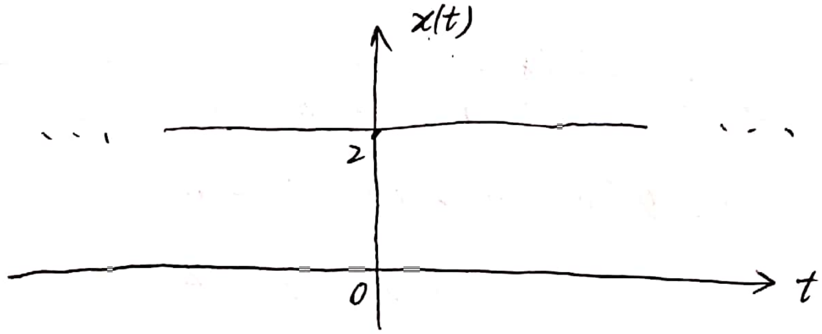
HW1 solution

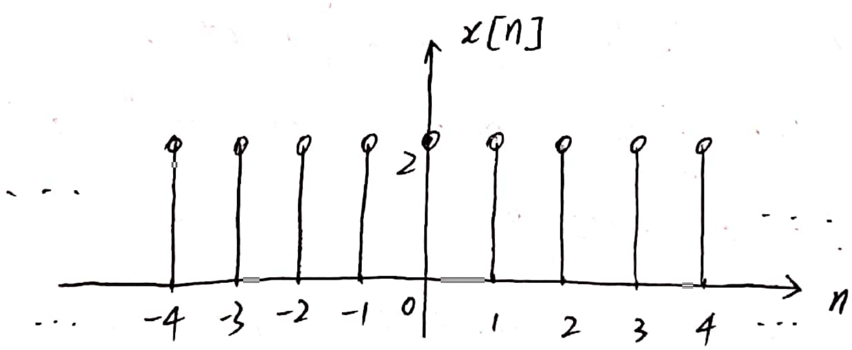
1. a) all possible real numbers $\mathbb{R}$.

   b) all possible integer numbers $\mathbb{Z}$.

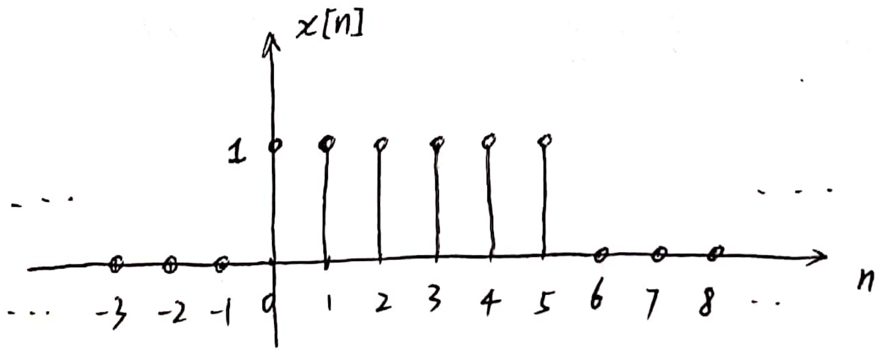
   c) Yes.



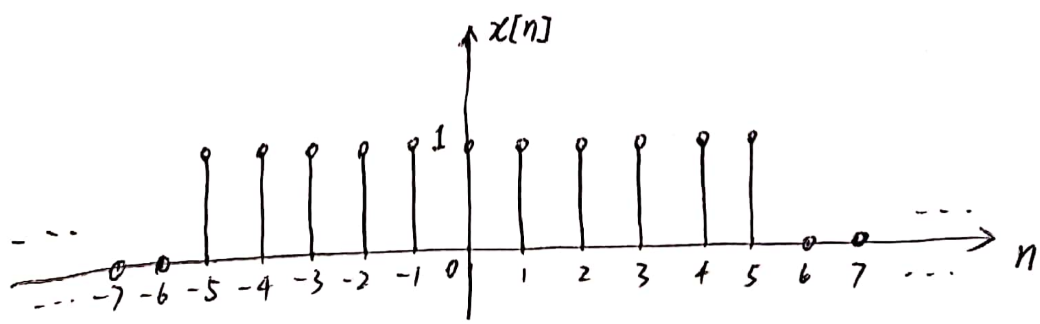
   d) Yes.



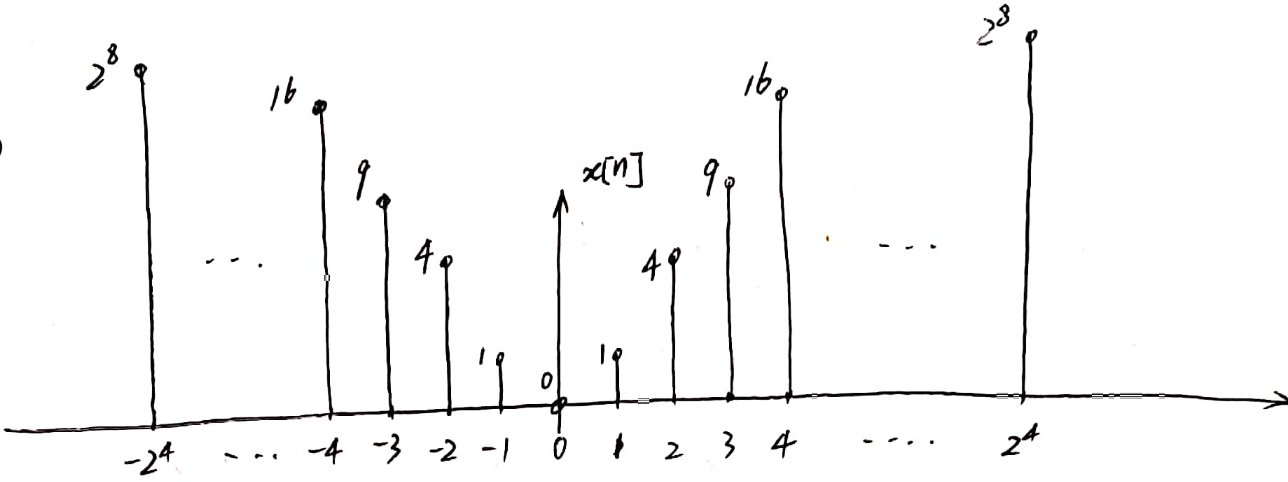
2. a)



   b)



3. a)



$x[0]$ = the height of the lollipop at $(x=0) = 0^2 = 0$

$x[2]$ = - - - - - - - - - $(x=2) = 2^2 = 4$

$x[4]$ = - - - - - - - - - $(x=4) = 4^2 = 16$

$(x[4])^2$ = the square of the height of the lollipop at $(x=4) = 16^2 = 256$

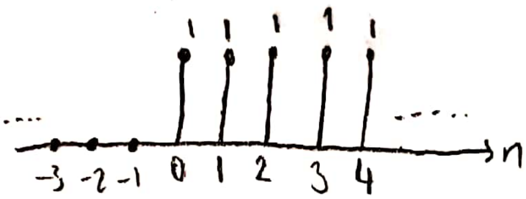
$x[2^4]$ = ~~the~~ the height of the lollipop at $(x=2^4) = (2^4)^2 = 2^8 = 256$

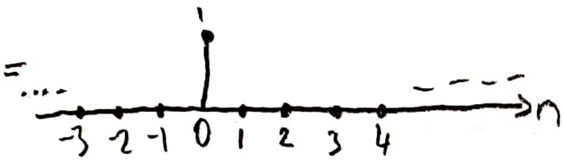
$x[-2^4] = x[2^4] = 256$ since this signal is even.

b) False; The index 1.5 is illegal for discrete-time signal/ $x[1.5]$ has no ~~value~~, meaning/ can't compute $x[1.5]$.

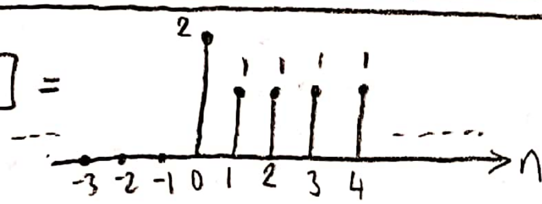
4)

a)

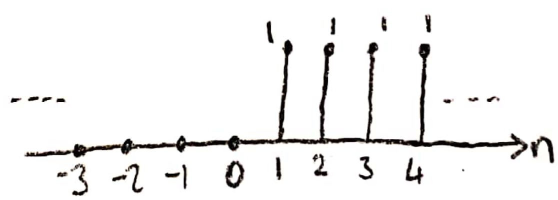
$u[n] =$ 

$\delta[n] =$ 

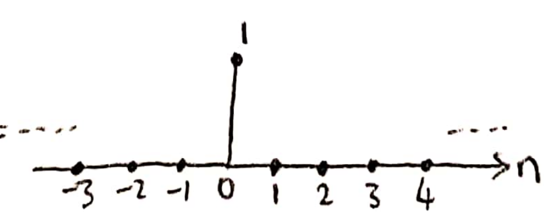
+

$u[n] + \delta[n] = \boxed{x[n] =}$ 

b)

$u[n] - \delta[n] = \boxed{x[n] =}$ 

c)

$u[n] \times \delta[n] = \boxed{x[n] =}$ 

5)

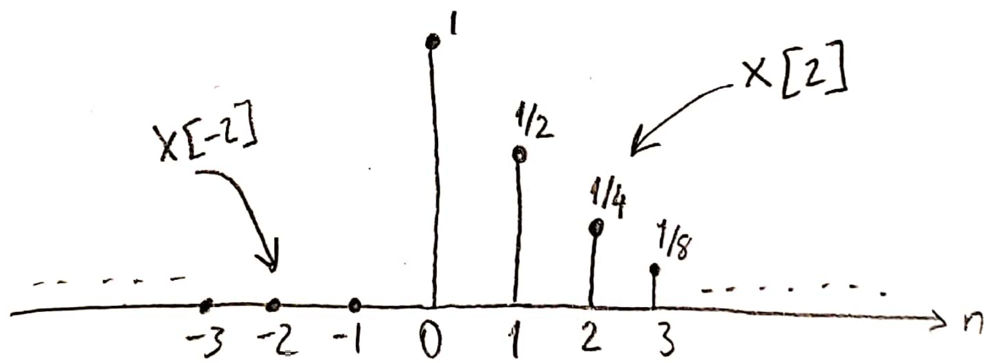
*Note that, $u[n]$ serves as a switch at $n=0$.

$u[n]=0$ for $n<0$

$u[n]=1$ for $n \geq 0$

So, $x[n] = \begin{cases} (0.5)^n \cdot 0 = 0 & , \text{for } n<0 \\ (0.5)^n \cdot 1 = (0.5)^n & , \text{for } n \geq 0 \end{cases}$

x[n]



a) $x[2] = (0.5)^2 \underbrace{u[2]}_{\text{equals to "1"}} = (0.5)^2 = \frac{1}{4} \Rightarrow \boxed{x[2] = \frac{1}{4}}$

$x[-2] = (0.5)^{-2} \underbrace{u[-2]}_{\text{equals to "0"}} = 0 \Rightarrow \boxed{x[-2]=0}$

b)

$$\alpha = x[0] + x[1] + x[2] + x[3] + \dots + x[99]$$

$$= (0.5)^0 + (0.5)^1 + (0.5)^2 + (0.5)^3 + \dots + (0.5)^{99}$$

$$\alpha(1-0.5) = (1-0.5)\left((0.5)^0 + (0.5)^1 + (0.5)^2 + (0.5)^3 + \dots + (0.5)^{99}\right)$$

$$= \left((0.5)^0 + \cancel{(0.5)^1} + \cancel{(0.5)^2} + \cancel{(0.5)^3} + \cancel{\dots} + \cancel{(0.5)^{99}}\right) - \left(\cancel{(0.5)^1} + \cancel{(0.5)^2} + \cancel{(0.5)^3} + \cancel{\dots} + \cancel{(0.5)^{99}} + (0.5)^{100}\right)$$

$$= (0.5)^0 - (0.5)^{100} = 1 - (0.5)^{100}$$

$$\boxed{\alpha(1-0.5) = 1 - (0.5)^{100}}$$

c)

In question it is asked to find the sum of all heights of the lollipops in $x[n]$, we can show this mathematically as follows,

$$\begin{matrix}\text{sum of all} \\ \text{heights of lollipops} \\ \text{in } x[n]\end{matrix} = \sum_{n=-\infty}^{\infty} x[n] = \sum_{n=-\infty}^{\infty} (0.5)^n u[n] = \sum_{n=0}^{\infty} (0.5)^n$$

This is zero when $n<0$.
So, I can change the lower limit of summation

$$\sum_{n=0}^{\infty} (0.5)^n = (0.5)^0 + (0.5)^1 + (0.5)^2 + (0.5)^3 + \text{------} + (0.5)^{\infty}$$

In previous part we showed that,

$$(0.5)^0 + (0.5)^1 + (0.5)^2 + (0.5)^3 + \text{----} + (0.5)^{99} = \frac{1-(0.5)^{100}}{1-0.5}$$

For our case, the summation will go up to ∞, so,

This will go to "0"

$$(0.5)^0 + (0.5)^1 + (0.5)^2 + (0.5)^3 + \text{----} + (0.5)^{\infty} = \lim_{N\to\infty} \frac{1-(0.5)^{N+1}}{1-0.5} = \frac{1}{1-0.5}$$

$$= \frac{1}{0.5} = \boxed{2}$$

$$\boxed{\begin{matrix}\text{sum of all heights} \\ \text{of lollipops in } x[n]\end{matrix} = 2}$$

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