EC401 FINAL EXAM (Spring 2020)
Exam Duration: 60 minutes
Open Book
No Collaboration *with anyone allowed,* **Formula Sheet** *provided on BB Learn.*
Please keep Zoom ***video and audio on****.*
At end of exam, you will have 15 minutes to ***upload your answers*** *to BB Learn in a* ***single*** *PDF file.*

Throughout this test, $\delta[n]$ and $u[n]$ denote the discrete-time unit impulse and unit step respectively. Also $\delta(t)$ and $u(t)$ respectively denote the unit impulse and the unit step in continuous time.

Problem 1 (10 points)
Let $x(t) = \cos(\pi t)\{u(t+1) - u(t-1)\}$.

a) Sketch $x(t)$.
b) Sketch a signal $g(t)$ such that $x(t) = g(1-t)$.

Justify your answers.

Problem 2 (10 points)
Let S be an LTI system with impulse response $h[n] = u[n] - u[n-3]$ and let the input signal to S be $x[n] = 2^n u[-n-1]$. Let the output signal for that input be denoted by $y[n]$. Determine the value of $y[100]$. *Justify your answer*.

Problem 3 (10 points)
Let $x[n] = \sum_{k=0}^{7} e^{j\frac{2\pi k(n-2)}{8}}$

a) Sketch $x[n]$.
b) Sketch $|X(e^{j\omega})|$, the magnitude of the DTFT of $x[n]$.

Justify your answers.

Problem 4 (10 points)

Throughout this problem, let $x(t)$ be a *real-valued signal* whose CTFT $X(j\omega)$ satisfies the following two equations.
1) $X(j\omega) = 0$ for $0 \le \omega \le 3000\pi$ and for $\omega \ge 6000\pi$.
2) $X(j\omega) = \omega - 3000\pi$ for $3000\pi < \omega < 6000\pi$

a) Sketch $X(j\omega)$.
b) If $y(t) = x(t) \times \sum_{k=-\infty}^{\infty} \delta(t - k/3000)$, sketch $Y(j\omega)$ for $-6000\pi \le \omega \le 6000\pi$

Justify your answers.

Problem 5 (10 points)

Consider a *stable* and *causal* continuous-time LTI system S that has impulse response $h(t)$ and for which the input signal $x(t)$ and the output signal $y(t)$ are related by the following differential equation:

$$\frac{d^2}{dt^2}y(t) + 5\pi\frac{d}{dt}y(t) + 6\pi^2 y(t) = x(t)$$

Determine the value of ω for which $|H(j\omega)|$ has the greatest value. *Justify your answer.*

Problem 6 (10 points)

Consider a *stable* discrete-time LTI system S with impulse response $h[n]$. Let the DTFT of $h[n]$ be denoted by $H(e^{j\omega})$. The following information is given to you:

1) System S is causal.
2) $H(e^{j\omega}) = H^*(e^{-j\omega})$
3) $e^{j2\omega}H(e^{j\omega}) = e^{-j2\omega}H(e^{-j\omega})$
4) $h[0] = h[1]$=1
5) $H(e^{j\pi}) = 2$

Determine and sketch a signal $h[n]$ that is consistent with *__all__* the above information. *Justify your answer.*

Problem 7 (5 points)

Sketch a signal $x(t)$ such that:

$$X(j\omega) = \frac{4}{2\pi}\int_{-\infty}^{\infty}\left(\frac{\sin(1000\theta)}{\theta}\right)\left(\frac{\sin(500(\omega-\theta))}{(\omega-\theta)}\right)d\theta.$$

Justify your answer.