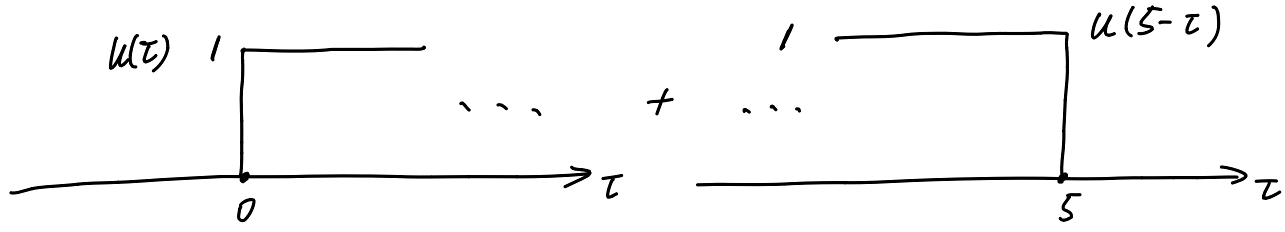
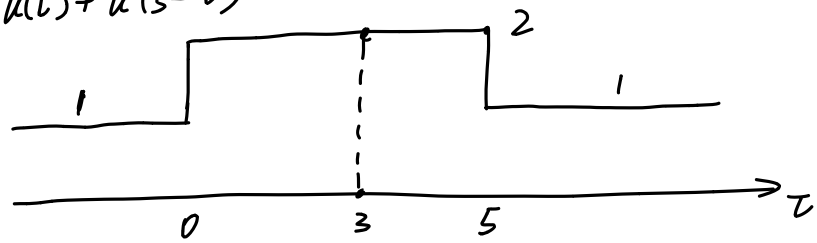
Test 2 solutions

1.



$= u(\tau) + u(5-\tau)$



Using the sifting property,

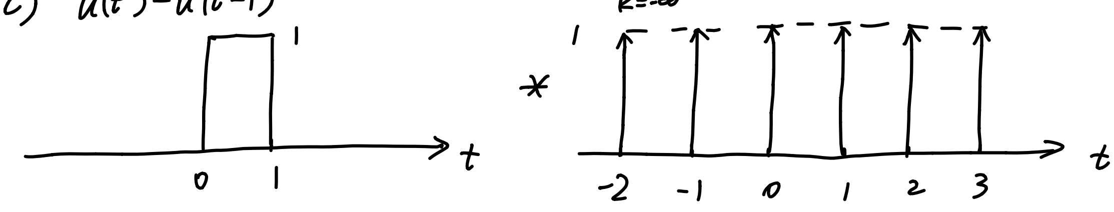
$$\int_{-\infty}^{+\infty} [u(\tau) + u(5-\tau)]\,\delta(\tau-3)\,d\tau = u(\tau) + u(5-\tau)\Big|_{\tau=3} = 2$$

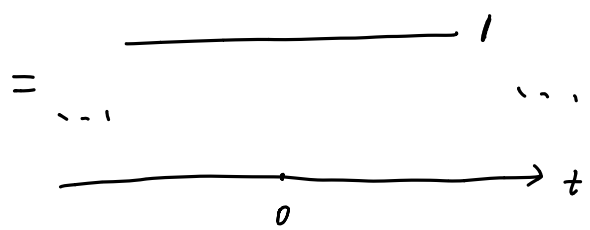
2. (a) $x_1(t) = j e^{j\frac{\pi}{4}t} = e^{j\frac{\pi}{2}} e^{j\frac{\pi}{4}t} = e^{j\frac{\pi}{4}(t+2)}$, it's a single complex exponential $e^{j\frac{\pi}{4}t}$ of frequency $\frac{\pi}{4}$ shifted to the left by 2.

(b) By the sifting property,

$x_3(t) = \int_{-\infty}^{+\infty} e^{j2000\tau}\,\delta(t-\tau)\,d\tau = e^{j2000t}$, a single complex exponential with frequency 2000

(c) $u(t) - u(t-1)$ $\quad \sum_{k=-\infty}^{+\infty} \delta(t-k)$



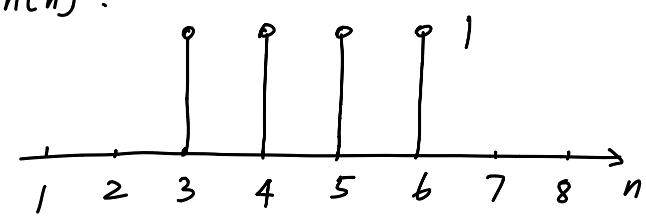


$x_4(t) = 1 = e^{j\cdot 0\cdot t}$, a single complex exponential with frequency 0.

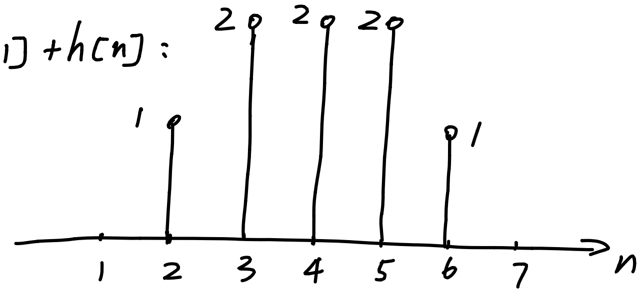
3. $x[n] = u[n+1] - u[n-2] = \delta[n+1] + \delta[n] + \delta[n-1]$

$$y[n] = x[n] * h[n] = (\delta[n+1] + \delta[n] + \delta[n-1]) * h[n]$$
$$= h[n+1] + h[n] + h[n-1]$$

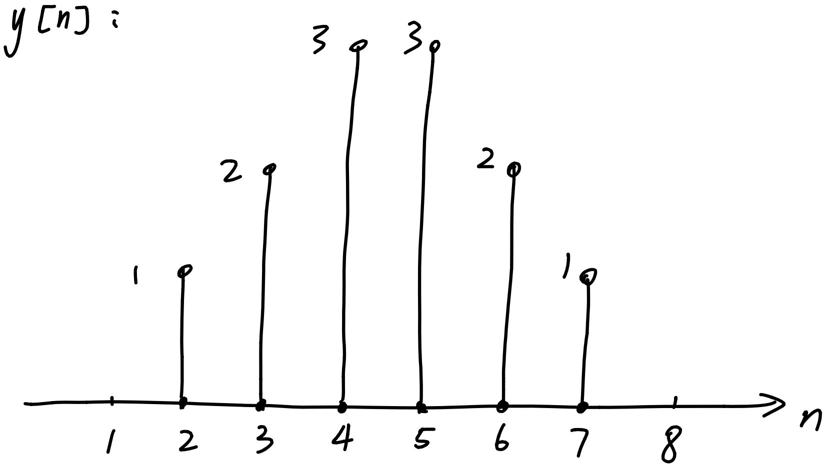
$h[n]$:



$h[n+1] + h[n]$:



$y[n]$:



4. The DTFT of a (discrete) box function is a (periodic) sinc function

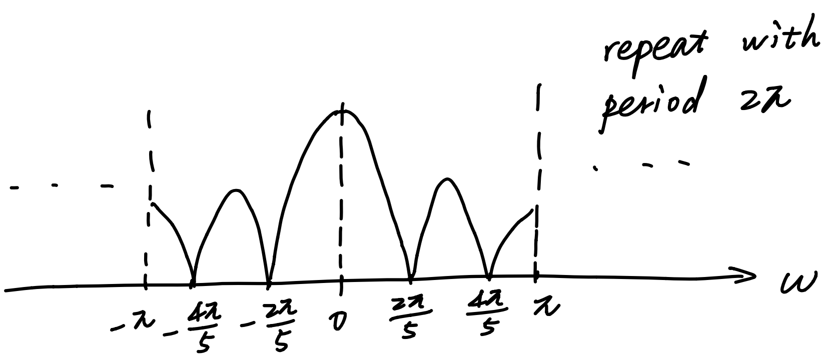
In this case $u[n] - u[n-5] \longleftrightarrow e^{-j2w} \cdot \dfrac{\sin(\frac{5w}{2})}{\sin(\frac{w}{2})}$

By convolution theorem,

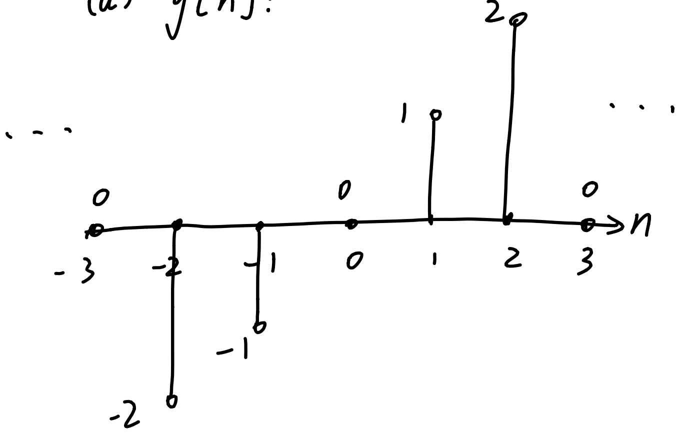
$$(u[n] - u[n-5]) * (u[n] - u[n-5]) \longleftrightarrow e^{-j4w} \cdot \left[\frac{\sin(\frac{5w}{2})}{\sin(\frac{w}{2})}\right]^2$$

$$|X(e^{jw})| = \left[\frac{\sin(\frac{5w}{2})}{\sin(\frac{w}{2})}\right]^2, \quad X(e^{j0}) = 25$$

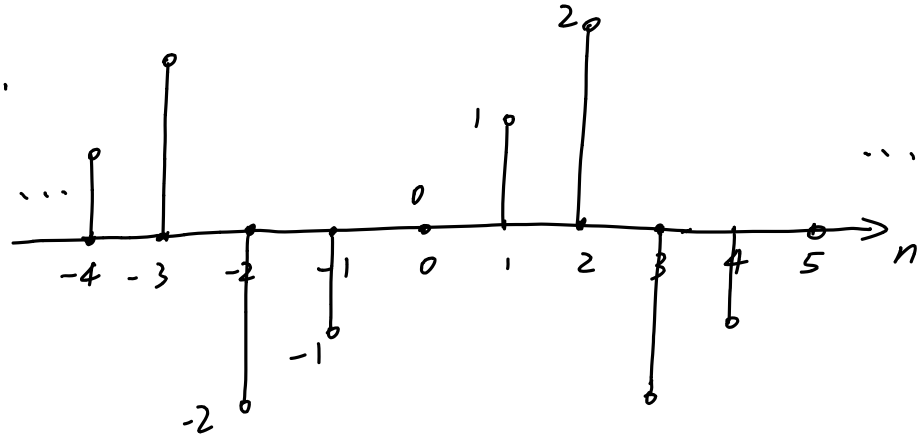
zero-crossings : $\frac{5w}{2} = k\pi \Rightarrow w = \frac{2k\pi}{5}$ ($k \neq 0$, integar)



5. (a) $g[n]$:



$x[n]$: $g[n]$ repeated every 5



OR $$x[n-5] = \sum_{k=-\infty}^{+\infty} g[n-5(k+1)] = \sum_{k=-\infty}^{+\infty} g[n-5k] = x[n]$$

so $x[n]$ is periodic with period 5

(b) Since $x[n]$ has a period of 5, it can be uniquely decomposed by complex exponentials with frequency $\frac{2\pi k}{5}$ (k is integar).

$$h[n] = u[n] - u[n-5] \longleftrightarrow H(e^{jw}) = e^{-j2w} \frac{\sin(\frac{5w}{2})}{\sin(\frac{w}{2})}$$

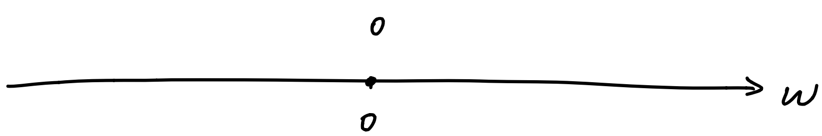
clearly the frequency response has zero-crossings at $\frac{2k\pi}{5}$ ($k \neq 0$, integar)

$H(e^{j0}) = 5$. so the output DTFT only has component of frequency 0.

Since the input signal $x[n]$ has a DC constant $= 0$ (0 frequency)

the output is 0

$$Y(e^{jw}) = H(e^{jw}) \cdot X(e^{jw}) = 0$$



6.

$$x[n] = u[n] - u[n-10] \longleftrightarrow X(e^{jw}) = e^{-j\frac{9}{2}w} \cdot \frac{\sin(5w)}{\sin(\frac{w}{2})}$$

it has zero-crossings at $5w = k\pi \Rightarrow w = \frac{k\pi}{5}$ ($k \neq 0$, integar)

$$y[n] = \cos(\frac{\pi n}{5}) \longleftrightarrow Y(e^{jw}) = \pi[\delta(w-\frac{\pi}{5}) + \delta(w+\frac{\pi}{5})] \quad (-\pi \leq w < \pi)$$

has two spikes at $w = -\frac{\pi}{5}$ and $\frac{\pi}{5}$.

$$Y(e^{j\frac{\pi}{5}}) = H(e^{j\frac{\pi}{5}}) \cdot X(e^{j\frac{\pi}{5}})$$

$\downarrow$ $+\infty$ $\quad$ $\downarrow$ impossible $\quad$ $\downarrow$ 0

So there's no G that produces such output.

7. $$q[n] = (-1)^n \cos(\frac{\pi}{5}n) = e^{j\pi n} \cdot \frac{1}{2}(e^{j\frac{\pi}{5}n} + e^{-j\frac{\pi}{5}n})$$

$$= \frac{1}{2}(e^{-j\frac{4\pi}{5}n} + e^{j\frac{4\pi}{5}n})$$

$\Updownarrow$

$$Q(e^{jw}) = \pi[\delta(w+\frac{4\pi}{5}) + \delta(w-\frac{4\pi}{5})] \quad (-\pi \leq w < \pi)$$

$|Q(e^{jw})|$: