TEST 2 FORMULA SHEET EC401 (SPRING 2020)

DT Unit Step (Switch)

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & otherwise \end{cases}$$

DT Unit Impulse (Atom):

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & otherwise \end{cases}$$

Discrete-Time Impulse decomposition of a Signal: $x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$

Even Signal: $x[n] = x[-n]$

Odd Signal: $x[n] = -x[-n]$

Even$\{x[n]\} = \frac{x[n]+x[-n]}{2}$

Odd$\{x[n]\} = \frac{x[n]-x[-n]}{2}$

Basic Signal Operations:

Shift: $y[n] = x[n-n_0]$

Flip: $y[n] = x[-n]$

Compress: $y[n] = x[Mn]$

Expand: $y[n] = \begin{cases} x\left[\frac{n}{M}\right] & \text{if } n > M \\ 0 & \text{otherwise} \end{cases}$

Multi-Signal Operations:

Linear Combination: $y[n] = \alpha_1 x_1[n] + \alpha_2 x_2[n]$

Product: $y[n] = g[n]h[n]$

Bounded Signal: A signal $x[n]$ is bounded if and only if $|x[n]| \leq B$ for some finite (positive) number B.

Linear System:

Suppose S:$x_1[n] \rightarrow y_1[n]$ and S:$x_2[n] \rightarrow y_2[n]$. The system S is linear *if and only if* S:$\alpha_1 x_1[n] + \alpha_2 x_2[n] \rightarrow \alpha_1 y_1[n] + \alpha_2 y_2[n]$ for all possible $x_1[n], x_2[n], \alpha_1$, and α_2.

Time-Invariant System:

Suppose S:$x_1[n] \rightarrow y_1[n]$. The system S is time-invariant if and only if S: $x_1[n-n_0] \rightarrow y[n-n_0]$ for all possible $x_1[n]$ and n_0.

Causal System:

A system S is causal *if and only if* the output at any given time is dependent only upon the input at the same time and/or past times.

Stable System:

A system S is stable *if and only* if bounded inputs always result in bounded outputs.

LTI System: S:$x[n] \rightarrow y[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$ where S:$\delta[n] \rightarrow h[n]$

Sifting property of the impulse:

$$\int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau = x(t)$$

Convolution Integral: $y(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)\,d\tau$

Convolution Sum: $y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$

Properties of Convolution (also true for discrete-time):

$x(t) * h(t) = h(t) * x(t)$ (commutatitive)
$x(t) * \delta(t) = x(t)$ (identity element)
$x(t) * \big(g(t) * h(t)\big) = \big(x(t) * g(t)\big) * h(t)$ (associative)
$x(t) * \big(g(t) + h(t)\big) = x(t) * g(t) + x(t) * h(t)$ (distributive over addition)

Complex Exponentials, Cosines, and Sines:

$e^{j\theta} = \cos(\theta) + j\sin(\theta)$

$\cos(\theta) = (1/2)(e^{j\theta} + e^{-j\theta})$

$\sin(\theta) = (1/2j)(e^{j\theta} - e^{-j\theta})$

The signal $x(t) = e^{j\omega_0 t}$ has fundamental period $T = 2\pi/|\omega_0|$

The signal $x[n] = e^{j2\pi na/b}$ has fundamental period $N = |b|$ provided the integers a and b don't have common factors.

Discrete Time Fourier Transform (DTFT)

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n} \qquad\qquad x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}d\omega$$

Basic DTFT Properties

$x[n-n_0] \Leftrightarrow e^{-j\omega n_0}X(e^{j\omega})$ $\qquad e^{j\omega_0 n}x[n] \Leftrightarrow X(e^{j(\omega-\omega_0)})$ $\qquad x^*[n] \Leftrightarrow X^*(e^{-j\omega})$

$x[-n] \Leftrightarrow X(e^{-j\omega})$ $\qquad x[n]*h[n] \Leftrightarrow X(e^{j\omega})H(e^{j\omega})$

Common DTFT Pairs

$$e^{j\omega_0 n} \Leftrightarrow 2\pi \sum_{k=-\infty}^{\infty} \delta(\omega - \omega_0 - 2\pi k)$$

$$\delta[n - n_0] \Leftrightarrow e^{-j\omega n_0}$$

$$u[n] - u[n-N] \Leftrightarrow \frac{\sin(\omega N / 2)}{\sin(\omega / 2)} e^{-j\omega(N-1)/2}$$