Sintaxis (λ^{BN})

```
M ::= x
    | λx:σ.M
    | M M
    | true
    | false
    | if M then M else M
    | zero
    | succ(M)
    | pred(M)
    | isZero(M)
```

```
σ ::= Bool
    | Nat
    | σ → σ
```

Tipado (λ^{BN})

$$\frac{}{\Gamma, x : \sigma \vdash x : \sigma}\ ax_v$$

$$\frac{\Gamma, x : \sigma \vdash M : \tau}{\Gamma \vdash \lambda x : \sigma.M : \sigma \to \tau} \to_i \qquad \frac{\Gamma \vdash M : \sigma \to \tau \quad \Gamma \vdash N : \sigma}{\Gamma \vdash M\ N : \tau} \to_e$$

$$\frac{}{\Gamma \vdash \mathsf{true} : \mathsf{Bool}}\ ax_{\mathsf{true}} \qquad \frac{}{\Gamma \vdash \mathsf{false} : \mathsf{Bool}}\ ax_{\mathsf{false}}$$

$$\frac{\Gamma \vdash M : \mathsf{Bool} \quad \Gamma \vdash P : \sigma \quad \Gamma \vdash Q : \sigma}{\Gamma \vdash \mathsf{if}\ M\ \mathsf{then}\ P\ \mathsf{else}\ Q : \sigma}\ \mathsf{if}$$

$$\frac{}{\Gamma \vdash \mathsf{zero} : \mathsf{Nat}}\ \mathsf{zero} \qquad \frac{\Gamma \vdash M : \mathsf{Nat}}{\Gamma \vdash \mathsf{succ}(M) : \mathsf{Nat}}\ \mathsf{succ}$$

$$\frac{\Gamma \vdash M : \mathsf{Nat}}{\Gamma \vdash \mathsf{pred}(M) : \mathsf{Nat}}\ \mathsf{pred} \qquad \frac{\Gamma \vdash M : \mathsf{Nat}}{\Gamma \vdash \mathsf{isZero}(M) : \mathsf{Bool}}\ \mathsf{isZero}$$

Semántica (λ^{BN}) (1)

V ::= true | false | λx:σ.M | zero | succ(V)

{β} **(λx:σ.M) V → M{x := V}**

{if_t} **if true then M else N → M**

{if_f} **if false then M else N → N**

{pred} **pred(succ(V)) → V**

{$isZero_0$} **isZero(zero) → true**

{$isZero_n$} **isZero(succ(V)) → false**

Semántica (λ^{BN}) (2)

Si $\mathbf{M} \rightarrow \mathbf{M'}$, entonces:

$$\{\mu\}\quad \mathbf{M\ N} \rightarrow \mathbf{M'\ N}$$

$$\{\nu\}\quad \mathbf{V\ M} \rightarrow \mathbf{V\ M'}$$

$$\{if_c\}\quad \mathbf{if\ M\ then\ N\ else\ O} \rightarrow \mathbf{if\ M'\ then\ N\ else\ O}$$

$$\{succ_c\}\quad \mathbf{succ(M)} \rightarrow \mathbf{succ(M')}$$

$$\{pred_c\}\quad \mathbf{pred(M)} \rightarrow \mathbf{pred(M')}$$

$$\{isZero_c\}\quad \mathbf{isZero(M)} \rightarrow \mathbf{isZero(M')}$$

Sustituciones (1)

$$\mathbf{x}\,\{\mathbf{x} := \mathbf{N}\} = \mathbf{N}$$

$$\mathbf{y}\,\{\mathbf{x} := \mathbf{N}\} = \mathbf{y}$$

$$(\lambda\mathbf{x}{:}\sigma.\mathbf{M})\,\{\mathbf{x} := \mathbf{N}\} = \lambda\mathbf{x}{:}\sigma.\mathbf{M}$$

$$(\lambda\mathbf{y}{:}\sigma.\mathbf{M})\,\{\mathbf{x} := \mathbf{N}\} = \lambda\mathbf{y}{:}\sigma.(\mathbf{M}\,\{\mathbf{x} := \mathbf{N}\})$$

si $\mathbf{y} \notin FV(\mathbf{N})$

$$(\lambda\mathbf{y}{:}\sigma.\mathbf{M})\,\{\mathbf{x} := \mathbf{N}\} = \lambda\mathbf{z}{:}\sigma.(\mathbf{M}\,\{\mathbf{y} := \mathbf{z}\}\,\{\mathbf{x} := \mathbf{N}\})$$

si $\mathbf{y} \in FV(\mathbf{N})$, con $\mathbf{z} \notin FV(\mathbf{N})$

$$(\mathbf{M}\,\mathbf{O})\,\{\mathbf{x} := \mathbf{N}\} = (\mathbf{M}\,\{\mathbf{x} := \mathbf{N}\})(\mathbf{O}\,\{\mathbf{x} := \mathbf{N}\})$$

Sustituciones (2)


```
    true {x := N} = true
   false {x := N} = false
    zero {x := N} = zero
(if M then O \
  else P) {x := N} = if (M {x := N}) then \
                        (O {x := N}) else (P {x := N})
  succ (M) {x := N} = succ (M{x := N})
  pred (M) {x := N} = pred (M{x := N})
isZero (M) {x := N} = isZero (M{x := N})
```

Juicio de tipado

$$\Gamma \vdash M : \sigma$$

con:

- Γ contexto $\{x_1{:}\sigma_1,\ x_1{:}\sigma_1,\ \ldots\ x_n{:}\sigma_n\}$
- M término de λ^{BN}
- σ tipo de λ^{BN}

Demostrar un juicio de tipado

- Es "dirigido por sintaxis" así que siempre sabemos qué regla usar
  - En general hay una única regla por término

`M ::= x`	ax_v
`| λx:σ.M`	$\to_i$
`| M M`	$\to_e$
`| true`	ax_t
`| false`	ax_f
`| if M then M else M`	if
`| zero`	ax_z
`| succ(M)`	succ
`| pred(M)`	pred
`| isZero(M)`	isZero