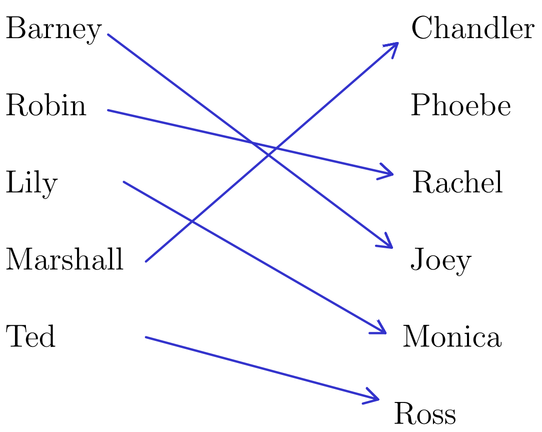
Problem 1.1 (F.1)

a) Please draw your arrows between the names provided. Please keep your work within this section even though there is no box surrounding it.



b)

Domain: {Barney, Robin, Lily, Marshall, Ted}   Range: {Chandler, Rachel, Joey, Monica, Ross}

c)

Joey

d)

Robin

Problem 1.2 (F.1)

a)

○ Well Defined ● Not Well Defined

Consider that particles interactions are the domain and the Feynmann diagrams are the target. We know that each interaction can have infinitely many Feynmann diagrams. Therefore, it is not one to one

∴ The function is not well defined.

b)

● Well Defined ○ Not Well Defined

Consider that stickers are the domain and the owners of the stickers are the target. We know that none of the stickers have been lost and each owner has exactly one sticker. We can say that $|D| = |T|$.

∴ The function is well defined.

Problem 2 (F.2)

a)

● Injective ○ Surjective ○ Bijective ○ None

Consider the domain is the students coming to the class and the target is the seats in the classroom. We know that every student has exactly one seat and no student shares seat with any other students. And there are more seats than students, so we can say that $|D| < |T|$.

∴ The function is injective only.

b)

○ Injective ○ Surjective ○ Bijective ● None

Consider the sets $A = \{a, b\}$ and $B = \{c, d\}$. $P(A)$ and $P(B)$ have the same average of cardinality of elements which is 1.

∴ The function is not injective.

Consider the average of cardinality of elements in $P(A)$ is an integer divided by another integer which is a rational number. However, the target is all real numbers which includes both rational and irrational numbers.

∴ The function is not surjective.

Problem 3 (F.3)

100 001
001 100
100 001

a)

Inverse (or simply "There is not an inverse"):

Yes, there is an inverse for function f.

The inverse of function f is f itself

Explanation:

We know that the input strings always have even size and the string can be break into two-half. Let say the first half of the string is a and the second half of the string is b, then we have the full string ab. And we call the reversed string of a as a' and the reversed string of b as b'. To reverses the full string ab, we have $f(ab) = a'b'$

In order to get back to ab from a'b', we simple use function f again we have $f(a'b') = ab$

We know that to reverse the same string twice will back the original string.

∴ The inverse of function f is f itself.

b)

Inverse (or simply "There is not an inverse"):

Yes, there is an inverse for function f

The inverse of f is f itself.

Explanation:

We know that the function f maps a string $a \oplus k$.

we express it as $f(a) = a \oplus k$.

We want to find the inverse of function f, and let's call it g.

We know k is a pre-defined string, then we use it to reverse the operation of xor.

Let g be $g(b) = b \oplus k$. Now we plug in $f(a)$ into $g(b)$

then we have $g(f(a))$

$= g(a \oplus k)$

$= (a \oplus k) \oplus k$

$= a \oplus (k \oplus k)$

$= a \oplus \emptyset$

$= a$ ∴ The inverse of f is f itself.

Problem 4 (F.4)

a)

Domain: $\{0,1\}^{10}$

Range: $\{0,1\}^{5}$

b)

Function Definition (or simply "Function is not well defined"):

$b: \{0,1\}^{10} \to \{0,1\}^{10}$, where b flips the value of all the odd bits in the string.

Explanation:

First we rotate the input strings to the right by 3. Rotating to the right by removing the last 3 bits and appending it to the start of the string. Then we flips all the even bits in the bit string, so we change 0 to 1 and 1 to 0. Finally, we rotate the bit string to the right by 7. Rotating to the right by removing the last 7 bits and appending it to the start of the string. Now, if we compare the output string to the original input string, we find that we can flip the value of all the odd bits of the input string to get the output string.

$\therefore$ The function can be described as flipping the value of all the odd bits in a 10 bits string.

c)

Function Definition (or simply "Function is not well defined"):

Function is not well defined.

Explanation:

Function k needs the input of a 10 bits string then output a 5 bits string and Function g_1 requires the input of a 10 bits string then output a 10 bits string. However, in the function $k \circ f_5 \circ g_1 \circ k$, g_1 takes the input from the output of k which is a 5 bits string, but g_1 requires a 10 bits string. The input does not match the domain of g_1.

$\therefore$ the function $k \circ f_5 \circ g_1 \circ k$ is not well defined.

d)

Inverse (or simply "There is not an inverse"):

The inverse of the function is the function itself

$d : \{0,1\}^5 \rightarrow \{0,1\}^5$, where d outputs the same bit string as the input using the same operation as $k \circ g_1 \circ f_{10} \circ h$.

Explanation:

First, we insert a duplicate of each bit immediately after that bit. After that, we flip the value of all the even bits in the string. Then we rotate the bit string to the right by 1. Lastly, we remove all the odd bits in the string. Now you may notice the output string is the same as the input string. The reason why this is the case is that the bits being remove are the duplicate of the original bits.

$\therefore$ the inverse of the function is the function itself.