Algorithms - Degree in Computer Science Engineering
Recurrence relations exercises - September the 30th, 2021

1. A system of signals allows to transmit zeros, ones and blank spaces. Zeros and ones go two by two. Find and solve the recurrence relation which calculates the number of signals of length n that can be transmitted.

2. The number a_n of Euro of a company's assets is incremented each year by five times last year's increment. If $a_0 = 3$ and $a_1 = 7$, calculate a_n.

3. Find and solve a recurrence relation a_n defined as the number of sequences of length n in the alphabet $\{0,1,2\}$ which have an even number of zeros.

4. Being a_r the number of edges of a complete graph of r vertices. Find a recurrence relation for a_r in terms of a_{r-1} and find its solution.

5. In many programming languages, valid arithmetic expressions, without brackets, are formed by digits $0,1,2,...,9$ and the binary operation symbols $+,*$ and . For example, $2+3/5$ is a valid arithmetic expression and $8+*9$ is not. Find a recurrence relation for the number of valid arithmetic expressions of length n.

6. Prove by induction, that if a_n is the sequence defined by $a_0 = 3, a_1 = 3$ and $a_{n+2} = 6a_{n+1} - 9a_n$ for $n \geq 0$, then it is true that $a_n = (3-2n)3^n$ for all natural n.

7. Prove by induction, that if a_n is the sequence defined by $a_1 = 3, a_2 = 5$ and $a_n = 6a_{n-1} - 2a_{n-2}$ for all $n \geq 3$, then it is true that $a_n = 2^n + 1$ for all natural n.

8. Solve each of the following linear homogeneous recurrence relations:
   - $4a_n - 5a_{n-1} = 0$ if $n \geq 1$ and $a_0 = 2$
   - $2a_{n+2} - 11a_{n+1} + 5a_n = 0$ if $n \geq 0$, $a_0 = 2$ and $a_1 = -8$
   - $a_n - 6a_{n-1} + 9a_{n-2} = 0$ if $n \geq 2$, $a_0 = 5$ and $a_1 = 12$

9. For each of the following linear homogeneous recurrence relations, find a general solution and indicate the relation that its coefficients should comply with in terms of the initial conditions given.
   - $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$ for $n \geq 3$, subject to initial conditions $a_0 = 2, a_1 = 5$ and $a_2 = 15$ (its characteristic roots are $r_1 = 1, r_2 = 2, r_3 = 3$.)
   - $a_{n+3} - 7a_{n+2} + 16a_{n+1} = 0$ for $n \geq 0$, subject to initial conditions $a_i = i, i = 0,1,2$ (its characteristic roots are $r_1 = r_2 = 2, r_3 = 3$.)
   - $a_n = 8a_{n-2} - 16a_{n-4}$, subject to initial conditions $a_0 = 1, a_1 = 4, a_2 = 28$ and $a_3 = 32$ (its characteristic roots are $r_1 = r_2 = 2, r_3 = r_4 = -2$.)

10. Solve the following recurrence relations:
    - $4a_n - 2a_{n-1} = 3^n$ for $n \geq 1$ and $a_0 = 6$
    - $a_n = 2a_{n-1} - a_{n-2} + 2^n$ for $n \geq 2$, $a_0 = 5$ and $a_1 = 3$
    - $a_n = 3a_{n-1} - 2a_{n-2} + 2^n$ for $n \geq 2$ with $a_0 = a_1 = 1$
    - $a_n = 2a_{n-1} + n$ for $n \geq 1$ and a_0 equal to true
    - $a_n = 4a_{n-1} - 3$ for $n \geq 1$ and $a_0 = 2$
    - $a_n = 4a_{n-1} - 4a_{n-2} - 1$ for $n \geq 2$ with $a_0 = a_1 = 3$