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COSC455.002

Regular Expressions for Languages over $\Sigma = \{0, 1\}$

(a) $\{w \in \Sigma^\star | w \text{ begins with a } 0 \text{ and ends with a } 1\}$

Regular Expression:
$0\Sigma^\star 1$

(b) $\{w \in \Sigma^\star | w \text{ contains at least four 0s}\}$

Regular Expression:
$\Sigma^\star 0 \Sigma^\star 0 \Sigma^\star 0 \Sigma^\star 0 \Sigma^\star$

(c) $\{w \in \Sigma^\star | w \text{ contains the substring } 1101\}$

Regular Expression:
$\Sigma^\star 1101 \Sigma^\star$

(d) $\{w \in \Sigma^\star | w \text{ has length at least 4 and its third symbol is a } 0\}$

Regular Expression: $\Sigma\Sigma 0 \Sigma\Sigma^\star$

(e)

$\{w \in \Sigma^\star | w \text{ begins with a } 0 \text{ and has even length, or begins with a } 1 \text{ and has odd length}\}$

Regular expression:
$0(\Sigma\Sigma)^\star | 1(\Sigma\Sigma)^\star\Sigma$

(f) $\{\epsilon, 0\}$

Regular Expression:
$\epsilon | 0$

(g) The empty set

Regular Expression: $\emptyset$

(h) All strings over Σ

Regular Expression: $\Sigma^\star$

(i) All strings over Σ except the empty string

Regular Expression: Σ^+

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2. Strings in Languages over $\Sigma = \{a, b\}$

For each of the following languages, give...

- two strings that `are` members and
- two strings that `are not` members
- **(a) $a^\star b^\star$
  - members
    - ϵ
    - $bbaa$
  - non-members
    - ab
    - $baab$
- **(b)** $a(ab)^\star b$
  - members
    - ab
    - abb
  - non-members
    - ba
    - aa
- **(c)** $a^+ \cup b^+$
  - members
    - a

        - bb
    - non-members
        - ϵ
        - ab
- **(d)** $(aba)^\star$
    - members
        - aba
        - ϵ
    - non-members
        - $abaa$
        - ab
- **(e)** $\Sigma a \Sigma^\star b \Sigma^\star a \Sigma$
    - members
        - $aabaa$
        - $babab$
    - non-members
        - $bbbaa$
        - $aaaaa$
- **(f)** $aba \cup bab$
    - members
        - aba
        - bab
    - non-members
        - ab
        - ba
- **(g)** $b(\epsilon \cup a)b$
    - members
        - bb
        - bab
    - non-members
        - ba
        - ab

- **(h)** $\Sigma^\star(a \cup ba \cup bb)\Sigma^\star$
  - members
    - a
    - aaa
  - non-members
    - none; this language allows any string over Σ , as long as it contains at least one of a, ba, or bb somewhere in it.

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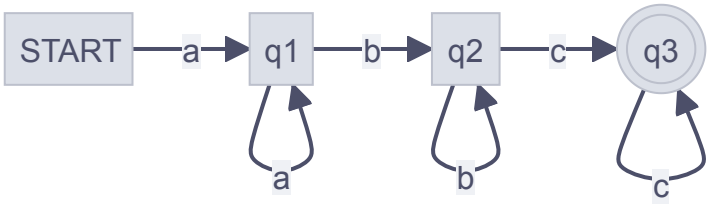
3. Regularity of Languages over $\Sigma = \{a, b, c\}$

(a) $\{a^{2i}b^j | i, j \geq 1\}$

- This language is **not** regular because it requires counting (to ensure the number of a's are in multiples of 2)
- Counting indicates a non-regular language. Finite automata does not have memory for this.

(b) $\{a^n b^m c^k | n, m, k \geq 1\}$

- This language is regular because it doesn't require counting. (It only needs one of each member)



(c) $\{a^n b^{n+m} c^k | n, m, m \geq 1\}$

- This language is **not** regular. It requires counting the number of a's and $b's$ to make sure that the number of b's is at least as many as the number of a's'.
- Counting indicates a non-regular language. Finite automata does not have memory for this.