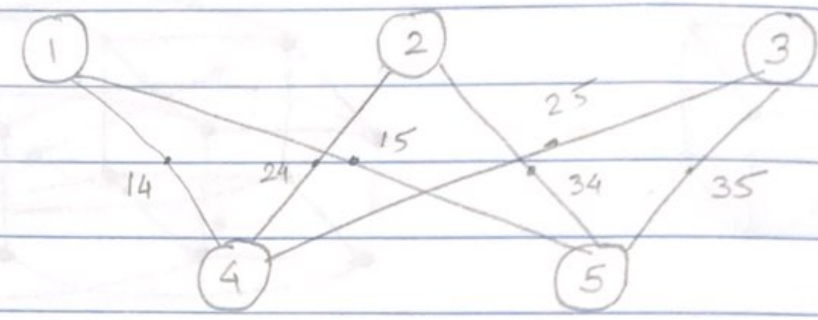
Assignment - 4

Q-1



$$M_{ij} = \begin{cases} 1 & \text{(if vertex is an endpoint of} \\ 0 & \text{edge } j\text{) otherwise} \end{cases}$$

$$MM^T = D + A$$

D = diagonal matrix

A = adjacency matrix

A)

$$(MM^T)_{ii} = \sum_{\substack{e=\{x,y\}\in E \\ i\in e}} m_{xy}^2 = \delta(i)$$

$(MM^T)_{ii}$ is the sum over all those edges that has i has one of its end points. Hence it is the degree of vertex i.

B) For $i \neq j$

The $(i,j)^{th}$ entry of MM^T is a sum of over all edges containing i and j as end vertices of which there is at most one.

When $\{i,j\} \in E$, the entry is 1 so, when $i \neq j$, the entry $(MM^T)_{ij}$ is the number of edges joining vertex i to vertex j.

c)

$$M = \begin{array}{c c} & \begin{array}{cccccc} 14 & 24 & 15 & 25 & 34 & 35 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

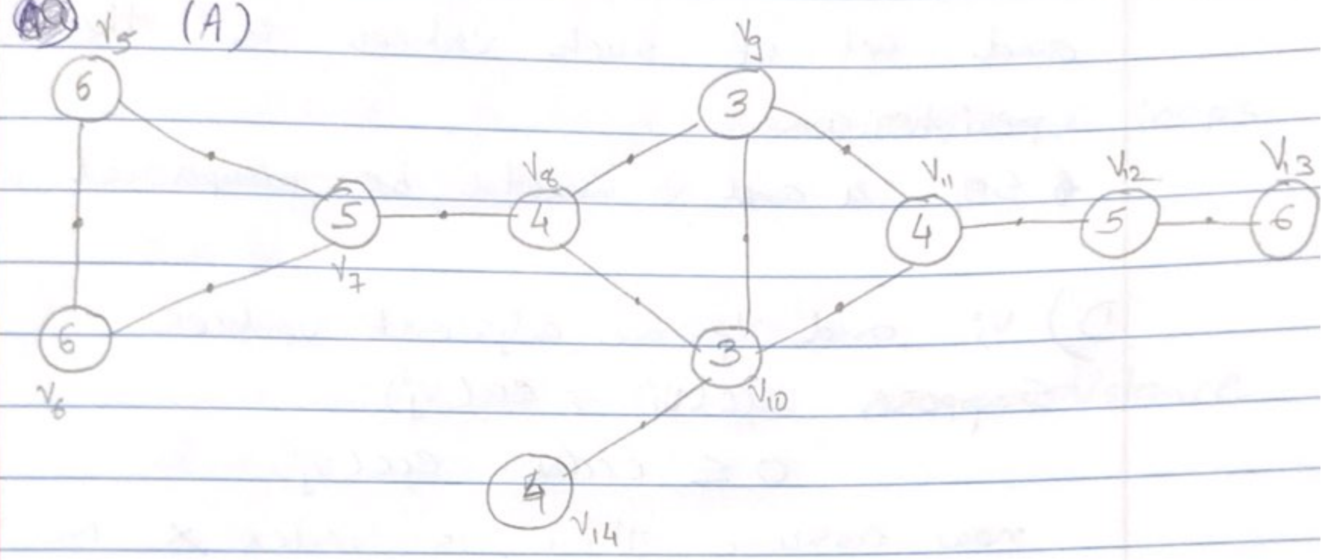
$$MM^T = \begin{array}{c c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 2 & 0 & 0 & 1 & 1 \\ 0 & 2 & 0 & 1 & 1 \\ 0 & 0 & 2 & 1 & 1 \\ 1 & 1 & 1 & 3 & 0 \\ 1 & 1 & 1 & 0 & 3 \end{bmatrix} \end{array}$$

$$D = \begin{array}{c c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 3 \end{bmatrix} \end{array}$$

$$= \operatorname{diag}(2, 2, 2, 3, 3)$$

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

Q-2 (A)



(B)

Diameter = 6

Radius = 3

central vertices = v_9, v_{10}

(c) peripheral vertices = v_5, v_6, v_{13}

u and v are vertices such that $d(u, v) = diameter(G)$

if distance ~~of~~ between vertices is equal to diameter then that becomes condition for peripheral vertices.

vertex of maximum eccentricity of Graph G and set of such vertices is the peripherian.

So, u and v must be peripheral vertices

D) v_i and v_j are adjacent vertices,

Suppose, $ecc(v_i) \geq ecc(v_j)$

$$0 \leq ecc(v_i) - ecc(v_j)$$

now assume there is vertex x in the graph

$$\therefore \; d(v_i, x) \leq d(v_j, x) + 1$$

$$ecc(u) = \max_{x \in V(G)} d(v_i, x) \leq \max_{x \in V(G)} (1 + d(v_j, x))$$

$$ecc(v_i) = 1 + \max_{x \in V(G)} d(v_j, x)$$

$$ecc(v_i) = 1 + ecc(v_j)$$

$$\therefore \; ecc(v_i) - ecc(v_j) = 1.$$

E) From definition of radius and diameter,

min eccentricity $\leq$ max eccentricity

$$\therefore \; radius(H) \leq diameter(H)$$

F) ω is a central vertex for graph H in part G.

$$d(u,v) \leq d(u,\omega) + d(\omega, v)$$

$d(u,\omega)$ and $d(\omega,v)$ ~~[illegible]~~ ~~be~~ are radius of graph because Every longest path has central vertex.

$d(u,v)$ is a diameter of graph.

$\therefore$ Diameter$(H) \leq 2$ radius (H)

Q-3

Given that G is a graph without loops and r ~~be~~ is vertex of G.

$u \approx w$

and $d(r, u) = d(r, \omega)$

u and ω are at the same distance of r.

A.) For each vertex $r \in V(G)$,

we have $d(r, u) = d(r, u)$

$\therefore \; u \approx u, \; \forall u \in V(G)$

so, $\approx$ is reflexive relation

B.) Suppose $u, \omega \in V(G)$ and $u \approx w$.

$\therefore d(r, u) = d(r, \omega)$

$\therefore d(r, \omega) = d(r, u)$

$\therefore \omega \approx u$.

So, $u \approx \omega \iff \omega \approx u, \; \forall u, w \in V(G)$

$\therefore \approx$ is symmetric relation.

C) Suppose $u, v, \omega \in V(G)$ and $u \approx \omega$ and $\omega \approx v$

$\therefore d(r, u) = d(r, \omega)$ and

$d(r, \omega) = d(r, v)$

$\therefore d(r, u) = d(r, v)$

$\therefore u \approx v$

Thus $u \approx w$ and $w \approx v \Rightarrow u \approx v$,
$\forall u, v, w \in V(G)$
$\therefore \approx$ is a transitive relation.

D)

equivalence class $[r] = \{u$ such that $r \sim u\}$
& $\{w$ such that $r \sim w\}$

$\therefore [r] = \{u : r \sim u\}$
$= \{w : r \sim w\}$

E) ru is an edge.
$\therefore d(r, u) = 1$
$\therefore [u] = \{v \in V(G) \mid D(r, v) = 1\}$
= The set of all vertices adjacent to the vertex u.
= $N(u)$, the set of all neighbouring vertices of u (different from u).