

## TEST 1 FORMULA SHEET EC401 (SPRING 2020)

### DT Unit Step (Switch)

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

### DT Unit Impulse (Atom):

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & \text{otherwise} \end{cases}$$

**Discrete-Time Impulse decomposition of a Signal:**  $x[n] = \sum_{k=-\infty}^{\infty} x[k] \delta[n-k]$

**Even Signal:**  $x[n] = x[-n]$

**Odd Signal:**  $x[n] = -x[-n]$

$$\text{Even}\{x[n]\} = \frac{x[n] + x[-n]}{2}$$

$$\text{Odd}\{x[n]\} = \frac{x[n] - x[-n]}{2}$$

### Basic Signal Operations:

*Shift:*  $y[n] = x[n - n_0]$

*Flip:*  $y[n] = x[-n]$

*Compress:*  $y[n] = x[Mn]$

*Expand:*  $y[n] = \begin{cases} x\left[\frac{n}{M}\right] & \text{if } n > M \\ 0 & \text{otherwise} \end{cases}$

### Multi-Signal Operations:

*Linear Combination:*  $y[n] = \alpha_1 x_1[n] + \alpha_2 x_2[n]$

*Product:*  $y[n] = g[n]h[n]$

**Bounded Signal:** A signal  $x[n]$  is bounded if and only if  $|x[n]| \leq B$  for some finite (positive) number  $B$ .

### Linear System:

Suppose  $S: x_1[n] \rightarrow y_1[n]$  and  $S: x_2[n] \rightarrow y_2[n]$ . The system  $S$  is linear *if and only if*  $S: \alpha_1 x_1[n] + \alpha_2 x_2[n] \rightarrow \alpha_1 y_1[n] + \alpha_2 y_2[n]$  for all possible  $x_1[n], x_2[n], \alpha_1$ , and  $\alpha_2$ .

### Time-Invariant System:

Suppose  $S: x_1[n] \rightarrow y_1[n]$ . The system  $S$  is time-invariant if and only if  $S: x_1[n - n_0] \rightarrow y_1[n - n_0]$  for all possible  $x_1[n]$  and  $n_0$ .

### Causal System:

A system  $S$  is causal *if and only if* the output at any given time is dependent only upon the input at the same time and/or past times.

### Stable System:

A system  $S$  is stable *if and only if* bounded inputs always result in bounded outputs.

**LTI System:**  $S: x[n] \rightarrow y[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$  where  $S: \delta[n] \rightarrow h[n]$