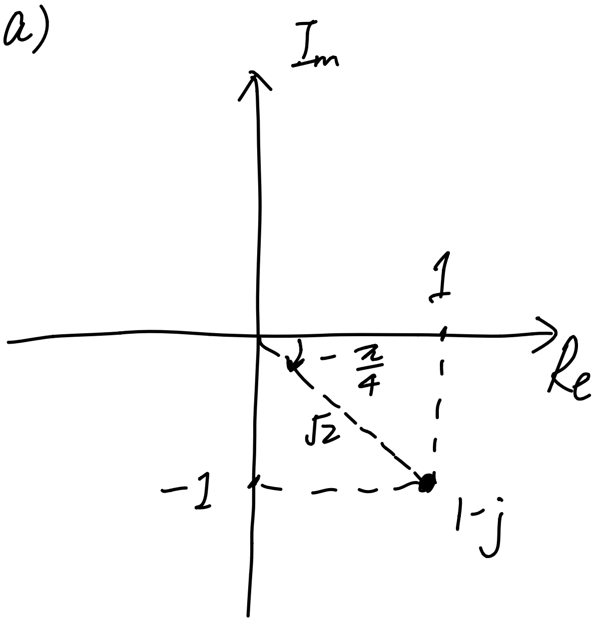
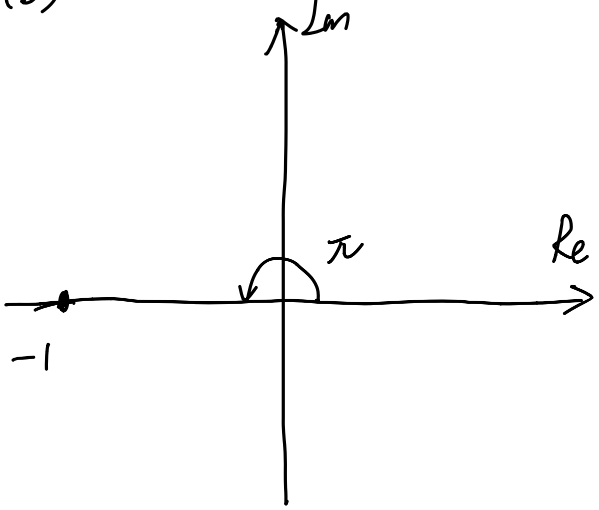
Homework 5 solution

1. (a)



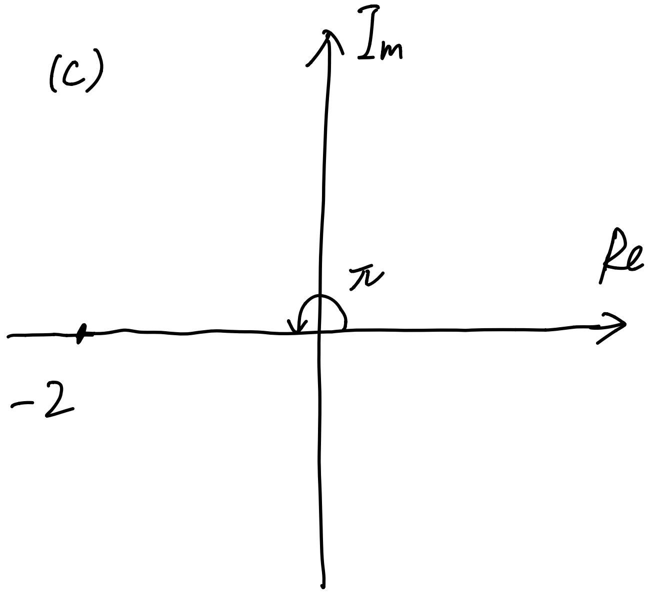
$$1-j = \underbrace{\sqrt{2}}_{\text{magnitude}}\, e^{j\underbrace{(-\frac{\pi}{4})}_{\text{phase}}}$$

(b)



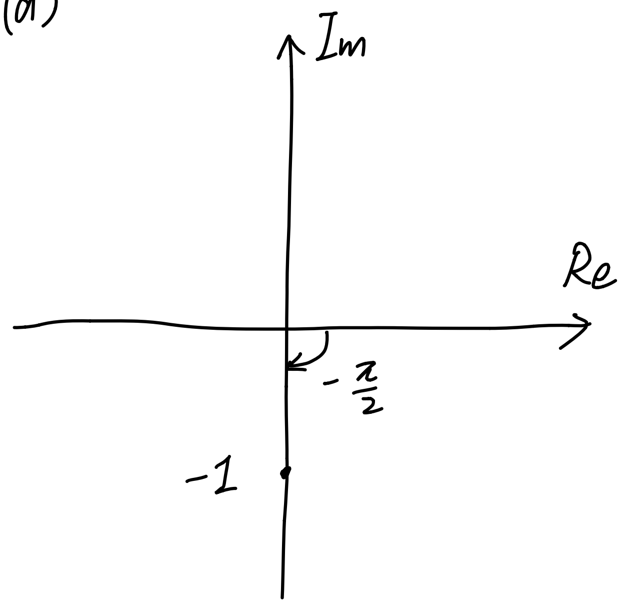
$$-1 = \underbrace{1}_{\text{magnitude}} \cdot e^{j\underbrace{\pi}_{\text{phase}}}$$

(c)



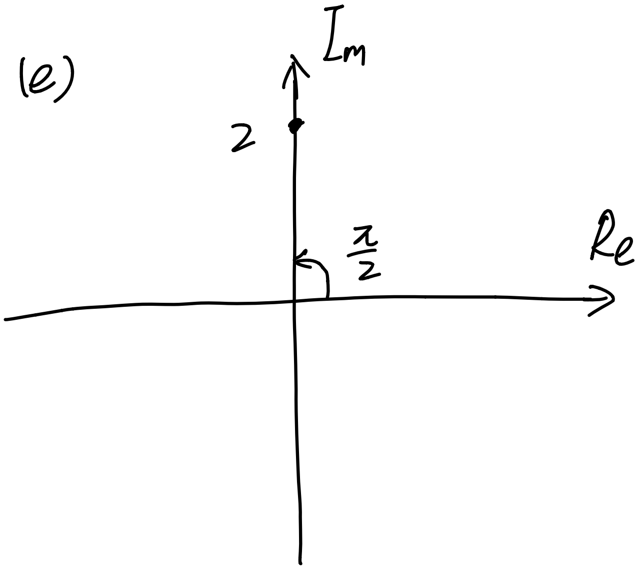
$$-2 = \underbrace{2}_{\text{magnitude}} \cdot e^{j\underbrace{\pi}_{\text{phase}}}$$

(d)



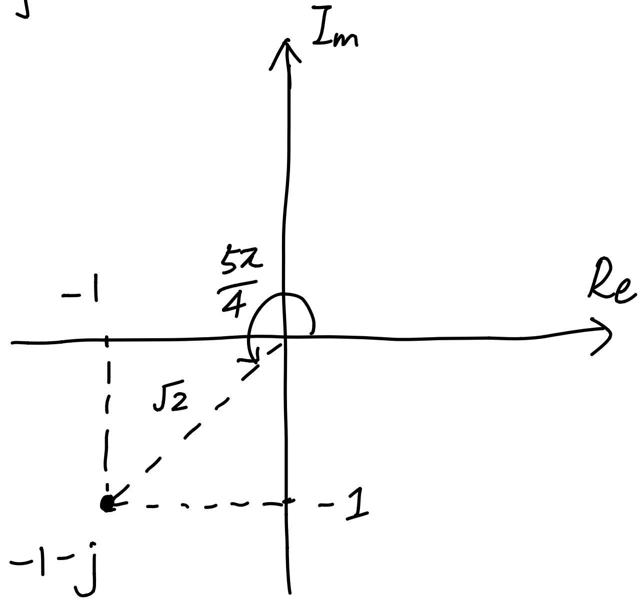
$$-j = \underbrace{1}_{\text{magnitude}} \cdot e^{j\cdot\underbrace{(-\frac{\pi}{2})}_{\text{phase}}}$$

(e)



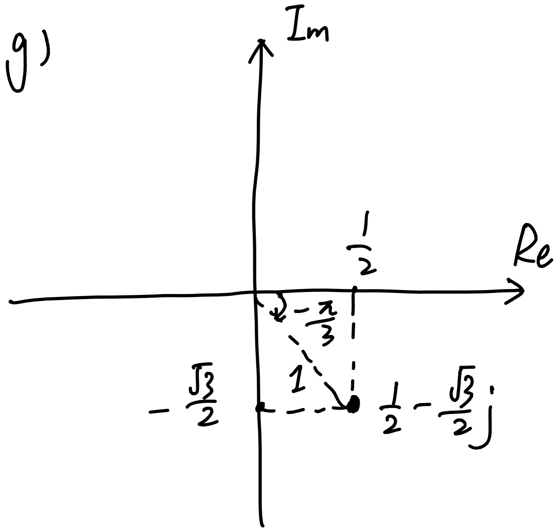
$$2j = \underbrace{2}_{\text{magnitude}} \cdot e^{j\underbrace{\frac{\pi}{2}}_{\text{phase}}}$$

(f)



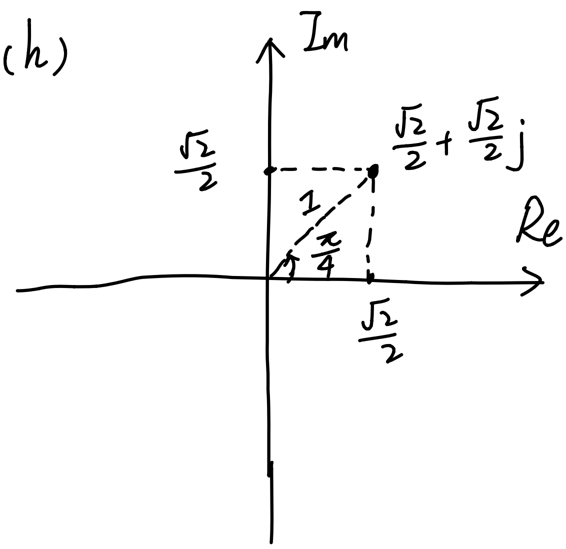
$$-1-j = \underbrace{\sqrt{2}}_{\text{magnitude}}\, e^{j\cdot\underbrace{\frac{5\pi}{4}}_{\text{phase}}}$$

(g)



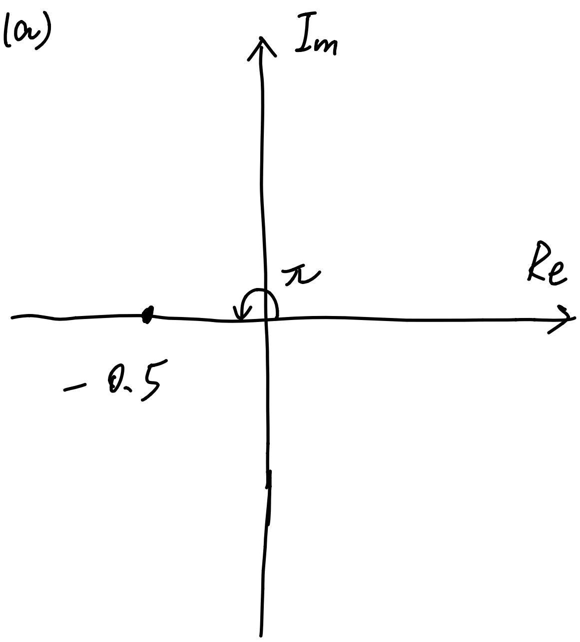
$$\frac{1}{2}-\frac{\sqrt{3}}{2}j = \underbrace{1}_{\text{magnitude}}\cdot e^{j\underbrace{\left(-\frac{\pi}{3}\right)}_{\text{phase}}}$$

(h)



$$\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}j = \underbrace{1}_{\text{magnitude}}\cdot e^{j\underbrace{\frac{\pi}{4}}_{\text{phase}}}$$

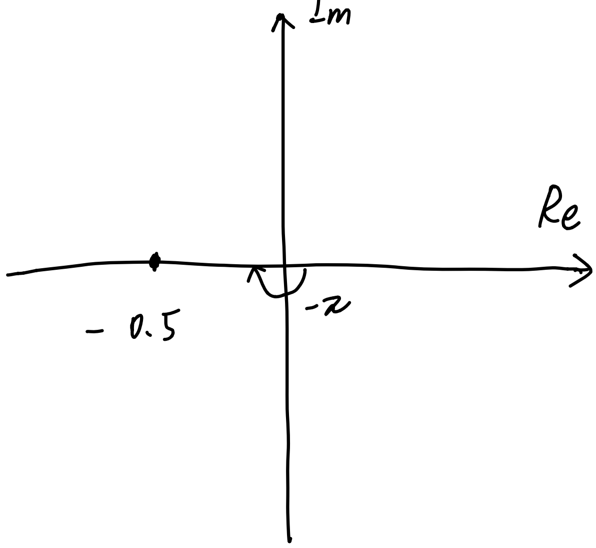
2. (a)



$$0.5e^{j\pi} = 0.5\times[\cos(\pi)+j\sin(\pi)]$$

$$= \underbrace{-0.5}_{\text{real part}} + j\underbrace{0}_{\text{imaginary part}}$$

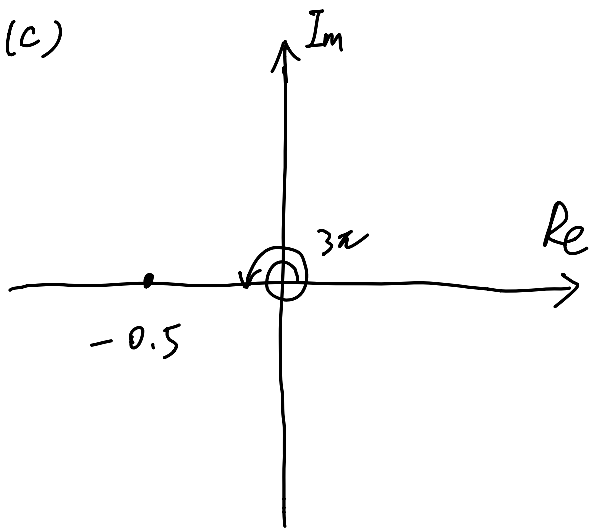
(b)



$$0.5e^{-j\pi} = 0.5\cdot[\cos(-\pi)+j\sin(-\pi)]$$

$$= \underbrace{-0.5}_{\text{real part}} + j\underbrace{0}_{\text{imaginary part}}$$

(c)

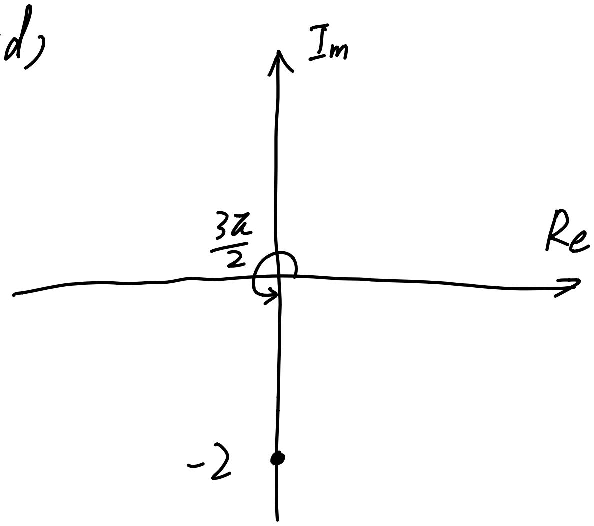


$$0.5e^{j3\pi} = 0.5e^{j(\pi+2\pi)}$$

$$= 0.5e^{j\pi}\cdot \underbrace{e^{j2\pi}}_{=1}$$

$$= \underbrace{-0.5}_{\text{real part}} + j\underbrace{0}_{\text{imaginary part}}$$

(d)

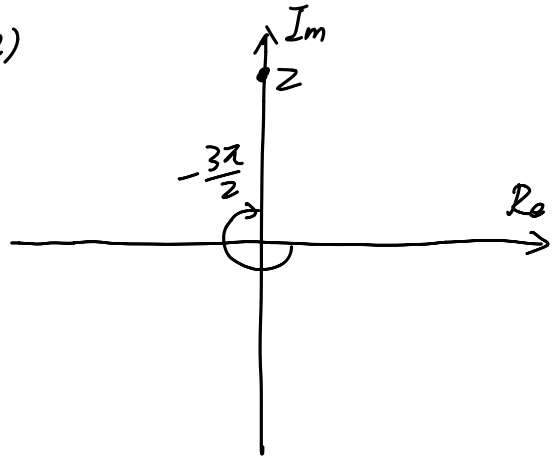


$$2e^{j\frac{3\pi}{2}} = 2\left[\cos\left(\tfrac{3\pi}{2}\right)+j\sin\left(\tfrac{3\pi}{2}\right)\right]$$

$$= -2j$$

$$= \underbrace{0}_{\text{real part}} + j\cdot(\underbrace{-2}_{\text{imaginary part}})$$

(e)

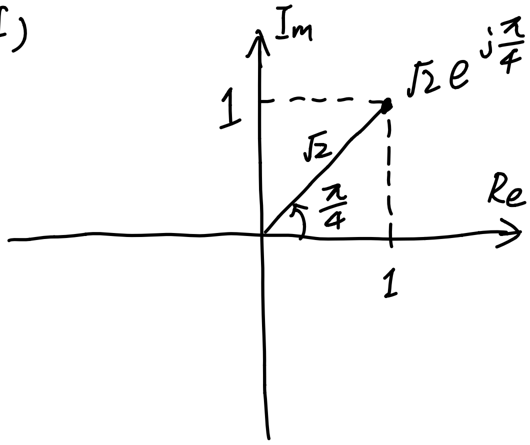


$$2e^{-j\frac{3\pi}{2}} = 2\left[\cos\left(-\frac{3\pi}{2}\right) + j\sin\left(-\frac{3\pi}{2}\right)\right]$$

$$= 2j$$

$$= \underbrace{0}_{\text{real part}} + j\cdot\underbrace{2}_{\text{imaginary part}}$$

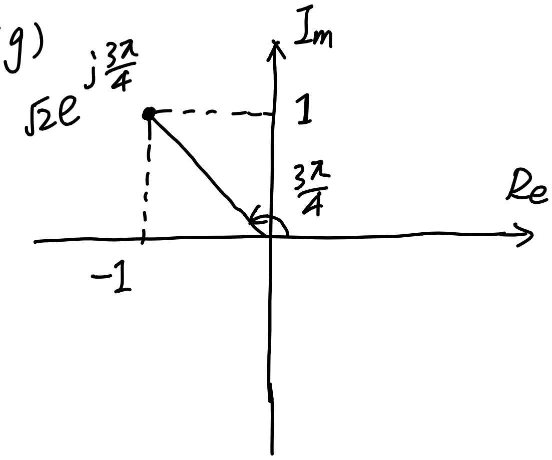
(f)



$$\sqrt{2}e^{j\frac{\pi}{4}} = \sqrt{2}\left[\cos\left(\frac{\pi}{4}\right) + j\sin\left(\frac{\pi}{4}\right)\right]$$

$$= \underbrace{1}_{\text{real part}} + j\cdot\underbrace{1}_{\text{imaginary part}}$$

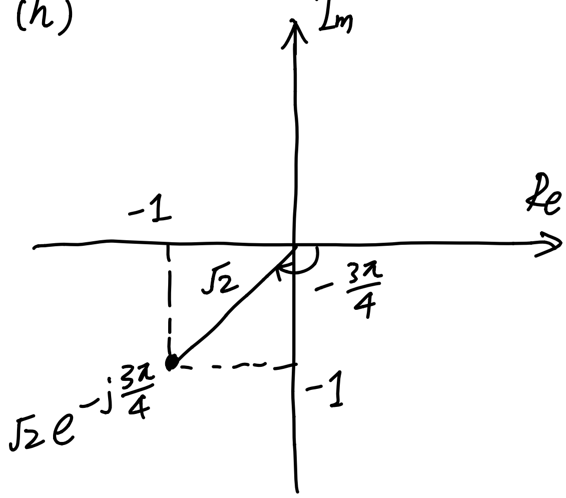
(g)



$$\sqrt{2}e^{j\frac{3\pi}{4}} = \sqrt{2}\left[\cos\left(\frac{3\pi}{4}\right) + j\sin\left(\frac{3\pi}{4}\right)\right]$$

$$= \underbrace{-1}_{\text{real part}} + j\cdot\underbrace{1}_{\text{imaginary part}}$$

(h)

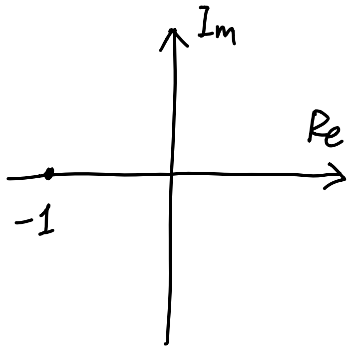


$$\sqrt{2}e^{-j\frac{3\pi}{4}} = \sqrt{2}\left[\cos\left(-\frac{3\pi}{4}\right) + j\sin\left(-\frac{3\pi}{4}\right)\right]$$

$$= \underbrace{-1}_{\text{real part}} + j\cdot\underbrace{(-1)}_{\text{imaginary part}}$$

3.

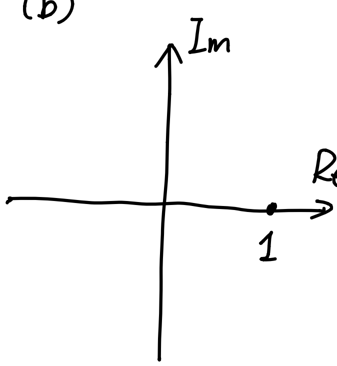
(a)



$$j = e^{j\frac{\pi}{2}}$$

$$je^{j\frac{\pi}{2}} = e^{j(\frac{\pi}{2}+\frac{\pi}{2})} = e^{j\pi} = -1$$

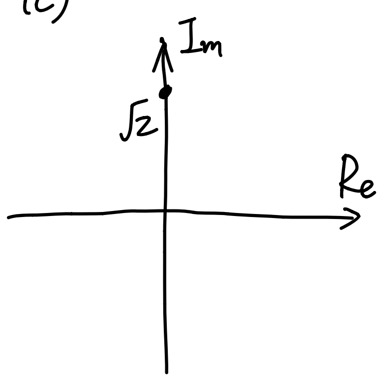
(b)



$$-j = e^{-j\frac{\pi}{2}}$$

$$-je^{j\frac{\pi}{2}} = e^{j(-\frac{\pi}{2}+\frac{\pi}{2})} = e^{j0} = 1$$

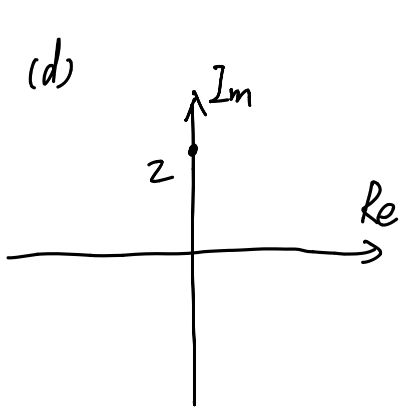
(c)



$$1+j = \sqrt{2}e^{j\frac{\pi}{4}}$$

$$(1+j)e^{j\frac{\pi}{4}} = \sqrt{2}e^{j(\frac{\pi}{4}+\frac{\pi}{4})} = \sqrt{2}e^{j\frac{\pi}{2}} = \sqrt{2}j$$

(d)

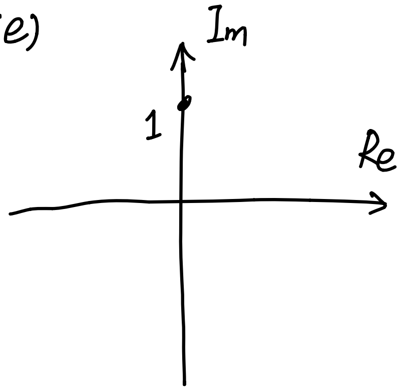


$$0.5+j0.5 = \frac{\sqrt{2}}{2}e^{j\frac{\pi}{4}}, \quad -2-j2 = 2\sqrt{2}e^{j\frac{5\pi}{4}}$$

$$e^{j3\pi}(0.5+j0.5)(-2-j2) = 2\sqrt{2}\times\frac{\sqrt{2}}{2}e^{j(3\pi+\frac{\pi}{4}+\frac{5\pi}{4})}$$

$$= 2e^{j\frac{9\pi}{2}} = 2e^{j\frac{\pi}{2}} = 2j$$

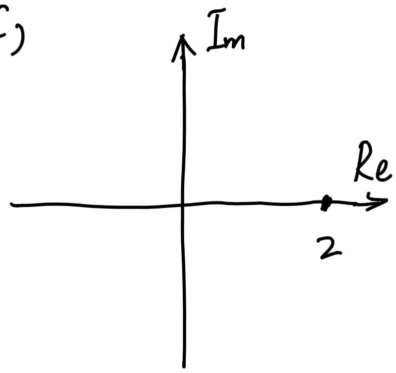
(e)



$1+j = \sqrt{2}e^{j\frac{\pi}{4}}$, $1-j = \sqrt{2}e^{-j\frac{\pi}{4}}$

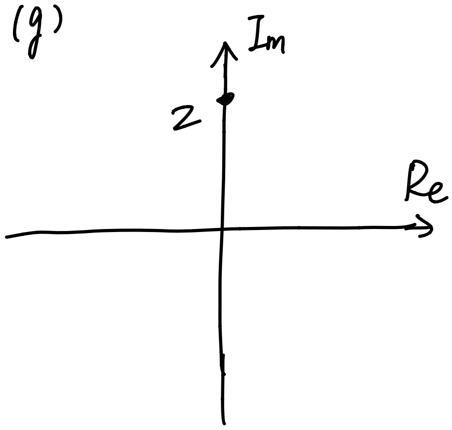
$$\frac{1+j}{1-j} = \frac{\sqrt{2}e^{j\frac{\pi}{4}}}{\sqrt{2}e^{-j\frac{\pi}{4}}} = e^{j(\frac{\pi}{4}+\frac{\pi}{4})} = e^{j\frac{\pi}{2}} = j$$

(f)



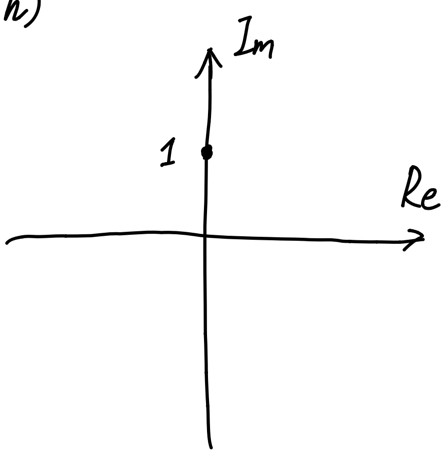
$$(1+j)(1-j) = \sqrt{2}e^{j\frac{\pi}{4}} \cdot \sqrt{2}e^{-j\frac{\pi}{4}}$$
$$= 2e^{j(\frac{\pi}{4}-\frac{\pi}{4})} = 2e^{j0} = 2$$

(g)



$$(1+j)^2 = (\sqrt{2})^2 e^{j(\frac{\pi}{4}+\frac{\pi}{4})} = 2e^{j\frac{\pi}{2}} = 2j$$

(h)



$$\frac{1}{32}(1+j)^{10} = \frac{1}{32} \times (\sqrt{2})^{10} \cdot e^{j(\frac{\pi}{4}\times 10)}$$
$$= \frac{2^5}{32} e^{j\frac{5\pi}{2}}$$
$$= e^{j\frac{\pi}{2}} = j$$

4)

a) $x(t) = e^{j1000\pi t} \longrightarrow$ PERIODIC

$$T = \frac{2\pi}{1000\pi} \Longrightarrow \boxed{T = 0.002}$$

b) $x(t) = e^{j2000\pi t} + e^{j1000\pi(t-1)} \longrightarrow$ PERIODIC

$$T_1 = \frac{2\pi}{2000\pi} \qquad T_2 = \frac{2\pi}{1000\pi}$$

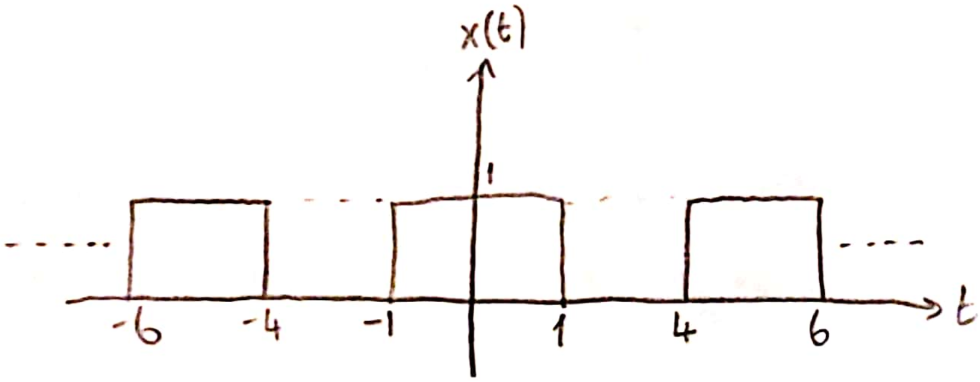
$$T_1 = 0.001 \qquad T_2 = 0.002$$

$T = \text{lcm}(T_1, T_2)$

least common multiple

$$\boxed{T = 0.002}$$

c) $x(t) = \sum_{k=-\infty}^{\infty} u(t+1+5k) - u(t-1+5k) \longrightarrow$ PERIODIC

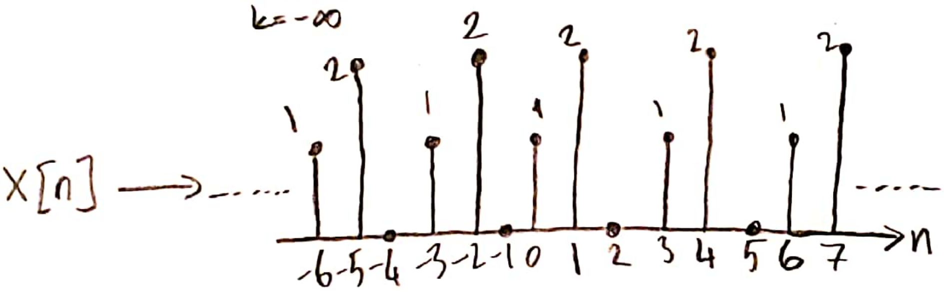


$$\boxed{T = 5}$$

d) $x(t) = \cos(1000\pi(t - 0.005)) \longrightarrow$ PERIODIC

$$T = \frac{2\pi}{1000\pi} \Longrightarrow \boxed{T = 0.002}$$

e) $x[n] = \sum_{k=-\infty}^{\infty} \delta[n-3k] + 2\delta[n-1-3k] \longrightarrow$ PERIODIC



$$\boxed{T = 3}$$

f) $x[n] = \cos(0.125\pi n) + \sin(0.25\pi n + 0.16\pi) \longrightarrow$ PERIODIC

$$T_1 = \frac{2\pi}{0.125\pi} \qquad T_2 = \frac{2\pi}{0.25\pi} \qquad T = \operatorname{lcm}(T_1, T_2)$$

$$T_1 = 16 \qquad T_2 = 8 \qquad \boxed{T = 16}$$

g) $x[n] = n^2 \cos(1000\pi n) \longrightarrow$ NOT PERIODIC

NOT a periodic function

h) $x[n] = e^{j\frac{\pi}{16}n} \longrightarrow$ PERIODIC

$$T = \frac{2\pi}{\frac{\pi}{16}} \Longrightarrow \boxed{T=32}$$

i) $x[n] = \sin(2n) \longrightarrow$ NOT PERIODIC

If I were to try calculate the period,

$$T = \frac{\cancel{2}\pi}{\cancel{2}} = \pi$$ $\longrightarrow$ It is a irrational number and it is not valid a period in discrete domain

5) $x(t) = u(t) - u(t-6)$

a) $$\int_{-\infty}^{\infty} x(\tau)\delta(\tau-3)\,d\tau = x(3) = \boxed{1}$$

b) $$\int_{-\infty}^{\infty} x(\tau)\,\delta(\tau-7)\,d\tau = x(7) = \boxed{0}$$

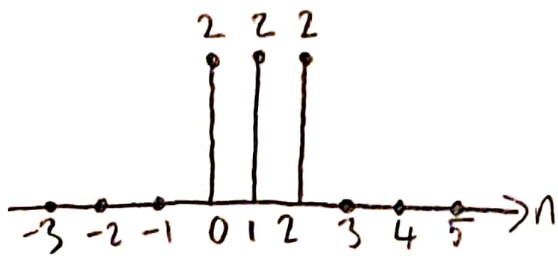
c) $$\int_{-\infty}^{\infty} x(\tau)\,x(-\tau-3)\,d\tau = x(-3) = \boxed{0}$$

d) $$\int_{-\infty}^{\infty} x(\tau)\,\delta(-\tau-7) = x(-7) = \boxed{0}$$

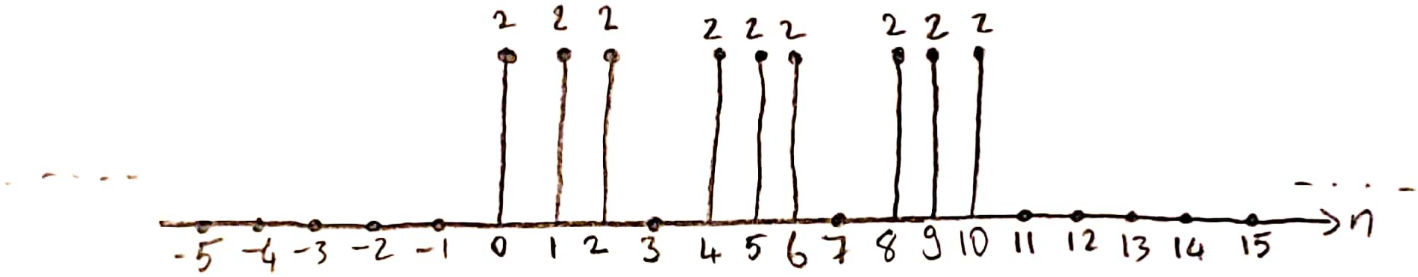
e) $$\int_{-\infty}^{\infty} \tau^2\,x(\tau)\,\delta(\tau-3) = 3^2\,x(3) = \boxed{9}$$

6)

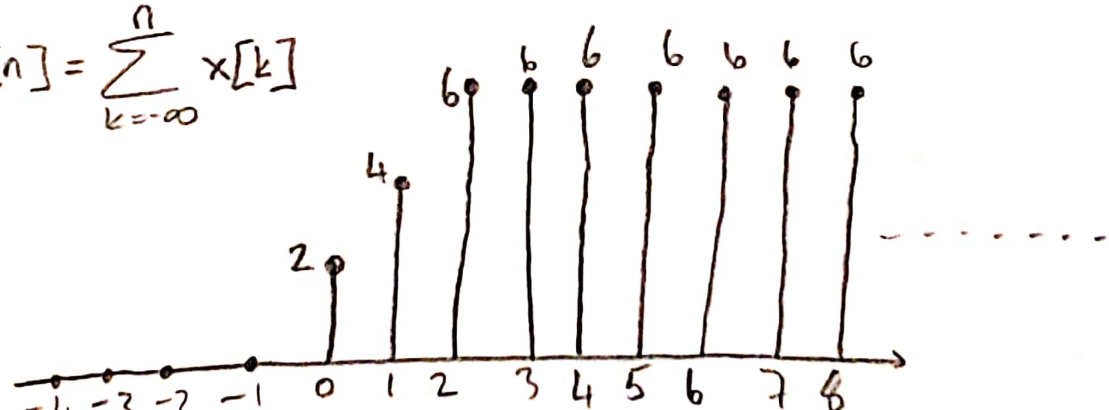
$$x[n] = 2\{u[n] - u[n-3]\} \rightarrow$$



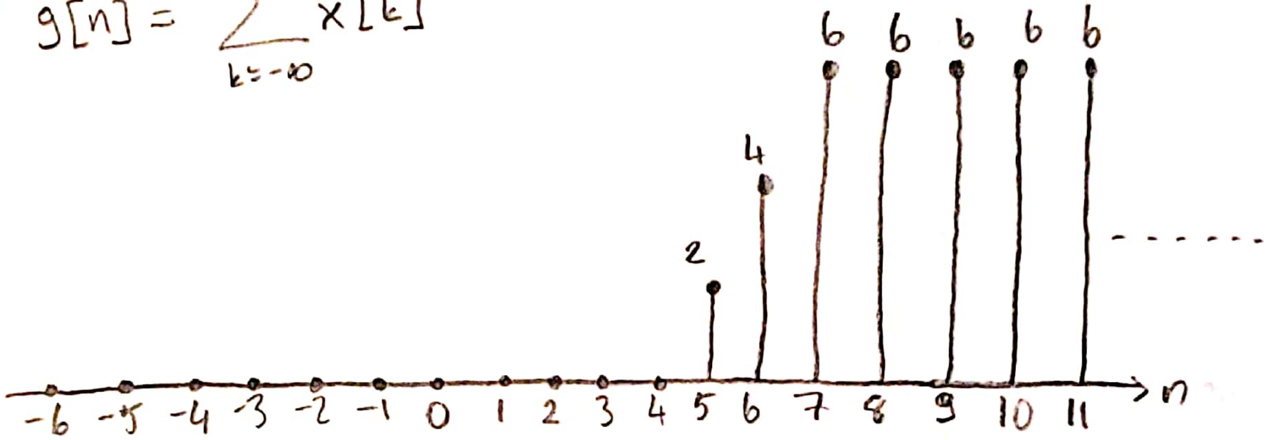
a) $$g[n] = \sum_{k=0}^{2} x[n-4k]$$



b) $$g[n] = \sum_{k=-\infty}^{n} x[k]$$

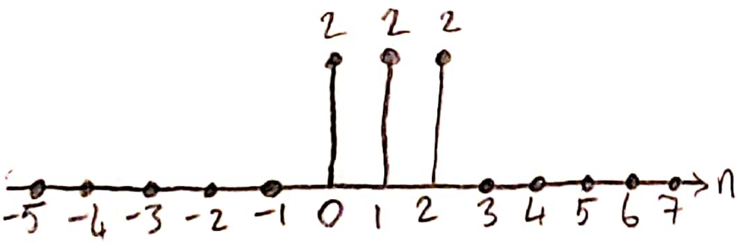


c) $g[n] = \sum_{k=-\infty}^{n-5} x[k]$



Impulse Decomposition

d) $g[n] = \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k] = x[n]$



e) $g[n] = \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k-2] = x[n-2]$