Cover Sheet for Submission of Maths Examinations Summer 2020

We would advise preparing your coversheets with your CID, Module Name and Code and Date, before the exams are due to take place.

CID: 01738166

Module Name: An Introduction to Applied Maths

Module Code: MATH40007

Date: 18/05/2020

Questions Answered (in the file):

Please tick next to the question or questions you have answered in this file.

Q1	✓
Q2	
Q3	
Q4	
Q5	
Q6	

(Note: this is a coversheet for all students - not all students will have exams with 6 questions. Please tick the boxes which are appropriate for your exam and/or the file you are submitting).

(Optional) Page Numbers for each question;

Page Number	Question Answered

If handwritten, please complete in CAPITAL Letters, in Blue or Black Ink, ensuring the cover sheet is legible.

(a) $$\begin{bmatrix} 2 & -1 & -1 & 0 & 0 & 0 \\ -1 & 4 & -1 & -1 & -1 & 0 \\ -1 & -1 & 4 & 0 & -1 & -1 \\ 0 & -1 & 0 & 2 & -1 & 0 \\ 0 & -1 & -1 & -1 & 4 & -1 \\ 0 & 0 & -1 & 0 & -1 & 2 \end{bmatrix} = K$$

(b) We have $K\underline{x} = \underline{d}$, where $\underline{x} = \begin{bmatrix} 1 \\ x_2 \\ x_3 \\ 0 \\ x_5 \\ x_6 \end{bmatrix}$ (unit volt. / grounded), $\underline{d} = \begin{bmatrix} d_1 \\ 0 \\ 0 \\ -d_1 \\ 0 \\ 0 \end{bmatrix}$ (KCL holds)

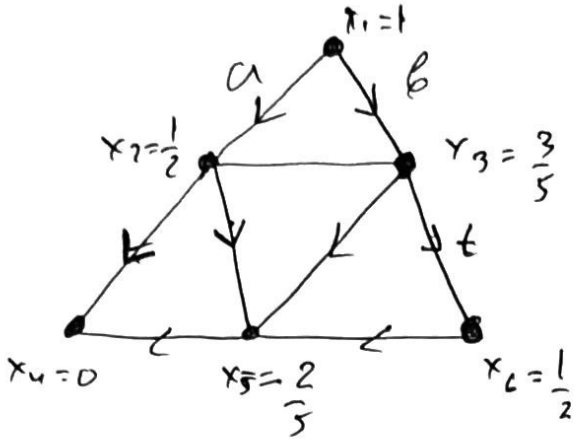
$$K\underline{x} = \begin{bmatrix} 2 - x_2 - x_3 \\ 4x_2 - x_3 - x_5 - 1 \\ -x_2 + 4x_3 - x_5 - x_6 - 1 \\ x_2 - x_5 \\ -x_2 - x_3 + 4x_5 - x_6 \\ -x_3 - x_5 + 2x_6 \end{bmatrix} = \begin{bmatrix} d_1 \\ 0 \\ 0 \\ -d_1 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{cases} 4x_2 = x_3 + x_5 + 1 \\ x_2 + x_5 + x_6 + 1 = 4x_3 \\ x_2 + x_3 + x_6 = 4x_5 \\ x_3 + x_5 = 2x_6 \end{cases} \overset{(*)}{\Rightarrow} \boxed{\begin{array}{l} x_2 = \frac{1}{2}, \; x_3 = \frac{3}{5}, \; x_5 = \frac{2}{5} \\ x_6 = \frac{1}{2} \end{array}}$$

(c) ~~(From (b), we have that 1 unit current enters node)~~

We have Ohm's Law: $w_i = -c_i(x_a - x_b)$ - current flowing through edge i connecting nodes a and b. Also, we know that the current flows from high voltage to low voltage.

We now draw the following diagram:



The arrows show the direction of the flow according to Ohms Law.

We have $I = -1 \cdot (x_6 - x_3) = -1 \cdot \left(\frac{1}{2} - \frac{3}{5}\right) = \frac{3}{5} - \frac{1}{2} = \frac{1}{10}$

$\Rightarrow \boxed{\frac{1}{10}}$ current flows from node 3 to node 6.

Current through node a: $\frac{1}{2}$, node b: $\frac{2}{5} \Rightarrow \frac{1}{2} + \frac{2}{5} = \frac{9}{10}$ → current into node 1. Scale by $\frac{10}{9}$, to make it unit $\Rightarrow$

(b) Solving (*): Scale our answer as well:

$\Rightarrow \frac{1}{10} \cdot \frac{10}{9} = \boxed{\frac{1}{9}}$ – current from node 3 to node 6.

$x_6 = \frac{x_3 + x_5}{2} \Rightarrow x_5 = \frac{x_2 + x_3 + (x_3 + x_5)/2}{4} \Rightarrow 4x_5 = x_2 + \frac{3}{2}x_3 + \frac{x_5}{2}$

$\Rightarrow x_5 \cdot \frac{7}{2} = x_2 + \frac{3x_3}{2} \Rightarrow x_5 = \frac{2x_2}{7} + \frac{3x_3}{7}$

$4x_3 = x_2 + \underbrace{\frac{2x_2}{7} + \frac{3x_3}{7}}_{x_5} + \underbrace{\frac{x_3}{2} + \frac{2x_2}{14} + \frac{3x_3}{14}}_{x_6} + 1 = \frac{10}{7}x_2 + \frac{7}{7} = \frac{46}{49}x_3$

$\Rightarrow x_3 = \frac{70}{46}x_2 + \frac{49}{46}$

$4x_2 = \frac{70}{46}x_2 + \frac{49}{46} + \frac{2x_2}{7} + \frac{3}{7}\left(\frac{70}{46}x_2 + \frac{49}{46}\right) + 1$

$\Rightarrow x_2 = \frac{1}{2}$, $\Rightarrow x_3 = \frac{3}{5}$, $x_5 = \frac{2}{5}$, $x_6 = \frac{1}{2}$