Imperial College London

DEPARTMENT OF MATHEMATICS
IMPERIAL COLLEGE LONDON
Academic Year 2019-2020

Introduction to University Mathematics **MATH40001/MATH40009**

Problem Sheet 1: Logic

To try these questions in Lean, go to https://tinyurl.com/Lean-M40001-Sheet1. Remember that this is completely optional, and there will be no Lean in any M40001/M40009 tests or exams.

1. In lectures, we proved that if P and Q were propositions, then $P \vee Q \implies Q \vee P$. This is called "symmetry of $\vee$"; we also say "$\vee$ is symmetric".

   Prove that $\wedge$ is also symmetric. That is, prove that if P and Q are propositions, then $P \wedge Q \implies Q \wedge P$.

2. (a) Is $\implies$ symmetric? In other words, is it true that for all propositions P and Q we have

   $$(P \implies Q) \implies (Q \implies P)?$$

   Give a proof or a counterexample.

   (b) Is $\iff$ symmetric? In other words, is it true that for all propositions P and Q we have

   $$(P \iff Q) \implies (Q \iff P)?$$

   Give a proof or a counterexample.

3. Suppose P, Q and R are propositions, and we know that if Q is true then P is true, and that if Q is false then R is false. Can we deduce that R implies P? Give a proof or a counterexample.

4. Your friend is thinking of three true-false statements P, Q and R, and they tell you the following facts:

   (a) $P \implies (Q \wedge R)$;
   (b) $Q \implies (R \wedge \neg P)$;
   (c) $R \implies (P \wedge \neg Q)$.

   What can you deduce?

5. An *integer* (or a "whole number") is an element of the set $\{\ldots, -2, -1, 0, 1, 2, 3, \ldots\}$. Say that for every integer n we have a true/false statement P_n. Say we know that $P_n \implies P_{n+8}$ for every integer n, and also that $P_n \implies P_{n-3}$ for every integer n. Prove that the P_n are either all true, or all false.