Exercise 7.1

1. What is the probability that a card selected at random from a standard deck of 52 cards is an ace?

p(Ace) = (1 + 1 + 1 + 1)/52 = 4/52 = 1/13 = 0.0769

3. What is the probability that a randomly selected integer chosen from the first 100 positive integers is odd?

There are 50 odd positive integers in the first 100 positive integers.

p(odd positive integers) = 50/100 = ½ = 0.5

24. Find the probability of winning a lottery by selecting the correct six integers, where the order in which these integers are selected does not matter, from the positive integers not exceeding

a) 30.

1/C(n, r) = 1/C(30, 6) = 1/(30!/24!6!) = (6 * 5 * 4 * 3 * 2 * 1) /(30 * 29 * 28 * 27 * 26 * 25) = 0.0000016841

33. What is the probability that Abby, Barry, and Sylvia win the first, second, and third prizes, respectively, in a drawing if 200 people enter a contest and

a) no one can win more than one prize.

No one can win more than one once = draw without replacement

1 / (200 * 199 * 198) = 1 / 7880400 = 0.00000012697

b) winning more than one prize is allowed.

Can win more than one prize = draw with replacement

1 / (200 * 200 * 200) = 1 / 8000000 = 0.000000125

Exercise 7.2

1. What probability should be assigned to the outcome of heads when a biased coin is tossed, if heads is three times as likely to come up as tails? What probability should be assigned to the outcome of tails?

p(H) = 3p(T)

p(H) + p(T) = 1

3p(T) + p(t) = 1

4p(T) = 1

p(T) = 1 / 4 = 0.25

∴ the probability of getting a head is 0.75, the probability of getting a tail is 0.25.

3. Find the probability of each outcome when a biased die is rolled, if rolling a 2 or rolling a 4 is three times as likely as rolling each of the other four numbers on the die and it is equally likely to roll a 2 or a 4.

Let k be the probability of rolling 2 or 4 and (1 – k) is the probability of rolling other numbers other 2 or 4.

$k = 3(1 - k)$

$k = 3 - 3k$

$4k = 3$

$K = 3/4$

$p(2) = p(4) = (3 / 4) / 2 = 3/8$

$p(1) = p(3) = p(5) = p(6) = (1 - 3/4) / 4 = (1/4) / 4 = 1/16$

23. What is the conditional probability that exactly four heads appear when a fair coin is flipped five times, given that the first flip came up heads?

Exactly four heads appear when the first flip is heads:

HHHHT

HHHTH

HHTHH

HTHHH

Total combinations of Heads and Tails = 16

$\therefore$ 4 / 16 = 1 / 4 = 0.25