ECE278C Imaging Systems

Lab 1

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0 Introduction

The main objective of this report is to visualize and analyze the spectral distribution of 2-D coherent wave-field patterns. We will also explore the phase-only concept, and make observations of changes due to the variation of wavelength, sample spacing, and aperture size.

1 Single Coherent Point Source

Figure 1 shows the scattered wave field pattern for a single point source at the origin. The point source scattered pattern is mathematically described by Green's function:

$$h(x, y) = \frac{1}{\sqrt{j\lambda_0 r}} exp \frac{j2\pi r}{\lambda_0})$$

$$r = \sqrt{x^2 + y^2}$$

Figure 2 shows the magnitude spectrum of a coherent point source scattered pattern. We can observe it takes its highest values along a ring of radius $\frac{1}{\lambda}$. We can confirm that this magnitude spectrum agrees with the spectrum derived from the differential equation associated with wave propagation also known a Helmholtz equation.

$$H(f_x, f_y) = \delta\left(|f| - \frac{1}{\lambda_0}\right)$$

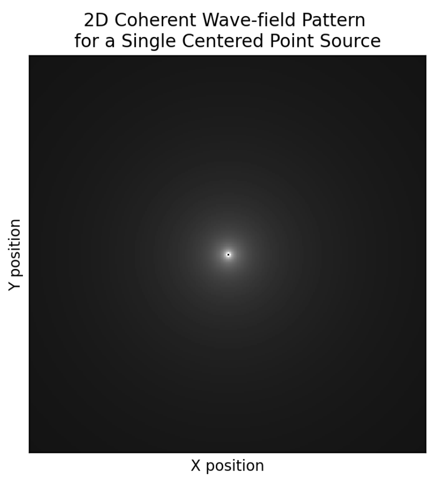

Figure 1: Scattered Wave-field

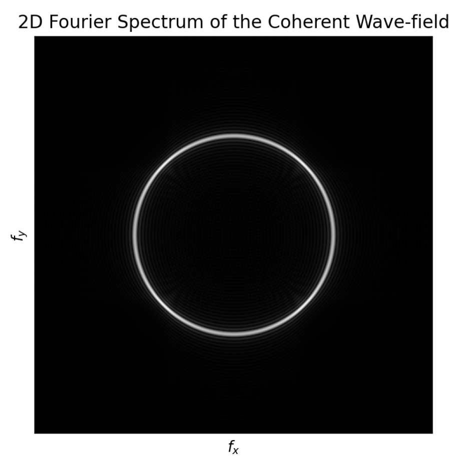

Figure 2: Magnitude of Spectrum

2 6 Coherent Point Sources

Figure 3 and 4 show the scattered wave-field and magnitude spectrum for 6 super imposed point sources with the same wavelength. Similarly to the single point source case, we observe that the spectrum takes it's highest values along a ring of radius $\frac{1}{\lambda_0}$. This reflects the linear shift invariant properties of our system. Furthermore, we notice that this ring has some lighter and darker areas along it. This could be attributed to the phase terms introduced by each shifted point sources.

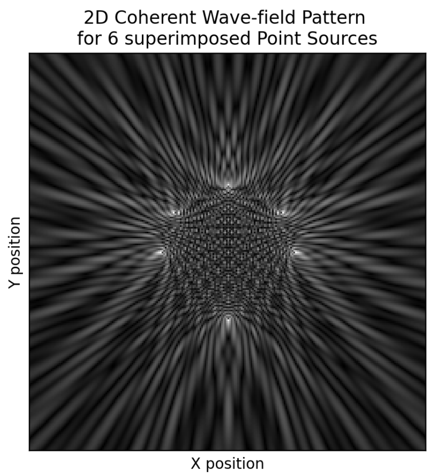

Figure 3: Scattered Wave-field

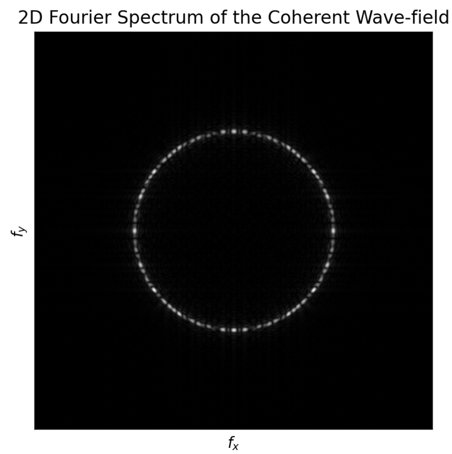

Figure 4: Magnitude of Spectrum

3 6 Point Sources with Different Wavelengths

Now we'll consider a different case where each of the 6 point sources is generating independent coherent wave-field patterns with a different operating wavelength described in figure 5.

	scatters	*scatter locations*	*wavelength*
1	(x_1, y_1)	$(0, +10\lambda_o)$	λ_o
2	(x_2, y_2)	$(+10\lambda_o, 0)$	$2\lambda_o$
3	(x_3, y_3)	$(0, -10\lambda_o)$	$3\lambda_o$
4	(x_4, y_4)	$(-10\lambda_o, 0)$	$4\lambda_o$
5	(x_5, y_5)	$(-8\lambda_o, -6\lambda_o)$	$5\lambda_o$
6	(x_6, y_6)	$(+8\lambda_o, -6\lambda_o)$	$6\lambda_o$

Figure 5: Point Source Parameters

Figure 6 and 7 show the resulting point scatter and magnitude spectrum. We can observe that the spectrum has its highest values along 6 rings with radii corresponding to their wavelengths as $\frac{1}{n\lambda_0}$.

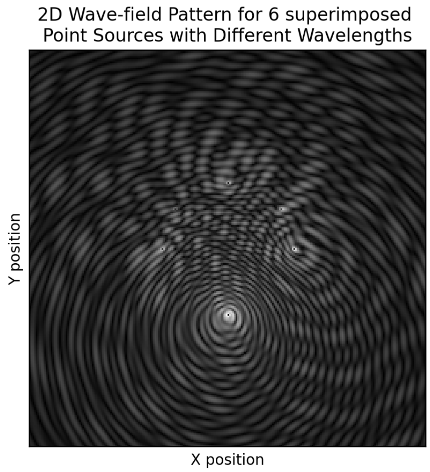

Figure 6: Scattered Wave-field

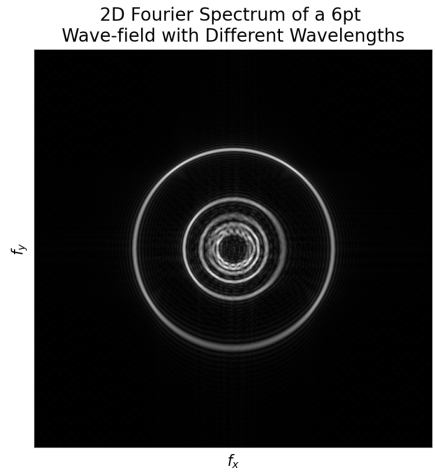

Figure 7: Magnitude of Spectrum

4 Phase Only Version

We will repeat the same exercises by using a modified version of the Green's function known as the phase only version given its constant amplitude.

$$h'(x, y) = Aexp(\frac{j2\pi r}{\lambda_0})$$

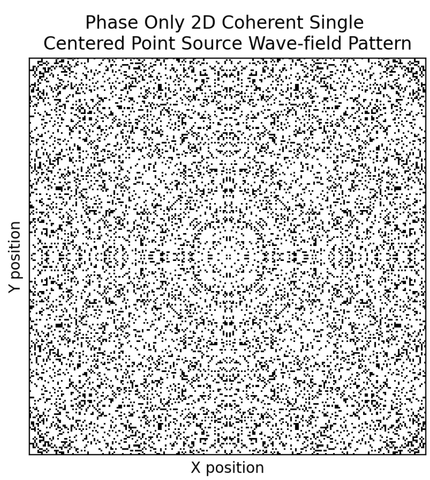

Figure 8: Scattered Wave-field

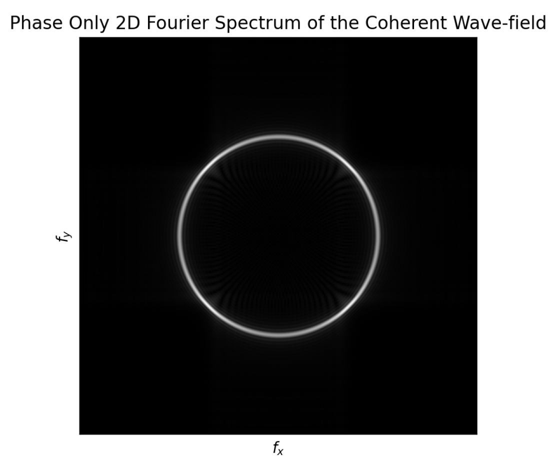

Figure 9: Magnitude of Spectrum

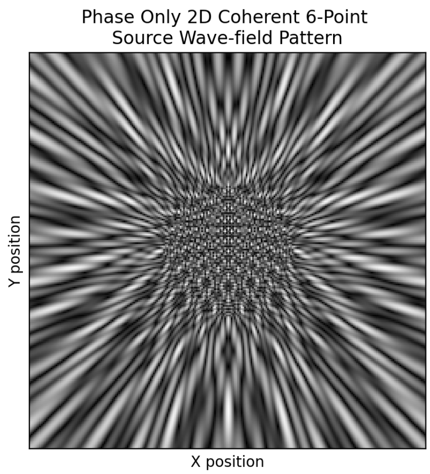

Figure 10: Scattered Wave-field

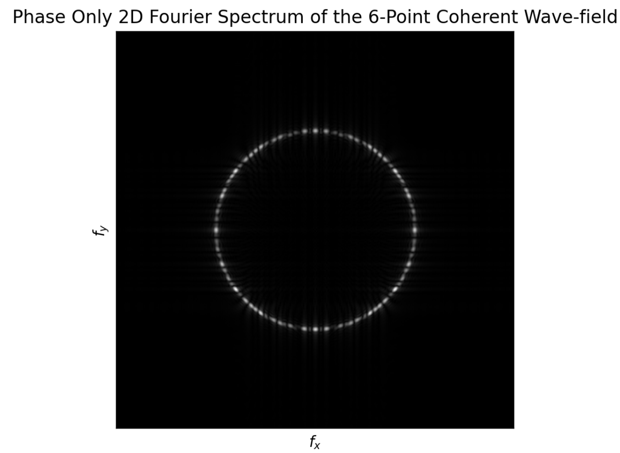

Figure 11: Magnitude of Spectrum

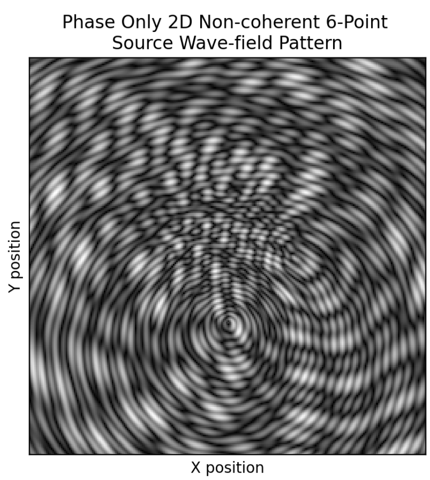

Figure 12: Scattered Wave-field

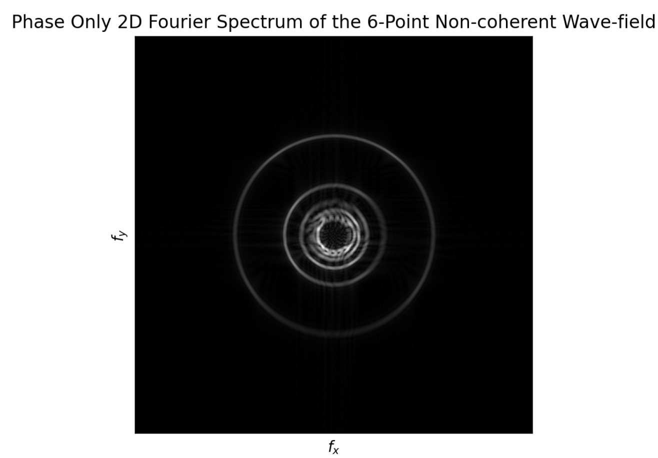

Figure 13: Magnitude of Spectrum

We can observe that the spectrum of the phase-only version of green's equation is a good approximation to the true source point wave-field equation with varying amplitudes. This demonstrates that most of the spectrum information is being carried in the phase rather than in the magnitude.

5 Effects of Sample Spacing

So far, we have conducted our experiments with a sample spacing of $\frac{\lambda_0}{4}$. We'll now repeat the phase-only 6 point with varying wavelength experiment and examine the change in the spectral distribution as we vary the sample spacing of the array. Figures 13-21 show the scattered wave-field and spectrum with sample spacing of λ_0, $\frac{\lambda_0}{2}$, $\frac{\lambda_0}{4}$, and $\frac{\lambda_0}{8}$.

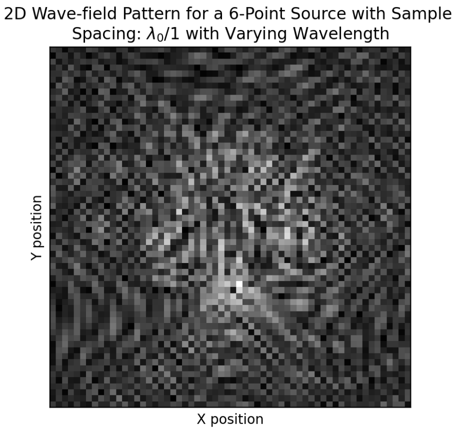

Figure 14: Scattered Wave-field

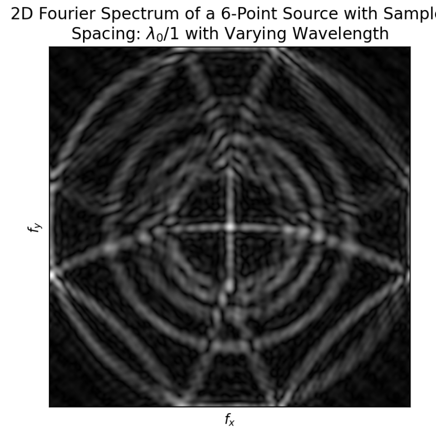

Figure 15: Magnitude of Spectrum

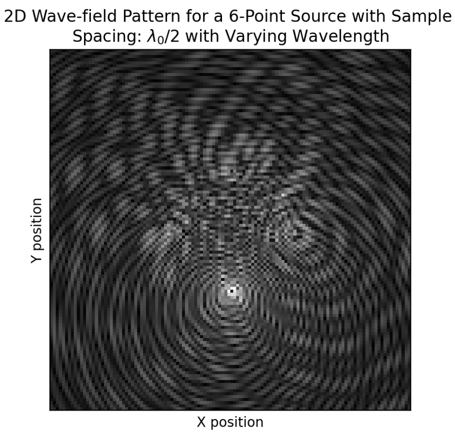

Figure 16: Scattered Wave-field

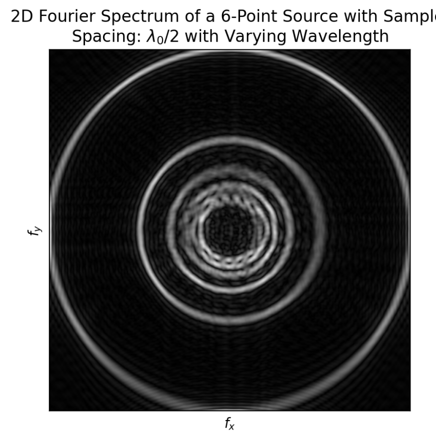

Figure 17: Magnitude of Spectrum

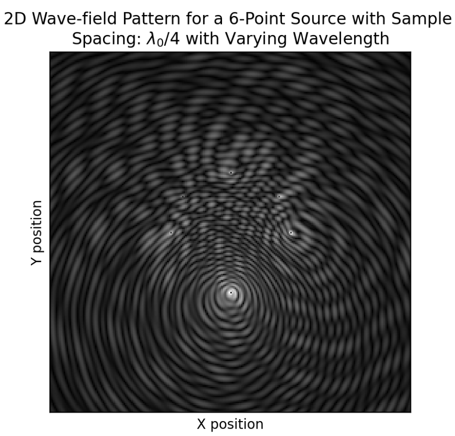

Figure 18: Scattered Wave-field

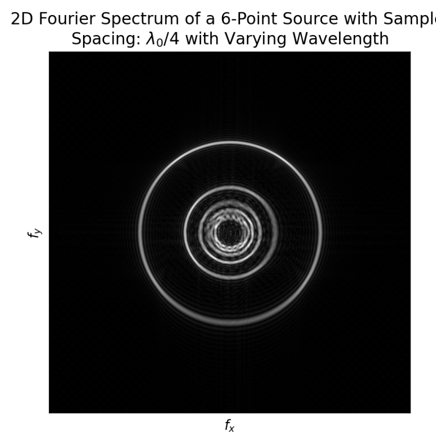

Figure 19: Magnitude of Spectrum

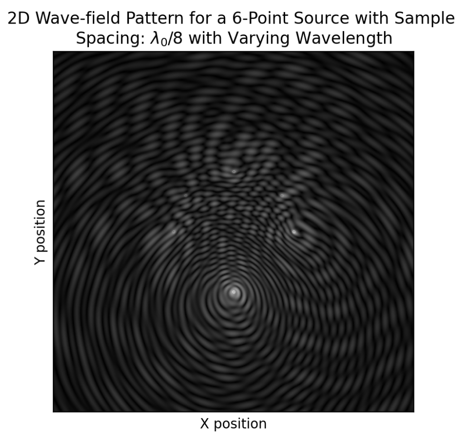

Figure 20: Scattered Wave-field

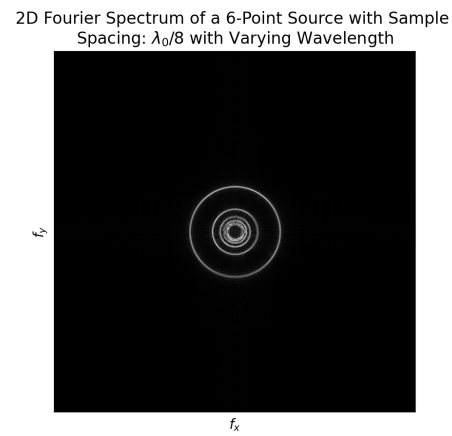

Figure 21: Magnitude of Spectrum

We can observe in figure 14 and 15 the effects of aliasing when we don't sample fast enough. Figure 16 and 17 illustrate that the Nyquist sampling rate (twice the largest frequency present) does not result in aliasing and contains the ring in its Fourier spectrum.

6 Effects of Aperture Radius

The radius of the circular aperture has been set to $30\lambda_0$ for the previous experiments. We'll now repeat the phase-only 6 point with varying wavelength experiment and examine the change of the spectral distribution as we vary the aperture size. Figures 22-27 show the effects of changing aperture radius for $15\lambda_0$, $30\lambda_0$, and $60\lambda_0$.

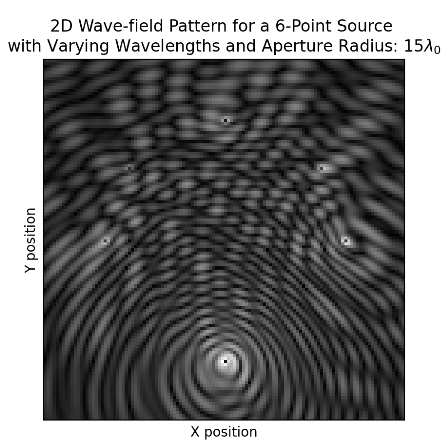

Figure 22: Scattered Wave-field

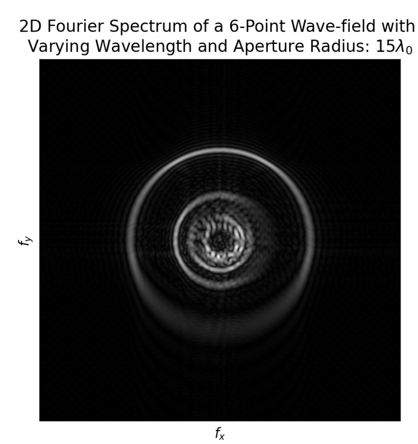

Figure 23: Magnitude of Spectrum

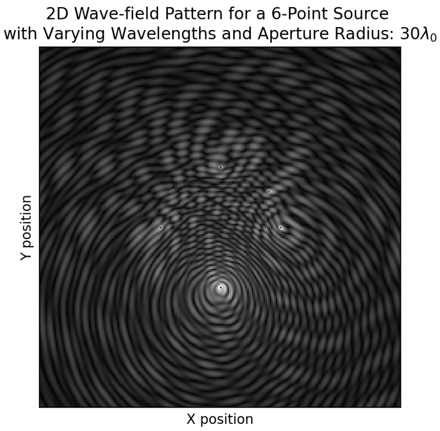

Figure 24: Scattered Wave-field

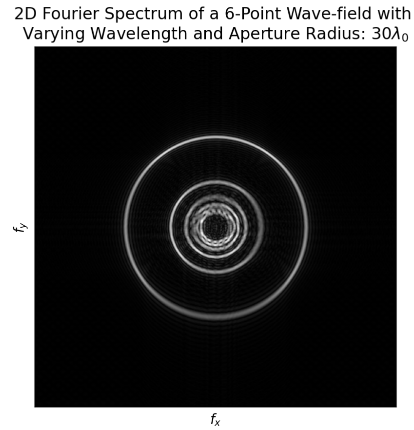

Figure 25: Magnitude of Spectrum

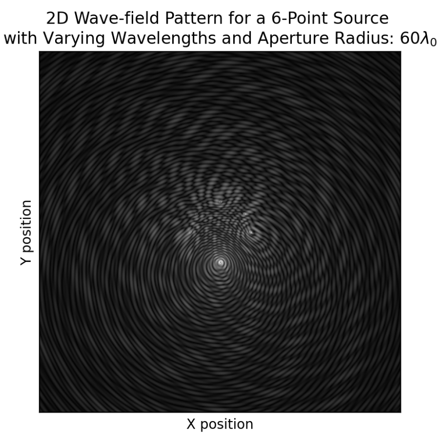

Figure 26: Scattered Wave-field

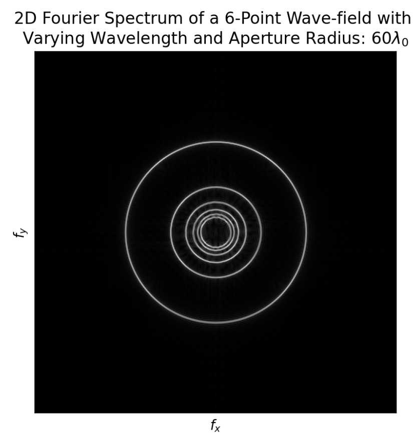

Figure 27: Magnitude of Spectrum

We can observe that as we increase the aperture radius, we can see more of the scattered wave-field and thus the resolution of our Fourier spectrum increases. Figure 27 shows much sharper rings than figure 23 for example. Given the same sample spacing, we capture more information with a larger aperture.

7 Conclusions

We showed that Green's equation describes the wave-field scattering of a single coherent source point and its Fourier spectrum is a ring with radius $1/\lambda$. This agrees with the spectrum derived from Helmholtz equation. Furthermore, we observed the linear shift-invariant properties of our system and how we can analyze sources by superposition of scaled and shifted point sources. From our phase-only version experiment, we can conclude that most of the spectrum information is stored in the phase rather the in the magnitude of the Fourier spectrum. From our sampling spacing experiment, we observed the Nyquist sampling theorem in order to avoid aliasing. Last, we concluded that the aperture radius controls the resolution of our Fourier spectrum

8 Code

```
import numpy as np
import matplotlib.pyplot as plt


def generate_wave_field_pattern(radius, sample_spacing, wavelength=1, xshift=0,
    yshift=0, greens_amp=None):

    def wave_field_tf(x,y):
        r = np.sqrt((x-xshift)**2 + (y-yshift)**2)
        if greens_amp:
            greens_amplitude = greens_amp
        else:
            greens_amplitude = (1/np.sqrt(1j*wavelength*r))
        h_xy = greens_amplitude * np.exp(1j*2*np.pi*r/wavelength)
        return h_xy

    v_wave_field_tf = np.vectorize(wave_field_tf)
    x = y = np.arange(-radius, radius, 1/sample_spacing)
    X, Y = np.meshgrid(x, y)
    wave_field_pattern = v_wave_field_tf(X,Y)

    return np.nan_to_num(wave_field_pattern)


def plt_plot(data, title='', xlabel='', ylabel=''):
    fig, ax1 = plt.subplots(1, 1, sharex=False)
    ax1.set_title(title)
    ax1.set_xlabel(xlabel)
    ax1.set_ylabel(ylabel)
    ax1.tick_params(
        axis='both', # changes apply to the x-axis
        which='both', # both major and minor ticks are affected
        bottom=False, # ticks along the bottom edge are off
        top=False,
        left=False,
        right=False, # ticks along the top edge are off
        labelbottom=False,
        labelleft=False) # labels along the bottom edge are off
    ax1.imshow(np.abs(data), cmap='gray')


def run_prob_1(greens_amp=None,
    wavelength=1,relative_radius=30,relative_sample_spacing=4, pat_title=None,
    spec_title=None):

    # Plot wave field pattern
    wave_field_pattern = generate_wave_field_pattern(relative_radius,
        relative_sample_spacing, wavelength=wavelength,
                                                greens_amp=greens_amp)
    pattern_title = pat_title if pat_title else '2D Coherent Wave-field Pattern \nfor a
        Single Centered Point Source'
```

```
    plt_plot(np.abs(wave_field_pattern), title=pattern_title,
             xlabel='X position', ylabel='Y position')

    # Plot wave field spectrum
    wave_field_spectrum = np.fft.fftshift(np.fft.fft2(wave_field_pattern, s=(512,512)))
    spectrum_title = spec_title if spec_title else '2D Fourier Spectrum of the Coherent
        Wave-field'
    plt_plot(np.abs(wave_field_spectrum), title=spectrum_title,
             xlabel='$f_x$', ylabel='$f_y$')
    plt.show()


def run_prob_2(greens_amp=None,
    wavelength=1,relative_radius=30,relative_sample_spacing=4, pat_title=None,
    spec_title=None):

    # Plot wave field pattern
    first_pt_flag = True
    for x,y in np.array([[0,10], [10,0], [0,-10], [-10,0], [-8,-6], [8,-6]]):
        wave_field_pattern = generate_wave_field_pattern(relative_radius,
            relative_sample_spacing, wavelength,
                                                  xshift=x, yshift=y,
                                                      greens_amp=greens_amp)
        if first_pt_flag:
            wave_field_6pt_pattern = wave_field_pattern
            first_pt_flag = False
        else:
            wave_field_6pt_pattern += wave_field_pattern

    pattern_title = pat_title if pat_title else '2D Coherent Wave-field Pattern \nfor 6
        superimposed Point Sources'
    plt_plot(np.abs(wave_field_6pt_pattern), title=pattern_title,
             xlabel='X position', ylabel='Y position')

    # Plot wave field spectrum
    wave_field_spectrum = np.fft.fftshift(np.fft.fft2(wave_field_6pt_pattern, s=(512,
        512)))
    spectrum_title = spec_title if spec_title else '2D Fourier Spectrum of the Coherent
        Wave-field'
    plt_plot(np.abs(wave_field_spectrum), title=spectrum_title,
             xlabel='$f_x$', ylabel='$f_y$')
    plt.show()


def run_prob_3(greens_amp=None, relative_radius=30,relative_sample_spacing=4,
    pat_title=None, spec_title=None):

    # Plot wave field pattern
    first_pt_flag = True
    for x,y,l in np.array([[0,10,1], [10,0,2], [0,-10,3], [-10,0,4], [-8,-6,5],
        [8,-6,6]]):
        wave_field_pattern = generate_wave_field_pattern(relative_radius,
            relative_sample_spacing, wavelength=l,
```

```
                                                   xshift=x, yshift=y,
                                                       greens_amp=greens_amp)
        if first_pt_flag:
            wave_field_6pt_pattern = wave_field_pattern
            first_pt_flag = False
        else:
            wave_field_6pt_pattern += wave_field_pattern

    patttern_title = pat_title if pat_title else '2D Wave-field Pattern for 6
        superimposed \nPoint Sources with Different Wavelengths'
    plt_plot(np.abs(wave_field_6pt_pattern), title=patttern_title, xlabel='X position',
        ylabel='Y position')

    # Plot wave field spectrum
    wave_field_spectrum = np.fft.fftshift(np.fft.fft2(wave_field_6pt_pattern, s=(512,
        512)))
    spectrum_title = spec_title if spec_title else '2D Fourier Spectrum of a 6pt
        \nWave-field with Different Wavelengths'
    plt_plot(np.abs(wave_field_spectrum), title=spectrum_title, xlabel='$f_x$',
        ylabel='$f_y$')
    plt.show()


def run_prob_4():
    run_prob_1(greens_amp=1, pat_title='Phase Only 2D Coherent Single \nCentered Point
        Source Wave-field Pattern',
              spec_title='Phase Only 2D Fourier Spectrum of the Coherent Wave-field')
    run_prob_2(greens_amp=1, pat_title='Phase Only 2D Coherent 6-Point \nSource
        Wave-field Pattern',
              spec_title='Phase Only 2D Fourier Spectrum of the 6-Point Coherent
                  Wave-field')
    run_prob_3(greens_amp=1, pat_title='Phase Only 2D Non-coherent 6-Point \nSource
        Wave-field Pattern',
              spec_title='Phase Only 2D Fourier Spectrum of the 6-Point Non-coherent
                  Wave-field')


def run_prob_5():
    for s in [1, 2, 4]:
        # run_prob_1(relative_sample_spacing=s, pat_title="2D Coherent Wave-field Pattern
            for a Single \n"
        #                                              "Centered Point Source with sample
            spacing: {}".format(s),
        #          spec_title="2D Fourier Spectrum of the Single-Point \nCoherent
            Wave-field with sample spacing {}".format(s))
        # run_prob_2(relative_sample_spacing=s, pat_title="2D Coherent Wave-field Pattern
            for a \n"
        #                                              "6-Point Source with sample spacing:
            {}".format(s),
        #          spec_title="2D Fourier Spectrum of the 6-Point \nCoherent Wave-field
            with sample spacing: {}".format(s))
        run_prob_3(relative_sample_spacing=s, pat_title="2D Wave-field Pattern for a
            6-Point Source with Sample \n"
```

```
                                                     "Spacing: $\lambda_0$/{} with Varying
                                                         Wavelength".format(s),
                  spec_title="2D Fourier Spectrum of a 6-Point Source with Sample \n"
                                                     "Spacing: $\lambda_0$/{} with Varying
                                                         Wavelength".format(s))


def run_prob_6():
    for r in [15,30,60]:
        # run_prob_1(relative_radius=r, pat_title="2D Coherent Wave-field Pattern for a
            Single \nCentered Point Source "
        #                                      "with aperture radius: {}".format(r),
        #           spec_title="2D Fourier Spectrum of the Single-Point Coherent
            Wave-field with Aperture Radius: {}".format(r))
        # run_prob_2(relative_radius=r, pat_title="2D Coherent Wave-field Pattern for a
            \n"
        #                                              "6-Point Source with Aperture
            Radius: {}".format(r),
        #           spec_title="2D Fourier Spectrum of the 6-Point Coherent Wave-field
            with Aperture Radius: {}".format(r))
        run_prob_3(relative_radius=r, pat_title="2D Wave-field Pattern for a 6-Point
            Source \n"
                                        "with Varying Wavelengths and Aperture
                                            Radius: {}$\lambda_0$".format(r),
                  spec_title="2D Fourier Spectrum of a 6-Point Wave-field with \n"
                             "Varying Wavelength and Aperture Radius:
                                 {}$\lambda_0$".format(r))


input = int(input("Run Problem: "))
if input==1:
    run_prob_1()
elif input==2:
    run_prob_2()
elif input==3:
    run_prob_3()
elif input==4:
    run_prob_4()
elif input==5:
    run_prob_5()
elif input==6:
    run_prob_6()
```