EC401 PRACTICE FINAL EXAM (Spring 2020)

60 minutes, Open Book, No Collaboration Allowed, Formula Sheet Provided.

Throughout this test, $\delta[n]$ and $u[n]$ denote the unit impulse and unit step respectively.

Problem 1 (10 points)

If $x(t) = t\{u(t+1) - u(t-1)\}$, sketch the signal $g(t) = x(-1-t)$. *Justify your answer.*

Problem 2 (10 points)

Let S be an LTI system with impulse response $h[n] = (0.5)^n u[n]$ and let the input signal to S be $x[n] = (0.5)^n u[n]$. If the output signal for that input is denoted by $y[n]$, determine the values of $y[0]$ and $y[2]$. *Justify your answers.*

Problem 3 (10 points)

Let $x[n] = \begin{cases} 1 & \text{for } (n-1) \text{ divisible by } 4 \\ 0 & \text{otherwise} \end{cases}$

a) Sketch $x[n]$. *Justify your answer.*
b) Sketch $|X(e^{j\omega})|$, the magnitude of the DTFT of $x[n]$. *Justify your answer.*

Problem 4 (10 points)

Consider a continuous-time LTI system S with impulse response

$$h(t) = \frac{\sin(400\pi t)}{\pi t}.$$

Determine and sketch the output signal of system S if the input signal is

a) $x_1(t) = \delta(t)$
b) $x_2(t) = \frac{\sin(4000\pi(t-1))}{\pi(t-1)}$

Justify your answers.

Problem 5 (10 points)

Consider a *causal* continuous-time LTI system S whose input and output are related by the following differential equation:

$$\frac{d^2}{dt^2}y(t) + 3\frac{d}{dt}y(t) + 2y(t) = x(t)$$

a) Is the system S *stable*? *Justify your answer*.
b) Sketch the output of system S if the input is $x(t) = 1$ for all t. *Justify your answer.*

Problem 6 (10 points)

Consider a discrete-time LTI system S with impulse response $h[n]$. Let the DTFT of $h[n]$ be denoted by $H(e^{j\omega})$. The following information is given to you:

1) $h[n]$ is a real-valued signal.
2) S is a causal system
3) $H\left(e^{j0.5\pi}\right) = H(e^{j\pi})=0$
4) $e^{j\left(\frac{3}{2}\right)\omega}H(e^{j\omega})$ is real.
5) $\sum_{n=-\infty}^{\infty} h[n] = 8$

Determine and sketch a signal $h[n]$ that is consistent with <u>*all*</u> the above information. *Justify your answer*.

Problem 7 (5 points)

Determine the numerical value of $A = \int_{-\infty}^{\infty}\{\frac{\sin{(1000\pi t)}}{\pi t}(\sum_{k=-\infty}^{\infty}\delta\left(t - \left(\frac{1}{750}\right)k\right))\}dt$