

Topic: Continuous random variables and their distributions

In today's problem class we will be studying properties of continuous random variables.

- For each of the function $f(x)$ given below determine whether $f(x)$ is a valid probability density function (p.d.f.). If $f(x)$ is not a valid p.d.f., determine if there exists a constant c such that $cf(x)$ is a valid p.d.f.. Note that in each case, $f(x) = 0$ for all x not in the interval specified.
 - $f(x) = 2x$, $0 < x < 1$.
 - $f(x) = |x|$, $|x| < \frac{1}{2}$.
 - $f(x) = 1 - |x|$, $|x| < 1$.
 - $f(x) = \log(x)$, $0 < x < 1$.
 - $f(x) = \log(x)$, $0 < x < 2$.
 - $f(x) = \frac{2}{3}(x - 1)$, $0 < x < 3$.
 - $f(x) = e^{-2x}$, $x > 0$.
 - $f(x) = 4e^{-2x} - e^{-x}$, $x > 0$.
 - $f(x) = e^{-|x|}$, $|x| < 1$.
- Let $Z \sim N(0, 1)$. Let $\mu \in \mathbb{R}$ and $\sigma > 0$. Find the c.d.f. and the p.d.f. of the random variable $X = \sigma Z + \mu$. Note that you can express the c.d.f. of X in terms of the c.d.f. Φ of Z .
- You are bidding against a competitor for an item on eBay. The amount, X , in pounds, of the bid placed by your competitor has probability density function given by:

$$f_X(x) = \begin{cases} c(20 - x), & 0 < x < 20; \\ 0, & \text{otherwise.} \end{cases}$$

You make a bid without knowing your competitor's bid.

- Determine the value of c .
- Find $F_X(x)$, the cumulative distribution function (cdf) of X .
- What is the probability that you lose the bid if you place a bid of £16?
- How much should you bid in order to have a 75% chance of winning?