15/12/20

EC 414

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Exam 3

Problem 1

a) $v_1 \rightarrow \frac{1}{\sqrt{3}} \begin{pmatrix} 6-\lambda & 2 & 1 \\ 2 & 6-\lambda & 1 \\ 1 & 1 & 7-\lambda \end{pmatrix} \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix} = 0$

$\cancel{\frac{1}{\sqrt{3}}} [-6+\lambda+2-1] = 0 \rightarrow \lambda = 5 \quad x$

$\cancel{\frac{1}{\sqrt{3}}} [-2+6-\lambda-1] = 0 \rightarrow \lambda = -3 \quad x$

v_1 is <u>not</u> an e-vec

$v_2 \rightarrow \frac{1}{\sqrt{6}} \begin{pmatrix} 6-\lambda & 2 & 1 \\ 2 & 6-\lambda & 1 \\ 1 & 1 & 7-\lambda \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = 0$

$\cancel{\frac{1}{\sqrt{6}}} [12-2\lambda+2+1] = 0 \rightarrow \lambda = 7.5 \quad x$

$\frac{1}{\sqrt{6}} [4+6-\lambda+1] = 0 \rightarrow \lambda = 11 \quad x$

v_2 is <u>not</u> an e-vec

$$V_3 \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 6-\lambda & 2 & 1 \\ 2 & 6-\lambda & 1 \\ 1 & 1 & 7-\lambda \end{pmatrix} \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} = 0$$

$$\cancel{\frac{1}{\sqrt{2}}}\left[6-\lambda-2\right] = 0 \rightarrow \lambda = 4$$

$$\frac{1}{\sqrt{2}}\left[2-6+\lambda\right] = 0 \rightarrow \lambda = 4$$

$$\frac{1}{\sqrt{2}}\left[1-1\right] = 0 \checkmark$$

$$V_4 \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 6-\lambda & 2 & 1 \\ 2 & 6-\lambda & 1 \\ 1 & 1 & 7-\lambda \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} = 0$$

$$\cancel{\frac{1}{\sqrt{2}}}\left[6-\lambda-1\right] = 0 \rightarrow \lambda = 5 \times$$

$$\frac{1}{\sqrt{2}}\left[1-7+\lambda\right] = 0 \rightarrow \lambda = 6 \times$$

V_4 is <u>not</u> an e-vec

$$V_5 \rightarrow \frac{1}{\sqrt{3}} \begin{pmatrix} 6-\lambda & 2 & 1 \\ 2 & 6-\lambda & 1 \\ 1 & 1 & 7-\lambda \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = 0$$

$$\cancel{1}\left[6-\lambda+2+1\right] = 0 \Rightarrow \lambda = 9$$

$\sqrt{3}\,[\ \cdots\ ]$

$\cancel{\frac{1}{\sqrt{3}}}\left[2+6-\lambda+1\right]=0 \Rightarrow \lambda = 9$

$\cancel{\frac{1}{\sqrt{3}}}\left[1+1+7-\lambda\right]=0 \rightarrow \lambda = 9$

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$$v_6 \rightarrow \frac{1}{\sqrt{6}}\begin{pmatrix} 6-\lambda & 2 & 1 \\ 2 & 6-\lambda & 1 \\ 1 & 1 & 7-\lambda \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} = 0$$

$\cancel{\frac{1}{\sqrt{6}}}\left[6-\lambda+2-2\right]=0 \rightarrow \lambda = 6$

$\cancel{\frac{1}{\sqrt{6}}}\left[2+6-\lambda-2\right]=0 \rightarrow \lambda = 6$

$\cancel{\frac{1}{\sqrt{6}}}\left[1+1-14+2\lambda\right]=0 \rightarrow \lambda = 6$

So, $v_1 = V_5$, $v_2 = V_6$, $v_3 = V_3$ #

b) $x_{test} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ $\qquad \mu_x = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$\hat{y}_i = v_i^T \left(x_{test} - \mu_x\right)$

$\hat{y}_1 = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ -2 \\ 0 \end{bmatrix} = \frac{-2}{\sqrt{3}}$

$\hat{y}_2 = \frac{1}{\sqrt{6}}\begin{bmatrix} 1 & 1 & -2 \end{bmatrix}\begin{bmatrix} 0 \\ -2 \\ 0 \end{bmatrix} = \frac{-2}{\sqrt{6}}$

$$\hat{y}_3 = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & -1 & 0\end{bmatrix}\begin{bmatrix}0\\-2\\0\end{bmatrix} = \frac{2}{\sqrt{2}}$$

c) $$\hat{x}_{test} = \sum_{i=1}^{2} v_i \hat{y}_i + \mu_x$$

$$= \frac{1}{\sqrt{3}}\begin{bmatrix}1\\1\\1\end{bmatrix}\left(-\frac{2}{\sqrt{3}}\right) + \frac{1}{\sqrt{6}}\begin{bmatrix}1\\1\\-2\end{bmatrix}\left(-\frac{2}{\sqrt{6}}\right) + \begin{bmatrix}1\\1\\1\end{bmatrix}$$

$$-2/3\begin{bmatrix}1\\1\\1\end{bmatrix} - \frac{1}{3}\begin{bmatrix}1\\1\\-2\end{bmatrix} + \begin{bmatrix}1\\1\\1\end{bmatrix}$$

$$= \begin{bmatrix}-1\\-1\\0\end{bmatrix} + \begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix}0\\0\\1\end{bmatrix} \#$$

Problem 2

$$L(\hat{y}, y) = (\hat{y} - y)^2$$

$$\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}1\\-1\end{bmatrix}, \quad y = 1$$

$$\begin{bmatrix}w_1\\w_2\\w_3\\w_4\\w_5\\w_6\\w_7\\w_8\end{bmatrix} = \begin{bmatrix}-1\\1/2\\1\\0\\-1\\4\\2\\1\end{bmatrix}$$

$$\text{Relu} = \max(0, x)$$

a) $U_1 = x_1 w_1 + x_2 w_4$

$= 1 \cdot (-1) + (-1)(0) = -1$

$S_1 = \max(0, -1) = 0$

$U_2 = x_1 w_3 + x_2 w_2$

$= 1(1) + (-1)(\tfrac{1}{2}) = \tfrac{1}{2}$

$S_2 = \max(0, \tfrac{1}{2}) = \tfrac{1}{2}$

$U_3 = S_1 w_5 + S_2 w_6 + w_7 x_1 + w_8 x_2$

$= 0 + \tfrac{1}{2}(4) + 2 \times 1 + 1(-1) = 3$

$S_3 = \max(0, 3) = 3$

$\hat{y} = 3$

$L(\hat{y}, y) = (3-1)^2 = 4$

\#

b) $\dfrac{\partial L}{\partial \hat{y}} = 2(\hat{y} - y) = 2(3-1) = 4$

$\sigma'(x) = 1(x > 0)$

$\dfrac{\partial L}{\partial w_8} = 2(\hat{y} - y) \cdot \sigma'(U_3) x_2$

$= 4 \cdot 1(3 > 0) \cdot (-1) = -4$

$= 4 \cdot 1(3>0) \times (-1) = -4$

$$\frac{\partial L}{\partial w_7} = 2(\hat{y}-y) \cdot \sigma'(v_3)\, x_1$$

$= 4 \cdot 1(3>0) \times (1) = 4$

$$\frac{\partial L}{\partial w_5} = 2(\hat{y}-y) \times \sigma'(v_3)\, s_1$$

$= 2(3-1) \times 1(3>0) \times 0 = 0$

$$\frac{\partial L}{\partial w_6} = 2(\hat{y}-y) \cdot \sigma'(v_3)\, s_2$$

$= 2(3-1) \cdot 1(3>0) \cdot \frac{1}{2} = 2$

$$\frac{\partial L}{\partial w_4} = 2(\hat{y}-y) \cdot \sigma'(v_3)\, w_5 \cdot \sigma'(v_1)\, x_2$$

$= 2(3-1) \cdot 1(3>0) \cdot (-1) \cdot 0 \cdot x_2 = 0$

$$\frac{\partial L}{\partial w_3} = 2(\hat{y}-y)\, \sigma'(v_3)\, w_6 \cdot \sigma'(v_2)\, x_1 =$$

$= 2(3-1)\, 1(3>0) \cdot 4 \cdot 1(\frac{1}{2}>0) \cdot 1 = 16$

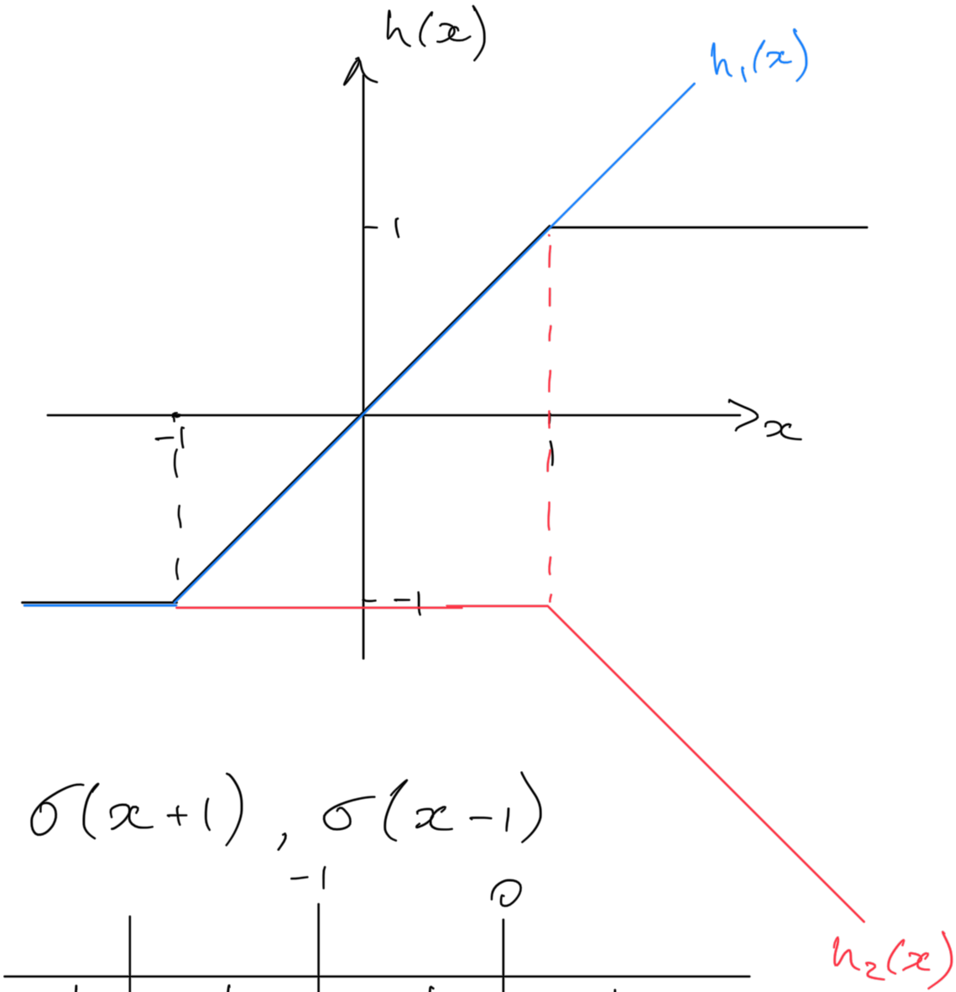
$$\frac{\partial L}{\partial w_2} = 2(\hat{y}-y)\, \sigma'(v_3)\, w_6\, \sigma'(v_2)\, x_2 =$$

$= 2(3-1)\, 1(3>0) \cdot 4 \cdot 1(\frac{1}{2}>0) \cdot (-1) = -16$

$$\frac{\partial L}{\partial w_1} = 0$$

Problem 3

$$h(x) = \begin{cases} -1 & x \le -1 \\ x & -1 < x \le 1 \\ 1 & \text{else} \end{cases}$$



$\sigma(x+1)$, $\sigma(x-1)$

		-1	0
-1	-1	-1	-1
$\sigma(x+1)$	0	$x+1$	$x+1$
$-\sigma(x-1)$	0	0	$-x+1$
	-1	x	1

so,

$\alpha_1 = 1$, $\beta_1 = 1$, $\gamma_1 = 1$ $\rightarrow h_1(x)$

$\alpha_2 = -1$, $\beta_2 = -1$, $\gamma_2 = 1$ $\rightarrow h_2(x)$

$\alpha_3 = -1$

Problem 4

a) F

b) T

c) T

d) F

e) F

f) F

g) F

h) T

Problem 5

a) T, because every e-vector measures the variance in a certain dimension, which are orthogonal

b) F, the variance property is only true only if you have centralised the data

c) F, $m \geq d \rightarrow m-d$ 0 e-vals in XX^T or $d \geq m \rightarrow d-m$ 0 e-vals in X^TX

d) F, they preserve the distances between samples, it doesn't specify anywhere that we use the labels

Problem 6

$U_1 = x_1 w_1 + x_2 w_4$

$S_1 =$

$U_2 = x_1 w_3 + x_2 w_2$

$S_2 =$

$U_3 = S_1 w_5 + S_2 w_6$

$S_3 = \hat{y} = 10 x_1 x_2$