

Question 1 (suggested for peer/personal tutorial)

Consider the roll of a fair six-sided die, and let X be the random variable that takes value 1 if the roll is an even number (i.e. $\{2, 4, 6\}$), and $X = 0$ otherwise. Let Y be the random variable that takes the value 1 if the roll is a number greater than 3 (i.e. $\{4, 5, 6\}$), and $Y = 0$ otherwise. Compute $E(Y|X = 1)$.

Question 2

Suppose that X_1, X_2, \dots, X_n are independent random variables that follow a $N(\mu, \sigma^2)$ distribution, and define $\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$ and $S^2 = \frac{1}{n-1} (\bar{X} - X_i)^2$, as usual. Show that the random variable T , where

$$T = \frac{\bar{X} - \mu}{S/\sqrt{n}},$$

can be written in the form

$$T = \frac{U}{\sqrt{V/p}},$$

where

- $U \sim N(0, 1)$,
- p is some function of n ,
- $V \sim \chi_p^2$, the chi-squared distribution with p degrees of freedom,
- U and V are independent random variables.

Question 3

Download the dataset `data_week18.csv` (link for dataset below link for problem sheet on Blackboard). This dataset contains 200 observations for each of the random variables X , Y and Z . Using different exploratory data analysis techniques or visualisations,

- (a) Investigate whether or not there is any relationship between any of the variables.
- (b) Guess the distributions of X , Y and Z .

Hints

- For Question 2, refer to Sections 1.6 and 1.7 in your notes.