(all limits are as $n \to \infty$)

1. **Asymptotic comparison**: 25 points, 3 points for each row that is *exactly* correct + 2 points for justification. Note: for partial credit, subtract 1 point for missing $\theta \Rightarrow (O \wedge \Omega)$ or $o \Rightarrow O$ or $\omega \Rightarrow \Omega$.

A	B	O	o	Θ	ω	Ω
$n^3 - 100n - 10,000$	$100n^2 + 5n + 3$	No	No	No	Yes	Yes
2^{2n+1}	2^n	No	No	No	Yes	Yes
2489^{200}	$\log_{2489} n$	Yes	Yes	No	No	No
n	$\sum_{i=1}^{n} \frac{50}{i}$	No	No	No	Yes	Yes
$200n^9$	e^n	Yes	Yes	No	No	No

   (a) $\lim \frac{100n^2+5n+3}{n^3-100n-10,000} = \frac{-3}{10,000} = 0$ so $A \in \omega(B)$ and therefore also $A \in \Omega(B)$.

   (b) $\lim \frac{2^n}{2^{2n+1}} = \lim \frac{2^n \times 2^n \times 2}{2^n} = \infty$ so $A \in \omega(B)$ and therefore also $A \in \Omega(B)$.

   (c) 2489^{200} is a constant. A constant is asymptotically smaller than anything that is not a constant.

   (d) $\lim \frac{\sum_{i=1}^{n} \frac{50}{i}}{n} = 0$ so $A \in \omega(B)$ and therefore also $A \in \Omega(B)$.

   (e) $\lim \frac{200n^9}{e^n} = \lim \frac{200 \cdot 9!}{n^9 e^n} = 0$ so $A \in o(B)$ and therefore also $A \in O(B)$.

2. **Asymptotics**: 25 points, 7 points times normalized Kendall tau distance to the correct order, 18 points for explanations (2 points for each bullet). See attached python script for Kendall tau distance computation.

$$n^{\frac{1}{\log n}} < 1000 < \log\log n < \sqrt{\log n} < \log n < (\sqrt{2})^{\log n} < \sqrt{n} < 5n+3 < 100^{\log n} < (\frac{3}{2})^n$$

- $n^{\frac{1}{\log n}} = e = 2.7182$ which is a constant
- 1000 is a constant.
- $\lim \frac{n^{\frac{1}{\log n}}}{\log\log n} = \lim \frac{e}{\log\log n} = 0$
- $\lim \frac{\log\log n}{\sqrt{\log n}} = 0$
- $\lim \frac{\sqrt{\log n}}{\log n} = \lim \frac{1}{\sqrt{\log n}} = 0$
- $\lim \frac{\log n}{\sqrt{2}^{\log n}} = 0$
- $\lim \frac{\sqrt{2}^{\log n}}{\sqrt{n}} = 0$
- $\lim \frac{\sqrt{n}}{5n+3} = 0$
- $\lim \frac{5n+3}{100^{\log n}} = 0$
- $\lim \frac{100^{\log n}}{(\frac{3}{2})^n} = 0$