Tarea 3

Para los siguientes sistemas. determine la solución para el vector de estado con las condiciones indicadas en cada caso.

Sistema 1. $\dot{x}_1 = x_2$ $\quad x_1(0) = 1$

$\dot{x}_2 = -2x_1 - 3x_2$ $\quad x_2(0) = 0$

$$\underbrace{\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}}_{\dot{z}} = \underbrace{\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{x} + Bu$$

$$\Phi(t) = e^{At}x(0) = \mathcal{L}^{-1}\{(sI-A)^{-1}\}x(0)$$

Resolvamos en Matlab:

$$\Phi(t) = \underbrace{\begin{bmatrix} 2e^{-t} - e^{-2t}, & e^{-t} - e^{-2t} \\ 2e^{-2t} - 2e^{-t}, & 2e^{-2t} + e^{-t} \end{bmatrix}}_{2\times 2} \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{2\times 1}$$

$$\Phi(t) = \begin{bmatrix} 2e^{-t} - e^{-2t} \\ 2e^{-2t} - 2e^{-t} \end{bmatrix}$$

Sistema 2.

$$\dot{x}_1 = x_2 \qquad\qquad x_1(0) = 0$$

$$\dot{x}_2 = x_1 - 6x_2 \qquad\qquad x_2(0) = 1$$

$$\underbrace{\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}}_{\dot{z}} = \underbrace{\begin{bmatrix} 0 & 1 \\ 1 & -6 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{x} + Bu \qquad x(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\Phi(t) = e^{At}x(0) = \mathcal{L}^{-1}\left\{(SI - A)^{-1}\right\}x(0)$$

Resolvamos en Matlab

$$\Phi(t) = \begin{bmatrix} \dfrac{\sqrt{10}\, e^{-3t}\, \text{senh}(\sqrt{10}\, t)}{10} \\[2ex] e^{-3t}\left[\cosh(\sqrt{10}\, t) - \dfrac{3\sqrt{10}\, \text{senh}(\sqrt{10}\, t)}{10}\right] \end{bmatrix}$$

Sistema 3.

$$\dot{X}_1 = -X_2 \qquad X_1(0) = 1$$
$$\dot{X}_2 = -X_1 + 3X_2 + u \qquad X_2(0) = 0.5$$

Sistema no homogeneo porque tiene entrada $u(t)$

$$\underbrace{\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix}}_{\dot{Z}} = \underbrace{\begin{bmatrix} 0 & -1 \\ -1 & 3 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} X_1 \\ X_2 \end{bmatrix}}_{X} + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{B} \underbrace{u}_{u}$$

$$X(0) = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}$$

$$\Phi(t) = \overbrace{\mathcal{L}^{-1}\left\{ (sI - A)^{-1} \right\}}^{e^{At}}$$

$$X(t) = \Phi(t)\, X(0) + \int_0^t \Phi(t-\tau)\, B u(\tau)\, d\tau$$

Resolvemos en matlab:

$$X(t) = \begin{bmatrix} \dfrac{\sqrt{13}\, e^{-\left[\frac{t(\sqrt{13}-3)}{2}\right]}}{26} - \dfrac{\sqrt{13}\, e^{\left[\frac{t(\sqrt{13}+3)}{2}\right]}}{26} + 1 \\[2ex] \dfrac{e^{-\frac{t(\sqrt{13}-3)}{2}} \left(13 e^{\sqrt{13}t} + 3\sqrt{13} e^{\sqrt{13}t} - 3\sqrt{13} + 13\right)}{52} \end{bmatrix}$$

Sistema 4.

$$\dot{X}_1 = X_2$$

$$\dot{X}_2 = -X_1 + 2u$$

$$X_1(0) = 0$$

$$X_2(0) = 1$$

$$\underbrace{\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix}}_{\dot{z}} = \underbrace{\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} X_1 \\ X_2 \end{bmatrix}}_{X} + \underbrace{\begin{bmatrix} 0 \\ 2 \end{bmatrix}}_{B} \underbrace{u}_{u}$$

$$X(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\Phi(t) = \mathcal{L}^{-1}\left\{ (sI - A)^{-1} \right\} = e^{At}$$

$$x(t) = \Phi(t)\, x(0) + \int_0^t \Phi(t-\tau)\, Bu(\tau)\, d\tau$$

Resolvemos en matlab

$$x(t) = \begin{bmatrix} \operatorname{sen}(t) - 2\cos(t) + 2 \\ \cos(t) + 2\operatorname{sen}(t) \end{bmatrix}$$