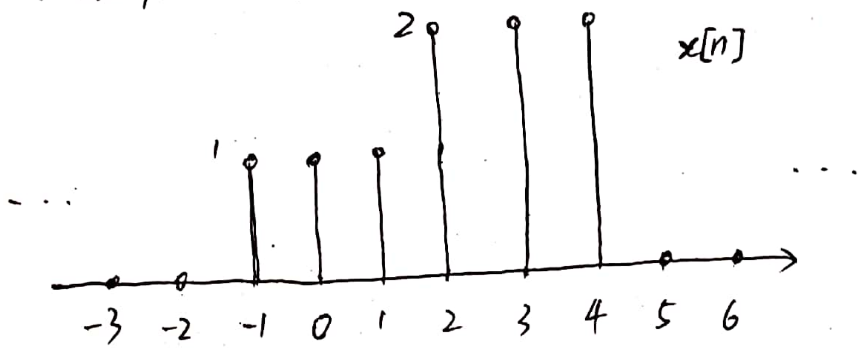
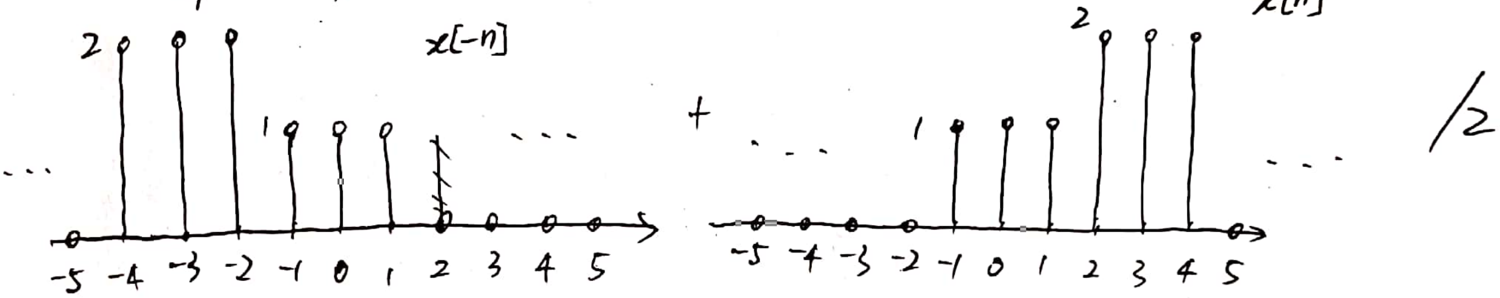
HW3 Solution

1. a) $x[n] = (u[n+1] - u[n-5]) + (u[n-2] - u[n-5])$

a box function from -1 to 4   a box function from 2 to 4.

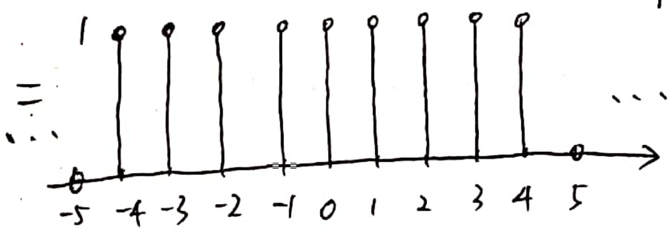


b) $Ep\{x[n]\} = \frac{x[n] + x[-n]}{2}$

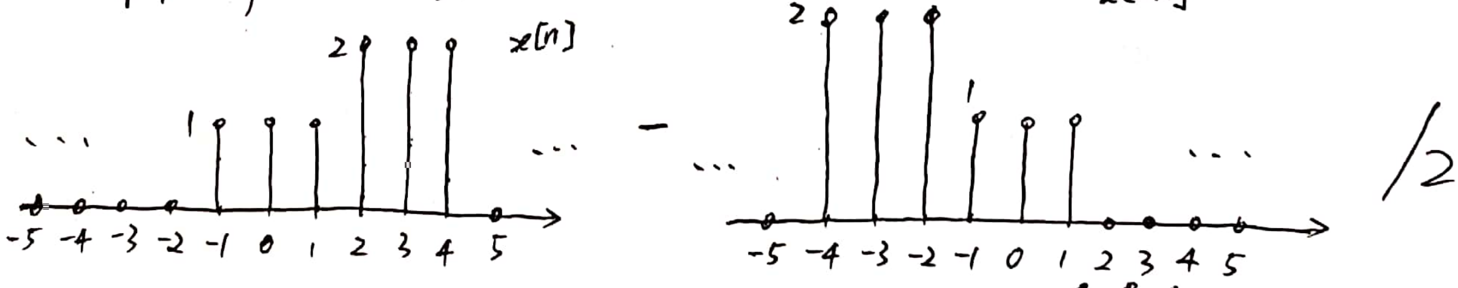


$Ep\{x[n]\}$

=



c) $Op\{x[n]\} = \frac{x[n] - x[-n]}{2}$



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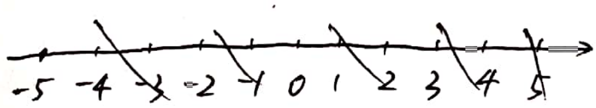
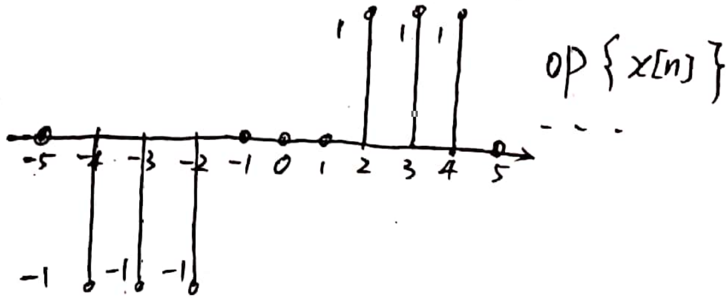
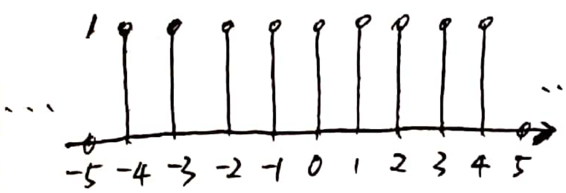




$Op\{x[n]\}$

d)

Ep{x[n]} op{x[n]}



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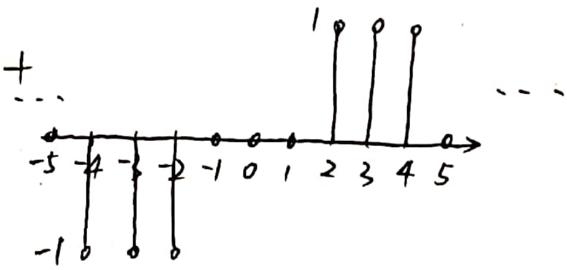

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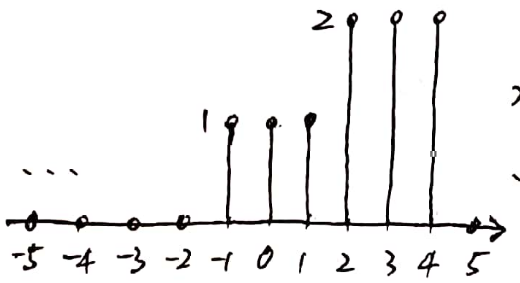


x[n]

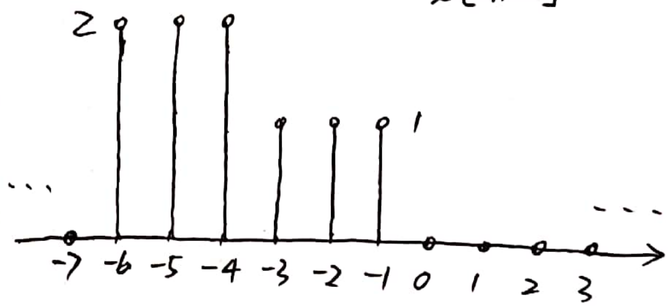
shifted to the right by 2

e) $Ep\{x[n-2]\} = \dfrac{x[n-2] + x[-n-2]}{2}$ → shf. to right by 2 then flipped

x[n-2]

+

x[-n-2]



/2

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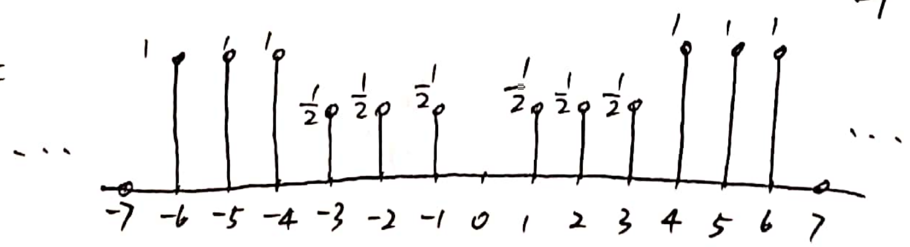
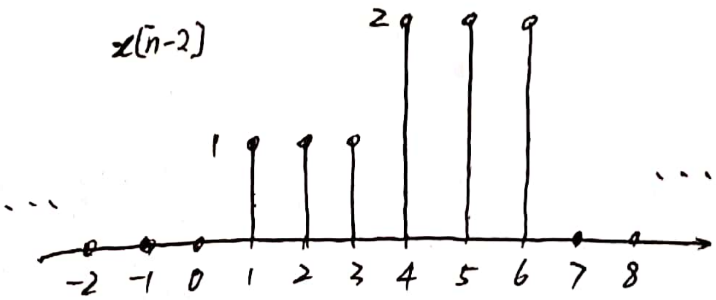
Ep{x[n-2]}



f) $op\{x[n-2]\} = \dfrac{x[n-2] - x[-n-2]}{2}$

x[n-2]



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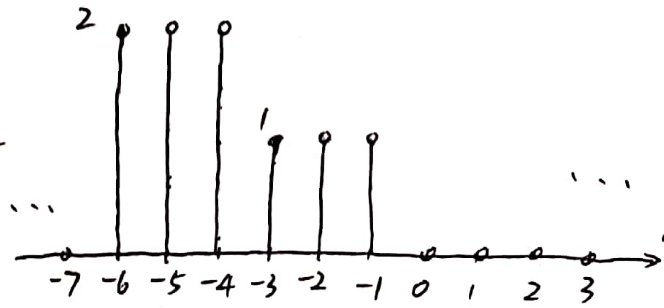

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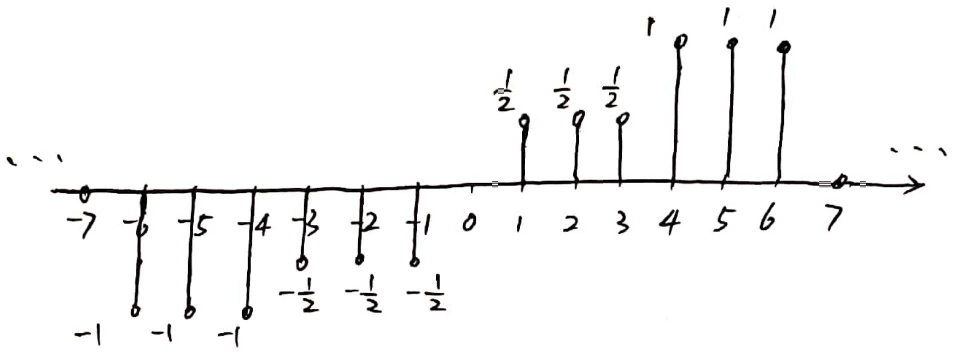
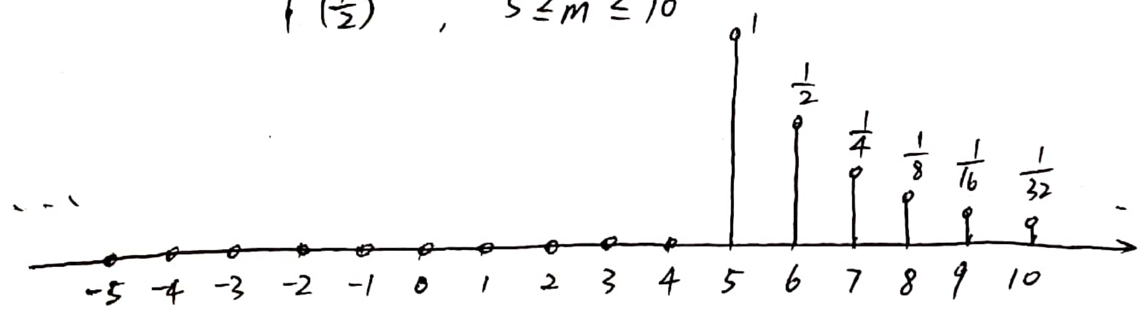


2. a) $x[n] = \sum_{k=5}^{\infty} \left(\frac{1}{2}\right)^{k-5} \delta[n-k]$ is the Impulse decomposition ~~of~~. where $\delta[n-k]$ represents the ~~impl~~ unit impulse that's shifted to the right by k and $\left(\frac{1}{2}\right)^{k-5}$ is the height of the lollipop at $\underline{n=k}$. Since the sum counts from $k=5$, the signal before $n=5$ is all-zero.

Then $x[m] = \begin{cases} 0, & -5 \le m \le 4 \\ \left(\frac{1}{2}\right)^{m-5}, & 5 \le m \le 10 \end{cases}$



b) $n_1 = n_2 = 5$

$$x[n] = \sum_{k=5}^{+\infty} (0.5)^{k-5} \delta[n-k] \xrightarrow{\text{since the equation is equivalent at } \underline{n=k}} \sum_{k=5}^{+\infty} (0.5)^{n-5} \delta[n-k]$$

$$\overset{\text{since the sum}}{\underset{\text{is over } k}{=\!=}} (0.5)^{n-5} \sum_{k=5}^{+\infty} \delta[n-k] = (0.5)^{n-5} u[n-5] \quad \left(\text{since } \sum_{k=5} \delta[n-k] = u[n-5]\right)$$

3. a) $S_1: \; x_1[n] \rightarrow y_1[n] = 1.5\, x_1[n]$

$S_2: \; x_2[n] \rightarrow y_2[n] = 1.5\, x_2[n]$

$S_3: \; x_3[n] = \cancel{x_1[n}\; \alpha_1 x_1[n] + \alpha_2 x_2[n] \longrightarrow y_3[n] = 1.5\, x_3[n]$

$$= 1.5\alpha_1 x_1[n] + 1.5\alpha_2 x_2[n]$$

$$= \alpha_1 y_1[n] + \alpha_2 y_2[n]$$

Then $\alpha_1 x_1[n] + \alpha_2 x_2[n] \longrightarrow \alpha_1 y_1[n] + \alpha_2 y_2[n]$, system S is linear.

b) $S_1: \; x_1[n] \rightarrow y_1[n] = x_1[n-3]$

$S_2: \; x_2[n] \longrightarrow y_2[n] = x_2[n-3]$

$S_3: \; x_3[n] = \alpha_1 x_1[n] + \alpha_2 x_2[n] \longrightarrow y_3[n] = x_3[n-3]$

$$= \alpha_1 x_1[n-3] + \alpha_2 x_2[n-3]$$

$$= \alpha_1 y_1[n] + \alpha_2 y_2[n]$$

Since $\alpha_1 x_1[n] + \alpha_2 x_2[n] \longrightarrow \alpha_1 y_1[n] + \alpha_2 y_2[n]$, system S is linear.

c) $S_1: \; x_1[n] \longrightarrow y_1[n] = n\, x_1[n]$

$S_2: \; x_2[n] \longrightarrow y_2[n] = n\, x_2[n]$

$S_3: \; x_3[n] = \alpha_1 x_1[n] + \alpha_2 x_2[n] \longrightarrow y_3[n] = n\, x_3[n]$

$$= n\alpha_1 x_1[n] + n\alpha_2 x_2[n]$$

$$= \alpha_1 y_1[n] + \alpha_2 y_2[n]$$

since $\alpha_1 x_1[n] + \alpha_2 x_2[n] \longrightarrow \alpha_1 y_1[n] + \alpha_2 y_2[n]$, system S is linear.

d) $S_1: \; x_1[n] \rightarrow y_1[n] = x_1[n+2] + x_1[n-3]$

$S_2: \; x_2[n] \longrightarrow y_2[n] = x_2[n+2] + x_2[n-3]$

$S_3: \; x_3[n] = \alpha_1 x_1[n] + \alpha_2 x_2[n] \longrightarrow y_3[n] = x_3[n+2] + x_3[n-3]$

$$= \alpha_1 x_1[n+2] + \alpha_2 x_2[n+2]$$
$$+ \alpha_1 x_1[n-3] + \alpha_2 x_2[n-3]$$

$$= \alpha_1 y_1[n] + \alpha_2 y_2[n]$$

Since $\alpha_1 x_1[n] + \alpha_2 x_2[n] \longrightarrow \alpha_1 y_1[n] + \alpha_2 y_2[n]$, system S is linear.

4)

a) Let $S: x_1[n] \longrightarrow y_1[n] = (x_1[n-3])^3$

$S: x_2[n] \longrightarrow y_2[n] = (x_2[n-3])^3$

$S: x_3[n] = \alpha_1 x_1[n] + \alpha_2 x_2[n] \longrightarrow y_3[n] = (x_3[n-3])^3 = (\alpha_1 x_1[n-3] + \alpha_2 x_2[n-3])^3$

Let's choose: $x_1[n] = 1, \ \alpha_1 = 1$

$x_2[n] = 1, \ \alpha_2 = 1$

So, $S: x_1[n] = 1 \longrightarrow y_1[n] = 1$

$x_2[n] = 1 \longrightarrow y_2[n] = 1$

$x_3[n] = 2 \longrightarrow y_3[n] = 8$

$y_3[n] \overset{?}{=} \alpha_1 y_1[n] + \alpha_2 y_2[n]$

$8 \neq 2$

$\boxed{\text{System is NOT linear}}$

b) Let $S: x_1[n] \longrightarrow y_1[n] = \sin(x_1[n])$

$x_2[n] \longrightarrow y_2[n] = \sin(x_2[n])$

$S: x_3[n] = \alpha_1 x_1[n] + \alpha_2 x_2[n] \longrightarrow y_3[n] = \sin(x_3[n]) = \sin(\alpha_1 x_1[n] + \alpha_2 x_2[n])$

Let's choose: $x_1[n] = \pi/2$, $\alpha_1 = 1$

$x_2[n] = \pi/2$, $\alpha_2 = 1$

So, $S: x_1[n] = \frac{\pi}{2} \longrightarrow y_1[n] = 1$

$x_2[n] = \frac{\pi}{2} \longrightarrow y_2[n] = 1$

$x_3[n] = \frac{\pi}{2} + \frac{\pi}{2} = \pi \longrightarrow y_3[n] = 0$

$y_3[n] \stackrel{?}{=} \alpha_1 y_1[n] + \alpha_2 y_2[n]$

$0 \neq 2$

$\boxed{\text{System is NOT linear}}$

c)

Let $\quad S: \; x_1[n] \longrightarrow y_1[n] = 2x_1[n-1] + x_1[n-3] + 2$

$\qquad\qquad x_2[n] \longrightarrow y_2[n] = 2x_2[n-1] + x_2[n-3] + 2$

$S: \; x_3[n] = \alpha_1 x_1[n] + \alpha_2 x_2[n] \longrightarrow y_3[n] = 2x_3[n-1] + x_3[n-3] + 2$

$$= 2\left(\alpha_1 x_1[n-1] + \alpha_2 x_2[n-1]\right) + \alpha_1 x_1[n-3] + \alpha_2 x_2[n-3] + 2$$

Let's choose; $x_1[n] = 1, \quad \alpha_1 = 1$

$\qquad x_2[n] = 1, \quad \alpha_2 = 1$

So, $\quad S: \; x_1[n] = 1 \longrightarrow y_1[n] = 5$

$\qquad x_2[n] = 1 \longrightarrow y_2[n] = 5$

$x_3[n] = 2 \longrightarrow y_3[n] = 8$

$$y_3[n] \stackrel{?}{=} \alpha_1 y_1[n] + \alpha_2 y_2[n]$$

$$8 \neq 10$$

$\boxed{\text{System is NOT linear}}$

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d)

Let $S: x_1[n] \rightarrow y_1[n] = \begin{cases} x_1[n]/|x_1[n]| & \text{if } x_1[n] \neq 0 \\ 0 & \text{if } x_1[n] = 0 \end{cases}$

$S: x_2[n] \rightarrow y_2[n] = \begin{cases} x_2[n]/|x_2[n]| & \text{if } x_2[n] \neq 0 \\ 0 & \text{if } x_2[n] = 0 \end{cases}$

$S: x_3[n] = \alpha_1 x_1[n] + \alpha_2 x_2[n] \rightarrow y_3[n] = \begin{cases} x_3[n]/|x_3[n]| & \text{if } x_3[n] \neq 0 \\ 0 & \text{if } x_3[n] = 0 \end{cases}$

Let's choose: $x_1[n] = 1, \quad \alpha_1 = 1$

$x_2[n] = 1, \quad \alpha_2 = 1$

So, $S: x_1[n] = 1 \rightarrow y_1[n] = 1$

$x_2[n] = 1 \rightarrow y_2[n] = 1$

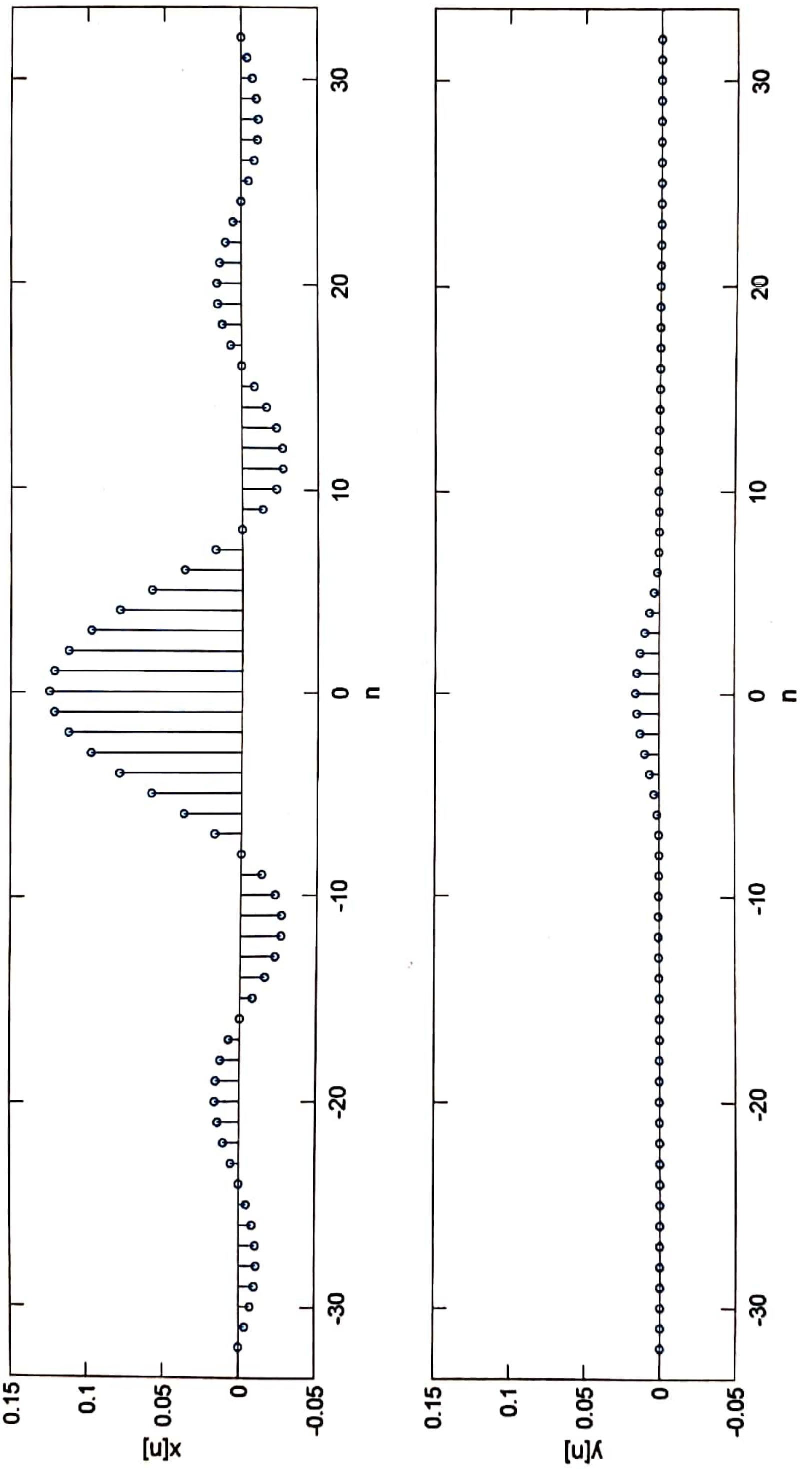
$x_3[n] = 2 \rightarrow y_3[n] = 1$

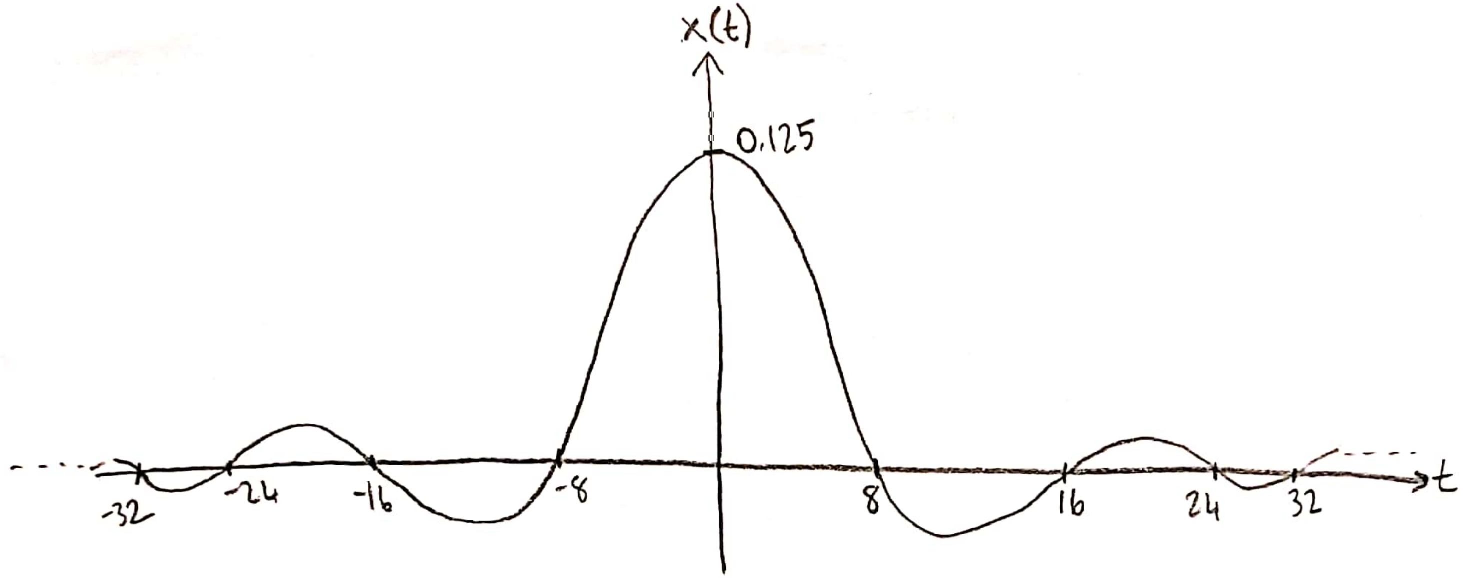
$$y_3[n] \stackrel{?}{=} \alpha_1 y_1[n] + \alpha_2 y_2[n]$$

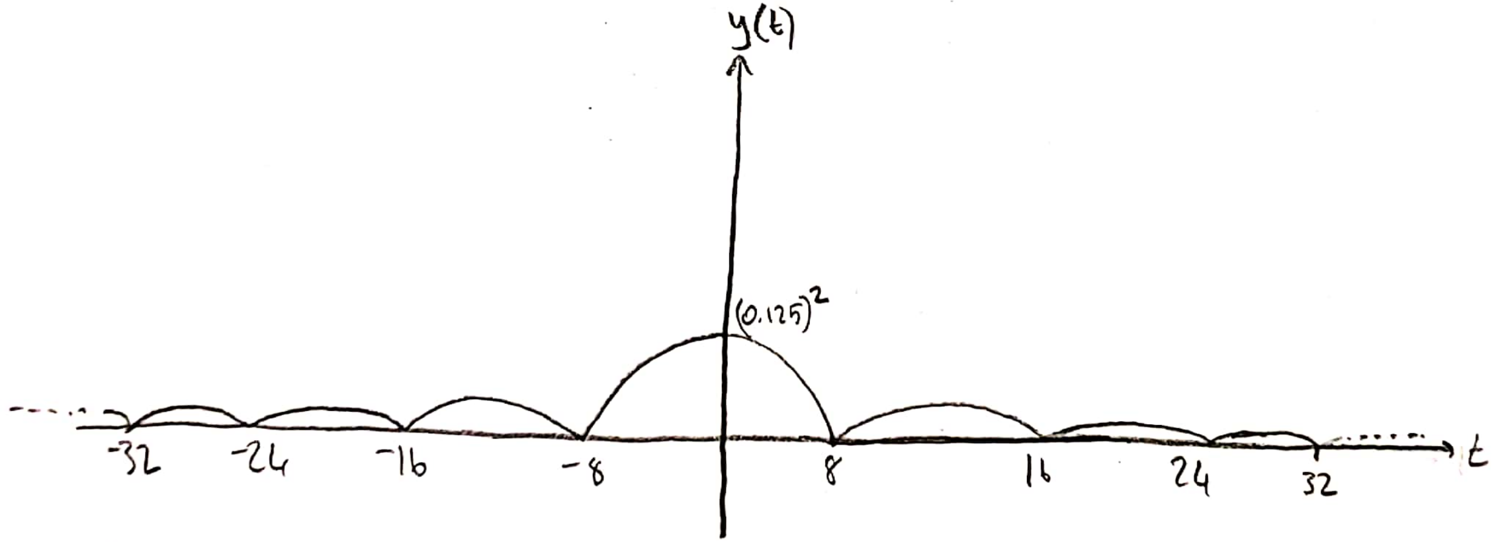
$$1 \neq 2$$

$\boxed{\text{System is NOT linear}}$

5)




x(t)
0.125
t
-32
-24
-16
-8
8
16
24
32


y(t)
(0.125)²
t
-32
-24
-16
-8
8
16
24
32

```
% Code for Homework 2 Problem 5
clc; clear all; close all;

x=zeros(65,1);

for n=-32:32

    ind=n+33;

    if n==0
        x(ind,1)= 0.125;
    else
        x(ind,1)= sin(0.125*pi*n) / (pi*n);
    end

end


y=zeros(65,1);

for n=-32:32

    ind=n+33;

    if n==0
        y(ind,1)= (0.125)^2;
    else
        y(ind,1)= (sin(0.125*pi*n) / (pi*n))^2;
    end

end




figure

subplot(2,1,1)
stem(-32:32, x);
xlabel('n')
ylabel('x[n]')
ylim([-0.05 0.15])
xlim([-33.5 33.5])
set(gca,'fontsize', 18);

subplot(2,1,2)
stem(-32:32, y);
xlabel('n')
ylabel('y[n]')
ylim([-0.05 0.15])
xlim([-33.5 33.5])
set(gca,'fontsize', 18);
```