

**Problem 1**

By the Hillas cryterion,

$$E_{\max} \approx 2\beta cZeBr. \quad (1.1)$$

With  $r = 4 \text{ km}$ ,  $B = 10 \text{ T}$ ,  $Z = 1$ , and the particles traveling at nearly the speed of light ( $\beta \rightarrow 1$ ),

$$E_{\max} \approx 2 \times 3 \times 10^8 \text{ m s}^{-1} \times 1 \times 1.602 \times 10^{-19} \text{ C} \times 10 \text{ T} \times 4 \text{ km} \approx \boxed{24 \text{ TeV}} \quad (1.2)$$

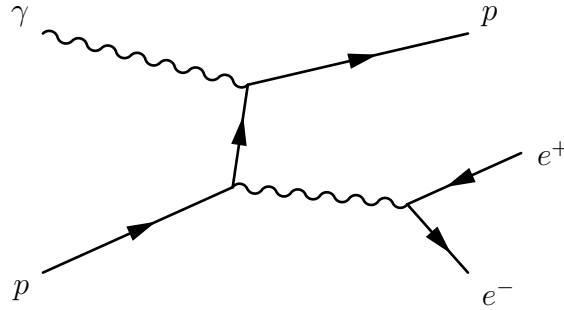
Discuss whether this places the LHC at about the right place on the Hillas diagram.

To accelerate particles to  $E = 1 \text{ ZeV} = 1 \times 10^{21} \text{ eV}$ , an accelerator of this size would need a magnetic field of

$$B_2 = \frac{E_2}{E_1} \times B_1 = \frac{1 \times 10^{21} \text{ eV}}{24 \times 10^{12} \text{ eV}} \times 10 \text{ T} = \boxed{4.16 \times 10^9 \text{ T}} \quad (1.3)$$

The maximum energy of an accelerator circling the earth ( $r = 6.4 \times 10^6 \text{ m}$ ) with a 10 T field would be

$$E_3 = \frac{r_3}{r_1} \times E_1 = \frac{6.4 \times 10^6 \text{ m}}{4 \times 10^3 \text{ m}} \times 24 \times 10^{12} \text{ eV} = \boxed{3.84 \times 10^4 \text{ TeV}} \quad (1.4)$$

**Problem 2**

$$P_\gamma + P_p = P_{p'} + P_{e^+} + P_{e^-} \quad (2.1)$$

Squaring both sides we get,

$$(P_\gamma + P_p)^2 = (m_p + 2m_e)^2. \quad (2.2)$$

$$(P_\gamma + P_p)^2 = P_\gamma^2 + P_p^2 + 2P_\gamma \cdot P_p = 0 + m_p^2 + 2P_\gamma \cdot P_p \quad (2.3)$$

Since the proton is ultra-relativistic,

$$P_\gamma = (E_\gamma \quad -E_\gamma \quad 0 \quad 0) \quad \text{and} \quad P_p \approx (E_p \quad E_p \quad 0 \quad 0). \quad (2.4)$$

$$\therefore P_\gamma \cdot P_p = 2E_p E_\gamma, \quad (2.5)$$

and

$$m_p^2 + 4E_p E_\gamma = m_p^2 + 4m_e m_p + 4m_e^2 \quad (2.6)$$

$$\boxed{E_p = \frac{m_e m_p + m_e^2}{E_\gamma}} \quad (2.7)$$

$m_e = 0.511$  MeV,  $m_p = 938$  MeV. The temperature of the CMB is 2.726 K which corresponds to  $E_\gamma = 0.235$  meV. Substituting, we get

$$E_p = \frac{0.511 \times 938 + 0.511^2}{0.235} \times \frac{10^{12}}{10^{-3}} \text{ eV} = \boxed{2.04 \times 10^{18} \text{ eV}} \quad (2.8)$$

### Problem 3

$$P_1 + P_2 = P_+ + P_- \quad (3.1)$$

After squaring both sides, since  $m_1 = m_2 = 0$ ,

$$2E_1 E_2 = 4m_e^2 \Rightarrow E_1 = \frac{2m_e^2}{E_2}. \quad (3.2)$$

For  $\lambda_2 = 1 \mu\text{m}$ ,

$$E_2 = \frac{hc}{\lambda} = 1.24 \text{ eV} \quad (3.3)$$

$$E_1 = \frac{2 \times (0.511 \times 10^6)^2}{1.24} \text{ eV} = 4.2 \times 10^{11} \text{ eV} = \boxed{0.42 \text{ TeV}} \quad (3.4)$$

### Problem 4

### Problem 5

### Problem 6

### Problem 7

### Problem 8