# CSCI 190 -- Homework 3



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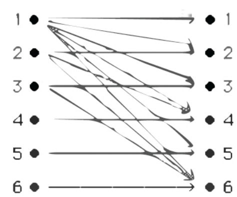
## Exercises from Book:

- Chapter 9
  - 9.1 –2, 4, 10

**2. a)** $(1,1)$, $(1,2)$, $(1,3)$, $(1,4)$, $(1,5)$, $(1,6)$, $(2,2)$, $(2,4)$, $(2,6)$, $(3,3)$, $(3,6)$, $(4,4)$, $(5,5)$, $(6,6)$

**b)** We draw a line from $a$ to $b$ whenever $a$ divides $b$, using separate sets of points; an alternate form of this graph would have just one set of points.




**c)** We put an $\times$ in the $i^{\text{th}}$ row and $j^{\text{th}}$ column if and only if $i$ divides $j$.

| R | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | × | × | × | × | × | × |
| 2 | | × | | × | | × |
| 3 | | | × | | | × |
| 4 | | | | × | | |
| 5 | | | | | × | |
| 6 | | | | | | × |

**4. a)** Being taller than is not reflexive (I am not taller than myself), nor symmetric (I am taller than my daughter, but she is not taller than I). It is antisymmetric (vacuously, since we never have $A$ taller than $B$, and $B$ taller than $A$, even if $A = B$). It is clearly transitive.

**b)** This is clearly reflexive, symmetric, and transitive (it is an equivalence relation—see Section 9.5). It is not antisymmetric, since twins, for example, are unequal people born on the same day.

**c)** This has exactly the same answers as part **(b)**, since having the same first name is just like having the same birthday.

**d)** This is clearly reflexive and symmetric. It is not antisymmetric, since my cousin and I have a common grandparent, and I and my cousin have a common grandparent, but I am not equal to my cousin. This relation is not transitive. My cousin and I have a common grandparent; my cousin and her cousin on the other side of her family have a common grandparent. My cousin's cousin and I do not have a common grandparent.

**10.** We give the simplest example in each case.

**a)** the empty set on $\{a\}$ (vacuously symmetric and antisymmetric)

**b)** $\{(a,b), (b,a), (a,c)\}$ on $\{a,b,c\}$

  - 9.3 – 2, 4, 8, 24

**2.** In each case we use a $4 \times 4$ matrix, putting a 1 in position $(i,j)$ if the pair $(i,j)$ is in the relation and a 0 in position $(i,j)$ if the pair $(i,j)$ is not in the relation.

**a)** $\begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ **b)** $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$ **c)** $\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$ **d)** $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

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## Exercises from Book:

- 9.3 – 4, 8, 24

**4. a)** Since the $(1,1)^{\text{th}}$ entry is a 1, $(1,1)$ is in the relation. Since $(1,3)^{\text{th}}$ entry is a 0, $(1,3)$ is not in the relation. Continuing in this manner, we see that the relation contains $(1,1)$, $(1,2)$, $(1,4)$, $(2,1)$, $(2,3)$, $(3,2)$, $(3,3)$, $(3,4)$, $(4,1)$, $(4,3)$, and $(4,4)$.

**b)** $(1,1)$, $(1,2)$, $(1,3)$, $(2,2)$, $(3,3)$, $(3,4)$, $(4,1)$, and $(1,4)$

**c)** $(1,2)$, $(1,4)$, $(2,1)$, $(2,3)$, $(3,2)$, $(3,4)$, $(4,1)$, and $(4,3)$

**8.** For reflexivity we want all 1's on the main diagonal; for irreflexivity we want all 0's on the main diagonal; for symmetry, we want the matrix to be symmetric about the main diagonal (equivalently, the matrix equals its transpose); for antisymmetry we want there never to be two 1's symmetrically placed about the main diagonal (equivalently, the meet of the matrix and its transpose has no 1's off the main diagonal); and for transitivity we want the Boolean square of the matrix (the Boolean product of the matrix and itself) to be "less than or equal to" the original matrix in the sense that there is a 1 in the original matrix at every location where there is a 1 in the Boolean square.

**a)** Since some 1's and some 0's on the main diagonal, this relation is neither reflexive nor irreflexive. Since the matrix is symmetric, the relation is symmetric. The relation is not antisymmetric—look at positions $(1,2)$ and $(2,1)$. Finally, the relation is not transitive; for example, the 1's in positions $(1,2)$ and $(2,3)$ would require a 1 in position $(1,3)$ if the relation were to be transitive.

**b)** Since there are all 1's on the main diagonal, this relation is reflexive and not irreflexive. Since the matrix is not symmetric, the relation is not symmetric (look at positions $(1,2)$ and $(2,1)$, for example). The relation is antisymmetric since there are never two 1's symmetrically placed with respect to the main diagonal. Finally, the Boolean square of this matrix is not itself (look at position $(1,4)$ in the square), so the relation is not transitive.

**24.** We list all the pairs $(x,y)$ for which there is an edge from $x$ to $y$ in the directed graph:
$\{(a,a),(a,c),(b,a),(b,b),(b,c),(c,c)\}$.

- 9.5 – 2

**2. a)** This is an equivalence relation by Exercise 9 ($f(x)$ is $x$'s age).

**b)** This is an equivalence relation by Exercise 9 ($f(x)$ is $x$'s parents).

**c)** This is not an equivalence relation, since it need not be transitive. (We assume that biological parentage is at issue here, so it is possible for $A$ to be the child of $W$ and $X$, $B$ to be the child of $X$ and $Y$, and $C$ to be the child of $Y$ and $Z$. Then $A$ is related to $B$, and $B$ is related to $C$, but $A$ is not related to $C$.)

**d)** This is not an equivalence relation since it is clearly not transitive.

**e)** Again, just as in part **(c)**, this is not transitive.

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## Exercises from Book:

- Chapter 10
  - 10.1 – 4, 6, 8, 18, 32

**4.** This is a multigraph; the edges are undirected, and there are no loops, but there are parallel edges.

**6.** This is a multigraph; the edges are undirected, and there are no loops, but there are parallel edges.

**8.** This is a directed multigraph; the edges are directed, and there are parallel edges.

**18.** Fred influences Brian, since there is an edge from Fred to Brian. Yvonne and Deborah influence Fred, since there are edges from these vertices to Fred.

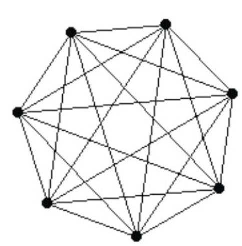
**32.** The model says that the statements for which there are edges to $S_6$ must be executed before $S_6$, namely the statements $S_1$, $S_2$, $S_3$, and $S_4$.

  - 10.2 – 6, 14, 20, 58

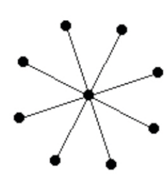
**6.** Model this problem by letting the vertices of a graph be the people at the party, with an edge between two people if they shake hands. Then the degree of each vertex is the number of people the person that vertex represents shakes hands with. By Theorem 1 the sum of the degrees is even (it is $2e$).

**14.** Since there is an edge from a person to each of the other actors with whom that person has appeared in a movie, the degree of $v$ is the number of other actors with whom that person has appeared. The neighborhood of $v$ is the *set* of actors with whom $v$ as appeared. An isolated vertex would be a person who has appeared only in movies in which he or she was the only actor, and a pendant vertex would be a person who has appeared with only one other actor in any movie (it is doubtful that there are many, if any, isolated or pendant vertices).

**20. a)** This graph has 7 vertices, with an edge joining each pair of distinct vertices.



**b)** This graph is the complete bipartite graph on parts of size 1 and 8; we have put the part of size 1 in the middle.



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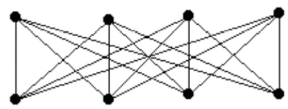
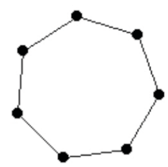
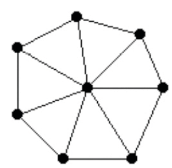
**c)** This is the complete bipartite graph with 4 vertices in each part.



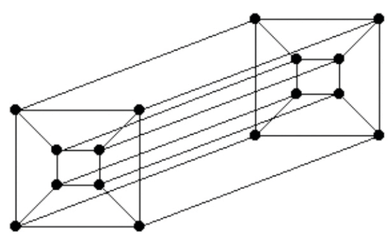
**d)** This is the 7-cycle.



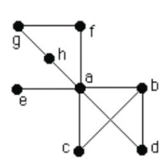
**e)** The 7-wheel is the 7-cycle with an extra vertex joined to the other 7 vertices. Warning: Some texts call this $W_8$, to have the consistent notation that the subscript in the name of a graph should be the number of vertices in that graph.



**f)** We take two copies of $Q_3$ and join corresponding vertices.



**58.** The union is shown here. The only common vertex is $a$, so we have reoriented the drawing so that the pieces will not overlap.



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## Exercises from Book:

- 10.3 – 2, 4, 6, 8, 12, 14, 26, 38, 42

**2.** This is similar to Exercise 1. The list is as follows.

| Vertex | Adjacent vertices |
|---|---|
| $a$ | $b, d$ |
| $b$ | $a, d, e$ |
| $c$ | $d, e$ |
| $d$ | $a, b, c$ |
| $e$ | $b, c$ |

**4.** This is similar to Exercise 3. The list is as follows.

| Initial vertex | Terminal vertices |
|---|---|
| $a$ | $b, d$ |
| $b$ | $a, c, d, e$ |
| $c$ | $b, c$ |
| $d$ | $a, e$ |
| $e$ | $c, e$ |

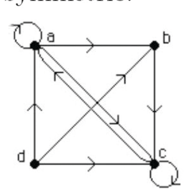
**6.** This is similar to Exercise 5. The vertices are assumed to be listed in alphabetical order.

$$\begin{bmatrix} 0 & 1 & 0 & 1 & \text{[redacted]} \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

**8.** This is similar to Exercise 7.

$$\begin{bmatrix} 0 & 1 & 0 & 1 & \text{[redacted]} \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

**12.** This graph is directed, since the matrix is not symmetric.




**14.** This is similar to Exercise 13.

$$\begin{bmatrix} 0 & 3 & 0 & 1 \\ 3 & 0 & 1 & 0 \\ 0 & 1 & 0 & 3 \\ 1 & 0 & 3 & 0 \end{bmatrix}$$

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## Exercises from Book:

- 10.3 – 26, 38, 42

**26.** Each column represents an edge; the two 1's in the column are in the rows for the endpoints of the edge.

Exercise 1 $\begin{bmatrix} 1 & 1 & 1 & 0 & \bullet \\ 1 & 0 & 0 & 1 & \bullet \\ \bullet & 1 & 0 & 0 & 1 \\ \bullet & 0 & 1 & 1 & 1 \end{bmatrix}$ Exercise 2 $\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & \bullet \\ 1 & 0 & 1 & 1 & 0 & \bullet \\ \bullet & 0 & 0 & 0 & 1 & 1 \\ \bullet & 1 & 1 & 0 & 1 & \bullet \\ \bullet & 0 & 0 & 1 & 0 & 1 \end{bmatrix}$

**38.** These two graphs are isomorphic. Each consists of a $K_4$ with a fifth vertex adjacent to two of the vertices in the $K_4$. Many isomorphisms are possible. One is $f(u_1) = v_1$, $f(u_2) = v_3$, $f(u_3) = v_2$, $f(u_4) = v_5$, and $f(u_5) = v_4$.

**42.** These graphs are not isomorphic. In the first graph the vertices of degree 4 are adjacent. This is not true of the second graph.

- 10.4 – 2, 8, 14

**2. a)** This is a path of length 4, but it is not a circuit, since it ends at a vertex other than the one at which it began. It is simple, since no edges are repeated.
**b)** This is a path of length 4, which is a circuit. It is not simple, since it uses an edge more than once.
**c)** This is not a path, since there is no edge from $d$ to $b$.
**d)** This is not a path, since there is no edge from $b$ to $d$.

**8.** A connected component of a collaboration graph represent a maximal set of people with the property that for any two of them, we can find a string of joint works that takes us from one to the other. The word "maximal" here implies that nobody else can be added to this set of people without destroying this property.

**14. a)** The cycle $baeb$ guarantees that these three vertices are in one strongly connected component. Since there is no path from $c$ to any other vertex, and there is no path from any other vertex to $d$, these two vertices are in strong components by themselves. Therefore the strongly connected components are $\{a, b, e\}$, $\{c\}$, and $\{d\}$.
**b)** The cycle $cdec$ guarantees that these three vertices are in one strongly connected component. The vertices $a$, $b$, and $f$ are in strong components by themselves, since there are no paths both to and from each of these to every other vertex. Therefore the strongly connected components are $\{a\}$, $\{b\}$ $\{c, d, e\}$, and $\{f\}$.
**c)** The cycle $abcdfghia$ guarantees that these eight vertices are in one strongly connected component. Since there is no path from $e$ to any other vertex, this vertex is in a strong component by itself. Therefore the strongly connected components are $\{a, b, c, d, f, g, h, i\}$ and $\{e\}$.

- 10.5 – 2

**2.** All the vertex degrees are even, so there is an Euler circuit. We can find one by trial and error, or by using Algorithm 1. One such circuit is $a, b, c, f, i, h, g, d, e, h, f, e, b, d, a$.

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## Exercises from Book:

- Chapter 11
  - 11.1 – 2, 4, 6, 10, 18, 20

**2.** **a)** This is a tree since it is connected and has no simple circuits.
**b)** This is a tree since it is connected and has no simple circuits.
**c)** This is not a tree, since it is not connected.
**d)** This is a tree since it is connected and has no simple circuits.
**e)** This is not a tree, since it has a simple circuit.
**f)** This is a tree since it is connected and has no simple circuits.

**4.** **a)** Vertex $a$ is the root, since it is drawn at the top.
**b)** The internal vertices are the vertices with children, namely $a$, $b$, $d$, $e$, $g$, $h$, $i$, and $o$.
**c)** The leaves are the vertices without children, namely $c$, $f$, $j$, $k$, $l$, $m$, $n$, $p$, $q$, $r$, and $s$.
**d)** The children of $j$ are the vertices adjacent to $j$ and below $j$. There are no such vertices, so there are no children.
**e)** The parent of $h$ is the vertex adjacent to $h$ and above $h$, namely $d$.
**f)** Vertex $o$ has only one sibling, namely $p$, which is the other child of $o$'s parent, $i$.
**g)** The ancestors of $m$ are all the vertices on the unique simple path from $m$ back to the root, namely $g$, $b$, and $a$.
**h)** The descendants of $b$ are all the vertices that have $b$ as an ancestor, namely $e$, $f$, $g$, $j$, $k$, $l$, and $m$.

**6.** This is not a full $m$-ary tree for any $m$. It is an $m$-ary tree for all $m \geq 3$, since each vertex has at most 3 children, but since some vertices have 3 children, while others have 1 or 2, it is not full for any $m$.

**10.** We describe the answers, rather than actually drawing pictures.
**a)** The subtree rooted at $a$ is the entire tree, since $a$ is the root.
**b)** The subtree rooted at $c$ consists of just the vertex $c$.
**c)** The subtree rooted at $e$ consists of $e$, $j$, and $k$, and the edges $ej$ and $ek$.

**18.** By Theorem 4($ii$), the answer is $mi + 1 = 5 \cdot 100 + 1 = 501$.

**20.** By Theorem 4($i$), the answer is $[(m-1)n+1]/m = (2 \cdot 100 + 1)/3 = 67$.

  - 11.3 – 8, 12, 14, 16, 22, 24

**8.** See the comments in the solution to Exercise 7 for the procedure. The only difference here is that some vertices have more than two children: after listing such a vertex, we list the vertices of its subtrees, in preorder, from left to right. The answer is $a, b, d, e, i, j, m, n, o, c, f, g, h, k, l, p$.

**12.** This is similar to Exercise 11. The answer is $k, e, l, m, b, f, r, n, s, g, a, c, o, h, d, i, p, j, q$.

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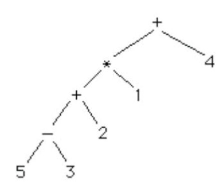
## Exercises from Book:

- 11.3 – 14, 16, 22, 24

**14.** The procedure is the same as in Exercise 13, except that some vertices have more than two children here: before listing such a vertex, we list the vertices of its subtrees, in postorder, from left to right. The answer is $d, i, m, n, o, j, e, b, f, g, k, p, l, h, c, a$.

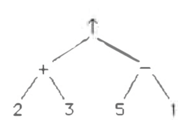
**16.** **a)** We build the tree from the top down while analyzing the expression by identifying the outermost operation at each stage. The outermost operation in this expression is the final subtraction. Therefore the tree has $-$ at its root, with the two operands as the subtrees at the root. The right operand is clearly $5$, so the right child of the root is $5$. The left operand is the result of a multiplication, so the left subtree has $*$ as its root. We continue recursively in this way until the entire tree is constructed.

**22.** We work from the beginning of the expression. In part **(a)** the root of the tree is necessarily the first $+$. We then use up as much of the rest of the expression as needed to construct the left subtree of the root. The root of this left subtree is the $*$, and its left subtree is as much of the rest of the expression as is needed. We continue in this way, making our way to the subtree consisting of root $-$ and children 5 and 3. Then the 2 must be the right child of the second $+$, the 1 must be the right child of the $*$, and the 4 must be the right child of the root. The result is shown here.

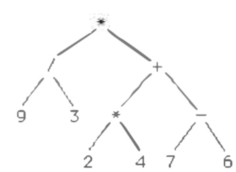


(a)

In infix form we have $((((5-3)+2)*1)+4)$. The other two trees are constructed in a similar manner.



(b)



(c)

The infix expressions are therefore $((2+3)\uparrow(5-1))$ and $((9/3)*((2*4)+(7-6)))$, respectively.

**24.** We exhibit the answers by showing with parentheses the operation that is applied next, working from left to right (it always involves the first occurrence of an operator symbol).

**a)** $5\,(2\,1\,-)-3\,1\,4+\,+\,*=(5\,1\,-)\,3\,1\,4+\,+\,*=4\,3\,(1\,4\,+)+\,*=4\,(3\,5\,+)\,*=(4\,8\,*)=32$

**b)** $(9\,3\,/)\,5+7\,2-*=(3\,5\,+)\,7\,2-*=8\,(7\,2\,-)\,*=(8\,5\,*)=40$

**c)** $(3\,2\,*)\,2\uparrow5\,3-8\,4\,/\,*\,-\;=\;(6\,2\uparrow)\,5\,3-8\,4\,/\,*\,-\;=\;36\,(5\,3\,-)\,8\,4\,/\,*\,-\;=\;36\;\;2\,(8\,4\,/)\,*\,-\;=\;36\,(2\,2\,*)\,-\;=\;(36\;4\,-)=32$

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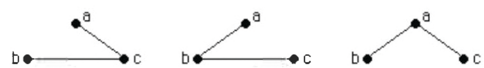
## Exercises from Book:

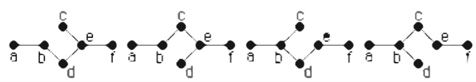
- 11.4 – 4, 8, 10, 14, 16

**4.** We can remove these edges to produce a spanning tree (see comments for Exercise 2): $\{a, i\}$, $\{b, i\}$, $\{b, j\}$, $\{c, d\}$, $\{c, j\}$, $\{d, e\}$, $\{e, j\}$, $\{f, i\}$, $\{f, j\}$, and $\{g, i\}$.

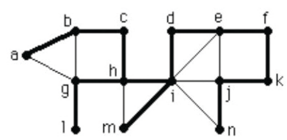
**8.** We can remove any one of the three edges to produce a spanning tree. The trees are therefore the ones shown below.



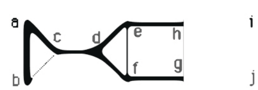

**10.** We can remove any one of the four edges in the middle square to produce a spanning tree, as shown.



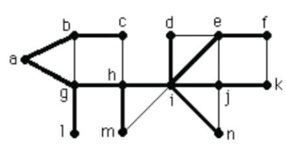
**14.** The tree is shown in heavy lines. It is produced by starting at $a$ and continuing as far as possible without backtracking, choosing the first unused vertex (in alphabetical order) at each point. When the path reaches vertex $l$, we need to backtrack. Backtracking to $h$, we can then form the path all the way to $n$ without further backtracking. Finally we backtrack to vertex $i$ to pick up vertex $m$.



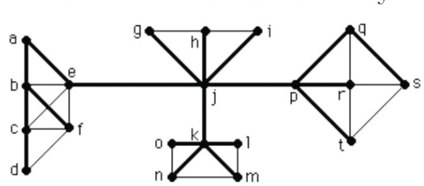

**16.** If we start at vertex $a$ and use alphabetical order, then the breadth-first search spanning tree is unique. Consider the graph in Exercise 13. We first fan out from vertex $a$, picking up the edges $\{a, b\}$ and $\{a, c\}$. There are no new vertices from $b$, so we fan out from $c$, to get edge $\{c, d\}$. Then we fan out from $d$ to get edges $\{d, e\}$ and $\{d, f\}$. This process continues until we have the entire tree shown in heavy lines below.



The tree for the graph in Exercise 14 is shown in heavy lines. It is produced by the same fanning-out procedure as described above.




The spanning tree for the graph in Exercise 15 is shown in heavy lines.



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## Exercises from Book:

- Chapter 12
  - 12.1 – 2, 6, 10, 12, 24, 28

**2.** **a)** Since $x \cdot 1 = x$, the only solution is $x = 0$.
**b)** Since $0 + 0 = 0$ and $1 + 1 = 1$, the only solution is $x = 0$.
**c)** Since this equation holds for all $x$, there are two solutions, $x = 0$ and $x = 1$.
**d)** Since either $x$ or $\overline{x}$ must be $0$, no matter what $x$ is, there are no solutions.

**6.** In each case, we compute the various components of the final expression and put them together as indicated. For part **(a)** we have simply

| $x$ | $y$ | $z$ | $\overline{z}$ |
|---|---|---|---|
| 1 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 1 |
| 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 1 |

For part **(b)** we have

| $x$ | $y$ | $z$ | $\overline{x}$ | $\overline{x}\,y$ | $\overline{y}$ | $\overline{y}\,z$ | $\overline{x}\,y + \overline{y}\,z$ |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 1 | 1 | 0 | 0 | 1 |
| 0 | 0 | 1 | 1 | 0 | 1 | 1 | 1 |
| 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 |

For part **(c)** we have

| $x$ | $y$ | $z$ | $\overline{y}$ | $x\,\overline{y}\,z$ | $xyz$ | $\overline{xyz}$ | $x\,\overline{y}\,z + \overline{xyz}$ |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 0 | 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 |
| 1 | 0 | 1 | 1 | 1 | 0 | 1 | 1 |
| 1 | 0 | 0 | 1 | 0 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 |
| 0 | 0 | 0 | 1 | 0 | 0 | 1 | 1 |

For part **(d)** we have

| $x$ | $y$ | $z$ | $\overline{x}$ | $\overline{y}$ | $\overline{z}$ | $xz$ | $\overline{x}\,\overline{z}$ | $xz + \overline{x}\,\overline{z}$ | $\overline{y}(xz + \overline{x}\,\overline{z})$ |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 1 |
| 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 1 |

# CSCI 190 -- Homework 3

Due: 7/31/2018



*It is required to be typed. Make sure to do the correct problems. Arrange your work in this order.Turn in your homework online using Canvas.*

## Exercises from Book:

- 12.1 – 10, 12, 24, 28

**10.** There are $2^{2^n}$ different Boolean functions of degree $n$, so the answer is $2^{2^7} = 2^{128} \approx 3.4 \times 10^{38}$.

**12.** The only way for the sum to have the value 1 is for one of the summands to have the value 1, since $0+0+0 = 0$. Each summand is 1 if and only if the two variables in the product making up that summand are both 1. The conclusion follows.

**24.** **a)** Since $0 \oplus 0 = 0$ and $1 \oplus 0 = 1$, this expression simplifies to $x$.
**b)** Since $0 \oplus 1 = 1$ and $1 \oplus 1 = 0$, this expression simplifies to $\overline{x}$.
**c)** Looking at the definition, we see that $x \oplus x = 0$ for all $x$.
**d)** This is similar to part **(c)**; this time the expression always equals 1.

**28.** In each case we simply change each 0 to a 1 and vice versa, and change all the sums to products and vice versa.
**a)** $xy$ **b)** $\overline{x} + \overline{y}$ **c)** $(x+y+z)(\overline{x}+\overline{y}+\overline{z})$ **d)** $(x+\overline{z})(x+1)(\overline{x}+0)$

- 12.2 – 2, 4, 6

**2.** **a)** We can rewrite this as $F(x,y) = \overline{x}\cdot 1 + \overline{y}\cdot 1 = \overline{x}(y+\overline{y}) + y(x+\overline{x})$. Expanding and using the commutative and idempotent laws, this simplifies to $\overline{x}\,y + \overline{x}\,\overline{y} + x\,y$.
**b)** This is already in sum-of-products form.
**c)** We need to write the sum of all products; the answer is $x\,y + x\,\overline{y} + \overline{x}\,y + \overline{x}\,\overline{y}$.
**d)** As in part **(a)**, we have $F(x,y) = 1\cdot\overline{y} = (x+\overline{x})y = x\,y + \overline{x}\,y$.

**4.** **a)** We need to write all the terms that have $\overline{x}$ in them. Thus the answer is $\overline{x}\,y\,z + \overline{x}\,y\,\overline{z} + \overline{x}\,\overline{y}\,z + \overline{x}\,\overline{y}\,\overline{z}$.
**b)** We need to write all the terms that include either $\overline{x}$ or $\overline{y}$. Thus the answer is $x\,\overline{y}\,z + x\,\overline{y}\,\overline{z} + \overline{x}\,y\,z + \overline{x}\,y\,\overline{z} + \overline{x}\,\overline{y}\,z + \overline{x}\,\overline{y}\,\overline{z}$.
**c)** We need to include all the terms that have both $\overline{x}$ and $\overline{y}$. Thus the answer is $\overline{x}\,\overline{y}\,z + \overline{x}\,\overline{y}\,\overline{z}$.
**d)** We need to include all the terms that have at least one of $\overline{x}$, $\overline{y}$, and $\overline{z}$. This is all the terms except $x\,y\,z$, so the answer is $x\,y\,\overline{z} + x\,\overline{y}\,z + x\,\overline{y}\,\overline{z} + \overline{x}\,y\,z + \overline{x}\,y\,\overline{z} + \overline{x}\,\overline{y}\,z + \overline{x}\,\overline{y}\,\overline{z}$.

**6.** We need to include all terms that have three or more of the variables in their uncomplemented form. This will give us a total of $1+5+10 = 16$ terms. The answer is

$$\begin{aligned} &x_1 x_2 x_3 x_4 x_5 + x_1 x_2 x_3 x_4 \overline{x}_5 + x_1 x_2 x_3 \overline{x}_4 x_5 + x_1 x_2 \overline{x}_3 x_4 x_5 + x_1 \overline{x}_2 x_3 x_4 x_5 + \overline{x}_1 x_2 x_3 x_4 x_5 \\ &\quad + x_1 x_2 x_3 \overline{x}_4 \overline{x}_5 + x_1 x_2 \overline{x}_3 x_4 \overline{x}_5 + x_1 x_2 \overline{x}_3 \overline{x}_4 x_5 + x_1 \overline{x}_2 x_3 x_4 \overline{x}_5 + x_1 \overline{x}_2 x_3 \overline{x}_4 x_5 \\ &\quad + x_1 \overline{x}_2 \overline{x}_3 x_4 x_5 + \overline{x}_1 x_2 x_3 x_4 \overline{x}_5 + \overline{x}_1 x_2 x_3 \overline{x}_4 x_5 + \overline{x}_1 x_2 \overline{x}_3 x_4 x_5 + \overline{x}_1 \overline{x}_2 x_3 x_4 x_5 . \end{aligned}$$

- 12.3 – 2, 4, 6, 16(a)(b)(c)

**2.** The inputs to the AND gate are $\overline{x}$ and $\overline{y}$. The output is then passed through the inverter. Therefore the final output is $\overline{(\overline{x}\,\overline{y})}$. Note that there is a simpler way to form a circuit equivalent to this one, namely $x+y$.

**4.** This is similar to the previous three exercises. The output is $\overline{(\overline{x}\,y\,z)}(\overline{x}+y+\overline{z})$.

# CSCI 190 -- Homework 3



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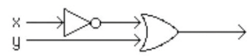
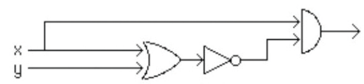
## Exercises from Book:

- 12.3 – 6, 16(a)(b)(c)

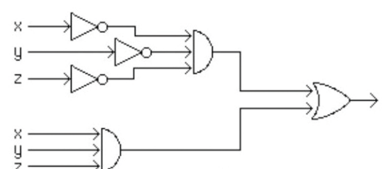
**6.** We build these circuits up exactly as the expressions are built up. In part **(b)**, for example, we use an AND gate to join the outputs of the inverter (which was applied to the output of the OR gate applied to $x$ and $y$) and $x$.



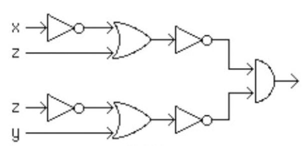

(a)




(b)



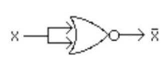

(c)



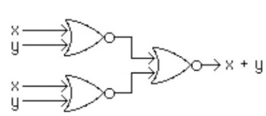

(d)

**16.** The answers here are duals to the answers for Exercise 15. Note that the usual symbol $\downarrow$ represents the *NOR* operation.

**a)** The circuit is the same as in Exercise 15a, with a NOR gate in place of a NAND gate, since $\overline{x} = x \mid x = x \downarrow x$.




**b)** Since $x + y = (x \downarrow y) \downarrow (x \downarrow y)$, the answer is as shown.




**c)** Since $xy = (x \downarrow x) \downarrow (y \downarrow y)$, the answer is as shown.