Math40002 Analysis 1 **Term 2 Unseen 3**

Definition 1. A function $f : A \to \mathbb{R}$, where $A \subseteq \mathbb{R}$ is *Lipschitz continuous* if there exists some $C \in \mathbb{R}$ such that $\forall x, y \in A : |f(x) - f(y)| \leq C|x - y|$.

For each of the following items, determine whether it is true or false and prove your answer. The items are totally independent from one another, so there is no need to solve them in any particular order.

1. If f is Lipschitz continuous, then it is uniformly continuous.
2. If f is uniformly continuous, then it is Lipschitz continuous.
3. If f is continuous and bounded on the interval (a, b) (meaning there exist $M, L \in \mathbb{R}$ such that $\forall x \in (a, b) : M \leq f(x) \leq L$), then f is uniformly continuous on (a, b).
4. If f is bounded on $\mathbb{R}$ and uniformly continuous on every interval $[a, b]$ where $a, b \in \mathbb{R}$, then f is uniformly continuous on $\mathbb{R}$.
5. If f is bounded, continuous and monotonic on $(0, 1)$, then f is uniformly continuous on $(0, 1)$.
6. If f is uniformly continuous on $(0, 1)$, then f is bounded on $(0, 1)$.