# Practice test 2 solution

1. since $\delta(t)$ is an even function

$$\int_5^{25} u(\tau)\,\delta(3-\tau)\,d\tau = \int_5^{25} u(\tau)\,\delta(\tau-3)\,d\tau$$

Using the sifting property and note $t=3$ lies out of the integral range $[5, 25]$,

$$\int_5^{25} u(\tau)\,\delta(\tau-3)\,d\tau = 0$$

2. (a) $x_1(t) = (1+0.5j)e^{j\frac{\pi}{4}t} = e^{j\frac{\pi}{4}t} + \frac{1}{2}e^{j\frac{\pi}{2}}e^{j\frac{\pi}{4}t}$

$$= e^{j\frac{\pi}{4}t} + \frac{1}{2}e^{j\frac{\pi}{4}(t+2)}$$

so it's a combination of the original complex exponential $e^{j\frac{\pi}{4}t}$ and $\frac{1}{2}e^{j\frac{\pi}{4}(t+2)}$, which is the original signal amplitude-scaled by $\frac{1}{2}$ and shifted to left by 2

(b) $x_2(t) = (-2)^t = (2\cdot e^{j\pi})^t = \underline{2^t}\cdot e^{j\pi t}$

The only complex exponential $e^{j\pi t}$ is multiplied by $2^t$, which is not a constant. So it can not be expressed as mentioned in the question.

(c) $x_3(t) = \delta(t-2)$ is a singular function, $x_3(2) = +\infty$, so can not be expressed as any version of a complex exponential.

(d) $x_4(t) = \int_{-\infty}^{+\infty} \delta(t-\tau)\,d\tau = 1 = e^{j\cdot 0\cdot t}$ (sifting property).

so it can be expressed as a complex exponential that has a zero frequency component.

3. $x[n] = u[n] - u[n-3] = \delta[n] + \delta[n-1] + \delta[n-2]$

Using the echo sum expression:

$$y[n] = x[n] * h[n] = \delta[n] * h[n] + \delta[n-1] * h[n] + \delta[n-2] * h[n]$$

$$= (u[n-3] - u[n-8]) + (u[n-4] - u[n-9]) + (u[n-5] - u[n-10])$$

"starting point" $= 3$

three overlapping box functions

"ending point" $= 9$

Then $m = 3$, $k = 9$

4. $y(t) = x(t) * h(t) = \left[\sum_{k=0}^{3} \delta(t-4k) - \delta(t-1-4k)\right] * h(t)$

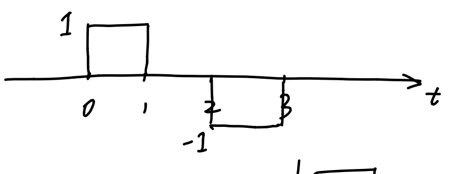
$$= \sum_{k=0}^{3} \delta(t-4k) * h(t) - \delta(t-1-4k) * h(t)$$

$$\overset{\text{sifting}}{\underset{\text{property}}{=}} \sum_{k=0}^{3} h(t-4k) - h(t-1-4k)$$

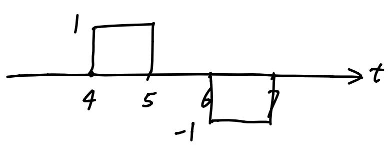
$$= \sum_{k=0}^{3} \left(u(t-4k) - u(t-2-4k) - [u(t-1-4k) - u(t-3-4k)]\right)$$

The term inside:

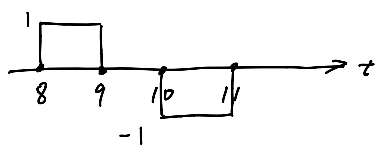
$k=0$:



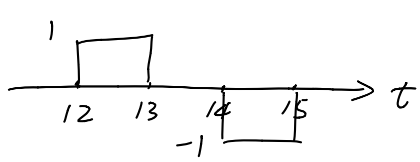

$k=1$:



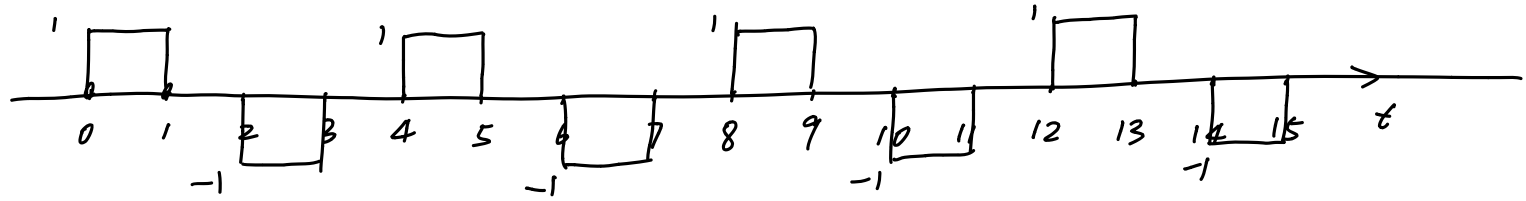

$k=2$:




$k=3$:



summing all over $\Rightarrow y(t)$ :




5. Using the DTFT pair $e^{j\omega_0 n} \longleftrightarrow 2\pi \sum_{k=-\infty}^{+\infty} \delta(\omega-\omega_0-2\pi k)$

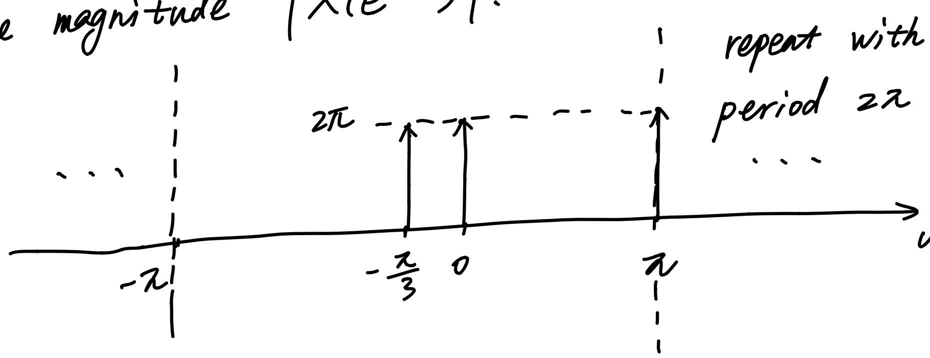
$$1 = e^{j\cdot 0\cdot n} \longleftrightarrow 2\pi \sum_{k=-\infty}^{+\infty} \delta(\omega-2\pi k)$$

$$(-1)^n = e^{j\pi n} \longleftrightarrow 2\pi \sum_{k=-\infty}^{+\infty} \delta(\omega-\pi-2\pi k)$$

$$e^{j\frac{5\pi}{3}n} = e^{-j\frac{\pi}{3}n} \longleftrightarrow 2\pi \sum_{k=-\infty}^{+\infty} \delta\left(\omega+\frac{\pi}{3}-2\pi k\right)$$

$$x[n] \longleftrightarrow 2\pi \sum_{k=-\infty}^{+\infty} \left[\delta(\omega-2\pi k)+\delta(\omega-\pi-2\pi k)+\delta\left(\omega-\frac{5\pi}{3}-2\pi k\right)\right]$$

The magnitude $|X(e^{j\omega})|$:




6. The DTFT of $h[n]$:

$$H(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} h[n]e^{-j\omega n} = \sum_{n=-\infty}^{+\infty} \left(\delta[n]+\delta[n-1]+\delta[n-2]\right)e^{-j\omega n}$$

$$= \sum_{n=0}^{2} e^{-j\omega n} = \frac{1\cdot(1-e^{-j3\omega})}{1-e^{-j\omega}} = \frac{e^{-j\frac{3}{2}\omega}\left(e^{j\frac{3}{2}\omega}-e^{-j\frac{3}{2}\omega}\right)}{e^{-j\frac{\omega}{2}}\left(e^{j\frac{\omega}{2}}-e^{-j\frac{\omega}{2}}\right)}$$

$$= e^{-j\omega}\frac{\sin(\frac{3\omega}{2})}{\sin(\frac{\omega}{2})}$$

$$x[n] = 1 + \cos\left(\frac{2\pi}{3}n + \frac{\pi}{13}\right) = e^{j\cdot 0\cdot n} + \frac{1}{2}\left(e^{j\frac{\pi}{13}} e^{j\frac{2\pi}{3}n} + e^{-j\frac{\pi}{13}} e^{-j\frac{2\pi}{3}n}\right)$$

has 3 frequency component $w_1 = 0$, $w_2 = \frac{2\pi}{3}$, $W_3 = -\frac{2\pi}{3}$

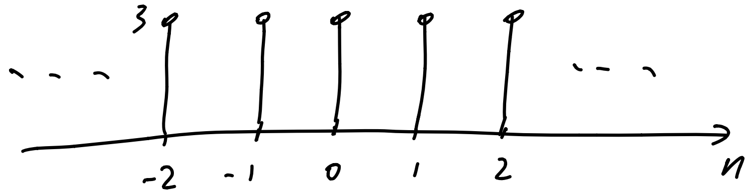
The corresponding frequency responses are

$$H(e^{jw_1}) = 3, \quad H(e^{jw_2}) = 0, \quad H(e^{jw_3}) = 0$$

So the output signal

$$y[n] = H(e^{jw_1})\cdot 1 + H(e^{jw_2})\cdot\frac{1}{2}e^{j\frac{\pi}{13}}e^{j\frac{2\pi}{3}n} + H(e^{jw_3})\cdot\frac{1}{2}e^{-j\frac{\pi}{13}}e^{-j\frac{2\pi}{3}n}$$

$$= 3$$




7. $$q[n] = \cos^2\left(\frac{\pi}{4}n\right) = \left[\frac{1}{2}\left(e^{j\frac{\pi}{4}n} + e^{-j\frac{\pi}{4}n}\right)\right]^2 = \frac{1}{4}\left(e^{j\frac{\pi}{2}n} + e^{-j\frac{\pi}{2}n} + 2\right)$$

$$= \frac{1}{2} + \frac{1}{4}\left(e^{j\frac{\pi}{2}n} + e^{-j\frac{\pi}{2}n}\right)$$

$$\frac{1}{2} \longleftrightarrow \pi \sum_{k=-\infty}^{+\infty} \delta(w - 2\pi k)$$

$$\frac{1}{4}\left(e^{j\frac{\pi}{2}n} + e^{-j\frac{\pi}{2}n}\right) \longleftrightarrow \frac{\pi}{2} \sum_{k=-\infty}^{+\infty} \left[\delta\left(w - \frac{\pi}{2} - 2\pi k\right) + \delta\left(w + \frac{\pi}{2} - 2\pi k\right)\right]$$

$$Q(e^{jw}) = \left[\pi\delta(w) + \frac{\pi}{2}\delta\left(w - \frac{\pi}{2}\right) + \frac{\pi}{2}\delta\left(w + \frac{\pi}{2}\right)\right] * \sum_{k=-\infty}^{+\infty} \delta(w - 2\pi k)$$

$|Q(e^{jw})|$ :