

## 1.20

$$\begin{aligned}
\nabla(fg) &= f\nabla g + g\nabla f \\
\frac{\partial}{\partial x}(fg) &= \frac{\partial f}{\partial x}g + f\frac{\partial g}{\partial x} \rightarrow \text{Lo mismo para } y, z \\
&= f\frac{\partial g}{\partial x}\hat{i} + g\frac{\partial f}{\partial x}\hat{i} + f\frac{\partial g}{\partial y}\hat{j} + g\frac{\partial f}{\partial y}\hat{j} + f\frac{\partial g}{\partial z}\hat{k} + g\frac{\partial f}{\partial z}\hat{k} \\
&= f\left(\frac{\partial g}{\partial x}\hat{i} + \frac{\partial g}{\partial y}\hat{j} + \frac{\partial g}{\partial z}\hat{k}\right) + g\left(\frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k}\right) \\
&= f\nabla g + g\nabla f \quad \text{QED.}
\end{aligned}$$

$$\begin{aligned}
\nabla \cdot (A \times B) &= \frac{\partial}{\partial x}(A_y B_z - A_z B_y) + \frac{\partial}{\partial y}(A_z B_x - A_x B_z) + \frac{\partial}{\partial z}(A_x B_y - B_y A_x) \\
&= A_y \frac{\partial B_z}{\partial x} + B_z \frac{\partial A_y}{\partial x} - \left(A_z \frac{\partial B_y}{\partial x} + B_y \frac{\partial A_z}{\partial x}\right) + A_z \frac{\partial B_x}{\partial y} + B_x \frac{\partial A_z}{\partial y} \\
&\quad - \left(A_x \frac{\partial B_z}{\partial y} + B_z \frac{\partial A_x}{\partial y}\right) + A_x \frac{\partial B_y}{\partial z} + B_y \frac{\partial A_x}{\partial z} - \left(A_y \frac{\partial B_x}{\partial z} + B_x \frac{\partial A_y}{\partial z}\right) \\
&= B_z \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right) + B_y \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + B_x \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) \\
&\quad - A_z \left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}\right) + A_y \left(\frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x}\right) + A_x \left(\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}\right) \\
&= B \cdot (\nabla \times A) - A \cdot (\nabla \times B) \quad \text{QED.}
\end{aligned}$$

$$\begin{aligned}
\nabla \times (fA) &= f(\nabla \times A) - A \times (\nabla f) \\
&= \left(\frac{\partial f A_z}{\partial y} - \frac{\partial f A_y}{\partial z}\right)\hat{i} + \left(\frac{\partial f A_x}{\partial z} - \frac{\partial f A_z}{\partial x}\right)\hat{j} + \left(\frac{\partial f A_y}{\partial x} - \frac{\partial f A_x}{\partial y}\right)\hat{k} \\
&= \left(f\frac{\partial A_z}{\partial y} + A_z\frac{\partial f}{\partial y} - f\frac{\partial A_y}{\partial z} - A_y\frac{\partial f}{\partial z}\right)\hat{i} + \left(f\frac{\partial A_x}{\partial z} + A_x\frac{\partial f}{\partial z} - f\frac{\partial A_z}{\partial x} - A_z\frac{\partial f}{\partial x}\right)\hat{j} \\
&\quad + \left(A_y\frac{\partial f}{\partial x} + f\frac{\partial A_y}{\partial x} - A_x\frac{\partial f}{\partial y} - f\frac{\partial A_x}{\partial y}\right)\hat{k} \\
&= f\left(\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{i} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{j} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{k}\right) \\
&\quad + \left(\left(A_z\frac{\partial f}{\partial y} - A_y\frac{\partial f}{\partial z}\right)\hat{i} + \left(A_x\frac{\partial f}{\partial z} - A_z\frac{\partial f}{\partial x}\right)\hat{j} + \left(A_y\frac{\partial f}{\partial x} - A_x\frac{\partial f}{\partial y}\right)\hat{k}\right) \\
&= f(\nabla \times A) - A \times (\nabla f)
\end{aligned}$$