Assignment 2 - Solutions

1. Give regular expressions generating the following languages. The alphabet is $\Sigma = \{0, 1\}$.

   (a)

   (b) $\{w \mid w$ begins with a 0 and ends with a 1$\}$
   $0\Sigma^*1$

   (c) $\{w \mid w$ contains at least four 0s$\}$
   $\Sigma^*0\Sigma^*0\Sigma^*0\Sigma^*0\Sigma^*$

   (d) $\{w \mid w$ contains the substring 1101$\}$
   $\Sigma^*1101\Sigma^*$

   (e) $\{w \mid w$ has length at least 4 and its third symbol is a 0$\}$
   $\Sigma\Sigma0\Sigma\Sigma^*$

   (f) $\{w \mid w$ begins with a 0 and has even length, or begins with a 1 and has odd length$\}$
   $(0\Sigma \cup 1)(\Sigma\Sigma)^*$

   (g) $\{\epsilon, 0\}$
   $\epsilon \cup 0$

   (h) The empty set
   $\emptyset$

   (i) All strings over $\Sigma$
   $\Sigma^*$

   (j) All strings over $\Sigma$ except the empty string
   $\Sigma^+$

2. For each of the following languages, give two strings that are members and two strings that are *not* members. The alphabet is $\{a, b\}$.

   (a) $b^*a^*$
   $ba, \epsilon$; $\quad ab, bab$

   (b) $a(ab)^*b$
   $ab, aabb$; $\quad \epsilon, ba$

   (c) $a^+ \cup b^+$
   $bbb, a$; $\quad \epsilon, ab$

   (d) $(aba)^*$
   $abaabaaba, \epsilon$; $\quad ab, ba$

   (e) $\Sigma a\Sigma^*b\Sigma^*a\Sigma$
   $aabaa, babbbab$; $\quad \epsilon, aba, abba$

(f) $aba \cup bab$

$aba, bab; \quad \epsilon, ababab$

(g) $b(\epsilon \cup a)b$

$bb, bab; \quad \epsilon, baab, ab$

(h) $\Sigma^*(a \cup ba \cup bb)\Sigma^*$

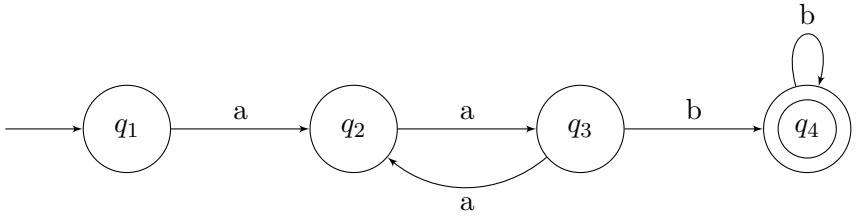
$a, ba, bb, aa, ab; \quad \epsilon, b$

3. For each of the following languages, specify if the language is regular or non-regular. You must support your answer: if the language is non-regular, explain why; if the language is regular design a finite automaton that accepts the language.

   The alphabet is $\{a, b, c\}$.

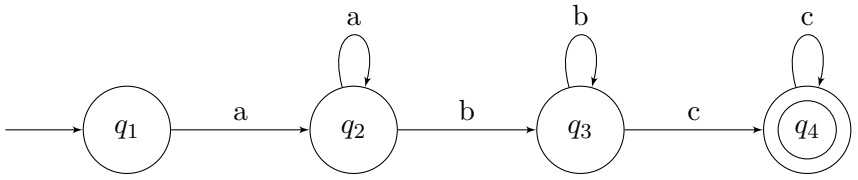
   (a) $\{a^{2i}b^j \mid i, j \geq 1\}$

   This language is regular. Its corresponding regular expression is $(aa)^+b^+$. The following DFA accepts/recognizes the language.




   (b) $\{a^n b^m c^k \mid n, m, k \geq 1\}$

   This language is regular. Its corresponding regular expression is $a^+b^+c^+$. The following DFA accepts/recognizes the language.




   (c) $\{a^n b^{n+m} c^m \mid n, m \geq 1\}$

   This language is non-regular: the automaton would need to keep track of the number of a's, b's and c's and check that the total number of a's and c's is equal to the number of b's.