

**Problem 1**

By the Hillas cryterion,

$$E_{\max} \approx 2\beta cZeBr. \quad (1.1)$$

With  $r = 4 \text{ km}$ ,  $B = 10 \text{ T}$ ,  $Z = 1$ , and the particles traveling at nearly the speed of light ( $\beta \rightarrow 1$ ),

$$E_{\max} \approx 2 \times 3 \times 10^8 \text{ m s}^{-1} \times 1 \times 1.602 \times 10^{-19} \text{ C} \times 10 \text{ T} \times 4 \text{ km} \approx \boxed{24 \text{ TeV}} \quad (1.2)$$

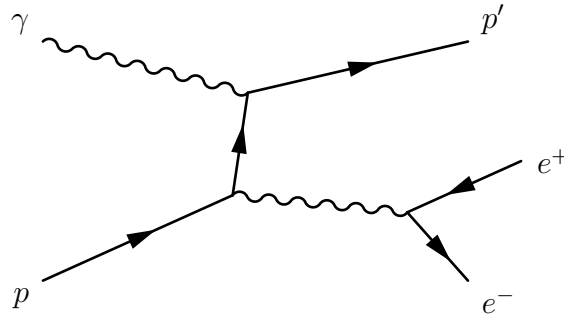
Discuss whether this places the LHC at about the right place on the Hillas diagram.

To accelerate particles to  $E = 1 \text{ ZeV} = 1 \times 10^{21} \text{ eV}$ , an accelerator of this size would need a magnetic field of

$$B_2 = \frac{E_2}{E_1} \times B_1 = \frac{1 \times 10^{21} \text{ eV}}{24 \times 10^{12} \text{ eV}} \times 10 \text{ T} = \boxed{4.16 \times 10^9 \text{ T}} \quad (1.3)$$

The maximum energy of an accelerator circling the earth ( $r = 6.4 \times 10^6 \text{ m}$ ) with a 10 T field would be

$$E_3 = \frac{r_3}{r_1} \times E_1 = \frac{6.4 \times 10^6 \text{ m}}{4 \times 10^3 \text{ m}} \times 24 \times 10^{12} \text{ eV} = \boxed{3.84 \times 10^4 \text{ TeV}} \quad (1.4)$$

**Problem 2**

$$P_\gamma + P_p = P_{p'} + P_{e^+} + P_{e^-} \quad (2.1)$$

Squaring both sides, and noting that in the center-of-mass frame the outgoing particles are produced at rest, we get

$$(P_\gamma + P_p)^2 = (m_p + 2m_e)^2. \quad (2.2)$$

$$(P_\gamma + P_p)^2 = P_\gamma^2 + P_p^2 + 2P_\gamma \cdot P_p = 0 + m_p^2 + 2P_\gamma \cdot P_p \quad (2.3)$$

Since the proton is ultra-relativistic,

$$P_\gamma = \begin{pmatrix} E_\gamma & -E_\gamma & 0 & 0 \end{pmatrix} \quad \text{and} \quad P_p \approx \begin{pmatrix} E_p & E_p & 0 & 0 \end{pmatrix}. \quad (2.4)$$

$$\therefore P_\gamma \cdot P_p = 2E_p E_\gamma, \quad (2.5)$$

and

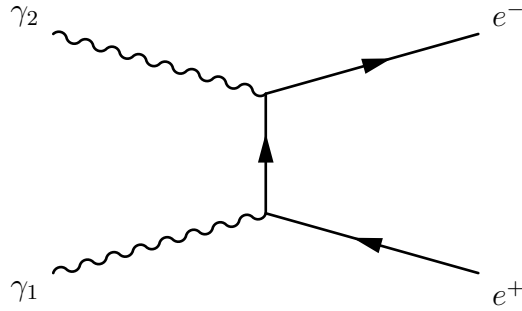
$$m_p^2 + 4E_p E_\gamma = m_p^2 + 4m_e m_p + 4m_e^2 \quad (2.6)$$

$$\boxed{E_p = \frac{m_e m_p + m_e^2}{E_\gamma}} \quad (2.7)$$

$m_e = 0.511$  MeV,  $m_p = 938$  MeV. The temperature of the CMB is 2.726 K which corresponds to  $E_\gamma = 0.235$  meV. Substituting, we get

$$E_p = \frac{0.511 \times 938 + 0.511^2}{0.235} \times \frac{10^{12}}{10^{-3}} \text{ eV} = \boxed{2.04 \times 10^{18} \text{ eV}} \quad (2.8)$$

### Problem 3



$$P_1 + P_2 = P_+ + P_- \quad (3.1)$$

After squaring both sides, since  $m_1 = m_2 = 0$ ,

$$2E_1 E_2 = 4m_e^2 \Rightarrow E_1 = \frac{2m_e^2}{E_2}. \quad (3.2)$$

For  $\lambda_2 = 1 \mu\text{m}$ ,

$$E_2 = \frac{hc}{\lambda} = 1.24 \text{ eV} \quad (3.3)$$

$$E_1 = \frac{2 \times (0.511 \times 10^6)^2}{1.24} \text{ eV} = 4.2 \times 10^{11} \text{ eV} = \boxed{0.42 \text{ TeV}} \quad (3.4)$$

which calculation is more precise

### Problem 4

$$v = \frac{d}{t} \Rightarrow \frac{\delta v}{v} = \sqrt{\left(\frac{\delta d}{d}\right)^2 + \left(\frac{\delta t}{t}\right)^2} \quad (4.1)$$

We may as well ignore  $\delta d/d$ , because the fractional uncertainty of high measurement is usually much better than that of time. So,

$$\frac{\delta v}{v} \approx \frac{\delta t}{t} \quad (4.2)$$

For a particle traveling at nearly the speed of light,

$$\delta t \approx t \frac{\delta v}{v} \approx \frac{d}{c} \frac{\delta v}{v} \quad (4.3)$$

To measure the particle's velocity to 5%,

$$\delta t = \frac{1 \text{ m}}{3 \times 10^8 \text{ m s}^{-1}} \times 0.05 = 1.7 \times 10^{-10} \text{ s} = \boxed{0.17 \text{ ns}} \quad (4.4)$$

How accurate to tell an upmoving particle from a downmoving by 10 standard deviations?

### Problem 5

The Larmor radius is given by

$$r = \frac{mv}{qB} \quad (5.1)$$

Substituting  $mv = 1 \text{ TeV}/c$ ,  $q = e$ , and  $B = 5 \text{ kG}$ ,

$$r = \frac{1 \times 10^{12} \text{ eV} / 3 \times 10^8 \text{ m s}^{-1}}{1.6 \times 10^{-19} \text{ C} \times 5 \times 10^3 \text{ G}} = \frac{1 \times 10^{12} \times 1.6 \times 10^{-15} \text{ m}}{1.6 \times 10^{-19} \times 5 \times 10^3 \times 3 \times 10^8} = 6671 \text{ m} \quad (5.2)$$

When the sagitta is small in comparison to the radius, as is the case here, it may be approximated by the formula

$$s \approx \frac{l^2}{2r}, \quad (5.3)$$

where  $s$  is the sagitta,  $r$  is the radius of curvature, and  $l$  is half the length of the chord.

$$\frac{\delta r}{r} = \sqrt{\left(2 \frac{\delta l}{l}\right)^2 + \left(\frac{\delta s}{s}\right)^2} = \sqrt{4 \frac{\delta l^2}{l^2} + 4 \frac{\delta l^2}{l^4/r^2}} = 2 \frac{\delta l}{l} \sqrt{1 + \frac{r^2}{l^2}} \approx 2 \frac{\delta l}{l} \frac{r}{l} \quad (5.4)$$

$$\therefore \delta l = \frac{1}{2} \left(\frac{l}{r}\right)^2 \delta r \quad (5.5)$$

If the momentum is to be measured with 1% accuracy,

$$0.01 = \frac{\delta mv}{mv} = \frac{\delta r}{r} = \frac{2r}{l^2} \delta l. \quad (5.6)$$

For  $l \approx 1 \text{ m}$ ,

$$\delta l = 0.01 \times \frac{(1 \text{ m})^2}{2 \times 6671 \text{ m}} = 7.5 \times 10^{-7} \text{ m} = \boxed{750 \text{ nm}} \quad (5.7)$$

what technology

### Problem 6

$$\frac{1 \times 3 \text{ GeV}}{\text{cm}^2 \text{ s}} \times \frac{\text{J}}{6.24 \times 10^9 \text{ GeV}} \times \left(\frac{100 \text{ cm}}{\text{m}}\right)^2 = 4.8 \times 10^{-6} \text{ W m}^{-2} \quad (6.1)$$

In comparison, the solar constant is  $1.36 \text{ kW m}^{-2}$ .

### Problem 7

The Larmor radius is given by

$$r = \frac{mv}{qB} \approx \frac{E}{qBc} \quad \Rightarrow \quad E = qBcr \quad (7.1)$$

$$E = 1.6 \times 10^{-19} \text{ C} \times 10^{-5.3} \text{ G} \times 3 \times 10^8 \text{ m s}^{-1} \times 10^{2.5} \text{ pc} \times 3.086 \times 10^{16} \frac{\text{m}}{\text{pc}} = \boxed{1.465 \times 10^{18} \text{ eV}} \quad (7.2)$$

### Problem 8

(a) There are

$$\int_{1 \text{ TeV}}^{5 \text{ TeV}} AE^{-2.7} dE \bigg/ \int_{5 \text{ TeV}}^{\infty} AE^{-2.7} dE = \frac{1^{-1.7} - 5^{-1.7}}{5^{-1.7}} = 14.4 \quad (8.1)$$

times fewer CR above 5 TeV than between 1 TeV and 5 TeV.