Question 1

Consider two discrete random variables X and Y with joint probability density function given by

$$f(x,y) = \begin{cases} c(2x+y), & \text{if } x \in \{0,1,2\} \text{ and } y \in \{0,1,2,3\}, \\ 0, & \text{otherwise.} \end{cases}$$

where c is an appropriately-chosen constant.

(a) Find the value of c.

(b) Find $\mathrm{P}(X = 2, Y = 1)$

(c) Find $\mathrm{P}(X \geq 1, Y = 1)$

(d) Find $\mathrm{P}(X \geq 1, Y \leq 1)$

(e) Find the marginal probability (mass) function of X.

(f) Find the marginal probability (mass) function of Y.

(g) Are X and Y independent random variables?

(h) Find the the probability mass function of Y given $X = 2$.

(i) Compute $\mathrm{P}(Y = 1|X = 2)$.

(j) Compute $\mathrm{E}(Y|X = 2)$.

Question 2

Suppose X is uniformly distributed on the interval $[0, 4]$, i.e. $X \sim \mathrm{Unif}(0, 4)$.

(a) Compute $\mathrm{P}(|X - 2| \geq 1)$.

(b) Use Chebyshev's inequality to bound the probability that $|X - 2| \geq 1$.

(c) Is the bound in (b) informative?

(d) For which values $\epsilon > 0$ can Chebyshev's inequality be used to obtain a nontrivial bound for $\mathrm{P}(|X - 2| \geq \epsilon)$?

Question 3

Prove Proposition 1.8.8 from the lecture notes:

Proposition 1.8.8. Given a sample of observations $x_1, x_2, \ldots, x_n$, with sample median m. Then, for any real value a,

$$\min_a \left(\sum_{i=1}^{n} |x_i - a| \right) = \sum_{i=1}^{n} |x_i - m|.$$