

**ASSIGNMENT 4: Multi-Frequency Backward Propagation**

Due: Tuesday, February 16

Name: \_\_\_\_\_

**Objective:** The objective of this programming assignment is to experience image formation by multi-frequency backward propagation and to observe the effects on image resolution especially in the *range direction*.

The system configuration is the same as that in Assignment 2. The *receiver aperture* is a centered linear receiver array with a span of  $60\lambda_0$  (from  $x = -30\lambda_0$  to  $x = +30\lambda_0$ ). This receiver array is located at the *plane*  $y = y_0 = -60\lambda_0$ , with quarter-wavelength spacing ( $\lambda_0/4$ ) spacing. For each coherent frequency, there are 241 wavefield samples in total over the  $60\lambda_0$ -long aperture.

In assignment 2, one single wavelength was applied. As we move into the multi-frequency operating mode, we collect wavefield samples over a wide range of spectrum corresponding to 40 different wavelengths, in the form

$$\lambda_n = 40\lambda_0/(n+20) \quad n = 1, 2, \dots, 40$$

Thus, this imaging modality operates with a sequence of wavelengths, from  $0.67\lambda_0$  to  $2\lambda_0$ , corresponding to the spatial-frequency band from  $0.5(1/\lambda_0)$  to  $1.5(1/\lambda_0)$  resulting a bandwidth of  $(1/\lambda_0)$ . This is to perform coherent image formation procedure 40 times with different operating wavelengths, with the same receiver array configuration. This process produces 40 coherent sub-images,  $\hat{s}_n(x, y)$ , for  $n = 1, 2, \dots, 40$ . (It should be noted that the locations of the source distribution and receivers are independent of the change of operating wavelength.)

The goal is to perform multi-frequency image reconstruction of the  $60\lambda_0 \times 60\lambda_0$  2D source region with the 40 sets of coherent wavefield samples. The source region is an area centered at  $(0, 0)$  and bounded by  $x = \pm 30\lambda_0$  and  $y = \pm 30\lambda_0$ . For consistency, use quarter-wavelength ( $\lambda_0/4$ ) spacing as the sample spacing in both directions.

- (A) By repeating the coherent backward propagation image formation procedure, produce the 40 coherent sub-images.
- (B) Observe the characteristics of the sub-images and compare to the results from Assignment 2.
- (C) Superimpose the 40 coherent sub-images *sequentially* and observe the convergence to the overall image,

$$\hat{S}_n(x, y) = \sum_{k=1}^n \hat{s}_k(x, y) \quad n = 1, 2, \dots, 40.$$

- (D) Plot the magnitude distribution of the 4 images  $\hat{S}_{10}(x, y)$ ,  $\hat{S}_{20}(x, y)$ ,  $\hat{S}_{30}(x, y)$ , and  $\hat{S}_{40}(x, y)$ .
- (E) Compile a 40-frame video of the image sequence  $\hat{S}_n(x, y)$ ,  $n = 1, 2, \dots, 40$ .
- (F) Compile a 40-frame video of the spectra of the image sequence  $\hat{S}_n(x, y)$ .

Report format:

1. Cover page.
2. Magnitude distribution of the final images from Part (D).
3. Video sequences from Parts (E) and (F).
4. Summary: (comments based on your observations)
5. Appendix: (computer code)