UNIVERSITY OF CALIFORNIA, SANTA BARBARA
Department of Electrical and Computer Engineering

ECE 178 **Image Processing** Fall 2020

Homework Assignment #5

(Due on Thursday 11/12/2020 by 11:59 pm)

Problem # 1. Given the continuous image $f(x, y) = 4\,\text{sinc}(2x)\text{sinc}(2y)$, perform 2D convolution, i.e., find $g(x, y) = f(x, y) * h(x, y)$, for the following examples of the 2D impulse response $h(x, y)$. You may perform convolution directly in the spatial domain, or use the convolution theorem to operate in the transform domain, as convenient.

a) $h(x, y) = \delta(x - 3)\delta(y - 1)$

b) $h(x, y) = \cos(\pi x)\cos(\pi y)$

c) $h(x, y) = 9\,\text{sinc}(3x)\,\text{sinc}(3y)$

Problem # 2. Consider the time signal $x(t) = \cos(3\pi t)$. A perhaps naive signal processing amateur decides to sample it using a sampling period $\Delta = \frac{1}{2}$ (and hence sampling rate $S = \frac{1}{\Delta} = 2$):

$$x_S(t) = x(t)\delta_S(t) = x(t)\sum_{n=-\infty}^{\infty}\delta(t - n\Delta) = x(t)\sum_{n=-\infty}^{\infty}\delta(t - \frac{n}{2})$$

Next, our SP amateur attempts to reconstruct the original signal using the sinc interpolation proposed by the sampling theorem:

$$\hat{x}(t) = x_S(t) * \text{sinc}(\frac{t}{\Delta}) = x_S(t) * \text{sinc}(2t)$$

where $\hat{x}(t)$ is the reconstructed signal.

a) First, we would like to find the exact reconstructed signal $\hat{x}(t)$ and compare it with $x(t)$. Use the following steps to do so:

   i) find and sketch $X(f)$, the Fourier transform of $x(t)$;

ii) then use it to find and sketch $X_S(f)$, the Fourier transform of $x_S(t)$;

iii) which you will then use to find and sketch $\hat{X}(f)$;

iv) finally, perform inverse transform to obtain the reconstructed signal $\hat{x}(t)$.

If all goes well you'll find that the reconstructed signal is also a sinusoid. How would you explain the differences between $\hat{x}(t)$ and $x(t)$, if any?

b) Our naive SP amateur turns to you for sampling advice. Determine how small the sampling interval Δ should be to ensure that the reconstruction is perfect. Sketch next, for such a better choice of Δ, the signals $x(t)$, $x_S(t)$ and $\hat{x}(t)$, accompanied by their respective Fourier transforms $X(f)$, $X_S(f)$ and $\hat{X}(f)$.

Problem # 3. *A hands-on problem*

In this problem we are going to explore how we can use the Fourier coefficients to reconstruct the original input image, through a "progressive inverse transform." Your task will be to perform a step by step reconstruction of the image, and create a short video demo showing the progression through the intermediate results. Your final output will be a video showing the progressively reconstructed image on the left, and the current basis function being added on the right (see the attached sample video for the format). To achieve this, you will need to perform the following steps:

1. Take the Fourier transform of the input image, *goldhill.png*, calculate the magnitude and phase, and sort the coefficients in descending order of magnitude (be sure to also keep the indexes as you will need them later to generate the corresponding basis functions/images).

2. Write a function that generates the real part of a Fourier basis function according to the frequency parameters supplied to it.

3. Multiply a given basis function/image by the magnitude of the corresponding Fourier transform coefficient. For example, if the $(1, 2)$ basis function has Fourier coefficient with magnitude $|F(1, 2)| = 2$, then this basis image will be multiplied by 2.

4. Adjust the phase of the basis image (2D sinusoid) to account for the phase of the corresponding Fourier coefficient. For example, if $(1, 2)$ basis function has Fourier coefficient with phase $\angle F(1, 2) = \pi/3$, then the real part of the basis image, this 2D sinusoid, will undergo a corresponding phase adjustment of $+\pi/3$ for both its horizontal and vertical cosines

5. Finally, accumulate the weighted, phase-adjusted basis images to obtain a progressive inverse Fourier transform, so you can visualize the steps of the reconstruction.
6. Generate subplots containing the current reconstruction (including all basis images added so far) and the current basis image being added.
7. Create the video using the subplots using *VideoWriter* on MATLAB or *ffmpy* on Python.

For your video, set the frame rate to 10 fps for better visualization. After adding up the top 100 basis functions, you can make your video run faster by just plotting the accumulation every 10 added basis functions, as lower energy coefficients will add diminishing amount of information to the reconstructed image. You can also choose to downsample the input image during debugging to make your code run faster, but once you believe everything runs correctly, please submit your results in full resolution.

Problem # 4. *A hands-on problem*

In this problem, you will observe the effects, on an image, of aliasing due to downsampling. Load the given *barbara.png* sample image, and visualize it side by side with the magnitude of its Fourier spectrum using a subplot. Next, downsample the image by a factor of 2 (per coordinate axis) and perform the same visualization, what to you see? Repeat the downsampling step two more times, so you ultimately have overall downsampling by a factor of 8 (per axis), how does the image looks now? Can you explain why it looks like this using the concepts of sampling theory and the Fourier transform?