# EC401 HW07 Spring 2020

## Due Date: Wednesday April 01, 2020

*You must submit your homework in pdf form to the EC401 Blackboard Learn site by 12:30pm on the due date. Please be sure to write your name on the first page of the homework you submit. Additionally, if you have collaborated on the homework with other individuals enrolled in EC401 this semester, please identify them as your collaborators on the first page of the submitted homework.*

## HW07.1

For each of the parts of this problem, determine and sketch the *magnitude* of the discrete-time Fourier transform of the given signal. *Show your work.*

(a) $x[n] = u[n] - u[n-3]$

(b) $x[n] = u[n-1] - u[n-4]$

(c) $x[n] = u[n+2] - u[n-1]$

(d) $x[n] = u[n] - u[n-10]$

## HW07.2

For each of the parts of this problem, determine the discrete-time Fourier transform (DTFT) of the given signal and sketch the *magnitude* of the DTFT. *Show your work.*

(a) $x[n] = (0.5)^{n-1}u[n]$

(b) $x[n] = 2^n u[-n-1]$

(c) $x[n] = (-1)^n$

(d) $x[n] = 1 + \cos\left(\frac{\pi n}{5}\right) + \cos\left(\frac{2\pi n}{5}\right) + \cos\left(\frac{3\pi n}{5}\right) + \cos\left(\frac{4\pi n}{5}\right)$

(e) $x[n] = \sin\left(\frac{\pi n}{5}\right) + \sin\left(\frac{2\pi n}{5}\right) + \sin\left(\frac{3\pi n}{5}\right) + \sin\left(\frac{4\pi n}{5}\right)$

(f) $x[n] = \cos\left(\frac{2\pi n}{5}\right)\sin\left(\frac{4\pi n}{5}\right)$

# HW07.3

Throughout this problem, let $x[n]$ be a signal whose discrete-time Fourier transform (DTFT) is $X(e^{j\omega})$.

(a) Show that the magnitude of the DTFT of x[n]=cos $(\frac{\pi n}{17})$ is an even function of frequency.
(b) Show that the magnitude of the DTFT of x[n]=sin $(\frac{\pi n}{17})$ is an even function of frequency.
(c) Show that if $x[n]$ is any *real* signal, then $|X(e^{j\omega})|$ is an *even* function of frequency. (HINT: Use the fact that a real number is one that is equal to its own conjugate).
(d) Show that if $x[n]$ is any *real* and *even* signal, then $X(e^{j\omega})$ is also *real* and *even*.
(e) Give an example of a real and even signal whose DTFT is a real and even function of frequency.
(f) Show that the DTFT of $(-1)^n x[n]$ is $X(e^{j(\omega-\pi)})$

# HW07.4

Let $x[n] = u[n-1] - u[n-4]$ and let $h[n] = u[n-1] - u[n-4]$

a) Use the convolution sum to determine and sketch $y[n] = x[n] * h[n]$
b) Determine and sketch the magnitude of the DTFT of $x[n]$
c) Determine and sketch the magnitude of the DTFT of $y[n]$ using the property of the DTFT that the DTFT of the convolution of two signals is the product of their DTFTs.

# HW07.5

Determine and sketch the magnitude of the DTFT of the following signals. Your sketch of the magnitude of the DTFT must be for $-\pi \leq \omega \leq \pi$.

(a) $x[n] = e^{j5\pi n/2}$

(b) $x[n] = e^{-\frac{j25\pi n}{4}}$

(c) $x[n] = \cos\left(\frac{17\pi}{3}n + \frac{\pi}{6}\right)$

(d) $x[n] = \sin\left(\frac{17\pi}{3}n + \frac{\pi}{6}\right)$

## HW07.6

Consider an LTI system **S** with impulse response $h[n] = u[n] - u[n - 6]$.

(a) Sketch and label the magnitude of the DTFT of $h[n]$

(b) Determine and sketch the output of S when the input signal is $x[n] = 1$.

(c) Determine and sketch the output of S when the input signal is $x[n] = (-1)^n$

(d) Determine and sketch the output of S when the input signal is $x[n] = \cos\left(\frac{\pi n}{3}\right)$

(e) Determine all values of $\omega_0$ in the range $-\pi < \omega_0 \leq \pi$ for which it is *guaranteed* that if the input to **S** is $x[n] = e^{j\omega_0 n}$, then the output of **S** is $y[n] = 0$. Justify your answer.