Assignment 6

CS-GY 6033 INET Fall 2024

Due date: Dec 16th 2024, 11:55pm

Question 1: Complexity classes

12 points

Short answers!

Consider the following problems. For each problem, determine if it is *possible* that there exists a polynomial-time algorithm for solving that problem. Justify your answer using what is currently know about their complexity classes.

- Travelling salesman problem
- $n \times n$ chess
- The Halting Problem
- Vertex Cover
- Integer Factorization
- Given a set of n items, where each item has a specific weight, can we pack them onto K trucks where each truck can hold at most weight B.

Question 2

12 points

Below are a list of runtimes for decision problems. For each runtime, determine if the corresponding problem is in P or EXP or both or neither.

1. $T(n) = (\log n)^6$
2. $T(n) = \log(n^6)$
3. $T(n) = (6n)^6$
4. $T(n) = n + 1000$
5. $T(n) = n^n$
6. $T(n) = 3^n + n^6$
7. $T(n) = 3^{n^2+6}$

Question 3

27 points

For each problem below, determine whether or not there is a known polynomial-time algorithm for solving the problem. You must justify why there is *no known poly-time algorithm* **OR** identify a *poly-time procedure* that solves the problem.

(a) Consider a the political meeting which has n participants. There are m issues which are to be discussed at the meeting. Each participant must list **exactly two** issues that interest them. The organisers would like select at most k issues, so that each person is interested in at least one of the selected issues.

(b) A graph G has n vertices and m edges. The problem is to determine if G contains a simple cycle of length at least 3.

(c) A graph G has n vertices and m edges. The problem is to determine if G contains a simple cycle of length at least k

(d) A *directed* graph G contains n vertices and m edges. The problem is to determine if there is path from vertex s to every other vertex in the graph.

(e) A *directed* graph G contains n vertices and m edges. The problem is to determine if there is path from vertex s to **and from** every other vertex in the graph.

(f) A *directed* graph G contains n vertices and m edges. The graph is not weighted. The problem is to determine if there is path from vertex s to every other vertex in the graph, where the number of edges in the path must be at most k.

(g) A directed graph contains n vertices and m edges. The problem is to determine if G is a DAG.

(h) An undirected graph has weighted edges. The problem is determine if there is a path that starts at vertex s and travels to vertex t where the sum of the edge weights is less than k.

(j) An undirected graph has weighted edges. The problem is determine if there is a path that starts at vertex s and visits all vertices exactly once, where the sum he edge weights is less than k.

Question 4

10 points

Prove that the following problem is NP-complete using a reduction from either : Vertex Cover, Independent Set, Dominating Set, or Clique. Recall the *two steps* that are necessary in order to show that a problem is NP complete.

A set of n people attend a political meeting, where m issues are to be discussed. Each person attending has created a sublist of issues (selected from the main set of m issues) that they are most interested in. The organisers would like to select at most k issues so that each person is interested in at least one of the selected issues. The problem is to determine if it possible or not.

Question 5

10 points

Prove that the following problem is NP-complete using a reduction from either: Vertex Cover, Indepednent Set, Dominating Set, Subset Sum, Hamiltonian cycle, or Clique. Recall the *two steps* that are necessary in order to show that a problem is NP complete.

A graph G consists of a set of n vertices and m edges. A specific vertex is labelled S. The problem is to determine if there is a simple path that starts at vertex S and visits all other vertices in the graph.