Cover Sheet for Submission of Maths Examinations Summer 2020

We would advise preparing your coversheets with your CID, Module Name and Code and Date, before the exams are due to take place.

CID: 01738166

Module Name: Linear Algebra and Groups

Module Code: MATH40003

Date: 07/05/2020

Questions Answered (in the file):

Please tick next to the question or questions you have answered in this file.

Q1	✓
Q2	
Q3	
Q4	
Q5	
Q6	

(Note: this is a coversheet for all students - not all students will have exams with 6 questions. Please tick the boxes which are appropriate for your exam and/or the file you are submitting).

(Optional) Page Numbers for each question;

Page Number	Question Answered

If handwritten, please complete in CAPITAL Letters, in Blue or Black Ink, ensuring the cover sheet is legible.

$$A = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix}$$

(a)

$$\left(\begin{array}{ccc|ccc} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 1 \end{array}\right) \xrightarrow[R_3 \times \sqrt{2}]{R_1 \times \sqrt{2}} \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & \sqrt{2} & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & \sqrt{2} \end{array}\right)$$

$$\xrightarrow[R_1 = (R_1 + R_3)/2]{R_3 = (R_3 - R_1)/2} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \sqrt{2}/2 & 0 & \sqrt{2}/2 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & -\frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \end{array}\right) \Rightarrow A^{-1} = \begin{bmatrix} \sqrt{2}/2 & 0 & \sqrt{2}/2 \\ 0 & 1 & 0 \\ -\sqrt{2}/2 & 0 & \sqrt{2}/2 \end{bmatrix}$$

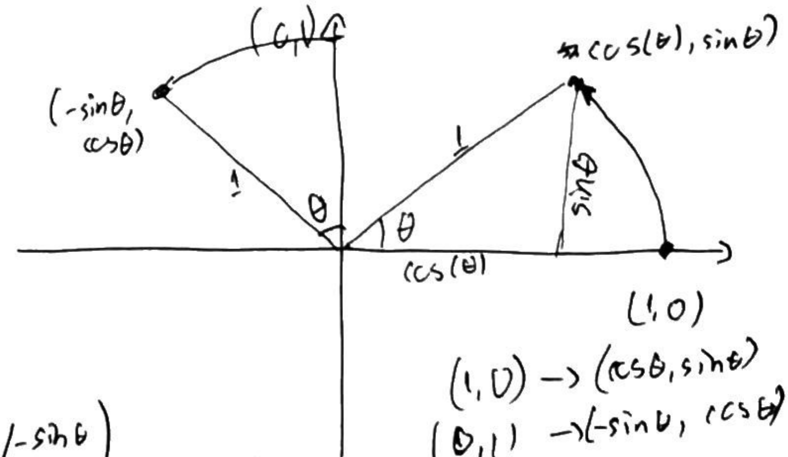
$\left(\frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}\right)$

(b) $A^T A = I_n$

(c) $A^T = \begin{pmatrix} 1/\sqrt{2} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ -1/\sqrt{2} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix} = A^{-1} \Rightarrow A^T A = A^{-1} A = I_3 \Rightarrow A$ is orthogonal.

(d) $R_\theta \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \to$ seen from the graph

$R_\theta \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix}$



$(1,0) \to (\cos\theta, \sin\theta)$
$(0,1) \to (-\sin\theta, \cos\theta)$

So we get:

$R_\theta \begin{pmatrix} x \\ y \end{pmatrix} = R_\theta\left(x\begin{pmatrix} 1 \\ 0 \end{pmatrix} + y\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right) = (R_\theta \text{ is linear})$

$= x R_\theta \begin{pmatrix} 1 \\ 0 \end{pmatrix} + y R_\theta \begin{pmatrix} 0 \\ 1 \end{pmatrix} = x \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} + y \begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix}$

$\Rightarrow \underline{R_\theta \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}}$

(e) For M, we have that $x_2 = 0$ for all $x_1 \Rightarrow$

$\tan\theta = 0 \Rightarrow \theta = n\pi,\ n \in \mathbb{Z} \Rightarrow$ ~~[illegible]~~ $\sin(2\theta) = 0$

$\cos(2\theta) = 1$

$$\Rightarrow M = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

For M', we have $x_1 = x_2 \Rightarrow x_2 = x_1 \tan\theta \Rightarrow \tan\theta = 1$

$\Rightarrow \sin\theta = \cos\theta \Rightarrow \theta = n\pi + \frac{\pi}{4},\ n \in \mathbb{Z}$

$\sin 2\theta = 1$

$\cos 2\theta = 0$

$$\Rightarrow M' = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$MM' = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} \cos(270^\circ) & -\sin(270^\circ) \\ \sin(270^\circ) & \cos(270^\circ) \end{pmatrix}$$ $\Rightarrow$ anticlockwise reflection on $\frac{3}{2}\pi$ radians

(f) A - orthogonal $2\times2 \Leftrightarrow$ its columns are perpendicular unit vectors.

~~[illegible]~~ Any unit vector can be written as $\begin{pmatrix} \cos x \\ \sin x \end{pmatrix}$ and there are only 2 unit vectors perpendicular to $\begin{pmatrix} \cos(x) \\ \sin(x) \end{pmatrix}$ ~~[illegible]~~ ~~[illegible]~~ : $\begin{pmatrix} -\sin x \\ \cos x \end{pmatrix}$ and $\begin{pmatrix} \sin(x) \\ -\cos(x) \end{pmatrix} \Rightarrow$ We get two matrices only $\rightarrow$ precisely M and M'.