CSCI 190 -- Homework 1

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Exercises from Book:

- Chapter 1
  - 1.1 – 2, 4, 6

2. Propositions must have clearly defined truth values, so a proposition must be a declarative sentence with no free variables.

a) This is not a proposition; it's a command.

b) This is not a proposition; it's a question.

c) This is a proposition that is false, as anyone who has been to Maine knows.

d) This is not a proposition; its truth value depends on the value of x.

e) This is a proposition that is false.

f) This is not a proposition; its truth value depends on the value of n.

4. a) Jennifer and Teja are not friends.

b) There are not 13 items in a baker's dozen. (Alternatively: The number of items in a baker's dozen is not equal to 13.)

c) Abby sent fewer than 101 text messages yesterday. Alternatively, Abby sent at most 100 text messages yesterday. Note: The first printing of this edition incorrectly rendered this exercise with "every day" in place of "yesterday." That makes it a much harder problem, because the days are quantified, and quantified propositions are not dealt with until a later section. It would be incorrect to say that the negation in that case is "Abby sent at most 100 text messages every day." Rather, a correct negation would be "There exists a day on which Abby sent at most 100 text messages." Saying "Abby did not send more than 100 text messages every day" is somewhat ambiguous—do we mean $\neg\forall$ or do we mean $\forall\neg$?

d) 121 is not a perfect square.

6. a) True, because $288 > 256$ and $288 > 128$.

b) True, because C has 5 MP resolution compared to B's 4 MP resolution. Note that only one of these conditions needs to be met because of the word *or*.

c) False, because its resolution is not higher (all of the statements would have to be true for the conjunction to be true).

d) False, because the hypothesis of this conditional statement is true and the conclusion is false.

e) False, because the first part of this biconditional statement is false and the second part is true.

  - 1.2 – 2, 4

2. Recall that p only if q means $p \to q$. In this case, if you can see the movie then you must have fulfilled one of the two requirements. Therefore the statement is $m \to (e \vee p)$. Notice that in everyday life one might actually say "You can see the movie *if* you meet one of these conditions," but logically that is not what the rules really say.

4. The condition stated here is that if you use the network, then either you pay the fee or you are a subscriber. Therefore the proposition in symbols is $w \to (d \vee s)$.

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Exercises from Book:

- 1.3 – 6, 8, 10, 12

6. We see that the fourth and seventh columns are identical.

p	q	$p \wedge q$	$\neg(p \wedge q)$	$\neg p$	$\neg q$	$\neg p \vee \neg q$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

8. We need to negate each part and swap "and" with "or."

a) Kwame will not take a job in industry and will not go to graduate school.

b) Yoshiko does not know Java or does not know calculus.

c) James is not young, or he is not strong.

d) Rita will not move to Oregon and will not move to Washington.

10. We construct a truth table for each conditional statement and note that the relevant column contains only T's. For part **(a)** we have the following table.

p	q	$\neg p$	$p \vee q$	$\neg p \wedge (p \vee q)$	$[\neg p \wedge (p \vee q)] \to q$
T	T	F	T	F	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	F	F	T

For part **(b)** we have the following table. We omit the columns showing $p \to q$ and $q \to r$ so that the table will fit on the page.

p	q	r	$(p \to q) \to (q \to r)$	$q \to r$	$[(p \to q) \to (q \to r)] \to (p \to r)$
T	T	T	T	T	T
T	T	F	F	T	T
T	F	T	T	T	F
T	F	F	F	F	T
F	T	T	T	T	T
F	T	F	F	T	F
F	F	T	T	T	F
F	F	F	T	T	T

For part **(c)** we have the following table.

p	q	$p \to q$	$p \wedge (p \to q)$	$[p \wedge (p \to q)] \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

For part **(d)** we have the following table. We have omitted some of the intermediate steps to make the table fit.

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Exercises from Book:

p	q	r	$(p \vee q) \wedge (p \to r) \wedge (p \to r)$	$[(p \vee q) \wedge (p \to r) \wedge (p \to r)] \to r$
T	T	T	T	T
T	T	F	F	T
T	F	T	T	T
T	F	F	F	T
F	T	T	T	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

12. We argue directly by showing that if the hypothesis is true, then so is the conclusion. An alternative approach, which we show only for part **(a)**, is to use the equivalences listed in the section and work symbolically.

a) Assume the hypothesis is true. Then p is false. Since $p \vee q$ is true, we conclude that q must be true. Here is a more "algebraic" solution: $[\neg p \wedge (p \vee q)] \to q \equiv \neg[\neg p \wedge (p \vee q)] \vee q \equiv \neg\neg p \vee \neg(p \vee q)] \vee q \equiv p \vee \neg(p \vee q) \vee q \equiv (p \vee q) \vee \neg(p \vee q) \equiv \mathbf{T}$. The reasons for these logical equivalences are, respectively, Table 7, line 1; De Morgan's law; double negation; commutative and associative laws; negation law.

b) We want to show that if the entire hypothesis is true, then the conclusion $p \to r$ is true. To do this, we need only show that if p is true, then r is true. Suppose p is true. Then by the first part of the hypothesis, we conclude that q is true. It now follows from the second part of the hypothesis that r is true, as desired.

c) Assume the hypothesis is true. Then p is true, and since the second part of the hypothesis is true, we conclude that q is also true, as desired.

d) Assume the hypothesis is true. Since the first part of the hypothesis is true, we know that either p or q is true. If p is true, then the second part of the hypothesis tells us that r is true; similarly, if q is true, then the third part of the hypothesis tells us that r is true. Thus in either case we conclude that r is true.

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Exercises from Book:

- 1.4 – 6, 8, 14

6. The answers given here are not unique, but care must be taken not to confuse nonequivalent sentences. Parts **(c)** and **(f)** are equivalent; and parts **(d)** and **(e)** are equivalent. But these two pairs are not equivalent to each other.

a) Some student in the school has visited North Dakota. (Alternatively, there exists a student in the school who has visited North Dakota.)

b) Every student in the school has visited North Dakota. (Alternatively, all students in the school have visited North Dakota.)

c) This is the negation of part **(a)**: No student in the school has visited North Dakota. (Alternatively, there does not exist a student in the school who has visited North Dakota.)

d) Some student in the school has not visited North Dakota. (Alternatively, there exists a student in the school who has not visited North Dakota.)

e) This is the negation of part **(b)**: It is not true that every student in the school has visited North Dakota. (Alternatively, not all students in the school have visited North Dakota.)

f) All students in the school have not visited North Dakota. (This is technically the correct answer, although common English usage takes this sentence to mean—incorrectly—the answer to part **(e)**. To be perfectly clear, one could say that every student in this school has failed to visit North Dakota, or simply that no student has visited North Dakota.)

8. Note that part **(b)** and part **(c)** are not the sorts of things one would normally say.

a) If an animal is a rabbit, then that animal hops. (Alternatively, every rabbit hops.)

b) Every animal is a rabbit and hops.

c) There exists an animal such that if it is a rabbit, then it hops. (Note that this is trivially true, satisfied, for example, by lions, so it is not the sort of thing one would say.)

d) There exists an animal that is a rabbit and hops. (Alternatively, some rabbits hop. Alternatively, some hopping animals are rabbits.)

14. **a)** Since $(-1)^3 = -1$, this is true.

b) Since $(\frac{1}{2})^4 < (\frac{1}{2})^2$, this is true.

c) Since $(-x)^2 = ((-1)x)^2 = (-1)^2x^2 = x^2$, we know that $\forall x((-x)^2 = x^2)$ is true.

d) Twice a positive number is larger than the number, but this inequality is not true for negative numbers or 0. Therefore $\forall x(2x > x)$ is false.

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Exercises from Book:

- 1.5 – 2, 6, 10

2. a) There exists a real number x such that for every real number y, $xy = y$. This is asserting the existence of a multiplicative identity for the real numbers, and the statement is true, since we can take $x = 1$.
b) For every real number x and real number y, if x is nonnegative and y is negative, then the difference $x - y$ is positive. Or, more simply, a nonnegative number minus a negative number is positive (which is true).
c) For every real number x and real number y, there exists a real number z such that $x = y + z$. This is a true statement, since we can take $z = x - y$ in each case.

6. a) Randy Goldberg is enrolled in CS 252.
b) Someone is enrolled in Math 695.
c) Carol Sitea is enrolled in some course.
d) Some student is enrolled simultaneously in Math 222 and CS 252.
e) There exist two distinct people, the second of whom is enrolled in every course that the first is enrolled in.
f) There exist two distinct people enrolled in exactly the same courses.

10. a) $\forall x F(x, \text{Fred})$ **b)** $\forall y F(\text{Evelyn}, y)$ **c)** $\forall x \exists y F(x, y)$ **d)** $\neg \exists x \forall y F(x, y)$ **e)** $\forall y \exists x F(x, y)$
f) $\neg \exists x (F(x, \text{Fred}) \wedge F(x, \text{Jerry}))$
g) $\exists y_1 \exists y_2 (F(\text{Nancy}, y_1) \wedge F(\text{Nancy}, y_2) \wedge y_1 \neq y_2 \wedge \forall y (F(\text{Nancy}, y) \rightarrow (y = y_1 \vee y = y_2)))$
h) $\exists y (\forall x F(x, y) \wedge \forall z (\forall x F(x, z) \rightarrow z = y))$ **i)** $\neg \exists x F(x, x)$
j) $\exists x \exists y (x \neq y \wedge F(x, y) \wedge \forall z ((F(x, z) \wedge z \neq x) \rightarrow z = y))$ (We do not assume that this sentence is asserting that this person can or cannot fool her/himself.)

- 1.6 – 2, 4, 8

2. This is modus tollens. The first statement is $p \rightarrow q$, where p is "George does not have eight legs" and q is "George is not a spider." The second statement is $\neg q$. The third is $\neg p$. Modus tollens is valid. We can therefore conclude that the conclusion of the argument (third statement) is true, given that the hypotheses (the first two statements) are true.

4. a) We have taken the conjunction of two propositions and asserted one of them. This is, according to Table 1, simplification.
b) We have taken the disjunction of two propositions and the negation of one of them, and asserted the other. This is, according to Table 1, disjunctive syllogism. See Table 1 for the other parts of this exercise as well.
c) modus ponens **d)** addition **e)** hypothetical syllogism

8. First we use universal instantiation to conclude from "For all x, if x is a man, then x is not an island" the special case of interest, "If Manhattan is a man, then Manhattan is not an island." Then we form the contrapositive (using also double negative): "If Manhattan is an island, then Manhattan is not a man." Finally we use modus ponens to conclude that Manhattan is not a man. Alternatively, we could apply modus tollens.

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Exercises from Book:

- 1.7 – 6, 18, 24, 28

6. An odd number is one of the form $2n+1$, where n is an integer. We are given two odd numbers, say $2a+1$ and $2b+1$. Their product is $(2a+1)(2b+1) = 4ab+2a+2b+1 = 2(2ab+a+b)+1$. This last expression shows that the product is odd, since it is of the form $2n+1$, with $n = 2ab+a+b$.

18. **a)** We must prove the contrapositive: If n is odd, then $3n+2$ is odd. Assume that n is odd. Then we can write $n = 2k+1$ for some integer k. Then $3n+2 = 3(2k+1)+2 = 6k+5 = 2(3k+2)+1$. Thus $3n+2$ is two times some integer plus 1, so it is odd.

b) Suppose that $3n+2$ is even and that n is odd. Since $3n+2$ is even, so is $3n$. If we add subtract an odd number from an even number, we get an odd number, so $3n-n=2n$ is odd. But this is obviously not true. Therefore our supposition was wrong, and the proof by contradiction is complete.

24. We give a proof by contradiction. If there were at most two days falling in the same month, then we could have at most $2 \cdot 12 = 24$ days, since there are 12 months. Since we have chosen 25 days, at least three of them must fall in the same month.

28. There are two things to prove. For the "if" part, there are two cases. If $m = n$, then of course $m^2 = n^2$; if $m = -n$, then $m^2 = (-n)^2 = (-1)^2 n^2 = n^2$. For the "only if" part, we suppose that $m^2 = n^2$. Putting everything on the left and factoring, we have $(m+n)(m-n) = 0$. Now the only way that a product of two numbers can be zero is if one of them is zero. Therefore we conclude that either $m+n=0$ (in which case $m=-n$), or else $m-n=0$ (in which case $m=n$), and our proof is complete.

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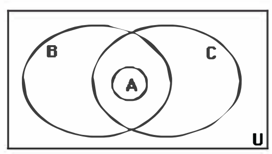
Exercises from Book:

- Chapter 2
  - 2.1 – 10, 16, 24, 32

10. **a)** true **b)** true **c)** false—see part **(a)** **d)** true
e) true—the one element in the set on the left is an element of the set on the right, and the sets are not equal
f) true—similar to part **(e)** **g)** false—the two sets are equal

16. We allow B and C to overlap, because we are told nothing about their relationship. The set A must be a subset of each of them, and that forces it to be positioned as shown. We cannot actually show the properness of the subset relationships in the diagram, because we don't know where the elements in B and C that are not in A are located—there might be only one (which is in both B and C), or they might be located in portions of B and/or C outside the other. Thus the answer is as shown, but with the added condition that there must be at least one element of B not in A and one element of C not in A.



24. **a)** The power set of every set includes at least the empty set, so the power set cannot be empty. Thus $\emptyset$ is not the power set of any set.
b) This is the power set of $\{a\}$.
c) This set has three elements. Since 3 is not a power of 2, this set cannot be the power set of any set.
d) This is the power set of $\{a, b\}$.

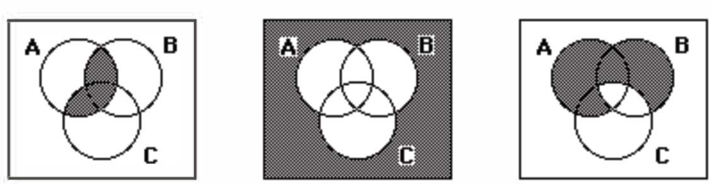
32. In each case the answer is a set of 3-tuples.
a) $\{(a,x,0),(a,x,1),(a,y,0),(a,y,1),(b,x,0),(b,x,1),(b,y,0),(b,y,1),(c,x,0),(c,x,1),(c,y,0),(c,y,1)\}$
b) $\{(0,x,a),(0,x,b),(0,x,c),(0,y,a),(0,y,b),(0,y,c),(1,x,a),(1,x,b),(1,x,c),(1,y,a),(1,y,b),(1,y,c)\}$
c) $\{(0,a,x),(0,a,y),(0,b,x),(0,b,y),(0,c,x),(0,c,y),(1,a,x),(1,a,y),(1,b,x),(1,b,y),(1,c,x),(1,c,y)\}$
d) $\{(x,x,x),(x,x,y),(x,y,x),(x,y,y),(y,x,x),(y,x,y),(y,y,x),(y,y,y)\}$

  - 2.2 – 2, 4, 26

2. **a)** $A \cap B$ **b)** $A \cap \overline{B}$, which is the same as $A - B$ **c)** $A \cup B$ **d)** $\overline{A} \cup \overline{B}$

4. Note that $A \subseteq B$.
a) $\{a,b,c,d,e,f,g,h\} = B$ **b)** $\{a,b,c,d,e\} = A$
c) There are no elements in A that are not in B, so the answer is $\emptyset$. **d)** $\{f,g,h\}$

26. The set is shaded in each case.



(a) (b) (c)

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Exercises from Book:

- 2.3 – 8, 10, 14, 20

8. We simply round up or down in each case.
a) 1 **b)** 2 **c)** -1 **d)** 0 **e)** 3 **f)** -2 **g)** $\lfloor \frac{1}{2} + 1 \rfloor = \lfloor \frac{3}{2} \rfloor = 1$
h) $\lceil 0 + 1 + \frac{1}{2} \rceil = \lceil \frac{3}{2} \rceil = 2$

10. a) This is one-to-one. **b)** This is not one-to-one, since b is the image of both a and b.
c) This is not one-to-one, since d is the image of both a and d.

14. a) This is clearly onto, since $f(0, -n) = n$ for every integer n.
b) This is not onto, since, for example, 2 is not in the range. To see this, if $m^2 - n^2 = (m-n)(m+n) = 2$, then m and n must have same parity (both even or both odd). In either case, both $m - n$ and $m + n$ are then even, so this expression is divisible by 4 and hence cannot equal 2.
c) This is clearly onto, since $f(0, n-1) = n$ for every integer n.
d) This is onto. To achieve negative values we set $m = 0$, and to achieve nonnegative values we set $n = 0$.
e) This is not onto, for the same reason as in part **(b)**. In fact, the range here is clearly a subset of the range in that part.

20. a) $f(n) = n + 17$ **b)** $f(n) = \lceil n/2 \rceil$
c) We let $f(n) = n - 1$ for even values of n, and $f(n) = n + 1$ for odd values of n. Thus we have $f(1) = 2$, $f(2) = 1$, $f(3) = 4$, $f(4) = 3$, and so on. Note that this is just one function, even though its definition used two formulae, depending on the the parity of n.
d) $f(n) = 17$

- 2.4 – 2, 4, 6

2. In each case we just plug $n = 8$ into the formula.
a) $2^{8-1} = 128$ **b)** 7 **c)** $1 + (-1)^8 = 0$ **d)** $-(-2)^8 = -256$

4. a) $a_0 = (-2)^0 = 1$, $a_1 = (-2)^1 = -2$, $a_2 = (-2)^2 = 4$, $a_3 = (-2)^3 = -8$
b) $a_0 = a_1 = a_2 = a_3 = 3$
c) $a_0 = 7 + 4^0 = 8$, $a_1 = 7 + 4^1 = 11$, $a_2 = 7 + 4^2 = 23$, $a_3 = 7 + 4^3 = 71$
d) $a_0 = 2^0 + (-2)^0 = 2$, $a_1 = 2^1 + (-2)^1 = 0$, $a_2 = 2^2 + (-2)^2 = 8$, $a_3 = 2^3 + (-2)^3 = 0$

6. These are easy to compute by hand, calculator, or computer.
a) 10, 7, 4, 1, -2, -5, -8, -11, -14, -17
b) We can use the formula in Table 2, or we can just keep adding to the previous term ($1 + 2 = 3$, $3 + 3 = 6$, $6 + 4 = 10$, and so on): 1, 3, 6, 10, 15, 21, 28, 36, 45, 55. These are called the triangular numbers.
c) 1, 5, 19, 65, 211, 665, 2059, 6305, 19171, 58025
d) 1, 1, 1, 2, 2, 2, 2, 2, 3, 3 (there will be $2k + 1$ copies of k) **e)** 1, 5, 6, 11, 17, 28, 45, 73, 118, 191
f) The largest number whose binary expansion has n bits is $(11\ldots1)_2$, which is $2^n - 1$. So the sequence is 1, 3, 7, 15, 31, 63, 127, 255, 511, 1023.
g) 1, 2, 2, 4, 8, 11, 33, 37, 148, 153 **h)** 1, 2, 2, 2, 2, 3, 3, 3, 3, 3

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Exercises from Book:

- 2.6 – 2, 4, 26

2. We just add entry by entry.

a) $\begin{bmatrix} 0 & 3 & 9 \\ 1 & 4 & -1 \\ 2 & -5 & -3 \end{bmatrix}$ **b)** $\begin{bmatrix} -4 & 9 & 2 & 10 \\ -4 & -5 & 4 & 0 \end{bmatrix}$

4. To multiply matrices $\mathbf{A}$ and $\mathbf{B}$, we compute the $(i,j)^{\text{th}}$ entry of the product $\mathbf{AB}$ by adding all the products of elements from the i^{th} row of $\mathbf{A}$ with the corresponding element in the j^{th} column of $\mathbf{B}$, that is $\sum_{k=1}^{n} a_{ik}b_{kj}$. This can only be done, of course, when the number of columns of $\mathbf{A}$ equals the number of rows of $\mathbf{B}$ (called n in the formula shown here).

a) $\begin{bmatrix} -1 & 1 & 0 \\ 0 & 1 & -1 \\ 1 & -2 & 1 \end{bmatrix}$ **b)** $\begin{bmatrix} 4 & -1 & -7 & 6 \\ -7 & -5 & 8 & 5 \\ 4 & 0 & 7 & 3 \end{bmatrix}$ **c)** $\begin{bmatrix} 2 & 0 & -3 & -4 & -1 \\ 24 & -7 & 20 & 29 & 2 \\ -10 & 4 & -17 & -24 & -3 \end{bmatrix}$

26. We follow the definitions.

a) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ **b)** $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ **c)** $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

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Exercises from Book:

- Chapter 3
  - 3.1 – 4, 12, 34

4. Set the answer to be $-\infty$. For i going from 1 through $n-1$, compute the value of the $(i+1)^{\text{st}}$ element in the list minus the i^{th} element in the list. If this is larger than the answer, reset the answer to be this value.

12. Four assignment statements are needed, one for each of the variables and a temporary assignment to get started so that we do not lose one of the original values.

$$\begin{aligned}&\mathit{temp} := x\\ &x := y\\ &y := z\\ &z := \mathit{temp}\end{aligned}$$

34. There are five passes through the list. After one pass the list reads $2, 3, 1, 5, 4, 6$, since the 6 is compared and moved at each stage. During the next pass, the 2 and the 3 are not interchanged, but the 3 and the 1 are, as are the 5 and the 4, yielding $2, 1, 3, 4, 5, 6$. On the third pass, the 2 and the 1 are interchanged, yielding $1, 2, 3, 4, 5, 6$. There are two more passes, but no further interchanges are made, since the list is now in order.

  - 3.2 – 2, 24

2. Note that the choices of C and k witnesses are not unique.

a) Yes, since $17x + 11 \leq 17x + x = 18x \leq 18x^2$ for all $x > 11$. The witnesses are $C = 18$ and $k = 11$.
b) Yes, since $x^2 + 1000 \leq x^2 + x^2 = 2x^2$ for all $x > \sqrt{1000}$. The witnesses are $C = 2$ and $k = \sqrt{1000}$.
c) Yes, since $x \log x \leq x \cdot x = x^2$ for all x in the domain of the function. (The fact that $\log x < x$ for all x follows from the fact that $x < 2^x$ for all x, which can be seen by looking at the graphs of these two functions.) The witnesses are $C = 1$ and $k = 0$.
d) No. If there were a constant C such that $x^4/2 \leq Cx^2$ for sufficiently large x, then we would have $C \geq x^2/2$. This is clearly impossible for a constant to satisfy.
e) No. If 2^x were $O(x^2)$, then the fraction $2^x/x^2$ would have to be bounded above by some constant C. It can be shown that in fact $2^x > x^3$ for all $x \geq 10$ (using mathematical induction—see Section 5.1—or calculus), so $2^x/x^2 \geq x^3/x^2 = x$ for large x, which is certainly not less than or equal to C.
f) Yes, since $\lfloor x \rfloor \lceil x \rceil \leq x(x+1) \leq x \cdot 2x = 2x^2$ for all $x > 1$. The witnesses are $C = 2$ and $k = 1$.

24. The first algorithm uses fewer operations because $n^2 2^n$ is $O(n!)$ but $n!$ is not $O(n^2 2^n)$. In fact, the second function overtakes the first function for good at $n = 8$, when $8^2 \cdot 2^8 = 16{,}384$ and $8! = 40{,}320$.

  - 3.3 – 2, 16, 18

2. The statement $t := t + i + j$ is executed n^2 times, so the number of operations is $O(n^2)$. (Specifically, $2n^2$ additions are used, not counting any arithmetic needed for bookkeeping in the loops.)

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Exercises from Book:

- Chapter 3
  - 3.3 – 16, 18

16. If each bit operation takes 10^{-11} second, then we can carry out 10^{11} bit operations per second, and therefore $60 \cdot 60 \cdot 24 \cdot 10^{11} = 864 \cdot 10^{13}$ bit operations per day. Therefore in each case we want to solve the inequality $f(n) = 864 \cdot 10^{13}$ for n and round down to an integer. Obviously a calculator or computer software will come in handy here.

a) If $\log n = 864 \cdot 10^{13}$, then $n = 2^{864 \cdot 10^{13}}$, which is an unfathomably huge number.
b) If $1000n = 864 \cdot 10^{13}$, then $n = 864 \cdot 10^{10}$, which is still a very large number.
c) If $n^2 = 864 \cdot 10^{13}$, then $n = \sqrt{864 \cdot 10^{13}}$, which works out to about $9.3 \cdot 10^7$.
d) If $1000n^2 = 864 \cdot 10^{13}$, then $n = \sqrt{864 \cdot 10^{10}}$, which works out to about $2.9 \cdot 10^6$.
e) If $n^3 = 864 \cdot 10^{13}$, then $n = (864 \cdot 10^{13})^{1/3}$, which works out to about $2.1 \cdot 10^5$.
f) If $2^n = 864 \cdot 10^{13}$, then $n = \lfloor \log(864 \cdot 10^{13}) \rfloor = 52$. (Remember, we are taking log to the base 2.)
g) If $2^{2n} = 864 \cdot 10^{13}$, then $n = \lfloor \log(864 \cdot 10^{13})/2 \rfloor = 26$.
h) If $2^{2^n} = 864 \cdot 10^{13}$, then $n = \lfloor \log(\log(864 \cdot 10^{13})) \rfloor = 5$.

18. We are asked to compute $(2n^2 + 2^n) \cdot 10^{-9}$ for each of these values of n. When appropriate, we change the units from seconds to some larger unit of time.

a) 1.224×10^{-6} seconds **b)** approximately 1.05×10^{-3} seconds
c) approximately 1.13×10^6 seconds, which is about 13 days (nonstop)
d) approximately 1.27×10^{21} seconds, which is about 4×10^{13} years (nonstop)

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It is required to be typed. Make sure to do the correct problems. Arrange your work in this order. Turn in your homework online using Canvas.

Exercises from Book:

- Chapter 4
  - 4.1 – 6, 10, 28

6. Under the hypotheses, we have $c = as$ and $d = bt$ for some s and t. Multiplying we obtain $cd = ab(st)$, which means that $ab \mid cd$, as desired.

10. In each case we can carry out the arithmetic on a calculator.

a) Since $8 \cdot 5 = 40$ and $44 - 40 = 4$, we have quotient $44 \textbf{ div } 8 = 5$ and remainder $44 \textbf{ mod } 8 = 4$.

b) Since $21 \cdot 37 = 777$, we have quotient $777 \textbf{ div } 21 = 37$ and remainder $777 \textbf{ mod } 21 = 0$.

c) As above, we can compute $123 \textbf{ div } 19 = 6$ and $123 \textbf{ mod } 19 = 9$. However, since the dividend is negative and the remainder is nonzero, the quotient is $-(6+1) = -7$ and the remainder is $19 - 9 = 10$. To check that $-123 \textbf{ div } 19 = -7$ and $-123 \textbf{ mod } 19 = 10$, we note that $-123 = (-7)(19) + 10$.

d) Since $1 \textbf{ div } 23 = 0$ and $1 \textbf{ mod } 23 = 1$, we have $-1 \textbf{ div } 23 = -1$ and $-1 \textbf{ mod } 23 = 22$.

e) Since $2002 \textbf{ div } 87 = 23$ and $2002 \textbf{ mod } 87 = 1$, we have $-2002 \textbf{ div } 87 = -24$ and $2002 \textbf{ mod } 87 = 86$.

f) Clearly $0 \textbf{ div } 17 = 0$ and $0 \textbf{ mod } 17 = 0$.

g) We have $1234567 \textbf{ div } 1001 = 1233$ and $1234567 \textbf{ mod } 1001 = 334$.

h) Since $100 \textbf{ div } 101 = 0$ and $100 \textbf{ mod } 101 = 100$, we have $-100 \textbf{ div } 101 = -1$ and $-100 \textbf{ mod } 101 = 1$.

28. We just subtract 3 from the given number; the answer is "yes" if and only if the difference is divisible by 7.

a) $37 - 3 \textbf{ mod } 7 = 34 \textbf{ mod } 7 = 6 \neq 0$, so $37 \not\equiv 3 \pmod 7$.

b) $66 - 3 \textbf{ mod } 7 = 63 \textbf{ mod } 7 = 0$, so $66 \equiv 3 \pmod 7$.

c) $-17 - 3 \textbf{ mod } 7 = -20 \textbf{ mod } 7 = 1 \neq 0$, so $-17 \not\equiv 3 \pmod 7$.

d) $-67 - 3 \textbf{ mod } 7 = -70 \textbf{ mod } 7 = 0$, so $-67 \equiv 3 \pmod 7$.

  - 4.2 – 2, 4, 6, 8, 22

2. To convert from decimal to binary, we successively divide by 2. We write down the remainders so obtained from right to left; that is the binary representation of the given number.

a) Since $321/2$ is 160 with a remainder of 1, the rightmost digit is 1. Then since $160/2$ is 80 with a remainder of 0, the second digit from the right is 0. We continue in this manner, obtaining successive quotients of 40, 20, 10, 5, 2, 1, and 0, and remainders of 0, 0, 0, 0, 1, 0, and 1. Putting all these remainders in order from right to left we obtain $(1\ 0100\ 0001)_2$ as the binary representation. We could, as a check, expand this binary numeral: $2^0 + 2^6 + 2^8 = 1 + 64 + 256 = 321$.

b) We could carry out the same process as in part **(a)**. Alternatively, we might notice that $1023 = 1024 - 1 = 2^{10} - 1$. Therefore the binary representation is 1 less than $(100\ 0000\ 0000)_2$, which is clearly $(11\ 1111\ 1111)_2$.

c) If we carry out the divisions by 2, the quotients are 50316, 25158, 12579, 6289, 3144, 1572, 786, 393, 196, 98, 49, 24, 12, 6, 3, 1, and 0, with remainders of 0, 0, 0, 1, 1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0, 1, and 1. Putting the remainders in order from right to left we have $(1\ 1000\ 1001\ 0001\ 1000)_2$.

4. **a)** $1 + 2 + 8 + 16 = 27$ **b)** $1 + 4 + 16 + 32 + 128 + 512 = 693$

c) $2 + 4 + 8 + 16 + 32 + 128 + 256 + 512 = 958$

d) $1 + 2 + 4 + 8 + 16 + 1024 + 2048 + 4096 + 8192 + 16384 = 31775$

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Exercises from Book:

- Chapter 4
  - 4.2 – 6, 8, 22

6. We follow the procedure of Example 7.

a) $(1111\ 0111)_2 = (011\ 110\ 111)_2 = (367)_8$

b) $(1010\ 1010\ 1010)_2 = (101\ 010\ 101\ 010)_2 = (5252)_8$

c) $(111\ 0111\ 0111\ 0111)_2 = (111\ 011\ 101\ 110\ 111)_2 = (73567)_8$

d) $(101\ 0101\ 0101\ 0101)_2 = (101\ 010\ 101\ 010\ 101)_2 = (52525)_8$

8. Following Example 7, we simply write the binary equivalents of each digit. Since $(\mathrm{A})_{16} = (1010)_2$, $(\mathrm{B})_{16} = (1011)_2$, $(\mathrm{C})_{16} = (1100)_2$, $(\mathrm{D})_{16} = (1101)_2$, $(\mathrm{E})_{16} = (1110)_2$, and $(\mathrm{F})_{16} = (1111)_2$, we have $(\mathrm{BADFACED})_{16} = (10111010110111111010110011101101)_2$. Following the convention shown in Exercise 3 of grouping binary digits by fours, we can write this in a more readable form as 1011 1010 1101 1111 1010 1100 1110 1101.

22. We can just add and multiply using the grade-school algorithms (working column by column starting at the right), using the addition and multiplication tables in base three (for example, $2 + 1 = 10$ and $2 \cdot 2 = 11$). When a digit-by-digit answer is too large to fit (i.e., greater than 2), we "carry" into the next column. Note that we can check our work by converting everything to decimal numerals (the check is shown in parentheses below). A calculator or computer algebra system makes doing the conversions tolerable. For convenience, we leave off the "3" subscripts throughout.

a) $112 + 210 = 1022$ (decimal: $14 + 21 = 35$)
$112 \cdot 210 = 101{,}220$ (decimal: $14 \cdot 21 = 294$)

b) $2112 + 12021 = 21{,}210$ (decimal: $68 + 142 = 210$)
$2112 \cdot 12021 = 111{,}020{,}122$ (decimal: $68 \cdot 142 = 9656$)

c) $20001 + 1111 = 21{,}112$ (decimal: $163 + 40 = 203$)
$20001 \cdot 1111 = 22{,}221{,}111$ (decimal: $163 \cdot 40 = 6520$)

d) $120021 + 2002 = 122{,}100$ (decimal: $412 + 56 = 468$)
$120021 \cdot 2002 = 1{,}011{,}122{,}112$ (decimal: $412 \cdot 56 = 23{,}072$)

  - 4.3 – 2, 4, 16, 24, 26

2. The numbers 19, 101, 107, and 113 are prime, as we can verify by trial division. The numbers $27 = 3^3$ and $93 = 3 \cdot 31$ are not prime.

4. We obtain the answers by trial division. The factorizations are $39 = 3 \cdot 13$, $81 = 3^4$, $101 = 101$ (prime), $143 = 11 \cdot 13$, $289 = 17^2$, and $899 = 29 \cdot 31$.

16. Since these numbers are small, the easiest approach is to find the prime factorization of each number and look for any common prime factors.

a) Since $21 = 3 \cdot 7$, $34 = 2 \cdot 17$, and $55 = 5 \cdot 11$, these are pairwise relatively prime.

b) Since $85 = 5 \cdot 17$, these are not pairwise relatively prime.

c) Since $25 = 5^2$, 41 is prime, $49 = 7^2$, and $64 = 2^6$, these are pairwise relatively prime.

d) Since 17, 19, and 23 are prime and $18 = 2 \cdot 3^2$, these are pairwise relatively prime.

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Exercises from Book:

- Chapter 4
  - 4.3 – 24, 26

24. We form the greatest common divisors by finding the minimum exponent for each prime factor.
a) $2^2 \cdot 3^3 \cdot 5^2$ **b)** $2 \cdot 3 \cdot 11$ **c)** 17 **d)** 1 **e)** 5 **f)** $2 \cdot 3 \cdot 5 \cdot 7$

26. We form the least common multiples by finding the maximum exponent for each prime factor.
a) $2^5 \cdot 3^3 \cdot 5^5$ **b)** $2^{11} \cdot 3^9 \cdot 5 \cdot 7 \cdot 11 \cdot 13 \cdot 17^{14}$ **c)** 17^{17} **d)** $2^2 \cdot 5^3 \cdot 7 \cdot 13$
e) undefined (0 is not a positive integer) **f)** $2 \cdot 3 \cdot 5 \cdot 7$

  - 4.6 – 2, 4

2. These are straightforward arithmetical calculations, as in Exercise 1.
a) WXST TSPPYXMSR **b)** NOJK KJHHPODJI **c)** QHAR RABBYHCAJ

4. We just need to "subtract 3" from each letter. For example, E goes down to B, and B goes down to Y.
a) BLUE JEANS **b)** TEST TODAY **c)** EAT DIM SUM