Kinematics for branch cuts in $gg \to hgg$

Kirtimaan A. Mohan

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1 Introduction

The process $gg \to hgg$ has 86 feynman diagrams in total. 38 have triangle loops, 36 have boxes and 12 are pentagons - all with top quarks flowing through the loop. As a direct probe for the $h\bar{t}t$ yukawa coupling, we would like to examine higgs production through these loop diagrams and isolate structures that exist due to the massive fermion in the loop. To this end we look for singularities in the amplitude where the top- quark goes on-shell and can potentially give rise to structures in an otherwise smooth amplitude. Our objective is to find observables/ distributions that clearly demarcate the presence of these thresholds. This can be done analytically by applying the Landau conditions to identify kinematic regions when the top goes on-shell.

In general the Landau conditions will identify all kinematic variables and conditions on them that will give rise to singularities, including soft/ collinear singularities as well singularities that are unphysical. By unphysical we mean here, thresholds that are not allowed due to the $2 \to 3$ kinematics. For example, a condition that we see is $t_{15} = 4m_t^2$. Since $t_{15} = (k_1 - k_5)^2 = -2k_1 \cdot k_5 < 0$, such a threshold cannot be reached. While I will omit such obvious ones from the list I provide, I will not do a careful job of removing more complicated expressions that are harder to analyze. We can discard these later after looking at their distributions through numerical simulation.

I provide below a list of all such observables in the form

$$x_i^n = f(\text{mandelstam invariants}, m_h, m_t) \tag{1}$$

, i.e. as a function of mandelstam invariants of the process and higgs mass and top mass.

Numerically we have seen that the triangles provide the largest contribution to the amplitude followed by the boxes and then the triangles. Hence I will initially only list invariants for the triangles and boxes - and later add in pentagons.

2 Kinematics

We look at the process

$$g(k_1) + g(k_2) \to h(k_3) + g(k_4) + g(k_5) \tag{2}$$

Here k_1 and k_2 are incoming four momenta and k_3, k_4 and k_5 are outgoing momenta. We introduce mandelstam s invariants as follows.

$$
\begin{aligned}
s_{12} &= (k_1+k_2)^2 = 2k_1\cdot k_2 \\
s_{34} &= (k_3+k_4)^2 = 2k_3\cdot k_4 + m_h^2 \\
s_{35} &= (k_3+k_5)^2 = 2k_3\cdot k_5 + m_h^2 \\
s_{45} &= (k_4+k_5)^2 = 2k_4\cdot k_5
\end{aligned}
$$

Similarly we introduce the mandelstam t invariants. Note all the t invariants that do not have 3 in their label are strictly negative for physical kinematics, whereas all the s invariants are strictly positive.

$$
\begin{aligned}
t_{15} &= (k_1-k_5)^2 = -2k_2\cdot k_5 \\
&\quad . \\
&\quad . \\
t_{23} &= (k_2-k_3)^2 = m_h^2 - 2k_2\cdot k_3 \\
t_{24} &= (k_2-k_4)^2 = -2k_2\cdot k_4 \\
&\quad . \\
&\quad . \\
t_{45} &= (k_4-k_5)^2 = -2k_4\cdot k_5
\end{aligned}
$$

The $\cdots$ denote all the invariants that have not been written explicitly here. Finally it is important to realize that all the invariants are not independent of each other and relations exist among them, so it is in fact possible to describe the entire kinematics using 5 invariants instead of 10 listed above. However, some expressions will turn out to be smaller if we use the entire set of invariants and so I will utilize all of them.

3 Triangles

$$
\begin{aligned}
x_0^3 &= \left(m_h^2 - 4m_t^2 - s_{12} + s_{45} - t_{23}\right) && (3)\\
x_1^3 &= s_{34} - 4m_t^2 && (4)\\
x_2^3 &= \left(m_h^2 - 4m_t^2 + s_{12} - s_{34} - s_{45}\right) && (5)\\
x_3^3 &= s_{45} - 4m_t^2 && (6)\\
x_4^3 &= s_{12} - 4m_t^2 && (7)\\
x_5^3 &= \left(4m_t^2 + s_{12} - s_{34} + t_{15}\right) && (8)\\
x_6^3 &= \left(4m_t^2 + s_{45} + t_{15} - t_{23}\right) && (9)\\
x_7^3 &= \left(m_h^2 - 4m_t^2 - s_{34} + t_{15} - t_{23}\right) && (10)\\
x_8^3 &= m_h^2\left(s_{12}s_{45} - 2\left(s_{12}+s_{45}\right)m_t^2\right) + m_h^4 m_t^2 + \left(s_{12}-s_{45}\right)^2 m_t^2 && (11)\\
x_9^3 &= m_h^2\left(2m_t^2\left(s_{12} - s_{34} + s_{45} + 2t_{15} - t_{23}\right) + \left(s_{12} - s_{34} + t_{15}\right)\left(s_{45} + t_{15} - t_{23}\right)\right) && (12)\\
&+ m_h^4 m_t^2 + m_t^2\left(s_{12} - s_{34} - s_{45} + t_{23}\right)^2 \\
x_{10}^3 &= t_{15}m_h^4 + m_t^2\left(s_{34}+t_{23}\right)^2 - t_{15}m_h^2\left(4m_t^2 + s_{34} - t_{15} + t_{23}\right) && (13)
\end{aligned}
$$

If we create histograms of the observables above , then a branch cut exists at $x_i^n = 0$. It remains to be seen whether the branch cut has a large effect. Additionally if the histograms we see are only populated for either positive only or negative only values of x, then we cannot reach the branch cut with physical kinematics.

4 Boxes

4.1 Double cut

Two propagators on shell

$$
\begin{aligned}
x_0^4 &= s_{12} - 4m_t^2 && (14)\\
x_1^4 &= s_{34} - 4m_t^2 && (15)\\
x_2^4 &= \left(m_h^2 - 4m_t^2 + s_{12} - s_{34} - s_{45}\right) && (16)\\
x_3^4 &= s_{45} - 4m_t^2 && (17)\\
x_4^4 &= \left(4m_t^2 + s_{45} + t_{15} - t_{23}\right) && (18)\\
x_5^4 &= 4m_t^2 - t_{23} && (19)\\
x_6^4 &= \left(4m_t^2 + s_{12} - s_{34} + t_{15}\right) && (20)\\
x_7^4 &= \left(m_h^2 - 4m_t^2 - s_{12} + s_{45} - t_{23}\right) && (21)\\
x_8^4 &= \left(m_h^2 - 4m_t^2 - s_{34} + t_{15} - t_{23}\right) && (22)\\
x_9^4 &= \left(-m_h^2 + 4m_t^2 + s_{12} - s_{45} + t_{23}\right) && (23)
\end{aligned}
$$

4.2 Triple Cut

Three propagators on-shell

$$
\begin{aligned}
x_{10}^4 &= m_h^2\left(s_{12}s_{45} - 2\left(s_{12} + s_{45}\right)m_t^2\right) && (24)\\
&+ m_h^4 m_t^2 + \left(s_{12} - s_{45}\right)^2 m_t^2 &&\\
x_{11}^4 &= m_h^2\left(2m_t^2\left(s_{12} - s_{34} + s_{45} + 2t_{15} - t_{23}\right) + \left(s_{12} - s_{34} + t_{15}\right)\left(s_{45} + t_{15} - t_{23}\right)\right) && (25)\\
&+ m_h^4 m_t^2 + m_t^2\left(s_{12} - s_{34} - s_{45} + t_{23}\right)^2 &&\\
x_{12}^4 &= t_{15} m_h^4 + m_t^2\left(s_{34} + t_{23}\right)^2 && (26)\\
&- t_{15} m_h^2\left(4m_t^2 + s_{34} - t_{15} + t_{23}\right) &&
\end{aligned}
$$

4.3 Quadruple Cut

Four propagators on-shell

$$
\begin{aligned}
x_{13}^4 &= \left(\left(s_{12}-s_{34}\right)\left(m_h^2-s_{34}\right)+s_{45}\left(s_{34}-4m_t^2\right)\right) && (27)\\
x_{13}^4 &= 4m_t^2\left(s_{34}-s_{12}\right)\left(m_h^2-s_{34}\right)+s_{34}s_{45}\left(s_{34}-4m_t^2\right) && (28)\\
x_{13}^4 &= -s_{45}\left(m_h^4+\left(-s_{12}+s_{34}+s_{45}\right)^2\right)+2m_h^2\left(2\left(s_{12}-s_{34}\right)m_t^2+s_{45}\left(-s_{12}+s_{34}+s_{45}\right)\right) && (29)\\
&+ 4s_{34}\left(-s_{12}+s_{34}+s_{45}\right)m_t^2 &&\\
x_{13}^4 &= \left(4m_h^2m_t^2\left(s_{45}+t_{15}\right)-t_{23}\left(4\left(-s_{12}+s_{34}+s_{45}\right)m_t^2+t_{23}\left(s_{12}-s_{34}+t_{15}\right)\right)\right) && (30)\\
x_{13}^4 &= \left(m_h^2\left(s_{45}+t_{15}\right)-4m_t^2\left(s_{12}-s_{34}+t_{15}\right)+\left(s_{12}-s_{34}-s_{45}\right)t_{23}\right) && (31)\\
x_{13}^4 &= m_h^4\left(s_{12}-s_{34}+t_{15}\right)+2m_h^2\left(\left(s_{12}-s_{34}-s_{45}\right)\left(s_{12}-s_{34}+t_{15}\right)-2m_t^2\left(s_{45}+t_{15}\right)\right) &&\\
&+ \left(s_{12}-s_{34}-s_{45}\right)\left(\left(s_{12}-s_{34}-s_{45}\right)\left(s_{12}-s_{34}+t_{15}\right)-4t_{23}m_t^2\right) && (32)\\
x_{13}^4 &= 4m_t^2\left(t_{15}m_h^2+\left(s_{12}-s_{34}-s_{45}\right)\left(s_{12}-s_{45}+t_{23}\right)\right) &&\\
&- t_{15}\left(m_h^2-s_{12}+s_{45}-t_{23}\right)^2 && (33)\\
x_{13}^4 &= \left(t_{15}\left(m_h^2-4m_t^2\right)+\left(s_{12}-s_{34}-s_{45}\right)\left(s_{12}-s_{45}+t_{23}\right)\right) && (34)\\
x_{13}^4 &= 4m_t^2\left(t_{15}m_h^2+\left(s_{12}-s_{34}-s_{45}\right)\left(s_{12}-s_{45}+t_{23}\right)\right) &&\\
&- t_{15}\left(m_h^2+s_{12}-s_{34}-s_{45}\right)^2 && (35)\\
x_{13}^4 &= \left(m_h^2\left(4t_{15}m_t^2-t_{23}^2\right)+t_{23}\left(t_{23}\left(s_{34}-t_{15}+t_{23}\right)-4s_{34}m_t^2\right)\right) && (36)\\
x_{13}^4 &= \left(m_h^2\left(t_{15}-4m_t^2\right)+4m_t^2\left(s_{34}-t_{15}+t_{23}\right)-s_{34}t_{23}\right) && (37)\\
x_{13}^4 &= \left(-s_{34}^2\left(m_h^2+t_{15}-t_{23}\right)+4t_{15}m_h^2m_t^2-4s_{34}t_{23}m_t^2+s_{34}^3\right) && (38)\\
x_{13}^4 &= m_h^4\left(s_{45}+t_{15}-t_{23}\right)-2m_h^2\left(2m_t^2\left(s_{12}+t_{15}\right)+\left(s_{45}+t_{15}-t_{23}\right)\left(s_{12}-s_{45}+t_{23}\right)\right) &&\\
&- \left(s_{12}-s_{45}+t_{23}\right)\left(-4s_{34}m_t^2-\left(s_{45}+t_{15}-t_{23}\right)\left(s_{12}-s_{45}+t_{23}\right)\right) && (39)\\
x_{13}^4 &= \left(m_h^2\left(s_{12}+t_{15}\right)-4m_t^2\left(s_{45}+t_{15}-t_{23}\right)-s_{34}\left(s_{12}-s_{45}+t_{23}\right)\right) && (40)\\
x_{13}^4 &= \left(4m_h^2m_t^2\left(s_{12}+t_{15}\right)+s_{34}\left(-4m_t^2\left(s_{12}-s_{45}+t_{23}\right)-s_{34}\left(s_{45}+t_{15}-t_{23}\right)\right)\right) && (41)\\
x_{13}^4 &= s_{12}m_h^4+\left(s_{12}-s_{45}+t_{23}\right)\left(s_{12}\left(s_{12}-s_{45}+t_{23}\right)-4t_{23}m_t^2\right) &&\\
&- 2m_h^2\left(2m_t^2\left(s_{45}-t_{23}\right)+s_{12}\left(s_{12}-s_{45}+t_{23}\right)\right) && (42)\\
x_{13}^4 &= 4m_h^2m_t^2\left(t_{23}-s_{45}\right)+t_{23}\left(s_{12}t_{23}-4m_t^2\left(s_{12}-s_{45}+t_{23}\right)\right) && (43)\\
x_{13}^4 &= \left(m_h^2\left(s_{45}-t_{23}\right)-4s_{12}m_t^2+t_{23}\left(s_{12}-s_{45}+t_{23}\right)\right) && (44)
\end{aligned}
$$