EC401 HW06 Spring 2020

Due Date: Wednesday March 25, 2020

You must submit your homework in pdf form to the EC401 Blackboard Learn site by 12:30pm on the due date. Please be sure to write your name on the first page of the homework you submit. Additionally, if you have collaborated on the homework with other individuals enrolled in EC401 this semester, please identify them as your collaborators on the first page of the submitted homework.

HW06.1

For each of the parts of this problem, perform a *wave decomposition* of the given continuous-time signal $x(t)$ as a *linear combination of complex exponentials* and specify the fundamental period of $x(t)$. *Justify your answers*.

(a) $x(t) = 2$

(b) $x(t) = -2$

(c) $x(t) = (-1)^t$

(d) $x(t) = \cos(0.25\pi t) + \sin(0.5\pi t)$

(e) $x(t) = \cos(0.25\pi t + 0.1\pi) + \sin(0.5\pi t - 0.2\pi)$

(f) $x(t) = \cos(0.25\pi t) \times \sin(0.5\pi t)$

HW06.2

For each of the parts of this problem, perform a *wave decomposition* of the given discrete-time signal $x[n]$ as a *linear combination of complex exponentials* and specify the fundamental period of $x[n]$.

(a) $x[n] = 5$
(b) $x[n] = 1 + (-1)^n$
(c) $x[n] = (-1)^n \cos(0.25\pi n)$
(d) $x[n] = 1 + \cos\left(\frac{\pi n}{5}\right) + \cos\left(\frac{2\pi n}{5}\right) + \cos\left(\frac{3\pi n}{5}\right) + \cos\left(\frac{4\pi n}{5}\right)$
(e) $x[n] = \sin\left(\frac{\pi n}{5}\right) + \sin\left(\frac{2\pi n}{5}\right) + \sin\left(\frac{3\pi n}{5}\right) + \sin\left(\frac{4\pi n}{5}\right)$
(f) $x[n] = \cos\left(\frac{2\pi n}{5}\right) \sin\left(\frac{4\pi n}{5}\right)$

HW06.3

Suppose the signal $x[n] = \cos(0.5\pi n)$ is provided as input to an LTI system with impulse response $h[n] = \delta[n] - \delta[n-2]$.

(a) Sketch $x[n]$

(b) Determine and sketch $y[n]$ by calculating the convolution of $x[n]$ with $h[n]$.

(c) Verify your answer to the previous part by performing *wave decomposition* on $x[n]$ and then determining how the LTI system with impulse response $h[n]$ responds to each of the complex exponentials contained within $x[n]$.

HW06.4

Determine the discrete-time Fourier transform (DTFT), denoted by $H(e^{j\omega})$, of each of the following signals and sketch the magnitude of each DTFT.

(a) $h[n] = \delta[n+2] + \delta[n-2]$

(b) $h[n] = \delta[n] + \delta[n-4]$

(c) $h[n] = \delta[n] + \delta[n-1]$

(d) $h[n] = \delta[n] - \delta[n-1]$

(e) $h[n] = \cos\left(\frac{\pi n}{4} + \frac{\pi}{2}\right)$

(f) $h[n] = \sin\left(\frac{\pi n}{8}\right) + \cos\left(\frac{\pi n}{16}\right)$

HW06.5

Consider an LTI system with impulse response $h[n] = 0.2\{u[n+2] - u[n-3]\}$. Determine and sketch the output of this system for each of the following input signals:

(a) $x[n] = 1$

(b) $x[n] = \sin\left(\frac{2\pi}{5}n\right)$

(c) $x[n] = \cos\left(\frac{2\pi}{5}n\right)$

(d) $x[n] = \sum_{k=-\infty}^{\infty} \delta[n-5k]$

HW06.6

Throughout this problem, let $x[n]$ be a signal whose discrete-time Fourier transform (DTFT) is $X(e^{j\omega})$.

(a) Show that $X(e^{j\omega}) = X(e^{j(\omega-2\pi k)})$ for any integer *k*.

(b) Show that $X(e^{j0}) = \sum_{n=-\infty}^{\infty} x[n] =$ sum of all the lollipops in $x[n]$

(c) Show that $X(e^{j\pi}) = \sum_{n=-\infty}^{\infty} x[2n] - \sum_{n=-\infty}^{\infty} x[2n+1] =$ sum of even indexed lollipops of $x[n]$ minus the sum of the odd indexed lollipops of $x[n]$.

(d) Show that $2\pi x[0] = \int_{-\pi}^{\pi} X(e^{j\omega})\,d\omega =$ area under 1 period of $X(e^{j\omega})$