6.1 The Basics of Counting

Ch 6.1 Q 1,3,7

1. There are 18 mathematics majors and 325 computer science majors at a college.

a) In how many ways can two representatives be picked so that one is a mathematics major and the other is a computer science major?

b) In how many ways can one representative be picked who is either a mathematics major or a computer science major?

a) $18 \cdot 325 = 5850$ ways to pick the two representatives.

b) $18 + 325 = 343$ ways to pick one representative.

3. A multiple-choice test contains 10 questions. There are four possible answers for each question.

a) In how many ways can a student answer the questions on the test if the student answers every question?

b) In how many ways can a student answer the questions on the test if the student can leave answers blank?

a) $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 4^{10} = 1{,}048{,}576$ ways

b) $5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 = 5^{10} = 9{,}765{,}625$ ways

7. How many different three-letter initials can people have?

$26 \cdot 26 \cdot 26 = 26^3 = 17{,}576$ possible three-letter initials

6.2 The Pigeonhole Principle

Ch.6.2 Q# 1,3

1. Show that in any set of six classes, each meeting regularly once a week on a particular day of the week, there must be two that meet on the same day, assuming that no classes are held on weekends.

NO weekend => MONDAY through FRIDAY

By the pigeonhole principle,

at least one day must contain at least two classes.

3. A drawer contains a dozen brown socks and a dozen black socks, all unmatched. A man takes socks out at random in the dark.

a) How many socks must he take out to be sure that he has at least two socks of the same color?

b) How many socks must he take out to be sure that he has at least two black socks?

a) There are 2 colors => these are the pigeonholes.

By the pigeonhole principle,

it should be 3 socks.

b) He need to take out 14 socks in order to ensure at least two black socks.

6.3 Permutations and Combinations

Ch. 6.3 Q# 1. 5(a). 9

1. List all the permutations of $\{a, b, c\}$.

$3! = 3 \cdot 2 \cdot 1 = 6$

a, b, c
a, c, b
b, a, c
b, c, a
c, a, b
c, b, a

5. Find the value of each of these quantities.

a) $P(6, 3)$

$P(n, r) = n(n-1)(n-2)\ldots(n-r+1)$

$P(n, r) = \frac{n!}{(n-r)!}$

$P(6, 3) = 6 \cdot 5 \cdot 4 = 120$

9. How many possibilities are there for the win, place, and show (first, second, and third) positions in a horse race with 12 horses if all orders of finish are possible?

$P(12, 3) = 12 \cdot 11 \cdot 10 = 1320$ possibilities.

Ch.6.4 Q# 5, 9, 13

5. How many terms are there in the expansion of $(x+y)^{100}$ after like terms are collected?

There is one term for each i from 0 to 100.
So there are 101 terms.

9. What is the coefficient of $x^{101}y^{99}$ in the expansion of $(2x-3y)^{200}$?

By binomial theorem,

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

$$(2x-3y)^{200} = (2x+(-3y))^{200}$$
$$= \binom{200}{99}(2x)^{101}(-3y)^{99}$$

Therefore, the coefficient is $\binom{200}{99} 2^{101}(-3)^{99}$
$$= -2^{101}3^{99}C(200,99)$$

13. What is the row of Pascal's triangle containing the binomial coefficients $\binom{9}{k}$, $0 \le k \le 9$?

Binomial expansion

$$(x+y)^n = ({}_nC_0)x^n + ({}_nC_1)x^{n-1}y + ({}_nC_2)x^{n-2}y^2 + \dots + ({}_nC_n)y^n$$

$n = 9$

$${}_nC_r = \frac{n!}{(n-r)!\,r!}$$

${}_9C_0$	${}_9C_1$	${}_9C_2$	${}_9C_3$	${}_9C_4$	${}_9C_5$	${}_9C_6$	${}_9C_7$	${}_9C_8$	${}_9C_9$
1	9	36	84	126	126	84	36	9	1