Problem 2. A mouse moves along a tiled corridor with $2m$ tiles, where $m > 1$. From each tile $i \neq 1, 2m$, it moves to either tile $i - 1$ or $i + 1$ with equal probability. From tile 1 or tile $2m$, it moves to tile 2 or $2m - 1$, respectively, with probability 1. Each time the mouse moves to a tile $i \leq m$ or $i > m$, an electronic device outputs a signal L or R, respectively. Can the generated sequence of signals L and R be described as a Markov chain with states L and R?

Problem 3. Consider the Markov chain in Example 7.2, for the case where $m = 4$, as in Fig. 7.2, and assume that the process starts at any of the four states, with equal probability. Let $Y_n = 1$ whenever the Markov chain is at state 1 or 2, and $Y_n = 2$ whenever the Markov chain is at state 3 or 4. Is the process Y_n a Markov chain?

Please turn in the solution to Problem 3 (you do not need to do problem 2).
Do not simply answer yes or no; rather, prove the Markov property holds if the process is a Markov chain, or give an example to show it doesn't if it is not a Markov chain.

- Let P be the probability transition matrix of a Markov chain on n states. Show that:

  (i) 1 is an eigenvalue of P.

  (ii) suppose $\sum_{i=1}^{n} p_{ij} = 1$ for every state j; in other words, every column of P adds up to one

  (for future information, a nonnegative matrix whose row and columns add up to one is called *doubly stochastic*).

  Show that if this Markov chain is initialized at the distribution where it is equally likely to be at any state, it stays at that distribution for all time.

  In other words, if the distribution at time zero is $\mathbf{p} = (1/n, \ldots, 1/n)$, then the distribution of the chain at any time k is also $(1/n, \ldots, 1/n)$.

  (iii) Show that any power of a doubly stochastic matrix is doubly stochastic.

- Suppose X_t is a Markov chain on the state-space $\{1, \ldots, n\}$ which behaves as follows: if $X_t = i$ then X_{t+1} is a uniformly random j such that $|j - i| = 1$. For example, if $X_1 = 3$, then X_2 is either 2 or 4, both with probability 1/2. Another example: if $X_3 = n$, then $X_4 = n - 1$ with probability one.
- This Markov chain is initialized at state n. We stop the process when it reaches state 1.
- Prove: $\lim_{t \to \infty} P($ process is stopped by time $t) = 1$.