Coursework 1

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Problem 1.

(a) Is $\implies$ associative? In other words, if P, Q and R are propositions, is $(P \implies Q) \implies R$ always logically equivalent to $P \implies (Q \implies R)$? Give a proof or a counterexample.

(b) Is $\iff$ associative? In other words, is $(P \iff Q) \iff R$ always logically equivalent to $P \iff (Q \iff R)$? Give a proof or a counterexample.

(a) No, $(P \implies Q) \implies R$ is not always logically equivalent to $P \implies (Q \implies R)$. To show this, lets consider the counterexample when all P, Q, and R are false. Then $P \implies Q$ would be true and $Q \implies R$ would also be true. However, $(P \implies Q) \implies R$ is then false, while $P \implies (Q \implies R)$ is true. Therefore, the two propositions are not always logically equivalent.

(b) Yes. To prove that $(P \iff Q) \iff R$ is always logically equivalent to $P \iff (Q \iff R)$, lets try all the possibilities:

P	Q	R	$P \iff Q$	$Q \iff R$	$(P \iff Q) \iff R$	$P \iff (Q \iff R)$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	F	F	F
T	F	F	F	T	T	T
F	T	T	F	T	F	F
F	T	F	F	F	T	T
F	F	T	T	F	T	T
F	F	F	T	T	F	F

Since the last two columns are the same, it is true that

$$((P \iff Q) \iff R) \iff (P \iff (Q \iff R)).$$