HW 4 Solution

1. a) $S: x[n] \longrightarrow y[n] = (x[n])^2$, take any $n_0 \in \mathbb{Z}$

$S: x_1[n] = x[n-n_0] \longrightarrow y_1[n] = (x_1[n])^2 = (x[n-n_0])^2$

$y_2[n] = y[n-n_0] = (x[n-n_0])^2 = y_1[n]$, so ~~$\nexists$~~ S is time-invariant

b) $S: x[n] \longrightarrow y[n] = x[n-1] + 2x[n+1]$, take any $n_0 \in \mathbb{Z}$

$S: x_1[n] = x[n-n_0] \longrightarrow y_1[n] = x_1[n-1] + 2x_1[n+1]$

$= x[n-n_0-1] + 2x[n+1-n_0]$

$y_2[n] = y[n-n_0] = x[n-n_0-1] + 2x[n-n_0+1] = y_1[n]$, So S is time-invariant

c) $S: x[n] \longrightarrow y[n] = \cos(x[n-1])$, take any $n_0 \in \mathbb{Z}$

$S: x_1[n] = x[n-n_0] \longrightarrow y_1[n] = \cos(x_1[n-1]) = \cos(x[n-n_0-1])$

$y_2[n] = y[n-n_0] = \cos(x[n-n_0-1]) = y_1[n]$, so S is time-invariant

2. a) Take $x[n] = \delta[n]$, $n_0 = 1$

$S:$ ~~$\delta[n] \longrightarrow$~~ $x[n] = \delta[n] \longrightarrow y[n] = (n-1)^2\delta[n+1] = 4\delta[n+1]$

$S: x_1[n] = \delta[n-1] \longrightarrow y_1[n] = (n-1)^2 x_1[n+1] = (n-1)^2\delta[n] = \delta[n]$

$y_2[n] = y[n-1] = 4\delta[n] \underset{\triangle}{\neq} \delta[n] = y_1[n]$, so S is not time-Invariant

b) Take $x[n] = \delta[n]$, $n_0 = -1$

$S: x[n] = \delta[n] \longrightarrow y[n] = u[n]\cdot x[n] = u[n]\cdot\delta[n] = \delta[n]$

$S: x_1[n] = x[n+1] = \delta[n+1] \longrightarrow y_1[n] = u[n]\cdot\delta[n+1] = 0$

$y_2[n] = y[n+1] = \delta[n+1] \neq 0 = y_1[n]$, So S is not time-Invariant

c) Take $x[n] = \delta[n]$, ~~$n_0 = \pm 1$~~ $n_0 = 1$

$S: x[n] = \delta[n] \longrightarrow y[n] = (-1)^n x[n] = (-1)^n \delta[n] = \delta[n]$

$S: x_1[n] = x[n-1] = \delta[n-1] \longrightarrow y_1[n] = (-1)^n x_1[n] = (-1)^n \delta[n-1] = -\delta[n-1]$

$y_2[n] = y[n-1] = \delta[n-1] \neq -\delta[n-1] = y_1[n]$, so S is not time-invariant

3. a) S is not causal;

$y[n] = \cancel{x} \cos(\underbrace{x[n+5]}_{n+5>n})$, the output depends on input $x[n+5]$, which is at future times, so it's not causal system.

b) S is not causal;

$$y[n] = \sum_{k=-1}^{100} (0.5)^k x[n-k] = 2\underbrace{x[n+1]}_{n+1>n.} + \sum_{k=0}^{100} (0.5)^k x[n-k]$$

the output depends on input $x[n+1]$, which is at future times, not causal.

c) S is not causal;

$y[n] = x[-n]$, take $n=-1$, then $y[-1] = \underbrace{x[+1]}_{+1>-1.}$ the output depends on future times' input, not causal.

d) S is causal;

$y[n] = \underbrace{2\cos(n+1)}_{\text{not input}} \underbrace{(x[n-3])^3}_{\text{input at past times}}$, the output depends on a known signal and the input at past times, so it's causal.

4)

For all parts of this question assume x[n] is bounded.

In other words, assume $|x[n]| \leqslant B$ → finite number

a)

$$|y(n)| = |(x[n-3])^3| = |\underbrace{x[n-3]}_{\leqslant B}|^3 \leqslant B^3 \leftarrow \text{finite number}$$

So, $\boxed{S \text{ is stable}}$

b)

$$|y[n]| = |\sin(x[n])| \leqslant 1$$

So, $\boxed{S \text{ is stable}}$

c)

$$|y[n]| = |2x[n-1] + x[n-3] + 2| \leqslant 3B + 2$$

So, $\boxed{S \text{ is stable}}$

d) $y[n] = \begin{cases} \frac{x[n]}{|x[n]|} & \text{if } x[n] \neq 0 \\ 0 & \text{if } x[n] = 0 \end{cases}$

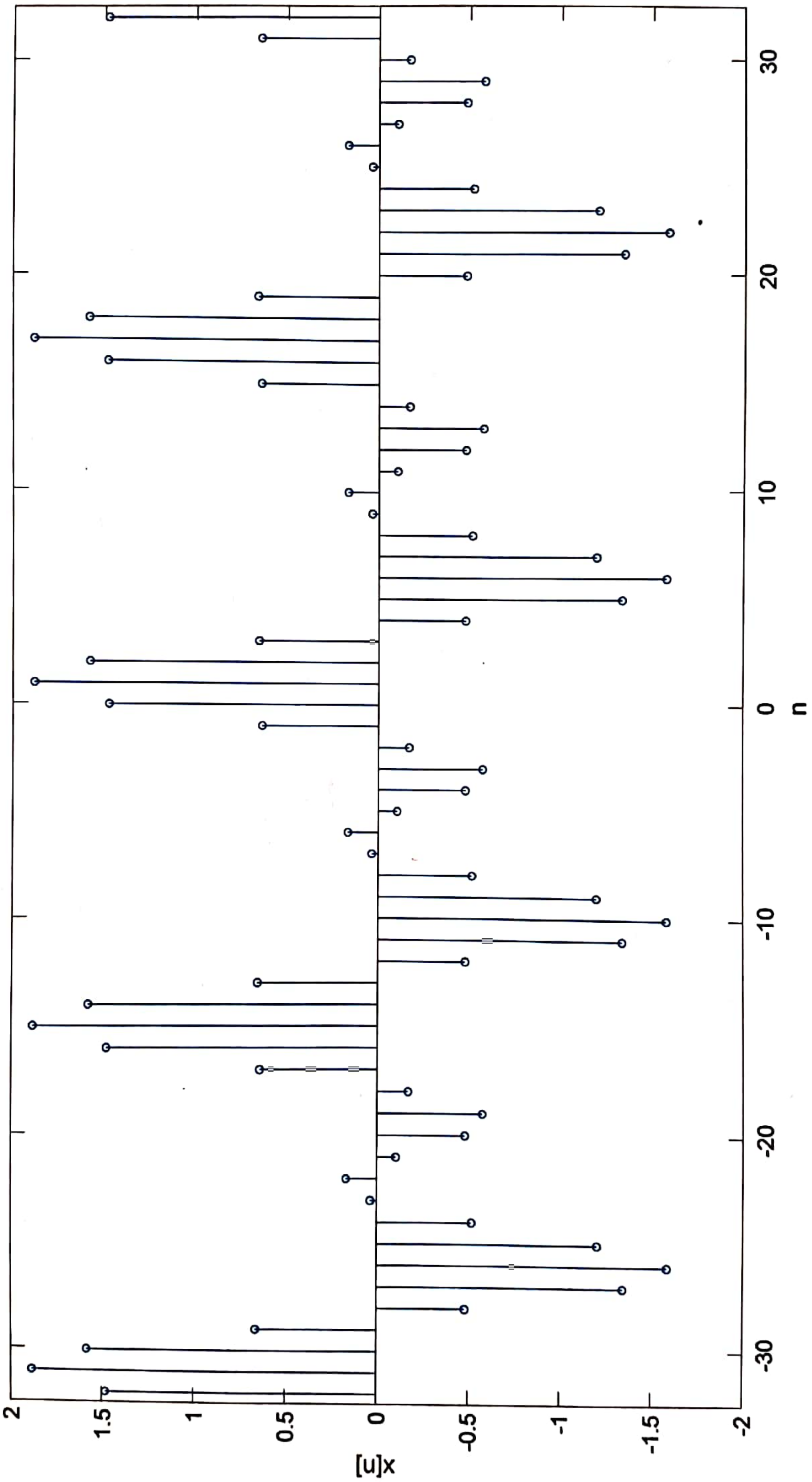
$|y[n]| = \begin{cases} 1 & \text{if } x[n] \neq 0 \\ 0 & \text{if } x[n] = 0 \end{cases}$

$|y[n]|$ can be either "1" or "0". So, it will be bounded.

$\boxed{S \text{ is stable}}$

e) $|y[n]| = |(0.5)^{n-1} x[n]| = \underbrace{|(0.5)^{n-1}|}_{\leq \infty} \cdot \underbrace{|x[n]|}_{\leq B} \leq \infty$

$\boxed{S \text{ is NOT stable}}$



5)

$$x[n] = \cos(0.125\pi n) + \sin(0.25\pi n + 0.16\pi)$$

$$T_1 = \frac{2\pi}{0.125\pi} = 16 \qquad T_2 = \frac{2\pi}{0.25\pi} = 8$$

Period of $x[n]$ will be equal to least common multiple of T_1 and T_2. So, $lcm(T_1, T_2) = 16$. This indicates $x[n]$ repeats every 16 units.

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% Code for Homework 4 Problem 5
clc; clear all; close all;

n = -32:32;

x = cos(0.125*pi*n) + sin( (0.25*pi*n) + 0.16*pi );


figure

stem(n, x);
xlabel('n')
ylabel('x[n]')
ylim([-2 2])
xlim([-32.5 32.5])
set(gca,'fontsize', 18);
```