Topic 7 - Search Trees
Lecture 7a
Binary Search Trees

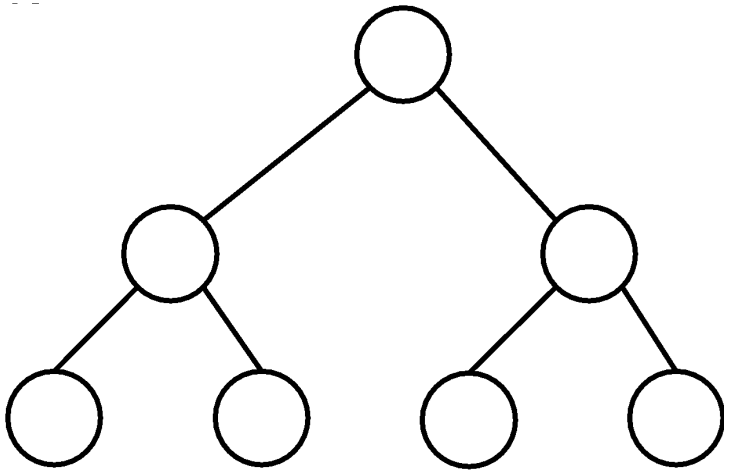
CSCI 240

Data Structures and Algorithms

Prof. Dominick Atanasio

Today

- This Class
  - Binary Search Tree (BST)
    - Definition
    - Operations in Binary Search Tree
      - Search for an Entry
      - Adding an Entry (Iterative Version)
      - Removing an Entry
    - Efficiency of Operations



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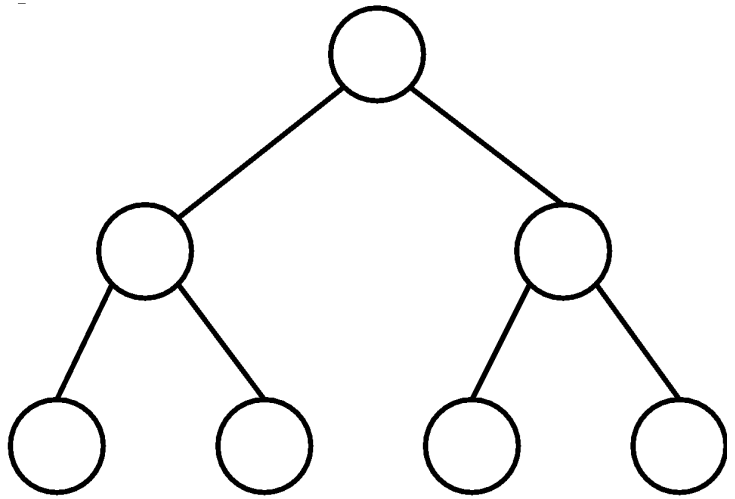
    - Both the left subtree and right subtree are BSTs.



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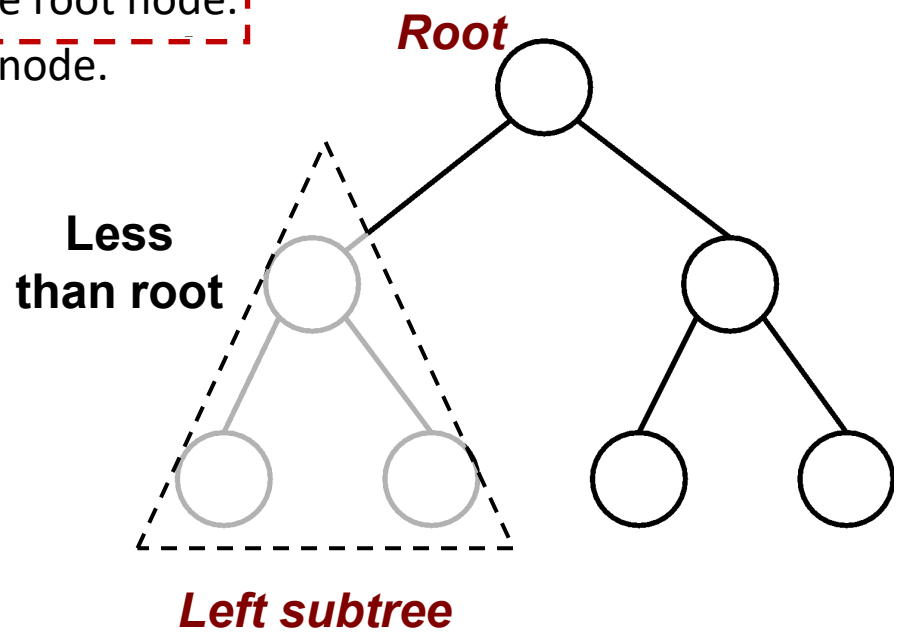

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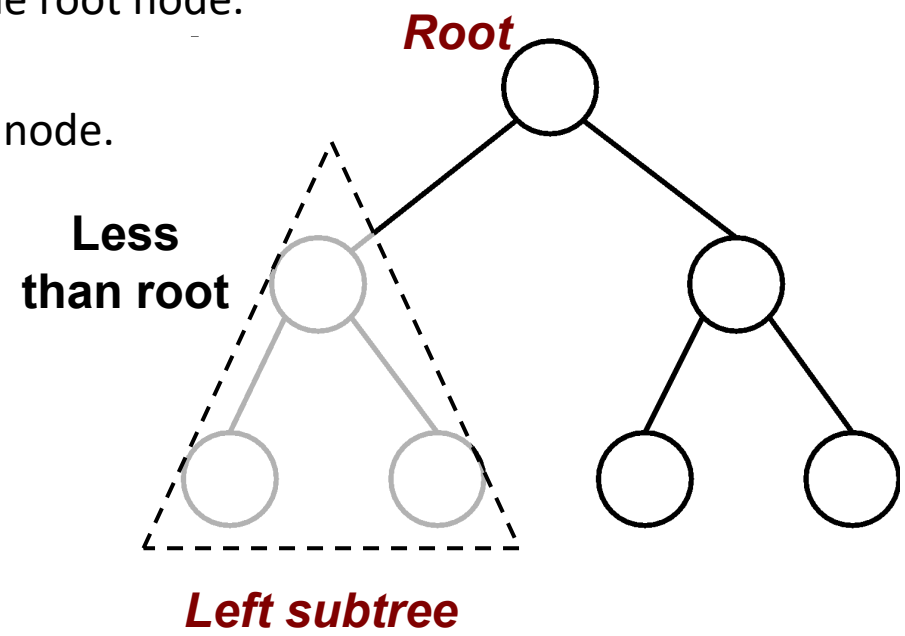

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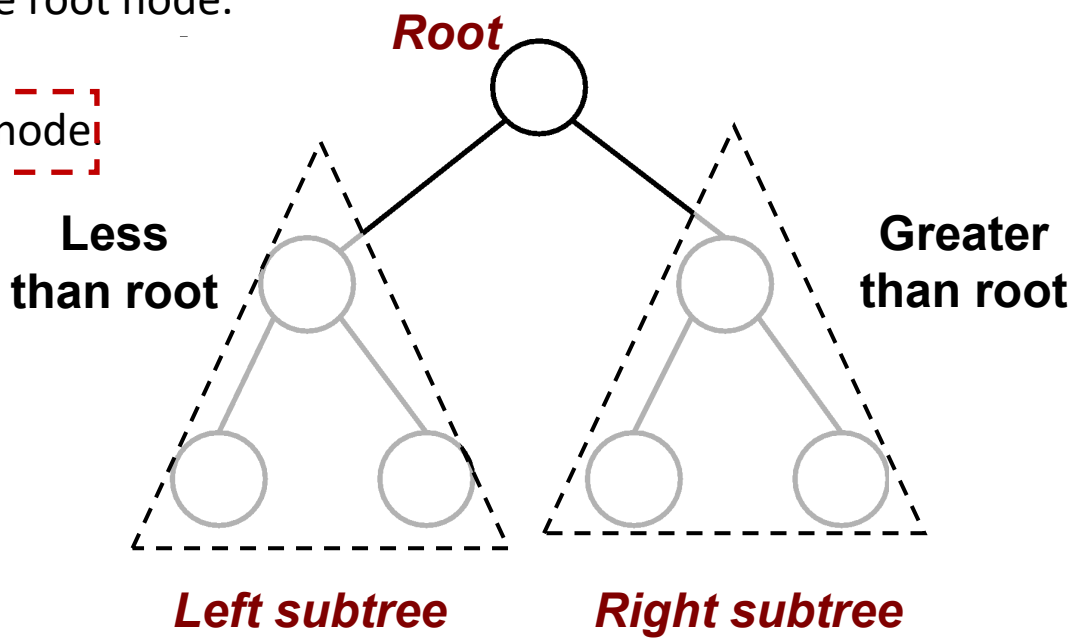

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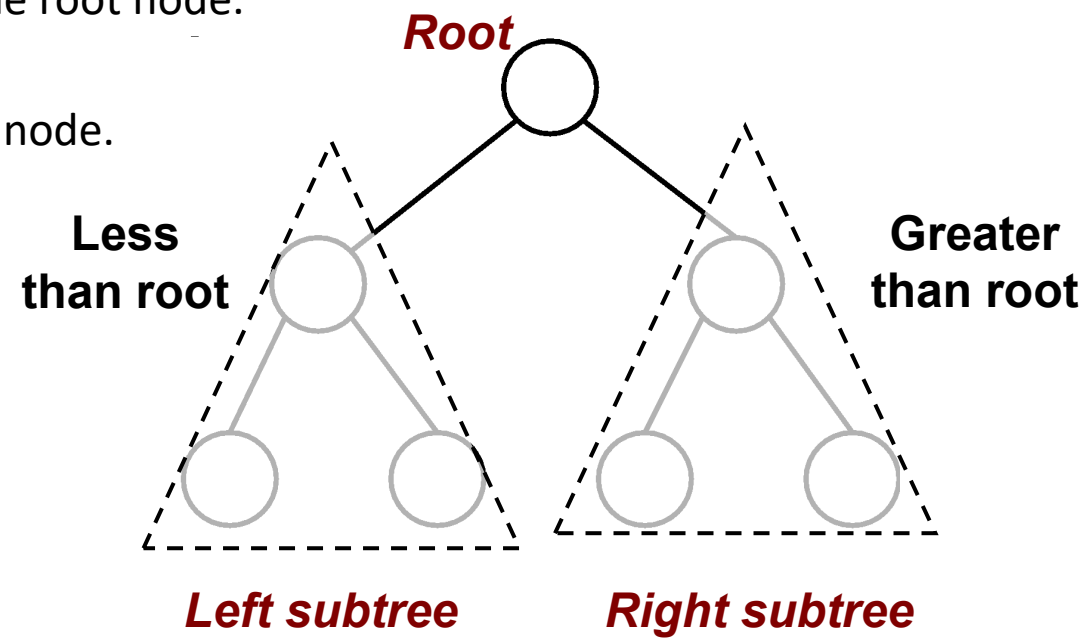

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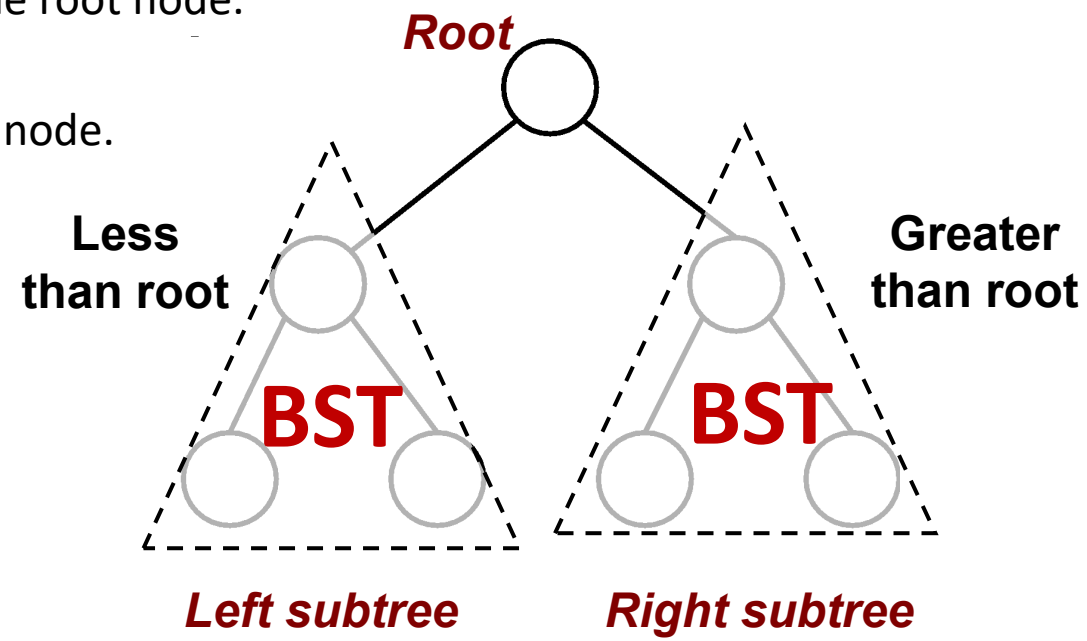

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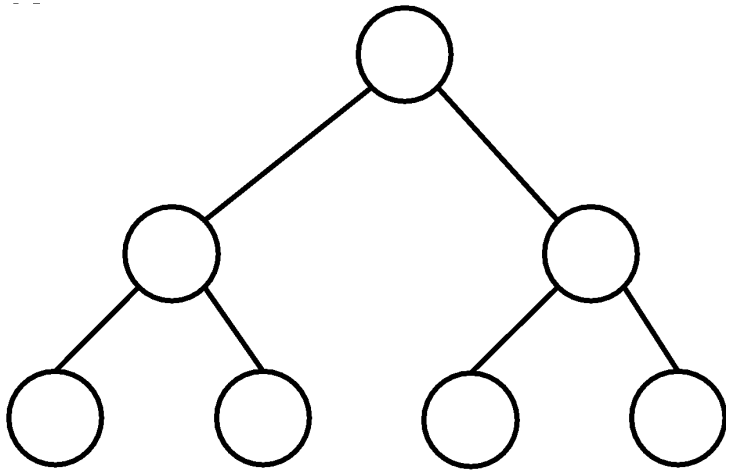

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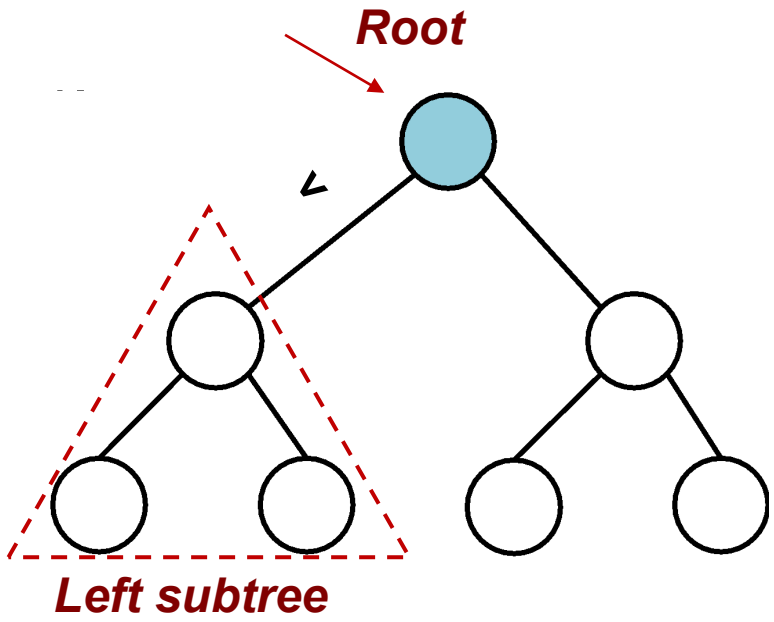

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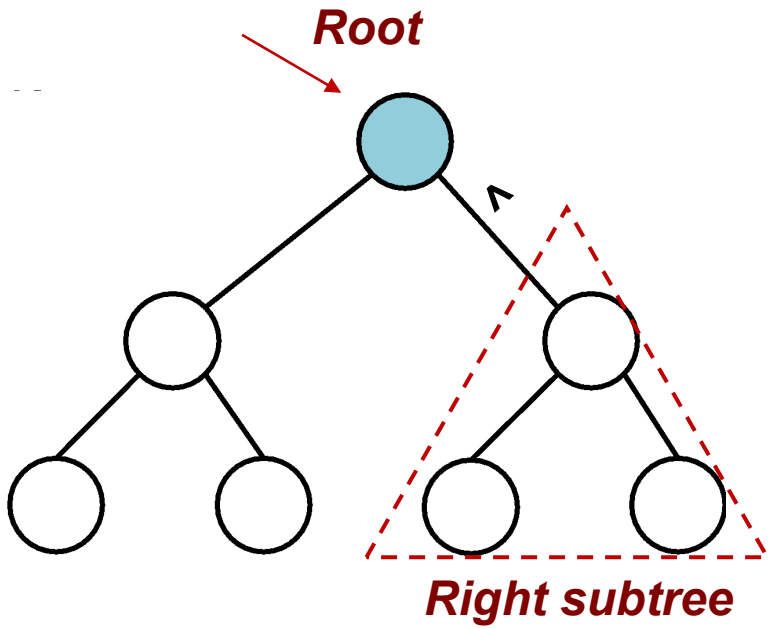

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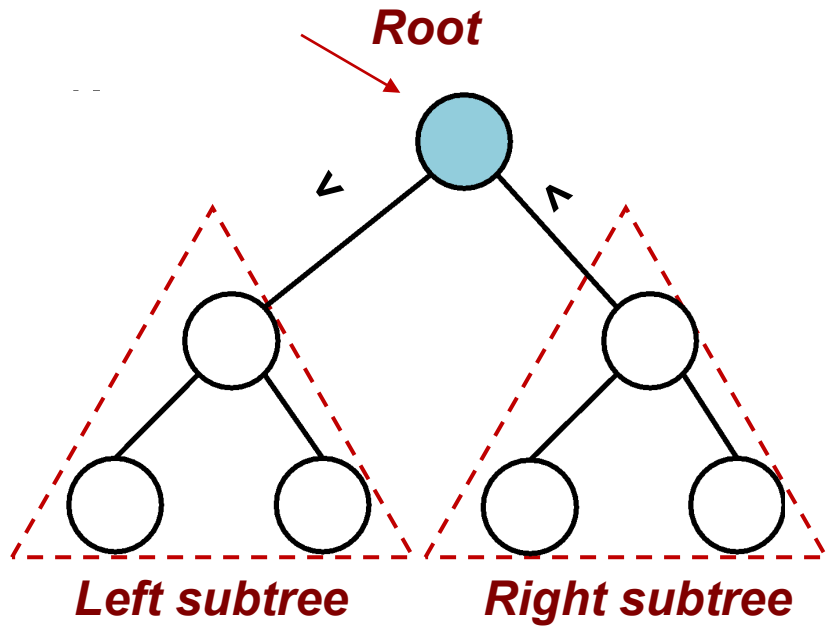

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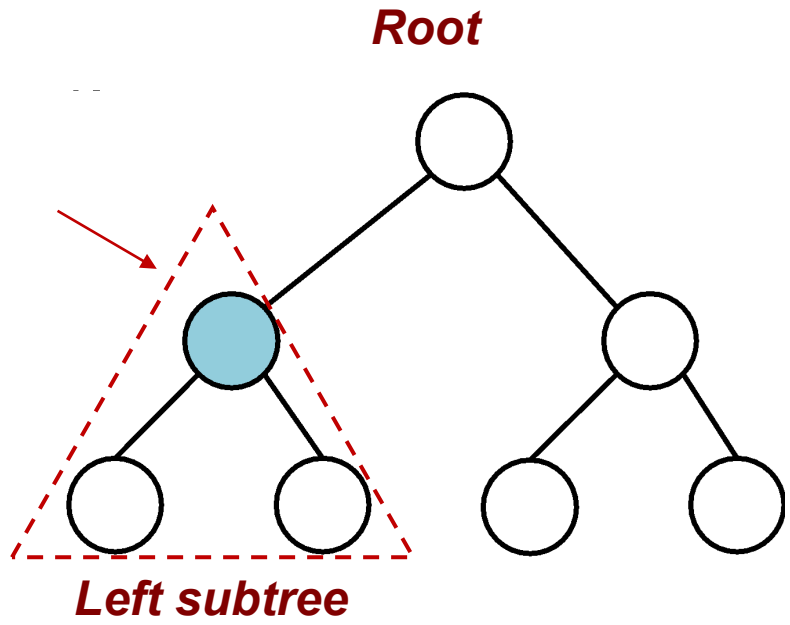

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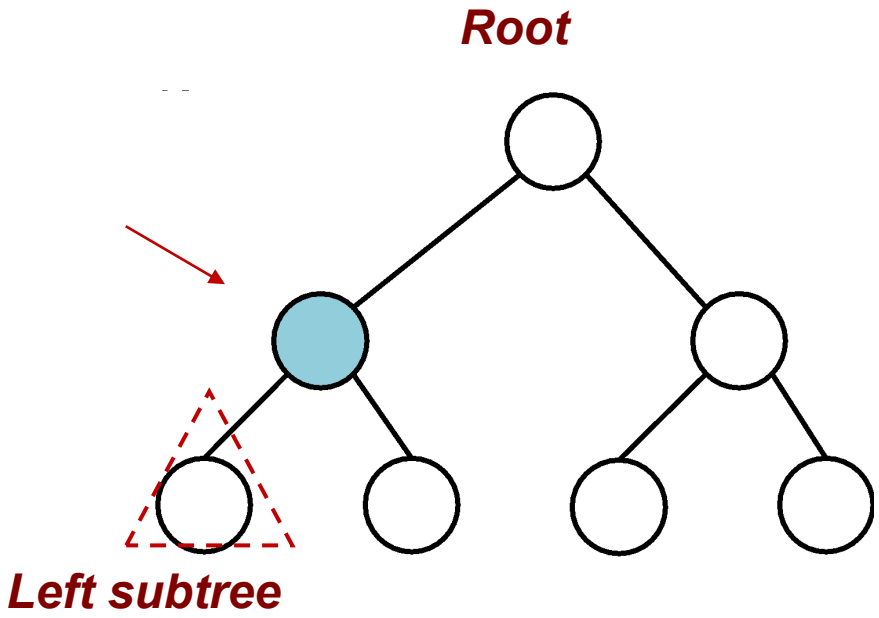

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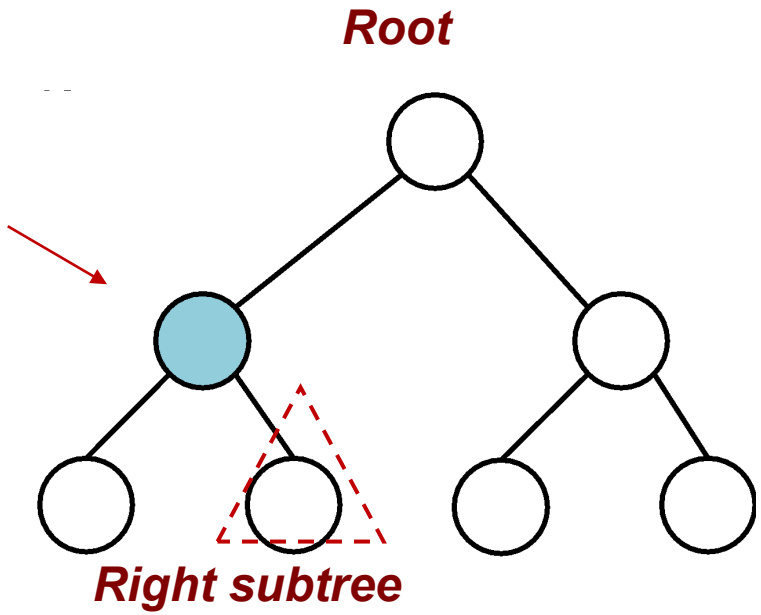

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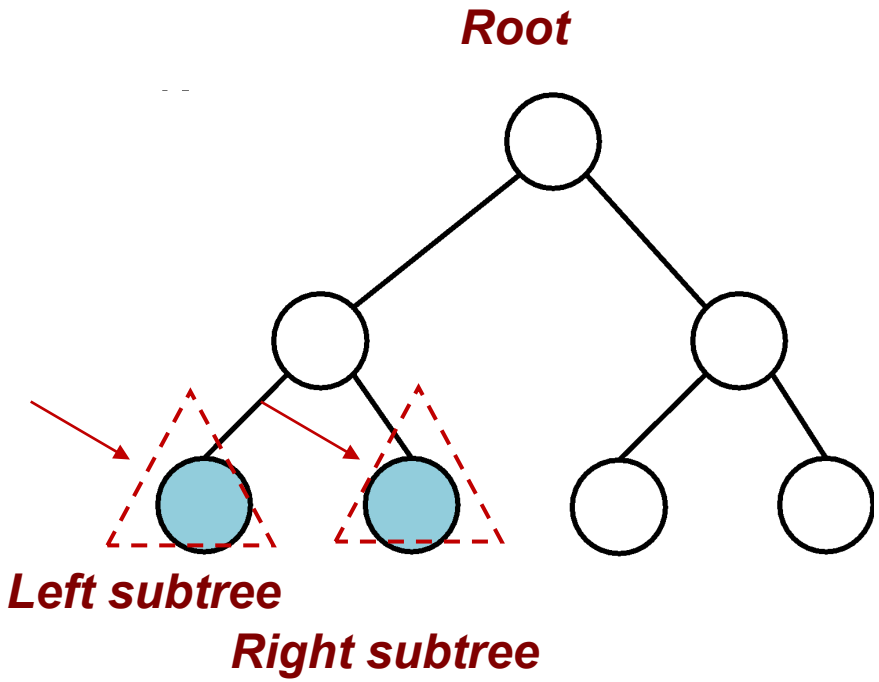

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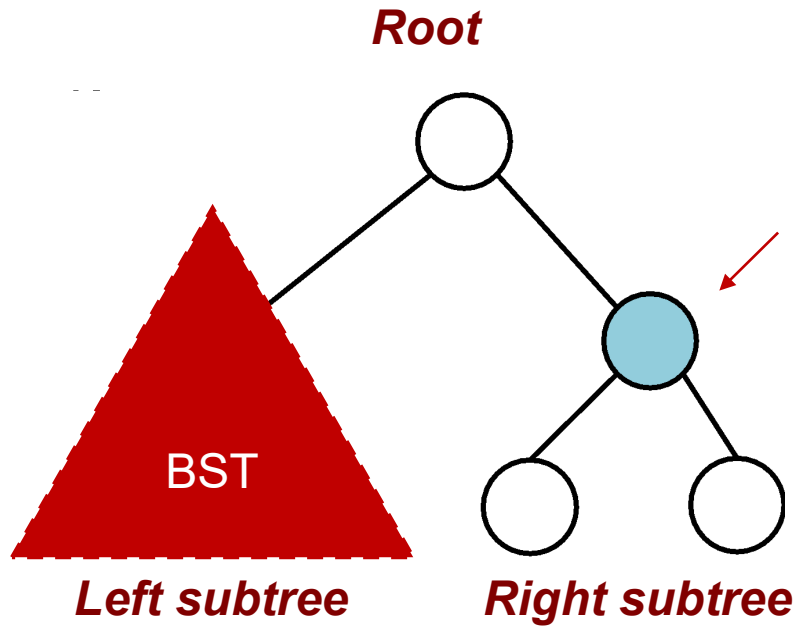

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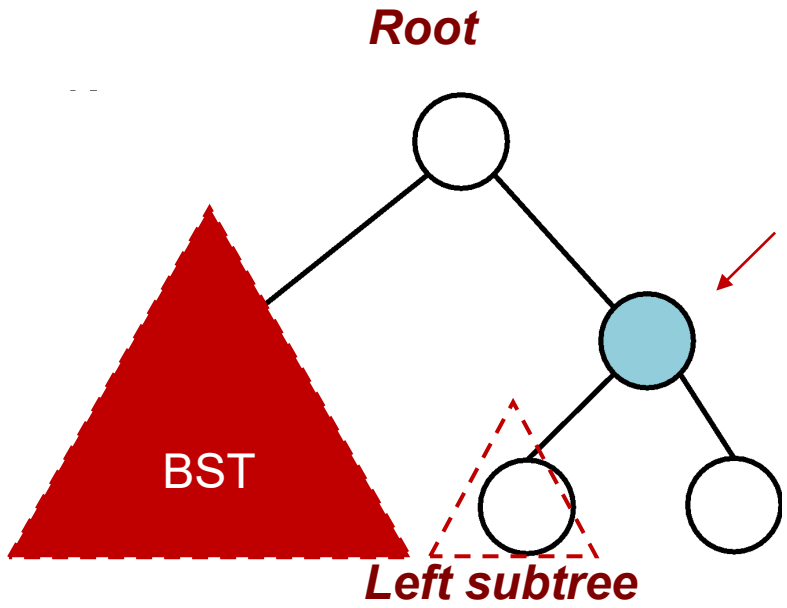

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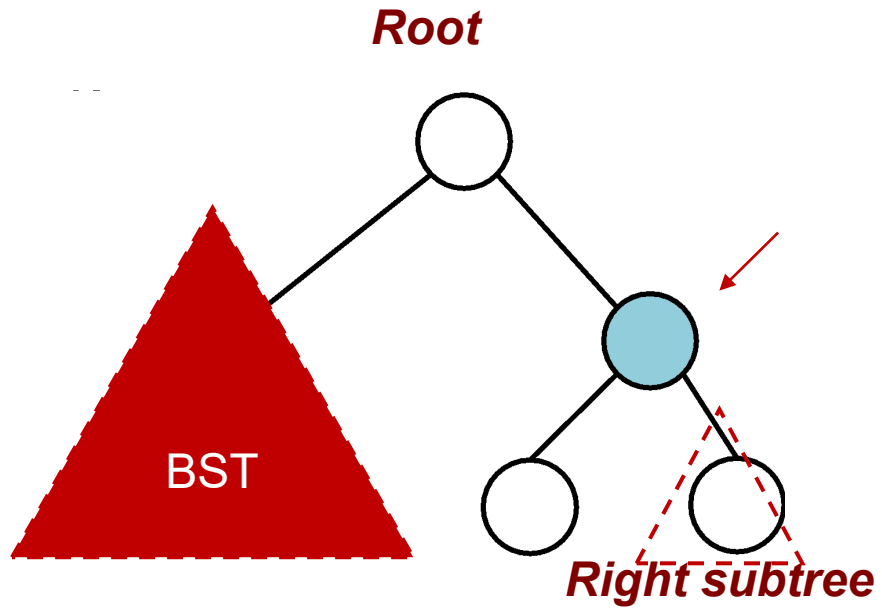

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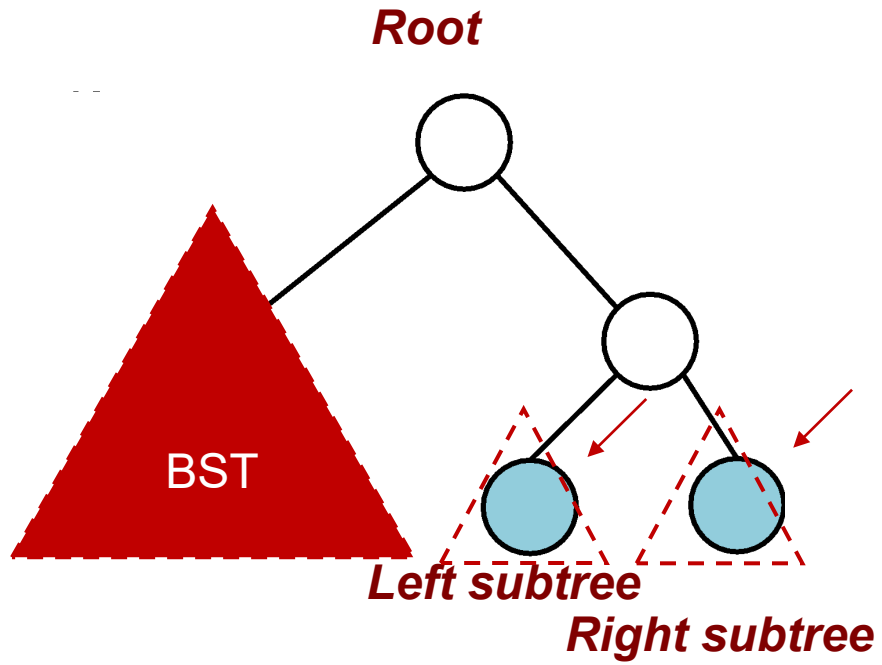

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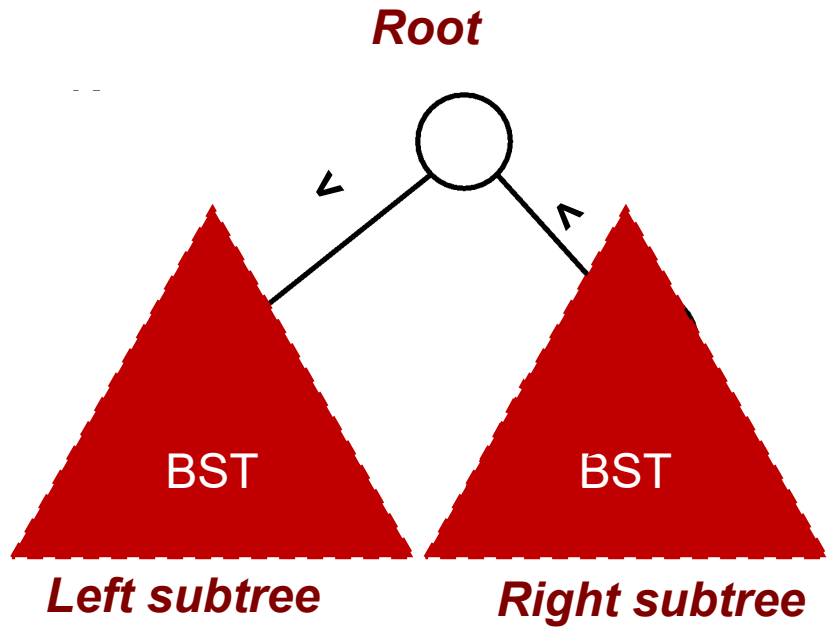

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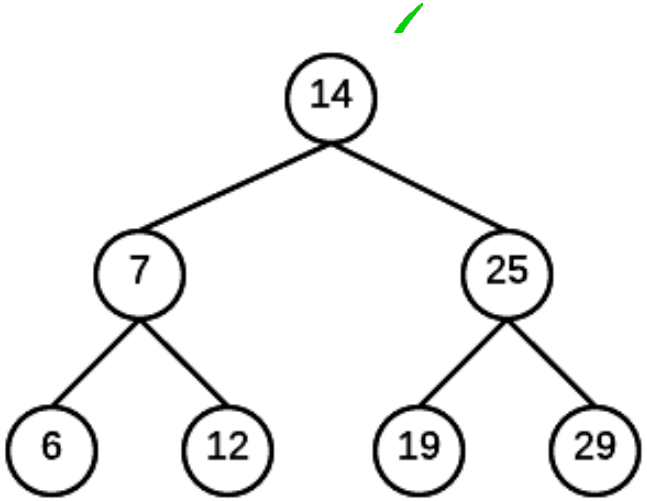

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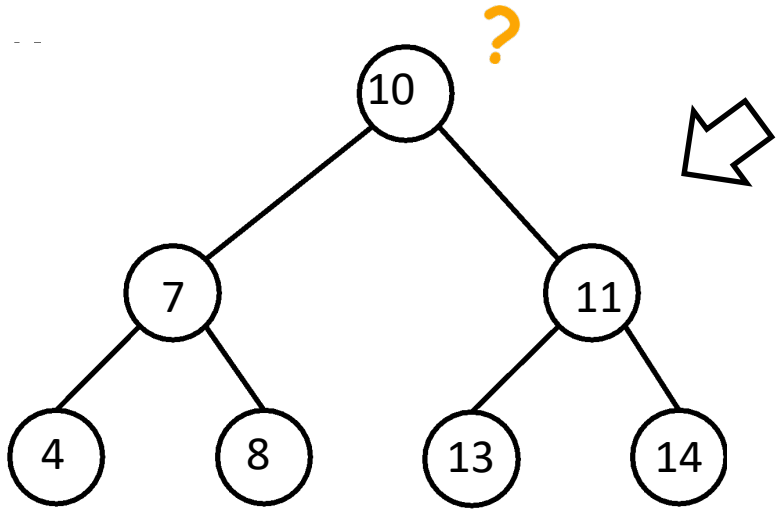

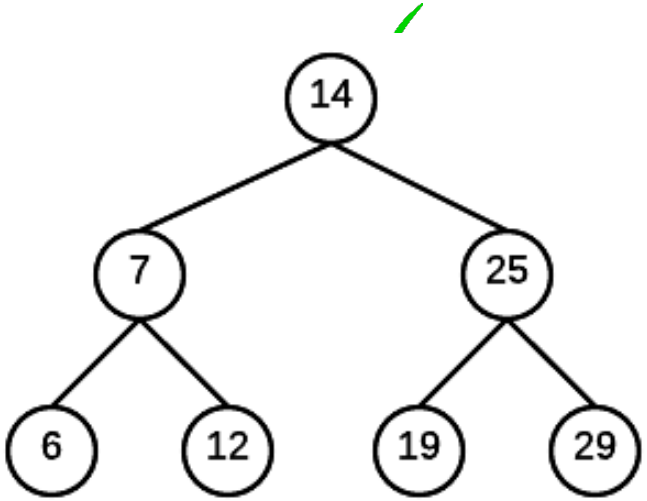

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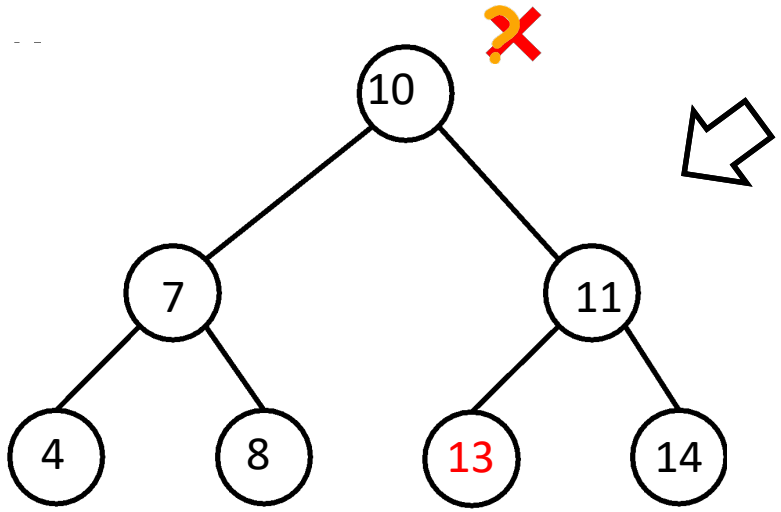
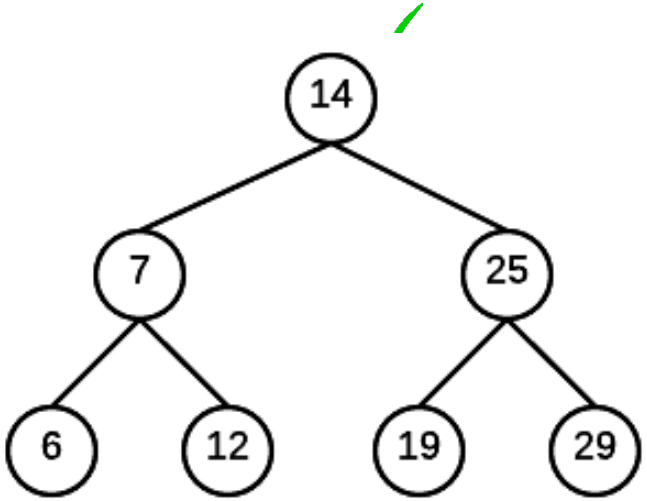




Operations in Binary Search Trees

- An interface for a search tree

- Operations
  - Searching
  - Adding an entry
  - Removing an entry
  - …

- Interface for the Search Tree

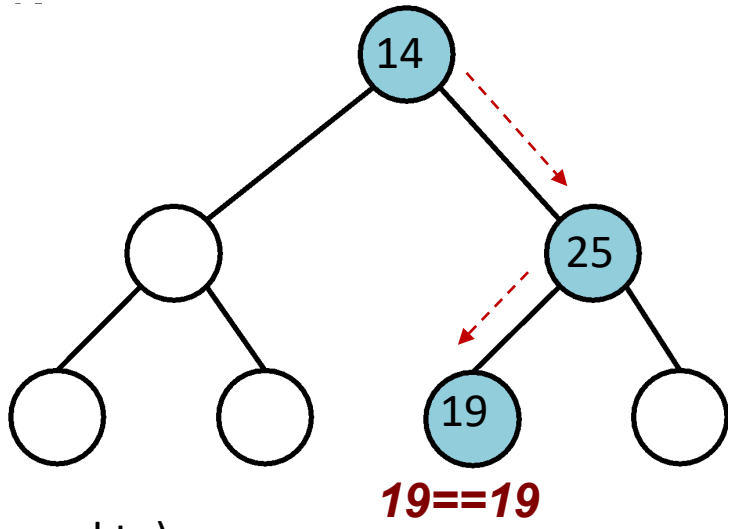
```
template<typename K, typename V>
struct SearchTree
{
    // find item in tree, if key exists
    // return const pointer to value
     virtual const V* find(const K&) = 0;

     // add Item to the tree,
     // if key already exists,
     // overwrite value with new
     virtual void put(const K&, const V&) = 0;

     // remove item from list,
     // return true of item existed
     virtual bool erase(const K&) = 0;
};
```

Searching a Binary Search Tree

- Search for 19



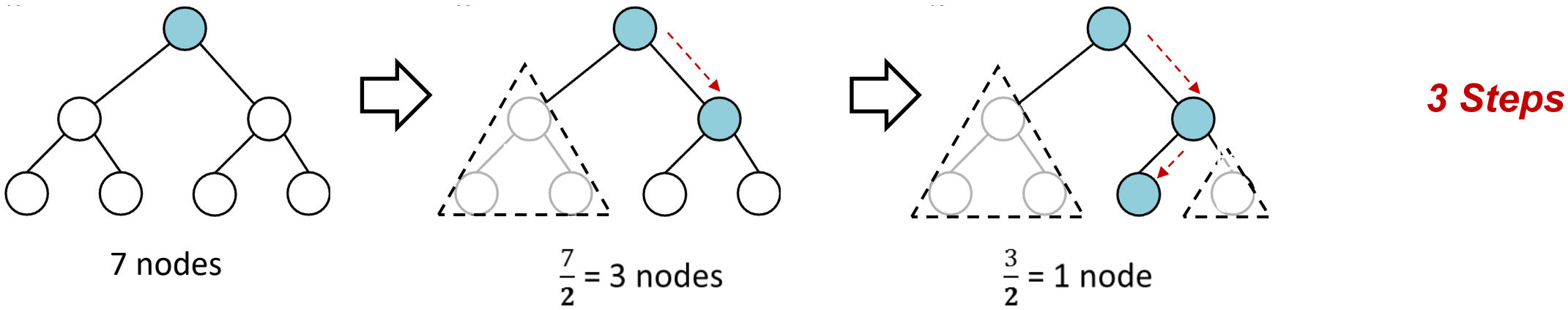
3 Steps

Left Less (or equal to)

Right Greater

Efficiency of a Search

- Search for 19



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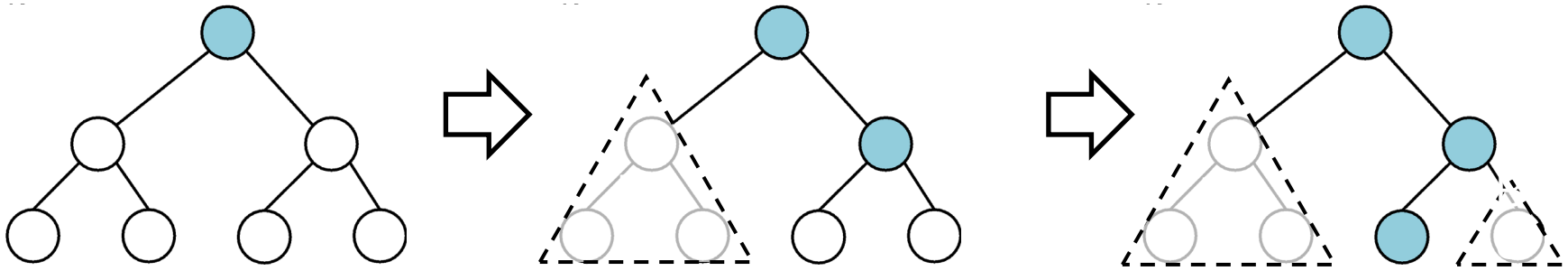


7 nodes

$\frac{7}{2}$ = 3 nodes

$\frac{3}{2}$ = 1 node

3 Steps

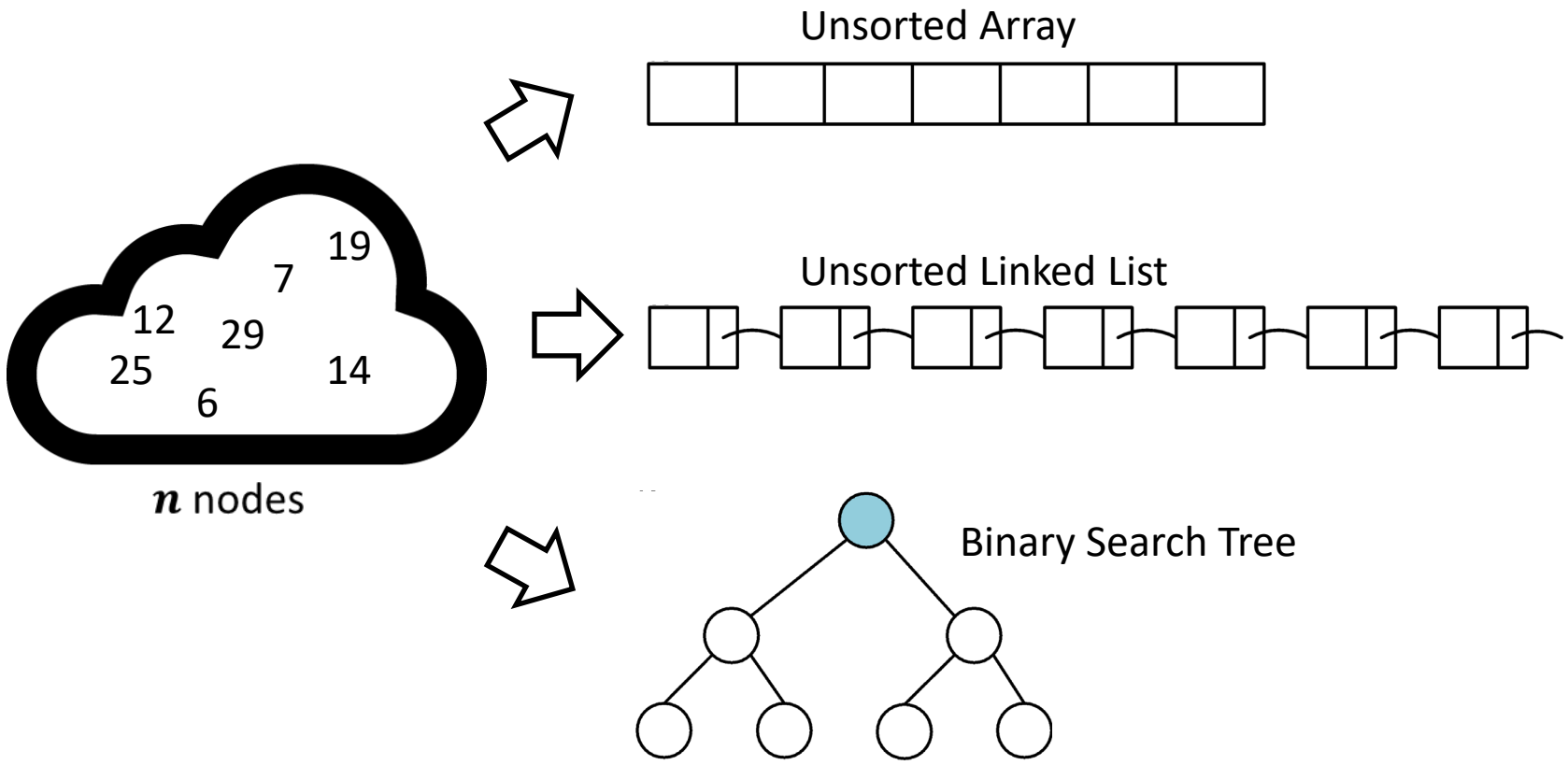
- If a BST has n nodes,

$\log_2(n)$ Steps

$\mathbf{n}$ nodes ⇨ $\frac{n}{2}$ nodes ⇨ $\frac{n}{4}$ nodes ⇨ ⋯ ⇨ 1 node

Efficiency of a Search

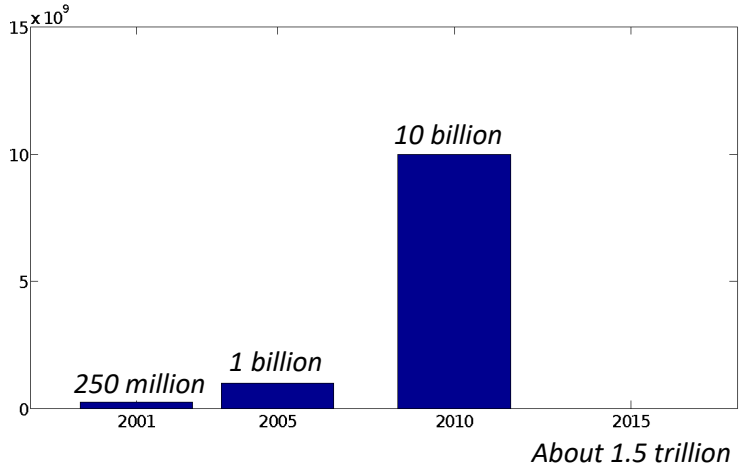
- Performance Comparisons



Example: Google image search

Google Image Search Service

Number of images indexed by Google



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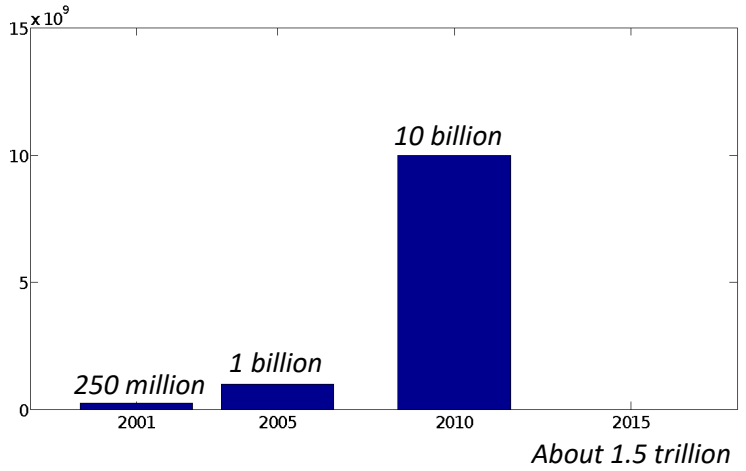

About 1.5 trillion

Search time (Binary Search Tree): *O(log₂(n))*

$$log_2(1.5 \times 10^{12}) \times 10^{-9} \approx 40.44 \times 10^{-9} seconds$$

Search time (Unsorted Array or Linked List) : *O(n)*

$$1.5 \times 10^{12} \times 10^{-9} = 1500\ seconds$$

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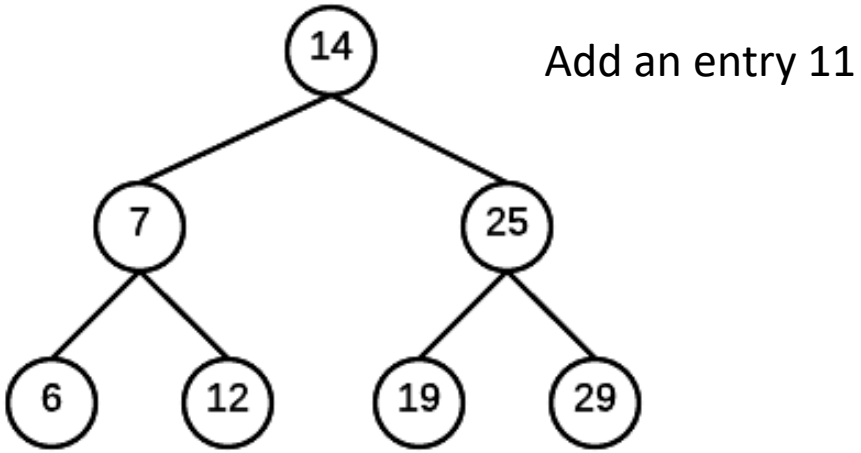
In-class Exercise

- Adding an Entry



Add an entry 11