Carrier Phase Uncertainty and Modeling a lamppost based broadband network

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ECE 146 Lab 2

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0 Introduction

This lab is designed to implement various tasks in MATLAB and practice using functions and syntax necessary to reinforce signals and systems concepts. Specifically we will be modelling receiver operations in complex base-band and undoing carrier phase mismatch between receiver and carrier. Furthermore, we will demonstrate how wireless multi-path channels can be modeled in complex base-band with a simple post to post and post to car scenario. Finally, we will explore the concept of diversity and how we can use it to mitigate multi-path fading.

1 Modeling Carrier-Phase Uncertainty

Let's consider a pair of independently modulated signals, $u_c(t) = \sum_{n-1}^{N} b_c[n]p(t-n)$ and $u_s(t) = \sum_{n-1}^{N} b_s[n]p(t-n)$, where the symbols $b_c[n]$ and $b_s[n]$ are chosen with equal probability to be $+1$ and —1, and $p(t)$ is a rectangular pulse from 0 to 1.

1.1 Let's see what a typical realization of $u_c(t)$ and $u_s(t)$ over 10 symbols might look like.

```
% Problem 1.1
dt = 0.01; % Sampling period
number_of_symbols = 10;
b_c = randsample([-1 1], number_of_symbols, true);
b_s = randsample([-1 1], number_of_symbols, true);
u_c = repelem(b_c, 1/dt); u_s = repelem(b_s, 1/dt);
figure('Name', 'Problem 1.1');
subplot(2, 1, 1); plot(u_c);
title('Problem 1.1 U_c(t)'); xlabel('Time'); ylabel('Magnitude');
subplot(2, 1, 2); plot(u_s);
title('Problem 1.1 U_s(t)'); xlabel('Time'); ylabel('Magnitude');
```

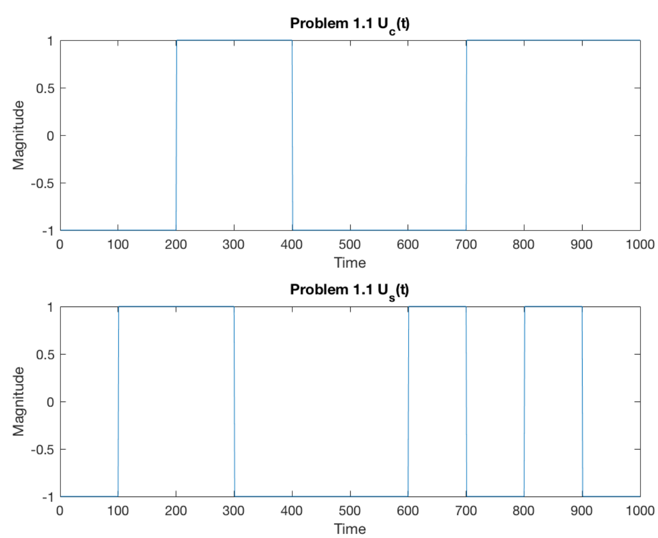

Figure 1: Typical realization of $u_c(t)$ and $u_s(t)$ over 10 symbols

1.2 Let's now upconvert this baseband waveform to get a binary phase-shift keyed signal (BPSK) of the form $u_{p,1}(t) = u_c(t)cos(40\pi t)$ and plot it over four symbols

```
t = 0:dt:length(b_c)-dt;
u_p1 = u_c.* cos(40*pi*t);
figure('Name', 'Problem 1.2');
plot(t, u_p1);
title('Problem 1.2 U_p1(t) = u_c* cos(40*pi*t)');
xlabel('Time'); ylabel('Magnitude');
```

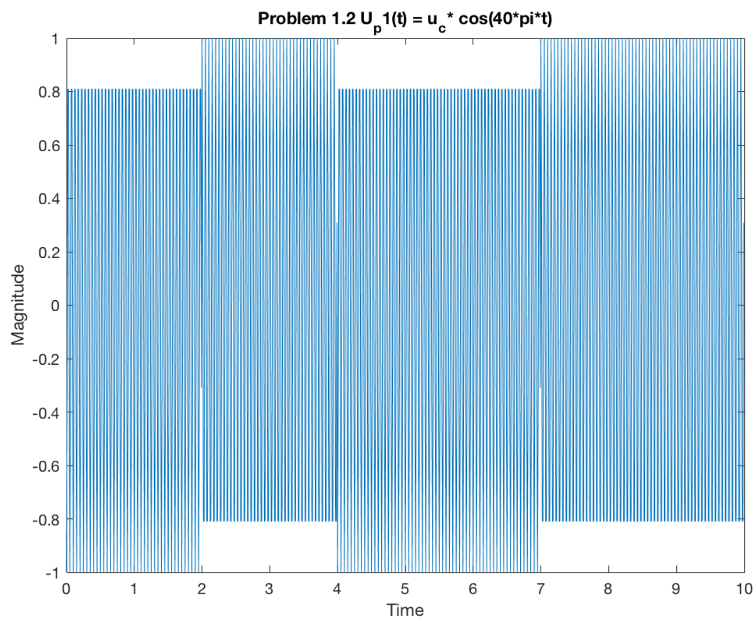

Figure 2: binary phase-shift keyed signal (BPSK) of the form $u_{p,1}(t) = u_c(t)cos(40\pi t)$

1.3 Let's now add in the the Q component and obtain the quaternary phase-shift keyed (QPSK) signal of the form $u_p(t) = u_c(t)cos(40\pi t) - u_s(t)sin(40\pi t)$

```
u_p = u_c .* cos(40*pi*t) - u_s .* sin(40*pi*t);
figure('Name', 'Problem 1.3');
plot(t, u_p);
title('Problem 1.2 U_p(t) = u_c * cos(40*pi*t) - u_s * sin(40*pi*t)');
xlabel('Time'); ylabel('Magnitude');
```

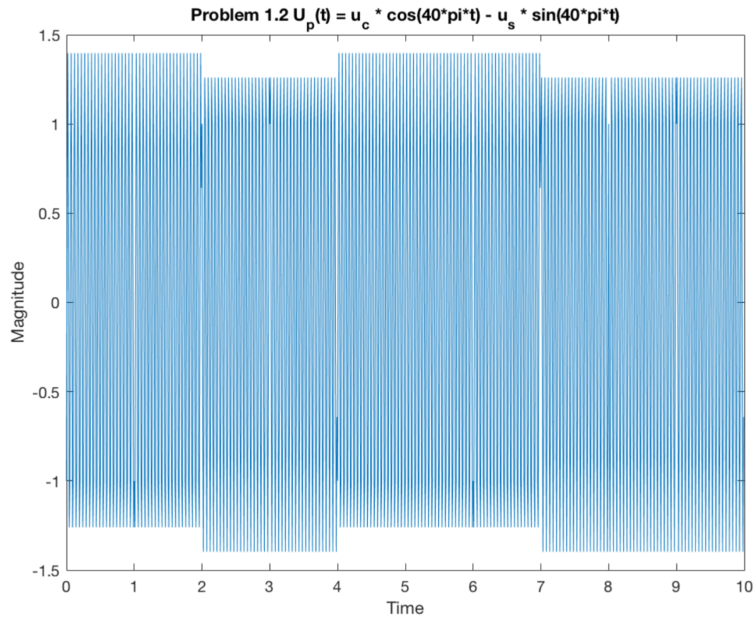

Figure 3: binary phase-shift keyed signal (BPSK) of the form $u_{p,1}(t) = u_c(t)cos(40\pi t)$

1.4 We can now downcovert $u_p(t)$ by passing $2u_p(t)cos(40\pi t+\theta)$ and $2u_p(t)sin(40\pi t+\theta)$ through low pass filters and plot the resulting I and Q components denotes as v_c and v_s respectively.

```
theta = pi/4;
v_c = 2 * u_p .* cos(40*pi*t + theta);
FT_v_p = fft(v_c);
low_pass = zeros(1, length(FT_v_p));
low_pass(t >= 0 & t <= 0.25) = 1;
recovered_v_c = (4/2500)*ifft(FT_v_p .* fft(low_pass));
figure('Name', 'Problem 1.5');
subplot(2,1,1);
plot(t, recovered_v_c);
title('Recovered v_c(t) with \theta = \pi/4');
xlabel('Time (s)');
ylabel('Magnitude');

v_s = -2 * u_p .* sin(40*pi*t + theta);
FT_v_s = fft(v_s);
low_pass = zeros(1, length(FT_v_s));
low_pass(t >= 0 & t <= 0.25) = 1;
recovered_v_s = (4/2500)*ifft(FT_v_s .* fft(low_pass));
subplot(2,1,2);
plot(t, recovered_v_s);
title('Recovered v_s(t) with \theta = \pi/4');
xlabel('Time (s)');
ylabel('Magnitude');
```

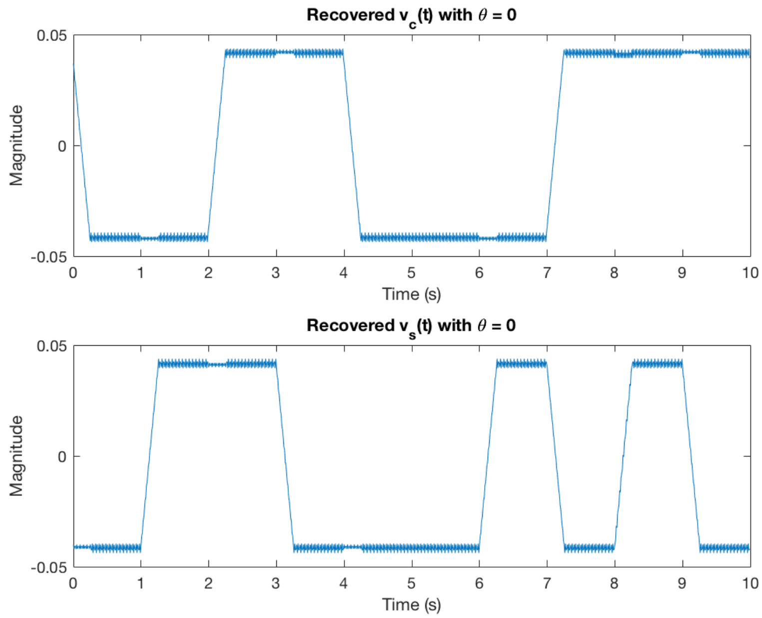

Figure 4: I and Q components of downcovert $u_p(t)$

We can clearly read off the corresponding bits to $b_c[n]$ and $b_s[n]$ by compating Figure 1 and Figure 4. Note that the transitions from high to low in the recovered I and Q components are not as sharp as that of the original bit stream.

1.5 If we now consider a value for θ of $\pi/4$, we observe how the recovered I and Q components compare to $b_c[n]$ and $b_s[n]$.

```
theta = pi/4;
v_c = 2 * u_p .* cos(40*pi*t + theta);
FT_v_p = fft(v_c);
low_pass = zeros(1, length(FT_v_p));
low_pass(t >= 0 & t <= 0.25) = 1;
recovered_v_c = (4/2500)*ifft(FT_v_p .* fft(low_pass));
figure('Name', 'Problem 1.5');
subplot(2,1,1);
plot(t, recovered_v_c);
title('Recovered v_c(t) with \theta = \pi/4');
xlabel('Time (s)');
ylabel('Magnitude');

v_s = -2 * u_p .* sin(40*pi*t + theta);
FT_v_s = fft(v_s);
low_pass = zeros(1, length(FT_v_s));
low_pass(t >= 0 & t <= 0.25) = 1;
recovered_v_s = (4/2500)*ifft(FT_v_s .* fft(low_pass));
subplot(2,1,2);
plot(t, recovered_v_s);
title('Recovered v_s(t) with \theta = \pi/4');
xlabel('Time (s)');
```

```
ylabel('Magnitude');
```

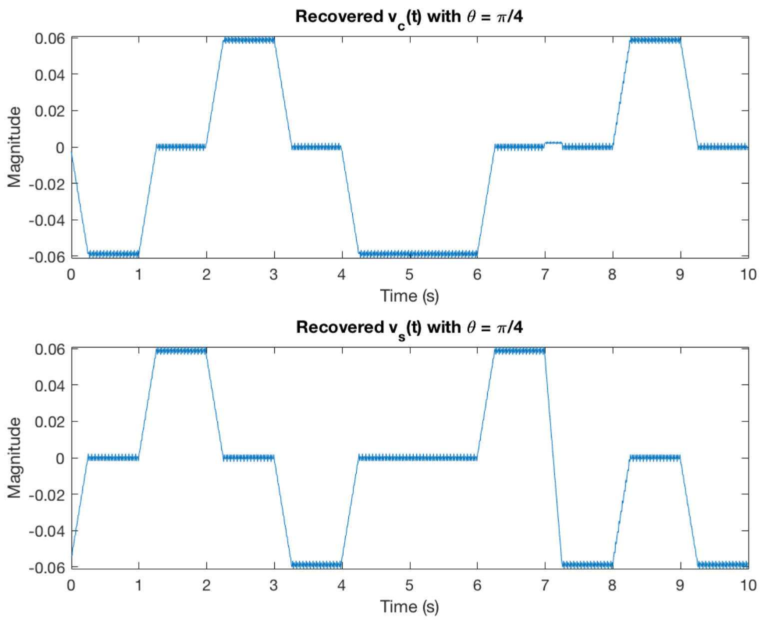

Figure 5: I and Q components of downcovert $u_p(t)$

We can observe that unlike the case where θ was 0, the recovered Q and I components don't correspond to the bit values originally sent in Figure 1.

1.6 Let's try to figure out how to recover u_c and u_s from the previous recovered I and Q factors in Figure 5 given θ.

Choosing the complex envelope of the signal with respect to the $40\pi t$ reference to be $u(t) = u_c(t) + ju_s(t)$, we can define the phase-shifted signal that arrived at the receiver as $v(t) = u(t)e^{-j\theta}$. We can now see how we can recover our desired $\tilde{u}(t)$ by undoing the phase shift of $v(t)$. Explicitly, $\tilde{u}(t) = v(t)e^{j\theta}$. Expanding each term, we get that $\tilde{u}(t) = (v_c(t) + jv_s(t))(cos(\theta) + jsin(\theta))$. Expanding this expression we arrive at $\tilde{u}(t) = v_c(t)(cos(\theta) - sin(\theta)) + jv_s(t)(cos(\theta) + sin(\theta))$ which correspond to $\tilde{u}_c(t)$ and $\tilde{u}_s(t)$ respectively. Let's plot the recovered $\tilde{u}_c(t)$ and $\tilde{u}_s(t)$ terms following this method.

```
phase_offset_v_c = cos(theta)*recovered_v_c - sin(theta)*recovered_v_s;
phase_offset_v_s = cos(theta)*recovered_v_s + sin(theta)*recovered_v_c;

figure('Name', 'Problem 1.6');
subplot(2,1,1);
plot(t, phase_offset_v_c);
title('Undoing phase offset of recovered v_c(t) with \theta = \pi/4');
xlabel('Time (s)'); ylabel('Magnitude');
subplot(2,1,2);
plot(t, phase_offset_v_s);
title('Undoing phase offset of recovered v_s(t) with \theta = \pi/4');
xlabel('Time (s)'); ylabel('Magnitude');
```

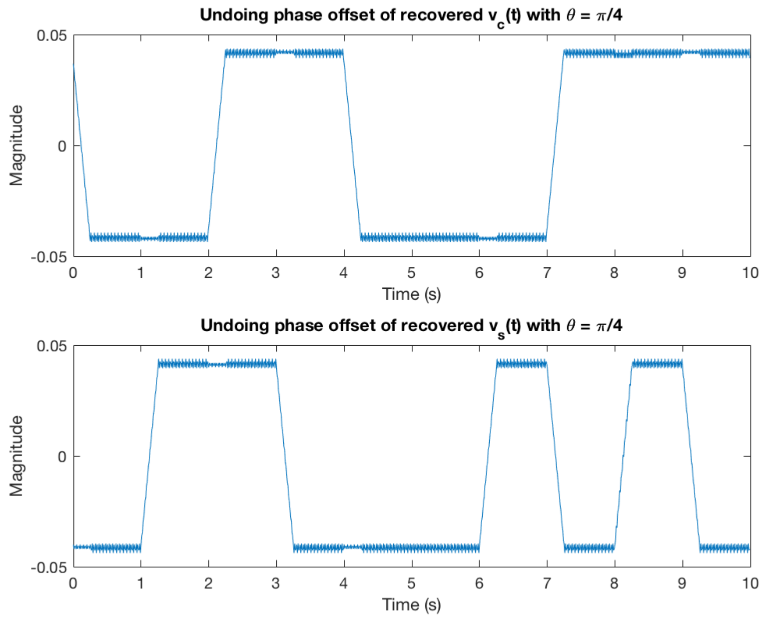

Figure 6: I and Q components of downcovert $u_p(t)$

It is now clear that the recovered $\tilde{u}_c(t)$ and $\tilde{u}_s(t)$ correspond to the same bit representation as that in Figure 1 and proved that given the phase offset of the arrived signal, we can recover the original I and Q components by undoing the phase offset.

2 Modeling a lamppost based broadband network

Let's now consider a lamppost-based network supplying broadband access using unlicensed spectrum at 60 GHz. Figure 7 shows two kinds of links: lamppost-to-lamppost for backhaul, and lamppost-to-mobile for access, where we show nominal values of antenna heights and distances. We explore simple channel models for each case, consisting only of the direct path and the ground reflection.

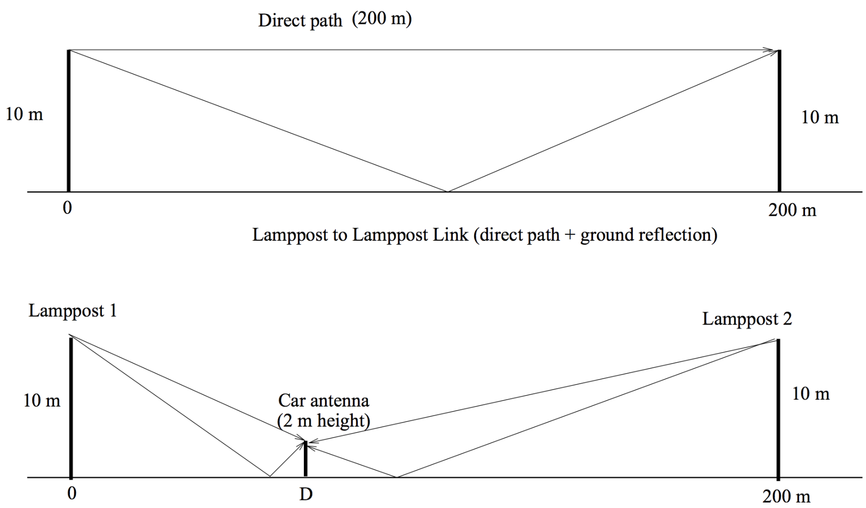

Figure 7: Links in lamp post based network.

2.1 Let's begin our discussion by finding the delay spread and coherence bandwidth for the lamppost-to-lamppost link, and determine if the signal is considered "narrowband" given that it has a 20 MHz bandwidth.

```
% For the case where both lampost are same height
c = 3*10^8; % Speed of light
r_min = 200; height_source = 10;
i_angle = atan((r_min/2)/height_source);
t_d = r_min*(1-sin(i_angle))/(c*sin(i_angle)); % Channel delay spread
B_c = 1/t_d; % Coherance Bandwidth

% A baseband signal of bandwidth W is said to be narrowband if WTau_d = W/Bc
W = 20*10^6;
if W/B_c < 0.1
    fprintf("This signal with BW = " + W + " and a channel coherance BW of magnitude " +
        B_c + " is narrowband");
else
    fprintf("This signal with BW = " + W + " and a channel coherance BW of magnitude " +
        B_c + " is not narrowband");
end
```

```
This signal with BW = 20000000 and a channel coherance BW of magnitude 300748134.3168 is
    narrowband
```

2.2 Let's now repeatthis for the lamppost-to-car link when the car is 100 m away from each lamppost

```
% For the case for lamppost-to-car link when the car is 100 m away from each
% lamppost.
c = 3*10^8; % Speed of light
height_source = 10;
height_receiver = 2;
distance_source_receiver = 100;
r_min = sqrt((height_source - height_receiver)^2+distance_source_receiver^2);
theta = atan(distance_source_receiver/(height_receiver+height_source));
r_max = ((height_source+height_receiver)/cos(theta));
t_d = (r_max/c)-(r_min/c); % Channel delay spread
B_c = 1/t_d; % Coherance Bandwidth

% A baseband signal of bandwidth W is said to be narrowband if WTau_d = W/Bc
W = 20*10^6;

if W/B_c < 0.1
    fprintf("This signal with BW = " + W + " and a channel coherance BW of magnitude " +
        B_c + " is narrowband");
else
    fprintf("This signal with BW = " + W + " and a channel coherance BW of magnitude " +
        B_c + " is not narrowband");
end
```

```
This signal with BW = 20000000 and a channel coherance BW of magnitude 753888435.4799 is
    narrowband
```

2.3 Let's now analyze the fading and diversity for the backhaul link by exploring the sensitivity of the lamppost to lamppost link to variations in height. We'll fix the height of the transmitter on lamppost 1 at 10 m and vary the height of the receiver on lamppost 2 from 9.8 to 10.2 m.

```
% For the case for lamppost-to-lampost link with varying receiver heights.
c = 3*10^8; % Speed of light
fc = 60*10^9; % lamppost-based network supplying broadband access at 60 GHz
height_source = 10;
height_receiver = 9.8:0.0001:10.2; % Varying receiver heights.
distance_source_receiver = 200;
r_min = sqrt((height_source - height_receiver).^2+distance_source_receiver.^2);
theta = atan(distance_source_receiver./(height_receiver+height_source));
r_max = ((height_source+height_receiver)./cos(theta));
t_min = r_min/c;
t_max = r_max/c;
h_nom = (1./r_min).*exp(-1i*(1*2*pi*fc.*t_min));

theta_t = asin((1/8)*sin(theta)); % Find transmitted angle using snells law.
A_r = (cos(theta)-8*cos(theta_t))./(cos(theta)+8*cos(theta_t)); % Reflected Magnitude
h = h_nom + (A_r./r_max).*exp(-1i*(1*2*pi*fc.*t_max));
```

```
figure('Name', 'Problem 2.3');
plot(height_source-height_receiver, 20*log10(abs(h)./abs(h_nom)));
title('Normalized power gain as a function of the variation in the receiver height');
xlabel('Variation in Receiver Height');
ylabel('Magnitude (dB)');
```

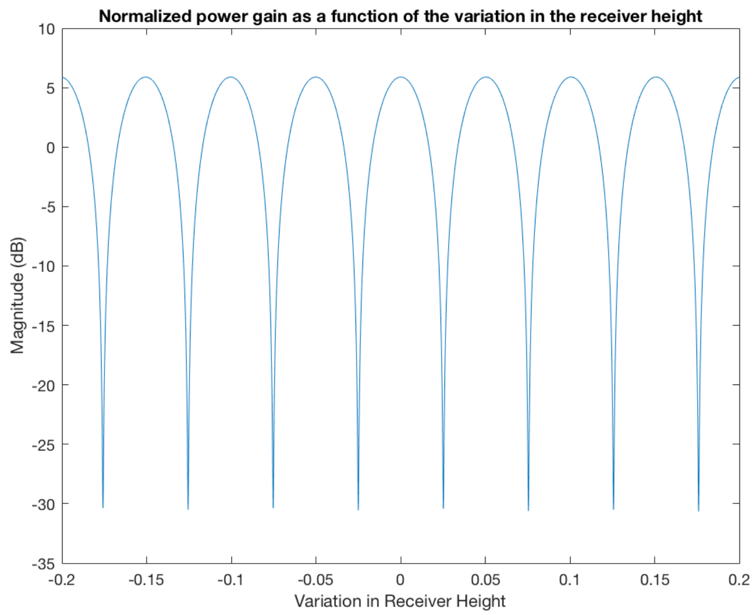

Figure 8: Normalized power gain in dB, $20log10\frac{|h|}{|hnom|}$, as a function of the variation in the receiver height.

The channel seems to be vary sensitive to receiver height. We can observe how fading is periodic with respect to the receiver height. This occurs because the phase for the signal that bounces off the ground has a certain phase shift relative to the line of sight ray due to the difference in distance travelled and attenuation coefficient dependent on the incidence angle at which the ray arrived at the floor.

2.4 We can now analyze our channel quality by identifying what the probability that the normalized power gain is smaller than -10 dB.

```
norm_h = 20*log10(abs(h./h_nom));
samples_under_10dB = norm_h(norm_h <= -10);
probability_of_10dB_attenuation = (length(samples_under_10dB)/length(norm_h))
```

```
probability_of_10dB_attenuation =

 0.1030
```

2.5 **Lets further explore the spatial diversity by supposing that the transmitter has two antennas, vertically spaced by 1 cm, with the lower one at a height of 10 m. Defining h1 and h2 as the channels from the two antennas to the receiver, we can plot the normalized power gains in dB for each antenna as a function of receiver height.**

```
c = 3*10^8; % Speed of light
fc = 60*10^9; % lamppost-based network supplying broadband access at 60 GHz

height_receiver = 9.8:0.0001:10.2; % Varying receiver heights.
distance_source_receiver = 200;
lambda = c/fc; % wavelength

height_source1 = 10;
r_min1 = sqrt((height_source1 - height_receiver).^2+distance_source_receiver.^2);
theta1 = atan(distance_source_receiver./(height_receiver+height_source1));
r_max1 = ((height_source1+height_receiver)./cos(theta1));
t_min1 = r_min1/c;
t_max1 = r_max1/c;
h_nom1 = (1./r_min1).*exp(-1i*(1*2*pi*fc.*t_min1));
theta_t1 = asin((1/8)*sin(theta1)); % Find transmitted angle using snells law.
A_r1 = (cos(theta1)-8*cos(theta_t1))./(cos(theta1)+8*cos(theta_t1)); % Reflected
    Magnitude
h1 = h_nom1 + (A_r1./r_max1).*exp(-1i*(1*2*pi*fc.*t_max1));

height_source2 = 10.01;
r_min2 = sqrt((height_source2 - height_receiver).^2+distance_source_receiver.^2);
theta2 = atan(distance_source_receiver./(height_receiver+height_source2));
r_max2 = ((height_source2+height_receiver)./cos(theta2));
t_min2 = r_min2/c;
t_max2 = r_max2/c;
h_nom2 = (1./r_min2).*exp(-1i*(1*2*pi*fc.*t_min2));
theta_t2 = asin((1/8)*sin(theta2)); % Find transmitted angle using snells law.
A_r2 = (cos(theta2)-8*cos(theta_t2))./(cos(theta2)+8*cos(theta_t2)); % Reflected
    Magnitude
h2 = h_nom2 + (A_r2./r_max2).*exp(-1i*(1*2*pi*fc.*t_max2));

figure('Name', 'Problem 2.5');
plot(height_source1-height_receiver, 20*log10(abs(h1./h_nom1))); hold on;
plot(height_source2-height_receiver, 20*log10(abs(h2./h_nom2)));
title('Normalized power gain as a function of the variation in the receiver height');
xlabel('Variation in Receiver Height');
ylabel('Magnitude (dB)');
legend('Transmitter at 10.00 m', 'Transmitter at 10.01 m')
```

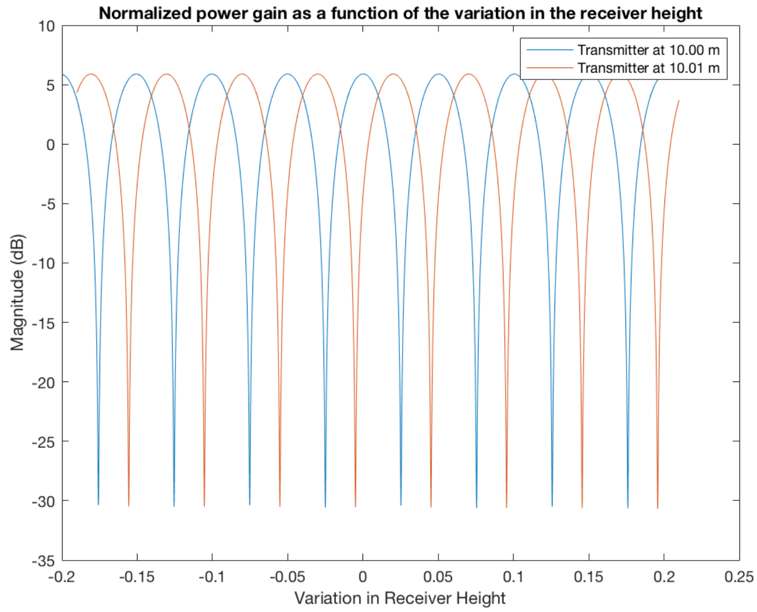

Figure 9: Normalized power gain in dB of two transmitter antennas spaced out by 1 cm, $20log10\frac{|h|}{|hnom|}$, as a function of the variation in the receiver height.

We can clearly see how the dips and peaks of the normalized power gain occur at different times. Specifically, we notice that if we were to pick the best channel given a variation in receiver height, we would always have a normalized power gain above -10 dB.

2.6 This strategy of switching to the transmit antenna which has the better channel, is called switched diversity. We can further analyze this technique by determining what the minimum guaranteed channel gain is for our specific example.

```
c = 3*10^8; % Speed of light
fc = 60*10^9; % lamppost-based network supplying broadband access at 60 GHz

height_receiver = 9.8:0.0001:10.2; % Varying receiver heights.
distance_source_receiver = 200;
lambda = c/fc; % wavelength

height_source1 = 10;
r_min1 = sqrt((height_source1 - height_receiver).^2+distance_source_receiver.^2);
theta1 = atan(distance_source_receiver./(height_receiver+height_source1));
r_max1 = ((height_source1+height_receiver)./cos(theta1));
t_min1 = r_min1/c;
t_max1 = r_max1/c;
t_d = (t_max1)-(t_min1); % Channel delay spread
B_c1 = 1./t_d; % Coherance Bandwidth
h_nom1 = (1./r_min1).*exp(-1i*(1*2*pi*fc.*t_min1));
theta_t1 = asin((1/8)*sin(theta1)); % Find transmitted angle using snells law.
```

```
A_r1 = (cos(theta1)-8*cos(theta_t1))./(cos(theta1)+8*cos(theta_t1)); % Reflected
    Magnitude
phi1 = mod((r_max1 - r_min1), lambda)*((2*pi)/lambda); % Calculate phi
h1 = h_nom1 + (A_r1./r_max1).*exp(-1i*(1*2*pi*fc.*t_max1));

height_source2 = 10.01;
r_min2 = sqrt((height_source2 - height_receiver).^2+distance_source_receiver.^2);
theta2 = atan(distance_source_receiver./(height_receiver+height_source2));
r_max2 = ((height_source2+height_receiver)./cos(theta2));
t_min2 = r_min2/c;
t_max2 = r_max2/c;
t_d2 = (t_max2)-(t_min2); % Channel delay spread
B_c2 = 1./t_d2; % Coherance Bandwidth
h_nom2 = (1./r_min2).*exp(-1i*(1*2*pi*fc.*t_min2));
theta_t2 = asin((1/8)*sin(theta2)); % Find transmitted angle using snells law.
A_r2 = (cos(theta2)-8*cos(theta_t2))./(cos(theta2)+8*cos(theta_t2)); % Reflected
    Magnitude
phi2 = mod((r_max2 - r_min2), lambda)*((2*pi)/lambda); % Calculate phi
h2 = h_nom2 + (A_r2./r_max2).*exp(-1i*(1*2*pi*fc.*t_max2));

figure('Name', 'Problem 2.6');
norm_h1 = abs(h1./h_nom1);
norm_h2 = abs(h2./h_nom2);
plot(height_source1-height_receiver, 20*log10(max(norm_h1, norm_h2)));
title('Normalized power gain as a function of the variation in the receiver height');
xlabel('Variation in Receiver Height');
ylabel('Magnitude (dB)');
```

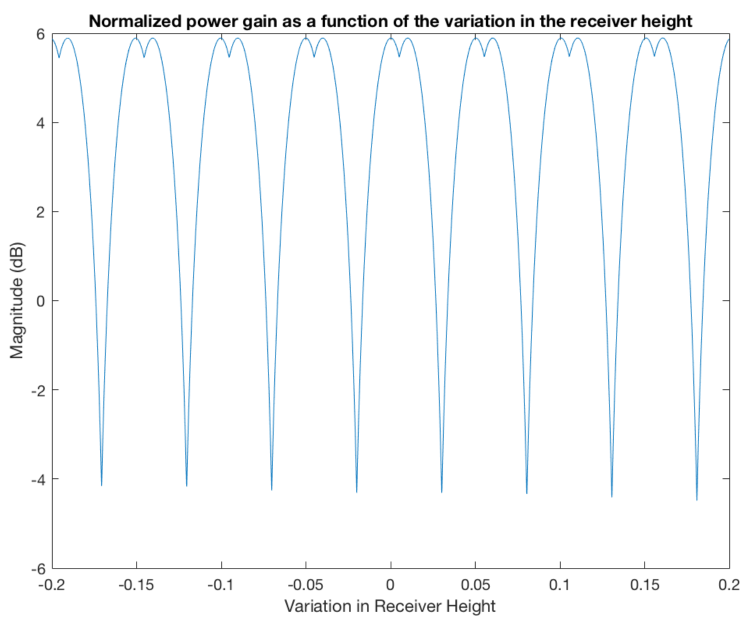

Figure 10: Minimum guaranteed channel gain with switched diversity.

2.7 Lets continue our analysis of this technique by finding the probability that the normalized power gain of the switched diversity scheme is smaller than -10 dB.

```
max_norm_h = 20*log10(max(norm_h1, norm_h2));
samples_under_10dB = max_norm_h(max_norm_h <= -10);
probability_of_10dB_attenuation = (length(samples_under_10dB)/length(max_norm_h))
```

```
probability_of_10dB_attenuation =

   0
```

2.8 Furthermore, by changing the antenna spacing from 0 cm and 5 cm in steps of 1 mm, we can determine which separation of transmitter antennas guarantees the strongest channel.

```
% For the case for lamppost-to-lampost link with varying receiver and transmitter
    heights.
c = 3*10^8; % Speed of light
fc = 60*10^9; % lamppost-based network supplying broadband access at 60 GHz
height_receiver = 9.8:0.001:10.2; % Varying receiver heights.
distance_source_receiver = 200;
height_source2 = 10:0.001:10.05;
min_gain = zeros(1, length(height_source2));

height_source1 = 10;
r_min1 = sqrt((height_source1 - height_receiver).^2+distance_source_receiver.^2);
theta1 = atan(distance_source_receiver./(height_receiver+height_source1));
r_max1 = ((height_source1+height_receiver)./cos(theta1));
t_min1 = r_min1/c;
t_max1 = r_max1/c;
h_nom1 = (1./r_min1).*exp(-1i*(1*2*pi*fc.*t_min1));
theta_t1 = asin((1/8)*sin(theta1)); % Find transmitted angle using snells law.
A_r1 = (cos(theta1)-8*cos(theta_t1))./(cos(theta1)+8*cos(theta_t1)); % Reflected
    Magnitude
h1 = h_nom1 + (A_r1./r_max1).*exp(-1i*(1*2*pi*fc.*t_max1));
norm_h1 = abs(h1./h_nom1);

for i = 1:length(height_source2)
r_min2 = sqrt((height_source2(i) - height_receiver).^2+distance_source_receiver.^2);
theta2 = atan(distance_source_receiver./(height_receiver+height_source2(i)));
r_max2 = ((height_source2(i)+height_receiver)./cos(theta2));
t_min2 = r_min2/c;
t_max2 = r_max2/c;
h_nom2 = (1./r_min2).*exp(-1i*(1*2*pi*fc.*t_min2));
theta_t2 = asin((1/8)*sin(theta2)); % Find transmitted angle using snells law.
A_r2 = (cos(theta2)-8*cos(theta_t2))./(cos(theta2)+8*cos(theta_t2)); % Reflected
    Magnitude
h2 = h_nom2 + (A_r2./r_max2).*exp(-1i*(1*2*pi*fc.*t_max2));
norm_h2 = abs(h2./h_nom2);
```

```
min_gain(i) = min(20*log10(max(norm_h1, norm_h2)));
end

figure('Name', 'Problem 2.8');
plot(height_source2-10, min_gain);
title('Minimum power gain as a function of transmitter antenna spacing');
xlabel('Variation in Transmitter spacing');
ylabel('Magnitude (dB)');
```

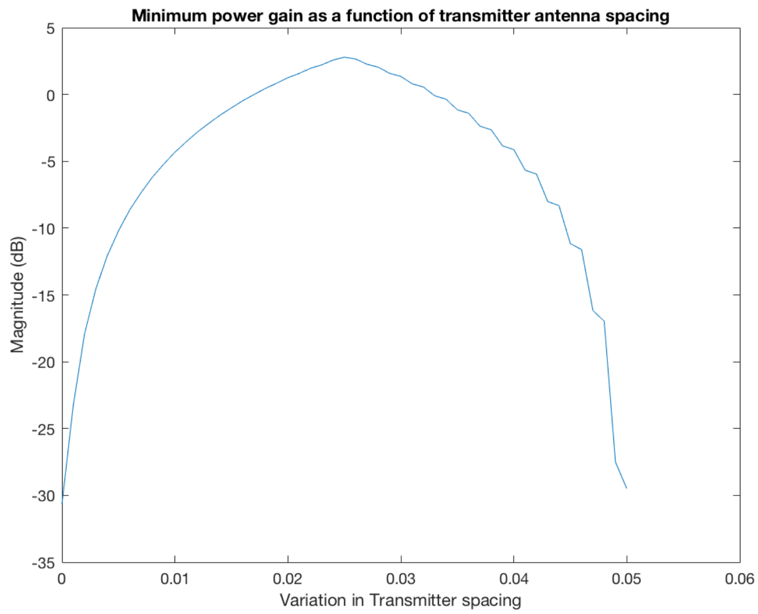

Figure 11: Minimum guaranteed channel gain with switched diversity as a function of transmitter antenna spacing given a signal of 60 GHz

We can observe from Figure 11 that the best antennas spacing for a the least amount of possible power gain attenuation given switched diversity capability is 2.5 cm.

2.9 It is worth noting the physical limitations of this switched diversity technique as the frequency of the transmitted signal decreases. As an example, lets repeat the previous step of determining the best antenna spacing to optimize our power gain at the receiver given a a frequency of 5 GHz.

```
% For the case for lamppost-to-lampost link with varying receiver and transmitter
    heights.
c = 3*10^8; % Speed of light
fc = 5*10^9; % lamppost-based network supplying broadband access at 60 GHz
height_receiver = 9:0.001:11; % Varying receiver heights.
distance_source_receiver = 200;
height_source2 = 10:0.01:10.6;
min_gain = zeros(1, length(height_source2));

height_source1 = 10;
```

```
r_min1 = sqrt((height_source1 - height_receiver).^2+distance_source_receiver.^2);
theta1 = atan(distance_source_receiver./(height_receiver+height_source1));
r_max1 = ((height_source1+height_receiver)./cos(theta1));
t_min1 = r_min1/c;
t_max1 = r_max1/c;
h_nom1 = (1./r_min1).*exp(-1i*(1*2*pi*fc.*t_min1));
theta_t1 = asin((1/8)*sin(theta1)); % Find transmitted angle using snells law.
A_r1 = (cos(theta1)-8*cos(theta_t1))./(cos(theta1)+8*cos(theta_t1)); % Reflected
    Magnitude
h1 = h_nom1 + (A_r1./r_max1).*exp(-1i*(1*2*pi*fc.*t_max1));
norm_h1 = abs(h1./h_nom1);

for i = 1:length(height_source2)
r_min2 = sqrt((height_source2(i) - height_receiver).^2+distance_source_receiver.^2);
theta2 = atan(distance_source_receiver./(height_receiver+height_source2(i)));
r_max2 = ((height_source2(i)+height_receiver)./cos(theta2));
t_min2 = r_min2/c;
t_max2 = r_max2/c;
h_nom2 = (1./r_min2).*exp(-1i*(1*2*pi*fc.*t_min2));
theta_t2 = asin((1/8)*sin(theta2)); % Find transmitted angle using snells law.
A_r2 = (cos(theta2)-8*cos(theta_t2))./(cos(theta2)+8*cos(theta_t2)); % Reflected
    Magnitude
h2 = h_nom2 + (A_r2./r_max2).*exp(-1i*(1*2*pi*fc.*t_max2));
norm_h2 = abs(h2./h_nom2);
min_gain(i) = min(20*log10(max(norm_h1, norm_h2)));
end

figure('Name', 'Problem 2.8');
plot(height_source2-10, min_gain);
title('Minimum power gain as a function of transmitter antenna spacing');
xlabel('Variation in Transmitter spacing');
ylabel('Magnitude (dB)');
```

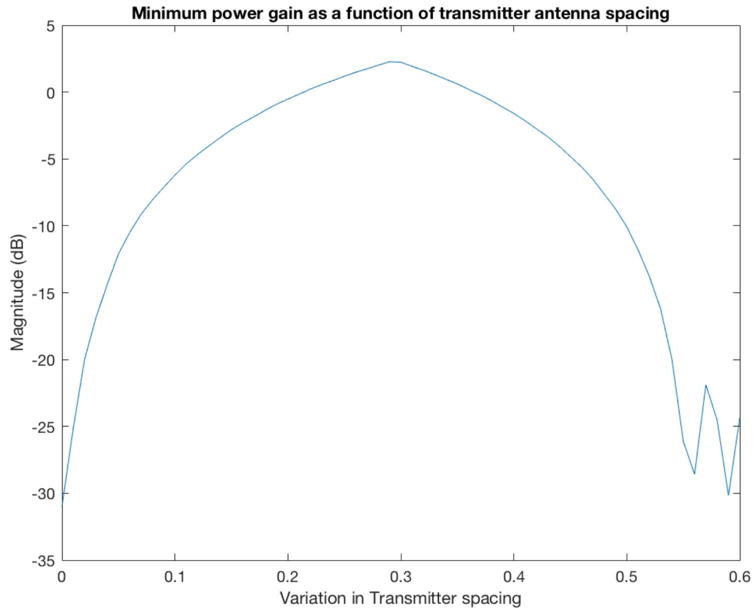

Figure 12: Minimum guaranteed channel gain with switched diversity as a function of transmitter antenna spacing given a signal frequency of 5 GHz

We can observe from Figure 12 that the best antennas spacing for a the least amount of possible power gain attenuation given switched diversity capability is 30 cm. This is in the order of a whole order of magnitude bigger than the optimal antenna spacing for a signal frequency of 60 Ghz. It is important to keep this relationship between signal frequency and optimal transceiver antenna spacing when designing any system.

2.10 Lets now explore the concept of frequency diversity. Assuming again that the transmitter has a single antenna at height of 10 m, we can compute and plot the normalized channel as a function of carrier frequency and observe how it influences the power gain at the receiver.

```
% For the case for lamppost-to-lampost link with varying receiver heights.
c = 3*10^8; % Speed of light
fc = 59*10^9:10^6:60*10^9; % lamppost-based network supplying broadband access in the
    range [59, 64] GHz with a step size of 10 MHz.
height_source = 10;
height_receiver = 10; % Stationary receiver height.
distance_source_receiver = 200;
r_min = sqrt((height_source - height_receiver).^2+distance_source_receiver.^2);
theta = atan(distance_source_receiver./(height_receiver+height_source));
r_max = ((height_source+height_receiver)./cos(theta));
t_min = r_min/c;
t_max = r_max/c;
h_nom = (1./r_min).*exp(-1i*(1*2*pi*fc.*t_min));
theta_t = asin((1/8)*sin(theta)); % Find transmitted angle using snells law.
A_r = (cos(theta)-8*cos(theta_t))./(cos(theta)+8*cos(theta_t)); % Reflected Magnitude
h = h_nom + (A_r./r_max).*exp(-1i*(1*2*pi*fc.*t_max));
```

```
figure('Name', 'Problem 2.10');
plot(fc, 20*log10(abs(h./h_nom)));
title('Normalized power gain as a function of the carrier frequency in GHz');
xlabel('Frequency (GHz)');
ylabel('Magnitude (dB)');
```

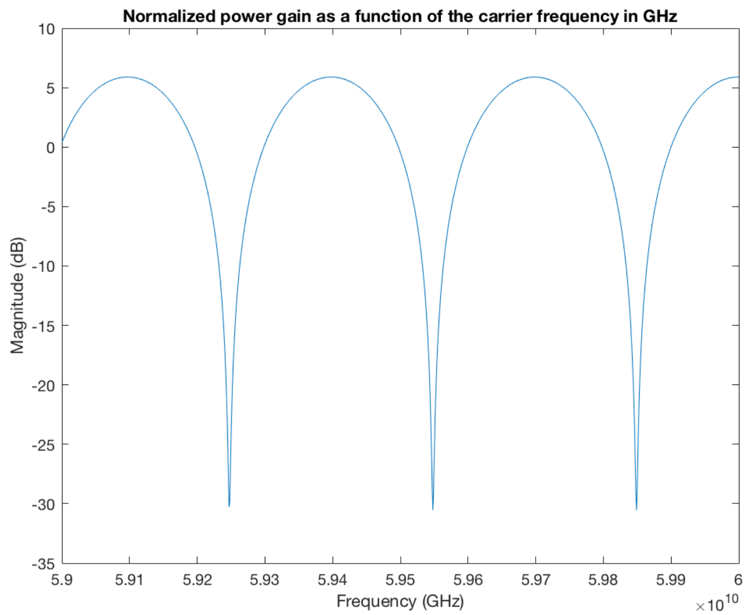

Figure 13: Normalized channel power again as a function of carrier frequency

2.11 Once again, lets determine what the probability that the channel gain is below -10 dB if the carrier frequency is chosen randomly with uniform distribution over this range.

```
norm_h = 20*log10(abs(h./h_nom));
samples_under_10dB = norm_h(norm_h <= -10);
probability_of_10dB_attenuation = (length(samples_under_10dB)/length(norm_h))
```

```
probability_of_10dB_attenuation =

    0.0919
```

2.12 Now, lets assume the carrier frequency is chosen randomly over [59, 64] GHz but transmitter is allowed to tune it by at most B, i.e., it is allowed to choose any $f_c \in [f_0, f_0 + B]$ where $f_0 U([59, 64]$ GHz). Lets determine how large B has to be so that the probability of dips below -10 and 0 dB is zero

```
norm_h_period = norm_h(fc < 59.4*10^9);
values_under_neg_10dB = norm_h_period(norm_h_period < -10);
B = length(values_under_neg_10dB)*(fc(2)-fc(1))

norm_h_period = norm_h(fc < 59.4*10^9);
values_under_neg_10dB = norm_h_period(norm_h_period < 0);
B2 = length(values_under_neg_10dB)*(fc(2)-fc(1))
```

```
B =
    30000000

B2 =
   102000000
```

Observing the output of the previous code, we can conclude that B would have to be at least 30 MHz to have 0 probability of having a dip under -10 dB and at least 102 MHz to have 0 probability of having a dip under 0 dB

2.13 Lets draw some conclusion about diversity and the extent to which it helps combat fading.

We can observe from Figures 10 and 13 how given a flexible choice of antenna spacing and tun-able bandwidth room we can theoretically eliminate, if not strongly mitigate, power gain fading.

2.14 Lets now return to the example of the access channel from lamppost 1 to the car, and observe the relationship between power fading and distance between the car and the base of the lamppost. Specifically, lets note the “long-term” variation due to range, and the “short-term” variation due to multi-path fading.

```
% For the case for lamppost-to-car link with varying car distance from transmitter.
c = 3*10^8; % Speed of light
fc = 60*10^9; % lamppost-based network supplying broadband access at 60 GHz
height_source = 10;
height_receiver = 2; % Car receiver heights.
distance_source_receiver = 0:0.01:200;
r_min = sqrt((height_source - height_receiver).^2+distance_source_receiver.^2);
theta = atan(distance_source_receiver./(height_receiver+height_source));
r_max = ((height_source+height_receiver)./cos(theta));
t_min = r_min/c;
t_max = r_max/c;
h_nom = (1./r_min).*exp(-1i*(1*2*pi*fc.*t_min));
theta_t = asin((1/8)*sin(theta)); % Find transmitted angle using snells law.
A_r = (cos(theta)-8*cos(theta_t))./(cos(theta)+8*cos(theta_t)); % Reflected Magnitude
h = h_nom + (A_r./r_max).*exp(-1i*(1*2*pi*fc.*t_max));


figure('Name', 'Problem 2.14');
plot(distance_source_receiver, 20*log10(abs(h_nom))); hold on;
```

```
plot(distance_source_receiver, 20*log10(abs(h)));
title('Normalized power gain as a function of the variation in Receiver Distance to
    Transmitter');
xlabel('Variation in Receiver Distance to Transmitter');
ylabel('Magnitude (dB)');
legend('h_{nom}(D)', 'h(D)');
```

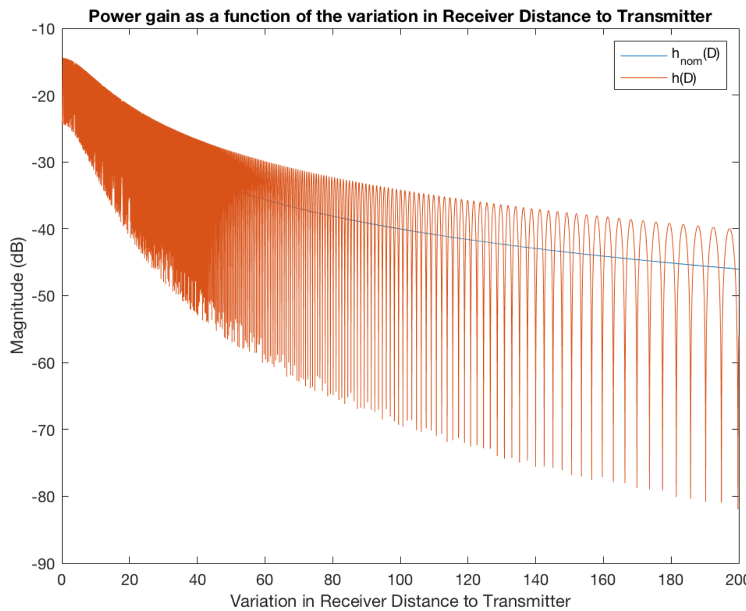

Figure 14: "Long-term" variation due to range, and the "short-term" variation due to multi-path fading.

Figure 14 shows a very interesting phenomenon of power gain as the receiver moves further away from the transmitter antenna. First, we notice that the nominal power gain magnitude decays as the receiver moves farther away from the transmitter. This agrees with our intuition that the power gain of the line of sight ray signal decays at a rate of $\frac{1}{r}$. Second, we observe that the channel gain including the ground reflected ray, has a smaller peak to peak gain fading when it is closer to the transmitter antenna and slowly increases as we move away from it. And third, we notice that gain fading is not a periodic phenomenon relative to horizontal distance between transmitter and receiver. The spacing for which the power gain peaks and dips gets bigger as we move away from the transmitter. This facts are important to keep in mind if we are to implement spatial or frequency diversity to optimize power gain at the receiver since it tells us how we need to choose our antenna spacing and bandwidth tunability as function of distance to the receiver.

2.15 **Lets end our discussion by mentioning the effects of surface roughness. So far, we've been using the Fresnel reflection coefficients to describe specular reflection from smooth surfaces. If surfaces are rough relative to the wavelength, then part of the wave power will be scattered in other directions and the reflected ray will be attenuated by some factor α_s. When the surface is not too rough, this attenuation factor can be approximated by $\alpha_s = exp(8(\pi\sigma(cos(\theta i/\lambda))))$. Where σ is the standard deviation of surface height. The reflected path amplitude will therefore be equal to $Ar = \alpha_s \rho_s$. Lets repeat our previous exercise assuming that the ground is rough with a σ value of 1mm and compare it to that of a smooth ground model.**

```
% For the case for lamppost-to-car link with varying car distance from transmitter.
c = 3*10^8; % Speed of light
fc = 60*10^9; % lamppost-based network supplying broadband access at 60 GHz
height_source = 10;
height_receiver = 2; % Car receiver heights.
distance_source_receiver = 0:0.01:200;
r_min = sqrt((height_source - height_receiver).^2+distance_source_receiver.^2);
theta = atan(distance_source_receiver./(height_receiver+height_source));
r_max = ((height_source+height_receiver)./cos(theta));
t_min = r_min/c;
t_max = r_max/c;
t_d = (t_max)-(t_min); % Channel delay spread
B_c = 1./t_d; % Coherance Bandwidth
h_nom = (1./r_min).*exp(-1i*(1*2*pi*fc.*t_min));
theta_t = asin((1/8)*sin(theta)); % Find transmitted angle using snells law.
sigma = 0.001;
rho_s = (cos(theta)-8*cos(theta_t))./(cos(theta)+8*cos(theta_t)); % Reflected Magnitude
lambda = c/fc; % wavelength
A_r = exp(-8*(pi*sigma*cos(theta)/lambda).^2).*rho_s;
h = h_nom + (A_r./r_max).*exp(-1i*(1*2*pi*fc.*t_max));


figure('Name', 'Problem 2.15');
plot(distance_source_receiver, 20*log10(abs(h_nom))); hold on;
plot(distance_source_receiver, 20*log10(abs(h)));
title('Power gain as a function of the variation in Receiver Distance to Transmitter');
xlabel('Variation in Receiver Distance to Transmitter');
ylabel('Magnitude (dB)');
legend('h_{nom}(D)', 'h(D)');
```

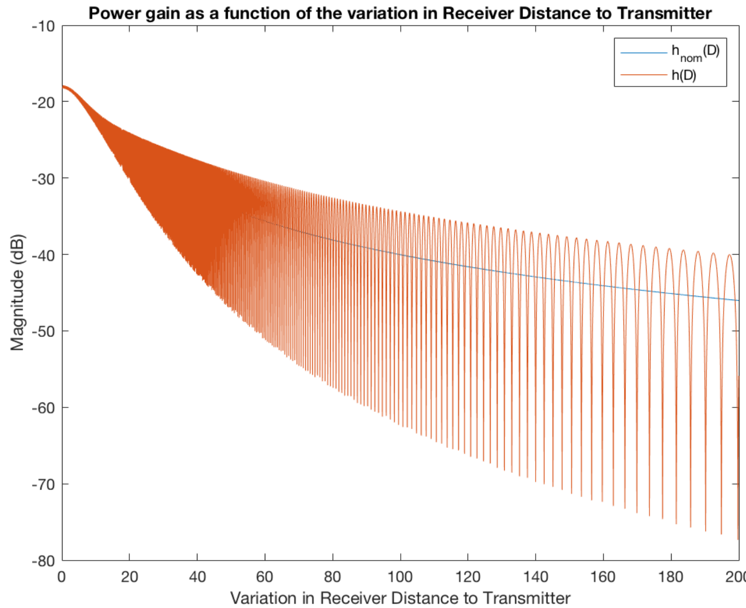

Figure 15: “Long-term” variation due to range, and the “short-term” variation due to multi-path fading assuming a rough ground model.

Surprisingly, our channel gain attenuation for the rough ground model displays more stable behaviour in terms that it has a very small peak to peak gain fading, meaning that our gain fading stays relatively constant with respect to distance when close to the transmitter. Furthermore, this model displays the same three characteristics mentioned of the smooth ground model shown in Figure 14.

3 Conclusion

In conclusion, we started this lab by modeling the process of acquiring a binary representation of some signal, up converting it to a pass-band signal that could be sent through a channel, and down converting it to a base-band signal where we can recover our message signal. Furthermore, we modeled carrier-phase uncertainty by assuming a case in which our carrier arrived at our receiver with some phase. We learned that we can’t directly recover our base-band signal by down-converting this phase shifted pass-band. Instead we must figure out the phase at which the message arrived at our receiver through training process and then undo the phase shift of the arrived signal to recover our original message. We then proceeded to model a lamppost based broadband network with a single line of sight and reflected ray to acquire some insight into how sensitive our channel gain is to small spatial and frequency perturbations. Moreover, we discussed the concept of diversity and how we can use this strategy to guarantee a minimum power-gain fading given some spatial and frequency freedom. We concluded by analyzing channel gain fading as a function of distance from transmitter and introduced a rough ground model that affects the reflected ray attenuation and compared it to the smooth ground model we used throughout this lab.