**Math40002 Analysis 1** **Unseen 6**

1. Let $(a_n)$ be a sequence of real numbers. Prove/Disprove:
   (a) If $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} a_n^2$ converges.
   (b) If $\sum_{n=1}^{\infty} |a_n|$ converges, then $\sum_{n=1}^{\infty} |a_n^2|$ converges.
   (c) If $\sum_{n=1}^{\infty} a_n^2$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.

2. Assume $\sum_{n=1}^{\infty} \frac{1}{n^2} = S$. Find $\sum_{n=1}^{\infty} \frac{1}{(2n+1)^2}$.

3. (a) Assume $a_n$ is a monotonically decreasing sequence. Prove that if $\sum_{n=1}^{\infty} a_n$ converges, then $na_n \to 0$.
   (b) Is it true that in general, if $a_n \geq 0$ for all $n \in \mathbb{N}$ and $\sum_{n=1}^{\infty} a_n$ converges, then $na_n \to 0$?