Cover Sheet for Submission of Maths Examinations Summer 2020

We would advise preparing your coversheets with your CID, Module Name and Code and Date, before the exams are due to take place.

CID: 01738166

Module Name: Linear Algebra and Groups

Module Code: MATH40003

Date: 07/05/2020

Questions Answered (in the file):

Please tick next to the question or questions you have answered in this file.

Q1	
Q2	
Q3	✓
Q4	
Q5	
Q6	

(Note: this is a coversheet for all students - not all students will have exams with 6 questions. Please tick the boxes which are appropriate for your exam and/or the file you are submitting).

(Optional) Page Numbers for each question;

Page Number	Question Answered

If handwritten, please complete in CAPITAL Letters, in Blue or Black Ink, ensuring the cover sheet is legible.

(a)

- U is not empty
- if $u_1, u_2 \in U$, then $u_1 \oplus u_2 \in U$ - closed under addition
- if $u \in U$ and $\lambda \in \mathbb{R}$, then $\lambda \odot u \in U$ - closed under multipl.

(b) Clearly U_n is a subset of V. Check subspace 'test':

- $f(x) = a_0 \Rightarrow U_n$ is not empty
- Let $f(x) = \sum_{i=0}^{n} a_i x^i$, $a_{i+1} = 2a_i$ for some a_0.

  $g(x) = \sum_{i=0}^{n} b_i x^i$, $b_{i+1} = 2b_i$ for some b_0.

Then $f(x) + g(x) = \sum_{i=0}^{n} a_i x^i + \sum_{i=0}^{n} b_i x^i = \sum_{i=0}^{n} (a_i x^i + b_i x^i) = \sum_{i=0}^{n} (a_i + b_i) x^i \in U_n$

- Let $f(x)$ be as above and $\lambda \in \mathbb{R}$. We have

$\lambda f(x) = \lambda \sum_{i=0}^{n} a_i x^i = \sum_{i=0}^{n} (\lambda a_i) x^i \in U_n \Rightarrow U_n$ is a subspace of V.

A basis for U_n is $\{1, x, x^2, \dots, x^n\}$:

- spans U_n
- is LI ~~...~~

(c)

* T is linear $\Leftrightarrow$
  - T preserves addition: $T(v_1 + v_2) = T(v_1) + T(v_2)$, $\forall v_1, v_2 \in V$
  - T preserves scalar mult.: $\forall v \in V$ and $\lambda \in \mathbb{R}$, $T(\lambda v) = \lambda T(v)$
* $Ker(T) = \{ v \in V : T(v) = 0 \}$
* $Im(T) = \{ T(v) : v \in V \}$
* - 0_V - the null vector $\in Ker(T) \Rightarrow Ker(T)$ is not empty
  - $u_1, u_2 \in Ker(T) \Rightarrow T(u_1) = T(u_2) = 0 \Rightarrow T(u_1) + T(u_2) = 0$

$\Rightarrow T(u_1 + u_2) = 0 \Rightarrow u_1 + u_2 \in Ker(T)$

(Linear)

- $\lambda \in \mathbb{R}$, $v \in Ker(T)$,

~~Proof~~ $\Rightarrow$ $T(\lambda v) = \lambda T(v) = \lambda . 0 = 0 \Rightarrow T(\lambda v) = 0$
(linear) $\Rightarrow \underline{\lambda v \in Ker(T)}$

$\Rightarrow$ Ker(T) is a subspace of V

(d) We have $a_n = 2^n a_0 \Rightarrow a_0 = 2^{-i} a_i$, $\forall i$

as in (b)

- $T_n(f(x) + g(x)) = T_n\left(\sum_{i=0}^{n}(a_i + b_i)x^i\right) = \sum_{i=0}^{n+1} 2^i \frac{(a_0 + b_0)}{2^i} x^i$

$= \sum_{i=0}^{n+1} a_0 x^i + \sum_{i=0}^{n+1} b_0 x^i = T_n(f(x)) + T_n(g(x))$

- $T_n(\lambda f(x)) = T_n\left(\lambda \sum_{i=0}^{n} a_i x^i\right) = T_n\left(\sum_{i=0}^{n} \lambda a_i x^i\right) = \sum_{i=0}^{n+1} 2^i (\lambda a_i x^i)$

$= \sum_{i=0}^{n+1} \cancel{2^i} \lambda \frac{a_0}{\cancel{2^i}} x^i = \lambda \sum_{i=0}^{n+1} a_0 x^i = \lambda T_n(f(x))$

- $Im(T) = \left\{ \sum_{i=0}^{n+1} 2^i a_0 x^i : a_0 \in \mathbb{R} \right\}$

- $Ker(T) = \left\{ a_0 \in \mathbb{R} : \sum_{i=0}^{n+1} 2^i a_0 x^i = 0, \forall x \right\} = \{ a_0 = 0 \}$

- ~~$\underbrace{dim(U_n)}_{=n+1} = \underbrace{dim(Im(T))}_{=n+1} + \underbrace{dim(Ker(T))}_{=0}$~~

Yes, it is a subspace by the subspace test: