

1. **Summations, 20 points** 5 parts, 4 points per part. 2 points for correct answer, 2 points for work/explanation.

(a)

$$\sum_{i=10}^n (7)^i = \sum_{i=0}^n (7)^i - \sum_{i=0}^9 (7)^i = \frac{7^{n+1} - 1}{7 - 1} - \frac{7^{9+1} - 1}{7 - 1} = \frac{7^{n+1} - 7^{10}}{6}$$

(b)

$$\sum_{i=0}^{\infty} \frac{6}{17^i} = 6 \sum_{i=1}^{\infty} \left(\frac{1}{17}\right)^i = \frac{6}{1 - \frac{1}{17}}$$

(c)

$$\sum_{i=1}^n (i^2 + ni) = \sum_{i=1}^n i^2 + \sum_{i=1}^n ni = \frac{n(n+1)(2n+1)}{6} + \frac{n^2(n+1)}{2} = \frac{n(n+1)(5n+1)}{6}$$

(d) The harmonic series demonstrates logarithmic growth.

$$\ln(315) - \ln(5) + \text{constant} = \ln(63) + \text{constant}$$

(e)

$$\sum_{i=0}^{\infty} \frac{i-1}{2^i} = -1 + \frac{1}{2} \sum_{i=0}^{\infty} i \left(\frac{1}{2}\right)^i = -1 + \frac{1}{2} \cdot \frac{\frac{1}{2}}{\frac{1}{4}} = -1 + 1 = 0$$

2. **Exponents and logs, 20 points** 5 parts, 4 points per part. 2 points for correct answer, 2 points for work/explanation.

(a) Using formula for sum of consecutive integers:

$$x^{11} \cdot x^{12} \cdots x^n = x^{\sum_{i=11}^n i} = x^{\frac{n \cdot (n+1)}{2} - \frac{10 \cdot 11}{2}} = x^{\frac{n \cdot (n+1)}{2} - 55}$$

(b) $\log_{99}(33 \cdot 33 \cdot 33 \cdot 33 \cdot 33) = 5 \log_{99}(33)$ or $5 \frac{\ln 99}{\ln 33}$

(c) $44^{\log_{44} 330} = 330$

(d) $\log_x((3x)^x) = x \log_x(3x) = x \log_x 3 + x \log_x x = x \log_x 3 + x$

(e) $\sum_{i=1}^{5^N} \log_{23} i = \log_{23} \left(\prod_{i=1}^{5^N} i \right) = \log_{23}(5^N!)$ or $\frac{\ln(5^N!)}{\ln 23}$

3. **Combinatorics, 10 points** 2 parts, 5 points per part. 3 points for correct answer, 2 points for work/explanation.

(a) There are 7 digits greater than or equal to 3. Now for each of the 6 decimal positions we have 7 digits to choose from. Therefore,

$$7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 = 7^6 = 117649$$

(b) There are 52 different numbers between 17 and 68 (both inclusive). Therefore,

$$\binom{52}{9} = 3679075400$$