M1GLA Geometry and Linear Algebra   Problem Sheet 3

1. * Let $\mathbb{R}[x]$ be the set of all polynomials with variable x and real coefficients, with standard addition and scalar multiplication. Show that this is a vector space over $\mathbb{R}$.

2. Decide whether the following sets together with the indicated operations of addition and scalar multiplication is a vector space:

   (a) The set $\mathbb{R}^2$, with the usual addition but with scalar multiplication defined by
   $$r \odot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ry \\ rx \end{pmatrix}.$$

   (b) The set $\mathbb{R}^2$, with the usual scalar multiplication but with addition defined by
   $$\begin{pmatrix} x \\ y \end{pmatrix} \oplus \begin{pmatrix} r \\ s \end{pmatrix} = \begin{pmatrix} y+s \\ x+r \end{pmatrix}.$$

   (c) The set $\mathbb{R}^2$, with addition and scalar multiplication defined by
   $$\begin{pmatrix} x \\ y \end{pmatrix} \oplus \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} x+a+1 \\ y+b \end{pmatrix} \quad \text{and} \quad r \odot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} rx+r-1 \\ ry \end{pmatrix}.$$

3. Show every F-vector space V with additive identity 0_V has the following properties:

   (a) The vector 0_V is the unique vector satisfying the equation $0_V \oplus v = v$ for all vectors v in V.

   (b) For 0 the additive identity in F, $0 \odot v = 0_V$ for all vectors v in V.

4. Describe all subspaces of $\mathbb{R}^3$

5. Let U, W be subspaces of a vector space V over F. Show that $U \cup W$ is a subspace of V iff either $U \subseteq W$ or $W \subseteq U$.