

Statistical Inference - Simulation Exercise

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```
library(ggplot2)
library(ggpubr)
```

Overview

In this project I will investigate the exponential distribution in R and compare it with the Central Limit Theorem. I will simulate 1000 random samples with 40 random exponentials and compare the mean and variance of the samples with the theoretical statistics.

Simulations

In the following code, I am creating 1000 samples of 40 random exponentials and putting them into a matrix 'x' so that there are 1000 rows each with 40 columns of random samples. Setting the seed to 188 so that the code can be reproduced.

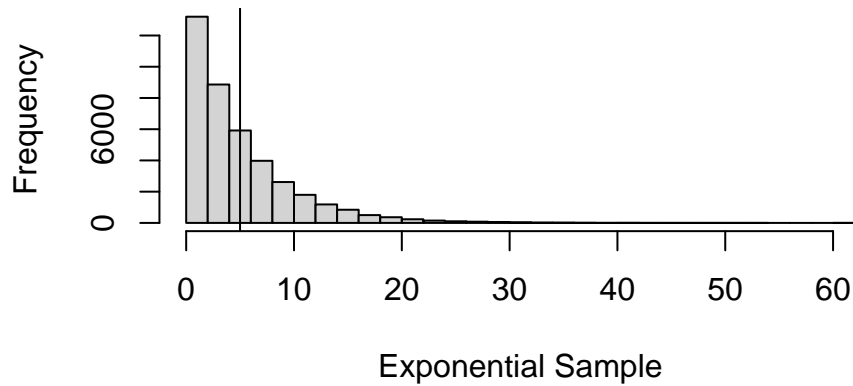
```
options(scipen=999)
set.seed(1888)
x<- matrix(rexp(40*1000, 0.2),ncol=40)
```

Sample Mean versus Theoretical Mean

Let's first take a look at the distribution in a histogram with all 40,000 values included.

```
hist(x,
      breaks = 40,
      xlab = "Exponential Sample",
      main = "Exponential Distribution",)
abline(v=mean(x))
```

Exponential Distribution



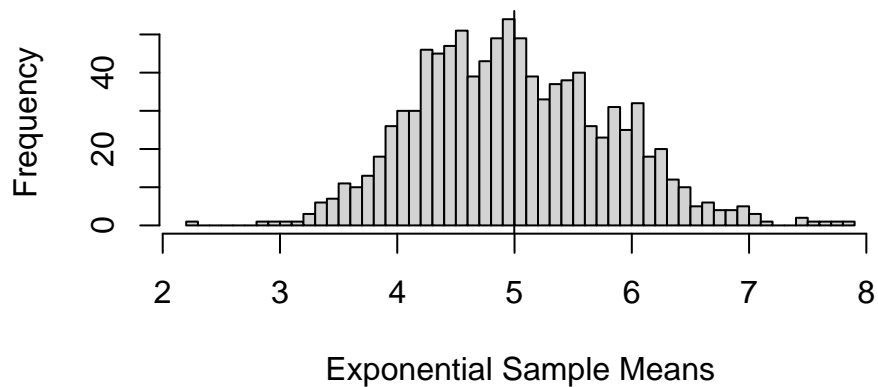
```
mean(x)
```

```
## [1] 4.997758
```

In the above figure we can see that the data is not normally distributed, with the mean sitting at just below 5 (I have added in a line with the mean) Let's compare this to the mean of the samples which as per the central limit theorem should be normally distributed.

```
hist(rowMeans(x),  
     breaks = 40,  
     xlab = "Exponential Sample Means",  
     main = "Exponential Sample Distribution",)  
abline(v=mean(rowMeans(x)))
```

Exponential Sample Distribution



```
mean(rowMeans(x))
```

```
## [1] 4.997758
```

We can see the above figure appears to be normally distributed with a mean of 4.985 as well which is also close to the theoretical mean of 5 ($1/\lambda(0.2)$)

Sample Variance versus Theoretical Variance:

Let's take a look at the sample variance compared to the theoretical variance. For the theoretical sample variance, this would be calculated as $1/n * \lambda^2$ which equals 0.625

```
var(rowMeans(x))
```

```
## [1] 0.6666267
```

We can see that the sample variance which we obtained is not too different from the theoretical variance above.

Distribution:

We can use the Shapiro-Wilk test to also see if the sample means are normally distributed.

```
shapiro.test(rowMeans(x))$p.value
```

```
## [1] 0.00007277564
```

We can see that the p-value is well below 5% we can conclude that the distribution is normal when broken into samples of 40 rather than using the entire 40,000 samples together. Our assumptions are that the `set.seed()` was set to 1888 and sampling method is exponential distribution using R's built in function `rexp()` with `rate = 0.2`