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Abstract: This paper illustrates the power of modern statistical modelling in estimating measures of market risk, here applied to the Brent and WTI spot price of oil. Both Value-at-Risk (VaR) and Expected Shortfall (ES) are cast in terms of conditional centiles based upon semi-parametric regression models. Using the GAMLSS statistical framework, we stress the important aspects of selecting a highly flexible parametric distribution (skewed Student's t-distribution) and of modelling both skewness and kurtosis as non-parametric functions of the price of oil futures. Furthermore, an empirical application characterises the relationship between spot oil prices and oil futures – exploiting the futures market to explain the dynamics of the physical market. Our results suggest that NYMEX WTI has heavier tails compared with the ICE Brent. Contrary to the common platitude of the industry, we argue that 'somebody knows something' in the oil business.

Keywords: semi-parametric regression; conditional market risks; oil prices; West Texas Intermediate; Brent.

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1 Introduction

Since the turn of this century, there is a renewed interest in the modelling of spot oil prices. The primary aim is to assess the market risks faced by oil producers and consumers, including financial organisations with an exposure in the energy markets. Despite that, the most common paradigm in the commodity markets is that 'nobody knows anything' and hence the spot price of oil cannot be predicted. We argue instead that 'somebody knows something'.

Estimating a correctly specified parametric probability density function (pdf) can in fact help market analysts to assess measures of market risks such as the Value-at-Risk (VaR) and Expected Shortfall (ES). There many methods to estimate measures of VaR and ES (e.g. Chernozhukov and Umantsev, 2001; De Rossi and Harvey, 2009; Aloui and Mabrouk, 2010). In the presence of heavy tails, non-parametric approaches can lead to very unreliable statistical inferences (Bahadur and Savage, 1956). As argued by Davidson (2012), statistical models with parametric theoretical distributions can perform better in the presence of heavy tails. Therefore, being the spot price of oil a prime example of time series presenting heavy tails distribution, we can argue that a semi-parametric approach is a plausible research avenue.

Despite the presence of heavy tails in the spot price of oil, conventional modelling approaches of spot oil prices is based on autoregressive or white noise models adopting the Gaussian distribution. Using different frequencies (daily, weekly, monthly), existing empirical models model the stochastic features of spot prices with autoregressive, GARCH and ARIMA models (Morana, 2001; Yang et al., 2002; Sadorsky, 2006; Agnolucci, 2009; Mirmirani and Li, 2004; Mohammadi and Su 2010), neural network (Yu et. al., 2008), and jump-diffusion process (Askari and Krichene, 2008). Non-Gaussian models have not been extensively used to model oil prices. Benth and Saltyte-Benth (2003), for example, use a parametric time series model where the assumption of the Gaussian distribution is relaxed. The maim argument here is that to model the dynamics of spot oil prices we need to employ flexible modelling frameworks that significantly depart from conventional approaches that only model the mean and variance. The modelling of skewness and kurtosis using non-parametric functions is important in order to properly approximate the true data generating process.

Since it is common for policy makers and market analysts to use the prices of oil futures to interpret developments in the global crude oil market, an alternative approach is to use oil futures to improve our understanding of the dynamics of spot oil prices. In our reasoning, oil futures, as predictors of the spot price of oil, reflect the price that both the buyer and the seller agree for a future transaction to happen. Therefore, oil futures are likely to provide direct information on the agents' expectations about the future price of oil. However, Alquist and Kilian (2010) argue that oil futures prices tend to be less accurate in the mean-squared prediction error sense than no-change forecasts. It is important to note that Alquist and Kilian employed the Gaussian distribution in the regression models that they tested.

Using the GAMLSS (Generalised Additive Models for Location Scale and Shape) framework (Rigby and Stasinopoulos, 2005), we re-examine the argument that oil futures fail to contain any significant information in terms of explaining the dynamics of spot oil prices. Our initial approach is to compare many competing models for spot oil prices. The flexible four-parameters skewed Student's *t* distribution, developed by Hansen (1994) and subsequently by Fernandez and Steel (1998), is used in Section 3 to provide a good fit of Brent and WTI spot prices when oil futures are included in the model as predictors.

As shown in Section 3, the important difference here is that we do not only model the conditional mean and variance but also the conditional skewness and kurtosis. In other words, using the flexibility of the GAMLSS framework for regression type of models for the distribution parameters μ, σ, ν and τ of the skew Student's *t* distribution, we are able to address the consequential difficulty in modelling the dispersion, skewness and kurtosis of spot oil prices. Following Voudouris et al. (2012), by setting the problem as a random effect model, we allow smooth modelling of distribution parameters of the response variable (spot oil prices). It is important to note that for the skew Student's *t* distribution, the μ is the mean of Y (spot oil prices), the σ is the standard deviation of Y while the ν controls the skewness of the distribution and the $\tau > 2$ controls the heaviness of the tails of the distribution (a lower τ implies heavier tails).

Given the estimated conditional probability density function, we are able to estimate measures of market risk such as VaR and ES. This is because VaR is given by the inverse of the cumulative distribution function (cdf) while the ES at q% level is the probability-weighted average of the $(1-q)$% cases in the tail of the distribution. It is important to note that here we estimate futures-varying VaR and ES based upon oil price levels rather than oil price returns. Therefore, the 99% level VaR might be interpreted as the ceiling of the spot price of oil while the 99% level ES might be considered as the expected spot price of oil if there are significant forces in the oil market that push the spot oil prices at unprecedented high levels – in the upper tail of the distribution. Following De Rossi and Harvey (2009), the proposed futures varying centiles for the estimation of the VaR and ES of the spot price of oil satisfy the defining property of fixed centiles in having the appropriate number of observations above and below.

In summary, the distribution of the Brent and WTI spot prices are better approximated by the parametric skewed Student's *t* distribution. Moreover, the resulting non-parametric model of distribution parameters as additive functions of the Brent and WTI oil futures reveal significant informational signals about the Brent and WTI spot price of oil. Thus, we argue that the proposed semi-parametric model is a robust and flexible tool for risk management in the oil industry.

Section 2 presents a discussion conducted on some of the most important properties of GAMLSS framework and the skewed Student's *t* distribution. Section 3 begins with an account of the empirical oil data, which is followed by an assessment of how the flexibility of our semi-parametric model captures the long tails of the spot oil prices using a distribution with finite moments. Section 4 presents a summary of the results together with the conclusion that 'somebody knows something' in the oil business.

2 The GAMLSS framework

The generalised additive models for location, scale and shape (GAMLSS) are semi-parametric regression type models. They are parametric, in that they require a parametric distribution assumption for the response variable, and ‘semi’ in the sense that the modelling of the parameters of the distribution, as functions of explanatory variables, may involve using non-parametric smoothing functions.

GAMLSS were introduced by Rigby and Stasinopoulos (2005) as a way of overcoming some of the limitations associated with the popular generalised linear models (GLM) and generalised additive models (GAM) (Nelder and Wedderburn, 1972 and Hastie and Tibshirani, 1990, respectively).

In GAMLSS the exponential family distribution assumption for the response variable (Y) is relaxed and replaced by a general distribution family, including highly skew and/or kurtotic continuous and discrete distributions. The systematic part of the model is expanded to allow modelling not only of the mean (or location) but other parameters of the distribution of Y as, linear and/or non-linear, parametric and/or smooth non-parametric functions of explanatory variables and/or random effects.

Hence GAMLSS is especially suited to modelling a response variable, such as WTI and Brent spot oil prices, which does not follow an exponential family distribution and which exhibit heterogeneity (e.g. where the scale or shape of the distribution of the response variable changes with explanatory variables(s)).

A GAMLSS model assumes that, for $i = 1, 2,\ldots, n$, independent observations Y_i have probability (density) function $f_Y(y_i|\boldsymbol{\theta}^i)$ conditional on $\boldsymbol{\theta}^i = (\theta_{1i}, \theta_{2i}, \theta_{3i}, \theta_{4i}) = (\mu_i, \sigma_i, \nu_i, \tau_i)$ a vector of four distribution parameters, each of which can be a function to the explanatory variables. This is denoted by $y_i|\boldsymbol{\theta}^i \sim D(\boldsymbol{\theta}^i)$, i.e. $Y_i|(\mu_i, \sigma_i, \nu_i, \tau_i) \sim D(\mu_i, \sigma_i, \nu_i, \tau_i)$ independently for $i = 1, 2,\ldots, n$, where D represent the distribution of Y. We shall refer to $(\mu_i, \sigma_i, \nu_i, \tau_i)$ as the *distribution parameters*. The first two population distribution parameters μ_i and σ_i are usually characterised as location and scale parameters, while the remaining parameter(s), if any, are characterised as shape parameters, e.g. skewness and kurtosis parameters, although the model may be applied more generally to the parameters of any population distribution, and can be generalised to more than four distribution parameters.

Let $\mathbf{Y}^{\mathrm{T}}> = (Y_1, Y_2, \ldots, Y_n)$ be the n length vector of the response variable. The original formulation of a GAMLSS model is defined as follows. For $k = 1, 2, 3, 4$, let $g_k(.)$ be a known monotonic link function relating the distribution parameter $\boldsymbol{\theta}_k$ to predictor $\boldsymbol{\eta}_k$:

$$g_k(\boldsymbol{\theta}_k) = \boldsymbol{\eta}_k = \mathbf{X}_k\boldsymbol{\beta}_k + \sum_{j=1}^{J_k} \mathbf{Z}_{jk}\boldsymbol{\gamma}_{jk}, \tag{1}$$

i.e.

$$g_1(\boldsymbol{\mu}) = \boldsymbol{\eta}_1 = \mathbf{X}_1\boldsymbol{\beta}_1 + \sum_{j=1}^{J_1} \mathbf{Z}_{j1}\boldsymbol{\gamma}_{j1}$$

$$g_2(\boldsymbol{\sigma}) = \boldsymbol{\eta}_2 = \mathbf{X}_2\boldsymbol{\beta}_2 + \sum_{j=1}^{J_2} \mathbf{Z}_{j2}\boldsymbol{\gamma}_{j2}$$

$$g_3(\boldsymbol{\nu}) = \boldsymbol{\eta}_3 = \mathbf{X}_3\boldsymbol{\beta}_3 + \sum_{j=1}^{J_3} \mathbf{Z}_{j3}\boldsymbol{\gamma}_{j3}$$

$$g_4(\tau) = \boldsymbol{\eta}_4 = \mathbf{X}_4\boldsymbol{\beta}_4 + \sum_{j=1}^{J_4} \mathbf{Z}_{j4}\boldsymbol{\gamma}_{j4}.$$

where $\boldsymbol{\mu}$, $\boldsymbol{\sigma}$, $\boldsymbol{\nu}$, $\boldsymbol{\tau}$, and, for k = 1, 2, 3, 4, $\boldsymbol{\theta}_k$ and $\boldsymbol{\eta}_k$ are vectors of length n, $\boldsymbol{\beta}_k^{\mathrm{T}} = \left(\beta_{1k}, \beta_{2k}, \ldots, \beta_{J'_k k}\right)$ is a parameter vector of length J'_k, $\mathbf{X}_k$ is a fixed known design matrix of order $n \times J'_k$, $\mathbf{Z}_{jk}$ is a fixed known $n \times q_{jk}$ design matrix and γ_{jk} is a q_{jk} dimensional random variable which is assumed to be distributed as $\gamma_{jk} \sim N_{qjk}\left(\mathbf{0}, \mathrm{G}_{jk}^{-1}\right)$, where G_{jk}^{-1} is the (generalised) inverse of a $q_{jk} \times q_{jk}$ symmetric matrix $\mathbf{G}_{jk} = \mathbf{G}_{jk}(\boldsymbol{\lambda}_{jk})$ which may depend on a vector of hyperparameters $\boldsymbol{\lambda}_{jk}$, and where if $\mathbf{G}_{jk}$ is singular then γ_{jk} is understood to have an improper prior density function proportional to $\exp\left(-\frac{1}{2}\gamma_{jk}^{\mathrm{T}}\mathbf{G}_{jk}\boldsymbol{\gamma}_{jk}\right)$, while if $\mathbf{G}_{jk}$ is non-singular then $\boldsymbol{\gamma}_{jk}$ has a q_{jk} dimensional multivariate normal distribution with mean 0 and variance-covariance matrix G_{jk}^{-1}.

As shown below to properly capture the dynamics of the WTI and Brent spot oil prices all distribution parameters of the selected skew Student's t distribution need to be modelled as smooth non-parametric functions of the Brent and WTI oil futures. Therefore, the proposed model to estimate the VaR and ES is a semi-parametric regression model.

Following Wurtz et al. (2006), the skewed Student's t distribution (denoted here by *SST*) has been used to allow for skewness and kurtosis in the conditional distribution of the Brent and WTI spot prices. Note that $SST(\mu, \sigma, \nu, \tau) \equiv ST3(\mu_1, \sigma_1, \nu, \tau)$ where $\mu_1 = \mu - \sigma m_1/s_1$, $\sigma_1 = \sigma/s_1$ and where $m_1 = 2\tau^{1/2}(\nu^2 - 1)/\ [(\tau - 1)\nu B(\frac{1}{2}, \frac{\tau}{2})]$, $s_1 = (m_2 - m_1^2)^{1/2}$ and $m_2 = \tau(\nu^3 + \nu^{-3})/[(\tau - 2)(\nu + \nu^{-1})]$.

Following Fernandez and Steel (1998), $ST3(\mu_1, \sigma_1, \nu, \tau)$ is a shifted and scaled t_τ distribution denoted here by $TF(\mu, \sigma, \tau)$ and splice together at μ, giving

$$\begin{aligned} f_{Y_0}(y) &= \frac{2}{\left(1+\nu^2\right)}\left\{f_{Y_1}(y)I(y<\mu) + \nu^2 f_{Y_2}(y)I(y \geq \mu)\right\} \\ f_Y(y|\mu,\sigma,\nu,\tau) &= \frac{c}{\sigma}\left\{1 + \frac{z^2}{\tau}\left[\nu^2 I(y<\mu) + \frac{1}{\nu^2}I(y \geq \mu)\right]\right\} \end{aligned} \tag{2}$$

for $-\infty < y < \infty$, where $Y_1 \sim TF(\mu, \sigma|\nu, \tau)$ and $Y_2 \sim TF(\mu, \sigma\nu, \tau)$, $-\infty < \mu < \infty$, $\sigma > 0$, $\nu > 0$, and $\tau > 0$, and where $z = (y - \mu)/\sigma$ and $c = 2\nu/[\sigma(1 + \nu^2)B(\frac{1}{2}, \frac{\tau}{2})\tau^{1/2}]$.

Note that *ST*3 is the skew t distribution and *TF* is the t-family distribution (see the R *gamlss.dist* package for details). The distribution parameters μ, σ, ν and τ of the *SST* distribution are modelled as smooth non-parametric functions of oil futures using the P-splines of Eilers and Marx (1996).

3 Empirical analysis: Brent and WTI oil prices

3.1 Data and construction

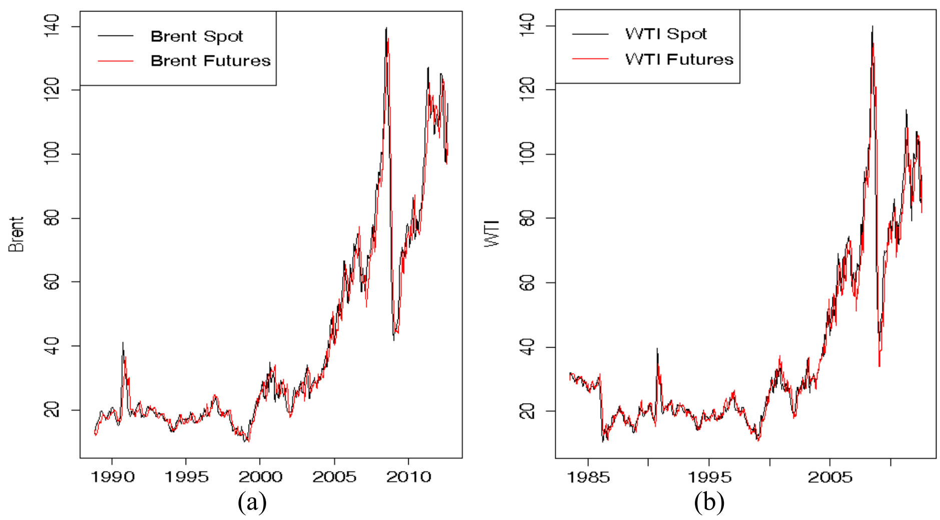
We analyse the Brent and WTI futures and spot prices as the leading world benchmarks of the oil industry. We only consider monthly observations. The spot price (our dependent variable *Y*) is the last traded price of the last date of the month. While the futures price (our explanatory variable) is the last traded price of the last traded day before the delivery month.

For WTI (crude oil futures traded on the NYMEX), the trading of front-month futures contract ends on the forth business day prior to the 25th calendar day preceding the delivery month. If the 25th is not a business day, trading terminates 4 business day before the last business day prior the 25th calendar day. For Brent (crude oil futures traded on the ICE), treading ceases on the business day immediately preceding the 15th calendar day before the first day of the delivery month. If the 15th calendar day is not a business day, the next preceding business day is used.

The time series of the WTI spot contracts begins on February 1983 and extends through July 2012. The time series of the WTI futures contracts begins on January 1983 and extends through June 2012. The time series of the Brent spot contracts begins on October 1988 and extends through July 2012. The time series of the Brent futures contracts begins on January 1988 and extends through July 2012.

Figure 1a plots the monthly prices of the Brent spot oil and front-month rolling contract of Brent futures. Similarly, Figure 1b plots WTI prices. Note that the presence of heavy tails (large spikes) in the datasets. Both Brent and WTI are obtained from Bloomberg.

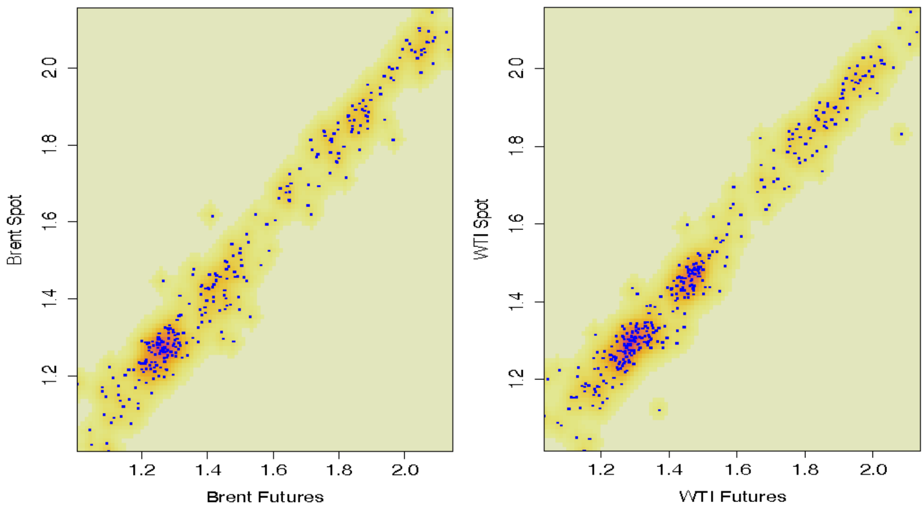
Figure 1 (a) ICE Brent spot and futures contracts, (b) NYMEX WTI spot and futures contracts (see online version for colours)



Source: Data available from Bloomberg

Given that the primary aim of the article is to propose a model for the estimation of VaR and ES using futures-varying centiles, Figure 2a shows the relationship between ICE Brent spot and futures contracts in the form of a density map. Similarly, Figure 2b shows the relationship between NYMEX WTI spot and futures contracts. Note the near linear relationship, the heterogeneous scattering around the linear relationship and the clusterings.

Figure 2 (a) ICE Brent spot against front-month futures contract, (b) NYMEX WTI spot against front-month futures contract (prices are in log10) (see online version for colours)



(a) (b)

Source: Data available from Bloomberg

3.2 The Brent and WTI GAMLSS models

The empirical GAMLSS-based model is $Y \sim SST(\mu, \sigma, \nu, \tau)$, where Y = log10 (Brent or WTI spot price of oil) and

$$
\begin{aligned}
\mu &= pb(x), \\
\log(\sigma) &= pb(x), \\
\log(\nu) &= pb(x), \\
\log(\tau) &= pb(x),
\end{aligned}
\tag{3}
$$

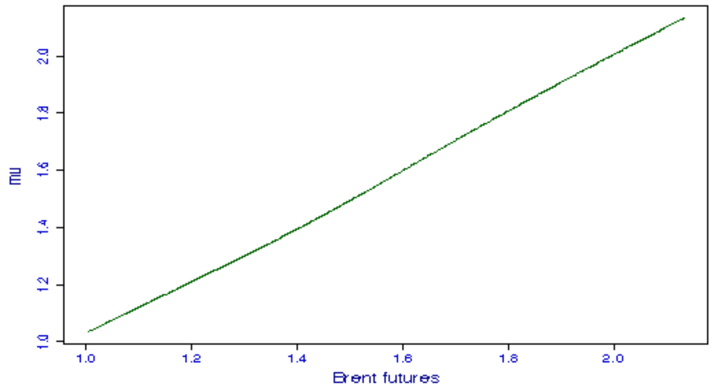
where x = log10 (Brent or WTI oil futures) and pb is the P-splines of Eilers and Marx (1996).

For the selection of the model, we used the model selection process suggested by Voudouris et al. (2012). Specifically, we use the Generalised Akaike Information Criterion (GAIC) and visual representations of quantile residuals such worm plots (Buuren and Fredriks, 2001). Thus, from the full list of the theoretical distributions in the R *gamlss.dist* package we selected the SST distribution and the regression-type models for the distribution parameters shown in equation (3).

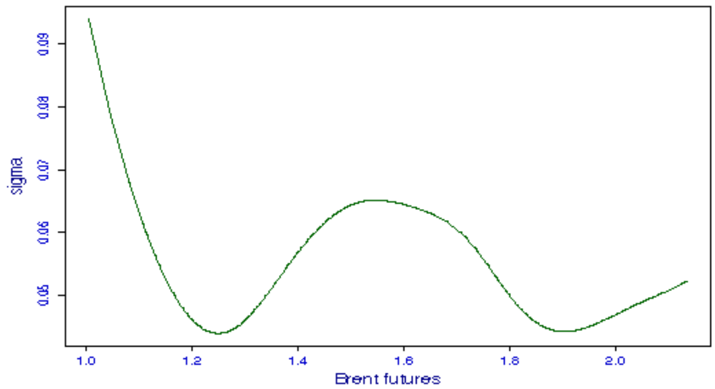
Figure 3 shows the fitted distribution parameters of the Brent model. The following key insights are observable:

- The expected value of the Brent spot price, the μ distribution parameter, increases almost linearly with the futures contracts.
- The standard deviation of the Brent spot price, the σ distribution parameter, declines rapidly until the price of the futures contract reach about \$15. Then a period of increase volatility is observed before a second decline from about \$30. As the price of the futures contract reaches \$80, the volatility increases again.
- The distribution becomes negatively skewed as the futures price increases. Note that when $v = 1$, the distribution of the Brent spot price of oil is symmetric.
- The distribution of Brent spot price of oil becomes more meso-kurtotic as the futures price increase.

Figure 3 Dynamics of the Brent model (see online version for colours)

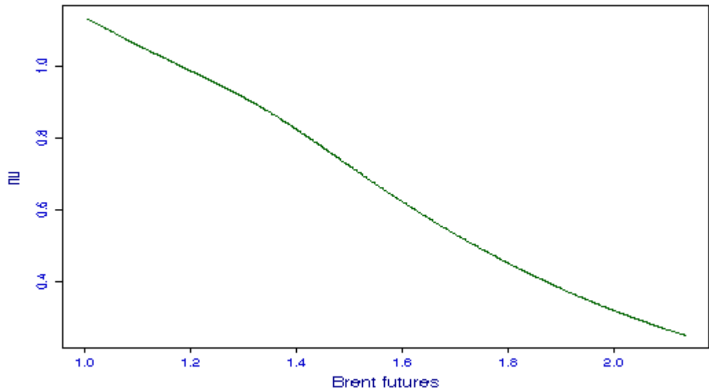


(a)

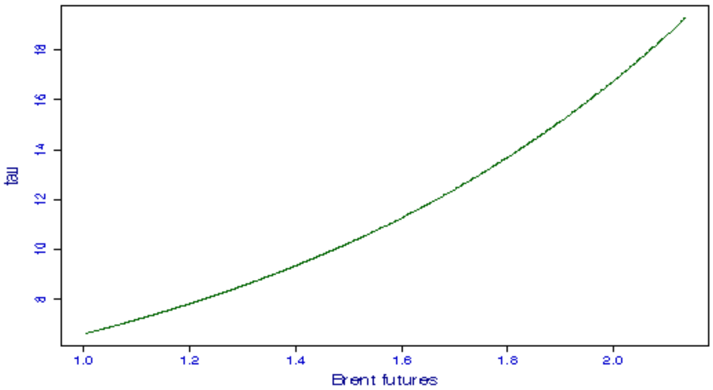


(b)

Figure 3 Dynamics of the Brent model (see online version for colours) (continued)



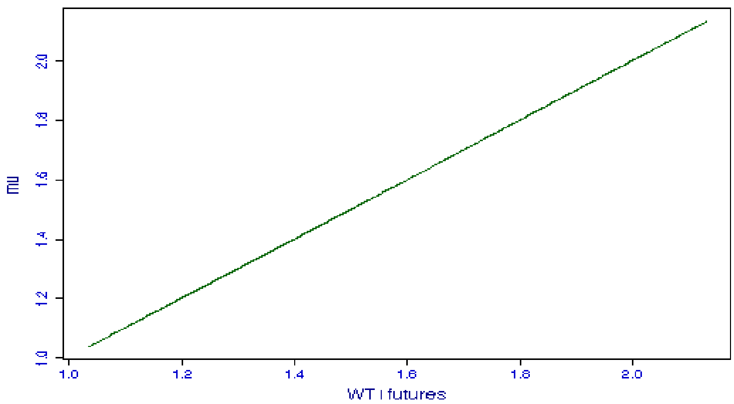
(c)



(d)

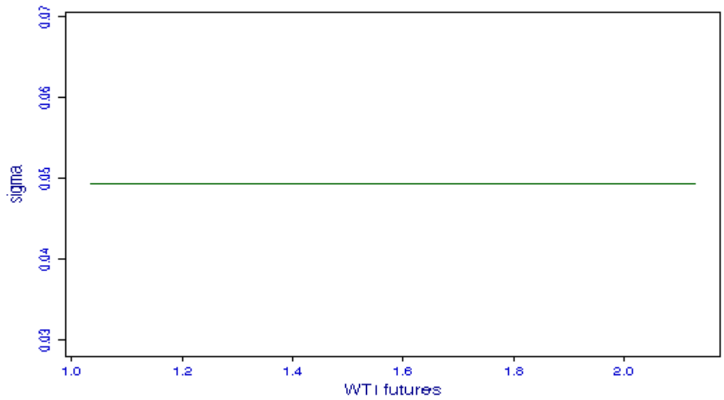
Similarly, Figure 4 shows the fitted distribution parameters of the WTI model.

Figure 4 Dynamics of the WTI model (see online version for colours)

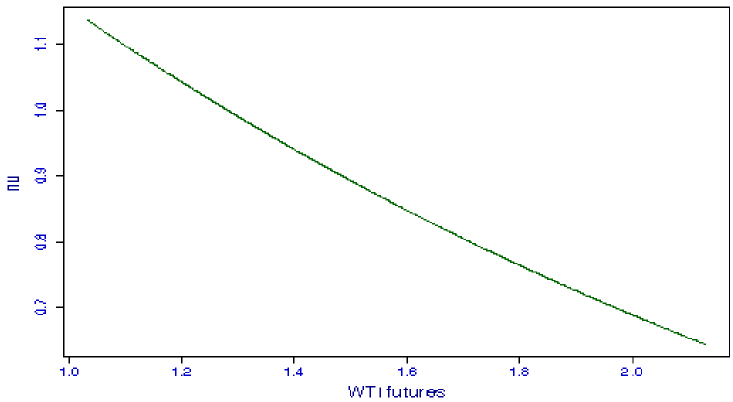


(a)

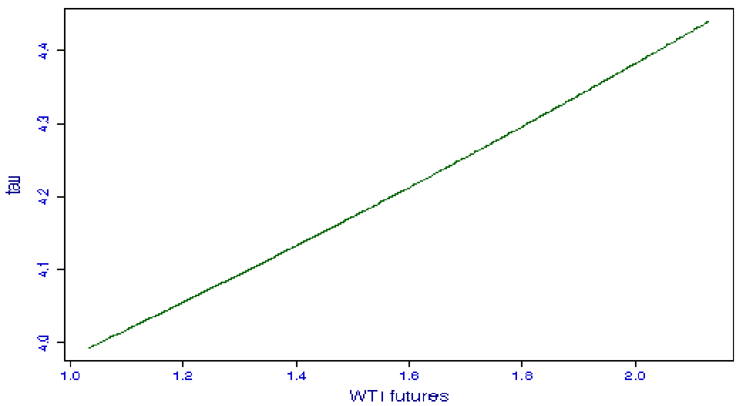
Figure 4 Dynamics of the WTI model (see online version for colours) (continued)



(b)



(c)



(d)

3.3 VaR and ES

Given the empirical GAMLSS model presented in equation (4), futures-varying VaR and ES of the Brent and WTI spot price of oil can be estimated. Figure 5 shows the futures-varying centile (or quantiles) of the Brent spot prices of oil. For example, the futures-varying 99.6% level VaR is graphically shown by the upper green curve. Different % levels are also shown. The graph gives a clear visual impression of the movements in level and dispersion of the Brent spot price of oil. Note the higher spread between the centiles as the futures price increases. Thus, the overall distribution of risk of the Brent spot price of oil is not constant. Similarly Figure 6 shows the futures-varying centile (or quantiles) of the WTI spot prices of oil.

Figure 5 Centiles as measures of futures-varying measures of Brent (see online version for colours)

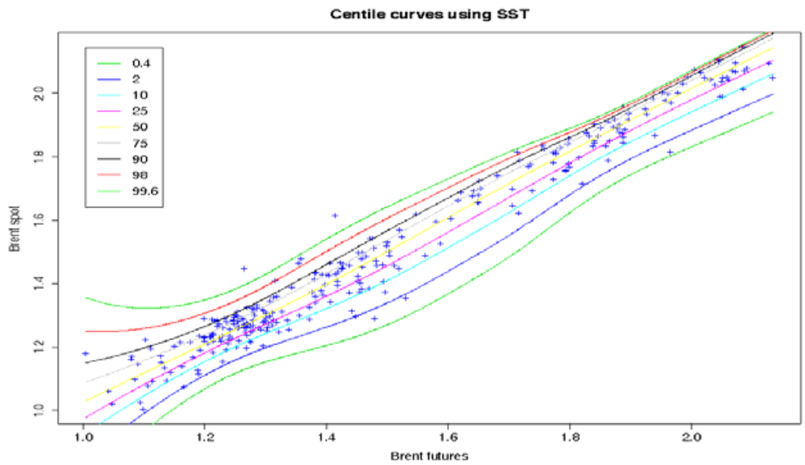
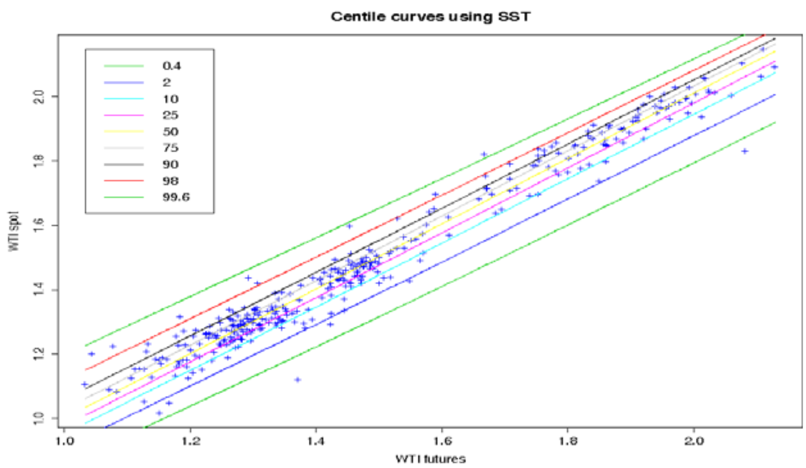


Figure 6 Centiles as measures of futures-varying measures of WTI (see online version for colours)



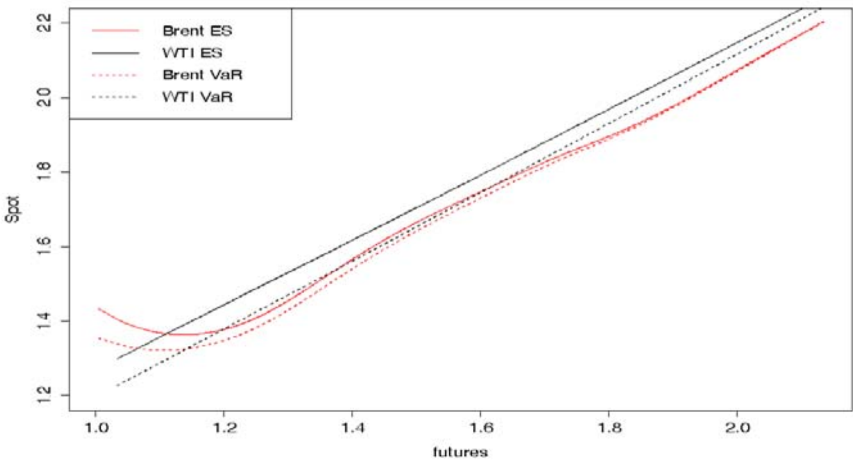
The ES can be estimated by

$$ES = \frac{1}{1 - F_{VaR}} \int_{VaR}^{\infty} x f(x) dx \tag{4}$$

where F_{VaR} is the *SST* cumulative density function at the given VaR level and $f(x)$ is the *SST* probability density function. Therefore, by utilising a flexible parametric distribution within a semi-parametric regression model that approximates the true data generating process, we can conveniently estimate the ES.

Figure 7 shows the 99.6% level futures-varying ES and VaR of the Brent and WTI spot price of oil. Note the distance between ES and VaR when the distribution is not symmetric and meso-kurtic. Contrary to what most practitioners support, the overall market risk, as measure by the ES and VaR, of the WTI is higher compared with the market risk of Brent. The reason for that is the estimated distribution parameter τ shown in Figure 3d and Figure 4d. Effectively the distribution of the Brent is more meso-kurtotic compared with the distribution of the WTI. In other words, the WTI spot price of oil has heavier tails compared with the distribution tails of the Brent spot price of oil.

Figure 7 99.6% level futures-varying ES and VaR of Brent and WTI (see online version for colours)



Following Kaufmann and Ullman (2009), one plausible explanation of the different conditional distribution shapes of the Brent and WTI spot price of oil might be that market participants first look at the NYMEX WTI and then at the ICE Brent. Therefore we argue that it is plausible that the spot price discovery process of Brent might be conditional on the spot price discovery process of WTI.

4 Conclusion

We introduced the skew Student's *t* distribution within the GAMLSS framework to model both the Brent and WTI spot markets using the information in the ICE Brent and

NYMEX WTI futures markets for crude oil. The model provides fresh insights on the interpretation of spot oil prices and related statistics such as futures-varying mean, standard deviation, skewness and kurtosis.

Using the estimated conditional distribution, we are able to extract estimates of market risks such as VaR and ES using futures-varying centiles. Although futures-varying centiles are of interest in themselves and provide information on various aspects of a time series, here we demonstrate how to estimate the VaR and ES of the spot price of oil.

Futures-varying centiles can be fitted by formulating a regression-type of model for the distribution parameters of the skew Student's *t* distribution, namely μ, σ, v and τ. The distribution parameters of the spot price of oil are modelled as non-parametric functions of oil futures. This non-parametric formulation allows us to characterise the movements in the time series of the Brent and WTI spot price of oil.

The resulting semi-parametric model is intended to be taken as a full description of the distribution of the Brent and WTI spot price of oil. Based upon the good statistical properties of our fitted model, we argue that in the oil business 'somebody knows something' in the sense that probabilistic statements about the spot price of oil can be made.

Our goal for the proposed model is to enhance the toolbox for risk management of commodities in a way that rings true to industry participants and policy makers and that can be used as a research and training tool for trading and investment processes.

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