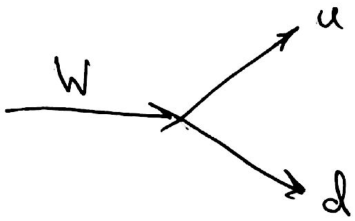

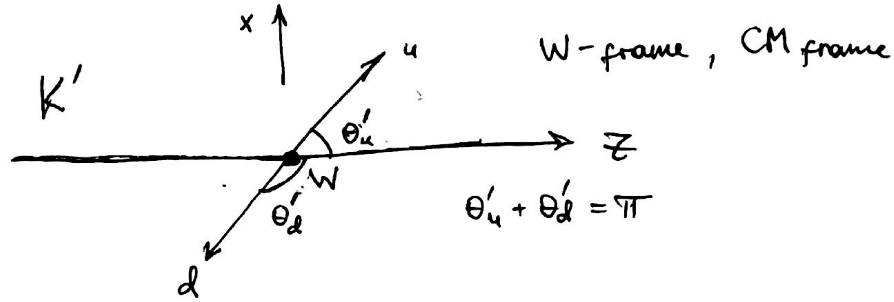

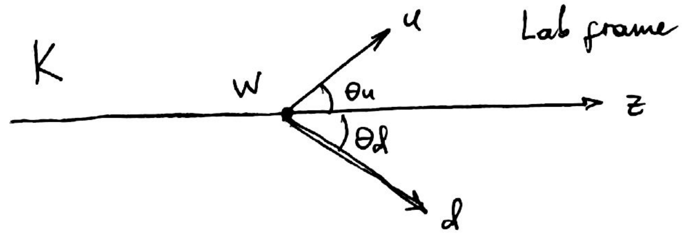

$$\cancel{W}\ P_W = (E_W, 0, 0, p_W)$$

$$P_u = (E_u, p_u \sin\theta_u, 0, p_u \cos\theta_u) \xrightarrow{m_u \ll E_u} (E_u, E_u \sin\theta_u, 0, E_u\cos\theta_u)$$

$$P_d = (E_d, -p_d \sin\theta_d, 0, p_d \cos\theta_d) \xrightarrow{m_d \ll E_d} (E_d, -E_d \sin\theta_d, 0, E_d \cos\theta_d)$$

$$E^2 = p^2 + m^2$$

$$E_d \approx p_d \ \& \ E_u \approx p_u$$

$$\begin{cases} E_W = E_u + E_d \\ E_u \sin\theta_u = E_d \sin\theta_d \\ p_W = E_u \cos\theta_u + E_d \cos\theta_d \end{cases}$$

$$E_d^2 \cos^2\theta_d = E_u^2 \cos^2\theta_u + p_W^2 - 2 p_W E_u \cos\theta_u$$

$$+ \quad E_d^2 \sin^2\theta_d = E_u^2 \sin^2\theta_u$$

$$E_d^2 = E_u^2 + p_W^2 - 2 p_W E_u \cos\theta_u \Rightarrow \cos\theta_u = \frac{p_W^2 + E_u^2 - E_d^2}{2 p_W E_u}$$

$$p_W = \sqrt{E_W^2 - m_W^2} = E_W \sqrt{1 - \left(\frac{m_W}{E_W}\right)^2} \approx E_W \left(1 - \frac{1}{2}\left(\frac{m_W}{E_W}\right)^2\right)$$

In the CM frame,

$$P'_W = (m_W, 0, 0, 0), \quad P'_{u,d} \rightarrow (E'_{u,d}, \pm E'_{u,d} \sin\theta'_{u,d}, 0, E'_{u,d} \cos\theta'_{u,d}), \quad E'_u = E'_d, \quad \theta'_u + \theta'_d = \pi$$

$$E'_u + E'_d = m_W \Rightarrow E'_u = E'_d = \frac{m_W}{2}$$

$$\begin{pmatrix} E_w \\ 0 \\ 0 \\ p_w \end{pmatrix} = \begin{pmatrix} \gamma\beta & 0 & 0 & \gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \gamma & 0 & 0 & \gamma\beta \end{pmatrix} \begin{pmatrix} m_w \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \gamma\beta m_w \\ 0 \\ 0 \\ \gamma m_w \end{pmatrix} \Rightarrow E_w = \gamma\beta m_w \Rightarrow \gamma\beta = \frac{E_w}{m_w}$$

$$\gamma = \frac{p_w}{m_w}$$

$$P_u = \Lambda P'_u \qquad P_{ud} = \Lambda \frac{m_w}{2}\begin{pmatrix} 1 \\ \pm \sin\theta'_{ud} \\ 0 \\ \cos\theta'_{ud} \end{pmatrix} = \frac{m_w}{2}\begin{pmatrix} \gamma\beta + \gamma\cos\theta'_{ud} \\ \pm \sin\theta'_{ud} \\ 0 \\ \gamma + \gamma\beta\cos\theta'_{ud} \end{pmatrix} = \begin{pmatrix} E_{ud} \\ \pm E_{ud}\sin\theta_{ud} \\ 0 \\ E_{ud}\cos\theta_{ud} \end{pmatrix}$$

$$E_{ud} = \left(\gamma\beta + \gamma\cos\theta'_{ud}\right)\frac{m_w}{2} = \frac{\gamma m_w}{2}\left(\beta + \cos\theta'_{ud}\right) = \frac{p_w}{2}\left(\beta + \cos\theta'_{ud}\right)$$

$$\beta = \frac{E_w}{m_w}\,\frac{m_w}{p_w} = \frac{E_w}{p_w}$$

$$\sin\theta_u = \sqrt{1-\cos^2\theta_u}\ , \quad \sin\theta_d = \frac{E_u}{E_d}\sin\theta_u = \frac{\beta+\cos\theta'_u}{\beta+\cos\theta'_d}\sin\theta_u$$

$$\cos\theta_u = \frac{p_w^2 + E_u^2 - E_d^2}{2 p_w E_u}$$

$$= \frac{p_w^2 + \frac{1}{4}p_w^2\left(\beta+\cos\theta'_u\right)^2 - \frac{1}{4}p_w^2\left(\beta+\cos\theta'_d\right)^2}{2p_w\cdot\frac{1}{2}p_w\left(\beta+\cos\theta'_u\right)} = \frac{1+\frac{1}{4}\left[\beta^2 + 2\beta\cos\theta'_u + \cos^2\theta'_u - \beta^2 - 2\beta\cos\theta'_d - \cos^2\theta'_d\right]}{\beta+\cos\theta'_u}$$

$$\beta = \frac{E_w}{p_w} = \frac{1}{\sqrt{1-\left(\frac{m_w}{E_w}\right)^2}} \approx 1 + \frac{1}{2}\left(\frac{m_w}{E_w}\right)^2$$

$$\boxed{\cos\theta_u = \frac{1+\frac{1}{4}\left[\left(\beta+\cos\theta'_u\right)^2 - \left(\beta-\cos\theta'_u\right)^2\right]}{\beta+\cos\theta'_u}}$$

$$\cos\theta'_d = \cos\left(\pi-\theta'_u\right) = -\cos\theta'_u$$

$$\Downarrow$$

$$\cos\theta_u = \frac{1+\beta\cos\theta'_u}{\beta+\cos\theta'_u}$$

$$\theta_u \approx \sin\theta_u = \sqrt{1-\cos^2\theta_u}\ , \quad \theta_d \approx \sin\theta_d = \frac{\beta+\cos\theta'_u}{\beta-\cos\theta'_u}\sin\theta_u\ , \quad \beta = \frac{1}{\sqrt{1-\left(\frac{m_w}{E_w}\right)^2}} \approx 1+\frac{1}{2}\left(\frac{m_w}{E_w}\right)^2$$

$$\cos\theta_u = \frac{1+\beta\cos\theta'_u}{\beta+\cos\theta'_u}$$