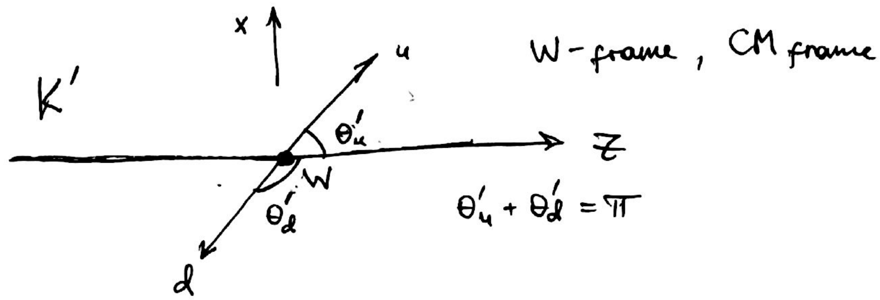

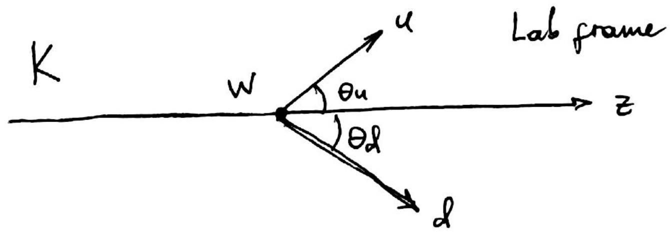

Boost from rest frame:

$$P = \Lambda P' \quad \Leftrightarrow \quad \begin{pmatrix} E \\ 0 \\ 0 \\ p \end{pmatrix} = \begin{pmatrix} \gamma & 0 & 0 & \gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \gamma\beta & 0 & 0 & \gamma \end{pmatrix} \begin{pmatrix} m \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \Rightarrow \quad \begin{cases} E = \gamma m \\ p = \gamma\beta m \end{cases} \tag{1}$$

$$\gamma = \frac{E}{m}, \quad \gamma\beta = \frac{p}{m}, \quad \beta = \frac{p}{E} \tag{2}$$

In the lab frame, K:

$$P_W = (E_W, 0, 0, p_W), \quad P_u = (E_u, p_u \sin\theta_u, 0, p_u \cos\theta_u), \quad P_d = (E_d, -p_d \sin\theta_d, 0, p_d \cos\theta_d) \tag{3}$$

By conservation of momentum,

$$\begin{cases} E_W = E_u + E_d, \\ p_u \sin\theta_u = p_d \sin\theta_d, \\ p_W = p_u \cos\theta_u + p_d \cos\theta_d. \end{cases} \qquad \begin{array}{rr} & p_d^2 \cos^2\theta_d = p_W^2 + p_u^2 \cos^2\theta_u - 2 p_W p_u \cos\theta_u \\ + & p_d^2 \sin^2\theta_d = p_u^2 \sin^2\theta_u \\ \hline & p_d^2 = p_W^2 + p_u^2 - 2 p_W p_u \cos\theta_u \end{array} \tag{4}$$

$$\therefore \cos\theta_u = \frac{p_W^2 + p_u^2 - p_d^2}{2 p_W p_u} \tag{5}$$

In the CM frame, K':

$$P'_W = (m_W, 0, 0, 0), \quad P'_u = (E'_u, p'_u \sin\theta'_u, 0, p'_u \cos\theta'_u), \quad P'_d = (E'_d, -p'_d \sin\theta'_d, 0, p'_d \cos\theta'_d). \tag{6}$$

By conservation of momentum,

$$\begin{cases} m_W = E'_u + E'_d, \\ p'_u \sin\theta'_u = p'_d \sin\theta'_d, \\ p'_u \cos\theta'_u = -p'_d \cos\theta'_d. \end{cases} \tag{7}$$

By the sum of squares of the last two equalities in Eq. (7), using $\sin^2\theta + \cos^2\theta = 1$,

$$p'_q = p'_u = p'_d, \quad \sin\theta'_d = \sin\theta'_u, \quad \cos\theta'_d = -\cos\theta'_u, \tag{8}$$

i.e. the momenta of the final state quarks in the W-frame are equal and oposite.

By the first equality in Eq. (7), using $E^2 = p^2 + m^2$,

$$m_W^2 = {E'_u}^2 + {E'_d}^2 + 2E'_uE'_d = {p'_u}^2 + {m_u}^2 + {p'_d}^2 + {m_d}^2 + 2\sqrt{({p'_u}^2 + {m_u}^2)({p'_d}^2 + {m_d}^2)} \tag{9}$$

Using Eq. (8), it can be shown that

$$p'_q = \frac{\sqrt{m_W^4 - 2m_W^2(m_u^2 + m_d^2) + (m_u^2 - m_d^2)^2}}{2m_W}, \tag{10}$$

and

$$E'_u = \sqrt{{p'_u}^2 + m_u^2} = \frac{m_W^2 + m_u^2 - m_d^2}{2m_W}. \tag{11}$$

For massless quarks, we obtain the expected result

$$\lim_{m_q\to 0} p'_q = \lim_{m_q\to 0} E'_q = \frac{m_W}{2}. \tag{12}$$

Applying Lorentz transformation:

$$\begin{pmatrix} E_q \\ \pm p_q\sin\theta_q \\ 0 \\ p_q\cos\theta_q \end{pmatrix} = P_q = \Lambda P'_q = \begin{pmatrix} \gamma & 0 & 0 & \gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \gamma\beta & 0 & 0 & \gamma \end{pmatrix} \begin{pmatrix} E'_q \\ \pm p'_q\sin\theta'_u \\ 0 \\ \pm p'_q\cos\theta'_u \end{pmatrix} = \begin{pmatrix} \gamma E'_q \pm \gamma\beta p'_q\cos\theta'_u \\ \pm p'_q\sin\theta'_u \\ 0 \\ \gamma\beta E'_q \pm \gamma p'_q\cos\theta'_u \end{pmatrix} \tag{13}$$

$$\therefore p_q^2 = \gamma^2(E'_q \pm \beta p'_q\cos\theta'_u)^2 - m_q^2 \tag{14}$$

Substituting into Eq. (5),

$$\cos\theta_u = \frac{p_W^2 + p_u^2 - p_d^2}{2p_Wp_u}, \quad \cos\theta_d = \frac{p_W^2 + p_d^2 - p_u^2}{2p_Wp_d} \tag{15}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\sqrt{1-\cos^2\theta}}{\cos\theta} \tag{16}$$

$$\boxed{\theta_{\text{opening}} = \tan^{-1}(\tan\theta_u) + \tan^{-1}(\tan\theta_d)} \tag{17}$$