Question 1

(a) Prove that for any random variable X and any constant $a \in \mathbb{R}$,

$$\mathrm{Cov}\left(X, a\right) = 0.$$

(b) Prove that for any random variable X, Y and Z, and constants $a, b \in \mathbb{R}$,

$$\mathrm{Cov}\left(aX + bY, Z\right) = a\mathrm{Cov}\left(X, Z\right) + b\mathrm{Cov}\left(Y, Z\right).$$

(c) For any random variables X and Y, and constants $a, b \in \mathbb{R}$, find an expression for

$$\mathrm{Cov}\left(aX + b, Y\right)$$

in terms of $\mathrm{Cov}\left(X, Y\right)$.

Question 2

Do Exercise 7.2.7 in the notes: show that the conjugate prior for the exponential distribution is the gamma distribution.

Hint:

Start by assuming that a sample of independent random variables $\mathbf{X} = (X_1, X_2, \ldots, X_n)$ is observed as $\mathbf{x} = (x_1, x_2, \ldots, x_n)$, and that each X_i follows an exponential distribution with the same unknown parameter θ. Derive an expression the likelihood. Write down the probability density function for the gamma distribution - this is the prior. Then, compute the posterior.

Question 3

Suppose we have the sample of observations

$$\mathbf{x} = \left(1.2, \quad 5.3, \quad 6.2, \quad 7.4, \quad 8.5\right).$$

Which of the following samples are/are not bootstrap samples of $\mathbf{x}$? Justify your answer in each case.

(a) (5.3, 1.2, 6.2, 8.5, 7.4)

(b) (6.2, 7.4, 7.4, 1.2, 5.3)

(c) (6.2, 7.4, 1.2, 7.4)

(d) (7.4, 5.3, 7.4, 9.9, 8.5)

(e) (5.3, 5.3, 5.3, 5.3, 5.3)

(f) (7.4, 5.3, 7.4, 8.5, 1.2, 6.2)