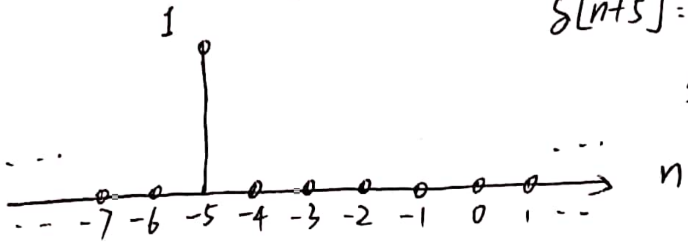
HW 2 solution

$\delta[n]$: start-time = end-time = 0

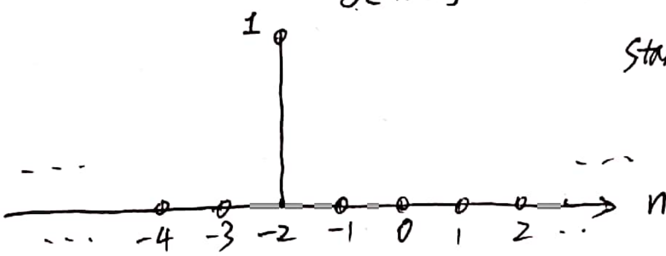
1. a) $\delta[n+5]$



$\delta[n+5]$: $n+5=0 \Rightarrow n=-5$

start-time = end-time = -5.

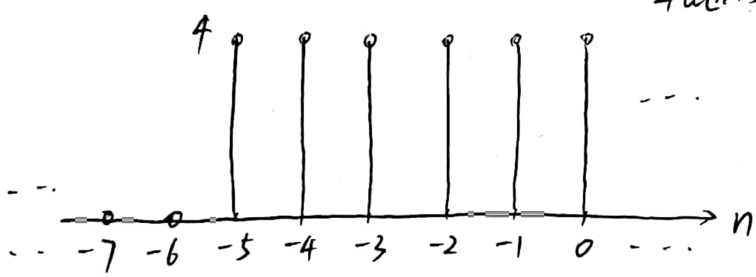
b) $\delta[-n-2]$



$\delta[-n-2]$: $-n-2=0 \Rightarrow n=-2$

start-time = end-time = -2.

c)



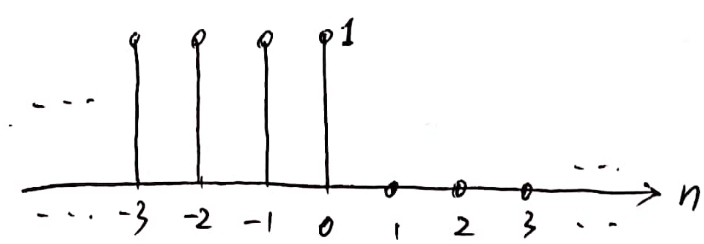
$4u[n+5]$: $n+5=0 \Rightarrow n=-5$

$n+5=\infty \Rightarrow n=\infty$

start-time = -5

end-time = ∞

d)



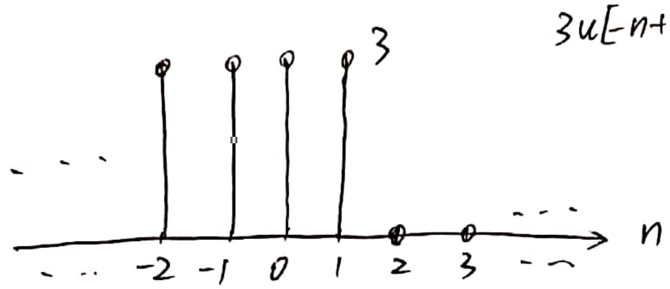
$u[-2n]$: $-2n=0 \Rightarrow n=0$

$-2n=\infty \Rightarrow n=-\infty$

start-time = 0

end-time = $-\infty$

e)



$3u[-n+1]$: $-n+1=0 \Rightarrow n=1$

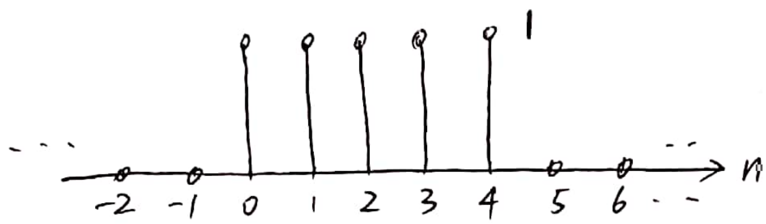
$-n+1=\infty \Rightarrow n=-\infty$

start-time = 1

end-time = $-\infty$, ✓

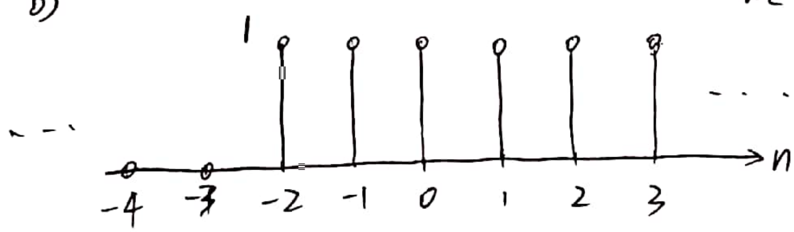
2. a)

$u[n]-\underline{u[n-5]}$ ← $u[n]$ shift to the right by 5



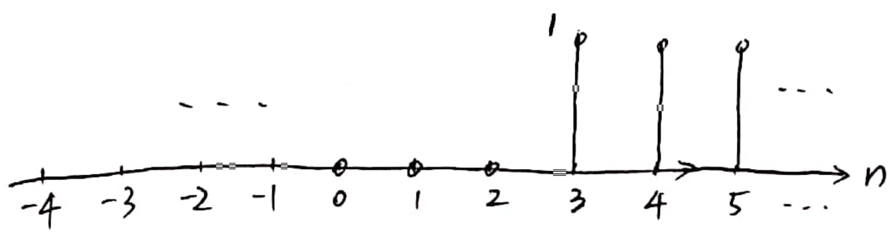
b)

$u[n+2]$ ← $u[n]$ shift to the left by 2



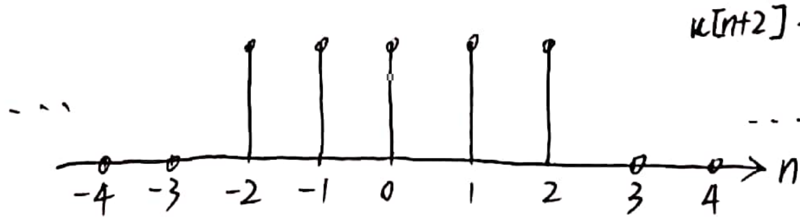
$-$

$u[n-3]$ ← $u[n]$ shift to the right by 3



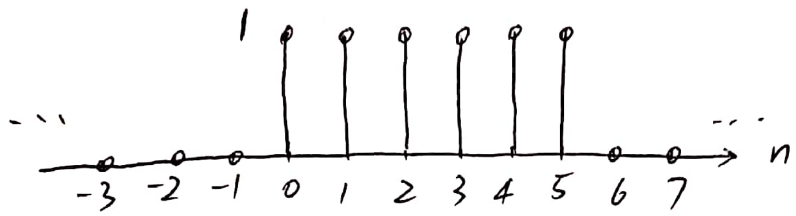
$=$

$u[n+2]-u[n-3]$



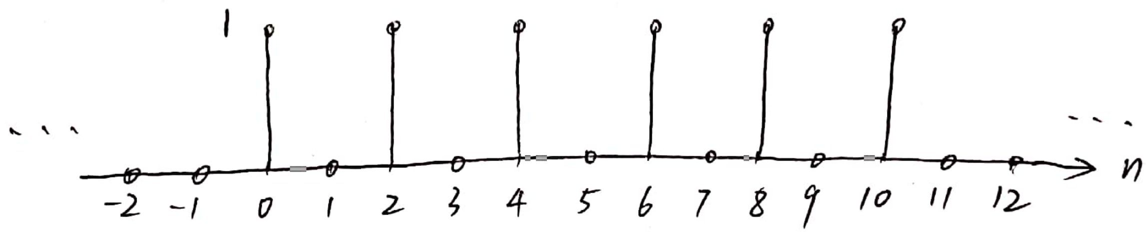
c) $\sum_{k=0}^{5} \delta[n-k] = \delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3] + \delta[n-4] + \delta[n-5]$

$\delta[n]$ shift to the right by 1 ... $\delta[n]$ shift to the right by 5



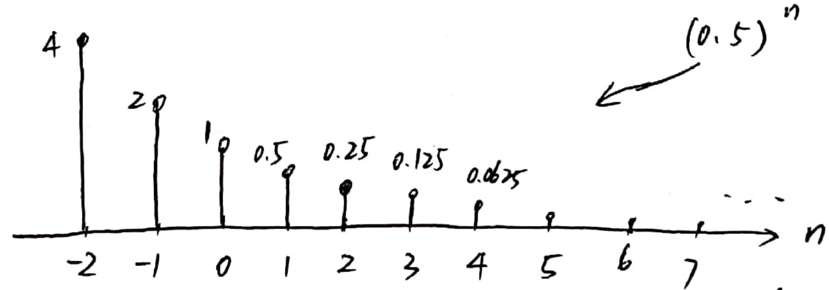
d) $\sum_{k=0}^{5} \delta[n-2k] = \delta[n] + \delta[n-2] + \delta[n-4] + \delta[n-6] + \delta[n-8] + \delta[n-10]$

$\delta[n]$ shift to the right by 2 ... $\delta[n]$ shift to the right by 10



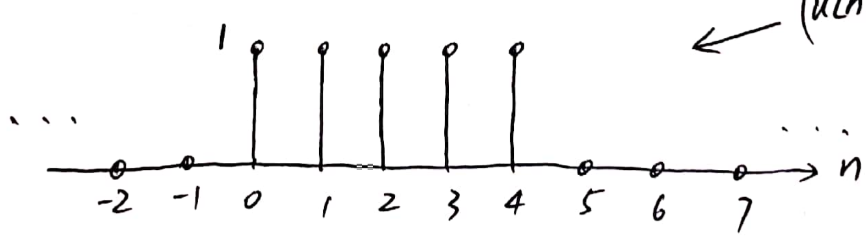
e)

$(0.5)^n$



$\times$

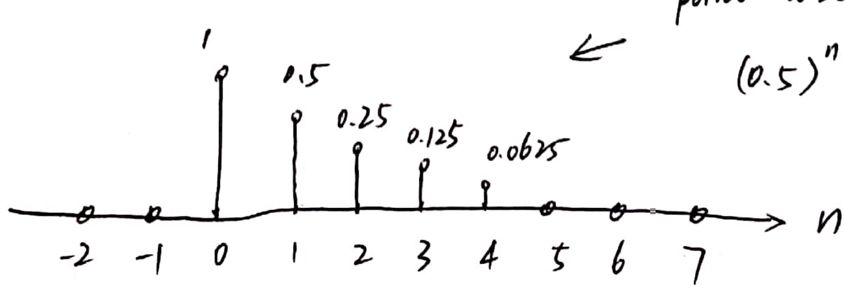
$(u[n] - u[n-5])$



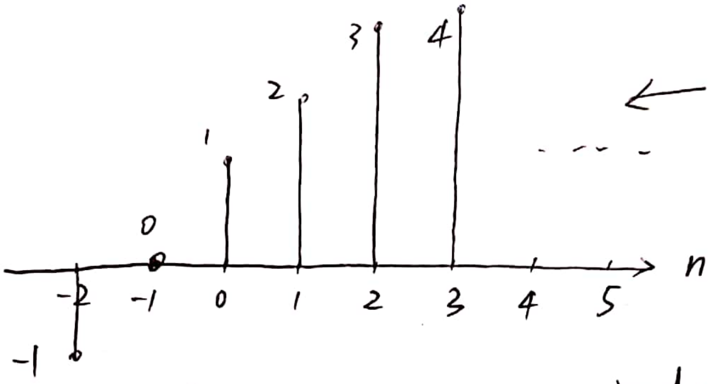
point-wise multiplication of

$(0.5)^n \times (u[n] - u[n-5])$

$=$

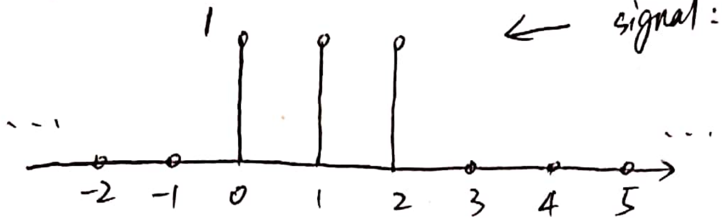


3. a)



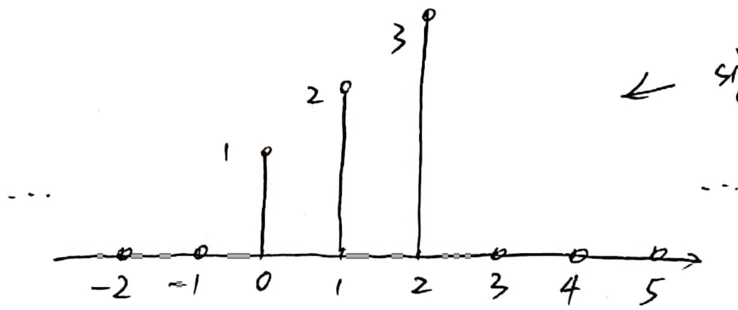
← signal : n+1

X

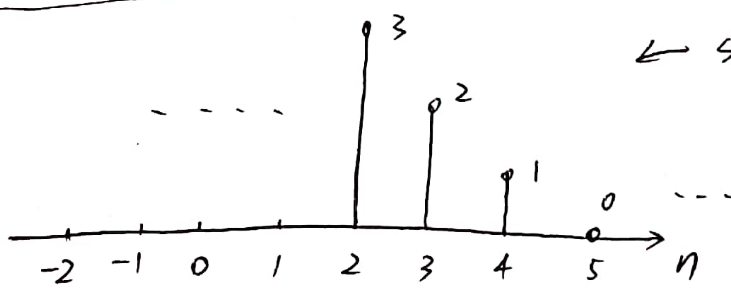


← signal: u[n] - u[n-3]

=

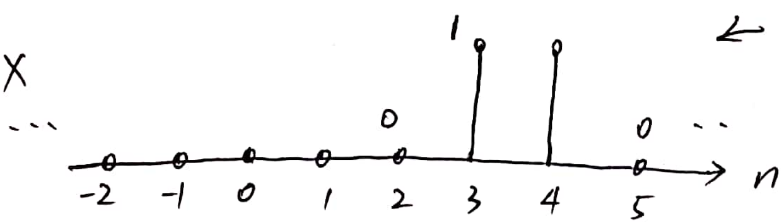


← signal : (n+1) · (u[n] - u[n-3])



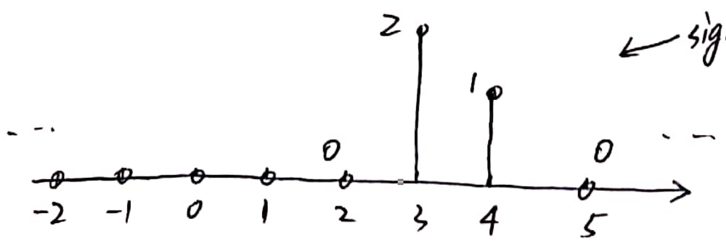
← signal : 5-n

X

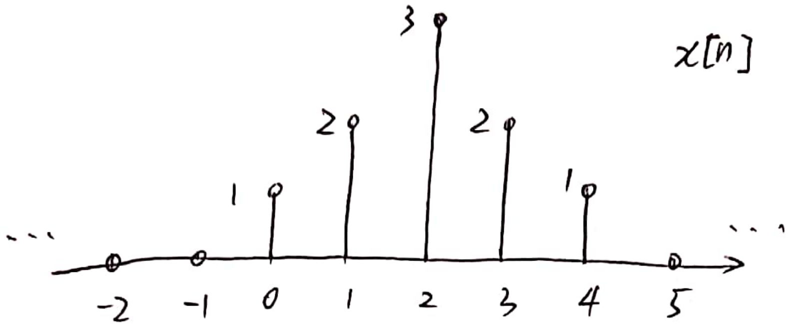


← signal: u[n-3] - u[n-5]

=

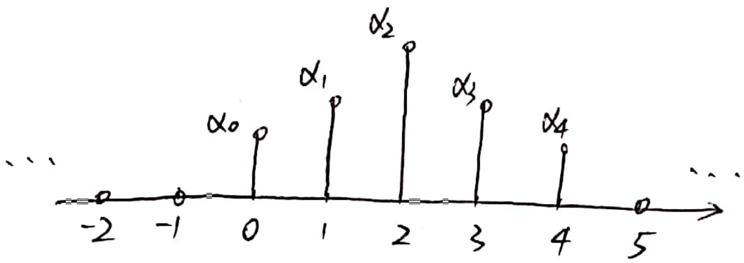


← signal: (5-n) (u[n-3] - u[n-5])



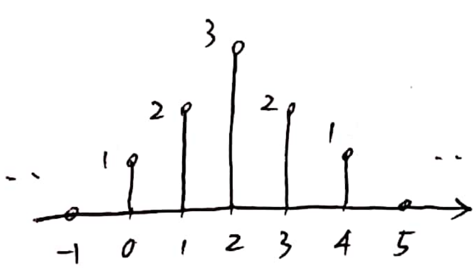
b) $x[n] = \sum_{k=0}^{4} \alpha_k \, \delta[n-k]$

$= \alpha_0 \, \delta[n] + \alpha_1 \, \delta[n-1] + \alpha_2 \, \delta[n-2] + \alpha_3 \, \delta[n-3] + \alpha_4 \, \delta[n-4]$

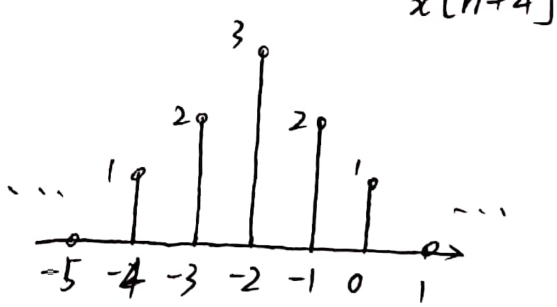


By comparing the two signals, and the uniqueness of the impulse decomposition, $\alpha_0 = 1$, $\alpha_1 = 2$, $\alpha_2 = 3$, $\alpha_3 = 2$, $\alpha_4 = 1$;

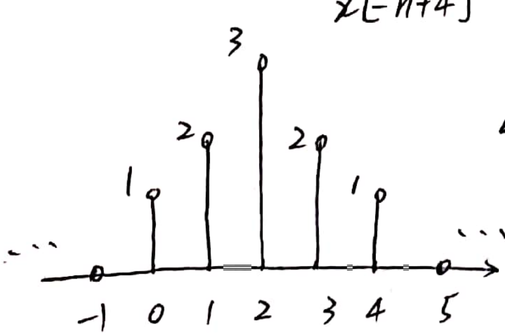
c)



shift 4 $\longrightarrow$ to the left



flip about $x=0$



$x[n]$: start-time $= 0$, end-time $= 4$

$x[-n+4]$: $-n+4 = 0 \Rightarrow n = 4$

$-n+4 = 4 \Rightarrow n = 0$

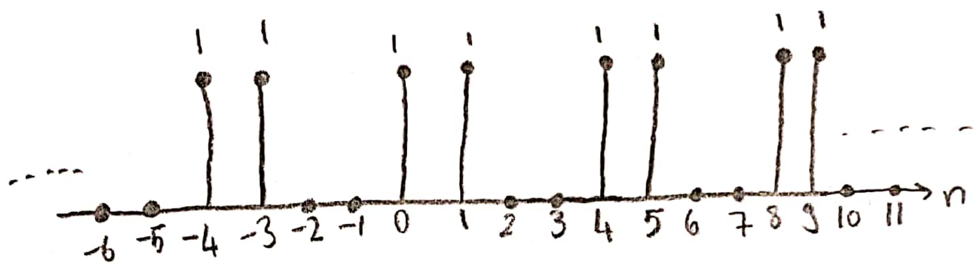
start-time $= 4$, end-time $= 0$, ✓

4)

$$x[n] = \sum_{k=-\infty}^{\infty} \delta[n-4k] + \delta[n-1-4k]$$

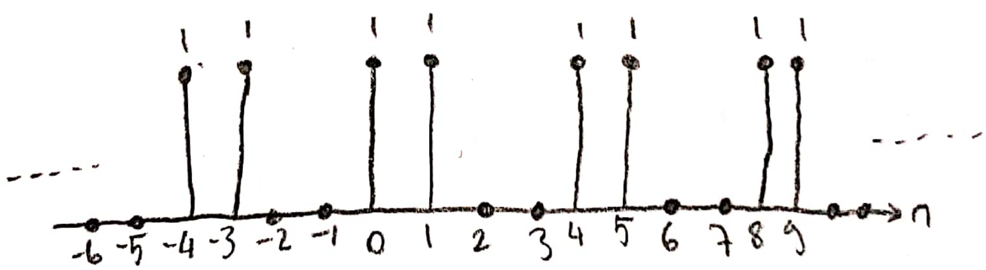
a)

$x[n] =$ ....



b) $x[n-4]$ → Shift the plot of $x[n]$ to right by 4

$x[n-4] =$ -----



Note that, plot $x[n]$ is same as $x[n-4]$.

Because $x[n]$ is a periodic signal and it has a period of 4.

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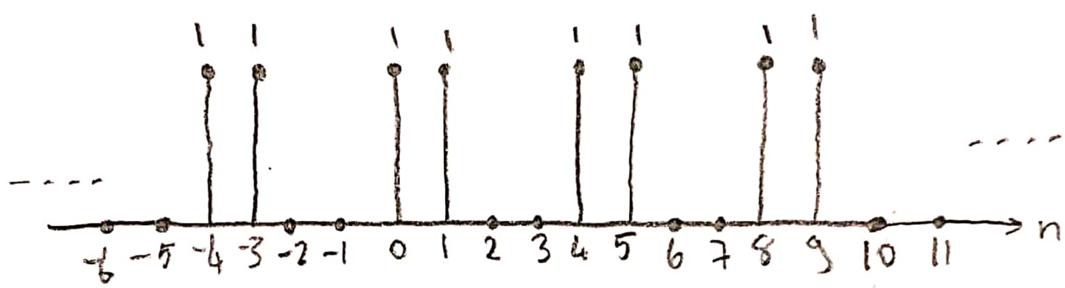
b) (continued...)

$x[n-8] \rightarrow$ shift $x[n]$ by 8 to the right

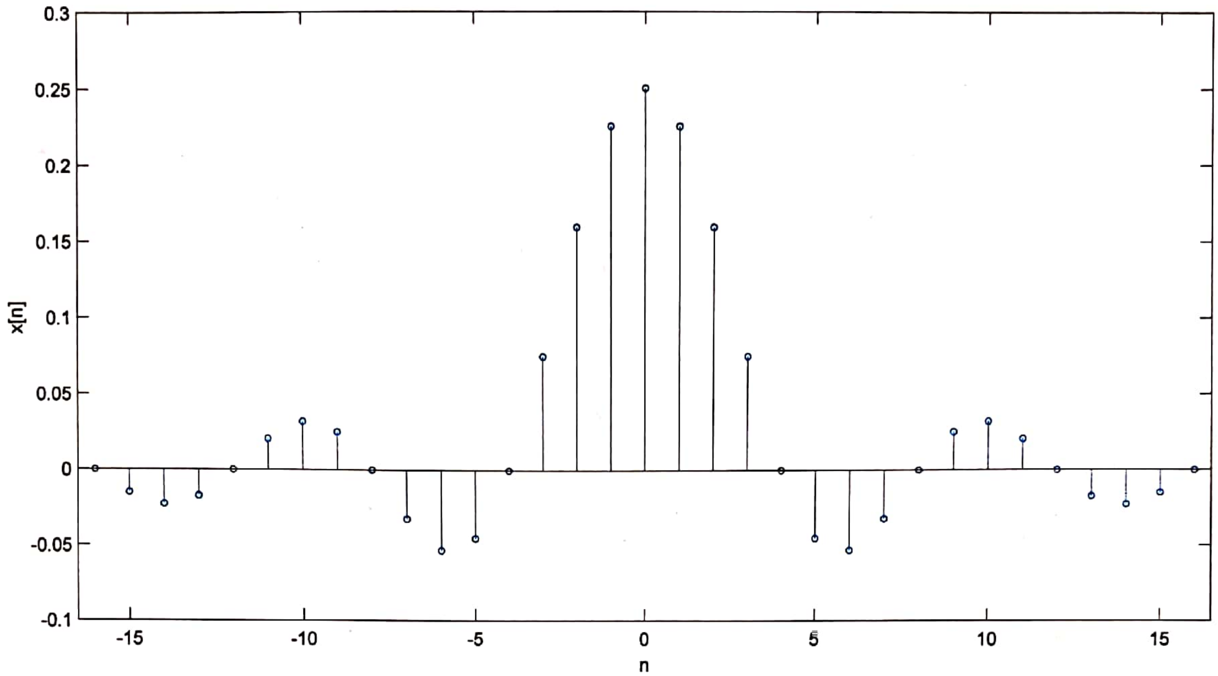
$x[n-8] =$



* Plot of $x[n-8]$ is same as $x[n]$.

$x[n]$ has a period of 4. So, if we shift $x[n]$ with an integer multiple of 4, resulting signal will be same as $x[n]$.

5)



x[n] equals to zero for $n = -16, -12, -8, -4, 4, 8, 12, 16$

In other words, x[n] is equal to zero when "n" is an integer multiple of 4 (except 0).

$$x[n] = \frac{\sin(0.25\pi n)}{\pi n}$$ → x[n] will be zero when $\sin(0.25\pi n)$ is zero.

* Note that, $n=0$, makes denominator equal to "0". So, we need to use L'hospital rule to evaluate the function at $n=0$

sin function equals to zero when its argument is integer multiple of π → $\sin(0.25\pi n) = 0$

$$0.25\pi n = \pi k, \quad k \in \mathbb{Z}$$

$$\boxed{n = 4k}$$

when "n" is an integer multiple of 4, x[n] will be zero. (Except zero)

```matlab
% Code for Homework 1 Problem 5
clc; clear all; close all;

x=zeros(33,1);

for n=-16:16

    ind=n+17;

    if n==0
        x(ind,1)= 0.25;
    else
        x(ind,1)= sin(0.25*pi*n) / (pi*n);
    end

end


figure
stem(-16:16, x);
xlabel('n')
ylabel('x[n]')
ylim([-0.1 0.3])
xlim([-16.5 16.5])
set(gca,'fontsize', 18);
```