Problem 1

By the Hillas cryterion,

$$E_{\text{max}} \approx 2\beta c Z e B r. \tag{1.1}$$

With $r = 4\,\text{km}$, $B = 10\,\text{T}$, $Z = 1$, and the particles traveling at nearly the speed of light ($\beta \to 1$),

$$E_{\text{max}} \approx 2 \times 3 \times 10^8\,\text{m}\,\text{s}^{-1} \times 1 \times 1.602 \times 10^{-19}\,\text{C} \times 10\,\text{T} \times 4\,\text{km} \approx \boxed{24\,\text{TeV}} \tag{1.2}$$

Discuss whether this places the LHC at about the right place on the Hillas diagram.

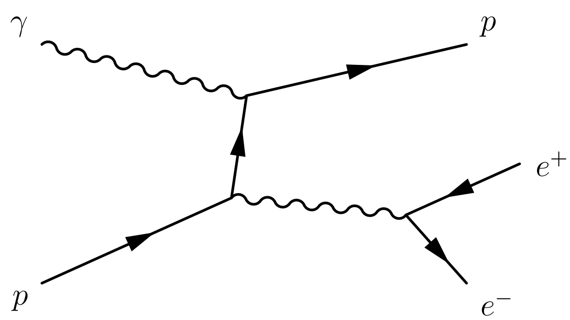
To accelerate particles to $E = 1\,\text{ZeV} = 1 \times 10^{21}\,\text{eV}$, an accelerator of this size would need a magnetic field of

$$B_2 = \frac{E_2}{E_1} \times B_1 = \frac{1 \times 10^{21}\,\text{eV}}{24 \times 10^{12}\,\text{eV}} \times 10\,\text{T} = \boxed{4.16 \times 10^9\,\text{T}} \tag{1.3}$$

The maximum energy of an accelerator circling the earth ($r = 6.4 \times 10^6\,\text{m}$) with a $10\,\text{T}$ field would be

$$E_3 = \frac{r_3}{r_1} \times E_1 = \frac{6.4 \times 10^6\,\text{m}}{4 \times 10^3\,\text{m}} \times 24 \times 10^{12}\,\text{eV} = \boxed{3.84 \times 10^4\,\text{TeV}} \tag{1.4}$$

Problem 2



$$P_\gamma + P_p = P_{p'} + P_{e^+} + P_{e^-} \tag{2.1}$$

Squaring both sides we get,

$$(P_\gamma + P_p)^2 = (m_p + 2m_e)^2. \tag{2.2}$$

$$(P_\gamma + P_p)^2 = P_\gamma^2 + P_p^2 + P_\gamma P_p = 0 + m_p^2 + 2P_\gamma \cdot P_p \tag{2.3}$$

Since the proton is ultra-relativistic,

$$P_\gamma = \begin{pmatrix} E_\gamma & -E_\gamma & 0 & 0 \end{pmatrix} \quad \text{and} \quad P_p \approx \begin{pmatrix} E_p & E_p & 0 & 0 \end{pmatrix}. \tag{2.4}$$

$$\therefore P_\gamma \cdot P_p = 2E_p E_\gamma, \tag{2.5}$$

and

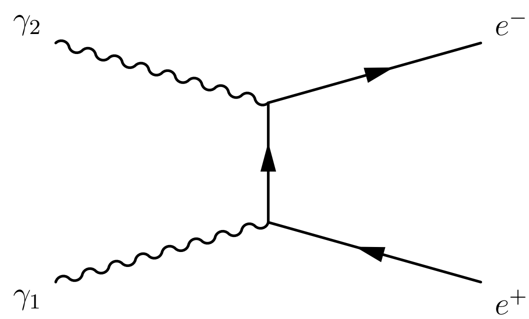
$$m_p^2 + 4E_pE_\gamma = m_p^2 + 4m_em_p + 4m_e^2 \tag{2.6}$$

$$\boxed{E_p = \frac{m_em_p + m_e^2}{E_\gamma}} \tag{2.7}$$

$m_e = 0.511$ MeV, $m_p = 938$ MeV. The temperature of the CMB is 2.726 K which corresponds to $E_\gamma = 0.235$ meV. Substituting, we get

$$E_p = \frac{0.511 \times 938 + 0.511^2}{0.235} \times \frac{10^{12}}{10^{-3}} \text{ eV} = \boxed{2.04 \times 10^{18} \text{ eV}} \tag{2.8}$$

Problem 3



$$P_1 + P_2 = P_+ + P_- \tag{3.1}$$

After squaring both sides, since $m_1 = m_2 = 0$,

$$2E_1E_2 = 4m_e^2 \quad \Rightarrow \quad E_1 = \frac{2m_e^2}{E_2}. \tag{3.2}$$

For $\lambda_2 = 1\,\mu\text{m}$,

$$E_2 = \frac{hc}{\lambda} = 1.24 \text{ eV} \tag{3.3}$$

$$E_1 = \frac{2 \times (0.511 \times 10^6)^2}{1.24} \text{ eV} = 4.2 \times 10^{11} \text{ eV} = \boxed{0.42 \text{ TeV}} \tag{3.4}$$

Problem 4

Problem 5

Problem 6

Problem 7

Problem 8