5 - Regresion lineal regularizada, sesgo y varianza

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1 Preprocesamiento de los datos

```
In [1]: import numpy as np

In [2]: from scipy.io import loadmat

In [3]: data = loadmat('datasets/ex5data1.mat')

        X_train = data['X']
        y_train = data['y']
        X_val = data['Xval']
        y_val = data['yval']
        X_test = data['Xtest']
        y_test = data['ytest']

        print("X_train.shape:{}y_train.shape:{}\nX_val.shape:{}y_val.shape{}\nX_test.shape:{}y_
```

```
X_train.shape:(12, 1)y_train.shape:(12, 1)
X_val.shape:(21, 1)y_val.shape(21, 1)
X_test.shape:(21, 1)y_test.shape:(21, 1)
```

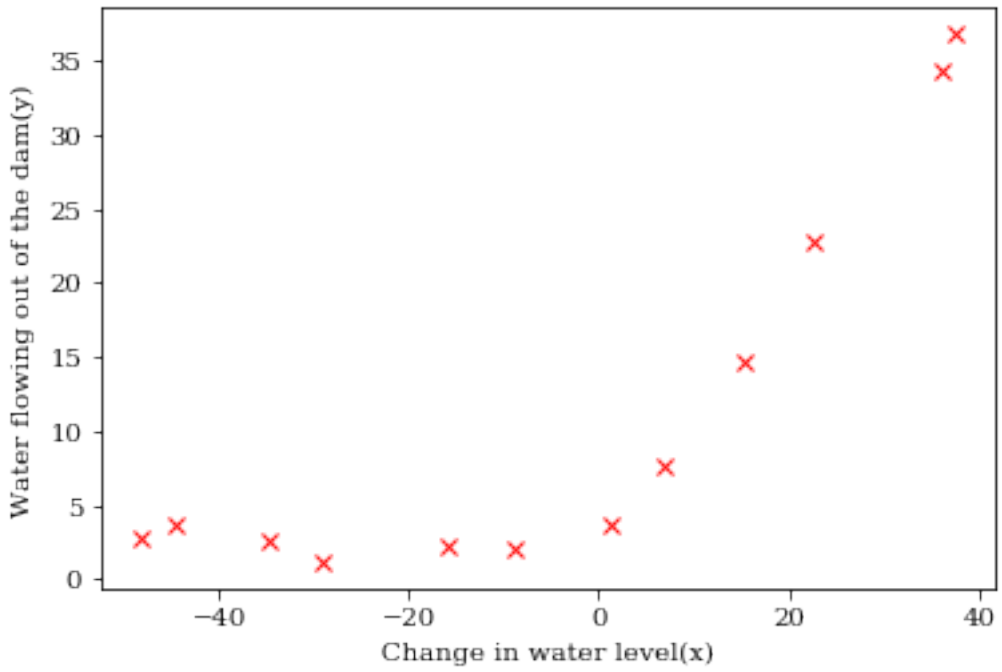
```
In [4]: def cost(th, X, y, reg):
            m = len(X)
            Xs = np.insert(X, 0, values=np.ones(m), axis=1)
            J = (1/(2*m))*np.sum((np.dot(Xs, th.T) - y)**2) + np.sum(th[1:]**2)*(reg/(2*m))
            return J

        def gradient(th, X, y, reg):
            m = len(X)
            Xs = np.insert(X, 0, values=np.ones(m), axis=1)
            gr = (1/m)*np.dot(Xs.T, (np.dot(Xs, th.T) - y))
            gr += np.insert(th[1:]*(reg/m), 0, values=0, axis=0)
            return gr
```

```
        def cg(th, X, y, reg):
            return (cost(th,X,y,reg), gradient(th,X,y,reg))
```

In [5]:
```
import matplotlib.pyplot as plt
plt.rc('text', usetex=False)
plt.rc('font', family='serif')
```

In [6]:
```
plt.figure()
plt.plot(X_train, y_train, 'rx')
plt.ylabel('Water flowing out of the dam(y)')
plt.xlabel('Change in water level(x)')
```

Out[6]: Text(0.5,0,'Change in water level(x)')



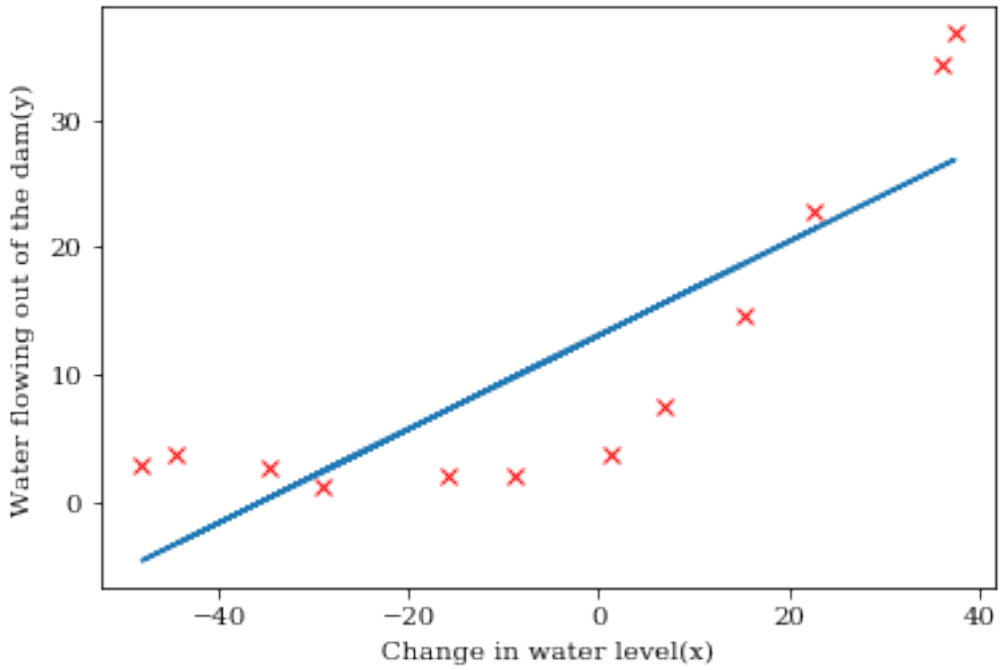
In [7]:
```
m_train = len(X_train)
m_val = len(X_val)
m_test = len(X_test)
th_initial = np.ones(len(X_train[0]) + 1)
reg = 1.0
```

In [8]:
```
cost(th_initial, X_train, np.ravel(y_train), reg)
```

Out[8]: 303.9931922202643

In [9]:
```
gradient(th_initial, X_train, np.ravel(y_train), reg)
```

```
Out[9]: array([-15.30301567, 598.25074417])

In [10]: import scipy.optimize as opt

In [11]: fmin  = opt.minimize(fun=cg, x0=th_initial, args=(X_train, np.ravel(y_train), 0),
                              method='TNC', jac=True, options={'maxiter':70})
         th_opt = fmin.x
         print ("Coste óptimo obtenido con scipy.optimize.fmin_tnc():{}"
                .format(cost(th_opt, X_train, np.ravel(y_train), 0)))

Coste óptimo obtenido con scipy.optimize.fmin_tnc():22.373906495108997


In [12]: X_train_ones = np.insert(X_train, 0, values=np.ones(m_train), axis=1)
         plt.plot(X_train, y_train, 'rx', X_train, np.dot(X_train_ones, th_opt.T))
         plt.ylabel('Water flowing out of the dam(y)')
         plt.xlabel('Change in water level(x)')

Out[12]: Text(0.5,0,'Change in water level(x)')
```



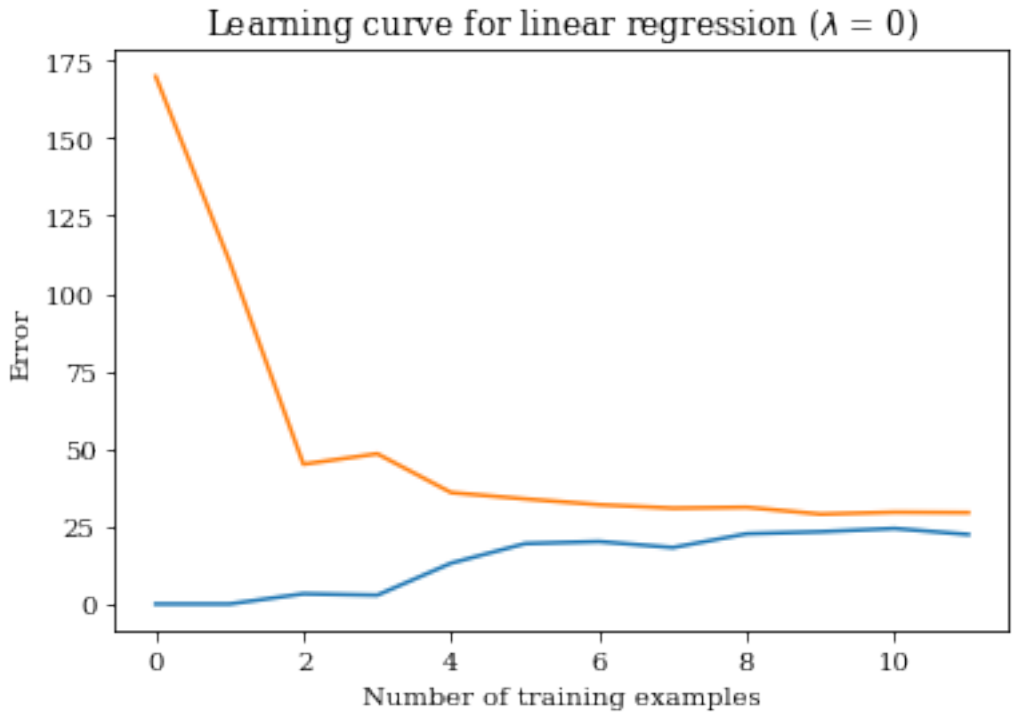
2 Curvas de aprendizaje

```
In [13]: def learning_analysis(X_train, y_train, X_val, y_val, reg):
             m_train = len(X_train)
             th_initial = np.ones(len(X_train[0]) + 1)
```

```
    lth_train = []
    for i in range(1,m_train + 1):
        x_i = X_train[:i,:]
        y_i = np.ravel(y_train[:i])
        fmin_i  = opt.minimize(fun=cg, x0=th_initial, args=(x_i, y_i, reg),
                               method='TNC', jac=True, options={'maxiter':100})
        lth_train.append(fmin_i.x)

    lcost_train = np.zeros(m_train)
    lcost_val = np.zeros(m_train)
    for i, th_i in enumerate(lth_train):
        xt_i = X_train[:i+1,:]
        yt_i = np.ravel(y_train[:i+1])
        lcost_train[i] = cost(th_i, xt_i, yt_i, reg)
        lcost_val[i] = cost(th_i, X_val, np.ravel(y_val), reg)
    return(lcost_train, lcost_val)
```

In [14]:
```
def show_learning_analysis(x1, y1, x2, y2, reg):
    lcost = learning_analysis(x1, y1, x2, y2, reg)
    plt.plot(lcost[0])
    plt.plot(lcost[1])
    plt.title('Learning curve for linear regression ($ \lambda $ = {})'.format(reg))
    plt.ylabel('Error')
    plt.xlabel('Number of training examples')
```

In [15]:
```
show_learning_analysis(X_train, y_train, X_val, y_val, 0)
```



3 Regresión polinomial

```
In [16]: def poly(X, p):
             xret = np.array(X)
             for i in range(0,p):
                 xret = np.c_[xret, xret[:,0]**(i+2)]
             return xret

         def norm(x):
             mu = [np.mean(x[:,i]) for i in range(0,len(x[0]))]
             sigma = [np.std(x[:,i]) for i in range(0,len(x[0]))]
             xnorm = (x - mu)/sigma
             return xnorm, np.ravel(mu), np.ravel(sigma)
```

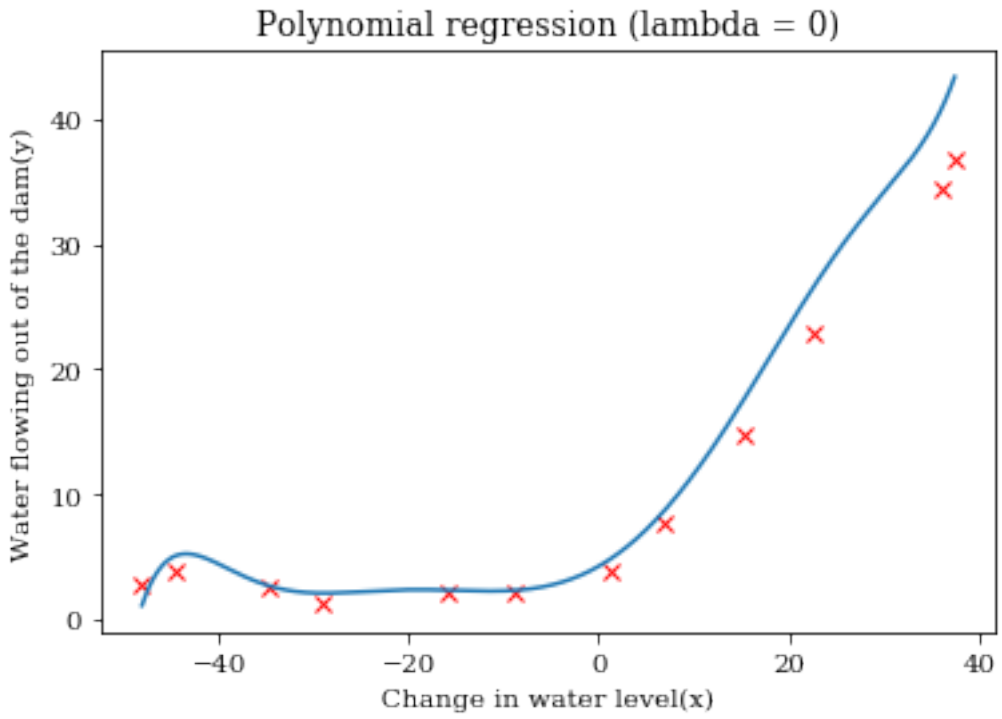
```
In [17]: p = 8
         X_train_poly = poly(X_train, p)
         X_train_poly, mu, sigma = norm(X_train_poly)
         th_initial = np.ones(len(X_train_poly[0]) + 1)
```

```
In [18]: fmin_poly  = opt.minimize(fun=cg, x0=th_initial, args=(X_train_poly, np.ravel(y_train
                                  method='TNC', jac=True, options={'maxiter':70})
         th_opt_poly = fmin_poly.x
```

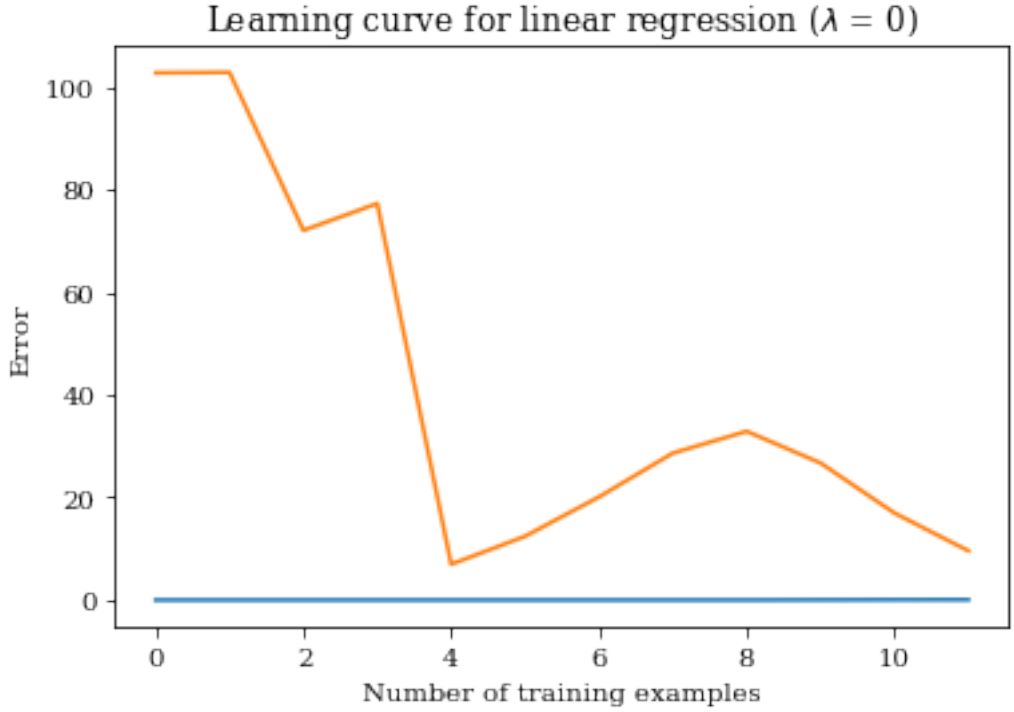
Usando los pesos obtenidos en el entrenamiento con lambda = 0, construimos un grafico que muestre como se ajusta a los ejemplos de entrenamiento

```
In [19]: X_train_ones = np.insert(X_train_poly, 0, values=np.ones(m_train), axis=1)
         plt.plot(X_train, y_train, 'rx')
         range_X_min_to_max = np.matrix(np.arange(np.min(X_train), np.max(X_train), 0.05)).resh
         curve_X_data = poly(range_X_min_to_max, p)
         curve_X_data, *_ = norm(curve_X_data)
         curve_X_data = np.insert(curve_X_data, 0, values=np.ones(curve_X_data.shape[0]), axis=

         plt.plot(range_X_min_to_max, np.dot(curve_X_data, th_opt_poly.T))
         plt.ylabel('Water flowing out of the dam(y)')
         plt.xlabel('Change in water level(x)')
         plt.title('Polynomial regression (lambda = 0)')
```

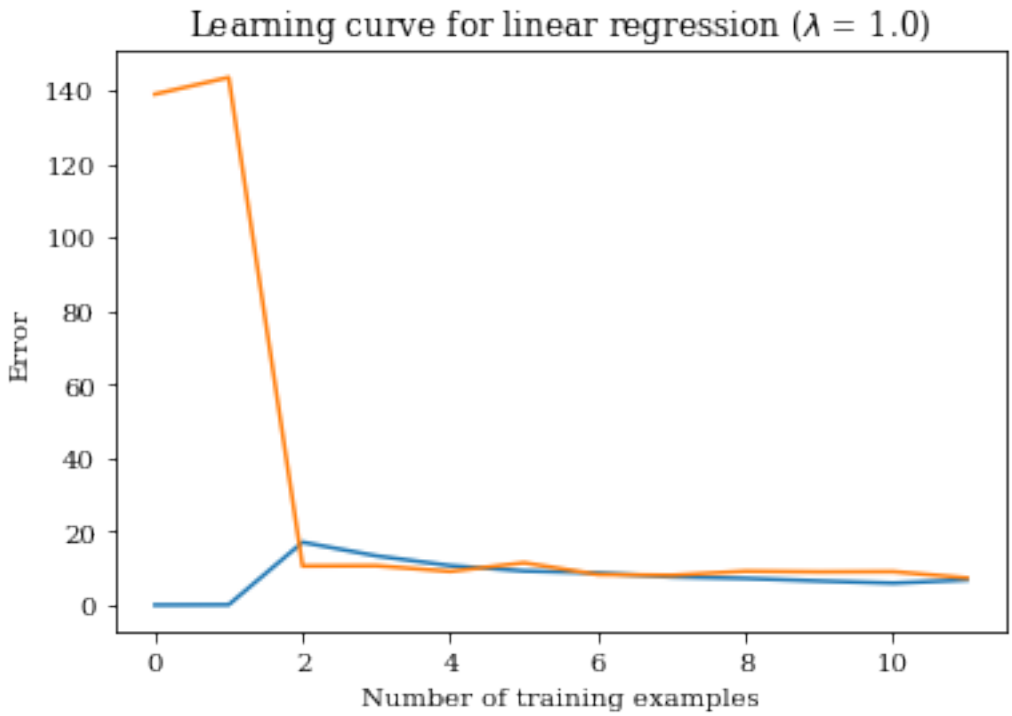
```
Out[19]: Text(0.5,1,'Polynomial regression (lambda = 0)')
```



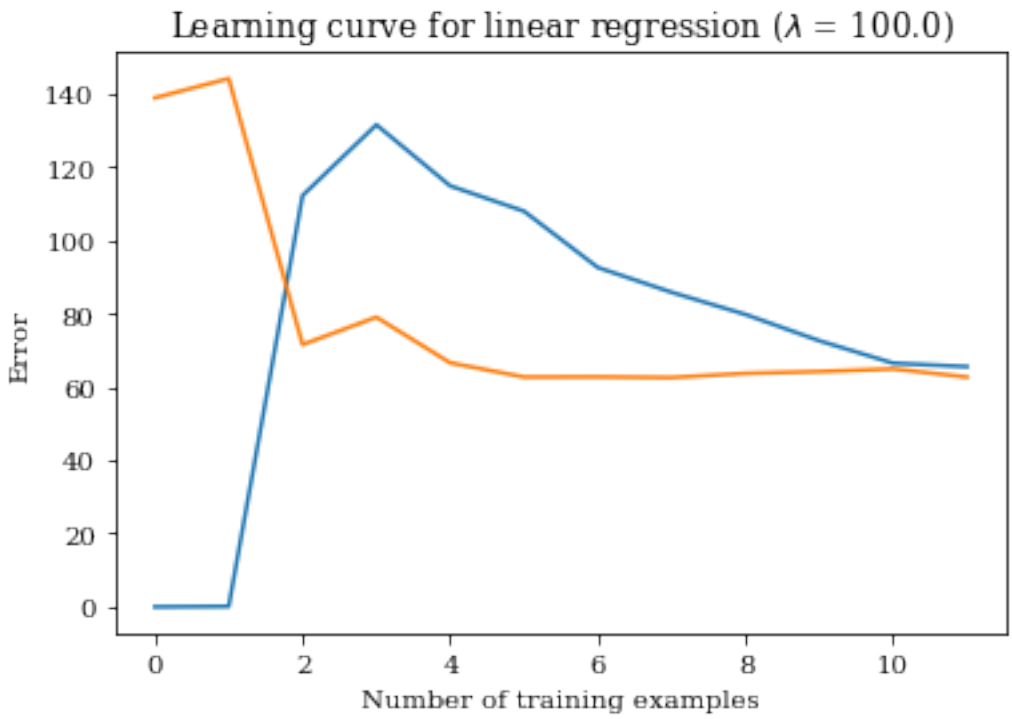
In [20]:
```
X_train_poly_val = poly(X_val, p)
X_train_poly_val, mu, sigma = norm(X_train_poly_val)
show_learning_analysis(X_train_poly, y_train, X_train_poly_val, y_val, 0)
```



In [21]: `show_learning_analysis(X_train_poly, y_train, X_train_poly_val, y_val, 1.0)`

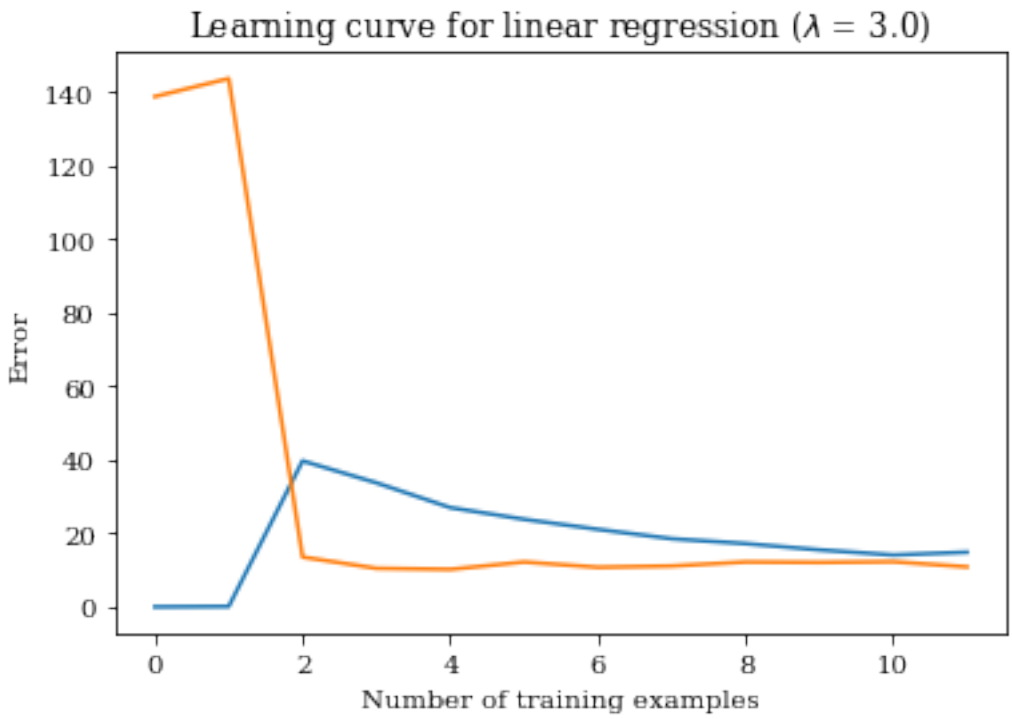


In [22]: `show_learning_analysis(X_train_poly, y_train, X_train_poly_val, y_val, 100.0)`

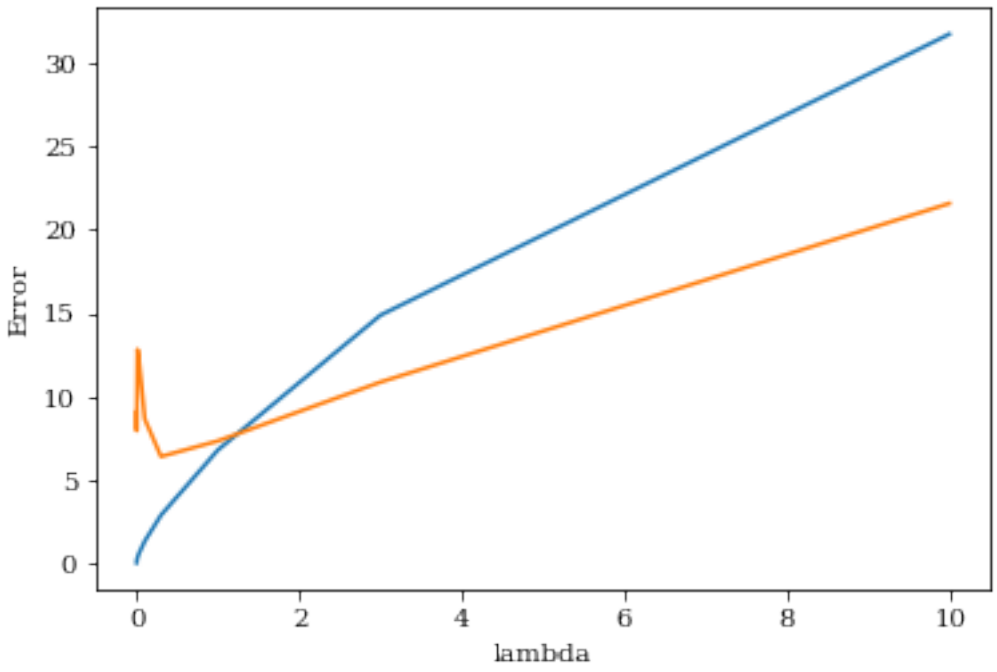


4 Selección del parámetro lambda

```
In [23]: reg_array = np.array([0, 0.001, 0.003, 0.01, 0.03, 0.1, 0.3, 1.0, 3.0, 10.0])

         th_initial = np.ones(len(X_train_poly[0]) + 1)
         cost_reg_array_train = []
         cost_reg_array_val = []
         for r in reg_array:
             fmin  = opt.minimize(fun=cg, x0=th_initial, args=(X_train_poly, np.ravel(y_train)
                                  method='TNC', jac=True, options={'maxiter':70})
             cost_reg_array_train.append(cost(fmin.x, X_train_poly, np.ravel(y_train), reg=r))
             cost_reg_array_val.append(cost(fmin.x, X_train_poly_val, np.ravel(y_val), reg=r))

In [24]: show_learning_analysis(X_train_poly, y_train, X_train_poly_val, y_val, 3.0)
```




```
In [25]: plt.plot(reg_array, cost_reg_array_train)
         plt.plot(reg_array, cost_reg_array_val)
         plt.ylabel('Error')
         plt.xlabel('lambda')

Out[25]: Text(0.5,0,'lambda')
```



In [26]:
```
# entrenamiento con el mejor lambda encontrado
th_initial = np.ones(len(X_train_poly[0]) + 1) # sustituir por los th optimos
fmin_poly_breg  = opt.minimize(fun=cg, x0=th_initial, args=(X_train_poly, np.ravel(y_t
                        method='TNC', jac=True, options={'maxiter':70})
th_opt_poly_breg = fmin_poly_breg.x

# coste usando los pesos entrenados sobre X_train, y_train,
X_test_poly, *_ = norm(X_test)
X_test_poly = poly(X_test_poly, p)
test_cost = cost(th_opt_poly_breg, X_test_poly, np.ravel(y_test), reg=3.0)
print(test_cost)
```

```
1071.0610999180983
```