Exercises 1.1

1. Which of these sentences are propositions? What are the
truth values of those that are propositions?

a) Boston is the capital of Massachusetts.

It is a proposition.
T

d) 5 + 7 = 10.

It is a proportion.
F

3. What is the negation of each of these propositions?

a) Mei has an MP3 player.

Mei does not have an MP3 player

b) There is no pollution in New Jersey.

There is pollution in New Jersey.

c) 2 + 1 = 3.

$2 + 1 \neq 3$

d) The summer in Maine is hot and sunny.

The summer in Maine is not hot or sunny

9. Let p and q be the propositions "Swimming at the New Jersey shore is allowed" and "Sharks have been spotted near the shore," respectively. Express each of these compound propositions as an English sentence.

c) ¬p ∨ q

Swimming at the New Jersey Shore is not allowed or sharks have been spotted near the shore.

Exercises 1.2

1. You cannot edit a protected Wikipedia entry unless you are an administrator. Express your answer in terms of e: "You can edit a protected Wikipedia entry" and a: "You are an administrator."

$\neg e$ unless a $\rightarrow \equiv \neg \neg e \lor \neg a$ $\rightarrow \equiv a \rightarrow e$
$\equiv \neg e$ unless $\neg \neg a$ $\equiv e \lor \neg a$
$\equiv \neg e \rightarrow \neg a$ $\equiv \neg a \lor e$

3. You can graduate only if you have completed the requirements of your major and you do not owe money to the university and you do not have an overdue library book. Express your answer in terms of g: "You can graduate," m: "You owe money to the university," r: "You have completed the requirements of your major," and b: "You have an overdue library book."

p only if q $p \equiv g$, $q \equiv r \land (\neg m) \land (\neg b)$
$\equiv p \rightarrow q$

$\equiv g \rightarrow (r \land (\neg m) \land (\neg b))$

Exercises 1.3

1. Use truth tables to verify these equivalences.

a) p ∧ T ≡ p

P	T	P∧T
T	T	T
F	T	F

∴ Yes, P∧T ≡ P

3. Use truth tables to verify the commutative laws

a) $p \lor q \equiv q \lor p$.

P	q	$p \lor q$	$q \lor p$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	F

∴ Yes, $P \lor q \equiv q \lor P$

7. Use De Morgan's laws to find the negation of each of the following statements.

a) Jan is rich and happy.

    P   q

$\neg(P \land q) \equiv \neg P \lor \neg q$

∴ Jan is not rich or not happy.

9. Show that each of these conditional statements is a tautology by using truth tables.

a) $(p \land q) \rightarrow p$

P	q	$P \land q$	$(P \land q) \rightarrow P$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

∴ Yes, $(p \land q) \rightarrow p$ is a tautology.

11. Show that each conditional statement in Exercise 9 is a tautology without using truth tables.

    9. Show that each of these conditional statements is a tautology by using truth tables.

    a) $(p \land q) \rightarrow p$

$(P \land q) \rightarrow P \equiv \neg(P \land q) \lor P$

$\equiv (\neg P \lor \neg q) \lor P$ 1st De Morgan's Law

$\equiv (\neg q \lor \neg P) \lor P$ Commutative Law

$\equiv \neg q \lor (\neg P \lor P)$ Associative Law

$\equiv \neg q \lor T$ Negation Law

$\equiv T$ Domination Law

∴ Yes, $(p \land q) \rightarrow p$ is a tautology.