11/11/20

EC 414

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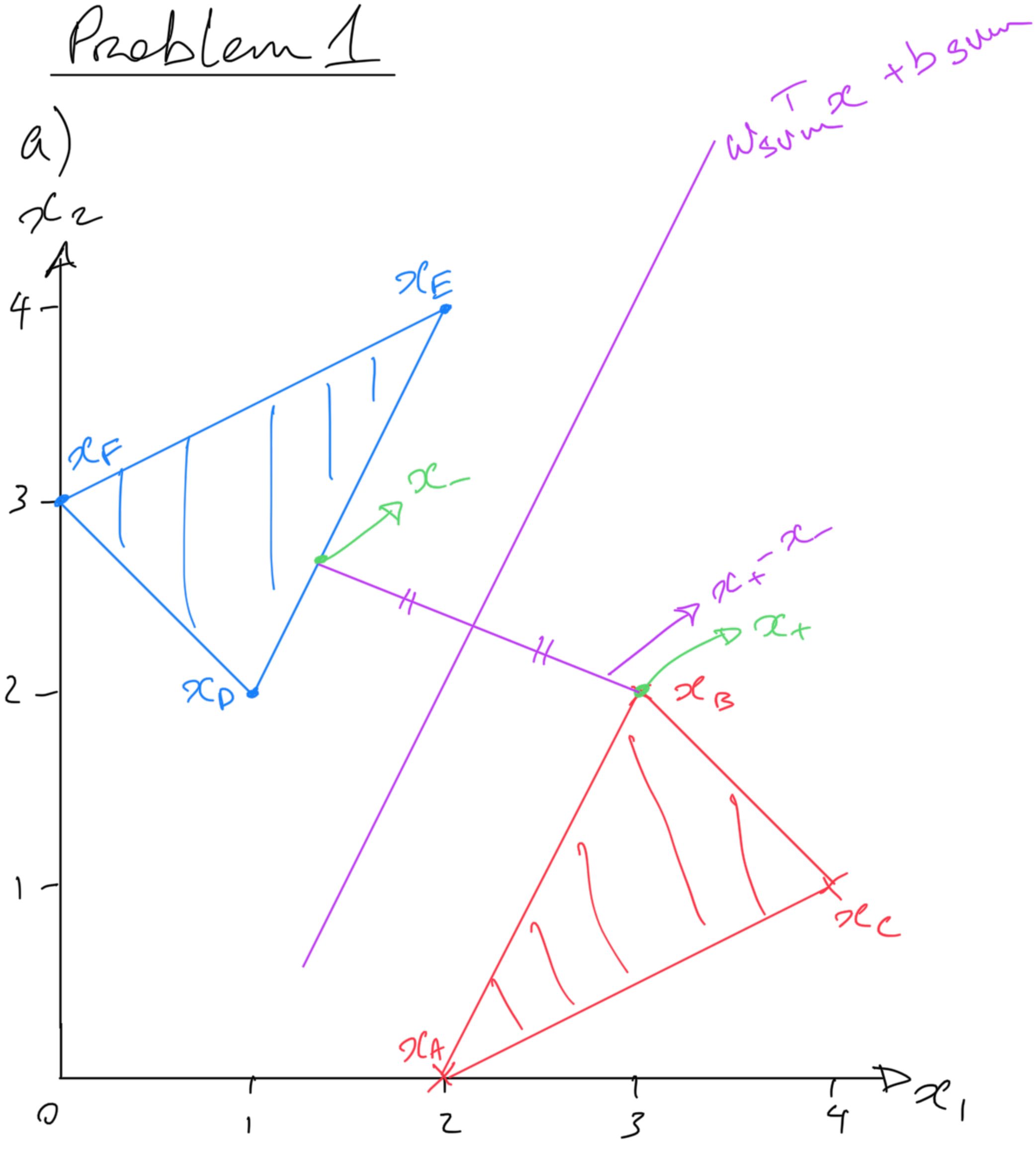
Midterm 2

Problem 1

a)



$\times = +1$, $\bullet = -1$

b) $x_- = \lambda x_D + (1-\lambda) x_E$

$$\lambda \begin{bmatrix} 1 \\ 2 \end{bmatrix} + (1-\lambda) \begin{bmatrix} 2 \\ 4 \end{bmatrix} = \begin{bmatrix} \lambda + 2 - 2\lambda \\ 2\lambda + 4 - 4\lambda \end{bmatrix} =$$

$$= \begin{bmatrix} 2-\lambda \\ 4-2\lambda \end{bmatrix}$$

$$(x_+ - x_-)^T (x_D - x_E) = 0$$

$$x_+ = x_B = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 3-2+\lambda & 2-4+2\lambda \end{bmatrix} \begin{bmatrix} 1-2 \\ 2-4 \end{bmatrix} =$$

$$= \begin{bmatrix} 1+\lambda & -2+2\lambda \end{bmatrix} \begin{bmatrix} -1 \\ -2 \end{bmatrix} = 0$$

$$-1 - \lambda + 4 - 4\lambda = 3 - 5\lambda = 0$$

$$\lambda \rightarrow 3/5$$

$$x_- = \begin{bmatrix} 7/5 \\ 14/5 \end{bmatrix}$$ #

c) $d = x_+ - x_-$

$\overline{x} = x_+ + x_-$

$$\bar{x} = \frac{x_+ + x_-}{2}$$

$$d = \begin{bmatrix} 3 \\ 2 \end{bmatrix} - \begin{bmatrix} 7/5 \\ 14/5 \end{bmatrix} = \begin{bmatrix} 8/5 \,,\; -4/5 \end{bmatrix}^T$$

$$\bar{x} = \frac{\begin{bmatrix} 3 \\ 2 \end{bmatrix} + \begin{bmatrix} 7/5 \\ 14/5 \end{bmatrix}}{2} = \begin{bmatrix} 11/5 \,,\; 12/5 \end{bmatrix}^T$$

$$w_{svm} = \alpha \begin{bmatrix} 8/5 \\ -4/5 \end{bmatrix} = \alpha \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$$w_{svm}^T \bar{x} + b_{svm} = 0$$

$$\alpha \begin{bmatrix} 2 & -1 \end{bmatrix} \begin{bmatrix} 11/5 \\ 12/5 \end{bmatrix} + b_{svm} = 0$$

$$2\alpha + b_{svm} = 0 \rightarrow b_{svm} = -2\alpha$$

$$w_{svm}^T x_+ + b_{svm} = 1$$

$$\alpha \begin{bmatrix} 2 & -1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix} - 2\alpha = 1$$

$$4\alpha - 2\alpha = 1 \rightarrow \alpha = 1/2$$

$$w_{svm} = \frac{1}{2} \begin{bmatrix} 2 \\ -1 \end{bmatrix}, \quad b_{svm} = -1$$

#

d) $$\frac{\|d\|_2}{2} = \frac{\sqrt{(8/5)^2 + (-4/5)^2}}{} =$$

$$= \frac{2\sqrt{5}}{5}$$

#

Problem 2

i)

a) T

b) F

c) F

d) F

ii)

a) F

b) T

c) F

d) F

Problem 3

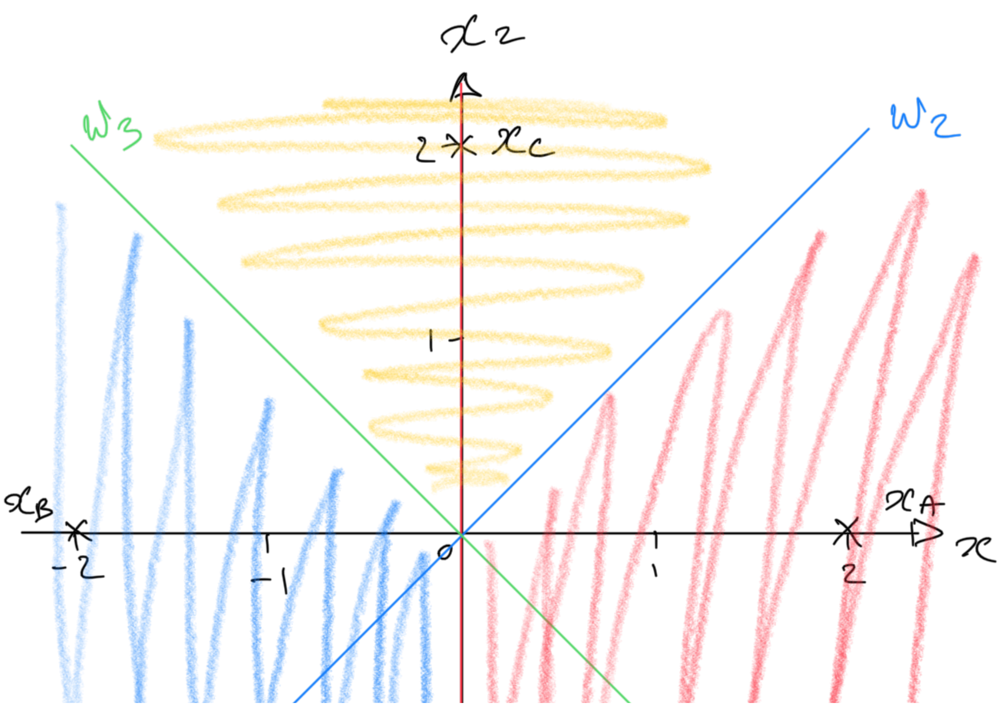
a) T, because it's an inner product between 2 vectors

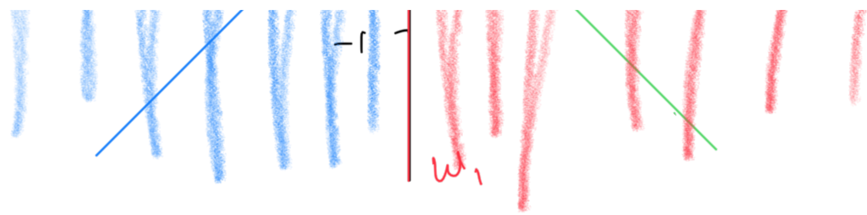
b) F, because inner products can have +ve or -ve values

c) T, since this is the unique transformation that transforms vectors $\mathbb{R}^d \to \mathbb{R}^D$

d) T, the inner product between a vector and itself is 1

<u>Problem 4</u>





① $w_1^T x_A + b = 0$

$w_1 // (x_A - x_B) = 0$

$(x_A - x_B) = \begin{bmatrix}2\\0\end{bmatrix} - \begin{bmatrix}-2\\0\end{bmatrix} = \begin{bmatrix}4\\0\end{bmatrix}$

$w_1 = \alpha \begin{bmatrix}4\\0\end{bmatrix}$

$\frac{x_A + x_B}{2} = \frac{\begin{bmatrix}0\\0\end{bmatrix}}{2} = \begin{bmatrix}0\\0\end{bmatrix} = \bar{x}$

$w^T \bar{x} + b = \alpha \begin{bmatrix}4 & 0\end{bmatrix} \begin{bmatrix}0\\0\end{bmatrix} + b_1 = 0$

$b_1 = 0 \ , \alpha = 0$

② $(x_A - x_C) = \begin{bmatrix}2\\0\end{bmatrix} - \begin{bmatrix}0\\2\end{bmatrix} = \begin{bmatrix}2\\-2\end{bmatrix}$

$w_2 = \alpha \begin{bmatrix}2\\-2\end{bmatrix}$

$\frac{x_A + x_C}{2} = \frac{1}{2}\begin{bmatrix}2\\2\end{bmatrix} = \begin{bmatrix}1\\1\end{bmatrix} = \bar{x}$

$$w_2^T \bar{x} + b_2 = 0$$

$$\alpha \begin{bmatrix} 2 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + b_2 = 0$$

$$\alpha (0) + b_2 = 0$$

$$\alpha = 1 \rightarrow b_2 = 0 \rightarrow w_2 = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

③ $$(x_B - x_C) = \begin{bmatrix} -2 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 2 \end{bmatrix} = \begin{bmatrix} -2 \\ -2 \end{bmatrix}$$

$$w_3 = \alpha \begin{bmatrix} -2 \\ -2 \end{bmatrix}$$

$$\frac{x_B + x_C}{2} = \frac{1}{2} \begin{bmatrix} -2 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \bar{x}$$

$$w_3^T \bar{x} + b_3 = 0$$

$$\alpha \begin{bmatrix} -2 & -2 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} + b_3 = 0$$

$$\alpha (0) + b_3 = 0$$

$$b_3 = 0 \ , \ \alpha = 1$$

$$w_3 = \begin{bmatrix} -2 \end{bmatrix}$$

$v_3 \quad \begin{bmatrix} -2 \end{bmatrix}$

Problem 5

i)

A) B, D, C, E, F

B) A, D, C, E, F

C) D, A, B, F, E

D) C, A, B, F, E

E) F, B, A, D, C

F) E, B, C, A, D

ii)

A) $k=1 \Rightarrow +1$ ✓

$k=3 \Rightarrow +1$ ✓

$k=5 \Rightarrow -1$ ×

B) $k=1 \Rightarrow +1$ ✓

$k=3 \Rightarrow +1$ ✓

$k=5 \Rightarrow -1$ ×

C) $k=1 \Rightarrow -1$ ×

$k=3 \rightarrow +1$ ✓

$k=5 \rightarrow -1$ ×

D) $k=1 \rightarrow +1$ ×

$k=3 \rightarrow +1$ ×

$k=5 \rightarrow +1$ ×

E) $k=1 \rightarrow -1$ ✓

$k=3 \rightarrow +1$ ×

$k=5 \rightarrow +1$ ×

F) $k=1 \rightarrow -1$ ✓

$k=3 \rightarrow +1$ ×

$k=5 \rightarrow +1$ ×

$k=1$ error $\rightarrow \frac{2}{6} = 1/3$

$k=3$ error $\rightarrow \frac{1}{2}$

$k=5$ error $\rightarrow 1$

So, best value for k in this set is $k=1$

Problem 6

Iteration	Clusters	μ_1	μ_2
initial	$\mu_1 = \{A\}$ $\mu_2 = \{B, C\}$	$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 3 \end{bmatrix}$
1	$\mu_1 = \{A, B\}$ $\mu_2 = \{C\}$	$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$	$\begin{bmatrix} 5/2 \\ 2 \end{bmatrix}$
2	$\mu_1 = \{A, B\}$ $\mu_2 = \{C\}$	$\begin{bmatrix} 0.5 \\ 2 \end{bmatrix}$	$\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

1. $\mu_1 = \{A\}$

$\mu_2 = \{B, C\}$

$$\rightarrow \mu_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\mu_2 = \left[\frac{1+4}{2}, \frac{2+2}{2} \right]^T = \left[\frac{5}{2}, 2 \right]^T$$

$\mu_1 = \{A, B\}$
$\mu_2 = \{C\}$

② $\mu_1 = \left[\frac{0+1}{2}, \frac{2+2}{2}\right]^T = \left[\frac{1}{2}, 2\right]^T$

$\mu_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$\mu_1 = \{A, B\}$
$\mu_2 = \{C\}$

$\therefore$ Convergence, since the labels haven't changed

Problem 7

$K(x,y) := \min(x,y)$
$\phi(x,y) = [x, y, -|y-x|]$

$\phi(x_1, x_2)^T \phi(y_1, y_2) =$