Markov Models Using Hanafuda

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To create a Markov model for the probability of scoring a set of "3 Brights" in Koi-Koi, we need to consider the possible states of the game and the transitions between them. The initial game state, as given in the problem, consists of 8 face up cards in the common area and 8 cards in each player's hand. We assume that the opponent's hand is not visible and that we do not know what cards they have.

To simplify the problem, we will only consider our own hand and the common area, and ignore the opponent's hand. We will also assume that the cards in the common area are randomly drawn from the deck and that the order in which they are drawn does not matter.

- Player 1 has 8 cards in their hand, with one card matching the bright card in the common area.
- Player 2 has 8 cards in their hand (not visible to us).
- The common area has 8 cards, with one bright card and one card matching the card in Player 1's hand.

There are five brights in a hanafuda deck, and there are a total of 48 cards in a deck. Therefore, the probability of drawing a bright card from the deck is: P(Bright) = 5/48

Player 1 has matching card for bright card in common are so he'll use that card and get one bright card. Now common are has 7 cards. Player 2 turn.

Player 2 can either draw a card from the deck or play a card from their hand. To get bright card Let's assume that Player 2 choose to draw a card from the deck.

Now, there are two possible states for the game:

1. Player 2 draws a bright card from the deck:

   Player 2 gets one bright card and the common area has cards remaining.

2. Player 2 draws a non-bright card from the deck:

   Player 2 gets no bright cards and the common area has 8cards remaining.

The transition probabilities for this Markov model would be:

Probability of drawing a bright card from the deck: 4/24

Probability of drawing a non-bright card from the deck: 20/24

At this point, Player 1's turn begins again. we can assume that the transitions would be similar to the previous steps, where Player 1 draw a card from the deck.

we have the following possible states and transitions:

After Player 2's turn, if Player 2 draws a non-bright card from the deck, the probabilities for Player 1's turn would be:

1. Player 1 draws a bright card from the deck:

   Player 1 gets two bright cards and now has a total of two bright cards.

2. Player 1 draws a non-bright card from the deck:

Player 1 still has one bright card.
Player 2 turn Again the same steps and this time he gets non bright card.

Player 1 turn He picks up card from deck and gets bright card now he has two bright card.
The transition probabilities for Player 1's turn would be:

1. Probability of drawing a bright card from the deck: 4/22 (if Player 2 does not have any bright cards).

2. Probability of drawing a bright card from the deck: 3/22 (if Player 2 has already drawn one bright card from the deck).

On Player 2's next turn, if Player 2 draws a non-bright card from the deck, the game would continue with Player 1's turn again.

However, if Player 2 draws a bright card from the deck on their turn, then Player 2 would have two bright card. On Player 1's turn, if Player 1 draws a bright card from the deck, Player 1 would have three bright cards and would win the game.

Furthermore, considering Player 2's cards, we can assume these possibilities:

- Player 2 does not have any bright cards. This scenario makes it more favorable for Player 1 to win the game by scoring a set of "3 Brights." As player 1 has matching card with common area bright card.

- Player 2 has at least one bright card in their hand, in which case they can create a set of "3 Brights" if they can obtain the other two bright cards from the common area or the deck. This scenario makes it more challenging for Player 1 to win the game by scoring a set of "3 Brights."

Scenario 1: Player 2 does not have any bright cards.

Player 1 has the option to play the matching card and collect the bright card from the common area, reducing the number of cards in the common area to 7. This would score one bright card for Player 1 and then it would be Player 2's turn.
If Player 2 does not have any bright cards, two scenarios are possible:

- Player 2 plays a non-matching card, adding it to the common area, which then has 8 cards again. Since the card does not match any cards in the common area, Player 2's turn ends, and it is Player 1's turn again.

- Player 2 plays a matching card and takes both the matching card and bright card from the common area, adding them to their own pile. Player 2 now has one matching card and the possibility of creating a set of "3 Brights" later in the game. The common area is now reduced to 6 cards.

It is then Player 1's turn again, and as there are no bright cards left in the common area, Player 1 must draw a card from the deck. Assuming that Player 2 does not have any bright cards, there are 4 bright cards remaining in the deck.
Player 1 draws one card from the deck, resulting in two possibilities:

- It is a bright card, in which case Player 1 now has two bright cards. It then becomes Player 2's turn again.

- It is not a bright card, which has two possibilities:

  - It matches with the common area, in which case Player 1 will collect it.
  - It does not match with the common area, in which case it will be added to the common area. It then becomes Player 2's turn again.

Assuming Player 2 then draws a card from the deck, which is assumed to be a non-bright card, and puts it in the common area. It is then Player 1's turn again.
Assuming Player 1 draws a card from the deck and receives a bright card, Player 1 now has three bright cards and scores a set of "3 Brights" worth 5 points, winning the game.

This scenario involves multiple outcomes, which depend on the actions of each player and the contents of the deck and common area. However, a Markov model can be developed to evaluate the probabilities of each possible outcome, which can be used to determine the likelihood of Player 1 winning the game by scoring a set of "3 Brights."

Scenario 2: Player 2 has at least one bright card in their hand.

Player 1 plays a matching card and takes the bright card from the common area: Player 1 has now scored one bright, and the game continues with Player 2's turn.

Assuming that Player 2 has one bright card in their hand, there are several possible scenarios:

- Player 2 plays a non-matching card.

  The card is added to the common area, which now has 7 cards.

  Player 2's turn ends, and it is Player 1's turn again.

- Player 2 plays their bright card.

  Player 2 takes both cards (the matching card and the bright card) from the common area and adds them to their own pile.

  Player 2 now has two bright cards and may be able to create a set of three brights later in the game.

  The common area now has 5 cards. It is now Player 1's turn again.

Player 1 takes a card from the deck and gets a non-bright card.

There are two possibilities:

- The card matches with the card in the common area: Player 1 takes both cards and adds them to their own pile.
- The card does not match with the card in the common area: The card is added to the common area, which now has 6 cards.

Player 2's turn again. Player 2 takes a card from the deck and gets a non-bright card.

The card is added to the common area, which now has 7 cards. Player 1's turn again. Player 1 takes a card from the deck and gets a non-bright card.

There are two possibilities:

- The card matches with the card in the common area: Player 1 takes both cards and adds them to their own pile.
- The card does not match with the card in the common area: The card is added to the common area, which now has 8 cards.

Player 2's turn again. Player 2 takes a card from the deck and gets a non-bright card.

The card is added to the common area, which now has 9 cards. Player 1's turn again. Player 1 takes a card from the deck and gets a non-bright card.

There are two possibilities:

- The card matches with the card in the common area: Player 1 takes both cards and adds them to their own pile.
- The card does not match with the card in the common area: The card is added to the common area, which now has 10 cards.

Player 2's turn again. Player 2 takes a card from the deck and gets a non-bright card.

The card is added to the common area, which now has 11 cards. Player 1's turn again. Player 1 takes a card from the deck and gets a bright card.

Player 1 now has two bright cards. Player 2's turn again. Player 2 takes a card from the deck and gets a non-bright card.

The card is added to the common area, which now has 12 cards. Player 1's turn again. Player 1 takes a card from the deck and gets a bright card.

Player 1 now has three bright cards, which is a set of "3 Brights" worth 5 points. Player 2's turn again. Since Player 1 has scored "3 Brights", they have won the game.

In below given figure:
B = Bright Card
C = In common are bright card

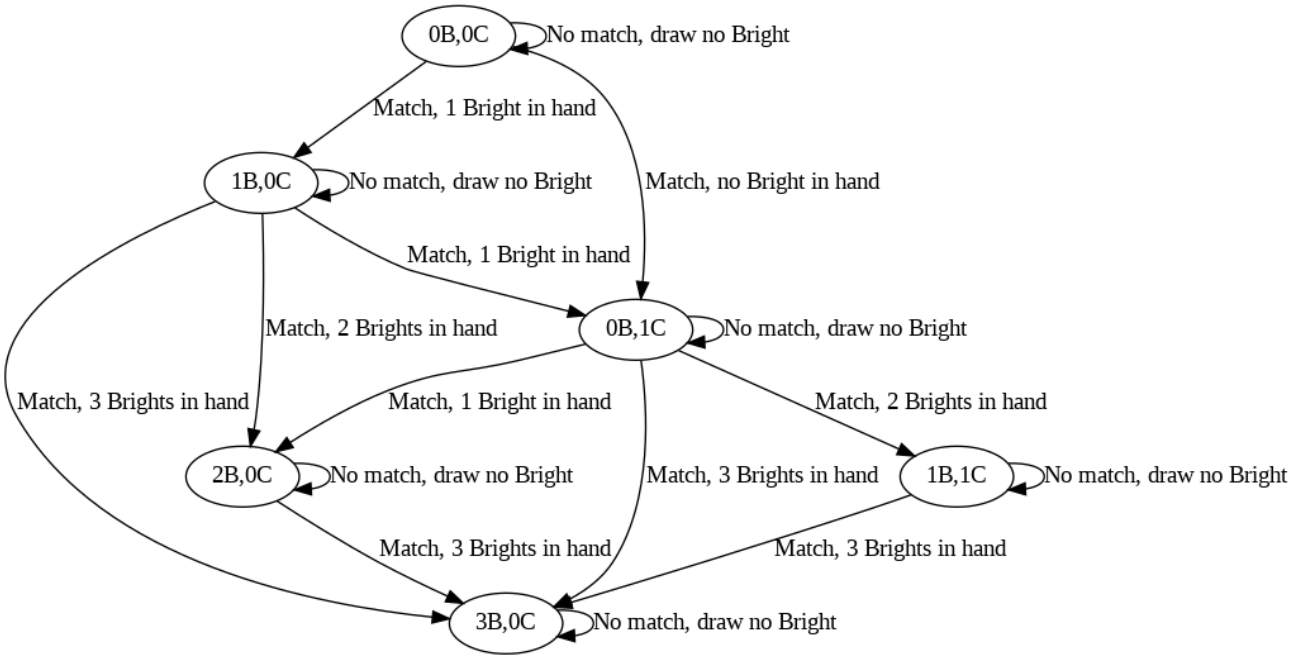

Figure 1: Markov Chain