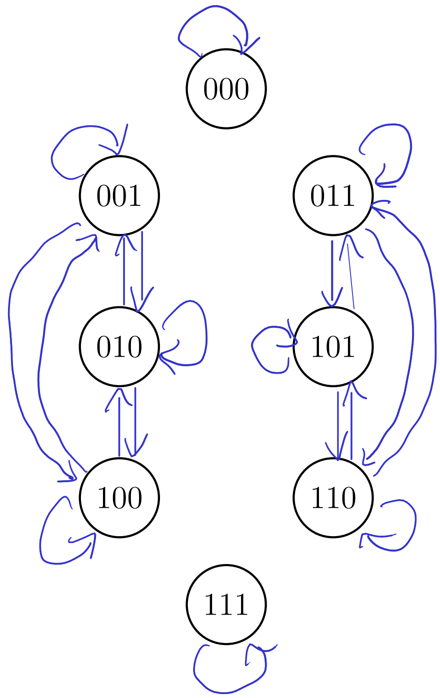
Problem 1 (R.1)

a) Please draw your arrows between the names provided. Please keep your work within this section even though there is no box surrounding it.

000 → 000
011 → 101, 110, 011
101 → 011, 110, 101
110 → 101, 011, 110
111 → 111
100 → 010, 001, 100
010 → 100, 001, 010
001 → 010, 100, 001



b)

S = {(α, α), (α, δ), (β, β), (β, γ), (δ, α), (δ, β), (δ, γ), (γ, δ)}

Problem 2 (R.2)

a) 2

b) 2

c)

A, C

d)

$\langle F, B, E, B \rangle$:

Explanation:

Since all vertices are connected by edges, it is a walk.
It is not a path because vertex B occured twice.
Since first and last vertices are not the same, it is not a closed walk, therefore it is not a circuit nor a cycle.

- [x] Walk
- [ ] Path
- [ ] Circuit
- [ ] Cycle

$\langle A, E, B, D, A \rangle$:

Explanation:

Since all vertices are connected by edges, it is a walk.
Since the first and last vertices are the same, it is a closed walk, therefore it cannot be a path.

- [x] Walk
- [ ] Path
- [x] Circuit
- [x] Cycle

$\langle A, A, E, B, D, A \rangle$:

Explanation:

Since all vertices are connected by edges, it is a walk, Since the first and last vertices are the same, it is a closed walk, therefore, it is not a path. Since the vertex A occured twice not including the last vertex, therefore, it is not a cycle.

- ☑ Walk
- ☐ Path
- ☑ Circuit
- ☐ Cycle

$\langle D \rangle$:

Explanation:

A single vertex is considered a closed walk with length 0. since the first and last vertices are the same and it has no edges repeated, it is a circuit. A single vertex is a trivial walk which is both a path and is closed, Since it has a length of 0, it cannot be a cycle because cycle requires a length of 1 or greater.

- ☑ Walk
- ☑ Path
- ☑ Circuit
- ☐ Cycle

$\langle F, E, B, E \rangle$:

Explanation:

Since (F, E) is not present in the graph, it is not a walk.
Since it is not a walk, it cannot be a path, circuit or cycle.

☐ Walk
☐ Path
☐ Circuit
☐ Cycle

Problem 3 (R.3)

a)

Explanation:

Reflexive / Anti-Reflexive : It is not reflexive, for example, (A, A) is not present in the graph. There are no self-loop present in the graph, therefore, it is anti-reflexive.

Symmetric / Anti-Symmetric : It is not symmetric, for example, (A, E) is present in the graph but (E, A) is not present in the graph. In fact, it is true for every pair of elements, therefore it is anti-symmetric.

Transitive : It is not transitive, for example, (D, C) and (C, A) are present in the graph but (D, A) is not present in the graph.

☐ Reflexive
☑ Anti-Reflexive
☐ Symmetric
☑ Anti-Symmetric
☐ Transitive

b)

- ☐ Reflexive
- ☐ Anti-Reflexive
- ☐ Symmetric
- ☑ Anti-Symmetric
- ☑ Transitive

Explanation:

Reflexive / Anti-Reflexive: It is neither reflexive or anti-reflexive, It is not reflexive because (W,W) and (Q,Q) are not present in the graph. It is not anti-reflexive because (A,A), (T,T), and (M,M) are present in the graph.

Symmetric / Anti-symmetric: It is not symmetric because (A,W) is true but (W,A) is false. It is anti-symmetric because none of the edges are bi-directional.

Transitive: It is transitive, since for all elements in the graph, they are all transitive.

c)

- ☐ Reflexive
- ☑ Anti-Reflexive
- ☑ Symmetric
- ☑ Anti-Symmetric
- ☑ Transitive

Explanation:

Reflexive / Anti-Reflexive: Since it is a empty relation, (Happy, Happy) is false, therefore, it is not reflexive. Since $E = \emptyset$, there are no elements relate to itself in the relation, therefore, it is anti-reflexive.

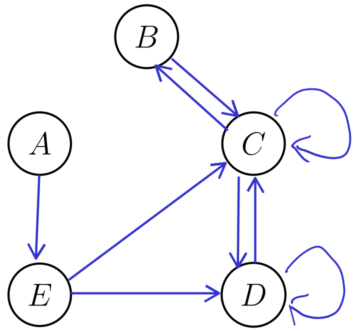
Symmetric / Anti-Reflexive: Since $E = \emptyset$, no elements is related to any other elements, it is vacuously true that it is both symmetric and anti-symmetric.

Transitive: Since $E = \emptyset$, there are no elements in the relation, therefore it is vacuously true that it is transitive.

Problem 4.1 (R.4)

~~$F \circ G$:~~

$G \circ F$



Explanation:

$G(F(D)) = (D,C) \in F,\ (C,C), (C,D) \in G,\ \therefore (D,C), (D,D) \in G \circ F$

$G(F(C)) = (C,A), (C,E) \in F,\ (A,C), (E,D), (E,B) \in G$

$\therefore (C,C), (C,D), (C,B) \in G \circ F$

$G(F(E)) = (E,C) \in F,\ (C,C), (C,D) \in G,\ \therefore (E,C), (E,D) \in G \circ F$

$G(F(A)) = (A,B) \in F,\ (B,E) \in G,\ \therefore (A,E) \in G \circ F$

$G(F(B)) = (B,A) \in F,\ (A,C) \in G,\ \therefore (B,C) \in G \circ F$

Problem 4.2 (R.4)

a)	b)	c)	d)
3	3	4	C

6^2 6^3

$A \to w \to x \to y \to z \to E$

1 2 3 4 5 6

Explanation (Not required, but putting an explanation may help you need to request a regrade):