Cover Sheet for Submission of Maths Examinations Summer 2020

We would advise preparing your coversheets with your CID, Module Name and Code and Date, before the exams are due to take place.

CID: 01738166

Module Name: An Introduction to Applied Maths

Module Code: MATH40007

Date: 18/05/2020

Questions Answered (in the file):

Please tick next to the question or questions you have answered in this file.

Q1	
Q2	✓
Q3	
Q4	
Q5	
Q6	

(Note: this is a coversheet for all students - not all students will have exams with 6 questions. Please tick the boxes which are appropriate for your exam and/or the file you are submitting).

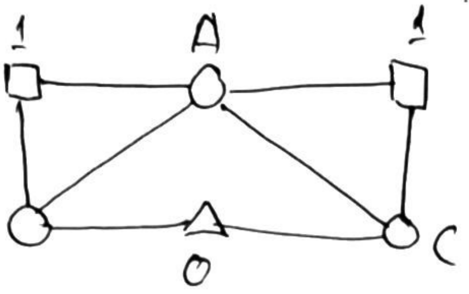
(Optional) Page Numbers for each question;

Page Number	Question Answered

If handwritten, please complete in CAPITAL Letters, in Blue or Black Ink, ensuring the cover sheet is legible.

01738166 MATH40007 Question 2 Page 1

(a) The problem is equivalent to finding the potentials at nodes A, B and C when the square nodes have voltage 1 and the Δ ~~box~~ ~~is~~ is grounded (in a circuit).



By using the harmonic property of the potentials, each potential is the average of ~~it~~ its neighbours $\Rightarrow$ we get the equations:

$$A = \frac{1+1+C+B}{4} \quad ; \quad \cancel{C = \frac{A+C}{2}} \;,\; \cancel{C = \frac{B+D}{2}}$$

$$C = \frac{A+1+0}{3} \quad ; \quad B = \frac{A+1+0}{3} \Rightarrow \begin{aligned} 3B &= A+1 \\ 3C &= A+1 \end{aligned} \Rightarrow \underline{B = C}$$

$4A = 2 + 2B \Rightarrow B + 1 = 2A$, and $A = 3B - 1 \Rightarrow$

$B + 1 = 6B - 2 \Rightarrow 5B = 3 \Rightarrow \boxed{B = \frac{3}{5}}$ $\boxed{C = \frac{3}{5}}$, $A = 3B - 1 \; \boxed{\frac{4}{5} = A}$

$\Rightarrow$ highest probability if he starts at A.

(b) Lets denote P_A, P_B, P_C - probabilities that he reaches $\square$ before Δ if he starts at A, B, C, respectively. We know from (a) that $P_A = \frac{4}{5}$, $P_B = P_C = \frac{3}{5}$.

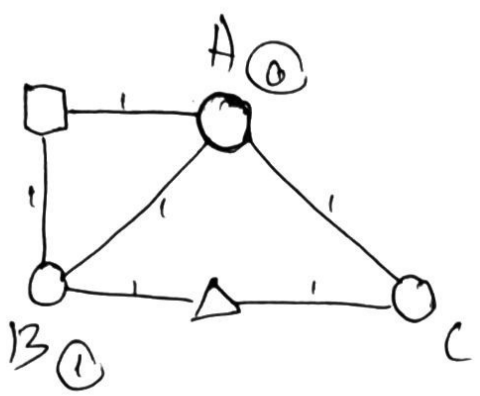
If he starts at the Δ, then there is $\frac{1}{2}$ probability that he goes to C and $\frac{1}{2}$ – to B. $\Rightarrow$

$$P_{esc} = \frac{1}{2} \cdot P_B + \frac{1}{2} P_C = \frac{1}{2} \cdot \frac{3}{5} + \frac{1}{2} \cdot \frac{3}{5} = \boxed{\frac{3}{5}}$$

Can also be calculated by $P_{esc} = \frac{C_{eff}}{2}$ → when voltage at $\square = 1$ at $\Delta = 0$

2 → nodes connected to Δ.

(c)



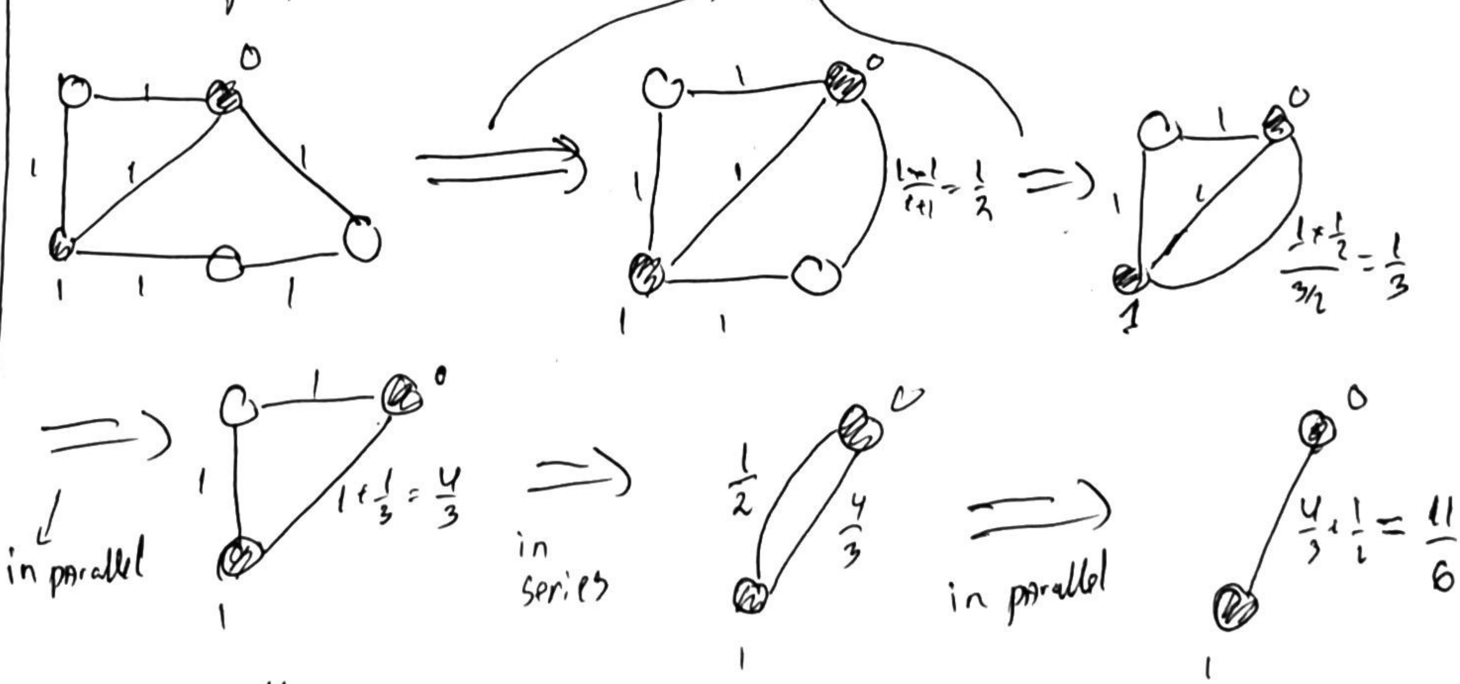
we have voltage at $A = 0$
at $B = 1$.

$$P_{esc} = \frac{C_{eff}}{3}$$

C_{eff} → effective conductance

3 – nodes connected to A

Reduce graph:



$$\Rightarrow C_{eff} = \frac{11}{6} \Rightarrow P_{esc} = \frac{11}{6} \cdot \frac{1}{3} = \boxed{\frac{11}{18}}$$