Assignment-7

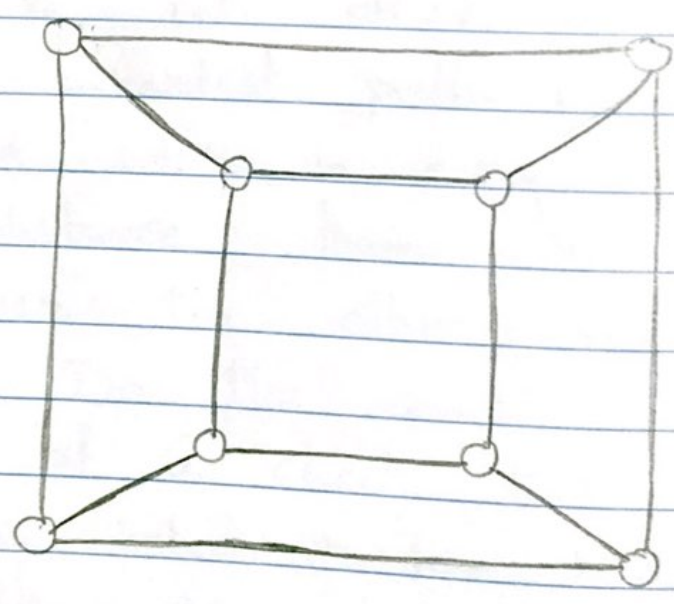
Q-1 A) Q_3

Q_3 have 8 vertices, 12 edges and 6 regions.

According to Euler's formula for Spherical graphs,

$$n - m + r = 8 - 12 + 6$$
$$= 2.$$

Thus, we can draw Q_3 on the plane.



B) Q_4

Suppose Q_4 is planer and let H be a spherical drawing of Q_4. The graph H has $n = 16$ vertices and $m = 32$ edges. From Euler,

$$n - m + r = 2$$

$$16 - 32 + r = 2$$

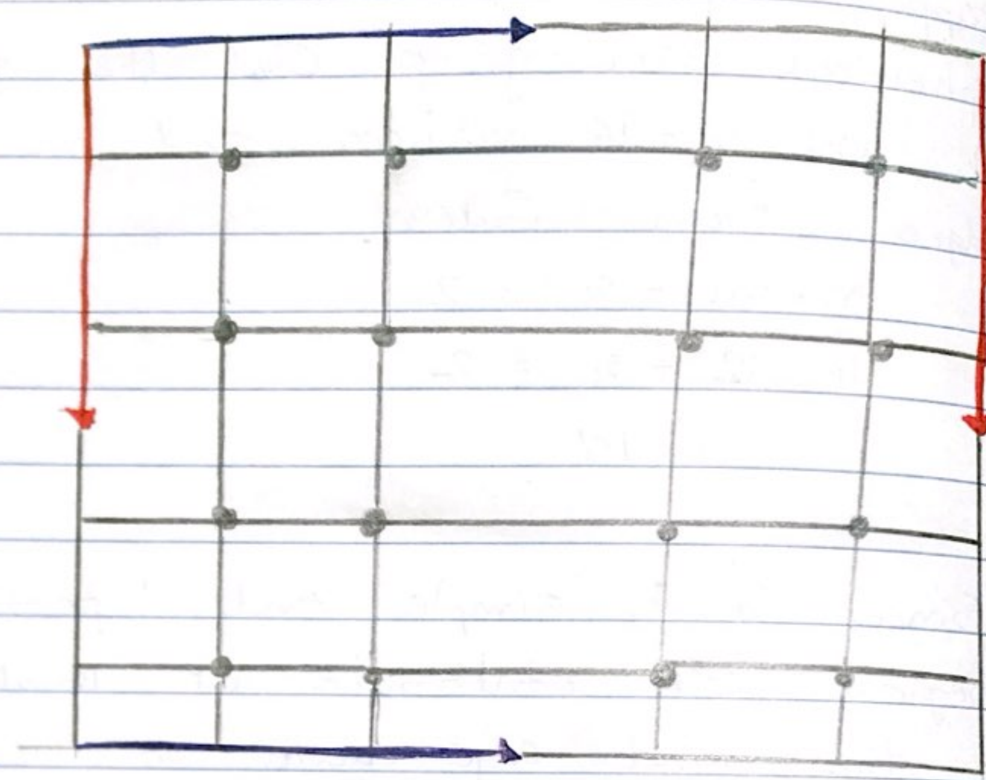
$$\therefore r = 18$$

Since G is simple and bipartite, each region will contribute at least 4 to the total edge count. So the total edge count must be at least $4 * 18 = 72$.

But the total edge count is $2m = 64$ which is not 72.

So, Q_4 is not planar.

c) The graph Q_4 is isomorphic to $C_4 \times C_4$



D) Q_5

For Q_5 we have $n = 32$ vertices and $m = 80$ edges. Q_5 is bipartite.
The total edge count is $2m = 160$.

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As Q_5 is bipartite & Simple graph the total edge count must be at least $4r$.

$\therefore \quad 160 \geq 4r$

$r \leq 40$

Using this information for Euler's formula,

$$n - m + r = 2 - 2h$$

$$\therefore 32 - 80 + r = 2 - 2h$$

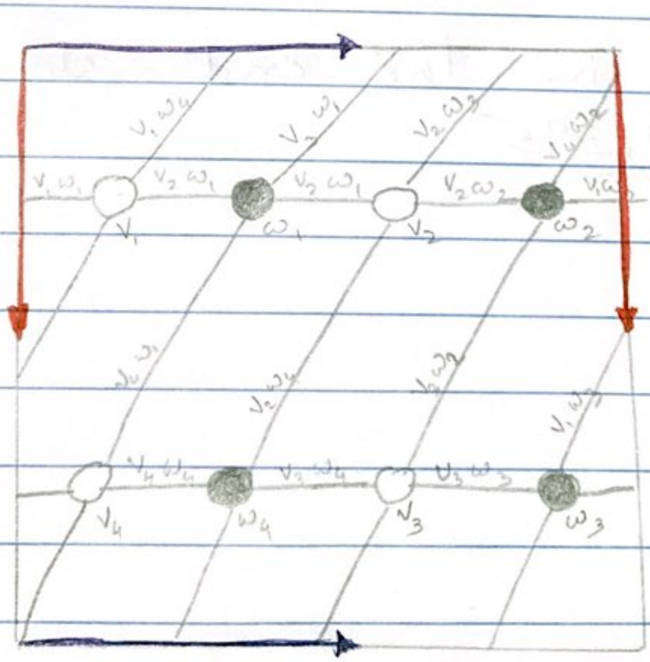
$$\therefore 2h = 50 - r$$

$$\therefore h = \frac{50 - r}{2}$$

since $r \leq 40$

$$\underline{h \geq 5}$$

Q-2 $K_{4,4}$



Q-3 A)

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix}$$

B)

$$\begin{matrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 3 & 2 & 3 & 2 \\ 0 & 5 & 3 & 4 & 2 \\ 0 & 6 & 5 & 5 & 3 \\ \vdots & \vdots & \vdots & \vdots & \vdots \end{matrix}$$

1	1	1	1	1
0	3	2	3	2
0	5	3	4	2
0	6	5	5	3
0	8	6	8	5
0	13	8	11	6
0	17	13	14	8
0	22	17	21	13
0	34	22	30	17
0	47	34	39	22
0	61	47	56	34
0	90	61	81	47
0	128	90	108	61
0	169	128	151	90
0	241	169	218	128
0	346	241	297	169
0	466	346	410	241
0	651	466	587	346

0	933	651	812	466
0	1278	933	1117	651
0	1768	1278	1584	933
0	2517	1768	2211	1278
0	3489	2517	3046	1768
0	4814	3489	4285	2517
0	6802	4814	6006	3489
0	9495	6802	8303	4814
0	13117	9495	11616	6802
0	18418	13117	16297	9495
0	25792	18418	22616	13117
0	35729	25792	31535	18418
0	49953	35729	44210	25792
0	70002	49953	61521	35729
0	97250	70002	85682	49953
0	135635	97250	119955	70002
0	189957	135635	167252	97250

	0	264502	189957	232885	135635
	0	368520	264502	325592	189957
	0	515549	368520	454459	264502
	0	718961	515549	633022	368520
	0	1001542	718961	884069	515549
	0	1399618	1001542	123510	718961
	0	1953471	1399618	1720503	1001542
	0	2722045	1953471	2401160	1399618
	0	3800778	2722045	3353089	1953471
	0	5306560	3800778	4675516	2722045
	0	7397561	5306560	6522823	3800778
	0	10323601	7397561	9107338	5306560
	0	14413898	10323601	12704121	7397561
	0	20101682	14413898	17721162	10323601
	0	28044763	20101682	24737499	14413898
	0	39151397	28044763	34515580	20101682
	0	54617262	39151397	48146445	28044763
	0	76191208	54617262	67196160	39151397

	0	106347557	76191208	93768659	54617262
	0	148385921	106347557	130808470	76191208
	0	206999678	148385921	182538765	106347557
	0	288886322	206999678	254733478	148385921
	0	403119399	288886322	355385599	206999678
	0	562385277	403119399	495886000	288886322
	0	784772322	562385277	692005721	403119399
	0	1.3954354	1.3950836	1.39549356	1.3954257

c

$$A = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \end{pmatrix}$$

Eigenvectors for the matrix A:

- $v = \begin{pmatrix} -1 \\ 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}$, eigenvalue $\lambda_1 = 0$
- $v \approx \begin{pmatrix} 0 \\ 0.225 \\ -0.475 \\ -1.107 \\ 1 \end{pmatrix}$, eigenvalue $\lambda_2 \approx -0.475$
- $v \approx \begin{pmatrix} 0 \\ 1.947 \\ 1.395 \\ 1.717 \\ 1 \end{pmatrix}$, eigenvalue $\lambda_3 \approx 1.395$

- $\boldsymbol{v} \approx \begin{pmatrix} 0 \\ -1.086 - i \cdot (1.049) \\ -0.460 + i \cdot (1.139) \\ \dfrac{1.390 - i \cdot (1.509)}{2} \\ 1 \end{pmatrix}$, eigenvalue $\lambda_4 \approx -0.460 + i \cdot (1.139)$

- $\boldsymbol{v} \approx \begin{pmatrix} 0 \\ -1.086 + i \cdot (1.049) \\ -0.460 - i \cdot (1.139) \\ \dfrac{1.390 + i \cdot (1.509)}{2} \\ 1 \end{pmatrix}$, eigenvalue $\lambda_5 \approx -0.460 - i \cdot (1.139)$

$$A = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \end{pmatrix}$$

Eigenvectors for the matrix A:

- $\boldsymbol{v} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$, eigenvalue $\lambda_1 = 0$

Q.-4 A)

$$M = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & -1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & -1 \end{bmatrix}$$

(B)

$$D = \begin{bmatrix} 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

$L = D - A$

$$L = \begin{bmatrix} 3 & -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 3 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 2 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ -1 & -1 & 0 & 4 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 3 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 4 & 0 & -1 & -1 & -1 \\ 0 & 0 & 0 & 0 & -1 & 0 & 2 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 & 0 & -1 & 0 & 3 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & -1 & 3 & -1 \\ 0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & -1 & 3 \end{bmatrix}$$