Cover Sheet for Submission of Maths Examinations Summer 2020

We would advise preparing your coversheets with your CID, Module Name and Code and Date, before the exams are due to take place.

CID: 01738166

Module Name: Linear Algebra and Groups

Module Code: MATH40003

Date: 07/05/2020

Questions Answered (in the file):

Please tick next to the question or questions you have answered in this file.

Q1	
Q2	
Q3	
Q4	✓
Q5	
Q6	

(Note: this is a coversheet for all students - not all students will have exams with 6 questions. Please tick the boxes which are appropriate for your exam and/or the file you are submitting).

(Optional) Page Numbers for each question;

Page Number	Question Answered

If handwritten, please complete in CAPITAL Letters, in Blue or Black Ink, ensuring the cover sheet is legible.

1(a)

$$A^2 - 3A + 2I_n = 0_n$$

(i) ~~Take the determinant on both sides~~:

~~det~~ Suppose $\det(A) = 0$. Then $\det(A^2) = \det(A) \cdot \det(A) = 0$

$\det(-3A) = (-3)^n \det(A) = 0$; $\det(2I_n) = 2^n \det I_n = 2^n$; $\det(0_n) = 0$

We have:

$$A(A - 3I_n) = -2I_n$$

Take the determinant on both sides:

$\det(A) \cdot \det(A - 3I_n) = \det(-2I_n)$. The LHS equals 0, but the RHS $= (-2)^n$

$\Rightarrow$ ✗, $\det(A) \neq 0$

$$A^{-1}(A^2 - 3A + 2I_n) = A^{-1}0_n = 0$$

$$\Rightarrow \underbrace{A^{-1}A}_{I_n}A - 3\underbrace{A^{-1}A}_{I_n} + 2A^{-1} = 0 \Rightarrow 2A^{-1} = 3I_n - A$$

$$\Rightarrow \boxed{A^{-1} = \frac{3}{2}I_n - \frac{1}{2}A}$$

(ii) Suppose $v \in E_1 \cap E_2 \Rightarrow v \in E_1$ and $v \in E_2 \Rightarrow$

$Av = v$ and $Av = 2v \Rightarrow v = 2v \Rightarrow v = 0$ (only sol) $\Rightarrow \boxed{E_1 \cap E_2 = \{0\}}$

(iii)

(b)

(i) $v^T C v = v^T B^T B v = (Bv)^T Bv$

As B is $(n\times n)$ and v is $(n\times 1)$, Bv is $(n\times 1)$ vector, say $\begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix}$. Then $(Bv)^T Bv = (v_1\ v_2 \dots v_n)\begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix} = v_1^2 + v_2^2 + \dots + v_n^2 > 0$,

as not all v_i-s are 0.

(ii) Since $C = B^T . B$ is a symmetric matrix, from Theorem ~~5~~ (6.5.4) in the lecture notes $\Rightarrow$ There exists an orthogonal matrix P_1, with $P_1^{-1} C P_1 = D$ - a ~~g~~ diagonal matrix.

We have $C = P_1 D P_1^{-1}$. Substitute $C = B^T B$ and multiply both sides on the left with $(B^T)^{-1}$ to get:

$$B = (B^T)^{-1} . P_1 D P_1^{-1}$$

Since P_1 is orthogonal, then $P_1^{-1} = P_1^T \Rightarrow P_1^{-1}$ is also orthogonal.
Denote $P_1^{-1} := P$. $\Rightarrow B = ((B^T)^{-1} . P_1) . D . P$

~~Now we are left to show that $(B^T)^{-1} P_1$ is orthogonal~~