UNIVERSITY OF CALIFORNIA, SANTA BARBARA
Department of Electrical and Computer Engineering

ECE 178 | **Image Processing** | Fall 2020

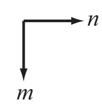
Homework Assignment #6

(Due before Thursday 11/26/2020, 11:59 pm)

Problem # 1. Let's do some "manual" 2D-DFT work in the very simple setting of 2×2 images. Consider the image

$$f[m,n] = \begin{bmatrix} \boxed{0} & 1 \\ 2 & 3 \end{bmatrix}$$

with the convention:



Of course, your favorite programming language may use a different convention but we are doing things manually here. To make sure we all agree, note that in the above matrix, $f[1,0] = 2$.

a) Manually calculate the 2D-DFT of image f, i.e., find each transform coefficient (without using a computer program), and write out the matrix $F[k,\ell]$ in the corresponding convention, with k the vertical coordinate and ℓ horizontal, and where $F[0,0]$ is at the top left corner.

b) Manually calculate each one of the basis functions, in other words, the 2D complex exponential $e^{j2\pi(\frac{km}{M}+\frac{\ell n}{N})}$, as a 2×2 matrix.

c) Now write out the inverse 2D-DFT as a weighted sum of your basis functions (remember that the weights look like $\frac{1}{MN}F[k,l]$), and verify that the weighted sum yields the original image f

Problem # 2. And now we are ready to do some manual circular convolution. Consider the now familiar image

$$f[m,n] = \begin{bmatrix} \boxed{0} & 1 \\ 2 & 3 \end{bmatrix}$$

which we would like to (circularly) convolve with a filter whose impulse response is

$$h[m,n] = \begin{bmatrix} \boxed{1} & 2 \\ 3 & 4 \end{bmatrix}$$

i.e., find $g = f \circledast h$.

a) First solve this in the 2D-DFT domain:

   i) Manually perform 2D-DFT on h to obtain $H[k,\ell]$

   ii) Multiply $H[k,\ell]$ with the $F[k,\ell]$ you already have, to obtain $G[k,\ell]$

   iii) Perform inverse 2D-DFT as a weighted sum of the basis functions you found in Problem 1, to obtain $g[m,n]$

b) Next, perform the same circular convolution directly in the spatial domain. Verify that you do get the same answer.

Problem # 3. *A hands-on problem.* In this problem, you will perform histogram equalization on a color image, using two different approaches. For this, consider the attached image: *tree-dark.png* and do the following:

i) *Approach 1*: Perform histogram equalization on each of the red, green, blue channels *separately*, (without using an inbuilt function). Save the output RGB image (suggested name: *tree-per-channel-hist-eq.png*).

ii) *Approach 2*: Now convert the provided RGB image into YIQ color space, using the linear transformation discussed in the lecture this week. Perform histogram equalization on only the Y channel and combine the equalized Y channel with the original I and Q channels. Convert the resulting image back to RGB space (again: do not use inbuilt functions). Save the output image. (suggested name: *tree-y-channel-hist-eq.png*).

Comment on the differences you see between the above two approaches. Submit your code and output images. Lastly, visualize the histograms of the red, green and blue channels for the above two approaches and comment on the differences you see. As always, please watch out for errors due to unmatched data types - a good rule of thumb for matlab is to always convert image to *double* for all the processing and convert it back to *uint8* before saving it out.