7 Topics: Moment generating functions, conditional distribution and conditional expectation

7.1 Prerequisites: Lecture 18

Exercice 7- 1: Use moment generating functions to find the mean and variance of

(a) $X \sim \mathrm{Poi}(\lambda)$,

(b) $X \sim \mathrm{Bin}(n, p)$,

(c) $X \sim \mathrm{Exp}(\lambda)$,

(d) $X \sim \mathrm{N}(\mu, \sigma^2)$.

Exercice 7- 2: (Suggested for personal/peer tutorial) Use moment generating functions to prove that for independent random variables $X \sim \mathrm{N}(\mu_X, \sigma_X^2), Y \sim \mathrm{N}(\mu_Y, \sigma_Y^2)$, we have that $X + Y \sim \mathrm{N}(\mu_X + \mu_Y, \sigma_X^2 + \sigma_Y^2)$.

(a)

Exercice 7- 3: Suppose that X_1 and X_2 are independent and identically distributed random variables, each having a standard normal distribution. Let random variable V be defined by

$$V = X_1^2 + X_2^2.$$

Find the pdf of V.

7.2 Prerequisites: Lecture 19

Exercice 7- 4: Consider tossing a coin repeatedly, where the probability of heads appearing in one toss is given by $p \in (0, 1)$. Let X denote the length of the initial run (i.e. if you toss heads first, how many heads do you toss before tossing tail and vice versa if you toss tail first). By conditioning on the outcome of the first coin toss and by using the law of total expectation, find $\mathrm{E}(X)$.