Question 1

Suppose $X_1, X_2, \ldots, X_n$ are independent and identically distributed random variables following a normal distribution with mean μ and variance σ^2. The value of μ is unknown, but σ^2 is known to be $\sigma^2 = 16$. Suppose we observe $\mathbf{X} = (X_1, X_2, \ldots, X_n)$ as $\mathbf{x} = (x_1, x_2, \ldots, x_n)$. Given that $\overline{x} = 7$ and $n = 50$, construct a 99% confidence interval for μ.

Question 2

Suppose $Y_1, Y_2, \ldots, Y_n$ are independent and identically distributed random variables following a normal distribution with mean μ and variance σ^2. The values of μ and σ^2 are both unknown. Suppose we observe $\mathbf{Y} = (Y_1, Y_2, \ldots, Y_n)$ as $\mathbf{y} = (y_1, y_2, \ldots, y_n)$. Given that the sample mean is $\overline{y} = 11$, the sample variance is $s^2 = 18$ and $n = 8$, construct a 90% confidence interval for μ.

Question 3

Suppose $Z_1, Z_2, \ldots, Z_n$ are independent and identically distributed random variables following an unknown distribution F_Z. The mean μ of the distribution F_Z is unknown, but the variance of F_Z is known to be $\sigma^2 = 7$. Suppose we observe $\mathbf{Z} = (Z_1, Z_2, \ldots, Z_n)$ as $\mathbf{z} = (z_1, z_2, \ldots, z_n)$. Given that the sample mean is $\overline{z} = 6$ and $n = 12$, construct a 95% confidence interval for μ.

Question 4

Suppose the heights of two groups of people are recorded. Group A consists of n people and their heights are recorded (in cm) as $x_1, x_2, \ldots, x_n$ with $n = 10$, sample mean $\overline{x} = 171.5$ and sample variance $s_x^2 = 2$. Group B consists of m people and their heights are recorded as $y_1, y_2, \ldots, y_m$, with $m = 12$, $\overline{y} = 170$ and sample variance $s_y^2 = 3$. We wish to test if the average heights of the two groups are significantly different or not. We start by assuming that the measurements $x_1, x_2, \ldots, x_n$ are observations of the independent random variables $X_1, X_2, \ldots, X_n$, respectively, which follow a normal distribution with unknown mean μ_1 and unknown variance σ_1^2. We also assume that the $y_1, y_2, \ldots, y_m$ are observations of the independent random variables $Y_1, Y_2, \ldots, Y_m$, respectively, following a normal distribution with unknown mean μ_2 and unknown variance σ_2^2. We also assume that although the variances are unknown, they are equal i.e. $\sigma_1^2 = \sigma_2^2 = \sigma^2$.

(a) What is the null hypothesis for this test?

(b) Assuming the null hypothesis is true, use Student's two-sample t-test to compute a p-value and decide whether or not the average heights of the two groups are signficantly different or not.

Question 5

A pharmaceutical company conducts a number of clinical trials simultaneously to test the effectiveness of different drug treatments for a particular disease. In each clinical trial $i \in \{1, 2, \ldots, n\}$, a group of patients is randomly divided into two subgroups, one of which is given drug treatment i while the other is given a placebo (a substance that has no effect on the disease, such as a sugar pill). After a period of time, the patients are examined and declared either to be cured or not to be cured. For each clinical trial, a statistical analysis is performed on the resulting data from the two subgroups.

(a) If the goal is to determine if a drug treatment is effective, what should the null hypothesis be for each statistical test?

(b) The results of the $n = 15$ statistical tests were the following p-values (in increasing order):

0.0001,	0.0004,	0.0019,	0.0095,	0.0201,	0.0278,	0.0298,	0.0344,
0.0459,	0.3240,	0.4262,	0.5719,	0.6528,	0.7590,	1.000.	

If the pharmaceutical company declared in advance that a significance level of $\alpha = 0.05$ would be used, which of the p-values should be considered as significant (and therefore, which corresponding hypotheses should be rejected)?

Hint:

If $X_1, X_2, \ldots, X_n \sim \mathrm{N}\left(\mu_1, \sigma^2\right)$ are independent and if $Y_1, Y_2, \ldots, Y_n \sim \mathrm{N}\left(\mu_2, \sigma^2\right)$ are independent (and each X_i is independent of each Y_j), then defining

$$T = \frac{\overline{X} - \overline{Y} - (\mu_1 - \mu_2)}{s_p \sqrt{\frac{1}{n} + \frac{1}{m}}},$$

where

$$s_p^2 = \frac{1}{n+m-2}\left(\sum_{i=1}^{n}(X_i - \overline{X})^2 + \sum_{j=1}^{m}(Y_j - \overline{Y})^2\right) = \frac{1}{n+m-2}\left((n-1)S_X^2 + (m-1)S_Y^2\right),$$

it can be shown that $T \sim t_{n+m-2}$.

Values of t for $\mathrm{P}(T < t)$, where T has Student's t-distribution with ν degrees of freedom

ν	0.60	0.667	0.75	0.80	0.87	0.90	0.95	0.975	0.99	0.995	0.999
1	0.325	0.577	1.000	1.376	2.414	3.078	6.314	12.706	31.821	63.657	318.31
2	0.289	0.500	0.816	1.061	1.604	1.886	2.920	4.303	6.965	9.925	22.327
3	0.277	0.476	0.765	0.978	1.423	1.638	2.353	3.182	4.541	5.841	10.215
4	0.271	0.464	0.741	0.941	1.344	1.533	2.132	2.776	3.747	4.604	7.173
5	0.267	0.457	0.727	0.920	1.301	1.476	2.015	2.571	3.365	4.032	5.893
6	0.265	0.453	0.718	0.906	1.273	1.440	1.943	2.447	3.143	3.707	5.208
7	0.263	0.449	0.711	0.896	1.254	1.415	1.895	2.365	2.998	3.499	4.785
8	0.262	0.447	0.706	0.889	1.240	1.397	1.860	2.306	2.896	3.355	4.501
9	0.261	0.445	0.703	0.883	1.230	1.383	1.833	2.262	2.821	3.250	4.297
10	0.260	0.444	0.700	0.879	1.221	1.372	1.812	2.228	2.764	3.169	4.144
11	0.260	0.443	0.697	0.876	1.214	1.363	1.796	2.201	2.718	3.106	4.025
12	0.259	0.442	0.695	0.873	1.209	1.356	1.782	2.179	2.681	3.055	3.930
13	0.259	0.441	0.694	0.870	1.204	1.350	1.771	2.160	2.650	3.012	3.852
14	0.258	0.440	0.692	0.868	1.200	1.345	1.761	2.145	2.624	2.977	3.787
15	0.258	0.439	0.691	0.866	1.197	1.341	1.753	2.131	2.602	2.947	3.733
16	0.258	0.439	0.690	0.865	1.194	1.337	1.746	2.120	2.583	2.921	3.686
17	0.257	0.438	0.689	0.863	1.191	1.333	1.740	2.110	2.567	2.898	3.646
18	0.257	0.438	0.688	0.862	1.189	1.330	1.734	2.101	2.552	2.878	3.610
19	0.257	0.438	0.688	0.861	1.187	1.328	1.729	2.093	2.539	2.861	3.579
20	0.257	0.437	0.687	0.860	1.185	1.325	1.725	2.086	2.528	2.845	3.552
21	0.257	0.437	0.686	0.859	1.183	1.323	1.721	2.080	2.518	2.831	3.527
22	0.256	0.437	0.686	0.858	1.182	1.321	1.717	2.074	2.508	2.819	3.505
23	0.256	0.436	0.685	0.858	1.180	1.319	1.714	2.069	2.500	2.807	3.485
24	0.256	0.436	0.685	0.857	1.179	1.318	1.711	2.064	2.492	2.797	3.467
25	0.256	0.436	0.684	0.856	1.178	1.316	1.708	2.060	2.485	2.787	3.450
26	0.256	0.436	0.684	0.856	1.177	1.315	1.706	2.056	2.479	2.779	3.435
27	0.256	0.435	0.684	0.855	1.176	1.314	1.703	2.052	2.473	2.771	3.421
28	0.256	0.435	0.683	0.855	1.175	1.313	1.701	2.048	2.467	2.763	3.408
29	0.256	0.435	0.683	0.854	1.174	1.311	1.699	2.045	2.462	2.756	3.396
30	0.256	0.435	0.683	0.854	1.173	1.310	1.697	2.042	2.457	2.750	3.385
35	0.255	0.434	0.682	0.852	1.170	1.306	1.690	2.030	2.438	2.724	3.340
40	0.255	0.434	0.681	0.851	1.167	1.303	1.684	2.021	2.423	2.704	3.307
45	0.255	0.434	0.680	0.850	1.165	1.301	1.679	2.014	2.412	2.690	3.281
50	0.255	0.433	0.679	0.849	1.164	1.299	1.676	2.009	2.403	2.678	3.261
55	0.255	0.433	0.679	0.848	1.163	1.297	1.673	2.004	2.396	2.668	3.245
60	0.254	0.433	0.679	0.848	1.162	1.296	1.671	2.000	2.390	2.660	3.232
∞	0.253	0.431	0.674	0.842	1.150	1.282	1.645	1.960	2.326	2.576	3.090

Table showing $P(Z < z)$ where $Z \sim N(0, 1)$ for values of z between 0.00 and 3.99

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7703	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Table showing selected values of z for $P(Z < z)$, where Z has a standard normal distribution

z	$P(Z < z)$
1.281	0.900
1.645	0.950
1.960	0.975
2.326	0.990
2.576	0.995