Khan exercises

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1 x4eabf393

Jimmy will be selling hot dogs at the football game. He bought hot dogs, buns and condiments for \$8 before the game and now he wants to calculate the profit he will make. The graph below shows how Sean's profit P depends on the number of hot dogs he sells at the game.

`![](https://ka-perseus-graphie.s3.amazonaws.com/2d882eb9739295abe5dc10510ecf8f2564fd633e.png)`

Write the equation which describes Jimmy's profit P as a function of the number n of hot dogs sold.

The equation which describes Jimmy's profit P as a function of the number of hot dogs sold n is [[? expression 1]]

Hint 1 The graph shows how the profit P varies as a function of the number of hot dogs sold n. Let's see how to use the graph of the function to find the equation of the function.

Hint 2 Looking at the graph, we see the initial value of the profit function is -8 dollars. If Jimmy doesn't sell any hot dogs ($n = 0$), his profit will be -8 dollars. A negative profit is a *loss*. Indeed, if Jimmy doesn't sell any hot dogs he will lose the money he invested to buy the ingredients.

Hint 3 Next, observe the slope of the graph is equal to 1. For each hot dog sold, the profit increases by \$1. This means Jimmy is selling the hot dogs for \$1 each.

Hint 4 Combining these facts about Jimmy's hot dog operation, we can now write his profit P as a function of the number of sales n as follows:

$$P = -8 + 1n.$$

Hint 5 Note that the profit P is described by a *linear equation* $P = m \cdot n + b$, where $b = -8$ represents Jimmy's initial investment and $m = 1$ represents sale price of each hot dog.

The graph of the function $P = -8 + 1n$ is a *line* which passes through the point $(0, -8)$ and has slope equal to 1. We can understand the function which describes Jimmy's profit either through its equation or through its graph.

Hint 6 The equation which describes Jimmy's profit P as a function of the number of hot dogs sold n is $P = -8 + 1 \cdot n$.

2 x4d8feb39

`https://www.khanacademy.org/devadmin/content/items/x4d8feb39`

Sean works in sales. His monthly salary S depends on his sales performance. The graph below shows his salary as a function of the number of sales he makes during the month.

`![](https://ka-perseus-graphie.s3.amazonaws.com/88f55c935fe40d38b0894522ee4c9ee3e18982d9.png)`

Find the equation which describes Sean's monthly salary S as a function of the number of sales n.

The equation which describes Sean's salary S as a function of the number of sales n is [[? expression 1]]

Hint 1 Were shown the graph of the function which describes how Sean's monthly salary S depends on the number of sales n. Let's use the information from the graph to figure out the equation which represents S as a function of n.

Hint 2 Looking at the graph we see that the initial value of the function S is \$1000. Even if Sean makes $n = 0$ sales, he will still receive a monthly salary of \$1000.

Hint 3 Next, let's look at the slope of the graph. If Sean makes $x = 2$ sales, his salary will be $S = \$1200$. Thus, a change of 2 in the number of sales n produces a change of \$200 in the salary S. The rate of change of the salary function is:

$$m = \frac{\text{change in } S}{\text{change in } n} = \frac{1200 - 1000}{2 - 0} = \frac{200}{2} = 100.$$

Sean's salary increases by \$100 for each sale he makes.

Hint 4 Combining all the information we have about Sean's monthly salary, we can now write his salary S as a function of the number of sales n as follows:

$$S = 1000 + 100n.$$

Hint 5 Note that the salary S is described by a *linear equation* $S = m \cdot n + b$, where $b = 1000$ represents the initial value of the function and $m = 100$ represents the rate of change of the function.

The graph of the function $S = 1000 + 100n$ is a *line* which passes through the point $(0, 1000)$ and has slope equal to 100. We can understand the function which describes Sean's salary either through its equation or through its graph.

Hint 6 The equation which describes Sean's salary S as a function of the number of sales n is $S = 1000 + 100n$.

3 x464f53e1

`https://www.khanacademy.org/devadmin/content/items/x464f53e1`

Jessica works in sales. Her monthly salary is calculated as a base amount of \$2000 plus a commission of \$100 for each sale she closes. Assume n represents the number of sales Jessica closes and S represents her monthly salary.

Find the equation which represents Jessica's monthly salary S as a function of the number of sales n.

The equation which describes S as a function of n is [[? expression 1]]

Hint 1 We are told Jessica's salary S increases by \$100 for each sale she makes. Also, we know that she receives a base amount of \$2000. Let's see how we can use the information provided to figure out the function which describes Jessica's monthly salary.

Hint 2 The initial value of the salary function is \$2000. This is the base amount Jessica earns even when she makes $n = 0$ sales.

The rate of change of the salary function is \$100 per sale because this is how much she makes per sale. If n increases by one, her salary S will increase by \$100.

We can combine these two facts to find Jessica's monthly salary S as a function of the number of sales n:

$$S = 100n + 2000.$$

Hint 3 Note that the salary S is described by a *linear equation* $S = m \cdot n + b$, where $b = 2000$ represents the initial value of the function and $m = 100$ represents the rate of change of the function.

Hint 4 The equation which describes S as a function of n is $S = 2000 + 100n$.

4 xd3aa970e

`https://www.khanacademy.org/devadmin/content/items/xd3aa970e`

Ron borrowed \$200 from his friend and promised to return the money by paying back \$20 each week. Assume x represents the time in weeks, and y represents the amount of money left to pay.

Find the equation which describes the loan remaining L as a function of time t.

Drag the two points to move the line into the correct position.

Hint 1 Were looking for the graph of the function which describes the amount of money y remaining for Ron to pay back to his friend as a function of the time x measured in weeks.

Hint 2 The initial value of Ron's debt is \$200. This is the amount Ron has to pay back. Ron's debt y decreases by \$20 each week. Ron's debt is described by the following linear equation:

$$\begin{aligned} y &= m \cdot x + b \\ &= -20 \cdot x + 200. \end{aligned}$$

The initial value of the loan is $b = 200$. This is the amount Ron owes to his friend when $x = 0$. The rate of change of the function is $m = -20$ because each week Ron pays back \$20 to his friend.

Hint 3 The graph of the function $y = m \cdot x + b$ is a line with y-intercept equal to b and slope equal to m.

Therefore, the graph of Ron's debt $y = 200 - 20x$ is a line which passes through the point $(0, 200)$ and has slope equal to -20: `![](https://ka-perseus-graphie.s3.amazonaws.com/bbe5bcc71845e5f1f2200362541175a6bb963515.png)`

5 xe3e0bf01

`https://www.khanacademy.org/devadmin/content/items/xe3e0bf01`

The cost of your annual visit to the dentist is calculated as a base price of \$50 dollars for the checkup and cleaning plus an additional \$100 per cavity the dentist has to fix.

What is the equation which describes the cost C of the visit if the dentist finds n cavities?

The cost of the visit C as a function of the number of cavities n is described by [[? expression 1]]

Hint 1 (draft)

Don't worry, you are not at the dentistthis is just a math question!

The cost of the visit, in dollars, is described by the math expression $50+100n$, where \$50 is the base price for the visit and \$100 is the price for repairing one cavity.

We have to evaluate this expression in the case of $n = 2$ cavities.

Hint 2 Plugging in the value $n = 2$ into the formula $50 + 100n$ we obtain the following expression:

$$\begin{aligned} 50 + 100n &= 50 + 100{\cdot}2 \\ &= 50{+}200 \\ &= 250 \end{aligned}$$

Note the order in which we performed the operations when evaluating the expression. We computed the product before carrying out the addition.

Hint 3 The cost of the visit to the dentist will be \$250.

6 x66af7067

`https://www.khanacademy.org/devadmin/content/items/x66af7067`

Covi is driving from Montreal to New York City. The graph below shows his position x, measured in kilometers from the Canada-US border, as a function of time t, measured in hours.

`![](https://ka-perseus-graphie.s3.amazonaws.com/e8fa2c2d06cffc6ef871bbc124814ca2d111357d.png)`

What is the equation that describes x as a function of t?

The equation which describes x as a function of t is [[? expression 1]]

Hint 1 We are shown the graph of Covi's position function, and we want to find the equation that corresponds to this graph. The graph we see is a line. Observe that the slope of this line is equal to 100 and its y-intercept is -200.

Hint 2 If the graph of a function looks like line, then the function is described by a linear equation $y = mx+b$, where m is the *rate of change* of the function and b is the *initial value* of the function. To find the equation of the function, we must figure out the values of m and b.

Hint 3 The graph of this line passes through the point $(0, -200)$. We say the y-intercept of the graph is equal to -200 because this is where the graph crosses the y-axis.

Lets now think in terms of the equation $x = mt+b$. When the input is $t = 0$ the output is $x = -200$:

$$
\begin{aligned}
y &= mx + b \\
4 &= m(0) + b \\
4 &= b
\end{aligned}
$$

So $b = 4$. The initial value of the function corresponds to the y-intercept of its graph.

Hint 4 If we can find the value of the rate of change m of the function we will be done. Observe that the graph passes through the point $(1, 6)$. When $x = 1$, $y = 6$. Lets plug these values into the equation:

$$
\begin{aligned}
y &= mx + b \\
6 &= m(1) + 4 \\
6 &= m + 4 \\
2 &= m
\end{aligned}
$$

So $m = 2$.

Now we can write the equation of the function whose graph we see in the figure:

$y = 2x + 4$

The rate of change $m = 2$ corresponds to the slope of the graph, and the initial value $b = 4$ corresponds to the the y-intercept of the graph.

Hint 5 The equation which describes y as a function of x is $y = 2x + 4$.

7 x288363ab

`https://www.khanacademy.org/devadmin/content/items/x288363ab`

Jane runs a company which manufactures bicycles. The operating costs of Janes company are of two types. Each month, the company must cover the fixed costs of \$3000 for rent and utilities. The production costs are \$100 per bicycle.

What is the equation which describes the costs C as a function of the number of bicycles produced x?

The equation that describes C as a function of x is [[? expression 1]]

Hint 1 Were trying to find the equation which corresponds to the operating costs C of Janes company as a function of the number of bicycles produced x. We know the fixed costs are \$3000 per month. Additionally, there is a production cost of \$100 per bicycle.

Hint 2 The *fixed costs* of a company is the part of the costs that are present even when Janes company does not produce any items $x = 0$. We know that each month Jane has to pay 3000 dollars for rent and utilities and that this number does not depend on the number of bicycles produced.

Hint 3 Now let's look at the variable costs. The *variable* costs of a company depend on the number of bicycles produced.

If Jane produces x bicycles this month, her variable costs will be $100x$ dollars, since each bicycle costs \$100 to produce.

Hint 4 The total of the costs of Janes company is the sum of the fixed costs and the variable costs:

$C = 100x + 3000$ dollars.

Hint 5 Note that the total costs of the company are described by a linear equation of the form $C = mx + b$, where the number m corresponds to the *unit cost of production* the function as x increases and b is the *initial value* of the function.

Hint 6 The equation that describes C as a function of x is $C = 100x + 3000$.

8 x0b401d79

`https://www.khanacademy.org/devadmin/content/items/x0b401d79`

David is visiting a new city. He wants to rent a bicycle for a couple of hours to be able to better explore the city. The price he has to pay is \$4 per hour plus a base charge of \$10.

Write the function which represents the price P he will have to pay for h hours?

$P =$ [[? expression 1]] dollars

Hint 1 The price function corresponds the equation of a line $P = mh + b$, where the m is the *slope* of the function and b is the *initial value*.

Hint 2 The number b corresponds to the fixed price of the bike rental, which is \$10.

Hint 3 The number m corresponds to the cost of the bike rental per hour. If David rents the bicycle for h hours, then the price will be $4h$ dollars plus the base charge.

Hint 4 The price function is $P = 4h + 10$ dollars.

9 x3d0ab2da

`https://www.khanacademy.org/devadmin/content/items/x3d0ab2da`

The graph below shows how the quantity y is related to the quantity x.

!\[\](https://ka-perseus-graphie.s3.amazonaws.com/c55256c02b190a7aba6f2a02fb7d6bf49a8652da.png)

What is the equation that describes y as a function of x?

The equation which describes y as a function of x is [[? expression 1]]

Hint 1 We are shown the graph of a function, and we want to find the equation of the function which corresponds to this graph. The graph we see is a line. Observe that the slope of this line is equal to 2 and its y-intercept is -4.

Hint 2 If the graph of a function looks like line, then the function is described by a linear equation $y = mx + b$, where m is the *rate of change* of the function and b is the *initial value* of the function. To find the equation of the function, we must figure out the values of m and b.

Hint 3 The graph of this line passes through the point $(0, 4)$. We say the y-intercept of the graph is equal to 4 because this is where the graph crosses the y-axis.

Lets now think in terms of the equation $y = mx + b$. When the input is $x = 0$ the output is $y = 4$:

$$\begin{aligned}
y &= mx + b \\
4 &= m(0) + b \\
4 &= b
\end{aligned}$$

So $b = 4$. The initial value of the function corresponds to the y-intercept of its graph.

Hint 4 If we can find the value of the rate of change m of the function we will be done. Observe that the graph passes through the point $(1, 6)$. When $x = 1$, $y = 6$. Lets plug these values into the equation:

$$\begin{aligned}
y &= mx + b \\
6 &= m(1) + 4 \\
6 &= m + 4 \\
2 &= m
\end{aligned}$$

So $m = 2$.

Now we can write the equation of the function whose graph we see in the figure:

$y = 2x + 4$

The rate of change $m = 2$ corresponds to the slope of the graph, and the initial value $b = 4$ corresponds to the the y-intercept of the graph.

Hint 5 The equation which describes y as a function of x is $y = 2x + 4$.

10 xfb15c7a4

`https://www.khanacademy.org/devadmin/content/items/xfb15c7a4`

Norm works in marketing. The table below shows the monthly price Norm pays for sending out emails to potential clients.

— n of emails — price P— — :-: — :-: — — 0 — \$10 — — 100 — \$11 — — 200 — \$12 — — 300 — \$13 — — 500 — \$15 — — 1000 — \$20 — — 3000 — \$30 —

The equation that describes P as a function of n is [[? expression 1]]

Hint 1 We want to find the equation which corresponds to the monthly price P as a function of the number of emails sent n. We can look at the values if the table to figure out the equation of the function.

Hint 2 The more emails Observe that the *change* in P is proportional to the *change* in n. Going from $n = 100$ emails to $n = 200$, a change of 100 in n, corresponds to a price increase of \$1 dollar. If Paul wants to send $n = 300$, then the monthly price will increase by an additional \$1 dollar.

The rate of change of the price P as the number of emails n increases is equal to:

$m = \frac{\$1}{100} = \frac{1}{100}$ dollars per email sent.

Lets make sure that the *price per email sent* remains the same when Norm sends more emails. Who knows, maybe there is a discount for large volumes of emails. When n changes from $n = 1000$ to $n = 2000$, the price increases from \$20 to \$30. This change in the price is is consistent with the rate of $m = \frac{1}{100}$ dollars per email:

$m \cdot 1000 = 1100 \cdot 1000 = 10 = 30 - 20 =$ change in price.

The price P increases at a rate of $m = \frac{1}{100}$ dollars per email sent.

Hint 3 So can we figure out what the price function is now? Since the change in the price P is proportional to the change in the input n we know the price is described by a linear equation $P = mn + b$. The constant m corresponds to the *rate of change* of the price and b is the *initial value* of the function.

We already figured out that $m = \frac{1}{100}$ dollars per email, so what remains to do is find b, the initial value of the function.

Hint 4 The number b corresponds to price Norm has to pay even when he sends $n = 0$ emails. When $x = 0$ the price is $P = 10 = m(0) + b$, so $b = 3000$.

Hint 5 The price (in dollars) of sending n emails is therefore described the following linear equation:

$$P = \frac{1}{100}n + 10.$$