Notes on interference channel outer bounds

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title says it all

I. INTRODUCTION

Definition 1 (Interference channel). *A two party interference channel $(\mathcal{X}_1 \otimes \mathcal{X}_2, p(y_1, y_2|x_1, x_2), \mathcal{Y}_1 \otimes \mathcal{Y}_2)$ is general model for communication networks with two inputs, two outputs and a probability transistion matrix $p(y_1, y_2|x_1, x_2)$.*

Definition 2 (Achievable rate pair). *We say that a rate (R_1, R_2) is achievable for a channel $(\mathcal{X}_1 \otimes \mathcal{X}_2, p(y_1, y_2|x_1, x_2), \mathcal{Y}_1 \otimes \mathcal{Y}_2)$ if there exists a code for n uses of the channel where the messages taken from respective sets $\{1, 2, \ldots, 2^{nR_1}\}$ and $\{1, 2, \ldots, 2^{nR_2}\}$ are transmitted with vanishing error probability.*

Definition 3 (Capacity). *The capacity region $\mathcal{C}$ of is the closure of the set of achievable rates (R_1, R_2).*

II. RELATION TO MULTIPLE ACCESS AND BROADCAST CHANNELS

We can think of the IC as either two multiple access channels or two two broadcast channels.

It is therefore important to review briefly the known capacity results for these simpler channels.

A. Multiple access

Let's say that using random coding to achieve the multiple access rate (R_1, R_2) to receiver Rx1. If you can also use random coding to acieve rates (R_1', R_2') for multiple access communication to Rx2, and if $R_1' > R_1$ and $R_2' > R_2$, then you can achieve the IC task at rate (R_1, R_2).

Note however that the decomposition of the IC in terms of two multiple access channels is too restrictive, since in fact Rx2 doesn't need to learn M_2 at all. Sure it can be useful side informaiton, but it is not a requirement.

A more direct generalization of MACs would be multiple multiple-access channels (MMAC) will be to require both messages M_1 and M_2 to be decodable at both receivers. The notion of interference will be then be much more interesting. In fact the code used for the Tx1-Rx1 communication will have to also be easily decodable at Rx2? Is it the same code? Surely with random coding it works, but what about more general inner-outer codes?

I wonder if there are no relations to network coding which we can do at this very low level of information theory.

The difference between the IC and the MMAC is that we guarantee that an extra ressource of

”cross communication” is available. Wouldn’t it be best to represent rates then as 4-tuples?

$$\begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix} \tag{1}$$

The IC problem is basically a promise about R_{11} and R_{22}, and no statement about the cross rates.

Are the cross rates not useful? These extra rates could be used to convert some other information and I feel they should be taken into account in general.

B. Broadcast

Both only depend on marginals $p(y_1|x_1x_2)$ and $p(y_2|x_1x_2)$ since if we manage to get both decoding errors low then we manage to get the error of the AND of the two events also. [*more details needed...*]

If a rate pair is impossible for a broadcast sub

III. NAIVE OUTER BOUND

Both broadcast channels and multiple access channel are special cases of the interference channel. In particular we can think of the interference channel as two separate multiple access channels.

We know that the region defined by

$$R_1 \leq I(X_1; Y_2|X_2) \tag{2}$$
$$R_2 \leq I(X_2; Y_2|X_1) \tag{3}$$

contains the capacity region $\mathcal{C}$.

This corresponds to a very loose rectangular bound on the true capacity region.

IV. LITERATURE REVIEW

[1]

[2]

V. SATO BOUND

We can describe a more precises outer bound to the capacity region by spcifying an inequality on the sum rate $R_1 + R_2$. This was done by Sato [Sato77].

The outer bound becomes:

$$R_1 \leq I(X_1;Y_2|X_2) \tag{4}$$
$$R_2 \leq I(X_2;Y_2|X_1) \tag{5}$$
$$R_1+R_2 \leq I(X_1X_2;Y_1Y_2) \tag{6}$$

Proof. b □

VI. CARLEIAL

A further development concerning an outer bound was obtained by Carleilal [Carleilal83]. Consider the two random variables Z_1, Z_2 such that

$$Y_1 \quad \text{is a degraded version of} \quad Z_1 \tag{7}$$
$$Y_2 \quad \text{is a degraded version of} \quad Z_2 \tag{8}$$
$$Y_2 \quad \text{is a degraded version of} \quad (X_1, Z_1) \tag{9}$$
$$Y_1 \quad \text{is a degraded version of} \quad (X_2, Z_2) \tag{10}$$

then we have the following outer bound

$$R_1 \leq I(X_1;Y_2|X_2) \tag{11}$$
$$R_2 \leq I(X_2;Y_2|X_1) \tag{12}$$
$$R_1+R_2 \leq I(X_1X_2;Z_1) \tag{13}$$
$$R_1+R_2 \leq I(X_1X_2;Z_2) \tag{14}$$

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[1] R. Ahlswede. The capacity region of a channel with two senders and two receivers. *The Annals of Probability*, 2(5):805–814, 1974.

[2] H. Sato. Two-user communication channels. *IEEE transactions on information theory*, 23(3):295–304, 1977.