Sudoku Game Plan Utilizing Graph Coloring

Ivani Patel

December 14, 2022

Abstract

Machines have forever been tackling the issues of humankind yet in each phase of the historical backdrop of the world they are unused potential. In the same way, we know that Graph coloring appreciates numerous practical applications as well as theoretical challenges. Besides the classical types of problems, different limitations can also be set on the graph or on how color is doled out, or even on the color itself. It has even arrived at fame with the general public in the form of the well-known number riddle Sudoku. Graph coloring is as yet an extremely dynamic field of examination.

1 Introduction

The Sudoku puzzle is perfect for whenever you have a couple spare minutes and want to indulge in a little bit of thinking power. Sudoku puzzles are likely to continue to grow in popularity as more people discover the fun and sense of mental stimulation that comes from solving these number puzzles. The specifics of graph theory are used to solve a 9×9 Sudoku problem using the concept of Graph Coloring.

What is Sudoku?: Sudoku is a single-player popular number-placement puzzle based on logic and combinatorics. The objective is to fill a 9×9 grid with digits such that each column, each row, and each of the nine 3×3 subgrids that compose the grid contain all of the digits from 1 to 9 (once and only once). Usually, the puzzle is partially filled in a way that guarantees a unique solution, as of now from what we know at least 17 cues are needed to create a puzzle with a unique solution.

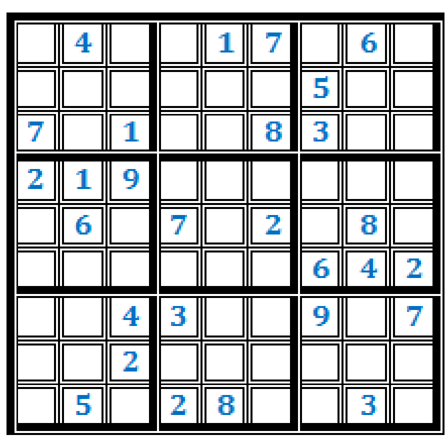


Figure 1: Sudoku

What is Graph?: A graph is a set of points, called nodes or vertices, which are interconnected by a set of lines called edges.

In figure 2 Graph is shown where a set of vertices is written as V and a set of edges is written as E.

$$V = \{A, B, C, D, E\}$$

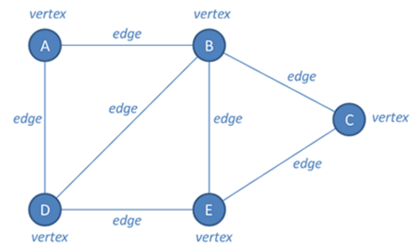


Figure 2: Graph

$$E = \{AB, AD, BD, BE, DE, BC, EC\}.$$

What is Graph Coloring?: Graph coloring is the procedure of assignment of colors to each vertex of a graph G such that no adjacent vertices get the same color. So, each vertex has a different color from its neighbors. The Graph Coloring algorithm can help you organize your time and/or be multitasking.

The objective is to minimize the number of colors used for the coloring of vertices.

In the graph coloring problem, we have to find if a graph can be colored with a minimum of 'G' colors. This 'G' is also known as the Chromatic Number of a Graph and is denoted as $\chi(G)$. Before starting to color the graph, one should know the minimum number of colors required to color the graph.

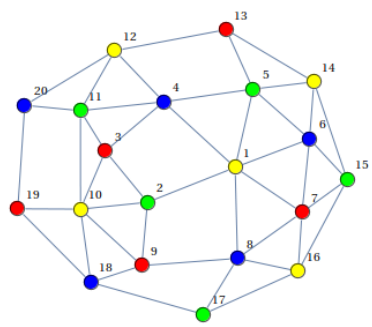


Figure 3: Graph Coloring

This paper aims to solve a 9 × 9 Sudoku puzzle using the concept of graph coloring. We assign 9 different colors to the 9 numbers and assign a sublattice for the known elements. Then we assign a node to each of the boxes in sudoku. The algorithm goes about checking the different colors and when it finds a wrong solution it backtracks, and it does this until it finds the correct solution for the given sudoku problem.

the

2 Solving Sudoku using Graph Coloring

The graph will have 81 vertices with each vertex relating to a cell in the grid. Two distinct vertices will be adjacent if and only if the corresponding cells in the grid are either in the same row, or same column, or the same sub-grid. Each completed Sudoku square then corresponds to a k-coloring of the graph.

The graph has 81 vertices in the standard sudoku where every vertex is adjacent to 8 vertices in its row + 8 vertices in its column and 4 more leftover cells in its block, hence the degree of every vertex in the graph is the same. Each vertex has a degree of 20, thus the number of edges is:

$$|H| = \frac{20 * 81}{2} = 810$$

Perception of the issue:

At first for settling the problem, we need to check for a superior description of the sudoku grid. Instead of simply observing that as a grid, we have to change our convention for simplifying the problem. For addressing we have two constraints:

1. Each component in the sublattice 3×3 is interlinked, that is they should be distinct from each other for solving the puzzle.

2. Similarly the row and columns of the main grid are interlinked since they should also have connectivity between the nodes for the correct solution.

2.1 Mathematical Model:

The initial step is to change over sudoku into a graph representation.

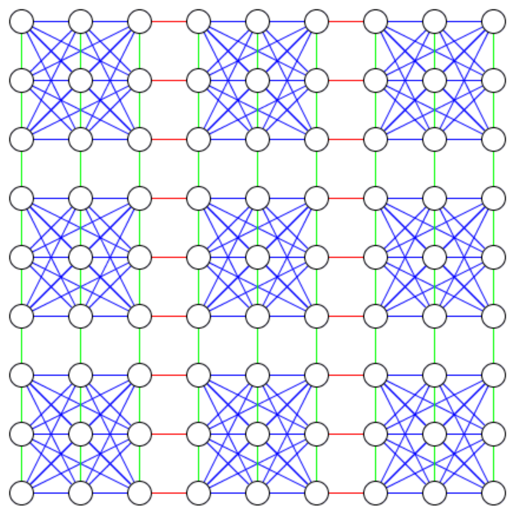


Figure 4: Graph Representation of Sudoku

1. Initialize the graph representing the 9×9 sudoku where each node represents a box.

2. Make the connections in the graph. As per the rule of the sudoku, we will connect a specisublattice every node in its 9×9 sudoku grid, also to all the from in its rows and columns.

Figure 4 shows one such task.

In this figure, all edges are not clearly shown due to 2D constraint consequently, the interlinking of the elements in the 1. same row/column 2. Overlapping edges are not shown.

Examining the figure:

- The circle represents any digit from 1-9 each.
- The green lines associate circles that can't be a similar digits since they're in the same column.
- The red lines associate circles that can't be a similar digits since they're in the same row.
- The blue lines associate circles that can't be a similar digit since they're in the same 3 x 3 square.
- Green and red are the most prominent. Only 4 blue lines from any single circle are visible because the other blue lines are occluded from either green or red since those circles are also from the same row or column in addition to being in the same 3 x 3 square.

We can change this into a more improved image which looks as figure 5.

We can notice the exact connectivity between each node clearly. We can see this issue as a graph in which we have to color such that no adjacent will have the same color. There are 9 distinct elements:1-9 hence we need 9 colors for satisfying this constraint. Consequently, the chromatic number = 9. So now sudoku can be viewed as a Graph.

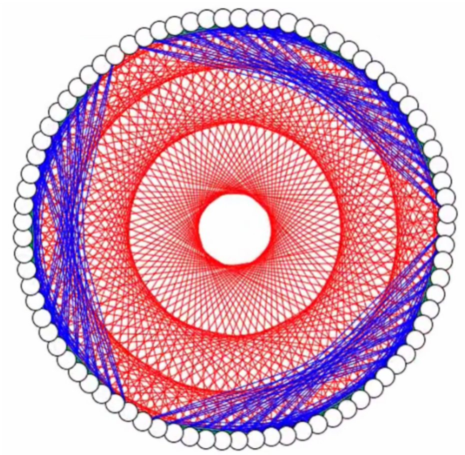


Figure 5: Improved representation of graph

The subsequent move toward changing over a Sudoku puzzle into a graph coloring problem is to assign colors to the numbers 1 through 9. This task is arbitrary and is not a priority ordering of the colors as in the greedy algorithm it's just a simple correspondence between numbers and colors.

1. Apply the coloring function to color the nodes of the graph. The coloring function will assign a color to each number from 1 to 9 and then assign the colors to the different nods.

In order to fully "color" the sudoku with the correct number in each box, we start by finding which node has the most colored neighbors. Whenever we have found this node, we narrow down the possible colors it can be by removing all the colors that its neighbors have. The algorithm inserts one of the possibilities into the graph and repeats the process of finding the next node to fill.

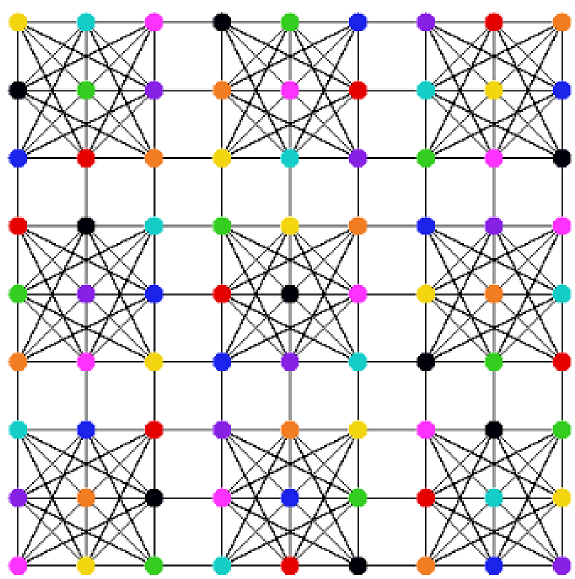


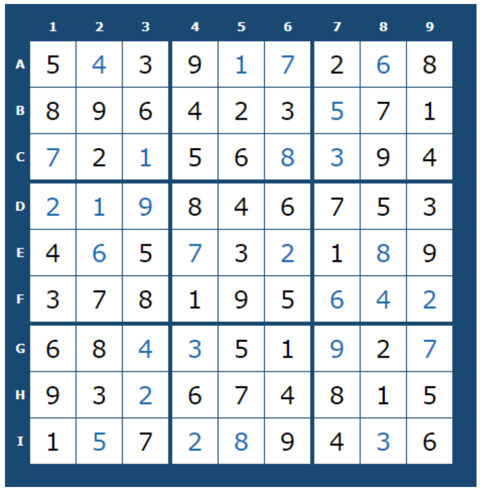
Figure 6: Solved Sudoku Graph

Assuming the color was right, it will continue to add colors until the whole riddle is tackled. On the off chance that for any node its neighbors as of now have 9 various colors, the arrangement is wrong since we realize that the diagram can be colored utilizing precisely 9 colors. For this situation where it ended up speculating an inaccurate solution, it will return to that decision point and attempt an alternate color, to check whether it works with that one. This is a common strategy called backtracking, used when you are slowly working towards a solution by making guesses to get closer.

After giving color to each node such that no two adjacent vertices receive the same color, we get the graph shown in figure 6.

2. Now when all the adjacent nodes have been assigned different colors, convert the colors back to their assigned numbers.

3. In the result we got solved Sudoku Puzzle. and get result as shown in figure 7.



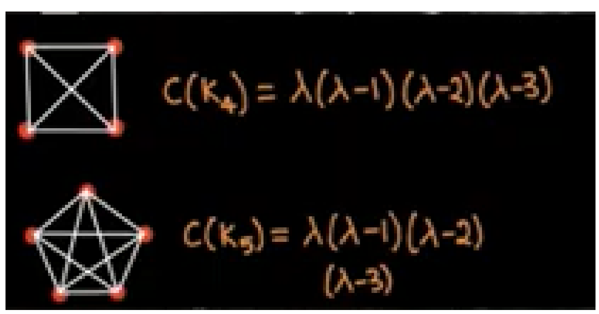
	1	2	3	4	5	6	7	8	9
A	5	4	3	9	1	7	2	6	8
B	8	9	6	4	2	3	5	7	1
C	7	2	1	5	6	8	3	9	4
D	2	1	9	8	4	6	7	5	3
E	4	6	5	7	3	2	1	8	9
F	3	7	8	1	9	5	6	4	2
G	6	8	4	3	5	1	9	2	7
H	9	3	2	6	7	4	8	1	5
I	1	5	7	2	8	9	4	3	6

Figure 7: Solved Sudoku Puzzle

2.2 Concept Connected to the Linear Algebra:

The chromatic number of graph is the minimum number of color the vertices of so that no two adjacent vertices share the same color. The chromatic polynomial is a graph polynomial studied in algebraic graph theory, a branch of mathematics. It counts the number of graph colorings as a function of the number of colors and was originally defined by George David Birkhoff.

This chromatic polynomial is connected to the characteristic polynomial which is used to find the eigenvalues. Similarly the chromatic number is connected to the rank of the matrix. The concept of adjacency matrix is used.



The relation between the chromatic number and eigen values:

Let G be a graph with n vertices and m edges. We always assume that G is nonempty, i.e. $m \geq 1$. We use $\lambda_1, \lambda_2, ..., \lambda_n$ where $\lambda_1 \geq \lambda_2 \geq ... \geq \lambda_n$ to denote the n eigenvalues of G(i.e., the n eigenvalues of the adjacency matrix A(G) of G). The following theorems are two classical spectral lower bounds for chromatic number $\chi(G)$. We note that both lower bounds are attained in any complete graph K_n with $n \geq 2$.

Theorem 1: (Hoffman's lower bound).

$$\chi(G) \geq 1 + \frac{\lambda_i}{|\lambda_n|}$$

Theorem 2: (Cvetkovic's lower bound).

$$\chi(G) \geq \frac{n}{n - \lambda_i}$$

3 Algorithm:

The algorithm for the Graph coloring problem is as below:

Step 1: Create a recursive function that takes the current vertex index, number of vertices, and output color as arguments.

Step 2: If the current vertex index is equal to the number of vertices. Return true and print the color configuration in the output array.

Step 3: Assign color to vertex(1 to 9).
For every assigned color, check if the configuration is safe, (i.e. check if the adjacent vertices do not have the same color) and recursively call the function with the next index and number of vertices.

Step 4: If any recursive function returns true break the loop and return true.

Step 5: If no recursive function returns true, then return false.

This will recursively solve the position of each color (number) on the board.

4 Future Updates

Graph coloring has gained some significant progress in new trends. The theory has grown to solve numerous current issues in the new century. However, there is still a lot that is yet to be discovered in the area. In the many sectors, especially in operating systems and computer science, graph coloring is also making huge advancements that will make considerable contribution in the future. data structures can be designed in the form of trees which utilizes vertices and edges. Similarly modeling of network topologies can be implemented through graph concepts. Other fields include image processing, software engineering, website designing and database designing, which lead to the development of new algorithm and theorems that can be used in tremendous applications in the coming years.

5 Conclusion

In this Project, the basic definitions of Graph, Graph coloring, and Sudoku Puzzles are discussed. The main aim of this study is to know where Graph coloring is used in various fields. This task is an astonishing utilization of the diagram shading methods we created in this venture and includes a significant genuine issue. This paper gives an outline of the applications of graph coloring in heterogeneous fields to some extent applications that use graph coloring concepts. Various papers based on graph coloring have been studied related to scheduling concepts, and computer science applications, and an overview has been presented here. We saw the method to solve the Sudoku puzzle using Graph coloring.

References

1. https://arxiv.org/pdf/1612.05026.pdf

2. https://www.reddit.com/r/dataisbeautiful/comments/6ty4vf/visualizing_the_sudoku_connectivity_graphand_more_in/

3. https://www.researchgate.net/publication/261513432_Recent_Advances_in_Graph_Vertex_Coloring

4. https://mathworld.wolfram.com/k-Coloring.html