4 Topics: Random variables and their distributions

4.1 Prerequisites: Lecture 10

Exercice 4- 1: (Suggested for personal/peer tutorial) Poisson approximation to the Binomial: If $X \sim \text{Bin}(n,p)$ and we have $n \to \infty$ and $p \to 0$ such that $\lambda = np$ remains constant, then the p.m.f. of X converges to the p.m.f. of a Poi(λ) random variable. The same result holds, when for $n \to \infty$ and $p \to 0$, we have that np converges to a positive constant λ.

Hint: Use the result that for all $t \in \mathbb{R}$,

$$\lim_{n\to\infty} \left(1 - \frac{t}{n}\right)^n = e^{-t}.$$

Exercice 4- 2: A company wishes to make two of a group of six employees, comprising three female and three male employees, redundant, by selecting two employees at random. Let X and Y be the random variables corresponding to the number of female and male employees made redundant, respectively.

Find the probability mass functions of X and Y.

Exercice 4- 3: Five balls numbered 1,2,3,4 and 5 are placed in a bag. Two balls are selected without replacement. Find the probability mass function of the following random variables:

(a) $X =$ the largest of the two selected numbers,

(b) $Y =$ the sum of the two selected numbers

Exercice 4- 4: A surgical procedure is successful with probability θ. The surgery is carried out on five patients, with the success or failure of each operation independent of all other operations. Let X be the discrete random variable corresponding to the number of successful operations.

Find the probability mass function of X, and evaluate the probability that

(a) all five operations are successful, if $\theta = 0.8$,

(b) exactly four operations are successful, if $\theta = 0.6$,

(c) fewer than two are successful, if $\theta = 0.3$.

Exercice 4- 5: If X has a Geometric distribution with parameter θ, so that

$$p_X(x) = (1-\theta)^{x-1}\theta, \quad x = 1, 2, 3, ...$$

and zero otherwise, show that, for $n, k \geq 1$,

$$\mathrm{P}(X = n + k | X > n) = \mathrm{P}(X = k).$$

This result is known as the *Lack of Memory* property (for a discrete random variable).

4.2 Prerequisites: Lecture 11

Exercice 4- 6: Suppose $X \sim \text{DUnif}(\{1, \ldots, n\})$. Find the c.d.f. of X.