Tarea 2.

1. Encuentre matriz de transferencia de:

$$L_a \cdot \frac{d i_a}{dt} + R_a \cdot i_a + B \cdot \frac{dq}{dt} = V \quad (1)$$

$$J_m \cdot \frac{d^2 q}{dt^2} + f_m \cdot \frac{dq}{dt} = k_a \cdot i_a \quad (2)$$

considerando como salida $y = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}$

$$z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} i_a \\ q \\ \dot{q} \end{bmatrix} \qquad \dot{z} = \begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{bmatrix} = \begin{bmatrix} \dot{i}_a \\ \dot{q} \\ \ddot{q} \end{bmatrix}$$

$\dot{i}_a$ de (1)

$\ddot{q}$ de (2)

$$\dot{z} = \begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{bmatrix} = \begin{bmatrix} \left(V - R_a i_a - B \frac{dq}{dt}\right) / L_a \\ z_3 \\ \left(K_a i_a - f_m \, dq/dt\right) / J_m \end{bmatrix}$$

$$\dot{z} = \begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{bmatrix} = \begin{bmatrix} \left(V - R_a z_1 - B z_3\right) / L_a \\ z_3 \\ \left(K_a z_1 - f_m z_3\right) / J_m \end{bmatrix}$$

$\dot{z} = A x + B u$

$$\underbrace{\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{bmatrix}}_{\dot{z}} = \underbrace{\begin{bmatrix} -\frac{R_a}{L_a} & 0 & -\frac{B z_3}{L_a} \\ 0 & 0 & 1 \\ \frac{K_a}{J_m} & 0 & -\frac{f_m}{J_m} \end{bmatrix}}_{A \; (3 \times 3)} \underbrace{\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}}_{x \; (3 \times 1)} + \underbrace{\begin{bmatrix} 1/L_a \\ 0 \\ 0 \end{bmatrix}}_{B \; (3 \times 1)} \underbrace{V}_{u}$$

Salidas:

$$Y = \begin{bmatrix} q \\ \dot{q} \end{bmatrix} = \begin{bmatrix} z_2 \\ z_3 \end{bmatrix}$$

$$Y = Cx + Du$$

$$Y = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\substack{2\times3 \\ C}} \underbrace{\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}}_{\substack{3\times1 \\ x}} + \underbrace{\begin{bmatrix} 0 \\ 0 \end{bmatrix}}_{\substack{2\times1 \\ D}} \underbrace{V}_{\substack{1\times1 \\ u}}$$

Una vez conocidas las matrices C, S, A, B y D

Calculemos con el modelo la matriz de transferencia.

$$G(s) = \left[C[SI - A]^{-1} B + D \right]$$

$$G(s) = \left[\underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\substack{2\times3 \\ C}} \left[\underbrace{\begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix}}_{\substack{3\times3 \\ SI}} - \underbrace{\begin{bmatrix} -\frac{R_a}{L_a} & 0 & -\frac{B}{L_a} \\ 0 & 0 & 1 \\ \frac{K_a}{J_m} & 0 & -\frac{f_m}{J_m} \end{bmatrix}}_{\substack{3\times3 \\ A}} \right]^{-1} \underbrace{\begin{bmatrix} 1/L_a \\ 0 \\ 0 \end{bmatrix}}_{B} + \underbrace{\begin{bmatrix} 0 \\ 0 \end{bmatrix}}_{D} \right]$$

Usemos matlab para resolver las operaciones.

$$G(s) = \begin{bmatrix} \dfrac{K_a}{B \cdot K_a \cdot s + R_a \cdot s \cdot f_m + J_m \cdot L_a \cdot s^3 + J_m \cdot R_a \cdot s^2 + L_a \cdot s^2 \cdot f_m} \\[2ex] \dfrac{K_a}{R_a \cdot f_m + B \cdot K_a + J_m \cdot R_a \cdot s + L_a \cdot s \cdot f_m + J_m \cdot L_a \cdot s^2} \end{bmatrix}$$

2. Encuentre matriz de transferencia de:

$\ddot{x} + 2(x+w) = 3u_1$ ①

$\dot{w} - 6(x-w) = 0.5u_2$ ②

considerando como salida $y = \begin{bmatrix} w \\ u_2 \\ \dot{x} \end{bmatrix}$

$$z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} x \\ \dot{x} \\ w \end{bmatrix} \qquad \dot{z} = \begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{bmatrix} = \begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{w} \end{bmatrix}$$

$\ddot{x}$ de ①
$\dot{w}$ de ②

$$\dot{z} = \begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{bmatrix} = \begin{bmatrix} z_2 \\ 3u_1 - 2(x+w) \\ 0.5u_2 + 6(x-w) \end{bmatrix}$$

$$\dot{z} = \begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{bmatrix} = \begin{bmatrix} z_2 \\ 3u_1 - 2(z_1+z_3) \\ 0.5u_2 + 6(z_1-z_3) \end{bmatrix}$$

$\dot{z} = Ax + Bu$

$$\underbrace{\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{bmatrix}}_{\dot{z}} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ -2 & 0 & -2 \\ 6 & 0 & -6 \end{bmatrix}}_{\substack{3\times 3 \\ A}} \underbrace{\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}}_{\substack{3\times 1 \\ x}} + \underbrace{\begin{bmatrix} 0 & 0 \\ 3 & 0 \\ 0 & 0.5 \end{bmatrix}}_{\substack{3\times 2 \\ B}} \underbrace{\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}}_{\substack{2\times 1 \\ u}}$$

Salidas:

$$Y = \begin{bmatrix} W \\ u_2 \\ \dot{X} \end{bmatrix} = \begin{bmatrix} Z_3 \\ u_2 \\ Z_2 \end{bmatrix} \qquad Y = CX + Du$$

$$Y = \underbrace{\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}}_{\substack{3\times 3 \\ C}} \underbrace{\begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \end{bmatrix}}_{\substack{3\times 1 \\ X}} + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}}_{\substack{3\times 2 \\ D}} \underbrace{\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}}_{\substack{2\times 1 \\ u}}$$

Hallemos la matriz de transferencia usando el modelo:

$$G(s) = \left[C[SI - A]^{-1} B + D \right]$$

Usemos matlab:

$$G(s) = \begin{bmatrix} \dfrac{18}{s^3+6s^2+2s+24} & , & \dfrac{s^2+2}{2(s^3+6s^2+2s+24)} \\ 0 & , & 1 \\ \dfrac{3s(s+6)}{s^3+6s^2+2s+24} & , & \dfrac{-s}{s^3+6s^2+2s+24} \end{bmatrix}$$