EC401 HW09 Spring 2020

Due Date: Wednesday April 22, 2020

You must submit your homework in pdf form to the EC401 Blackboard Learn site by 12:30pm on the due date. Please be sure to write your name on the first page of the homework you submit. Additionally, if you have collaborated on the homework with other individuals enrolled in EC401 this semester, please identify them as your collaborators on the first page of the submitted homework.

HW09.1

For a continuous-time LTI system S with impulse response $h(t)$ and input signal $x(t)$, the output signal $y(t)$ is given by the convolution integral:

$$y(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau$$

(a) If $x(t) = e^{j2000\pi t}$, show that the output of the LTI system S would be an amplitude-scaled and time-shifted version of $x(t)$.
(b) If $x(t) = \delta(t-2)$, show that the output of the LTI system S would be a shifted version of $h(t)$.
(c) If $x(t) = \sum_{k=-\infty}^{\infty} \delta(t-10k)$ and $h(t) = u(t) - u(t-5)$, sketch the output of system S.

HW09.2

Sketch the magnitude of the CTFT of each of the following signals using the fact that

$$\sum_{k=-\infty}^{\infty} \delta(t-kT) \leftrightarrow \sum_{k=-\infty}^{\infty} \frac{2\pi}{T}\delta(\omega - \frac{2\pi}{T}k)$$

(You may justify your answers using any of the CTFT properties discussed in lecture)

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(i) $x(t) = \sum_{k=-\infty}^{\infty} 2\delta(t-0.1k)$
(ii) $x(t) = \sum_{k=-\infty}^{\infty} \delta(t+0.1k)$
(iii) $x(t) = \sum_{k=-\infty}^{\infty} \delta\,(t-1-0.1k)$
(iv) $x(t) = \sum_{k=-\infty}^{\infty} 2\delta(t-2-0.1k)$
(v) $x(t) = \delta(t-0.15) * \sum_{k=-\infty}^{\infty} \delta(t-0.1k)$ where $*$ denotes convolution.
(vi) $x(t) = \int_{-\infty}^{\infty} \delta(\tau-0.15) \sum_{k=-\infty}^{\infty} \delta(t-\tau-0.1k)$

HW09.3

Consider (for $T > 0$) the following CTFT pair:

$$u(t+T) - u(t-T) \leftrightarrow \frac{2\sin(\omega T)}{\omega}$$

Use the *duality property* of the CTFT to show determine the CTFT of $g(t) = \frac{\sin(100\pi t)}{\pi t}$

Note: Duality can be summed up as: $2\pi x(-\omega) \leftrightarrow X(jt)$

HW 09.4

a) Starting with the time-shifting property of the CTFT (i.e., $x(t-t_0) \leftrightarrow e^{-j\omega t_0}X(j\omega)$, use *duality* of the CTFT to show the frequency-shifting property of the CTFT (i.e., $e^{j\omega_0 t}x(t) \leftrightarrow X(j(\omega-\omega_0))$.
b) Use the frequency-shifting property of the CTFT to determine and sketch the *magnitude* of the CTFT of the following signal:

$$g(t) = \cos(1000\pi t)\left\{\frac{\sin(500\pi t)}{\pi t}\right\}$$

HW09.5

Consider a real-valued signal $x(t)$ whose Fourier transform $X(j\omega)$ is such that $X(j\omega) = 0$ for $\omega \geq 10{,}000\pi$.

(a) If $x(t)$ is to be *sampled* with sampling period *T*, what is the *largest* value of *T* that can be used without causing aliasing. *Justify your answer*.
(b) Determine the *frequency response* (defined as the CTFT of the impulse response) of an LTI system that can convert the *sampled* $x(t)$ (the "dots") back into $x(t)$. Justify your answer.

HW09.6

Throughout this problem, the LTI system **S** has impulse response $h(t)$ whose CTFT is given as

$$H(j\omega) = \begin{cases} j\omega & for \ |\omega| \leq 1000\pi \\ 0 & Otherewise \end{cases}$$

(c) (10 points): Sketch and label one period of the output signal $y_c(t)$ from system **S** when the input signal to system **S** is given as $x_c(t) = \cos(800\pi t)$. *Justify your answer.*

(b) (10 Points): Sketch and label one period of the output signal $y_b(t)$ from system **S** when the input signal to system **S** is given as $x_b(t) = \sum_{k=-\infty}^{\infty} \delta(t - k/400)$. *Justify your answer.*

(a) (5 Points): If the input $x_a(t)$ to system **S** is given to be an odd signal, is the output $y_a(t)$ of the system **S** guaranteed to be even, odd, or neither? *Justify your answer.*