

Topic: Expectation of random variables

In today's problem class we will be computing expectations of various discrete and continuous random variables.

1. Compute the mean of the following random variables.

(a) $X \sim \text{Poi}(\lambda)$,

(b) $X \sim \text{Exp}(\lambda)$,

2. Let X be a continuous random variable with the following p.d.f.:

$$f_X(x) = \begin{cases} \theta \lambda e^{-\lambda x}, & x \geq 0; \\ (1 - \theta) \lambda e^{\lambda x}, & x < 0. \end{cases}$$

where λ and θ are constants such that $\lambda > 0$ and $0 \leq \theta \leq 1$.

(a) Show that $f_X(x)$ is a valid p.d.f..

(b) Find $E(X)$. *Hint: You might find it useful to look up the definition of the Gamma function.*

(c) Find $\text{Var}(X)$