Pumping Lemma

#fall2023 #COSC-455

The Basics

The Pumping Lemma is **a tool used to prove that a given language is not regular**. It says that for any regular language L, there exists a constant p (the "pumping length") such that any string s in L with length at least p can be divided into three parts $s = xyz$, satisfying:

1. xy^iz is in L for all $i \geq 0$.
2. $|y| > 0$ (i.e., y is not empty).
3. $|xy| \leq p$.

If you find a string s in L that violates any of these conditions, then L is not a regular language.

Examples

Example 1: $L = \{a^nb^n | n \geq 1\}$

1. Choose a string $s = a^pb^p$ where p is the pumping length.
2. Try to divide s into xyz to satisfy the conditions.
3. You'll find that no matter how you divide it, you can't satisfy all three conditions.
4. Conclusion: L is not a regular language.

Example 2: $L = \{a^n | n \geq 0\}$

1. Choose a string $s = a^p$ where p is the pumping length.
2. Divide $s = xyz = a^{p-1}a^1$ where $x = a^{p-1}$, $y = a^1$, $z = \epsilon$.
3. All conditions are met:
   - $xy^iz = a^{p-1}a^i$ is in L for all $i \geq 0$.
   - $|y| = 1 > 0$.
   - $|xy| = p \leq p$.
4. Conclusion: L could be a regular language (Pumping Lemma is not violated).

The Pumping Lemma is mainly used for proving non-regularity. If a language satisfies the Pumping Lemma, it doesn't necessarily mean it's regular; it just means it hasn't been proven non-regular by this method.