln(f(x))\,g'(x)\right)$$

PROFESOR MUNDIAL

PROFESOR MUNDIAL

LIMIT EVALUATION METHOD FACTOR AND CANCEL

$$\lim_{x\to -3}\frac{x^2-x-12}{x^2+3x} = \lim_{x\to -3}\frac{(x+3)(x-4)}{x(x+3)} = \lim_{x\to -3}\frac{(x-4)}{x} = \frac{7}{3}$$

L'HOPITALS RULE

If $\lim_{x\to a}\frac{f(x)}{g(x)} = \frac{0}{0}$ or $\frac{\pm\infty}{\pm\infty}$ then $\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$

PROPERTIES OF LIMITS

These properties require that the limit of $f(x)$ and $g(x)$ exist

$$\lim_{x\to a}[cf(x)] = c\lim_{x\to a} f(x)$$

$$\lim_{x\to a}[f(x) \pm g(x)] = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x)$$

$$\lim_{x\to a}[f(x)g(x)] = \lim_{x\to a} f(x)\lim_{x\to a} g(x)$$

$$\lim_{x\to a}\left[\frac{f(x)}{g(x)}\right] = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)} \text{ if } \lim_{x\to a} g(x) \neq 0$$

$$\lim_{x\to a}[f(x)]^n = \left[\lim_{x\to a} f(x)\right]^n$$

PROFESOR MUNDIAL

LIMIT EVALUATIONS AT +-∞

$\lim_{x\to\infty} e^x = \infty$ and $\lim_{x\to -\infty} e^x = 0$

$\lim_{x\to\infty} \ln(x) = \infty$ and $\lim_{x\to 0^+} \ln(x) = -\infty$

If $r > 0$ then $\lim_{x\to\infty}\frac{c}{x^r} = 0$

If $r > 0$ & x^r is real for $x < 0$ then $\lim_{x\to -\infty}\frac{c}{x^r} = 0$

$\lim_{x\to\pm\infty} x^r = \infty$ for even r

$\lim_{x\to\infty} x^r = \infty$ & $\lim_{x\to -\infty} x^r = -\infty$ for odd r

PROFESOR MUNDIAL

CALCULUS II

DEFINITE INTEGRAL DEFINITION

$$\int_a^b f(x)dx = \lim_{n\to\infty}\sum_{k=1}^{n} f(x_k)\Delta x$$

where $\Delta x = \frac{b-a}{n}$ and $x_k = a + k\Delta x$

FUNDAMENTAL THEOREM OF CALCULUS

$$\int_a^b f(x)dx = [F(x)]_a^b = F(b) - F(a)$$

where f is continuous on [a,b] and $F' = f$

INTEGRATION PROPERTIES

$$\int_a^b cf(x)dx = c\int_a^b f(x)dx$$

$$\int_a^b f(x) \pm g(x)dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$$

$$\int_a^a f(x)dx = 0 \text{ and } \int_a^b f(x)dx = -\int_b^a f(x)dx$$

$$\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$$

COMMON INTEGRALS

$$\int k\ dx = kx + C$$

$$\int x^n\ dx = \frac{1}{n+1}x^{n+1} + C, n \neq -1$$

$$\int x^{-1}\ dx = \int \frac{1}{x}\ dx = \ln|x| + C$$

$$\int \frac{1}{ax+b}\ dx = \frac{1}{a}\ln|ax+b| + C$$

$$\int \ln(x)\ dx = x\ln(x) - x + C$$

$$\int e^x\ dx = e^x + C$$

$$\int \cos x\ dx = \sin x + C$$

$$\int \sin x\ dx = -\cos x + C$$

$$\int \sec^2 x\ dx = \tan x + C$$

$$\int \sec x \tan x\ dx = \sec x + C$$

$$\int \csc x \cot x\ dx = -\csc x + C$$

$$\int \csc^2 x\ dx = -\cot x + C$$

$$\int \tan x\ dx = \ln|\sec x| + C$$

$$\int \sec x\ dx = \ln|\sec x + \tan x| + C$$

$$\int \frac{1}{a^2+u^2}\ dx = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$$

$$\int \frac{1}{\sqrt{a^2-u^2}}\ dx = \sin^{-1}\left(\frac{u}{a}\right) + C$$

APPROXIMATING DEFINITE INTEGRALS

Left-hand and right-hand rectangle approximations

$$L_n = \Delta x \sum_{k=0}^{n-1} f(x_k) \qquad R_n = \Delta x \sum_{k=1}^{n} f(x_k)$$

Midpoint Rule

$$M_n = \Delta x \sum_{k=0}^{n-1} f(\frac{x_k + x_{k+1}}{2})$$

Trapezoid Rule

$$T_n = \frac{\Delta x}{2}(f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + f(x_n))$$

TRIGNOMETRIC SUBSTITUTION

EXPRESSION	SUBSTITUTION	EXPRESSION EVALUATION	IDENTITY USED
$\sqrt{a^2 - x^2}$	$x = a\sin\theta$ $dx = a\cos\theta\, d\theta$	$\sqrt{a^2 - a^2\sin^2\theta}$ $= a\cos\theta$	$1 - \sin^2\theta$ $= \cos^2\theta$
$\sqrt{x^2 - a^2}$	$x = a\sec\theta$ $dx = a\sec\theta\tan\theta\, d\theta$	$\sqrt{a^2\sec^2\theta - a^2}$ $= a\tan\theta$	$\sec^2\theta - 1$ $= \tan^2\theta$
$\sqrt{a^2 + x^2}$	$x = a\tan\theta$ $dx = a\sec^2\theta\, d\theta$	$\sqrt{a^2 + a^2\tan^2\theta}$ $= a\sec\theta$	$1 + \tan^2\theta$ $= \sec^2\theta$

APPROXIMATION BY SIMPSON RULE FOR EVEN N

$$S_n = \frac{\Delta x}{3}(f(x_0) + 4f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n))$$

INTEGRATION BY SUBSTITUTION

$$\int_a^b f(g(x))\, g'(x)dx = \int_{g(a)}^{g(b)} f(u)du$$

where $u = g(x)$ and $du = g'(x)dx$

INTEGRATION BY PARTS

$$\int u\ dv = uv - \int v\ du \quad \text{where } v = \int dv$$

or

$$\int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)dx$$