Assignment - 2

1. Simple bipartite graph with $n = 200$ Obtain the bounds on the possible values of m for such a graph.

Let G be a simple bipartite graph with n vertices. It has two partitions X and Y.

We can assume partitions X and Y have vertices m and $n-m$ respectively.

~~Graph~~ Graph has maximum number of edges when it is complete graph.

$\therefore$ Number of Edges,

$$E = m(n-m) \quad \text{(i)}$$

Suppose,

$$m = x$$

$$\therefore E = x(n-x)$$

For a maximum number of edge,

$$E'(x) = 0$$

$$\therefore n - x + x(-1) = 0$$

$$\therefore n - x - x = 0$$

$$\therefore n - 2x = 0$$

$$\therefore n = 2x$$

$$\therefore \frac{n}{2} = x.$$

$\therefore$ We'll get max number of edge max E at $x = n/2$

$$\therefore \max E = \frac{n}{2}\left(n - n/2\right)$$

$$= \frac{n}{2}\left(\frac{n}{2}\right)$$

$$= \frac{n^2}{4}$$

$\therefore$ max number of Edge E is $\frac{n^2}{4}$

$$\therefore m \leq \frac{n^2}{4}$$

and $n = 200$

$\therefore \quad m \le \dfrac{(200)^2}{4} = \dfrac{40000}{4} = 10000$

$\therefore \quad m \le 10000$ ——— (ii)

As per (i)

$$E = m(n-m)$$

$$\& \quad E \le \frac{n^2}{4}$$

$$\therefore m(n-m) \le \frac{n^2}{4}$$

$n = 200$

$$m(200-m) \le \frac{(200)^2}{4}$$

$$200m - m^2 \le 10000$$

$$m^2 - 200m + 10000 \ge 0$$

$$\therefore (m-100)^2 \ge 0$$

$\therefore$ m being positive value

$$m - 100 \ge 0$$

$\therefore m \ge 100.$ ——— (iii)

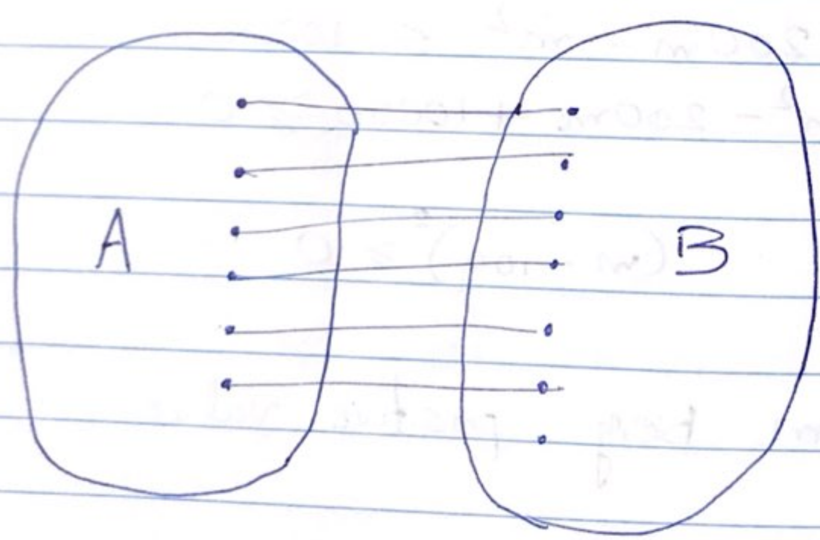
from (ii) & (iii)
we can say,
$$100 \leq m \leq 10000$$

$\therefore$ The lower bound of m is 100 and the upper bound of m is 10000.

$\therefore$ The bounds on the possible values of m are 100 & 10000.

2. p, q are positive integers.

$\therefore$ $p, q > 0$ & $p < q$



$|A| = p$ $\qquad$ $|B| = q$

As $p < q$,

Any largest path in $K_{p,q}$ will use all the vertices of A.

Also, as graph is bipartite, the vertices of the ~~graph~~ path with alternate between A and B.

Hence, Length of the max path

$$= p + p - 1$$

$$= 2p - 1$$

and the smallest path is an edge of length 1.

$\therefore$ The Tight Lower and upper bounds are

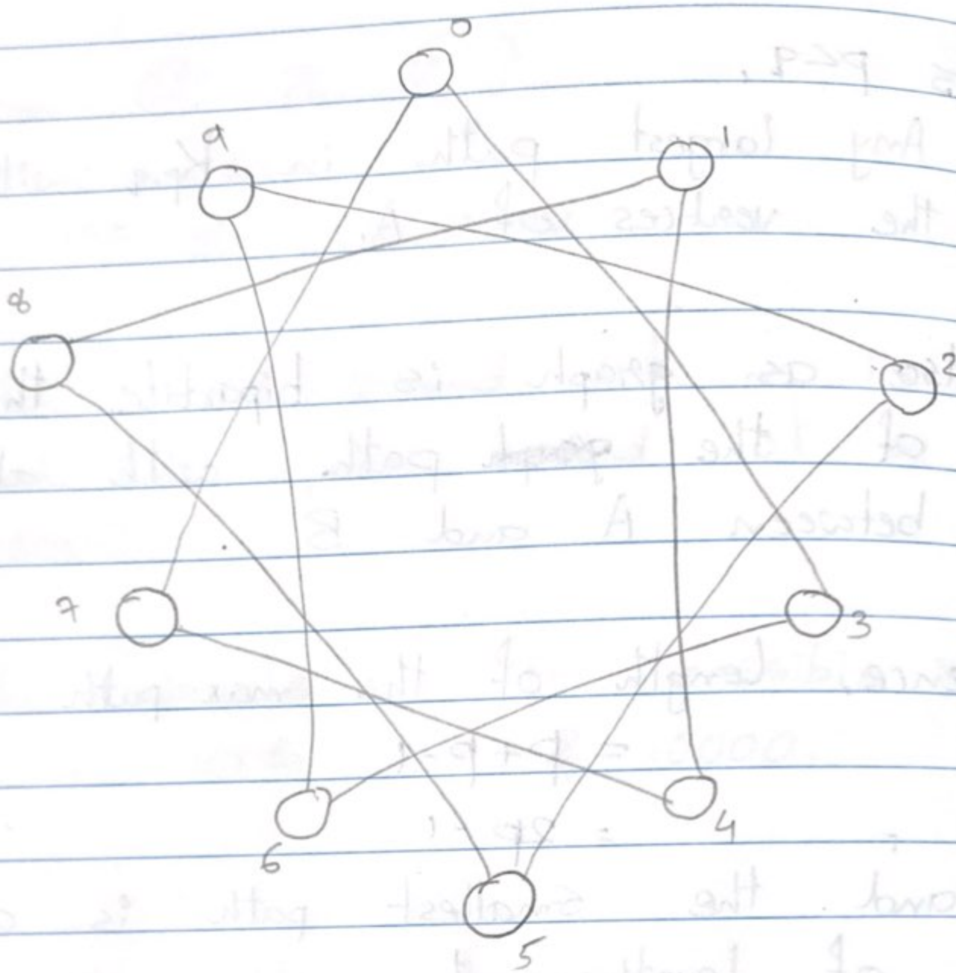
$$1 \leq l(P) \leq 2p - 1$$

(Or) $0 < l(P) < 2p$,

where $l(P)$ is length of a path P.

Tight lower bound on the length of path in $K_{p,q}$ is 1 & Upper bound is $2p - 1$.

3



[Sketch of G_3]

$Z_{10} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$V = Z_{10}$.

i and j are joined by an edge if and only if i and j differ by k mod 10.

Cosider $k = 0$.

$\therefore$ Each element of the set Z_{10} will be connected to itself only &

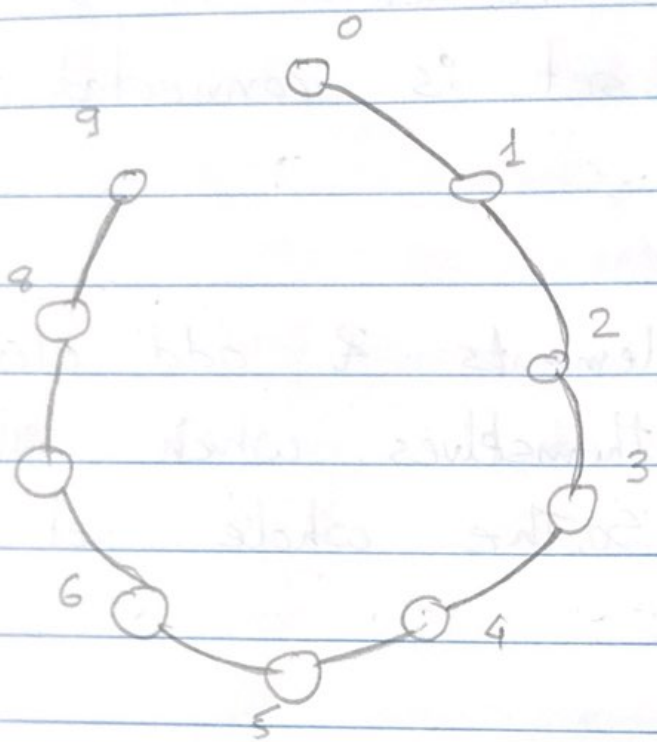
the whole set is disconnected.

Consider $k = 1$.
Then each element of the set Z_{10} will be connected to the next and previous element of the ordered set
$Z_{10} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and the whole set is connected.

Consider $k = 2$
Then even elements $\{0, 2, 4, 6, 8\}$ and odd elements $\{1, 3, 5, 7, 9\}$ of the set Z_{10} will be connected to themsleves and that will make two partition of set. so whole set is disconnected.

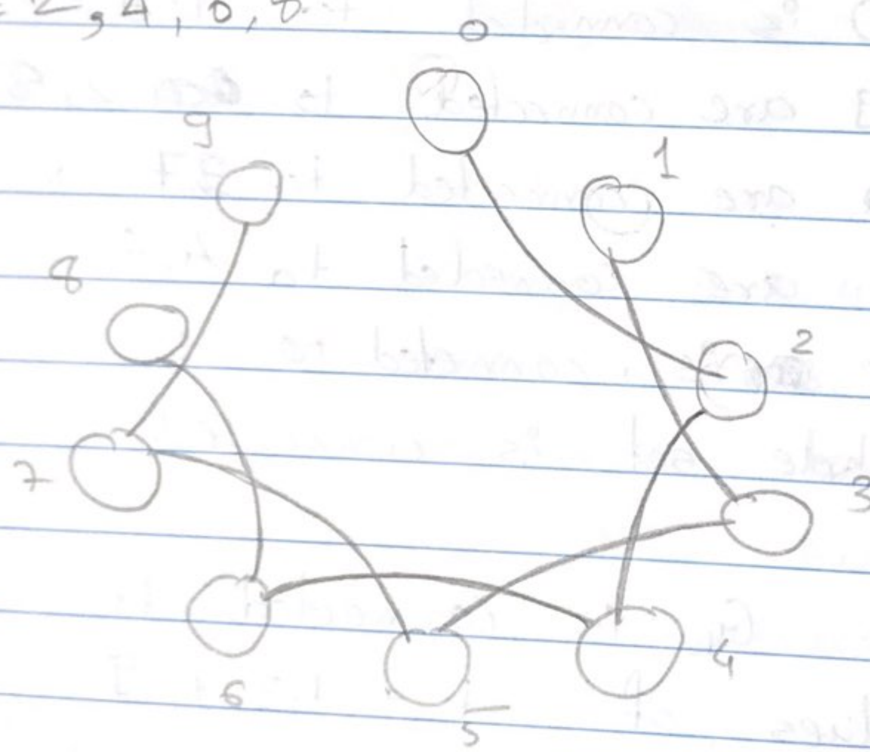
Consider $k = 3$
Then 0 is connected to $3, 7$
$3, 7$ are connected to $6, 4$
$6, 4$ are connected to $9, 1$
$9, 1$ are connected to $2, 8$
$2, 8$ are connected to 5.
$\therefore$ The whole set is connected.

k = 1



disconnected

k = 2, 4, 6, 8



disconnected

Consider $k=4$
Then even elements $\{0,2,4,6,8\}$ and odd elements of set Z_{10} will be connected to themselves and this will make two partitions of the set. So, the whole set is disconnected.

Consider $k=5$
Then $\{0,5\}$ are connected to themselves only and not connected with other elements. So, the whole set is disconnected

Consider $k=6$
Then even elements $\{0,2,4,6,8\}$ and odd elements ~~will b~~ $\{1,3,5,7,9\}$ of Z_{10} set will be connected to themseleves which will make two partitions. So, The whole set is disconnected.

Consider $k=7$
then 0 is connected to 3, 7
3, 7 are connected to 6, 4
6, 4 are connected to 9, 1
9, 1 are connected to 2, 8

2, 8 are connected to 5.
$\therefore$ The whole set is connected.

Consider $k = 8$

Then, even elements & odd elements are connected to themselves, which will form two partitions. So, The whole set is disconnected.

Consider $k = 9$

Then 0 is connected to 1, 9

1, 9 are connected to 2, 8

2, 8 are connected to 3, 7

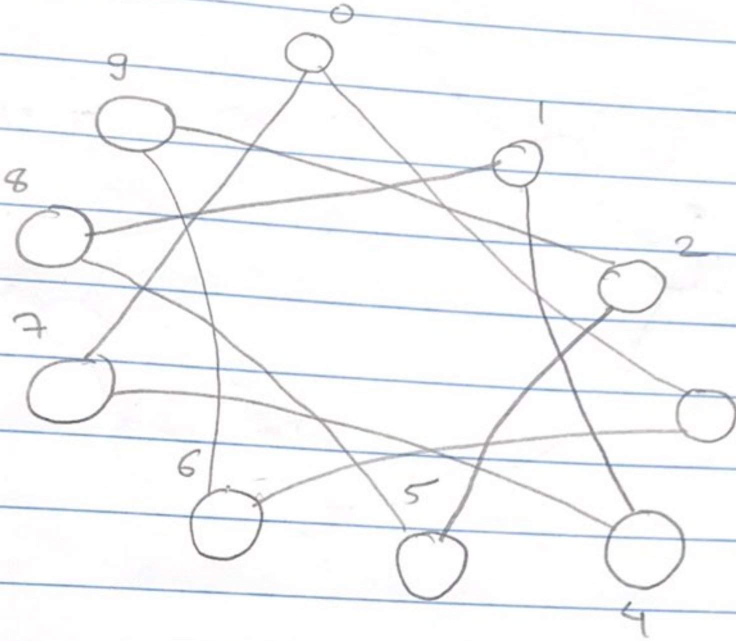
3, 7 are connected to 4, 6

4, 6 is connected to 5.

$\therefore$ The whole set is connected.

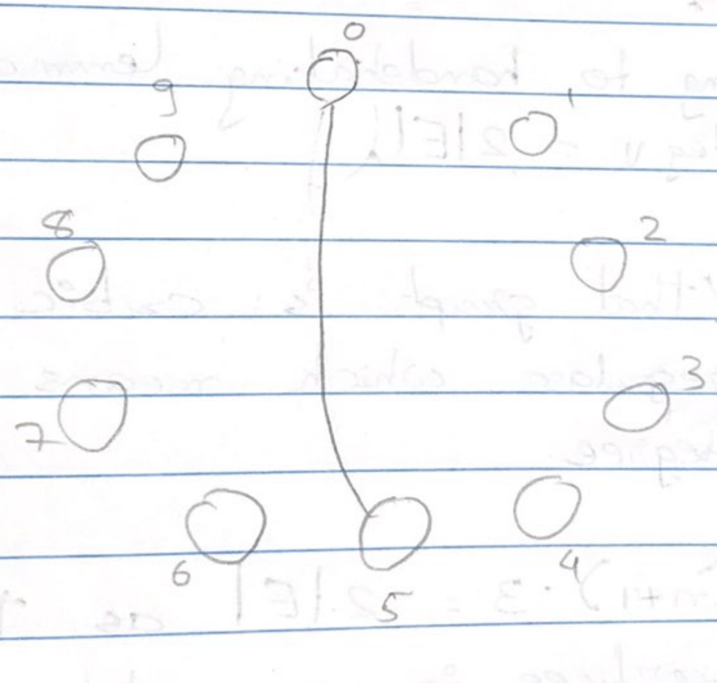
→ The set G_k is connected for all the values of $k = 1, 3, 7, 9$

K=3, 7



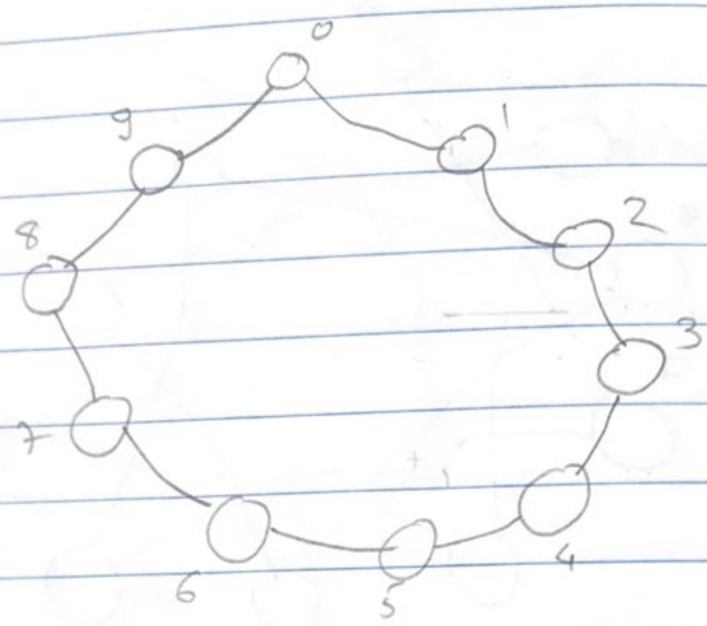
connected.

K=5



disconnected

$K = 9$



4 (A)

According to handshaking lemma

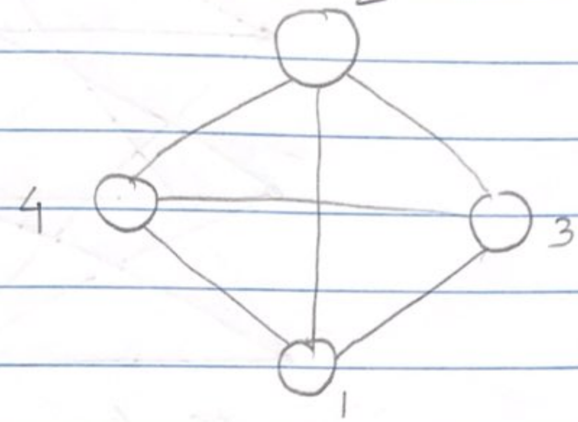
$\sum_{v \in V} \deg v = 2|E|$,

given that graph is cubic & it is 3-regular. which means it has 3 degree.

$\Rightarrow (2n+1) \cdot 3 = 2|E|$ as the number of vertices is an odd numbers.

$(2n+1) \cdot 3$ is not divisible by 2. so both sides are not equal. Therefore, no such graph exists.

(B)

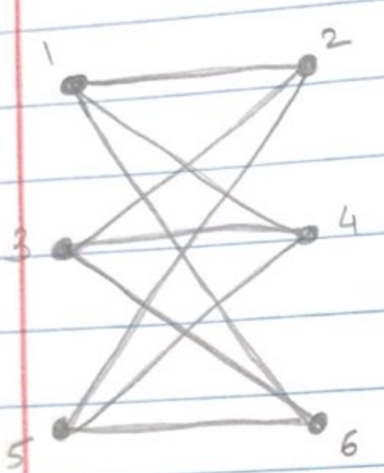


above shown graph is simple Cubic graph with 4 vertices.
$\therefore$ Simple cubic graph with 4 vertices exists.

(C)

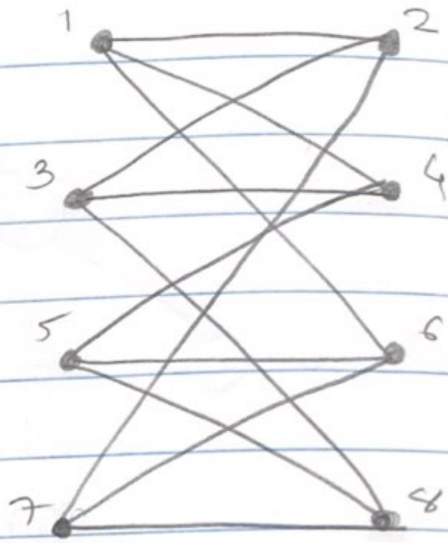
For any integer $n \geq 3$.
Construct simple cubic graph with $2n$ vertices.

For $n = 3 \Rightarrow 6$ vertices
$n = 4 \Rightarrow 8$ vertices
$n = 5 \Rightarrow 10$ vertices



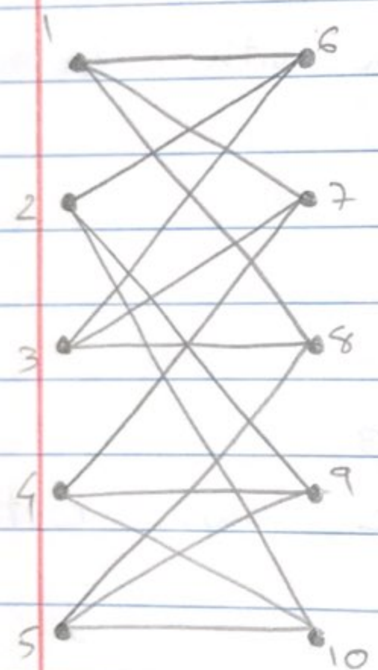
$n = 3$

6 vertices



$n = 4$

8 vertices.



$n = 5$

10 vertices

For $n \geq 3$ to construct simple cubic graph with $2n$ vertices, we divide the vertices in two parts of n vertices each and then draw edges between them such that $\deg(v) = 3$. for each vertex and complete the graph.