Tarea 1.

Determine la representación en espacio de estados de los siguientes sistemas, en caso de que sea lineal el sistema determine las matrices A y B.

1) $\ddot{y} - \mu(1 - y^2)\dot{y} + y = V$

$\ddot{y} - (\mu - \mu y^2)\dot{y} + y = V$

$$Z = \begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix} = \begin{bmatrix} y \\ \dot{y} \end{bmatrix} \qquad \dot{Z} = \begin{bmatrix} \dot{Z}_1 \\ \dot{Z}_2 \end{bmatrix} = \begin{bmatrix} Z_2 \\ \ddot{y} \end{bmatrix}$$

cambio de variable

$\frac{d}{dt}$

$\ddot{y} = V + (\mu - \mu y^2)\dot{y} - y \therefore$

$$\dot{Z} = \begin{bmatrix} \dot{Z}_1 \\ \dot{Z}_2 \end{bmatrix} = \begin{bmatrix} Z_2 \\ V + (\mu - \mu y^2)\dot{y} - y \end{bmatrix}$$

$$\dot{Z} = \begin{bmatrix} \dot{Z}_1 \\ \dot{Z}_2 \end{bmatrix} = \begin{bmatrix} Z_2 \\ V + (\mu - \mu y^2) Z_2 - Z_1 \end{bmatrix}$$

Sistema lineal $\therefore$ $\dot{Z} = Az + BU$

$$\begin{bmatrix} \dot{Z}_1 \\ \dot{Z}_2 \end{bmatrix} = \underbrace{\underbrace{\begin{bmatrix} 0 & 1 \\ -1 & (\mu - \mu y^2) \end{bmatrix}}_{2\times 2}}_{A} \underbrace{\underbrace{\begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix}}_{2\times 1}}_{Z} + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{B} \underbrace{V}_{U}$$

2) $(M+m)\ddot{x} + ml\ddot{\theta} = u$

$ml^2\ddot{\theta} + ml\ddot{x} = mgl\theta$

$$z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{bmatrix} \xrightarrow{\frac{d}{dt}} \dot{z} = \begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{z}_4 \end{bmatrix} = \begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{\theta} \\ \ddot{\theta} \end{bmatrix}$$

cambro de variables

$$\dot{z} = \begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{z}_4 \end{bmatrix} = \begin{bmatrix} z_2 \\ (u - ml\ddot{\theta})/(M+m) \\ z_4 \\ (mgl\theta - ml\ddot{x})/ml^2 \end{bmatrix}$$

$$\dot{z} = \begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{z}_4 \end{bmatrix} = \begin{bmatrix} z_2 \\ (u - ml\ddot{\theta})/(M+m) \\ z_4 \\ \{mgl\theta - ml[(u - ml\ddot{\theta})/(M+m)]\}/ml^2 \end{bmatrix}$$

$$\dot{z} = \begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{z}_4 \end{bmatrix} = \begin{bmatrix} z_2 \\ (u - ml\dot{z}_4)/(M+m) \\ z_4 \\ \{mglz_3 - ml[(u - ml\dot{z}_4)/(M+m)]\}/ml^2 \end{bmatrix}$$

3) $m_1\ddot{x}_1 + b\dot{x}_1 + (k_1+k_2)x_1 = b\dot{x}_2 + k_2x_2 + u$

$m_2\ddot{x}_2 + b\dot{x}_2 + (k_2+k_3)x_2 = b\dot{x}_1 + k_2x_1$

$$z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix} \qquad \dot{z} = \begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{z}_4 \end{bmatrix} = \begin{bmatrix} \dot{x}_1 \\ \ddot{x}_1 \\ \dot{x}_2 \\ \ddot{x}_2 \end{bmatrix}$$

$$\dot{z} = \begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{z}_4 \end{bmatrix} = \begin{bmatrix} z_2 \\ \left(b\dot{x}_2 + k_2x_2 + u - b\dot{x}_1 - (k_1+k_2)x_1\right)/m_1 \\ z_4 \\ \left[b\dot{x}_1 + k_2x_1 - b\dot{x}_2 - (k_2+k_3)x_2\right]/m_2 \end{bmatrix}$$

$$\dot{z} = \begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{z}_4 \end{bmatrix} = \begin{bmatrix} z_2 \\ \left[bz_4 + k_2z_3 + u - bz_2 - (k_1+k_2)z_1\right]/m_1 \\ z_4 \\ \left[bz_2 + k_2z_1 - bz_4 - (k_2+k_3)z_3\right]/m_2 \end{bmatrix}$$

$$\dot{z} = Az + Bu$$

$$\dot{z} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ -(k_1+k_2)/m_1 & -b/m_1 & k_2/m_1 & b/m_1 \\ 0 & 0 & 0 & 1 \\ k_2/m_2 & b/m_2 & -(k_2+k_3)/m_2 & -b/m_2 \end{bmatrix}}_{4\times4} \underbrace{\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix}}_{4\times1} + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} u$$

4) $$0 = \left(\frac{J_b}{R^2} + m\right)\ddot{r} + mg\,\text{sen}\,\theta - mr\dot{\theta}^2$$

$$\tau = (mr^2 + J + J_b)\ddot{\theta} + 2mr\dot{r}\dot{\theta} + mgr\cos\theta$$

$$Z = \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \end{bmatrix} = \begin{bmatrix} r \\ \dot{r} \\ \theta \\ \dot{\theta} \end{bmatrix} \qquad \dot{Z} = \begin{bmatrix} \dot{Z}_1 \\ \dot{Z}_2 \\ \dot{Z}_3 \\ \dot{Z}_4 \end{bmatrix} = \begin{bmatrix} \dot{r} \\ \ddot{r} \\ \dot{\theta} \\ \ddot{\theta} \end{bmatrix}$$

$$\dot{Z} = \begin{bmatrix} \dot{Z}_1 \\ \dot{Z}_2 \\ \dot{Z}_3 \\ \dot{Z}_4 \end{bmatrix} = \begin{bmatrix} Z_2 \\ (mr\dot{\theta}^2 - mg\,\text{sen}\,\theta)/\left(\frac{J_b}{R^2}+m\right) \\ Z_4 \\ (\tau - 2mr\dot{r}\dot{\theta} - mgr\cos\theta)/(mr^2+J+J_b) \end{bmatrix}$$

$$\dot{Z} = \begin{bmatrix} \dot{Z}_1 \\ \dot{Z}_2 \\ \dot{Z}_3 \\ \dot{Z}_4 \end{bmatrix} = \begin{bmatrix} Z_2 \\ (mZ_1Z_4^2 - mg\,\text{sen}\,Z_3)/\left(\frac{J_b}{R^2}+m\right) \\ Z_4 \\ (\tau - 2mZ_1Z_2Z_4 - mgZ_1\cos Z_3)/(mZ_1^2+J+J_b) \end{bmatrix}$$

$\dot{z} = Az + Bu$ pero no lineal