ECE 178 HW 6 Ivan Arevalo

Problem 1

$$f[m,n] = \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}$$

a) $$F[k,l] = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f[m,n] \, e^{-j2\pi\left(\frac{km}{M} + \frac{ln}{N}\right)}$$

$k = 0, \ldots, M-1$
$l = 0, \ldots, N-1$

$$= \sum_{m=0}^{M-1} e^{-j2\pi\frac{km}{M}} \sum_{n=0}^{N-1} f[m,n] e^{-j2\pi\frac{ln}{N}}$$

$$F[0,0] = \sum_{m=0}^{1} \overbrace{e^{-j2\pi\frac{0m}{2}}}^{1} \left(\sum_{n=0}^{N-1} f[m,n] \overbrace{e^{-j2\pi\frac{0n}{2}}}^{1} \right)$$

$$= f[0,0] + f[0,1] + f[1,0] + f[1,1] = \boxed{6}$$

$$F[0,1] = \sum_{m=0}^{1} \overbrace{e^{-j2\pi\frac{0m}{M}}}^{1} \left(\sum_{n=0}^{1} f[m,n] e^{-j2\pi\cdot\frac{1\cdot n}{2}} \right)$$

$$= f[0,0] + f[0,1] \overbrace{e^{-j\pi}}^{-1} + f[1,0] + f[1,1] \overbrace{e^{-j\pi}}^{-1} = \boxed{-2}$$

$$F[1,0] = \sum_{m=0}^{1} e^{-j2\pi\frac{1m}{2}} \sum_{n=0}^{1} f[m,n] \overbrace{e^{-j2\pi\frac{0\cdot n}{2}}}^{1}$$

$$= f[0,0] + f[0,1] + f[1,0] \overbrace{e^{-j\pi}}^{-1} + f[1,1] \overbrace{e^{-j\pi}}^{-1} = \boxed{-4}$$

$$F[1,1] = \sum_{m=0}^{1} e^{-j2\pi\frac{1m}{2}} \sum_{n=0}^{1} f[m,n] e^{-j2\pi\frac{1n}{2}}$$

$$= f[0,0] + f[0,1] e^{-j\pi} + f[1,0] e^{-j\pi} + f[1,1] \overbrace{e^{-j\pi} e^{-j\pi}}^{1} = \boxed{0}$$

$$\boxed{F[k,l] = \begin{bmatrix} 6 & -2 \\ -4 & 0 \end{bmatrix}}$$

b) Basis Functions $\quad e^{j2\pi\left(\frac{km}{M}+\frac{\ell n}{N}\right)}$

$$\begin{bmatrix} e^{j2\pi\left(\frac{0\cdot m}{2}+\frac{0n}{2}\right)} & e^{j2\pi\left(\frac{0m}{2}+\frac{1\cdot n}{2}\right)} \\ e^{j2\pi\left(\frac{1m}{2}+\frac{0n}{2}\right)} & e^{-j2\pi\left(\frac{1\cdot m}{2}+\frac{1n}{2}\right)} \end{bmatrix}$$

$$\text{Basis Functions} = \begin{bmatrix} 1 & e^{j\pi n} \\ e^{j\pi m} & e^{j\pi(m+n)} \end{bmatrix}$$

c) $f[m,n] = \frac{1}{MN}\sum_{k=0}^{M-1}\sum_{\ell=0}^{N-1} F[k,\ell]\, e^{j2\pi\left(\frac{km}{M}+\frac{\ell n}{N}\right)}$

$$f[0,0] = \frac{1}{4}\left(F[0,0]\cdot 1 + F[0,1]\cdot \overset{\nearrow 1}{e^{j\pi 0}} + F[1,0]\cdot \overset{\nearrow 1}{e^{j\pi 0}} + F[1,1]\overset{\nearrow 1}{e^{j\pi 0}}\right) = 0$$

$$f[0,1] = \frac{1}{4}\left(F[0,0]\cdot 1 + F[0,1]\overset{\nearrow -1}{e^{j\pi}} + F[1,0]\cdot \overset{\nearrow 1}{e^{j\pi 0}} + F[1,1]\overset{\nearrow -1}{e^{j\pi}}\right) = 1$$

$$f[1,0] = \frac{1}{4}\left(F[0,0]\cdot 1 + F[0,1]\overset{\nearrow 1}{e^{j\pi 0}} + F[1,0]\overset{\nearrow -1}{e^{j\pi}} + F[1,1]\overset{\nearrow -1}{e^{j\pi}}\right) = 2$$

$$f[1,1] = \frac{1}{4}\left(F[0,0]\cdot 1 + F[0,1]\overset{\nearrow -1}{e^{j\pi}} + F[1,0]\cdot \overset{\nearrow -1}{e^{j\pi}} + F[1,1]\cdot \overset{\nearrow 1}{e^{j2\pi}}\right) = 3$$

$$\boxed{f[m,n] = \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}}$$

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Problem 2

$$f[m,n] = \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}$$

$$h[m,n] = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

find $f \circledast h$

a) 2D-DFT of h to get $H[k,l]$

$$H[k,l] = \sum_{m=0}^{1} e^{-j2\pi \frac{km}{2}} \sum_{n=0}^{1} h[m,n]\, e^{-j2\pi \frac{ln}{2}}$$

$$H[0,0] = h[0,0] + h[0,1] + h[1,0] + h[1,1] = \boxed{10}$$

$$H[0,1] = h[0,0] + h[0,1]\overset{-1}{e^{-j\pi}} + h[1,0] + h[1,1]\overset{-1}{e^{-j\pi}} = \boxed{-2}$$

$$H[1,0] = h[0,0] + h[0,1] + h[1,0]\overset{-1}{e^{-j\pi}} + h[1,1]\overset{-1}{e^{-j\pi}} = \boxed{-4}$$

$$H[1,1] = h[0,0] + h[0,1]e^{-j\pi} + h[1,0]e^{-j\pi} + h[1,1] = \boxed{0}$$

$$G[k,l] = F[k,l] \cdot H[k,l]$$

$$= \begin{bmatrix} 6 & -2 \\ 4 & 0 \end{bmatrix} \cdot \begin{bmatrix} 10 & -2 \\ -4 & 0 \end{bmatrix} = \begin{bmatrix} 60 & 4 \\ -16 & 0 \end{bmatrix}$$

$$g[m,n] = \frac{1}{MN} \sum_{k=0}^{M-1} \sum_{l=0}^{N-1} F[k,l]\, e^{j2\pi\left(\frac{km}{M} + \frac{ln}{N}\right)}$$

$$g[0,0] = \frac{1}{4}\left(G[0,0] + G[0,1]\cdot\overset{1}{e^{j\pi 0}} + G[1,0]\cdot\overset{1}{e^{j\pi 0}} + G[1,1]\overset{1}{e^{j\pi 0}}\right) = \boxed{12}$$

$$g[0,1] = \frac{1}{4}\left(G[0,0] + G[0,1]\cdot\overset{-1}{e^{j\pi}} + G[1,0]\overset{1}{e^{j\pi 0}} + G[1,1]\cdot\overset{-1}{e^{j\pi}}\right) = \boxed{10}$$

$$g[1,0] = \frac{1}{4}\left(G[0,0] + G[0,1]\cancelto{1}{e^{j\pi 0}} + G[1,0]\cancelto{-1}{e^{j\pi}} + G[1,1]\cancelto{-1}{e^{j\pi}} \right) = \boxed{20}$$

$$g[1,1] = \frac{1}{4}\left(G[0,0] + G[0,1]\cancelto{-1}{e^{j\pi}} + G[1,0]\cdot\cancelto{-1}{e^{j\pi}} + G[1,1]\cancelto{1}{e^{j2\pi}} \right) = \boxed{18}$$

$$g[m,n] = \begin{bmatrix} 12 & 10 \\ 20 & 18 \end{bmatrix}$$

b) $f[m,n] \circledast h[m,n] = g[m,n]$

$$\begin{bmatrix} 1 & 0 & 1 \\ 3 & 2 & 3 \\ 1 & 0 & 3 \end{bmatrix} * \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 4+6+2 & 3+4+3 \\ 12+6+2 & 8+9+1 \end{bmatrix} = \boxed{\begin{bmatrix} 12 & 10 \\ 20 & 18 \end{bmatrix}} \checkmark$$