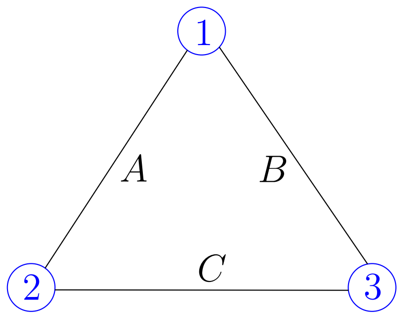
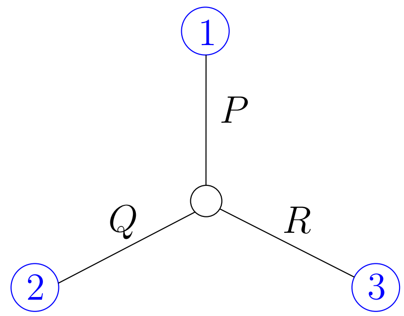
Consider three nodes labelled 1,2 and 3 appearing in two different electrical circuits shown below as Graph I and Graph II each having 3 different edges.



Graph I



Graph II

The *resistances* of each edge are indicated on the figure (note: the resistances, not the conductances, are shown): the three edges of Graph I have resistances A, B and C; the three edges of Graph II have resistances P, Q and R.

Graph II has a fourth internal node, but this node is **always** taken to be such that Kirchhoff's current law holds there. Consequently, there are three different effective conductances, or equivalently, effective resistances, that can be associated with each graph. They correspond to the following scenarios:

(1) when node 1 is at unit voltage, and node 2 is grounded (KCL at node 3);

(2) when node 1 is at unit voltage, and node 3 is grounded (KCL at node 2);

(3) when node 2 is at unit voltage, and node 3 is grounded (KCL at node 1).

Show that, if we require these three effective conductances to be the **same** for **both** circuits – so that they are "equivalent" circuits – then the following relations between the resistances must hold:

$$
\begin{aligned}
A &= \frac{PQ+PR+QR}{R}, & P &= \frac{AB}{A+B+C},\\
B &= \frac{PQ+PR+QR}{Q}, & Q &= \frac{AC}{A+B+C},\\
C &= \frac{PQ+PR+QR}{P}, & R &= \frac{BC}{A+B+C}.
\end{aligned}
$$