ICS 6B Sample Midterm 2

October 28, 2023

Name: ____________________

UCI NetID : | | | | | | | | |

(**alpha-numeric; NOT your student ID**)

- Read the instructions of each question carefully.
- Place your answers in the boxed regions. Writing outside of the boxes will not be considered as part of your final answer.
- Your scratch work should stay on the exam booklet. This means no extra scratch paper– even if you have work on scratch paper, it won't be considered.
- Show your work. Your work will be considered as part of your answer.
- Write your name and UCI NetID on every page.
- The core learning objectives are marked with asterisks at the problem title.
- This test is intended to take 45 minutes.
- This is a closed-book, closed-notes test. No notes or electronic devices are allowed (aside from the provided rules of inference sheet).
- Please keep in mind the academic honesty guidelines.
- Good luck!

Seat

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P.2: Proof Framework (*Core Objective)

Statement: For any odd integer x, $3x + 2$ must be odd.

Suppose we want to use a proof by contrapositive to show that the statement above is true. What assumption would you expect to see at the beginning of this proof?

Now, suppose we want to prove the statement above by contradiction. What assumption would you expect to see at the beginning of this proof?

S.1: Set Notation and Relationships (*Core Objective)

Consider the set S, defined as follows: $S = \{\ x \in \mathbb{Z}^+ : x \text{ is even and } x < 9\ \}$

Represent S in roster notation:

Which of the following are subsets of S? Fill in the squares for all answers that apply.

- ☐ S
- ☐ $\emptyset$
- ☐ $\{6\}$
- ☐ $\{\{4\}, 6\}$
- ☐ $\{2, 4, 6\}$
- ☐ $\{0, 1\}$

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S.2: Set Operations (*Core Objective)

Consider the sets A, B, C given below:
$A = \{1, 2, 5, 11, 12\}$, $B = \{1, 3, 7, 8\}$, $C = \{1, 7, 14\}$

Find each of the following and express them in roster notation:

$A \cap C$

$A \cup B$

$A - B$

$\{6\} \times C$

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P.1: Inference Analysis

Consider the argument in the box below:

Every item on the menu is either an entree or a side (not both).
Tomato soup is an item on the menu, but it's not an entree.
You can order tomato soup.
∴ Tomato soup is a side.

Is the argument above valid or invalid? If it is valid, state the laws and rules used to arrive at the conclusion from the hypotheses. If it is invalid, provide a justification.

Show your work with the following predicates:

M(x): Item x is on the menu

E(x): Item x is an entree

O(x): You can order item x

S(x): Item x is a side

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P.3: Proof Analysis

Find and explain one mistake in the proof below.

Statement: For any integer x, $x \cdot y$ will be non-positive for some integer y we choose.

(We will prove this statement by contradiction)

Given: The product of two integers is positive if and only if both of those integers are positive, or both of those integers are negative.

Assume: For some integer x, there exists some integer y where $x \cdot y$ is positive.

Proof: Consider the case where $x = 3$. We observe that $3 \cdot y$ will be positive if and only if either 3 and y are both positive, or 3 and y are both negative. We know that 3 is positive, so y must be positive as well. However, if we have $y = 0$, it is impossible for $3 \cdot y$ to be positive, since 0 is neither positive nor negative. This is a contradiction.

Conclusion: Therefore, for any integer x, there exists some integer y where $x \cdot y$ is non-positive.

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P.4: Written Proof

Prove that for any $x, y \in \mathbb{Z}$, $2x + y$ is odd only if $y = 2a - 3$ for some $a \in \mathbb{Z}$.

In your proof, clearly label sections Given, Assume, WTP, Proof, and Conclusion.

S.3: Set Partitions

Which of the following are partitions of $\mathbb{Z} \times \{0, 1\}$? Fill in the bubble next to the best answer. For each option that isn't a partition, briefly explain why it isn't.

$\mathbb{Z} \times \{1\}, \mathbb{Z} \times \{0\}$

- ○ Partition of $\mathbb{Z} \times \{0, 1\}$
- ○ NOT a partition of $\mathbb{Z} \times \{0, 1\}$

$\mathbb{Z}^- \times \{1\}, \mathbb{Z}^+ \times \{0\}, \mathbb{Z}^- \times \{0\}, \mathbb{Z}^+ \times \{1\}$

- ○ Partition of $\mathbb{Z} \times \{0, 1\}$
- ○ NOT a partition of $\mathbb{Z} \times \{0, 1\}$

$\mathbb{Z}^- \times \{0, 1\}, \mathbb{Z}^+ \times \{0, 1\}, \{(0, 0), (0, 1), (1, 1)\}$

- ○ Partition of $\mathbb{Z} \times \{0, 1\}$
- ○ NOT a partition of $\mathbb{Z} \times \{0, 1\}$