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Q1. [20 pts] Propositional Logic

Determine using a truth table whether the following sentence is valid, satisfiable or unsatisfiable:

- (a) $(P \wedge Q) \vee \neg Q$
- (b) $((P \wedge Q) \Rightarrow R) \Longleftrightarrow ((P \Rightarrow R) \vee (Q \Rightarrow R))$

(a) [10 pts] Is (a) valid, satisfiable or unsatisfiable?

Answer: (P∧Q) ∨¬Q is satisfiable

p	q	¬q	(p ∧ q)	((p ∧ q) ∨ ¬q)
F	F	**T**	**F**	**T**
F	T	**F**	**F**	**F**
T	F	**T**	**F**	**T**
T	T	**F**	**T**	**T**

(b) [10 pts] Is (b) valid, satisfiable or unsatisfiable?

Answer: $((P \wedge Q) \Rightarrow R) \Longleftrightarrow ((P \Rightarrow R) \vee (Q \Rightarrow R))$ is valid

P	Q	R	((P ∧ Q) → R)	((P → R) ∨ (Q → R))	(((P ∧ Q) → R) ↔ ((P → R) ∨ (Q → R)))
F	F	F	**T**	**T**	**T**
F	F	T	**T**	**T**	**T**
F	T	F	**T**	**T**	**T**
F	T	T	**T**	**T**	**T**
T	F	F	**T**	**T**	**T**
T	F	T	**T**	**T**	**T**
T	T	F	**F**	**F**	**T**
T	T	T	**T**	**T**	**T**

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Q2. [20 pts] Propositional Logic

Assume that a knowledge base KB contains the following rules:

- $P \Rightarrow \neg W$
- $R \Rightarrow S$
- $\neg R \Rightarrow P$

(a) [10 pts] Show that KB entails ($W \Rightarrow S$) using the truth table enumeration approach.

Answer:

P	R	S	W	$P \Rightarrow \neg W$	$R \Rightarrow S$	$\neg R \Rightarrow P$	$W \Rightarrow S$
0	0	0	0	1	1	0	1
0	0	0	1	1	1	0	0
0	0	1	0	1	1	0	1
0	0	1	1	1	1	0	1
0	1	0	0	1	0	1	1
0	1	0	1	1	0	1	0
0	1	1	0	1	1	1	1
0	1	1	1	1	1	1	1
1	0	0	0	1	1	1	1
1	0	0	1	0	1	1	0
1	0	1	0	1	1	1	1
1	0	1	1	0	1	1	1
1	1	0	0	1	0	1	1
1	1	0	1	0	0	1	0
1	1	1	0	1	1	1	1
1	1	1	1	0	1	1	1

KB entails ($W \Rightarrow S$)

KB $|=$ ($W \Rightarrow S$)

(b) [10 pts] Show that KB entails ($W \Rightarrow S$) using resolution.

Answer:

$P \Rightarrow \neg W$ $\quad$ $\neg P \vee \neg W$ $\quad$ $\neg(P \wedge W)$

$R \Rightarrow S$ $\quad$ $\neg R \vee S$

$\neg R \Rightarrow P$ $\quad$ $R \vee P$

$\neg R \Rightarrow \neg W$ $\quad$ $R \vee \neg W$

$W \Rightarrow R$ $\quad$ $\neg W \vee R$

$W \Rightarrow S$ $\quad$ $\neg W \vee S$

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Q3. [20 pts] Propositional Logic

Consider the following statement:

On either Saturday or Sunday, if I am free, I will go to the concert.

Using propositional logic, we can represent it as:

(saturday ∨ sunday) ⇒ (free ⇒ concert)

(a) [10 pts] Convert the above sentence into conjunctive normal form, using the logical equivalence table in Fig. 1.

$(\alpha \land \beta)$	$\equiv$	$(\beta \land \alpha)$	commutativity of $\land$
$(\alpha \lor \beta)$	$\equiv$	$(\beta \lor \alpha)$	commutativity of $\lor$
$((\alpha \land \beta) \land \gamma)$	$\equiv$	$(\alpha \land (\beta \land \gamma))$	associativity of $\land$
$((\alpha \lor \beta) \lor \gamma)$	$\equiv$	$(\alpha \lor (\beta \lor \gamma))$	associativity of $\lor$
$\neg(\neg\alpha)$	$\equiv$	α	double-negation elimination
$(\alpha \Rightarrow \beta)$	$\equiv$	$(\neg\beta \Rightarrow \neg\alpha)$	contraposition
$(\alpha \Rightarrow \beta)$	$\equiv$	$(\neg\alpha \lor \beta)$	implication elimination
$(\alpha \Leftrightarrow \beta)$	$\equiv$	$((\alpha \Rightarrow \beta) \land (\beta \Rightarrow \alpha))$	biconditional elimination
$\neg(\alpha \land \beta)$	$\equiv$	$(\neg\alpha \lor \neg\beta)$	De Morgan
$\neg(\alpha \lor \beta)$	$\equiv$	$(\neg\alpha \land \neg\beta)$	De Morgan
$(\alpha \land (\beta \lor \gamma))$	$\equiv$	$((\alpha \land \beta) \lor (\alpha \land \gamma))$	distributivity of $\land$ over $\lor$
$(\alpha \lor (\beta \land \gamma))$	$\equiv$	$((\alpha \lor \beta) \land (\alpha \lor \gamma))$	distributivity of $\lor$ over $\land$

Answer:

(Saturday ∨ Sunday) ⇒ (Free ⇒ Concert)

¬(Saturday ∨ Sunday) ∨ (Free ⇒ Concert)

(¬Saturday ∧ ¬Sunday) ∨ (¬Free ∨ Concert)

(¬Saturday ∨ ¬Free ∨ Concert) ∧ (¬Sunday ∨ ¬Free ∨ Concert)

(b) [10 pts] Continue to convert the above into implication form of Horn clause.

Answer:

(Saturday ∨ Sunday) ⇒ (Free ⇒ Concert)

¬(Saturday ∨ Sunday) ∨ (Free ⇒ Concert)

(¬Saturday ∧ ¬Sunday) ∨ (¬Free ∨ Concert)

(¬Saturday ∨ ¬Free ∨ Concert) ∧ (¬Sunday ∨ ¬Free ∨ Concert)

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Q4. [20 pts] Propositional Logic

Given the following:

If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned

(a) [10 pts] Can we prove that the unicorn is mythical?

Answer:
Propositional logic statements of the given facts:
(1) Mythical → ¬Mortal
(2) ¬Mythical → Mortal ∧ Mammal
(3) ¬Mortal ∨ Mammal →Horned
(4) Horned →Magical

from this I can write
(5) ¬Mythical ∨¬Mortal From (1)
(6) (Mythical ∨ Mortal) From (2)
(7) (Mythical ∨ Mammal) From (2)
(8) ¬Mortal ∨ Mammal Resolution on (5) and (7)
(9) Horned Modus Ponens on (3) and (8)
(10) Magical Modus Ponens on (4) and (9)
We can only prove the unicorn is horned and magical. Think of 2 cases, mythical or not. In both cases we have that the unicorn ios either immortal or a mammal which is a necessary condition for horned.

(b) [5 pts] Can we prove that the unicorn is magical?

Answer:

(1) Mythical → ¬Mortal
(2) ¬Mythical → Mortal ∧ Mammal
(3) ¬Mortal ∨ Mammal →Horned
(4) Horned →Magical
from this I can write
(5) ¬Mythical ∨¬Mortal From (1)
(6) (Mythical ∨ Mortal) From (2)
(7) (Mythical ∨ Mammal) From (2)
(8) ¬Mortal ∨ Mammal Resolution on (5) and (7)
(9) Horned Modus Ponens on (3) and (8)
(10) Magical Modus Ponens on (4) and (9)

Yes, from above given explanation we can prove unicorn is magical.

(c) [5 pts] Can we prove that the unicorn is horned?

Answer:

(1) Mythical → ¬Mortal
(2) ¬Mythical → Mortal ∧ Mammal
(3) ¬Mortal ∨ Mammal →Horned
(4) Horned →Magical

from this I can write
(5) ¬Mythical ∨¬Mortal From (1)

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(6) (Mythical ∨ Mortal) From (2)
(7) (Mythical ∨ Mammal) From (2)
(8) ¬Mortal ∨ Mammal Resolution on (5) and (7)
(9) Horned   Modus Ponens on (3) and (8)
(10) Magical   Modus Ponens on (4) and (9)

Yes, from above given explanation we can prove unicorn is horned.

Syntax	Meaning
Student(X)	X is a student
Takes(X, History)	X takes history
Takes(X, Biology)	X takes Biology
Fails(X, History)	X fails History
Fails(X, Biology)	X fails Biology
score(X, Biology)	X score Biology
score(X, History)	X score History
Person(X)	X is a person
Vegetarian(X)	X is a vegetarian
Dislikes(X, Y)	X dislikes Y
Likes(X, Y)	X likes Y
Smart(X)	X is smart
Woman(X)	X is a woman
Man(X)	X is a man
Town(X)	X in town
Barber(X)	X is a barber
Shaves(X, Y)	X shaves Y
Professor(X)	X is a professor
Politician(X)	X is a politician
Fools(X, Y, t)	X fools Y at time t

Q5. [20 pts] First-Order Logic

Given the vocabulary syntax below, translate each of the following sentences into the corresponding first-order logic statements:

(a) [2 pts] Not all students take both History and Biology

Answer: ¬∀ (X) [Student(X) → [Takes(X, History) ∧ Likes(X, Biology)]]

(b) [2 pts] Only one student failed History

Answer: ∃X [Student(X) ∧ Fails(X, History)]

(c) [2 pts] Only one student failed both History and Biology

Answer: ∃X [Student(X) ∧ Fails(X, History) ∧ Fails(X, Biology)]

(d) [2 pts] The best score in History was better than the best score in Biology.

Answer: ∀ (X)[Score(X, History) > Score(X, Biology)]

(e) [2 pts] Every person who likes all vegetarians is smart.

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Answer: $\forall(X)$ [Person(X) $\wedge$ [Vegetarian(X) $\rightarrow$ Likes(X, Y)] $\rightarrow$ Smart(X)]

(f) [2 pts] No person dislikes a smart vegetarian.

Answer: $\forall$ (X) [Person(X) $\wedge$ Vegetarian(X) $\wedge$ Smart(X) $\rightarrow$ $\neg$Likes(X, Y)

(g) [2 pts] There is a woman who likes all men who are not vegetarians.

Answer: $\exists$X [Woman(X) $\wedge$ [$\forall$X [Man(X) $\wedge$ $\neg$Vegetarian(X) $\rightarrow$ Likes(X, Y)]]]

(h) [2 pts] There is a barber who shaves all men in town who do not shave themselves.

Answer: $\exists$X [Barber(X) $\wedge$ Man(X) $\wedge$ $\neg$Shaves(X, Y) $\rightarrow$ Shaves(X, Y)]

(i) [2 pts] No person likes a professor unless the professor is smart.

Answer: $\forall$(X)[Person(X) $\wedge$ Professor(X)] $\rightarrow$ [Likes(X, Y) $\rightarrow$ Smart(X)]

(j) [2 pts] Politicians can fool some of the people all the time, and they can fool all of the people some of the time, but they cannot fool all of the people all of the time.

Answer: $\forall$(X)Politician(X) $\rightarrow$ [$\exists$X $\forall$T [Person(X) $\wedge$ [Fools(X, Y, T)]] $\wedge$ [$\exists$T $\forall$X [Person(X) $\wedge$ [Fools(X, Y, T)] $\wedge$ $\neg$[$\forall$T $\forall$X [Person(X) $\rightarrow$ Fools(X, Y, T)]]]]