

Assignment 1 Answers

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Question 1: Asymptotic Notation

Question A (8 points)

Rank the following functions in order (non-decreasing) of their asymptotic growth. Next to each function, write its big-Theta value, (ie. write the correct $\Theta(g(n))$ next to each function but you are not required to prove the big-Theta value).

Function to be Examined	Big Theta Value
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Question B (6 points)

Determine if each of the following statements are true or false. If the statement is false, provide a counter example. If the statement is true, justify the statement using the formal definitions from class.

1.

False. $O(n)$ being just means that f has an upper bound of n ; however, it does not describe the lower bound. The lower bound could be much smaller. For example, binary search has a $O(\log n)$ however, anything higher is a valid upper bound; hence, it is also $O(n)$.

2.

True. The exponential function will grow larger asymptotically than the polynomial. Since the algorithm runs in $\Theta(n^2)$ then we can say that there exists a positive constant c , such that after some threshold k we have: $\Theta(n^2) \leq c \cdot 2^n$. It would also be $\Theta(n^2)$ since there will exist some k such that for all further n 's, $\Theta(n^2) \geq c \cdot 2^n$. Hence,

3.

False. $\Theta(n^2)$ does not necessarily imply that it is also $\Theta(n)$. Since big O describes an upper bound, it asserts that there exists some constant c , such that after a certain threshold, k , $\Theta(n^2) \leq c \cdot n$. However, this is not a strict upper bound. Our function could actually be $\Theta(n)$ and the above would still be valid, but it would be $\Theta(n)$ as well—this being the tighter lower bound. In that scenario, $\Theta(n^2) \leq c \cdot n$ for all n after a large enough n .

Question C (16 points)

For each of the following, show that $\Theta(n^2)$ is for the correct function. Prove your result using the definitions from class, justifying your statement is true for all $n \geq k$. (provide the value of k).