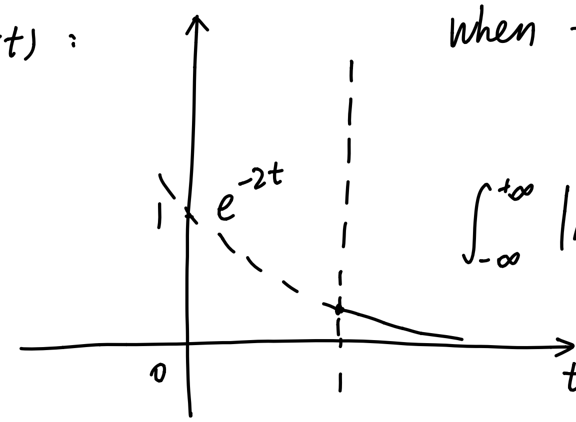
Homework 10 solutions

1. a) $h(t)$:



when $t < 0$, $h(t) = e^{-2t} u(t-1) = 0$

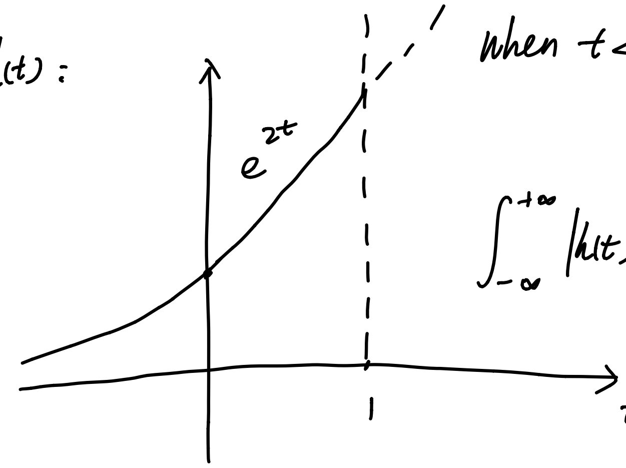
$\Rightarrow$ causal

$$\int_{-\infty}^{+\infty} |h(t)|\,dt = \int_{1}^{+\infty} e^{-2t}\,dt = -\frac{1}{2} e^{-2t} \Big|_{1}^{\infty}$$

$$= -\frac{1}{2}\left(0 - e^{-2}\right) = \frac{e^{-2}}{2} < \infty$$

$\Rightarrow$ stable

b) $h(t)$:



when $t < 0$, $h(t) = e^{2t} u(-t+1) = e^{2t} \neq 0$

$\Rightarrow$ not causal

$$\int_{-\infty}^{+\infty} |h(t)|\,dt = \int_{-\infty}^{1} e^{2t}\,dt = \frac{1}{2} e^{2t} \Big|_{-\infty}^{1}$$

$$= \frac{1}{2}\left(e^{2} - 0\right) < \infty$$

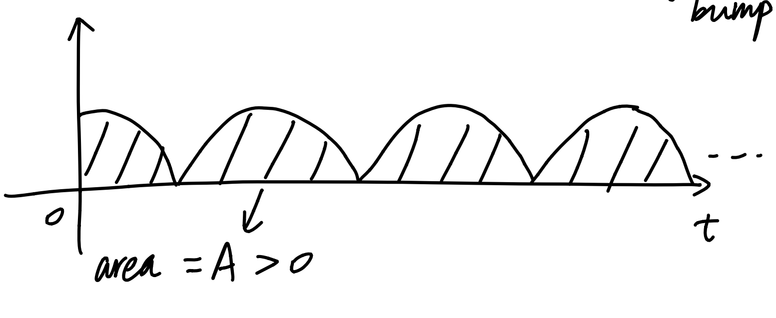
$\Rightarrow$ stable

c)

when $t<0$, $h(t) = e^{4t}\cos(2t)\underbrace{u(t)}_{0} = 0 \Rightarrow$ causal

$$\int_{-\infty}^{+\infty} |h(t)|\,dt = \int_{0}^{+\infty} e^{4t}|\cos(2t)|\,dt \quad (\text{note } e^{4t} > 1 \text{ when } t > 0)$$

$$\Rightarrow \quad > \int_{0}^{+\infty} |\cos(2t)|\,dt = \underbrace{A}_{\text{area of each bump}}\ \underbrace{N}_{\text{number of all bumps = infinite}} = +\infty$$

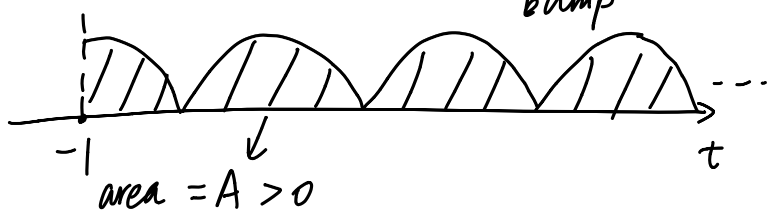


$\Rightarrow$ not stable

(d) when $t<0$, $\begin{cases} -1 \le t < 0, & h(t) = \cos(100\pi t)\, u(t+1) \neq 0 \\ t < -1, & h(t) = 0 \end{cases}$

$\Rightarrow$ not causal ;

$$\int_{-\infty}^{+\infty} |h(t)|\,dt = \int_{-1}^{+\infty} |\cos(100\pi t)|\,dt = \underbrace{A}_{\text{area of each bump}} \underbrace{N}_{\text{number of all bumps = infinite}} = +\infty$$



$\Rightarrow$ not stable.

2. a) when $n<0$, $\begin{cases} -2 \le n < 0, & h[n] = (0.9)^n u[n+2] \neq 0 \\ n < -2, & h[n] = 0 \end{cases}$ $\Rightarrow$ not causal

$$\sum_{n=-\infty}^{+\infty} |h[n]| = \sum_{n=-2}^{+\infty} (0.9)^n = \left(\frac{9}{10}\right)^{-2} \cdot \frac{1}{1-0.9} = \frac{1000}{81} < \infty,$$

$\Rightarrow$ stable

b) when $n<0$, $u[-n+2] = 1$, $h[n] \neq 0 \Rightarrow$ not causal

$$\sum_{n=-\infty}^{+\infty} |h[n]| = \sum_{n=-\infty}^{2} (0.9)^{-n} = \sum_{n=-2}^{+\infty} (0.9)^n = \frac{1000}{81} < \infty \Rightarrow \text{stable}$$

c) when $n<0$ $\begin{cases} -2 < n < 0, & h[n] = (-1)^n u[-n-2] = 0 \\ n \le -2 & h[n] \neq 0 \end{cases}$ $\Rightarrow$ not causal

$$\sum_{n=-\infty}^{+\infty} |h[n]| = \sum_{n=-\infty}^{+\infty} u[-n-2] = \sum_{n=-\infty}^{-2} 1 = \infty \Rightarrow \text{not stable}$$

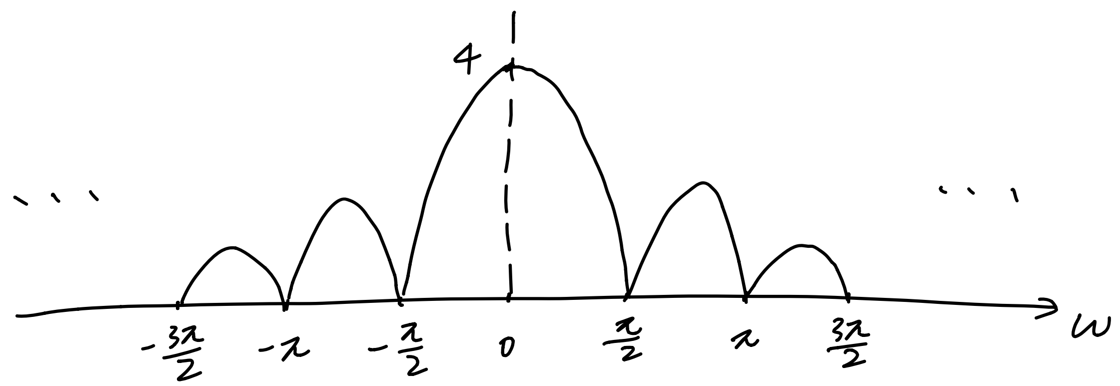
3. By the differentiation property of CTFT

$$g(t) = \frac{d\,x(t)}{dt} \longleftrightarrow G(jw) = jw\,X(jw)$$

$$\Rightarrow X(jw) = \frac{G(jw)}{jw} = \frac{2\sin(2w)}{jw}$$

$$|X(jw)| = \left|\frac{2\sin(2w)}{w}\right|, \quad |X(j0)| = 4$$

zero-crossings: $2w = k\pi \Rightarrow w = \frac{k\pi}{2}$ ($k \neq 0$, integar)



4. By the similar differentiation property of Laplace transform,

$$\frac{d^2y(t)}{dt^2} + \frac{5\,dy(t)}{dt} + 6\,y(t) = 2\,x(t)$$

$$\updownarrow$$

$$s^2 Y(s) + 5sY(s) + 6\,Y(s) = 2\,X(s)$$

$$\Rightarrow H(s) = \frac{Y(s)}{X(s)} = \frac{2}{s^2+5s+6} = \frac{2}{(s+2)(s+3)}$$

with 2 poles: $s=-2$, $s=-3$

Since the system is causal and all the poles of $H(s)$ lie in the left half of the complex plane $\Rightarrow$ It's stable.