# Cover Sheet for Submission of Maths Examinations Summer 2020

We would advise preparing your coversheets with your CID, Module Name and Code and Date, before the exams are due to take place.

**CID: 01738166**

**Module Name: Analysis 1**

**Module Code: MATH40002**

**Date: 04/05/2020**

## Questions Answered (in the file):

Please tick next to the question or questions you have answered in this file.

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|---|---|
| **Q1** | |
| **Q2** | |
| **Q3** | |
| **Q4** | |
| **Q5** | |
| **Q6** | ✓ |

(Note: this is a coversheet for all students - not all students will have exams with 6 questions. Please tick the boxes which are appropriate for your exam and/or the file you are submitting).

## (Optional) Page Numbers for each question;

| Page Number | Question Answered |
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If handwritten, please complete in CAPITAL Letters, in Blue or Black Ink, ensuring the cover sheet is legible.

(a)

(i) $\boxed{\text{FALSE}}$

$f(x) = 0$

$g(x) = \begin{cases} 0 & \text{if } x \neq \frac{1}{2} \\ 1 & \text{if } x = \frac{1}{2} \end{cases}$

(ii) $\boxed{\text{TRUE}}$

$f$ and $g$ - differentiable $\Rightarrow$ $fg$ - differentiable

$\frac{d}{dx}\left(f(x)g(x)\right) = f'(x)\,g(x) + f(x)\cdot g'(x)$ - ~~is differentiable~~ ~~continuous~~ integrable

By Fund. thrm of calc.

We have ~~$f'$~~ $fg'$ is integrable $\Rightarrow$

$f'(x)\,g(x) = \frac{d}{dx}\left(f(x)g(x)\right) - f(x)\,g'(x)$ is also integrable.

(iii) $g(x) = \ln(x)$ - integrable

$h(f) = \cancel{\ln f}\, e^{f}$ - integrable $\Rightarrow$ $g \circ h$ is integrable

$g \circ h = \ln(e^{f}) = f$ - integrable ; $\boxed{\text{TRUE}}$

(iv) $\boxed{\text{FALSE}}$ $f$ needs to converge uniformly

Counterexample:

$\phi(x) = \begin{cases} e^{-\frac{1}{x^2} - \frac{1}{(1-x)^2}} & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$

$f_n(x) = \frac{n}{c}\cdot \phi(nx)$

$c = \int_0^1 \phi(x)\,dx.$

(v) $\boxed{\text{TRUE}}$

$\max(f(x), g(x)) = \dfrac{f+g+|f-g|}{2}$. Since the sum of integrable functions is integrable and the absolute value of an integrable func. is integrable $\Rightarrow$ $\max(f, g)$ is also integrable.

(b)

(i) $f: [a,b] \to \mathbb{R}$ - continuous which has a continuous derivative on $(a,b)$. Then

$$\int_a^b f'(x)\,dx = f(b) - f(a).$$

(ii) $f(x) = \dfrac{1}{\sqrt{x+1}}$, $x \in [0; n]$

$$L(f,P) = \sum_{i=0}^{n-1} m_i(x_{i+1} - x_i).$$

$x_{i+1} - x_i = 1$; $m_i = \inf_{x_i \le t \le x_{i+1}} f(t) = f(x_{i+1}) = \dfrac{1}{\sqrt{x_{i+1}+1}}$

$$L(f,P) = \sum_{i=0}^{n-1} \frac{1}{\sqrt{i+2}}$$

$$U(f,P) = \sum_{i=0}^{n-1} M_i = \sum_{i=0}^{n-1} \frac{1}{\sqrt{i+1}}.$$

(iii) We know $U(f,P) \ge \displaystyle\int_0^n \frac{1}{\sqrt{x+1}}\,dx$ and $U(f,P) - 1 \le \displaystyle\int_0^n \frac{1}{\sqrt{x+1}}\,dx$

so $\left| U(f,P) - \displaystyle\int_0^{10^6-1} \frac{1}{\sqrt{x+1}}\,dx \right| \le 1$.

But $\int_0^{10^6-1} \frac{1}{\sqrt{x+1}} dx = 2\sqrt{x+1}\Big|_0^{10^6-1} = 1998$

So $|U(f,P) - 1998| \le 1$

(c) We know $h(x) := f(x) - g(x)$ is integrable on $[0,1]$ and $h(x) = 0$ for $x \in \mathbb{Q}$

Assume $\int_0^1 h(x)dx = a > 0$.

So there is a partition $P$ of the interval $[0,1]$ such that $|L(f,P) - a| < \frac{a}{2} \Rightarrow L(f,P) > \frac{a}{2} > 0$.

But since the rational are dense, $L(f,P) \le 0$ as any subinterval of the partition will contribute at most $0$ to the sum. #

Similarly if $\int_0^1 h(x)dx \le 0$ with $U(f,P)$.

So $\int_0^1 h(x)dx = 0$. So

$$\int_0^1 f(x)dx = \int_0^1 g(x)dx$$