Cover Sheet for Submission of Maths Examinations Summer 2020

We would advise preparing your coversheets with your CID, Module Name and Code and Date, before the exams are due to take place.

CID: 01738166

Module Name: Calculus and Applications

Module Code: MATH40004

Date: 12/05/2020

Questions Answered (in the file):

Please tick next to the question or questions you have answered in this file.

Q1	
Q2	✓
Q3	
Q4	
Q5	
Q6	

(Note: this is a coversheet for all students - not all students will have exams with 6 questions. Please tick the boxes which are appropriate for your exam and/or the file you are submitting).

(Optional) Page Numbers for each question;

Page Number	Question Answered

If handwritten, please complete in CAPITAL Letters, in Blue or Black Ink, ensuring the cover sheet is legible.

(a)

let $s = 1-p$

Consider $I = \int_0^\infty x^{s-1} e^{-x}\,dx$

I converges for $s \in (0,\infty) \iff p \in (0,1)$

Now $\int_0^1 x^{s-1} e^{-x}\,dx$ converges, since $x^{s-1} e^{-x} \leq x^{s-1}$ and $\int_0^1 x^{s-1}\,dx$ converges for $s > 0$

On the other hand,

$\int_1^\infty x^{s-1} e^{-x}\,dx$ is bounded above by $\int_1^\infty c\, x^{s-1-k}\,dx$, for $k >> 0$ as $e^t \geq \frac{t^k}{k!}$ for any k.

Thus I converges for $p \in (0,1)$.

(b)

$\cos(x^2) - \frac{1}{2}\frac{\sin(x^2)}{x^2} = \left[\frac{1}{2}\frac{\sin(x^2)}{x}\right]'$

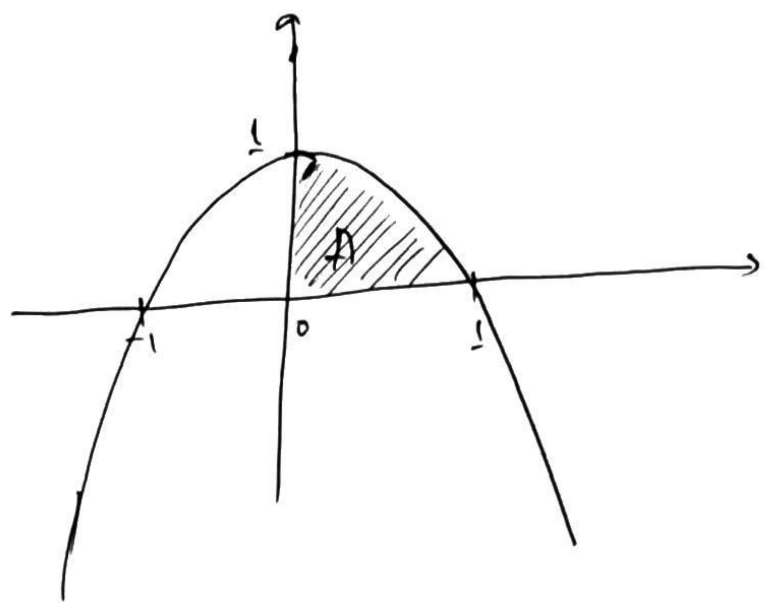
So $\int_0^\infty \cos(x^2)\,dx = \lim_{\substack{t\to\infty \\ s\to 0}} \left[\frac{1}{2}\left(\frac{\sin t^2}{t} - \frac{\sin s^2}{s} + \int_0^\infty \frac{\sin(x^2)}{x^2}\,dx\right)\right]$

$= 0 + I,$

where I exists as $\left|\frac{\sin(x^2)}{x^2}\right| \leq \frac{1}{x^2}$ and $\int_0^\infty \frac{1}{x^2}\,dx = 1$

(c)

(i) $f(x) = 1 - x^2$



(ii) $\text{Area} = \int_0^1 f(x)\,dx = \int_0^1 (1-x^2)\,dx = \left[x - \frac{x^3}{3}\right]_0^1 = 1 - \frac{1}{3} = \boxed{\frac{2}{3}}$

(iii) Coordinates of center of mass: $(\bar{x}, \bar{y})$

We have: $\bar{x} = \dfrac{\int_0^1 x \cdot f(x)\,dx}{\int_0^1 f(x)\,dx}$; $\bar{y} = \dfrac{\frac{1}{2}\int_0^1 f^2(x)\,dx}{\int_0^1 f(x)\,dx}$

$\int_0^1 x - x^3\,dx = \left(\frac{x^2}{2} - \frac{x^4}{4}\right)_0^1 = \frac{1}{2} - \frac{1}{4} = \underline{\frac{1}{4}}$

$\frac{1}{2}\int_0^1 (1-x^2)^2\,dx = \frac{1}{2}\int_0^1 (x^4 - 2x^2 + 1)\,dx = \frac{1}{2}\left[\frac{x^5}{5} - \frac{2x^3}{3} + x\right]_0^1 = \frac{1}{2}\left(\frac{1}{5} - \frac{2}{3} + 1\right)$

$= \frac{4}{15}$

$\Rightarrow \bar{x} = \dfrac{1/4}{2/3} = \frac{1}{4} \cdot \frac{3}{2} = \frac{3}{8}$

$\bar{y} = \dfrac{4/15}{2/3} = \frac{\cancel{4}^{2}}{\cancel{15}_{5}} \cdot \frac{\cancel{3}}{\cancel{2}} = \frac{2}{5}$

$\Rightarrow (\bar{x}, \bar{y}) = \left(\frac{3}{8}, \frac{2}{5}\right)$