UNIVERSITY OF CALIFORNIA, SANTA BARBARA
Department of Electrical and Computer Engineering

ECE 178 **Image Processing** Fall 2020

Homework Assignment #4

(Due on Wednesday 11/4/2020 by 11:59 pm)

Problem # 1. Use the continuous Fourier Transform properties covered in the lecture, or direct derivation if preferred, to find the Fourier transform of the following continuous images denoted $g(x, y)$. Whenever $g(x, y)$ is defined in terms of $f(x, y)$, express $G(\mu, \nu)$ in terms of $F(\mu, \nu)$. Some of these could provide insight into the experiments you will perform in the hands-on part.

a) $g(x, y) = f(2x, -3y)$

b) $g(x, y) = f(2y, -x)$

c) $g(x, y) = f(x + y, x - y)$

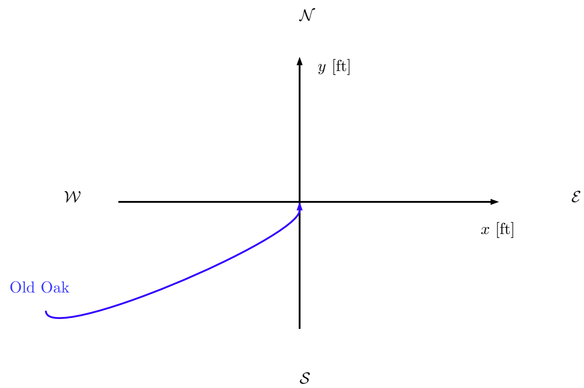
d) $g(x, y) = \cos(6\pi x) \sin(4\pi y)$

e) $g(x, y) = \cos(6\pi x) + \sin(4\pi y)$

Problem # 2. On his death bed, an old pirate, Captain Hook, decides to leave his treasure to another pirate, his old pal Captain Crook (also a fellow image processing enthusiast!). Hook writes a farewell letter to his friend, in which he wants to disclose where the treasure chest is buried. But he worries that the letter might be intercepted by other parties...

Captain Crook is reading the letter he just received: "...The treasure chest is buried in Treasure Island. I attach a map showing its exact location, relative to the old oak tree..." Crook wipes a tear from his eye and opens the attached map, which is blank, other than showing the coordinate system!

Let's take a look at the map on the next page.



Crooks stares at the empty map in disbelief, and then notices a small scribble on the margin:

$$\boxed{\text{Treasure Chest}} \qquad \stackrel{\mathcal{F}}{\longleftrightarrow} \qquad 6e^{-j20\pi\mu}e^{-j10\pi\nu}\text{sinc}(3\mu)\text{sinc}(2\nu)$$

Captain Crook laughs, drinks a quick bottle of rum in honor of his old friend, and hops on his ship to sail to treasure island.

Can you identify on the map where the treasure chest is buried?

Problem # 3. Recall the 1D discrete Fourier transform definition as in the lecture:

$$F[k] = \sum_{n=0}^{N-1} f[n]e^{-j2\pi\frac{kn}{N}}, \quad k = 0, \ldots, N-1$$

$$f[n] = \frac{1}{N}\sum_{k=0}^{N-1} F[k]e^{j2\pi\frac{kn}{N}}, \quad n = 0, \ldots, N-1$$

In this problem you are asked to verify that the forward and inverse transforms work as advertised by using the orthogonality property of complex exponential sequences:

$$\left\langle e^{j2\pi\frac{kn}{N}}, e^{j2\pi\frac{\ell n}{N}} \right\rangle = \sum_{n=0}^{N-1} e^{j2\pi\frac{n(\ell-k)}{N}} = N\delta[k-\ell]$$

In other words, show by plugging in the formal definition that applying the forward transform and than the inverse transform indeed produces the original signal:

$$\mathcal{F}^{-1}\left\{\mathcal{F}\left\{f[\cdot]\right\}\right\} = f[\cdot]$$

Note: It may be wise to make explicit the fact that summations always employ dummy variables.

Problem # 4. *A hands-on problem.*

Fourier Transform: In this problem you'll be exploring the Fourier transform and its properties: You can use the inbuilt function of fft2 in Matlab or its python counterpart numpy.fft.fft2.

Visualization of the transform domain: Note that output of the fft2 function maps the zero frequency to the upper left corner of the matrix. For visualization it is preferable to center the transform matrix, such that the zero frequency is at the center of the matrix. Use the Matlab fftshift function or numpy.fft.fftshift in python to do so. Note that *you need fftshift only for visualization purposes.*

a) *Signals and their transforms*: Visualize the magnitude of the Fourier transform of the following signals:

   i. $f[m,n] = 1$

   ii. $f[m,n] = sin(\frac{20\pi m}{M}) + cos(\frac{6\pi n}{N})$

   iii. $f[m,n] = sin(\frac{20\pi m}{M})cos(\frac{6\pi n}{N})$

   The image dimensions are $M = 512$ and $N = 256$. For convenience, python users will define the domain as $0 \leq m \leq M-1, 0 \leq n \leq N-1$, while Matlab users will define it as $1 \leq m \leq M, 1 \leq n \leq N$. Verify that the transform domain magnitude you obtained meet your expectations. This should should also build up your confidence while using the inbuilt functions.

b) *Gaussian filtering* : Consider the image and the Gaussian filter in problem 3 of homework 3.

   i) Zero pad the filter to match the size of the input image and visualize its transform.

   ii) Obtain the filtered image by multiplying the transforms of input image and the filter followed by inverse transform. Compare it with your result from previous assignment and see if you have similar effect.

c) *Edge detection*: Consider the following Sobel operator for vertical edge detection:

$$S = \begin{matrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{matrix}$$

i) Consider the same input image as the previous problem. *Perform periodic padding* for the input image and convolve it with Sobel operator. Observe the resulting image by taking its absolute value.

ii) Get the transform of the Sobel operator from the transform of the original image and the filtered image. Observe the magnitude of the obtained transform of Sobel operator and see if its intuitively satisfying.