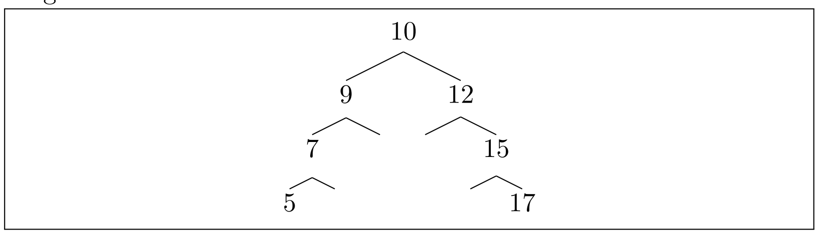
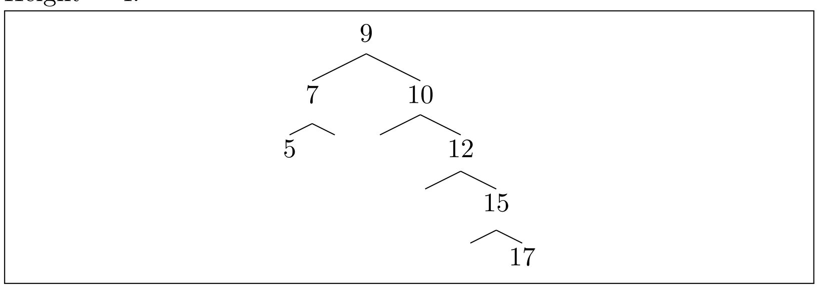
1. 6 points.

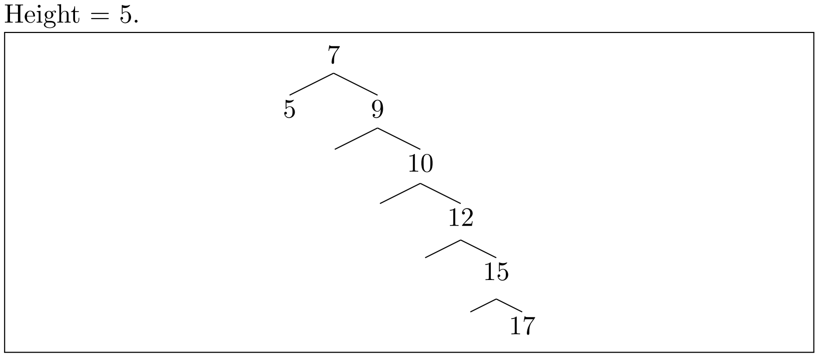
   (a) Height = 3.



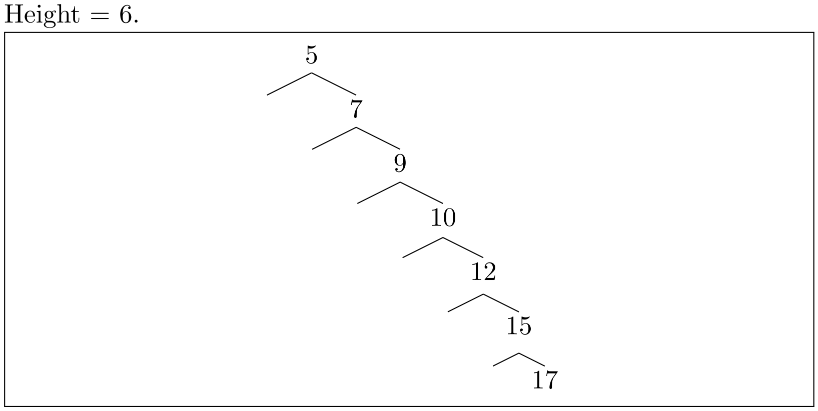
   (b) Height = 4.



   (c) Height = 5.

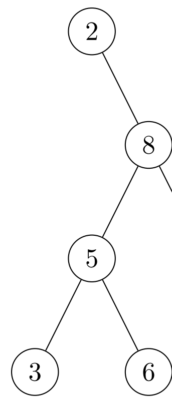


   (d) Height = 6.



2. 24 points.

   (a) 8 points.
   No. This claim is not correct. Suppose A = all keys on the left of the search path, B = all keys on the search path, and C = all keys on the right of the search path.
   In this example, we are searching for 6 and the sets A, B, and C are given by:
   $$A = \{3\}, B = \{2, 8, 5, 6\}, C = \{9\} \tag{1}$$



   As we can see, $3 \in A$, $2 \in B$ and $3 \geq 2$. Therefore, the property doesn't hold.

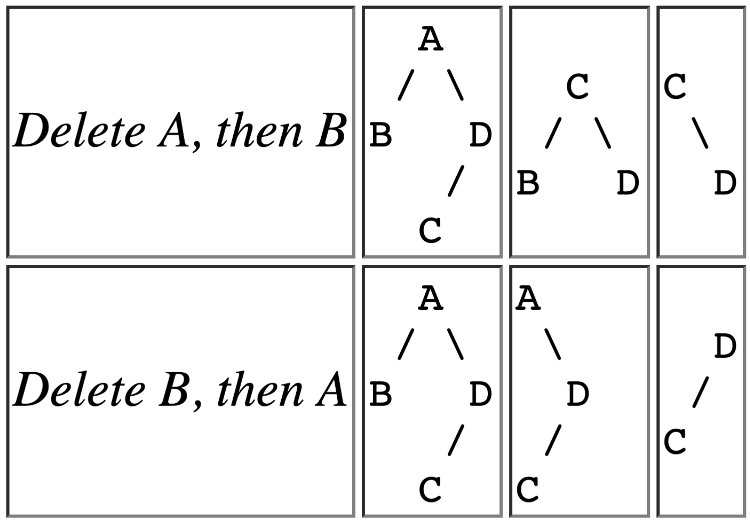
   (b) 8 points.
   Suppose there is a node x that has two children and suppose its predecessor is p and its successor is s.

   i. We will show by contradiction that the successor of x has no left child.
   Suppose s has a left child. Then the key of s is $>$ than $left[s]$. The key of s is also $>$ than the key of x, and since s has a left child the key of $left[s]$ is $>$ than that of x.
   Thus, $key[s] \geq key[left[s]] \geq key[x]$,
   which is a contradiction, since s is the successor of x. Hence the successor of x has no left child.

   ii. We will show by contradiction that the predecessor of x has no right child.
   Suppose p has a right child. Then the key of p is $<$ than that of $right[p]$. The key of p is also $<$ than the key of x, and since p has a right child the key of $right[p]$ is $<$ than x.
   Thus $key[p] \leq key[right[p]] \leq key[x]$,
   which is a contradiction, since p is the predecessor of x. Hence the predecessor of x has no right child.

   (c) 8 points.
   No, if we delete node x and then node y from a binary search tree,

we will **NOT** end up with the same resulting tree that we will get if we first delete node y and then node x. Deletion is not commutative. We will show this by an example:



3. 20 points.

(a) 5 points.

0	1	2	3	4	5	6	7	8	9	10
	89			26		50		19		
				70						
				59						

(b) 5 points.

0	1	2	3	4	5	6	7	8	9	10
	89			59	70	50	26	19		

(c) 5 points.

0	1	2	3	4	5	6	7	8	9	10
	89			59	26	50		19		70

(d) 5 points.

0	1	2	3	4	5	6	7	8	9	10
	89			59	26	50		19	70	