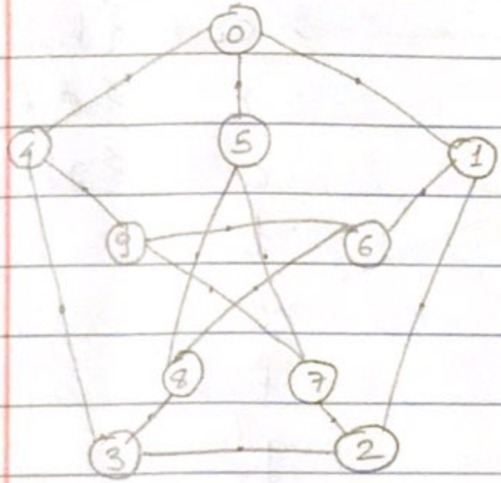
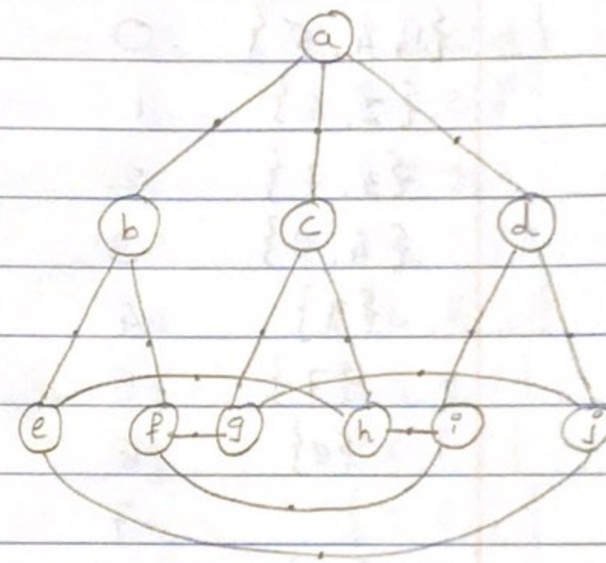
Assignment - 3

Q-1.





no. of vertices = 10
no. of edges = 15

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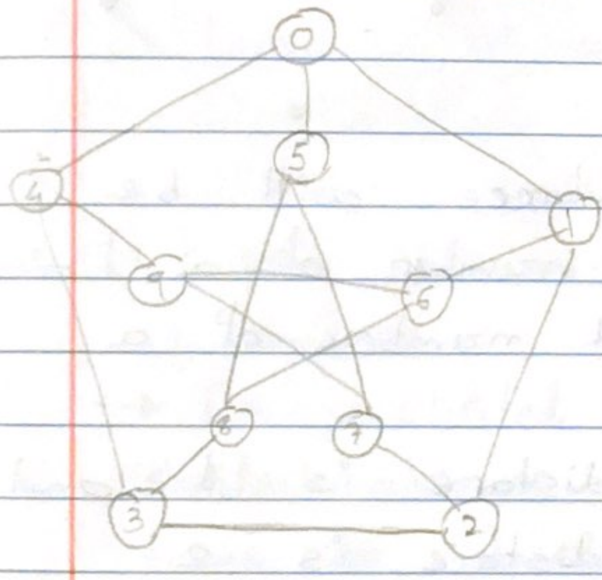
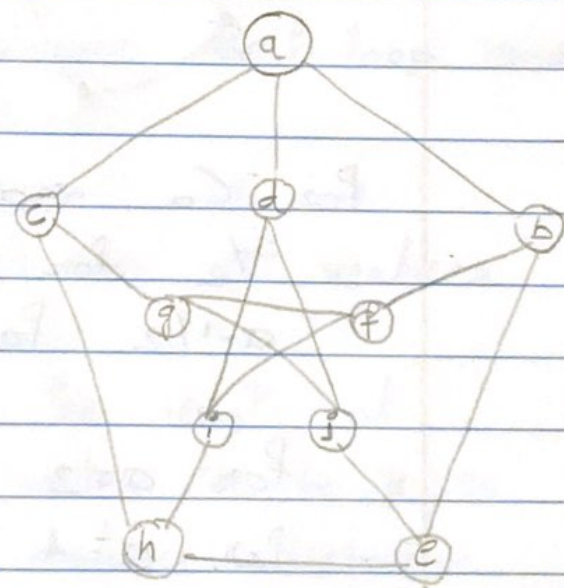
* degree of vertices

d[0] = 3	d[7] = 3	d[a] = 3	d[h] = 3
d[1] = 3	d[8] = 3	d[b] = 3	d[i] = 3
d[2] = 3	d[9] = 3	d[c] = 3	d[j] = 3
d[3] = 3		d[d] = 3	
d[4] = 3		d[e] = 3	
d[5] = 3		d[f] = 3	
d[6] = 3		d[g] = 3	

adjacent				adjacent
{1, 4, 5}	0 → a		a	{b, c, d}
{0, 2, 6}	1 → b		b	{a, e, f}
{1, 3, 7}	2 → e		c	{a, g, h}
{2, 4, 8}	3 → h		d	{a, i, j}
{0, 3, 9}	4 → c		e	{b, h, j}
{0, 7, 8}	5 → d		f	{b, g, i}
{1, 8, 9}	6 → f		g	{c, f, j}
{2, 5, 9}	7 → j		h	{c, e, j}
{3, 5, 6}	8 → i		i	{d, f, h}
{4, 6, 7}	9 → g		j	{d, e, g}





$0 \rightarrow a$

$1 \rightarrow b$

$2 \rightarrow e$

$3 \rightarrow h$

$4 \rightarrow c$

$5 \rightarrow d$

$6 \rightarrow f$

$7 \rightarrow j$

$8 \rightarrow i$

$9 \rightarrow g$

$\therefore$ These two graphs are isomorphic

Q-2 a & b be integers at least 2

(a) P_a

$P_2 \rightarrow$

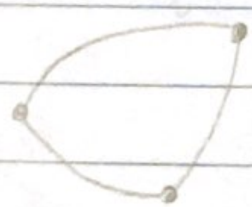
P_3

Max distance between vertices for P_a is $a-1$

$\therefore$ for $a=2$ max distance is 1 and for $a=3$ max distance is 2.

(b) C_a

C_3



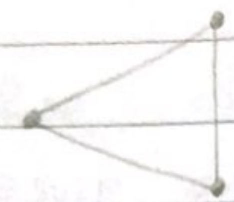
C_4

For C_a max distance will be $a/2$ for even number of a and $a-1/2$ for odd numbers of a.

$\therefore$ For $a=3$ max distace is 1 and for $a=4$ max distace is 2

(c) K_a K_4

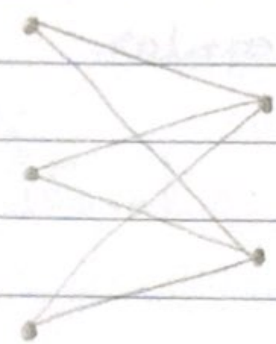
K_3

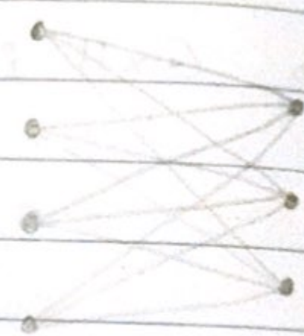


For K_a all vertices are adjacent to each other so max distance is 1.

(d) $K_{a,b}$ $K_{4,3}$

$K_{3,2}$

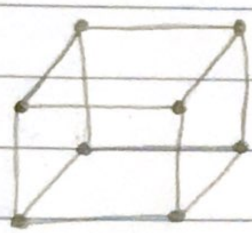


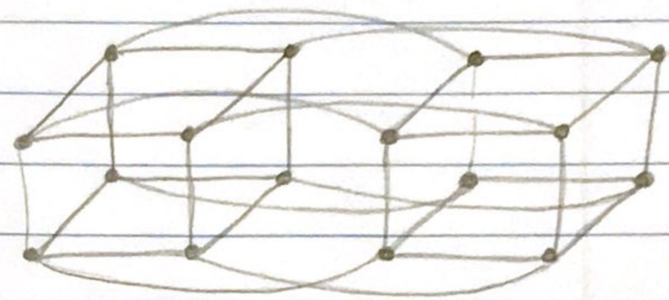


For $K_{a,b}$ max distace is 2 because it is bipatite and all possible edges exists.

(e) Q_a

Q_3 Q_4





Q_n max distance is when all n entries of two vertices differ and hence max values of distance is n.

ex distance between

00... 0 to 11.....1

for Q_3 max distance is 3

Q_4 max distance is 4.

Q-3 [A.] why $0 \leq d(u, v)$ for every vertices u and v.

→ If there is a path between vertices u and v, then we have some value of $d(u, v) > 0$. But if there is no path between vertices u and v, then $d(u, v) = \infty$.

In both of the cases $0 \leq d(u, v)$.

- [B] Under what condition u and v would the equation $d(u, v) = 0$ be true?

→ If $u = v$,

then $d(u, v) = 0$

because it will become self loop and for self loop $d(u, v) = 0$.

- [C] why $d(u, v) = d(v, u)$ for all vertices u and v

→ For undirected graph, the length of shortest distance from vertex u to v is same as ~~a~~ shortest distance between vertex v to vertex u.

$\therefore d(u, v) = d(v, u)$

• [D] $d(u,v) < \infty$ and $d(v,w) < \infty$.
why must $d(u,w) < \infty$, hold? shortest u, v.
path is P and shortest v, w path is Q
What can you do with P & Q?
→ If $d(u,v) < \infty$ and $d(v,w) < \infty$,
then,
the shortest path between vertices u and
v and shortest path between v and w
∴ we can move from vertex u to w
via first path P and then Q.
ex u - v - w
P Q

• [E] $d(u,w) = \infty$, what can you say
about $d(u,v)$ or $d(v,w)$?
→ If $d(u,w) = \infty$, then there is
no path between vertices u and w.
so, either $d(u,v) = \infty$ or $d(v,w) = \infty$.

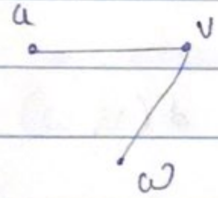
• [F] If $d(u,v) < \infty$, and $d(v,w) < \infty$, then
$d(u,w) \le d(u,v) + d(v,w)$ ← show.
→ If $d(u,v) < \infty$ and $d(v,w) < \infty$
then must be a shortest path
between vertices u and v and also

a shortst path between v and ω.
$\therefore$ there is a also a shortest path between vertices u and ω either directly or via v.

so, if path between u and ω is directly then,

$$d(u,\omega) < d(u,v) + d(v,\omega)$$

and, if path between u and ω is via v, then,



$$d(u,\omega) = d(u,v) + d(v,\omega)$$

Combining the both cases we get,

$$d(u,\omega) \le d(u,v) + d(v,\omega)$$

- (G) If $d(u,v) < \infty$ and $d(v,\omega) = \infty$, then $d(u,\omega) = ?$

$\rightarrow$ If ~~d(v,ω~~ $d(u,v) < \infty$ that means there is a shortest path between vertices u and v and $d(v,\omega) = \infty$, it means there is no shortest path between vertices v and ω.

But it is possible that there can be a shortest path between u and ω and in that case it doesn't matter where there is a path between v and ω.

But if there is no other path between vertices u and ω

Then, $d(u,v) < \infty$ and $d(v,\omega) = \infty$

that means $d(u,\omega) = \infty$.

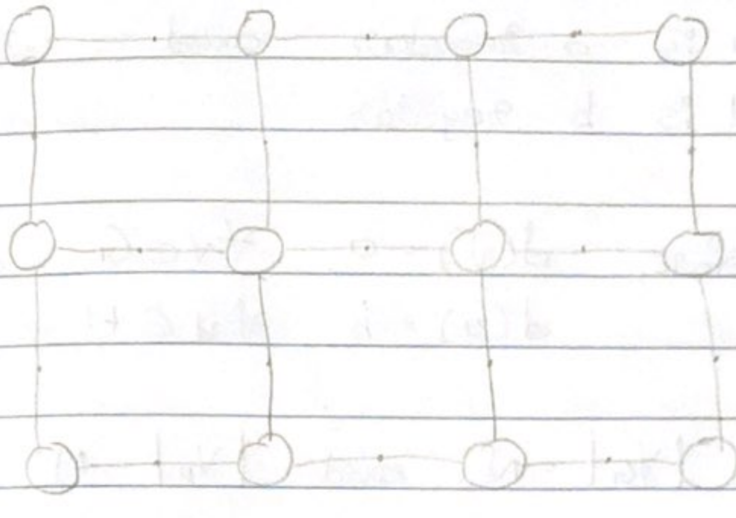
- (H) If $d(u,v) = \infty$ and $d(v,\omega) = \infty$, then $d(u,\omega) = ?$

→ If $d(u,v) = \infty$ that means there is no shortest path between vertices u and v. and $d(v,\omega) = \infty$ that means there is no shortest path between vertices v and ω.

But it does not indicate that there is no shortest path between vertices u and ω. There may be a path between vertices u and ω.

$\therefore$ $d(u,\omega) < \infty$.

Q-4



$P_a \times P_b \implies P_4 \times P_3$

A) It is clear that

$|V_{P_4}| = 4$

$|V_{P_3}| = 3$

Also, it is clear that

$\cancel{|V_{P_4} \times V_{P_3}|} =$

$|V_{P_4 \times P_3}| = 12$

So, in general, we can say that

$|V_{G \times H}| = |V_G| \cdot |V_H|$

B)

$|E_{G \times H}| = |V_G| \cdot |E_H| + |V_H| \cdot |E_G|$

c) G is a-regular and
H is b-regular

Therefore, $d(v) = a, \ \forall v \in G$ and
$d(u) = b, \ \forall u \in H.$

let, $|V_G| = N$ and $|V_H| = M$

$\therefore |E_G| = \frac{aN}{2}$ and $|E_H| = \frac{bM}{2}$

now, we saw that
$$|E_{G \times H}| = |V_G| \cdot |E_H| + |V_H| \cdot |E_G|$$

So,
$$|E_{G \times H}| = \frac{bNM}{2} + \frac{aNM}{2}$$

$$= (a+b)\frac{NM}{2}$$

$$= \frac{a+b}{2} |V_G| \cdot |V_H| \qquad \text{—— (1)}$$

using Euler's theorem, the sum of the degree of the vertices of a graph is equal to twice the number of its edges.

We saw that

$$|V_{G \times H}| = |V_G||V_H| = MN.$$

Therefore, Euler's theorem gives, for any $v \in G \times H$, we have,

$$d(v)\,|V_G|\,|V_H| = d(v)\,MN = 2 \times |E_{G \times H}| = 2 \times \frac{(a+b)}{2}\,|V_G| \cdot |V_H|$$

$$\Rightarrow d(v) = \underline{(a+b)}, \quad \forall v \in V(G \times H)$$

So, $G \times H$ is $(a+b)$-regular.