## Cover Sheet for Submission of Maths Examinations Summer 2020

We would advise preparing your coversheets with your CID, Module Name and Code and Date, before the exams are due to take place.

**CID: 01738166**

**Module Name: Analysis 1**

**Module Code: MATH40002**

**Date: 04/05/2020**

### Questions Answered (in the file):

Please tick next to the question or questions you have answered in this file.

| | |
|---|---|
| **Q1** | |
| **Q2** | |
| **Q3** | |
| **Q4** | ✓ |
| **Q5** | |
| **Q6** | |

(Note: this is a coversheet for all students - not all students will have exams with 6 questions. Please tick the boxes which are appropriate for your exam and/or the file you are submitting).

### (Optional) Page Numbers for each question;

| Page Number | Question Answered |
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If handwritten, please complete in CAPITAL Letters, in Blue or Black Ink, ensuring the cover sheet is legible.

(a) (i) $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x^3}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$

~~$f(x)$ is continuous at 0 $\iff$ $\forall \varepsilon > 0$ $\exists \delta > 0$ such that $|x - 0| < \delta \Rightarrow |f(x) - 0| < \varepsilon$~~

Since $-x^2 \leq x^2 \sin\left(\frac{1}{x^3}\right) \leq x^2$ for all $x$ and $\lim_{x \to 0^-} -x^2 = 0$, $\lim_{x \to 0^-} x^2 = 0$,

squeeze thrm $\Rightarrow \lim_{x \to 0^-} x^2 \sin\left(\frac{1}{x^3}\right) = 0$.

Similarly $\lim_{x \to 0^+} x^2 \sin\left(\frac{1}{x^3}\right) = 0 \Rightarrow \lim_{x \to 0} x^2 \sin\left(\frac{1}{x^3}\right) = 0 = f(0)$

$\Rightarrow f$ is continuous at $x = 0$

Continues next page

(c)

(i) $f: [a, b] \longrightarrow \mathbb{R}$ is continuous $\Rightarrow \forall d \in [f(a), f(b)]$ $\exists c \in [a, b]$ such that $d = f(c)$

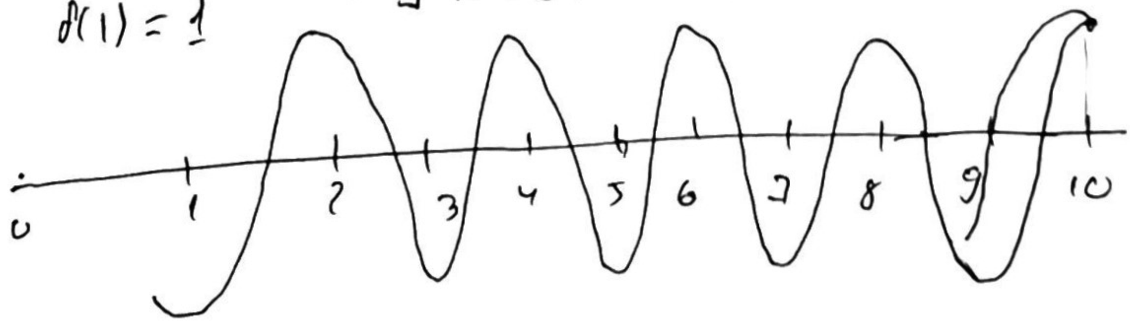
(ii) Consider $f(x) = 4^x - \cos x - 2$

$f(x)$ - continuous

$f(0) = -2$ $\overset{IVT}{\Rightarrow} \exists x \in (0, 1): f(x) = 0$

$f(1) = 1$

(iii)




For $k = 0, 1, 2, 3, 4$:

$f(2k+1) = -1$ $\overset{IVT}{\Rightarrow} \exists x_k \in (2k+1, 2k+2): f(x) = 0$

$f(2k+2) = 1$

Similarly $k = 1, 2, 3, 4$

$f(2k) = 1$, $f(2k+1) = -1$ $\overset{IVT}{\Longrightarrow}$ $\exists\, x_k \in (2k, 2k+1)$

(c) Suppose $f(x) > 0$ for all $x \in (0;1)$

Consider $x_0 \in (0,1)$.

By def. $\exists\, x_1 : |f(x_1)| \le |f(x_0)| \cdot \frac{99}{100}$

$\exists\, x_2 : |f(x_2)| < |f(x_1)| \cdot \frac{99}{100}$

Define the sequence $\{x_n\}$.

Notice $|f(x_n)| \le \left(\frac{99}{100}\right)^n \cdot |f(x_0)|$.

Now $\{x_n\}$ is bounded, $\exists$ convergent subsequence $\{x_{n_p}\}$.

As $f$ is countable at $\lim x_{n_p}$, $f(\lim x_{n_p}) = \lim(f(x_{n_p})) = 0$.

(a)

(ii) $x^2$ is continuous

Also $\frac{1}{x^3}$ is cont. for $x \neq 0$ and $\sin(x)$ is cont

$\Rightarrow x^2 \sin\left(\frac{1}{x^3}\right)$ is also continuous.