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A DCM Based Orientation Estimation Algorithm with an Inertial Measurement Unit and a Magnetic Compass

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Abstract: In this paper, Direction Cosine Matrix (DCM) method for attitude and orientation estimation is discussed. DCM method was chosen due to some advantages over the popular methods such as namely Euler Angle, Quaternion in light of reliability, accuracy and computational efforts. Proposed model for each method is developed for methodology comparison. It is shown that normal Kalman Filter in DCM method is better than extended Kalman Filter in Euler and Quaternion based method because it helps avoid the first order approximation error. Methodology errors are verified using Aerospace Blockset of Matlab Simulink.

Keywords: Orientation Estimation, DCM, Quaternion, Euler Representation
Categories: I.2.9, I.3.7, I.3.5

1 Introduction

Orientation estimation using accurate inertial sensors and magnetic compasses was first introduced in navigation area, but along with the development of MEMS technology, low-cost, small-size inertial sensors and magnetic compass sensors recently appeared in various kinds of consumer electronics, game consoles, virtual reality applications, etc. In this field, orientation representations and sensor fusion are the challenges to overcome. The paper addresses a sensor fusion technique based on

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three different orientation representations to find out the best one in light of their accuracy and computational efficiency.

Orientation determination with inertial sensors includes a propagating procedure with gyro sensor data and an updating procedure with accelerometer data. There are generally three principal methods to propagate the orientation information from the differential form such as Euler, Direction Cosine Matrix and Quaternion approaches.

It has been known that the Euler approach of propagating procedure is conceptually easy to understand but it is the most computationally expensive and the state may reach to singularity [Suh, 06] [Nguyen, 06] [Gebre-Egziabher, 04]. Quaternion approach generally has the least computations with only four variables propagated. Therefore, it is very helpful in some applications which strictly demand fast computation. But, it normally uses the first order approximation for its extended Kalman Filter to deal with its nonlinear relationship [Sabatini, 06] [Gebre-Egziabher, 04] so that its accuracy is traded off its computational efficiency. Conversely, the unscented Kalman filter has been used in order to improve its accuracy [Crassidis, 03]. In addition, Quaternion parameters have no physical interpretation about the motion. This leads to the difficulty in connecting the practical measurements with quaternion states in orientation estimator. The Direction Cosine Matrix (DCM) method of propagation transferring matrix has been known to show the performance in-between compared with Euler and Quaternion approaches [Choukroun, 03] [Nebot, 99].

DCM method has been widely used in attitude estimation but when extending to orientation, the number of DCM's parameters is much more than those of Euler and Quaternion parameters, In addition, the conventional State Matrix Kalman filter was used [Choukroun, 03] to keep the natural dynamics of DCM matrix which causes the big computing burden. In this paper, a two-step Kalman filter algorithm is applied to avoid estimating the whole DCM's parameters for its computational efficiency.

In this paper, we develop orientation determination algorithms corresponding to the three approaches with an IMU and a magnetic compass and perform computer simulation to compare them in light of their accuracy and computational efficiency. Here, for our application of a low cost, strapdown Inertial Measurement Unit and a magnetic compass, all sensor data are sampled at the same frequency, therefore direct Kalman filter was used. From the simulation results, the DCM method is selected as the best that all three methods show almost the same performance in computational efficiency, but DCM method outperforms the others in terms of the accuracy.

2 Nomenclature

Mechanical relationships in this paper are written on two coordinate systems: navigation frame, "*n*", which has coordinates are North, East and Down. And body fixed frame, "*b*'', which is attached to the object. And here are some notations:

${}^{n}_{b}\mathbf{C}$: DCM from frame n to b

${}^{n}_{b}C_{31}$: Component of DCM at position (3,1)

$^{b}\omega_{nb}$: Angular rate of the b-frame with respect to the n-frame in b-frame

$\left[{}^{b}\omega_{nb}\times\right]$: Skew matrix which is made by the three components of $^{b}\omega_{nb}$

$^{b}\omega_{x}$: Angular rate on axis x in b-frame

$^{b}f_{x}$: Acceleration on axis x in b-frame

3 DCM based Orientation Estimation

In this section, DCM based Orientation Estimation is divided into the two steps in cascade, estimates the attitude first and then heading. By this way, computational expense for the solution is smaller than estimating the whole at the same time and becomes comparable with the computational expense of Quaternion method.

3.1 DCM based Attitude Estimation

Attitude estimation involves roll and pitch angle determination that used the gyrometers and accelerometers. In our derivation, gyrometers and accelerometers play as the process model and the measurement model, respectively. The gravity vector computed with accelerometers is considered to be fixed or be easily compensated according to the location of the object.

DCM is written in term of rotation matrix that describes the orientation of coordinates frames *"b"* with respect to navigation frame *"n"*. Rotation order is about *zz'*, *yy'* and then *xx'* corresponding to Euler angles: yaw(ψ), pitch(θ), roll(ϕ). Rotation matrix ${}_{b}^{n}\mathbf{C}$ can be expressed as [Nebot, 99]:

$$ {}_{b}^{n}\mathbf{C} = \begin{bmatrix} \theta_c\psi_c & -\phi_c\psi_s + \phi_s\theta_s\psi_c & \phi_s\psi_s + \phi_c\theta_s\psi_c \\ \theta_c\psi_s & \phi_c\psi_c + \phi_s\theta_s\psi_s & -\phi_s\psi_c + \phi_c\theta_s\psi_s \\ -\theta_s & \phi_s\theta_c & \phi_c\theta_c \end{bmatrix} \qquad (1) $$

The notation *"s"* refers to sine and *"c"* refers to cosine. The transformation matrix ${}_{b}^{n}\mathbf{C}$ can be obtained with the following integration [Nebot, 99]:

$$ {}_{b}^{n}\dot{\mathbf{C}} = {}_{b}^{n}\mathbf{C}\left[{}^{b}\omega_{nb}\times\right] \qquad (2) $$

$$[{}^{b}\omega_{nb}\times]=\begin{bmatrix} 0 & -{}^{b}\omega_z & {}^{b}\omega_y \\ {}^{b}\omega_z & 0 & -{}^{b}\omega_x \\ -{}^{b}\omega_y & {}^{b}\omega_x & 0 \end{bmatrix} \tag{3}$$

Normally, the whole DCM could be updated. However, only three components of DCM will be selected and updated as below:

$$\begin{bmatrix} {}^{n}_{b}\dot{C}_{31} \\ {}^{n}_{b}\dot{C}_{32} \\ {}^{n}_{b}\dot{C}_{33} \end{bmatrix} = \begin{bmatrix} 0 & -{}^{n}_{b}C_{33} & {}^{n}_{b}C_{32} \\ {}^{n}_{b}C_{33} & 0 & -{}^{n}_{b}C_{31} \\ -{}^{n}_{b}C_{32} & {}^{n}_{b}C_{31} & 0 \end{bmatrix} \begin{bmatrix} {}^{b}\omega_x \\ {}^{b}\omega_y \\ {}^{b}\omega_z \end{bmatrix} \tag{4}$$

Where ${}^{b}\omega_x, {}^{b}\omega_y, {}^{b}\omega_z$ are approximated angular rates that measured by gyrometers when earth rotation velocity is neglected. Measurements model using accelerometers is constructed as:

$$\begin{bmatrix} {}^{b}f_x \\ {}^{b}f_y \\ {}^{b}f_z \end{bmatrix} = {}^{n}_{b}\mathbf{C}^{T} \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} = \begin{bmatrix} {}^{n}_{b}C_{31} \\ {}^{n}_{b}C_{32} \\ {}^{n}_{b}C_{33} \end{bmatrix} g \tag{5}$$

In this equation, ${}^{b}f_x$, ${}^{b}f_y$, ${}^{b}f_z$ are measured by accelerometers, g is the gravitational acceleration.

The relationships in “Eq. (4)” and “Eq. (5)” are used as the process model and the measurement model, respectively so that the reliable output would be expected due to its automatic adjustment of the measurement covariance values in Kalman Filter, which will be explained.

Applying the above mechanical relationship to the conventional Kalman Filter form:

$$\dot{\mathbf{x}}(t) = \Phi_t \mathbf{x}(t) + \mathbf{w}(t) \tag{6}$$

$$\mathbf{z}(t) = \mathbf{H}\mathbf{x}(t) + \mathbf{v}(t) \tag{7}$$

where,

$$\Phi_t = \begin{bmatrix} 0_{3\times3} & [C_3\times] \\ 0_{3\times3} & 0_{3\times3} \end{bmatrix}, \mathbf{H} = \begin{bmatrix} 0_{6\times6} & I_6 \end{bmatrix}$$

In which, $[C_3\times]$ represents for:

$$[C_3\times]=\begin{bmatrix} 0 & -{}_b^n C_{33} & {}_b^n C_{32} \\ {}_b^n C_{33} & 0 & -{}_b^n C_{31} \\ -{}_b^n C_{32} & {}_b^n C_{31} & 0 \end{bmatrix}$$

The state-variables are chosen as:

$$\mathbf{x}(t)=\begin{bmatrix} {}_b^n C_{31} & {}_b^n C_{32} & {}_b^n C_{33} & {}^b\omega_x & {}^b\omega_y & {}^b\omega_z \end{bmatrix}$$

And measurements are:

$$\mathbf{z}(t)=\begin{bmatrix} {}^b f_x & {}^b f_y & {}^b f_z & {}^b\omega_x & {}^b\omega_y & {}^b\omega_z \end{bmatrix}$$

The noise covariance value of process model and measurement model follow:

$$\mathbf{Q}=E\{\mathbf{w}(t)\,\mathbf{w}^T(t)\}=\begin{bmatrix} 0_{9\times 9} & 0_{9\times 3} \\ 0_{3\times 3} & q.I_{3\times 3} \end{bmatrix} \tag{8}$$

$$\mathbf{R}=E\{\mathbf{v}(t)\,\mathbf{v}^T(t)\}=\begin{bmatrix} r_a\ I_{3\times 3} & 0_{3\times 3} \\ 0_{3\times 3} & r_g\ I_{3\times 3} \end{bmatrix} \tag{9}$$

Accelerometer measurement noise covariance values can be adjusted to compensate the effect of external acceleration [Suh, 06], [Rehbinder, 04]. Here, q value is also chosen to maintain the smooth of output response.

$$r_a = r_{a0} \rightarrow \frac{1}{\alpha} r_{a0}\ , r_g = r_{g0} \rightarrow \alpha r_{g0},\ q = \mathrm{const}$$

In which, r_{a0}, r_{g0} are variance of static accelerometer and gyrometer data , respectively. The adjustment based on the following criteria:

$$f(a_x,a_y,a_z)=\left|a_x^2+a_y^2+a_z^2-1\right|>\sigma \tag{10}$$

Normalization is necessary for preserving the unit property.

$$\left.{}_b^n C_{3i}\right|_{i=1:3}=\frac{\mathbf{x}(i)}{\sqrt{\sum_{i=1:3}\mathbf{x}^2(i)}} \tag{11}$$

In this method, the process model matrix Φ_t is varied as the function of estimated value $[C_3\times]$. Due to its slow dynamic, state matrices can be considered as constant in the interval between two sampling times. The state matrices are in explicit form so the filter can be prevented from error of first order approximation.

3.2 DCM based Orientation Estimation

For orientation estimation, two filters in cascade are applied. The first filter is the attitude estimator described in previous section which gives roll and pitch angle as outputs. Now, we construct the second Kalman filter to find the yaw angle using a magnetic compass. Ladetto directly determined the yaw angle (ψ) as follows [Ladetto, 02]:

$$X_h = X\cos(\theta) + Y\sin(\theta)\sin(\phi) + Z\sin(\theta)\cos(\phi) \qquad (12)$$

$$Y_h = Y\cos(\phi) - Z\sin(\phi) \qquad (13)$$

$$\psi_s = -\frac{Y_h}{\sqrt{X_h^2 + Y_h^2}} \qquad (14)$$

$$\psi_c = \frac{X_h}{\sqrt{X_h^2 + Y_h^2}} \qquad (15)$$

Where X, Y, Z are three measured components from the magnetic compass and (ϕ, θ) are the previously determined roll and pitch angles.

The above equations will be used as the measurement model for our second Kalman filter and the required process model will be constructed in the same way in [Section 3.1] except that $[C_2 \times]$ is evaluated instead of $[C_3 \times]$ with the state-variables:

$$\mathbf{x}(t) = \begin{bmatrix} {}^n_b C_{21} & {}^n_b C_{22} & {}^n_b C_{23} & {}^b\omega_x & {}^b\omega_y & {}^b\omega_z \end{bmatrix}$$

And measurement model is constructed using "Eq. (1)" as follows:

$$\mathbf{z}(t) = [\theta_c\psi_s \quad \phi_c\psi_c + \phi_s\theta_s\psi_s \quad -\phi_s\psi_c + \phi_c\theta_s\psi_s \quad {}^b\omega_{ibx} \quad {}^b\omega_{iby} \quad {}^b\omega_{ibz}]$$

Measurement noise covariance is given by:

$$R_2 = E\{\mathbf{v}(t)\,\mathbf{v}^T(t)\} = \begin{bmatrix} \mu\, I_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & r_g\, I_{3\times3} \end{bmatrix} \qquad (16)$$

Gyrometer measurement noise covariance parameter r_g is variance of static gyrometer data. Measurement noise covariance for $[C_2 \times]$, μ, is changed with given orientation, its value is taken by experience.

After $[C_3 \times]$ and $[C_2 \times]$ is known, the remaining row of DCM can be calculated through the two ones:

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} \end{bmatrix} = \begin{bmatrix} C_{21} & C_{22} & C_{23} \end{bmatrix} \times \begin{bmatrix} C_{31} & C_{32} & C_{33} \end{bmatrix} \tag{17}$$

Here, the orthogonality of DCM is preserved by “Eqs. (11), (17)” and by the way $[C_2 \times]$ is updated.

The magnetic compass measures a full earth magnetic vector. But only direction information of this vector is preferred to be used for yaw determination. Because this way is more reliable in the uncertainty environment where, specially, magnitude of the earth magnetic field can be greatly changed

4 Euler based Attitude Estimation and Quaternion based Orientation Estimation

4.1 Euler based Attitude Estimation

For attitude estimation purposes only, Quaternion method is hardly used because it is burden to update its four variables while DCM method needs to update three and Euler method even only two. Here, we briefly describes Euler based attitude estimation for the comparison in [Section 5].

This method is described in [Suh, 06]. The propagation of Euler angles is given by:

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\phi\sec\theta & \cos\phi\sec\theta \end{bmatrix} \begin{bmatrix} {}^b\omega_x \\ {}^b\omega_y \\ {}^b\omega_z \end{bmatrix} \tag{18}$$

In this attitude estimation, the former two lines of transfer matrix corresponding to the former two variables are needed only. Notice that this relationship means whenever pitch angle reaches $\pi/2$, the state will be singularity($\tan\theta$, $\sec\theta \rightarrow \infty$). This is the first drawback of Euler method.

Measurement model using accelerometers:

$$f(\theta,\phi) = \begin{bmatrix} g\sin\theta \\ g\sin\phi\cos\theta \end{bmatrix} \tag{19}$$

In order to establish the filter, Extended Kalman Filter in this case, first derivation is applied for variable $x = [\theta \;\; \phi]^T$

$$\mathbf{H}_k(x) = \left.\frac{\partial f(x)}{\partial x}\right|_{x=[\hat{\theta}_k^-, \hat{\phi}_k^-]} \tag{20}$$

Error from the first order approximation is another limitation of Euler method which DCM doesn’t have.

4.2 Quaternion based Orientation Estimation

Orientation estimation is an extension of attitude estimation with the additional magnetic compass data. In this section, Quaternion based orientation estimation are briefly derived

In this estimator, gyrometers still play as process model, accelerometers which measure gravity vector and magnetic compass which measures earth field magnetic vector play as measurements.

For the quaternion $\mathbf{q} = [\vec{\mathbf{e}}^T, q_4]^T$ where, $\vec{\mathbf{e}} = [q_1\, q_2\, q_3]^T$, propagation of quaternion use measured angular velocity:

$$\dot{\mathbf{q}} = \Omega \mathbf{q} \tag{21}$$

Where,
$$\Omega = \frac{1}{2}\begin{bmatrix} [{}^b\omega_{nb}\times] & {}^b\omega_{nb} \\ -{}^b\omega_{nb}^T & 0 \end{bmatrix} \tag{22}$$

Changing to other form which shows the relationship between derivative of quaternion and angular velocity:

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} q_4 & q_3 & -q_2 \\ -q_3 & q_4 & q_1 \\ q_2 & -q_1 & q_4 \\ -q_1 & -q_2 & -q_3 \end{bmatrix}\begin{bmatrix} {}^b\omega_x \\ {}^b\omega_y \\ {}^b\omega_z \end{bmatrix} \triangleq \frac{1}{2}\Theta\, {}^b\omega_{nb} \tag{23}$$

Establish process model as:

$$\begin{bmatrix} \dot{\mathbf{q}} \\ {}^b\dot{w}_{nb} \end{bmatrix} = \begin{bmatrix} 0_{(4\times4)} & \Theta_{(4\times3)} \\ 0_{(3\times4)} & 0_{(3\times3)} \end{bmatrix}\begin{bmatrix} \mathbf{q} \\ {}^b w_{nb} \end{bmatrix} + \begin{bmatrix} 0 \\ \mathbf{w}(t) \end{bmatrix} \tag{24}$$

Here, we have the process model with seven states, three quaternion states and three angular velocities.

Measurement model use three accelerometers, ${}^b\mathbf{a}$, and three components of magnetic compass, ${}^b\mathbf{m}$. And ${}^b_n\mathbf{C}(\mathbf{q})$ is well-known conversion between DCM and Quaternion as [Sabatini, 06]:

$$\begin{bmatrix} {}^b\mathbf{a} \\ {}^b\mathbf{m} \end{bmatrix} = \begin{bmatrix} {}^b_n\mathbf{C}(\mathbf{q}) & 0_{(3\times3)} \\ 0_{(3\times3)} & {}^b_n\mathbf{C}(\mathbf{q}) \end{bmatrix}\begin{bmatrix} {}^n\mathbf{g}_o \\ {}^n\mathbf{m}_o \end{bmatrix} + \begin{bmatrix} \mathbf{v}_a(t) \\ \mathbf{v}_g(t) \end{bmatrix} \tag{25}$$

Because of the nonlinear nature of “Eq.(25)”, extended Kalman filter approach requires first order derivation around the current estimated state to get the Jacobian matrix:

$$\mathbf{H}_k = \frac{\partial}{\partial \mathbf{x}} \begin{bmatrix} {}^b\mathbf{a} \\ {}^b\mathbf{m} \end{bmatrix} \Bigg|_{\mathbf{x}=\hat{\mathbf{x}}_k^-} \quad (26)$$

This complexity leads to the large computational expense even though this method needs to update only four quaternion parameters. Another drawback of this method is using earth magnetic vector ${}^n\mathbf{m}$ which magnitude always be changed in the large range of time [Roetenberg, 05]. To overcome this problem, initialization must be done carefully to find out exactly the magnitude and orientation of magnetic field and this vector would be used during the experiment.

5 Matlab Simulation and Comparison

In this section, simulation system is described for the corresponding hardware of Strapdown Inertial Measurement Unit with a Magnetic Compass. Reference motion is generated by Matlab Aerospace Blockset. The motion is measured by virtual IMU and virtual magnetic compass which is developed as [Section 5.1]. Comparisons between the giving methods are described then.

5.1 Matlab Simulink Model for Inertial Measurement Unit

For algorithm's illustration, the simulink model of Strapdown Inertial Measurement Unit (Strapdown IMU) with Magnetic Compass has been made. This model is modified from the Gimbal IMU of Matlab Aerospace Blockset. The most difference between the two kinds of IMU is at the platform where accelerometers are attached. In Strapdown IMU, the platform rotates with respect to the moving object while that of Gimball IMU is kept stablely.
So, the nominal measure value of Strapdown IMU is:

$${}^bf_{ideal} = {}^b_nC\left[{}^nf + {}^n\omega \times ({}^n\omega \times d) + {}^n\dot{\omega} \times d - {}^ng \right] \quad (27)$$

Where, the ideal measured acceleration include the acceleration in the body axes at the center of gravity, lever arm effects due to the accelerometer not being at the center of gravity, and the Earth gravity vector. All of them are considered in body fixed coordinate by multiplied with DCM matrix.

The real measured value contains error sources such as: uncertainty scale factor, cross coupling, bias state and sensors noise:

$${}^bf_{real} = {}^bf_{ideal}K_{sf\_cc}\, k_{bias} + v_{noise} \quad (28)$$

The three axis gyroscope measures body angular rates include the bias, inaccuracy of scale factor, cross coupling, measurement noise

$${}^b\omega_{meas} = {}^b\omega.K_{sfcc} + \omega_{bias} + v_{noise} \quad (29)$$

Assuming we have the initial magnetic vector with respect to the word coordinate. The ideal magnetic vector can be calculated:

$$^{b}M_{ideal} = {}^{b}_{n}C\,{}^{n}V_{mag} \tag{30}$$

Other sensors model for magnetic compass and gyroscope such as the real measured value contains error sources, uncertainty scale factor, cross coupling, bias state and sensors noise, discretization dynamics model and saturation are similar to the corresponding of accelerometers.

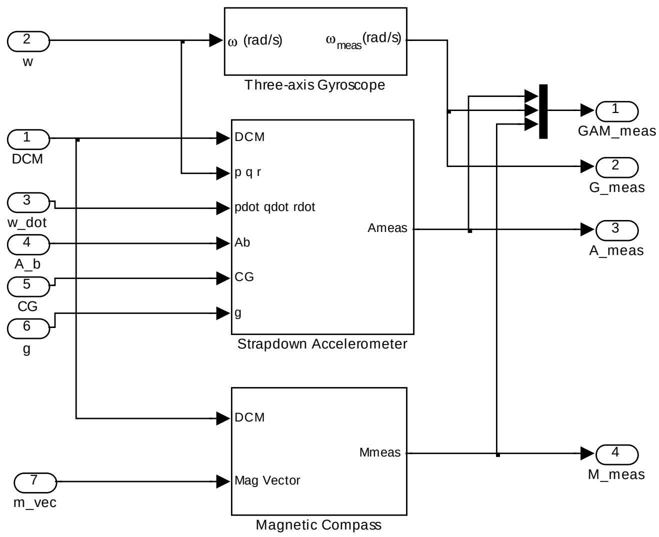


Figure 1: Strapdown Inertial Measurement Unit with a Magnetic Compass.

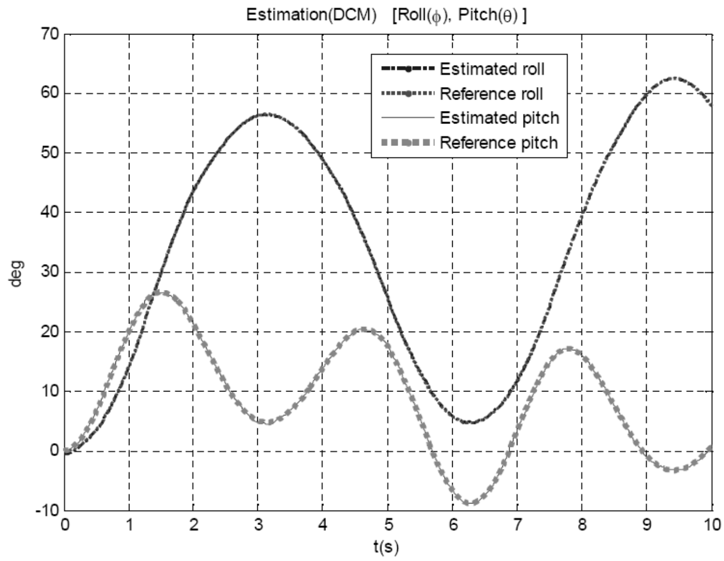
Matlab Aerospace Blockset is used as a tool to examine the designed filter. Motion is generated the by apply angular velocity and acceleration to the Euler 6dof motion planning and Quaternion 6dof motion planning. After that, use the virtual Strapdown IMU model to collect data. With the support of this model, algorithm testing is much easier, singularity points can be showed out clearly.

To adjust the accuracy of algorithm, average absolute error and maximum error performance indices have been used:

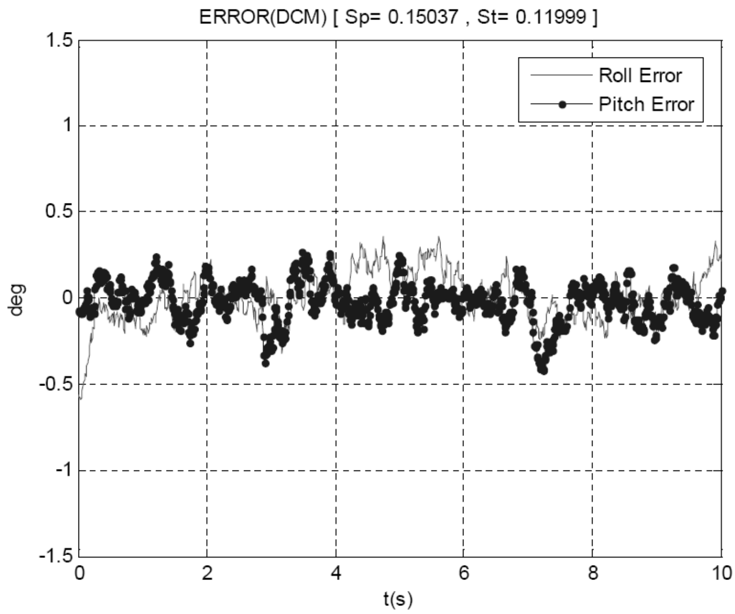
$$S_e \triangleq \frac{1}{N}\sum_{k=1}^{N} e^2(kT) \tag{31}$$

$$M_e \triangleq \max_k \left| e(kT) \right| \tag{32}$$

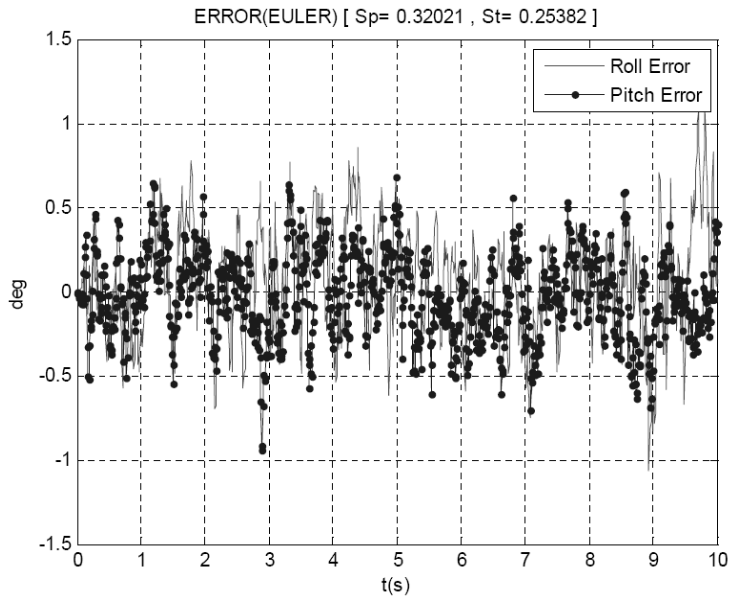
In which, $e(kT)$ is the error between the estimated angles and the true angle.


Estimation(DCM) [Roll(ϕ), Pitch(θ)]
Estimated roll
Reference roll
Estimated pitch
Reference pitch
deg
t(s)
70
60
50
40
30
20
10
0
-10
0
1
2
3
4
5
6
7
8
9
10

(a)


ERROR(DCM) [Sp= 0.15037 , St= 0.11999]
Roll Error
Pitch Error
deg
t(s)
1.5
1
0.5
0
-0.5
-1
-1.5
0
2
4
6
8
10

(b)



(c)

Figure 2: Attitude estimation, ordinary case, error compared between Euler and DCM method.
(a): Estimated Pitch and Roll angle compared with reference using DCM based method. (b) and (c): Angular error of DCM's method and corresponding error of Euler's. Average absolute errors are attached with each figure title ("p" denoted for Roll," t" for Pitch)

5.2 Comparison between Euler and DCM based Attitude Estimation

First, we take the simulation in singularity free case. We try to examine the error of each filter: DCM-based and Euler-based. Reference signal is received from motion generated model. Euler reference is given by Euler 6dof motion planning of Matlab Aerospace Blockset. DCM reference is given by Quaternion one.

As verified in [Fig. 2], the errors of DCM are shown to be smaller than that of the Euler based attitude estimation. This aspect fits the discussion about the method error in [Section 1]. In this experiment, roll and pitch angle changing in the range of $[-\pi/2 : \pi/2]$ so, there is no singularity happens to Euler-based method.

Second simulation involves of singularity case of Euler-based method. Whenever pitch angle reach exactly 90 degrees, there is a singularity occurs. This may cause variable change unwillingly. With the same rotation, DCM based attitude estimation has no bad effect even though the displays angular is ambiguous as the result of the conversion between the DCM states and the displayed angle.

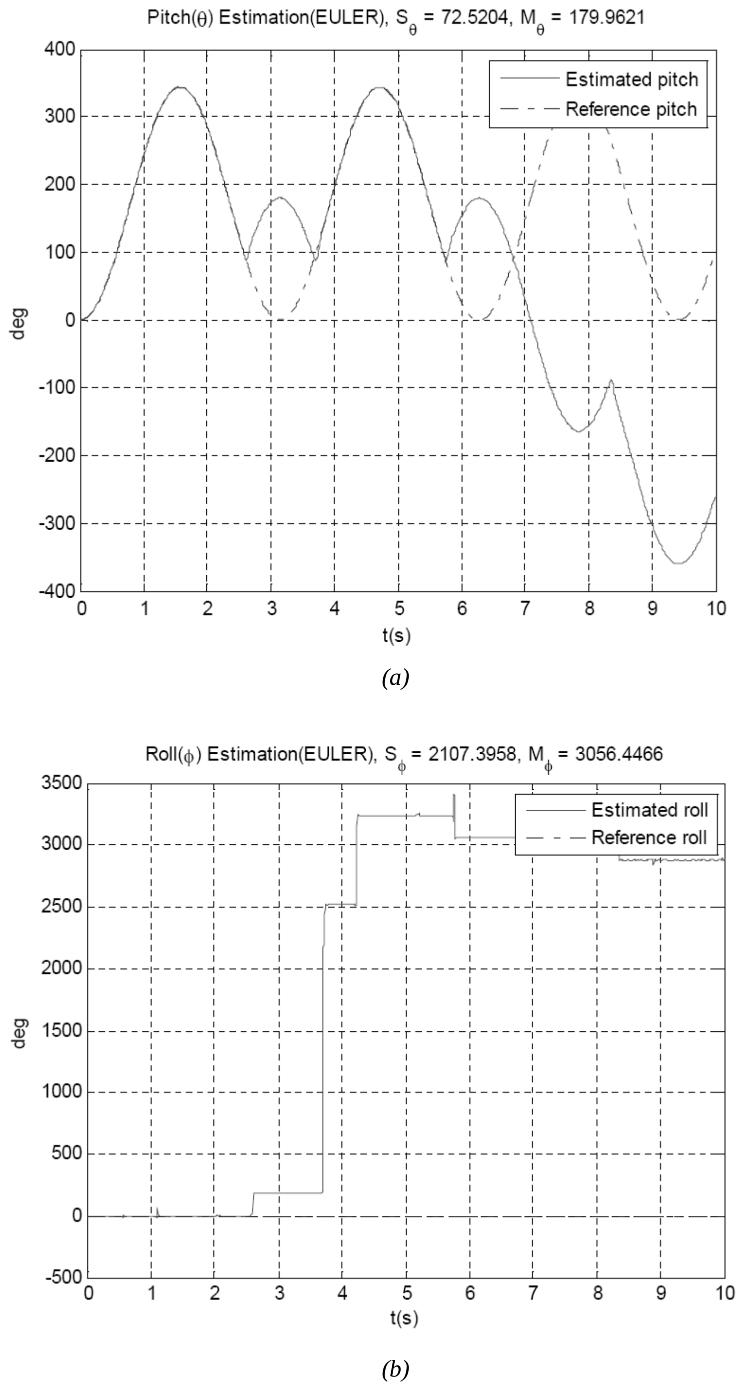


Figure 3: Attitude estimation in singularity case. (a) and (b) are Euler based method Estimated Pitch and Roll angle, respectively, compared with reference

In [Fig. 3(a)], the Euler estimated Pitch angle starts to be wrong when it reaches 90 degrees. As the co-effect, roll angle, in Fig 3(b), is wrong too. Because of the quantization reason, this case not happens in every time pitch angle goes through singularity points.

The [Tab. 1] below summarizes executing speed and accuracy analysis of DCM based attitude estimation compared with Euler based method (The algorithm was tested with Matlab Ver. 7.1, on CPU P4, 2.8 GHz), for more detail, see [Fig. 2]. and [Fig. 3]

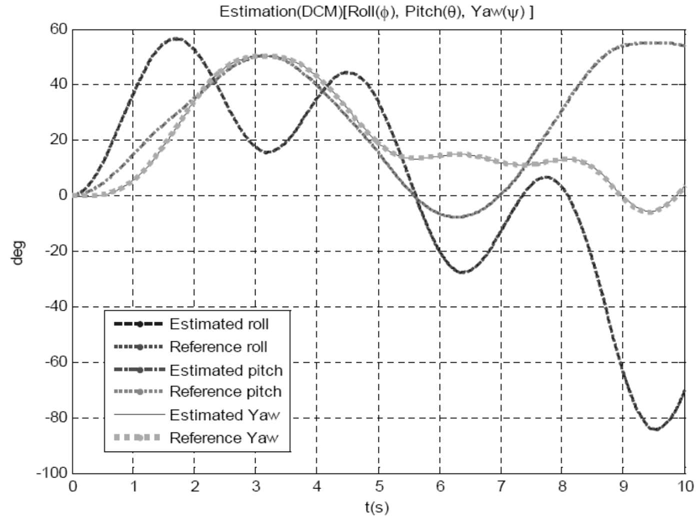
Method	**Euler**	**DCM**	**Ratio (Euler/DCM)**
Process model matrix	25 (5x5)	36 (6x6)	0.69/1
Executed time	0.31 sec	0.37 sec	0.84/1
Roll abs. error	0.32 deg	0.15deg	2.13/1
Pitch abs. error	0.25 deg	0.12deg	2.08/1
Singularity	Yes	No	

Table 1: Compare executing speed and accuracy of Euler and DCM attitude estimation for 10 seconds of measured data

As conclusion for the comparison, DCM method is more accuracy and reliability compared with Euler method. However, it shows a bit higher in computational expense.

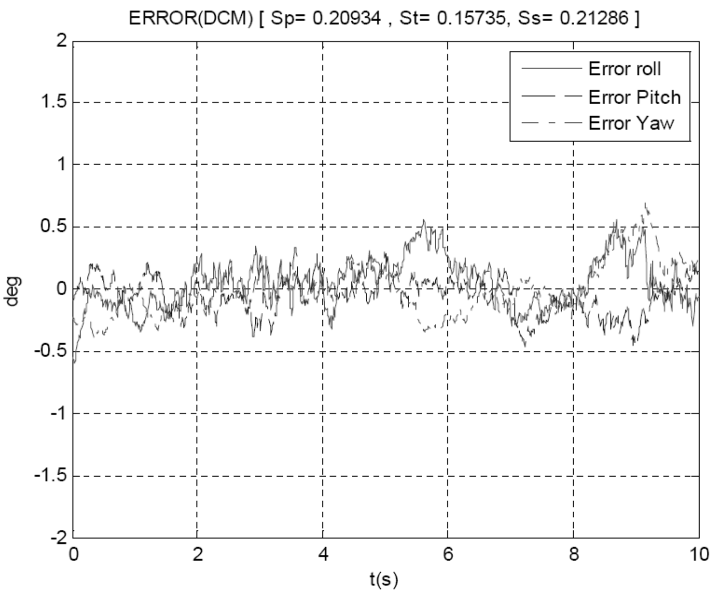
5.3 Comparison between DCM based and Quaternion based Orientation Estimation

Now, we take the simulation of orientation estimation. We try to examine the error of each filter: DCM based and Quaternion based method. Reference signal is given by Quaternion 6dof motion planning of Matlab Aerospace Blockset.



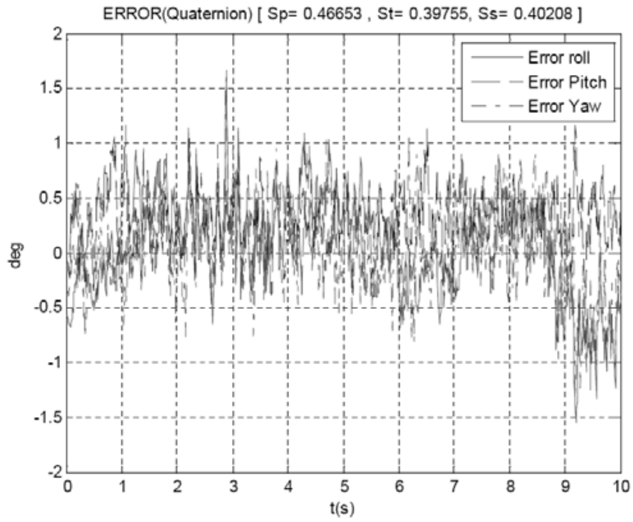
Estimation(DCM)[Roll(ϕ), Pitch(θ), Yaw(ψ)]
deg
t(s)
Estimated roll
Reference roll
Estimated pitch
Reference pitch
Estimated Yaw
Reference Yaw

(a)



ERROR(DCM) [Sp= 0.20934 , St= 0.15735, Ss= 0.21286]
deg
t(s)
Error roll
Error Pitch
Error Yaw

(b)



(c)

Figure 4: Orientation estimation, compared between DCM and Quaternion method. (a) DCM's Estimated Roll, Pitch and Yaw angles compared with reference. (b) and (c) Angular error of DCM method and corresponding of Quaternion's. Average absolute errors are attached with each figure title ("p" denoted for Roll, " t" for Pitch, "s" for Yaw)

As verified in [Fig. 4], the errors of DCM are shown to be smaller than that of the Quaternion based attitude estimation. This aspect fits the discussion about the method error in [Section 1]. About the computational expensive, DCM based method is compatible with that of Quaternion method. Both methods have no singularity.

The [Tab. 2] below summarizes executing speed and accuracy analysis of DCM based orientation estimation compared with Quaternion based method. See [Fig. 4] for more detail.

Method	**Quaternion**	**DCM**	**Ratio (Quat./DCM)**
Process model matrix	49 (7x7)	72 (2x6x6)	0.68/1
Executed time	0.96 sec	1.00 sec	0.96/1
Roll abs. error	0.47 deg	0.21 deg	2.24/1
Pitch abs. error	0.40 deg	0.16 deg	2.50/1
Yaw abs. error	0.40 deg	0.21 deg	1.90/1
Singularity	No	No	

Table 2: Compare executing speed and accuracy of Quaternion and DCM orientation estimation for 10 seconds of measured data

As conclusion for comparison, DCM based method using normal Kalman filter is more accuracy than Quaternion method which uses extended Kalman filter. However, it shows a bit higher in computational expense.

6 Conclusion

Although Euler-based attitude estimation method is a little faster, DCM based is the better choice in case the singularity points are needed to be completely preserved. The given DCM algorithm that needs three DCM's states updated only - out of nine states- is efficient to save the computational effort.

When extending to orientation estimation, Quaternion with the advantage of four variables updating only and easy normalizing then becomes the most selected method. This work presents the new method that using DCM but reaches to the Quaternion about the computational effort and exceeds the normal Quaternion based method about accuracy. In the given method, estimator is divided into two steps for saving computational expense.

Instead of extended Kalman Filter in Euler-based and Quaternion-based method, the Kalman Filter that developed in DCM-based method can help avoid error of first order approximation. That why DCM method leads others about accuracy.

The still limitation of this method is that measurement noise covariance of magnetic compass could not be accessed directly. However it gives another advantage is that the algorithm can give result at large range of earth magnetic field magnitude without initialization and calibration.

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