$\Gamma_1 = \forall x.\neg P(x), \exists x.P(x)$

$$\frac{\Gamma \vdash \exists X.\sigma \quad \Gamma, \sigma \vdash \tau \quad X \notin \mathsf{fv}(\Gamma, \tau)}{\Gamma \vdash \tau}\ \exists_e$$

tivos

$$\dfrac{\dfrac{\dfrac{}{\Gamma_1 \vdash \exists x.P(x)} \quad \dfrac{}{\Gamma_1, P(x) \vdash P(x)}}{\forall x.\neg P(x), \exists x.P(x) \vdash P(x)}\ \exists_e \quad \dfrac{\dfrac{}{\forall x.\neg P(x), \exists x.P(x) \vdash \forall x.\neg P(x)}\ Ax}{\forall x.\neg P(x), \exists x.P(x) \vdash \neg P(x)}\ \forall_e}{\dfrac{\forall x.\neg P(x), \exists x.P(x) \vdash \bot}{\dfrac{\forall x.\neg P(x) \vdash \neg \exists x.P(x)}{\vdash \forall X.\neg P(X). \implies \neg \exists X.P(X)}\ \Rightarrow_i}\ \neg_i}\ \neg_e$$

τ $\{x := x\}$

$$\frac{\Gamma \vdash \forall X.\sigma}{\Gamma \vdash \sigma\{X := t\}}\ \forall_e$$

$$\frac{\Gamma \vdash \tau \quad \Gamma \vdash \neg\tau}{\Gamma \vdash \bot}\ \neg_e$$

$$\frac{\Gamma, \tau \vdash \bot}{\Gamma \vdash \neg\tau}\ \neg_i$$