Ivan Arevalo ECE 178 HW4

P1) $F(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y) e^{-j2\pi(ux+vy)} dx dy$

a) $g(x,y) = f(2x,-3y) \longleftrightarrow \frac{1}{|2|}\frac{1}{|3|} F\left(\frac{u}{2}, \frac{v}{-3}\right)$

b) $g(x,y) = f(2y,-x) \longleftrightarrow \frac{1}{|2|} F\left(\frac{v}{2}, -u\right)$

c) $g(x,y) = f(x+y, x-y) \longleftrightarrow F(u+v, u-v)$

d) $g(x,y) = \cos(6\pi x)\sin(4\pi y)$

$$= \frac{e^{j6\pi x} + e^{-j6\pi x}}{2} \cdot \frac{e^{j4\pi y} - e^{-j4\pi y}}{2j}$$

$$= \frac{e^{j(6\pi x+4\pi y)} + e^{j(4\pi y - 6\pi x)} - e^{j(6\pi x - 4\pi y)} - e^{-j(6\pi x + 4\pi y)}}{4j}$$

$$= \frac{e^{j(6\pi x+4\pi y)} - e^{-j(6\pi x+4\pi y)}}{2\cdot 2j} + \frac{e^{j(6\pi x-4\pi y)} - e^{-j(6\pi x-4\pi y)}}{2\cdot 2j}$$

$$= \frac{1}{2}\sin(\underbrace{6\pi x}_{2\pi(3x)} + \overbrace{4\pi y}^{2\pi(2y)}) - \frac{1}{2}\sin(\underbrace{6\pi x}_{2\pi(3x)} - \underbrace{4\pi y}_{2\pi(2y)})$$

$$G(u,v) = \frac{1}{4j}\left[\delta(u-3, v-2) - \delta(u+3, v+2)\right] - \frac{1}{4j}\left[\delta(u-3, v+2) - \delta(u+3, v-2)\right]$$

e) $g(x,y) = \cos(\underbrace{6\pi x}_{2\pi(3x)}) + \sin(\underbrace{4\pi y}_{2\pi(2y)})$

$$G(u,v) = \frac{1}{2}\left[\delta(u-3) + \delta(u+3)\right] + \frac{1}{2j}\left[\delta(v-2) - \delta(v+2)\right]$$

2) $f(x, y) \xleftrightarrow{\mathcal{F}} 6e^{-j20\pi u} e^{-j10\pi v} \operatorname{Sinc}(3u)\operatorname{Sin}(2v)$

$$\underbrace{6e^{-j20\pi u}\operatorname{Sinc}(3u)}_{H(u)} \underbrace{e^{-j10\pi v}\operatorname{Sinc}(2v)}_{B(v)}$$

$-j20\pi u \rightarrow -j2\pi(10u)$ $\quad$ $-j10\pi v \rightarrow -j2\pi(5v)$

$$\boxed{\begin{array}{l} T\operatorname{Sinc}(fT) = \operatorname{rect}(t/T) \\ \operatorname{Sinc}(fT) = \frac{1}{T}\operatorname{rect}(t/T) \end{array}}$$

$$\xleftrightarrow{\mathcal{F}^{-1}} \cancel{6}\cdot\cancel{\tfrac{1}{3}}\operatorname{rect}\left(\frac{t-10}{3}\right)\cdot\cancel{\tfrac{1}{2}}\operatorname{rect}\left(\frac{t-5}{2}\right)$$

relative to oak tree at $(0,0)$.

treasure is at $(10/3, 5/2)$

3) $F[K] = \sum_{n=0}^{N-1} f[n] e^{-j2\pi \frac{Kn}{N}} \quad K = 0, \dots, N-1$

$f[n] = \frac{1}{N} \sum_{K=0}^{N-1} F[K] e^{j2\pi \frac{Kn}{N}}, \quad n = 0, \dots, N-1$

$\frac{1}{N} \sum_{K=0}^{N-1} \left(\sum_{m=0}^{N-1} f[m] e^{-j2\pi \frac{Km}{N}} \right) \cdot e^{j2\pi Kn}$

$= \frac{1}{N} \sum_{K=0}^{N-1} \sum_{m=0}^{N-1} f[m] e^{\frac{j2\pi K}{N}(n-m)}$

$= \frac{1}{N} \sum_{m=0}^{N-1} f[m] \sum_{K=0}^{N-1} e^{\frac{j2\pi K(n-m)}{N}}$ $\longrightarrow$ orthogonality property.

$\boxed{f[n] = \frac{1}{N} \sum_{m=0}^{N-1} f[m] \, N \, \delta[m-n] = \frac{1}{\cancel{N}} \cdot \cancel{N} f[n]}$

Hw4P4a.m* +

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% Part i)
M = 512;
N = 256;

image_i = ones(M, N);
image_i_fft = fft2(image_i);
figure;
image(fftshift(image_i_fft), 'CDataMapping','scaled');

% Part ii)
image_ii = zeros(M,N);
a = 0;
for m = 1:M
    for n = 1:N
        image_ii(m,n) = sin(20*pi*m/M) + cos(6*pi*n/N);
    end
end
figure;
image(image_ii, 'CDataMapping','scaled')
image_ii_fft = fft2(image_ii);
figure;
image(fftshift(abs(image_ii_fft)), 'CDataMapping','scaled');

% Part iii)
image_iii = zeros(M,N);
a = 0;
for m = 1:M
    for n = 1:N
        image_iii(m,n) = sin(20*pi*m/M)*cos(6*pi*n/N);
    end
end
figure;
image(image_iii, 'CDataMapping','scaled')
image_iii_fft = fft2(image_iii);
figure;
image(fftshift(abs(image_iii_fft)), 'CDataMapping','scaled');
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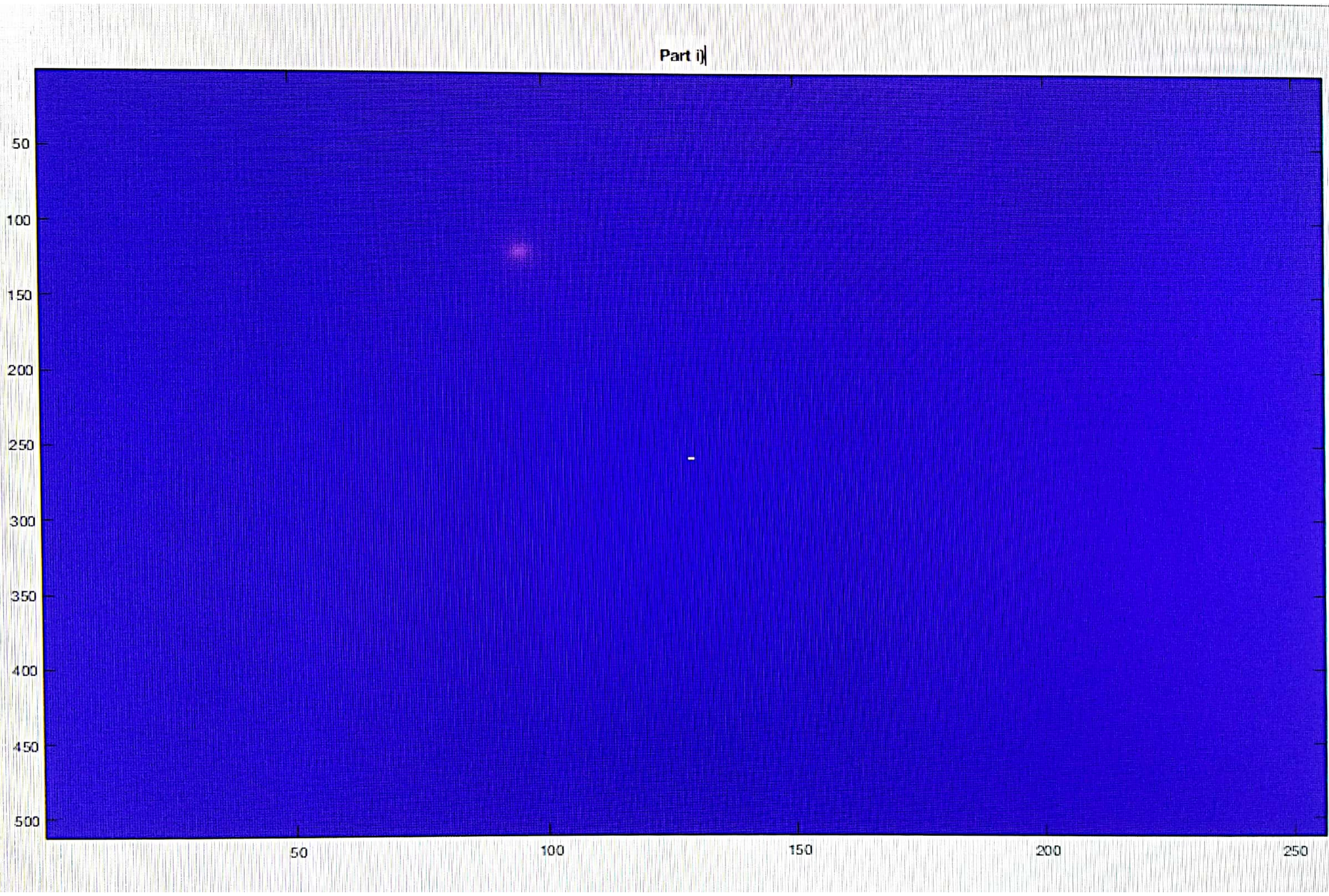
Part i)
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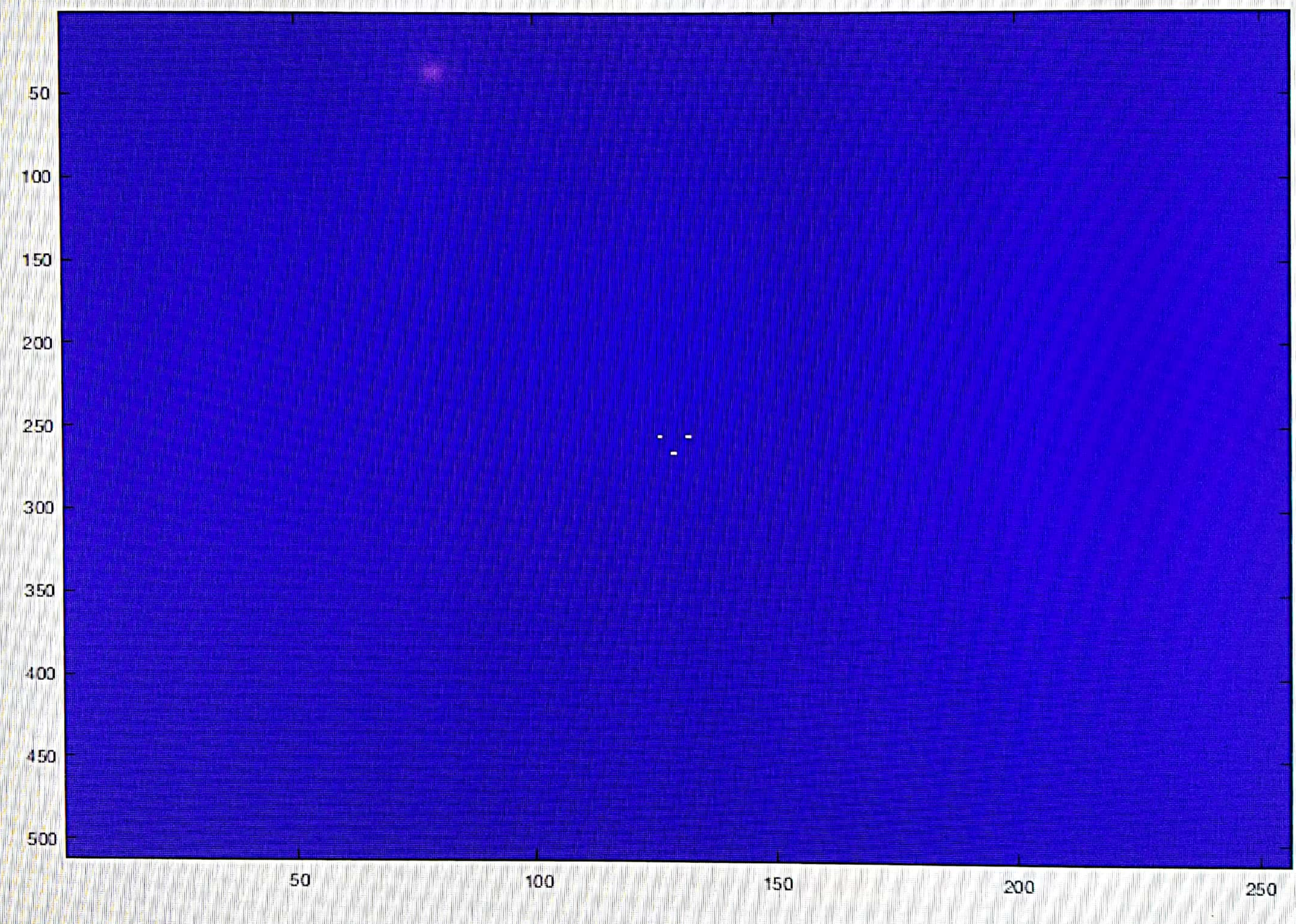
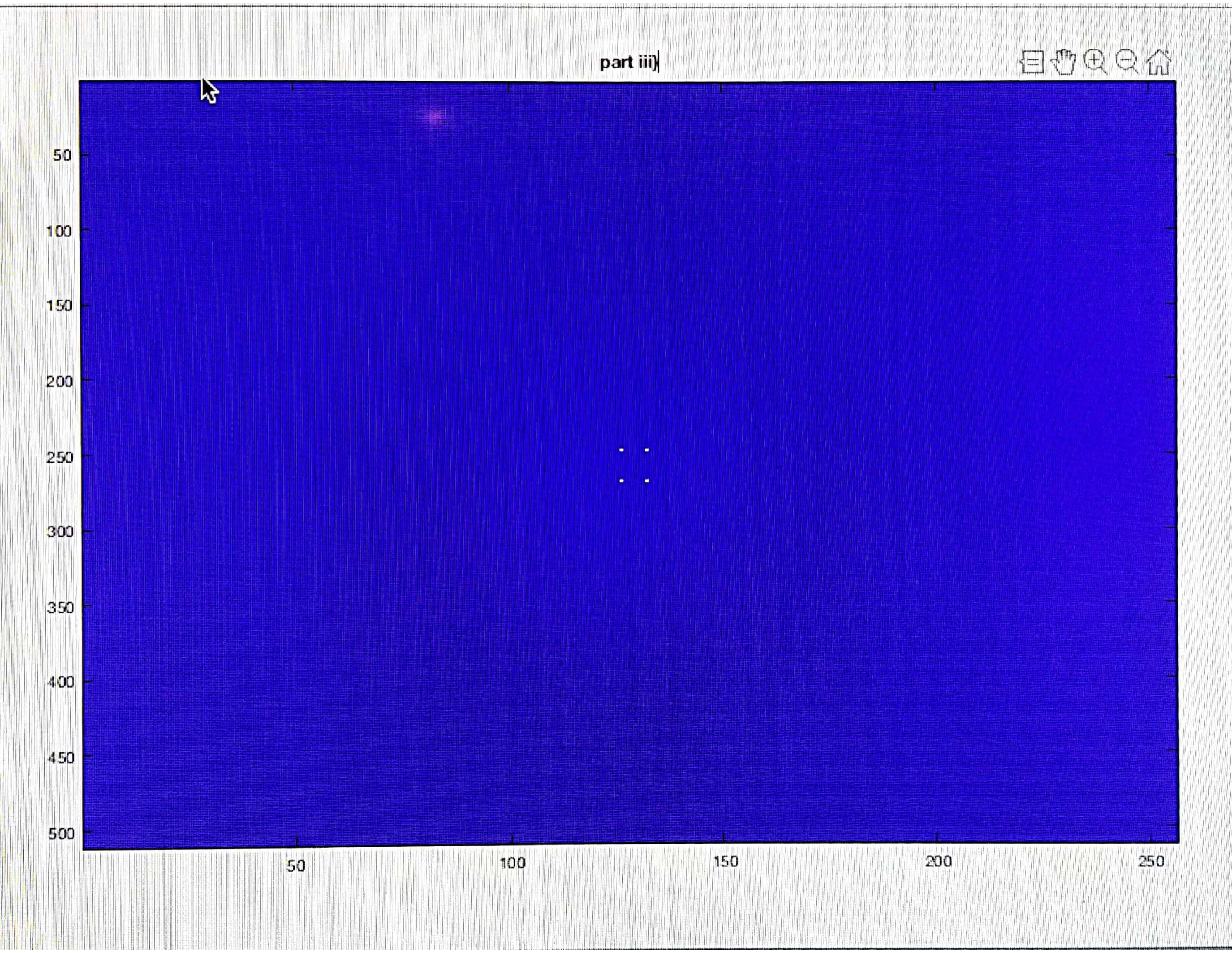
part ii)




part iii)
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