Graphing linear equations and word problems

August 7, 2013

Link to full list of exercises

1 x091b77a5

Phil sells hot dogs at football games. For last week's game, he bought hot dogs, buns and condiments for \$8 at the store, and sold the hot dogs for \$1 each at the game. The graph below shows the profit y he made as a function of the number x of hot dogs sold.
⟨ img ⟩
This week, Phil buys less hot dogs and buns, spending only \$6 at the store. **What will be the graph of the profit for this week?**

[[interactive-graph 1]]

Drag the two points to move the line into the correct position.

Hint 1 The first graph shows how the profit y varies as a function of the number of hot dogs sold x. The slope of the graph is equal to 1, since last week Phil sold the hot dogs for \$1 each. The y-intercept of the graph is $y = -8$, since he invested \$8 for the food.

How will the graph of the profit change if Phil invests only \$6 for the food?

Hint 2 First let's **find the equation** that describes the profit of last week's sales. The equation that describes the profit y as a function of the number of hot dogs sold x is

$$\begin{aligned} y &= mx + b \\ &= 1 \cdot x - 8 \end{aligned}$$

The slope $m = 1$ corresponds to the price \$1 per hot dog, and the initial value $b = -8$ corresponds to Phil's initial investment of \$8.

Hint 3 If Phil invests only \$6 for the food this week, then the equation of the profit y as a function of the number of hot dogs sold x will become

$$\begin{aligned} y &= mx + b \\ &= 1 \cdot x - 6 \end{aligned}$$

The initial value changed from $b = -8$ to $b = -6$ to reflect the fact that Phil spent only \$6 at the store this week. The slope remained the same: $m = 1$.

Hint 4 The graph of the new profit function $y = 1 \cdot x - 6$, passes through the point $(0, -6)$ and has slope equal to 1:
⟨ img ⟩

2 x1019b658

The graph below describes the cycles of the moon as observed from the Earth during the first 60 days of a year. The percentage of moon's surface which we see varies as a function of time. A *full moon* corresponds to 100% visibility and a *new moon* corresponds to 0% visibility.
⟨ img ⟩
Complete the sentences based on the graph of the function.

There was a new moon on the [[input-number 1]]$^{\text{th}}$ day of the year. There was also a new moon on the [[input-number 2]]$^{\text{st}}$ day of the year. Therefore, the period between two new moons is approximately [[input-number 3]] days long.

Complete the sentences.

Hint 1 The percentage of the moon which is visible from earth varies as a function of time. The figure shows the graph of this function. We can complete the description by *reading off* the values from the graph.

Hint 2 A *new moon* corresponds to $M = 0\%$ visibility. We see that this occurs when $t = 24$ and $t = 51$. Therefore there was a new moon on the 24^{th} and 51^{st} days of the year.

Hint 3 To find the *period* between two new moons, we can calculate the difference between the two times when the new moon occurred:

$$\text{period} = 51 - 24 = 27 \text{ days}$$

Hint 4 Note that the word *period* can be used to describe any graph that shows a repeating pattern. The period is the length of the pattern that repeats**.**

3 x150506e4

Alberto and Bianca are having a 50 km race. The illustration below shows the graph of the position x of the two runners as a function of time t.
⟨ img ⟩

Complete the sentences based on the graph of the function.

Initially, [[dropdown 4]] runs faster. Alberto maintains a speed of [[input-number 1]] km/h during the first hour of the race. However, after one hour he gets tired and must take a brake. Unfortunately he falls asleep! Meanwhile, Bianca's running speed [[dropdown 2]]. The first person to cross the finish line is [[dropdown 1]].

Complete the sentences.

Hint 1 The graph shows the position of the two runners as a function of time. The steepness of each graph represents the speed at which the person is running. Let's use the graph to complete the sentences one by one.

Hint 2 Looking at the graph, we see that initially Alberto's position changes faster. The graph of Alberto's position is steeper than that of Bianca's.

Hint 3 We can calculate Alberto's speed by calculating the slope of the graph of Alberto's position function as follows:

$$\frac{\text{rise}}{\text{run}} = \frac{20-0 \ \ [\text{km}]}{1-0 \ \ [\text{h}]} = \frac{20 \ [\text{km}]}{1 \ [\text{h}]} = 20 \text{ km/h}.$$

Alberto maintains a speed of 20 km/h during the first hour of the race.

Hint 4 Let's now look at the graph of Bianca's position as a function of time. She starts out running *really* slowly, but then she picks up speed with time. The steepness of the graph of Bianca's position keeps **increasing**.

Hint 5 The first person to cross the finish line is Bianca. We know this because her position graph is the first to reach the finish line at $d = 50$ km.

Hint 6 The correct way to complete the sentences is as follows.

Initially, **Alberto** runs faster. Alberto maintains a speed of **20** km/h during the first hour of the race. Meanwhile, Bianca's running speed **is increasing**. The first person to cross the finish line is **Bianca**.

4 x160049eb

Your friend describes to you the graph of a linear function over the phone. She says the y-intercept of the graph is equal to 6 and the slope of the graph is equal to -2.

Draw the graph of this function.

[[interactive-graph 1]]

Drag the two points to move the line into the correct position.

Hint 1 Let's draw the graph of the function which corresponds to the verbal description.

Hint 2 The y-intercept corresponds to the value of the function when $x = 0$. Since your friend told you the y-intercept of the graph is $y = 6$, you know the graph of the function passes through the point $(0, 6)$.

Drag one of the points on the graph placing it at the coordinates $(0, 6)$.

Hint 3 Since your friend told you the slope of the function is -2, you must position the second point on the graph to produce a line which decreases by two vertical steps for each horizontal step.

The graph of the function that corresponds to your friend's verbal description is ⟨ img ⟩

Hint 4 Note the linear equation that corresponds to this graph is $y = -2 \cdot x + 6$.

Hint 5

5 x16750bc5

The maximum speed that a sailboat can reach depends on the size of the boat. The graph below shows the maximum speed v that a sailboat can reach as a function of its length ℓ.

⟨ img ⟩

Complete the sentences based on the graph of the function.

The longer the sailboat is, the [[dropdown 1]] it can go. For a boat to reach the speed of 10 kilometers per hour, it needs to be at least [[input-number 1]] feet long. The maximum velocity that an 8 foot boat can reach is [[input-number 2]] kilometers per hour. A 16 foot sailboat can be [[dropdown 2]] faster than a 4 foot sailboat.

Complete the sentences.

Hint 1 The illustration shows the graph of the function that describes how the top speed of a boat (in km/h) depends on the length of the boat (in ft). We can complete the description by *reading off* the values from the graph.

Hint 2 The maximum velocity of the boat increases with size. The longer the sailboat is, the faster it can go.

Hint 3 Looking at the graph, we see that only boats which are $\ell = 16$ ft or longer can reach the velocity $v = 10$ km/h.

Hint 4 To complete the third sentence, we must read off the value of the function when $\ell = 8$. The graph passes through the point $(8, 7.1)$, which means that the maximum speed for a 8 ft boat is $v = 7.1$ km/h.

Hint 5 Observe that the graph of the function passes through the points $(16, 10)$ and $(4, 5)$. This means the maximum speed of a $\ell = 16$ ft boat is $v = 10$ km/h and the maximum speed of a $\ell = 4$ ft boat is $v = 5$ km/h.

Therefore, the speed of a 16 foot boat is **2 times** faster than the speed of a 4 foot boat.

Hint 6 The correct way to complete the sentences is as follows.

The longer the sailboat is, the **faster** it can go. For a boat to reach the speed of 10 kilometers per hour, it needs to be at least **16** feet long. The maximum velocity that an 8 foot boat can reach is **7.1** kilometers per hour. A 16 foot sailboat can be **2** **times** faster than a 4 foot sailboat.

6 x45bd9418

Ron borrowed \$200 from his friend and promised to return the money by paying back \$20 each week. Assume x represents the time in weeks, and y represents the amount of money left to pay.

Draw the graph of y as a function of x.

[[interactive-graph 1]]

Drag the two points to move the line into the correct position.

Hint 1 Were looking for the graph of the function which describes the amount of money y remaining for Ron to pay back to his friend as a function of the time x measured in weeks.

Hint 2 The initial value of Ron's debt is \$200. This is the amount Ron has to pay back. Ron's debt y decreases by \$20 each week. Ron's debt is described by the following linear equation:

$$\begin{aligned} y &= m \cdot x + b \\ &= -20 \cdot x + 200. \end{aligned}$$

The initial value of the loan is $b = 200$. This is the amount Ron owes to his friend when $x = 0$. The rate of change of the function is $m = -20$ because each week Ron pays back \$20 to his friend.

Hint 3 The graph of the function $y = m \cdot x + b$ is a line with y-intercept equal to b and slope equal to m.

Therefore, the graph of Ron's debt $y = 200-20x$ is a line which passes through the point $(0, 200)$ and has slope equal to -20: ⟨ img ⟩

7 x572138ad

The illustration below shows the graph of y as a function of x.
⟨ img ⟩

Complete the sentences based on the graph.

Initially, as x increases, y [[dropdown 1]]. Afterward, the slope of the graph of the function is equal to [[input-number 1]] for all x between $x = 3$ and $x = 5$. The slope of the graph is equal to [[input-number 2]] for x between $x = 5$ and $x = 9$. The greatest value of y is $y =$ [[input-number 3]] and it occurs when $x =$ [[input-number 4]].

Complete the sentences.

Hint 1 Let's use the graph to complete the sentences one by one.

Hint 2 Initially, the graph of the function is flat. The quantity y remains constant as x increases from $x = 0$ to $x = 3$.

The linear equation $y = mx + b$ that corresponds to the function in that region is $y = 0 \cdot x + 0$. The slope is $m = 0$ (since the graph is a horizontal line) and the initial value is $b = 0$.

Hint 3 Starting at $x = 3$ the graph of the function starts to increase. By counting the squares of the grid, we can calculate the slope of the graph:

$$\frac{\text{rise}}{\text{run}} = \frac{4-0}{5-3} = \frac{4}{2} = 2.$$

The slope of the graph is 2 for x between 3 and 5.

Hint 4 After $x = 5$, y starts to decrease as x increases. The slope of the graph between $x = 5$ and $x = 9$ is equal to:

$$\frac{\text{rise}}{\text{run}} = \frac{0-4}{9-5} = \frac{-4}{4} = -1.$$

The slope of the graph is equal to -1 for x between 5 and 9.

Hint 5 The maximum value of y in the graph occurs at the "peak" of the bump in the graph. The coordinates of this "peak" are $(5, 4)$. Therefore, the maximum value is $y = 4$ and it occurs when $x = 5$.

Hint 6 After completing the sentences we obtain the following description of the graph.

Initially, as x increases, y **stays constant**. Afterward, the slope of the graph of the function is equal to **2** for all x between $x = 3$ and $x = 5$. The slope of the graph is equal to $-\mathbf{1}$ for x between $x = 5$ and $x = 9$. The greatest value of y is $y = \mathbf{4}$ and it occurs when $x = \mathbf{5}$.

8 x58aec930

Miriam overhears her teacher talking about a function described by a linear equation. The rate of change of this function is 3 and its initial value is -6.

What is the graph of this function?

[[interactive-graph 1]]

Drag the two points to move the line into the correct position.

Hint 1 We know the function is described by a linear equation of the form:

$y = m \cdot x + b$.

The number m is called the rate of change of the function. The number b is the initial value of the function.

Hint 2 Miriam heard the teacher say the rate of change of this function is $m = 3$ and its initial value is $b = -6$, therefore the equation which describes this function is:

$y = 3 \cdot x - 6$.

Hint 3 The graph of the function $y = m \cdot x + b$ is a line with y-intercept equal to b and slope equal to m.

Therefore, the graph of the function Miriam's teacher is talking about is a line which passes through the point $(0, -6)$ and has slope equal to 3: ⟨ img ⟩

9 x59b75739

The illustration below shows the graph of y as a function of x.
⟨ img ⟩

Complete the sentences based on the graph of the function.

This is the graph of a [[dropdown 3]] function. The y-intercept of the graph is $y =$ [[input-number 1]]. The x-intercepts of the graph are at $x =$ [[input-number 2]] and $x =$ [[input-number 5]]. The greatest value of y is $y =$ [[input-number 3]] and it occurs when $x =$ [[input-number 4]]. For x between $x = 0$ and $x = 6$, y [[dropdown 1]] 0.

Complete the sentences.

Hint 1 Let's use the graph to complete the sentences one by one.

Hint 2 A function which has a constant rate of change produces a graph that is a line and we say the function is *linear*. Otherwise, if the rate of change of the function is not constant, the graph will not be a line, and we say the function is *nonlinear*.

The graph shown is not a line. Therefore, we are looking at the graph of a **nonlinear** function.

Hint 3 The y-intercept of the graph corresponds to the value of the function when $x = 0$. The y-intercept of the graph is at $y = 0$.

Hint 4 The x-intercepts of the graph correspond to the values of x for which $y = 0$. The x-intercepts of this graph are at $x = 0$ and $x = 6$.

Hint 5 The maximum value of y in the graph occurs at the top of the bump. The coordinates of the top of the bump are $(3, 9)$. Therefore, the maximum value is $y = 9$ and it occurs when $x = 3$.

Hint 6 The graph of the function is above the x-axis for values of x between $x = 0$ and $x = 6$. For these values of x, $y \geq 0$.

Hint 7 We can now complete the sentences.

This is the graph of a **nonlinear** function. The y-intercept of the graph is $y = \mathbf{0}$. The x-intercepts of the graph are at $x = \mathbf{0}$ and $x = \mathbf{6}$. The greatest value of y is $y = \mathbf{9}$ and it occurs when $x = \mathbf{3}$. For x between $x = 0$ and $x = 6$, $\mathbf{y} \geq \mathbf{0}$.

10 x6ad99bbe

The variable y depends on the variable x. When x increases by one unit, y increases by 3 units. Also, when $x = 1$, $y = 5$.

Draw the graph of y as a function of x.

[[interactive-graph 1]]

Drag the two points to move the line into the correct position.

Hint 1 We want to draw the graph of the function which corresponds to the verbal description. Let's see how to find the graph of the function given the information provided.

Hint 2 We are told that $y = 5$ when $x = 1$. In other words, the graph of the function passes through the point $(1, 5)$.

You can drag one of the points on the graph to the coordinates $(1, 5)$ since we know the graph passes through there.

Hint 3 We are also told that when x increases by one unit, y increases by 3 units. This is another way of saying the slope of the graph is equal to 3.

Hint 4 **We now have enough information to draw the graph of y as a function of x. Drag the second point to produce a line with slope equal to 3. You can do this by placing the second point at $(2, 8)$, which is 1 unit to the right and 3 units higher than the point $(1, 5)$. The result will look like this:**
⟨ img ⟩

Hint 5 Note that this is the graph of the function $y = 3 \cdot x + 2$**.**

11 x797d9dd8

Korey is organizing a music concert. She plans to invest \$2000 to rent the venue and to pay the musicians and then sell the tickets for \$20 each. The graph below shows the profit y she will make from the concert as a function of the number of tickets sold x.
⟨ img ⟩

If instead Korey decides to sell the tickets for \$40 each, **what will be the new graph of Korey's profit?**

[[interactive-graph 1]]

Drag the two points to move the line into the correct position.

Hint 1 The first graph shows how the profit y varies as a function of the number of tickets sold x. The slope of the graph is equal to 20, since Korey plans to sell the tickets for \$20. The y-intercept of the graph is $y = -2000$, since she plans to invest \$2000 to organize the concert.

How will the graph of the profit change if Korey decides to sell the tickets for \$40 each?

Hint 2 Let's **find the equation** that describes Korey's profit when she invests \$2000 and sells the tickets for \$20. In that case, the equation that describes the profit y as a function of the number of tickets sold x is

$$\begin{aligned} y &= mx + b \\ &= 20 \cdot x - 2000 \end{aligned}$$

The rate of change $m = 20$ corresponds to the price \$20 per ticket, and the initial value $b = -2000$ corresponds to Korey's initial investment of \$2000.

Hint 3 If instead Korey sells the tickets for \$40 each, then the equation of the profit y as a function of the number of tickets sold x will become

$$\begin{aligned} y &= mx + b \\ &= 40 \cdot x - 2000 \end{aligned}$$

The slope changed from $m = 20$ to $m = 40$ since Korey now sells the tickets for \$40.

Hint 4 The graph of the new profit function $y = 40 \cdot x - 2000$, passes through the point $(0, -2000)$ and has slope equal to 40:

⟨ img ⟩

Note the slope of the graph changed but the y-intercept of the graph remained the same.

12 x81f41d07

Jimmy will be selling hot dogs at the football game. He bought hot dogs, buns and condiments for \$8, and plans to sell the hot dogs for \$1 each at the game. The graph below shows the profit y he will make as a function of the number x of hot dogs sold.

⟨ img ⟩

If Jimmy sells the hot dogs for \$2 instead, what will be the new graph of the profit?

[[interactive-graph 1]]

Drag the two points to move the line into the correct position.

Hint 1 The first graph shows how the profit y varies as a function of the number of hot dogs sold x. The slope of the graph is equal to 1, since Jimmy sells the hot dogs for \$1 each. The y-intercept of the graph is $y = -8$.

How will the graph of the profit change if Jimmy sells the hot dogs for \$2 each?

Hint 2 First let's **find the equation** that describes the profit. The equation that describes the profit y as a function of the number of hot dogs sold x is

$$\begin{aligned} y &= mx + b \\ &= 1 \cdot x - 8 \end{aligned}$$

The slope $m = 1$ corresponds to the price \$1 per hot dog, and the y-intercept $b = -8$ corresponds to Jimmy's initial investment of \$8.

Hint 3 If Jimmy sells the hot dogs for \$2 each, then the equation of the profit y as a function of the number of hot dogs sold x will become

$$\begin{aligned} y &= mx + b \\ &= 2 \cdot x - 8 \end{aligned}$$

The slope changed to $m = 2$ since Jimmy sells the hot dogs for \$2 each. Jimmy's initial investment is still \$8, so the y-intercept remains $b = -8$.

Hint 4 The graph of the new profit function $y = 2 \cdot x - 8$, passes through the point $(0, -8)$ and has slope equal to 2:

⟨ img ⟩

13 x85e40c41

The illustration below shows the graph of y as a function of x.

⟨ img ⟩

Complete the sentences based on the graph of the function.

Initially, as x increases, y [[dropdown 4]]. The slope of the graph is equal to [[input-number 1]] for all x between $x = 0$ and $x = 3$. Starting at $x = 3$, y [[dropdown 2]] as x increases. The slope of the graph is equal to [[input-number 2]] for x between $x = 3$ and $x = 5$. For x between $x = 0$ and $x = 4$, y [[dropdown 3]] 0. For x between $x = 4$ and $x = 8$, y [[dropdown 1]] 0.

Complete the sentences.

Hint 1 Let's use the graph to complete the sentences one by one.

Hint 2 Initially, the graph of the function is decreasing. The quantity y decreases as x increases from $x = 0$ to $x = 3$.

By counting the squares of the grid, we can calculate the slope of the graph:

$$\frac{\text{rise}}{\text{run}} = \frac{-3-0}{3-0} = \frac{-3}{3} = -1.$$

So the slope of the line is $m = -1$.

Hint 3 Starting at $x = 3$ the graph of the function starts to increase. By counting the squares of the grid again, we can calculate the slope of the graph:

$$\frac{\text{rise}}{\text{run}} = \frac{3-(-3)}{5-3} = \frac{6}{2} = 3.$$

The slope of the graph is 3 for x between 3 and 5.

Hint 4 Now we're done completing the sentences which talk about the slope of the function. Next we have to complete the sentences that describe the values of the function. For what values of the input x is y positive and for what values of x is y negative?

The value of y is positive whenever the graph of the function is above the x-axis. The value of y is negative whenever the graph is below the x-axis.

Hint 5 The graph of the function is below the x-axis for x between $x = 0$ and $x = 4$, which means the value of y is negative. For x between $x = 0$ and $x = 4$, $y \leq 0$.

Hint 6 The graph of the function is above the x-axis for x between $x = 4$ and $x = 8$, which means the value of y is positive. For x between $x = 4$ and $x = 8$, $y \geq 0$.

Hint 7 We now know how to complete the sentences.

Initially, as x increases, y **decreases**. The slope of the graph is equal to $\mathbf{-1}$ for all x between $x = 0$ and $x = 3$. Starting at $x = 3$, y **increases** as x increases. The slope of the graph is equal to $\mathbf{3}$ for x between $x = 3$ and $x = 5$. For x between $x = 0$ and $x = 4$, $\mathbf{y \leq 0}$. For x between $x = 4$ and $x = 8$, $\mathbf{y \geq 0}$.

14 x906998f8

Jimmy will be selling hot dogs at the football game. He bought hot dogs, buns and condiments for $8, and now wants to calculate how many hot dogs he has to sell to make a profit. He makes a graph of the profit P he will make as a function of the number n of hot dogs sold.

⟨ img ⟩

Complete the sentences based on the graph of the function.

As Jimmy sells more hot dogs, his profit will [[dropdown 1]]. Jimmy sells the hot dogs for $[[input-number 1]] each. Jimmy needs to sell [[input-number 2]] hot dogs to recover the money he invested. If Jimmy sells 15 hot dogs, he will make $[[input-number 3]] in profit. To make $12 in profit, he would have to sell [[input-number 4]] hot dogs.

Complete the sentences.

Hint 1 The graph shows how the profit P varies as a function of the number of hot dogs sold n. We can use the graph of the function to complete the sentences.

Hint 2 The graph of the function is increasing. As n increases, the profit P will **increase**.

The slope of the graph is equal to 1. For each hot dog sold, the profit increases by $\$1$. This means Jimmy is selling the hot dogs for $\$1$ each.

Hint 3 To complete the remaining three sentences, we can *read off* the appropriate values from the graph of the function. Observe that the graph of the function passes through the points $(8, 0)$, $(15, 7)$, and $(20, 12)$. Therefore, Jimmy needs to sell 8 hot dogs to recover his investment (to break even), 15 hot dogs to make a profit of $P = \$7$, and 20 hot dogs to make a profit of $P = \$12$.

Hint 4 Let's check the answers by using the **equation** which corresponds to the function. Since the graph shows a linear relationship between the profit P and number of hot dogs sold, the equation which describes the profit must be of the form $P = mn + b$, where m is the slope and b is the initial value. The equation that describes the profit P as a function of the number of hot dogs sold n is

$$\begin{aligned} P &= mn + b \\ &= 1 \cdot n - 8. \end{aligned}$$

The slope $m = 1$ corresponds to the price per hot dog, and the y-intercept $b = -8$ corresponds to Jimmy's initial investment.

The slope is $m = 1 \geq 0$, therefore P increases with n. To find how many hot dogs Jimmy needs to sell to recover the money he invested, we must solve for n the equation $P = 0$:

$$\begin{aligned} P &= 0 \\ 1 \cdot n - 8 &= 0 \\ n &= 8. \end{aligned}$$

Similarly, to find the number of sales required to make $\$12$ in profit, we must find n when $P = 12$:

$$\begin{aligned} P &= 12 \\ 1 \cdot n - 8 &= 12 \\ n &= 12 + 8 \\ n &= 20. \end{aligned}$$

To find the profit Jimmy will make if he sells 15 hot dogs, he plug the value $n = 15$ into the equation for P to obtain

$$\begin{aligned} P &= mn + b \\ &= 1 \cdot 15 - 8 \\ &= 7. \end{aligned}$$

Hint 5 The correct way to complete the sentences is as follows.

As Jimmy sells more hot dogs, his profit will **increase**. Jimmy sells the hot dogs for $1 each. Jimmy needs to sell **8** hot dogs to recover the money he invested. If Jimmy sells 15 hot dogs, he will make $7 in profit. To make $12 in profit, he would have to sell **20** hot dogs.

15 x93a49507

David wants to rent a bicycle for a couple of hours to explore the city. The price of the bike rental P depends on the time the bike was out t. The price consists of a base charge of $8 and a variable hourly cost. The graph of the function is shown below.
⟨ img ⟩

Complete the sentences based on the graph of the function.

The hourly charge is $[[input-number 1]] per hour for the first 3 hours. The rate then drops to $[[input-number 2]] per hour until the end of the 6^{th} hour. The hourly rate drops further to $[[input-number 3]] per hour between 6^{th} and the 10^{th} hour. The maximum price of the bike rental is [[input-number 4]] dollars.

Complete the sentences.

Hint 1 The slope of the graph corresponds to the hourly rate for the bike rental.

Hint 2 During the first three hours of the bike rental, the price increases by $4 each hour.

Hint 3 Between third hour and the sixth hour, the slope of the graph is 2, which means the hourly rate of the bike rental is $2 per hour.

Hint 4 For the hours between the 6^{th} and 10^{th} hour, the price is $1 per hour.

Hint 5 After the 10^{th} hour, the price P stops increasing. The maximum price of the bike rental is $30.

Hint 6 The graph of the price function is horizontal after $t = 10$ hours. The slope of the graph is 0.

This means the price function is constant. After $t = 10$ hours, the equation which describes the price is:

$P = 0 \cdot t + 30 = 30$

This is the equation of a line with a slope $m = 0$.

Hint 7 The correct way to complete the sentences is as follows.

The hourly charge is $**4** per hour for the first 3 hours. The rate then drops to $**2** per hour until the end of the 6^{th} hour. The hourly rate drops further to $**1** per hour between 6^{th} and the 10^{th} hour. The maximum price of the bike rental is **30** dollars**.**

16 xa0f88ec4

You are downloading a file from the Internet. Your download rate is equal to 10 megabytes per minute. Assume x represents the time in minutes, and y represents the amount of downloaded data in megabytes.

Draw the graph that represents the amount of downloaded data as a function of time.

[[interactive-graph 1]]

Drag the two points to move the line into the correct position.

Hint 1 Were looking for the graph of the function which describes how the amount of downloaded data y depends on the time x.

Hint 2 The amount of downloaded data y is *proportional* to the time x. We can think of y as a function of x. The equation of this function is

$$\begin{aligned} y &= m \cdot x \\ &= 10 \cdot x \end{aligned}$$

The constant of proportionality is $m = 10$ megabytes per minute, since this is the download rate. Indeed, a download speed of 10 megabytes per minute means the download data increases by 10 megabytes each minute.

Hint 3 The graph of the function $y = 10x$ is a line which passes through the origin and has slope equal to 10: ⟨ img ⟩

Hint 4

17 xa79c5488

Jeff works as a waiter at a fancy restaurant. He receives a base amount of $70 each day, plus approximately $10 of tip for each client he serves. Assume x represents the number of clients Jeff will serve in a day and y represents his daily salary.

Draw the graph which represents Jeff's daily salary as a function of the number of clients he serves.

[[interactive-graph 1]]

Drag the two points to move the line into the correct position.

Hint 1 Were looking for the graph of the function which describes how Jeff's daily salary y depends on the number of clients he serves x.

Hint 2 We know his salary y increases by $10 for each client he serves. Also, we know that he receives a base daily salary of $70. Jeff's daily salary y as a function of the number of clients he serves x is described by the following linear equation:

$$y = m \cdot x + b$$
$$= 10 \cdot x + 70.$$

The rate of change is $m = 10$ because this is how much tip he makes per client. If x increases by one, his salary y will increase by \$10.

The initial value is $b = 70$. This is the base amount Jeff earns even when he serves $x = 0$ clients.

Hint 3 The graph of the function $y = m \cdot x + b$ is a line with y-intercept equal to b and slope equal to m.

Therefore, the graph of Jeff's daily salary $y = 10x + 70$ is a line which passes through the point $(0, 70)$ and has slope equal to 10:
⟨ img ⟩

Hint 4

18 xadd760c2

Korey is organizing a music concert. She plans to invest \$2000 to rent the venue and to pay the musicians and then sell the tickets for \$20 each. The graph below shows the profit y she will make from the concert as a function of the number of tickets sold x.
⟨ img ⟩

If instead Korey decides to invest only \$1200 for the concert and sells the tickets for \$40 each, **what will be the new graph of Korey's profit?**

[[interactive-graph 1]]

Drag the two points to move the line into the correct position.

Hint 1 The first graph shows how the profit y varies as a function of the number of tickets sold x. The slope of the graph is equal to 20, since Korey plans to sell the tickets for \$20. The y-intercept of the graph is $y = -2000$, since she plans to invest \$2000 to organize the concert.

How will the graph of the profit change if Korey invests \$1200 and sells the tickets for \$40 each?

Hint 2 Let's **find the equation** that describes Korey's profit when she invests \$2000 and sells the tickets for \$20. In that case, the equation that describes the profit y as a function of the number of tickets sold x is

$$y = mx + b$$
$$= 20 \cdot x - 2000$$

The rate of change $m = 20$ corresponds to the price \$20 per ticket, and the initial value $b = -2000$ corresponds to Korey's initial investment of \$2000.

Hint 3 If instead Korey invests \$1200 for the concert and sells the tickets for \$40 each, then the equation of the profit y as a function of the number of tickets sold x will become

$$y = mx + b$$
$$= 40 \cdot x - 1200$$

The initial value changed from $b = -2000$ to $b = -1200$ to reflect Korey's smaller up-front investment. The slope changed to $m = 40$ since Korey now sells the tickets for \$40.

Hint 4 The graph of the new profit function $y = 40 \cdot x - 1200$, passes through the point $(0, -1200)$ and has slope equal to 40:
⟨ img ⟩

19 xb754f5a4

The illustration below shows the graph of y as a function of x.
⟨ img ⟩

Complete the sentences based on the graph of the function.

As x increases, y [[dropdown 1]]. The rate of change of y as x changes is [[dropdown 2]], therefore the function is [[dropdown 3]]. For all values of x, y [[dropdown 4]] 0. The y-intercept of the function is $y =$ [[input-number 1]]. When $x = 1$, $y =$ [[input-number 2]].

Complete the sentences.

Hint 1 Let's use the graph to complete the sentences one by one.

Hint 2 Looking at the graph of the function, we see that y always **decreases** as x increases.

Hint 3 A function which has a constant rate of change produces a graph that is a line and we say the function is *linear*. Otherwise, if the rate of change of the function is not constant, the graph will not be a line, and we say the function is *nonlinear*.

In the graph shown, the rate of change of y as x changes is **not constant**. This means the function whose graph we are viewing is **nonlinear**.

Hint 4 Observe that the graph of the function lies entirely above the x-axis. This means the values of y are always positive. For all values of x, $y \geq 0$.

Hint 5 The y-intercept of the graph corresponds to the value of the function when $x = 0$. The y-intercept of the function is $y = 8$.

Hint 6 To complete the last sentence, we can read off the value of the function for $x = 1$. When $x = 1$, $y = 4$.

Hint 7 We can now complete the sentences.

As x increases, y **decreases**. The rate of change of y as x changes is **not constant**, therefore the function is **nonlinear**. For all values of x, $\mathbf{y \geq 0}$. The y-intercept of the function is $y = \mathbf{8}$. When $x = 1$, $y = \mathbf{4}$.

20 xc78ba6a5

The illustration below shows the graph of y as a function of x.
⟨ img ⟩

Complete the sentences based on the graph of the function.

As x increases, y [[dropdown 1]]. The rate of change of y as x changes is [[dropdown 2]], therefore the function is [[dropdown 3]]. For all values of x, y [[dropdown 4]] 0. The y-intercept of the function is $y =$ [[input-number 1]]. When $x = 6$, $y =$ [[input-number 2]].

Complete the sentences.

Hint 1 Let's use the graph to complete the sentences one by one.

Hint 2 Looking at the graph of the function, we see that y always **increases** as x increases.

Hint 3 A function which has a constant rate of change produces a graph that is a line and we say the function is *linear*. Otherwise, if the rate of change of the function is not constant, the graph will not be a line, and we say the function is *nonlinear*.

In the graph shown, the rate of change of y as x changes is **not constant**. This means the function whose graph we are viewing is **nonlinear**.

Hint 4 Observe that the graph of the function lies entirely above the x-axis. This means the values of y are always positive. For all values of x, $y \geq 0$.

Hint 5 The y-intercept of the graph corresponds to the value of the function when $x = 0$. The y-intercept of the function is $y = 1$.

Hint 6 To complete the last sentence, we can read off the value of the function for $x = 6$. When $x = 6$, $y = 7$.

Hint 7 We can now complete the sentences.

As x increases, y **increases**. The rate of change of y as x changes is **not constant**, therefore the function is **nonlinear**. For all values of x, $\mathbf{y \geq 0}$. The y-intercept of the function is $y = \mathbf{1}$. When $x = 6$, $y = \mathbf{7}$.

21 xcb3ee656

Jessica works in sales. Her monthly salary is calculated as a base amount of \$2000 plus a commission of \$100 for each sale she closes. Assume x represents the number of sales Jessica closes and y represents her monthly salary.

Draw the graph which represents Jessica's monthly salary as a function of the number of sales.

[[interactive-graph 1]]

Drag the two points to move the line into the correct position.

Hint 1 Were looking for the graph of the function which describes how Jessica's monthly salary y depends on the number of sales x.

Hint 2 We know her salary y increases by \$100 for each sale she makes. Also, we know that she receives a base amount of \$2000. Jessica's monthly salary y as a function of the number of sales x is described by the following linear equation:

$$\begin{aligned} y &= m \cdot x + b \\ &= 100 \cdot x + 2000. \end{aligned}$$

The rate of change is $m = 100$ because this is how much she makes per sale. If x increases by one, her salary y will increase by \$100.

The initial value is $b = 2000$. This is the base amount Jessica earns even when she makes $x = 0$ sales.

Hint 3 The graph of the function $y = m \cdot x + b$ is a line with y-intercept equal to b and slope equal to m.

Therefore, the graph of Jessica's monthly salary $y = 100x + 2000$ is a line which passes through the point $(0, 2000)$ and has slope equal to 100: ⟨ img ⟩

Hint 4

22 xccd47986

Today, Sean left home at 8AM, drove to work, and worked from 9AM until 5PM. On the drive back home, Sean got stuck in traffic for 2 hours, then stopped to have dinner and finally got home at 9:30PM.

The graph below shows Sean's distance from home d as a function of the time t.
⟨ img ⟩

Complete the sentences based on the graph of the function.

Sean's work is located at a distance [[input-number 1]] km from his home. Sean's speed when he was driving to work was [[input-number 2]] km/h. Sean's speed between 5PM and 7PM was [[input-number 3]] km/h.

Complete the sentences.

Hint 1 The graph shows Sean's position as a function of time. The slope of the graph represents the speed at which he is moving. Let's use the graph to complete the sentences one by one.

Hint 2 Looking at the graph, we see that Sean's position stays constant at $d = 20$ km, between 9AM and 5PM. This represents the time Sean spends at work. Therefore, his work place is located at a distance of 20 km from his home.

Hint 3 We can calculate the slope of the graph of Sean's position function between 8AM and 9AM as follows:

$$\frac{\text{rise}}{\text{run}} = \frac{20 - 0 \ \ [\text{km}]}{9 - 8 \ \ [\text{h}]} = \frac{20 \ [\text{km}]}{1 \ [\text{h}]} = 20 \text{ km/h}.$$

Sean maintains a speed of 20 km per hour on his drive to work.

Hint 4 We can also calculate Sean's speed when he leaves work:

$$\frac{\text{rise}}{\text{run}} = \frac{10 - 0 \ \ [\text{km}]}{19 - 17 \ \ [\text{h}]} = \frac{10 \ [\text{km}]}{2 \ [\text{h}]} = 5 \text{ km/h}.$$

Sean maintains a speed of 5 km/h while stuck in traffic.

Hint 5 The correct way to complete the sentences is as follows.

Sean's work is located at a distance **20** km from his home. Sean's speed when he was driving to work was **20** km/h. Sean's speed between 5PM and 7PM was **5** km/h**.**

23 xd764f378

The illustration below shows the graph of y as a function of x.

⟨ img ⟩

Complete the sentences based on the graph of the function.

This is the graph of a [[dropdown 3]] function. The y-intercept of the graph is $y =$ [[input-number 1]]. The x-intercepts of the graph are at $x =$ [[input-number 2]] and $x =$ [[input-number 5]]. The greatest value of y is $y =$ [[input-number 3]] and it occurs when $x =$ [[input-number 4]]. For x between $x = 2$ and $x = 6$, y [[dropdown 1]] 0.

Complete the sentences.

Hint 1 Let's use the graph to complete the sentences one by one.

Hint 2 A function which has a constant rate of change produces a graph that is a line and we say the function is *linear*. Otherwise, if the rate of change of the function is not constant, the graph will not be a line, and we say the function is *nonlinear*.

The graph shown is not a line. Therefore, we are looking at the graph of a **nonlinear** function.

Hint 3 The y-intercept of the graph corresponds to the value of the function when $x = 0$. The y-intercept of the graph is $y = -6$.

Hint 4 The x-intercepts of the graph correspond to the values of x for which $y = 0$. The x-intercepts of this graph are at $x = 2$ and $x = 6$.

Hint 5 The maximum value of y in the graph occurs at the top of the bump. The coordinates of the top of the bump are $(4, 2)$. Therefore, the maximum value is $y = 2$ and it occurs when $x = 4$.

Hint 6 The graph of the function is above the x-axis for values of x between $x = 2$ and $x = 6$. For these values of x, $y \geq 0$.

Hint 7 We can now complete the sentences.

This is the graph of a **nonlinear** function. The y-intercept of the graph is $y = \mathbf{-6}$. The x-intercepts of the graph are at $x = \mathbf{2}$ and $x = \mathbf{6}$. The greatest value of y is $y = \mathbf{2}$ and it occurs when $x = \mathbf{4}$. For x between $x = 2$ and $x = 6$, $\mathbf{y \geq 0}$.

24 xdba32ac1

Korey is organizing a music concert. She plans to invest \$2000 to rent the venue and to pay the musicians and then sell the tickets for \$20 each. The graph below shows the profit y she will make from the concert as a function of the number of tickets sold x.

⟨ img ⟩

If instead Korey decides to invest only \$1200 to organize the concert, **what will be the new graph of Korey's profit?**

[[interactive-graph 1]]

Drag the two points to move the line into the correct position.

Hint 1 The first graph shows how the profit y varies as a function of the number of tickets sold x. The slope of the graph is equal to 20, since Korey plans to sell the tickets for \$20. The y-intercept of the graph is $y = -2000$, since she plans to invest \$2000 to organize the concert.

How will the graph of the profit change if Korey invests only \$1200 for the concert organization?

Hint 2 Let's **find the equation** that describes Korey's profit when she invests \$2000 and sells the tickets for \$20. In that case, the equation that describes the profit y as a function of the number of tickets sold x is

$$\begin{aligned} y &= mx + b \\ &= 20 \cdot x - 2000 \end{aligned}$$

The rate of change $m = 20$ corresponds to the price \$20 per ticket, and the initial value $b = -2000$ corresponds to Korey's initial investment of \$2000.

Hint 3 If instead Korey invests \$1200 for the concert, then the equation of the profit y as a function of the number of tickets sold x will become

$$y = mx + b$$
$$= 20 \cdot x - 1200$$

The initial value changed from $b = -2000$ to $b = -1200$ to reflect Korey's smaller up-front investment.

Hint 4 The graph of the new profit function $y = 20 \cdot x - 1200$, passes through the point $(0, -1200)$ and has slope equal to 20:
⟨ img ⟩
Note the slope of the graph remained the same and only the y-intercept of the graph changed.

25 xe4b4df77

The variable y is proportional to the variable x. The constant of proportionality is 4.
Draw the graph of y as a function of x.
[[interactive-graph 1]]
Drag the two points to move the line into the correct position.

Hint 1 Let's find the equation which corresponds to the verbal description and then draw the graph.
We are told the variable y is *proportional* to the variable x which means we are looking for an equation of the form $y = m \cdot x$, where m is the constant of proportionality.

Hint 2 We are told the constant of proportionality is $m = 4$. Therefore, the equation which describes the relationship between y and x is $y = 4 \cdot x$.

Hint 3 Note that the equation $y = m \cdot x$ is a special case of the linear equation $y = m \cdot x + b$, with the y-intercept $b = 0$.
The graph of y versus x, for the equation $y = 4 \cdot x$ is a line which passes through the origin and has slope equal to 4:
⟨ img ⟩

Hint 4

26 xe931d345

Jane sells hot dogs at football games. For last week's game, she bought hot dogs, buns and condiments for \$6 at the store, and sold the hot dogs for \$1 each at the game. The graph below shows the profit y she made as a function of the number x of hot dogs sold.
⟨ img ⟩
This week, Jane wants to change things up. She will buy bigger hot dogs, investing \$9 at the store, and sell the hot dogs for \$2 each.
What will be the graph of her profit for this week?
[[interactive-graph 1]]
Drag the two points to move the line into the correct position.

Hint 1 The first graph shows how the profit y varies as a function of the number of hot dogs sold x. The slope of the graph is equal to 1, since Jane sold the hot dogs for \$1 each last week. The y-intercept of the graph is $y = -6$, since she invested \$6 for the food.
How will the graph of the profit change if Jane invests \$9 for the food and sells the hot dogs for \$2 each?

Hint 2 First let's **find the equation** that describes the profit of last week's sales. The equation that describes the profit y as a function of the number of hot dogs sold x is

$$y = mx + b$$
$$= 1 \cdot x - 6$$

The slope $m = 1$ corresponds to the price \$1 per hot dog, and the initial value $b = -6$ corresponds to Jane's initial investment of \$6.

Hint 3 If this week Jane invests \$9 for the food and sells the hot dogs for \$2 each, then the equation of the profit y as a function of the number of hot dogs sold x will become

$$y = mx + b$$
$$= 2 \cdot x - 9$$

The initial value changed from $b = -6$ to $b = -9$ to reflect the fact that Jane spent \$9 at the store this week. The slope changed to $m = 2$ since Jane sells the hot dogs for \$2 each this week.

Hint 4 The graph of the new profit function $y = 2 \cdot x - 9$, passes through the point $(0, -9)$ and has slope equal to 2:
⟨ img ⟩

27 xf16c46c4

A doctor observes a graph that shows the electrical activity (in Volts) of the heart of a patient over a period of time. Each spike corresponds to one heart beat.
⟨ img ⟩
The doctor needs to calculate the heart rate of the patient in *beats per minute*. **What is the heart rate of this patient?**
[[input-number 1]] beats per minute

Hint 1 The graph shown is called an *electrocardiogram*: a diagram which shows the electric voltage of the heart beat. Each spike in the graph represents one heart beat.
To find the heart rate in *beats per minute*, we need to calculate how many heart beats will occur in 60 seconds.

Hint 2 Looking at the graph, we observe that 6 heart beats occur in each period of 6 seconds. To find the number of heart beats in one minute we proceed as follows:

$\frac{6 \text{ beats}}{6 \cancel{\text{sec}}} \cdot \frac{60 \cancel{\text{sec}}}{1 \text{ min}} = \frac{\cancel{6} \cdot 60 \text{ beats}}{\cancel{6} \text{ min}} = 60$ beats per minute.

This makes sense: 60 beats per minute is equivalent to 1 beat every second, which is what we see in the graph.

Hint 3 The heart rate of the patient is 60 beats per minute.

28 xf17c366e

It takes two slices of bread to make a sandwich. When you are making sandwiches, the number of slices of bread you will need is a function of the number of sandwiches you want to prepare. Assume x represents the number of sandwiches you want to make, and y represents the number of slices of bread.

Draw the graph that represents this function.

[[interactive-graph 1]]

Drag the two points to move the line into the correct position.

Hint 1 Were looking for the graph of the function which describes how the number of slices of bread y depends on the number of sandwiches you want to make x.

Hint 2 The number of slices you will need y is *proportional* to the number of sandwiches you want to make x. We can think of y as a function of x. The equation of this function is

$$\begin{aligned} y &= m \cdot x \\ &= 2 \cdot x \end{aligned}$$

The constant of proportionality is $m = 2$ since it takes two slices of bread to make one sandwich.

Hint 3 The graph of the function $y = 2x$ is a line which passes through the origin and has slope equal to 2: ⟨ img ⟩

Hint 4

29 xf4f6c84c

A doctor observes a graph that shows the electrical activity (in Volts) of the heart of a patient over a period of time. Each spike corresponds to one heart beat.

⟨ img ⟩

The doctor needs to calculate the heart rate of the patient in *beats per minute*. **What is the heart rate of this patient?**

[[input-number 1]] beats per minute

Hint 1 The graph shown is called an *electrocardiogram*: a diagram which shows the electric voltage of the heart beat. Each spike in the graph represents one heart beat.

To find the heart rate in *beats per minute*, we need to calculate how many heart beats will occur in 60 seconds.

Hint 2 Looking at the graph, we observe that 9 heart beats occur in each period of 6 seconds. To find the number of heart beats in one minute we proceed as follows:

$\frac{9 \text{ beats}}{6 \cancel{\text{sec}}} \cdot \frac{60 \cancel{\text{sec}}}{1 \text{ min}} = \frac{9 \cdot 60 \text{ beats}}{6 \text{ min}} = \frac{9 \cdot \cancel{6} \cdot 10 \text{ beats}}{\cancel{6} \text{ min}} = 90$ beats per minute.

Hint 3 The heart rate of the patient is 90 beats per minute.

30 xf8bb6f6a

The variable y is related to the variable x through a linear equation. The graph of y as a function of x passes through the points $(1, 3)$ and $(2, 5)$.

Draw the graph of this function.

[[interactive-graph 1]]

Drag the two points to move the line into the correct position.

Hint 1 To draw the graph of y versus x, drag one of the points of the graph placing it at the point $(1, 3)$ and drag the second point to the coordinates $(2, 5)$.

⟨ img ⟩

Wow, that was easy!

Hint 2 The graph of this line corresponds to some linear equation: $y = m \cdot x + b$, where m is the slope and b is the y-intercept.

Looking at the graph of the line that passes through the points $(1, 3)$ and $(2, 5)$, we can easily read off the value of the slope m and the y-intercept b. The equation of the line in the graph is $y = 2 \cdot x + 1$.