EC401 TEST 2 (Spring 2020)

***60 minutes**, Open Book, Formula Sheet Provided.*

*You will have **15 minutes to upload your answers** on BB after finishing Exam.*

***Please don't consult anyone while taking this Exam**.*

*Please be on **Zoom with Video** on while taking this Exam*

Total of Seven Problems on This Exam.

Throughout this test, $\delta[n]$ and $u[n]$ denote the unit impulse and unit step respectively in discrete time. Also $\delta(t)$ and $u(t)$ denote the unit impulse and unit step respectively in continuous time.

Problem 1 (10 points)

Determine the numerical value of the integral specified as:

$$\int_{-\infty}^{\infty} \{u(\tau) + u(5-\tau)\}\delta(\tau - 3)d\tau$$

Justify your answer.

Problem 2 (10 points)

For the signal given in each part of this problem, determine whether or not it can be written as a *time-shifted* version of a *single continuous-time complex exponential signal of a particular frequency.* Justify your answer in each case.

(a) $x_1(t) = je^{j0.25\pi t}$
(b) $x_3(t) = \int_{-\infty}^{\infty} e^{j2000\tau}\delta(t-\tau)d\tau$
(c) $x_4(t) = \{u(t) - u(t-1)\} * \sum_{k=-\infty}^{\infty} \delta(t-k)$, where * denotes convolution.

Problem 3 (10 points)

If $x[n] = u[n+1] - u[n-2]$ and $h[n] = u[n-3] - u[n-7]$, sketch the convolution specified as

$$y[n] = x[n] * h[n].$$

Justify your answer.

Problem 4 (10 points)

Given that $*$ denotes convolution, sketch the magnitude of the DTFT of the signal specified as

$$x[n] = \{u[n] - u[n-5]\} * \{u[n] - u[n-5]\}$$

Justify your answer.

Problem 5 (10 points)

Consider a discrete-time LTI system *S* with impulse response

$$h[n] = u[n] - u[n-5].$$

You are given that $g[n] = n\{u[n+2] - u[n-3]\}$, and the input signal to system *S* is

$$x[n] = \sum_{k=-\infty}^{\infty} g[n-5k]$$

(a) Is the given input signal $x[n]$ periodic? *Justify your answer*.
(b) Sketch the magnitude of the DTFT of the output signal $y[n]$ of *S* for the input signal $x[n]$ given above in this problem. *Justify your answer*

Problem 6 (10 points)

Does there exist a Linear Time-Invariant (LTI) system *G* that produces output $y[n] = \cos\left(\frac{\pi n}{5}\right)$ when the input to *G* is $x[n] = u[n] - u[n-10]$? *Justify your answer*.

Problem 7 (5 points)

Sketch the magnitude of the DTFT of $q[n] = (-1)^n \cos\left(\frac{\pi n}{5}\right)$ for $-\pi \leq \omega < \pi$.