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Instructions

Please read the following instructions carefully:

1. Please show all notation for probability statements.
2. Box your final answers.
3. Please verify that your scans are legible.
4. Please assign pages to the questions when submitting to Gradescope.
5. This assignment is due via Gradescope on the due date.

1. In each part of the problem, state the support, $\mathbb{S}_X$, of X.

   (a) You are playing basketball and during the game you are given 5 free throws to shoot. Let X be the number of made shots you scored when you did the free throws.

   $\boxed{S_x = \{0, 1, 2, 3, 4, 5\}}$

   (b) You ask 8 people whether they prefer hats, beanies, or neither. Let X be the number of individuals that preferred neither.

   $\boxed{S_x = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}}$

   (c) You are visiting the department store, you are interested in purchasing a t-shirt for \$30, pants for \$50 and some socks for \$15. Let X represent the amount of money you spent if you purchased 2 items.

   $\boxed{S_x = \{45, 65, 80\}}$

   (d) Continuing from the previous part. Let X represent the amount of money you spent if you purchased 2 items at most.

   $\boxed{S_x = \{0, 15, 30, 50, 45, 65, 80\}}$

   (e) Continuing from the previous part. Let X represent the amount of money you spent if you purchased 2 or 3 items.

   $\boxed{S_x = \{45, 65, 80, 95\}}$

2. It is recommended that adults eat at least 4 pieces of fruit or vegetables each day. Let X be the number of pieces of fruit or veg that a random person, Paul, eats in a given day. The following table shows the **pmf** of X:

x	1	2	3	4	5
$P(X = x)$	0.23	0.20	0.19	?	.21

(a) Assuming the pmf is a valid one, what is the probability that Paul eats 4 pieces of fruit or veg in a given day?

$$P(X=4) = 1 - 0.23 - 0.20 - 0.19 - 0.21$$
$$= \boxed{0.17}$$

(b) Now that you have solved for the probability that Paul ate 4 pieces of fruit or veg, make an argument as to why this is a valid pmf.

The total of pmf of X is equal to 1.

(c) What is the probability that Paul ate less than 3 pieces of fruit or veg a day?

$$P(X < 3) = P(X \leq 2) = P(X=1) + P(X=2)$$
$$= 0.23 + 0.20$$
$$= \boxed{0.43}$$

(d) What is the probability that ~~Pete~~ Paul ate at least 4 pieces of fruit or veg on a random day?

$$P(X \geq 4) = P(X=4) + P(X=5)$$
$$= 0.17 + 0.21$$
$$= \boxed{0.38}$$

(e) What is the expected number of pieces of fruit or veg that Paul eats a day?

$$E(X) = 1(0.23) + 2(0.20) + 3(0.19) + 4(0.17) + 5(0.21)$$

$$= \boxed{2.93}$$

(f) What is the standard deviation of the number of pieces of fruit or veg that Paul eats a day?

$$E(X^2) = 1(0.23) + 4(0.20) + 9(0.19) + 16(0.17) + 25(0.21)$$

$$= 10.71$$

$$\sigma_X = \sqrt{VAR(X)} = \sqrt{E(X^2) - (E(X))^2}$$

$$= \sqrt{10.71 - (2.93)^2}$$

$$= \sqrt{2.1251}$$

$$= \boxed{1.4578}$$

3. You are just starting Fall Quarter 2023 at UCI. Some days of the week you have classes that start at 9am. Let Y be the number of days each week where your day starts at 9am. The following table shows the **cdf** of Y:

y	0	1	2	3	4	5
$P(Y \le y)$	0.09	0.25	0.47	0.61	0.82	1
$P(Y=y)$	0.09	0.16	0.22	0.14	0.21	0.18

(a) What is the probability that you have 2 or fewer days that start at 9am?

$P(Y \le 2) = \boxed{0.47}$

(b) What is the probability that you have exactly 3 days that start at 9am each week?

$P(Y \le 3) - P(Y \le 2) = 0.61 - 0.47$

$= \boxed{0.14}$

(c) What is the probability that you start at 9am more than 1 days a week?

$P(Y > 1) = 1 - P(Y \le 1)$

$= 1 - 0.25$

$= \boxed{0.75}$

(d) What is the expected number of days a week that you will start at 9am?

$E(X) = 0(0.09) + 1(0.25 - 0.09) + 2(0.47 - 0.25) + 3(0.61 - 0.47)$
$+ 4(0.82 - 0.61) + 5(1 - 0.82)$

$= 0 + 1(0.16) + 2(0.22) + 3(0.14) + 4(0.21) + 5(0.18)$

$= \boxed{2.76}$

(e) What is the standard deviation of the ranking given by the students?

$E(X^2) = 0(0.09) + 1(0.16) + 4(0.22) + 9(0.14) + 16(0.21) + 25(0.18)$

$= 10.16$

$\sigma_x = \sqrt{E(X^2) - (E(X))^2}$

$= \sqrt{10.16 - (2.76)^2}$

$= \sqrt{10.16 - 7.6176}$

$= \sqrt{2.5424}$

$= \boxed{1.5945}$

4. You are an urban planning student and are tasked with a project looking at the different ways students travel around campus around Aldrich Park. You survey students around 2 areas of campus, outer ring road and inner ring road. You also note down whether students are walking or not (e.g. using a skateboard/scooter/bike.) You surveyed a total of 420 students.

	Inner	Outer	Total
Walking	55	102	157
Not Walking	66	197	263
Total	121	299	420

Suppose a random participant is selected at random from the 430 students that you surveyed. Use the following notation for events: **W** = Walking, **NW** = Not Walking, **I** = Inner Ring Road, **O** = Outer Ring Road.

(a) Fill in the missing cells.

(b) Find the probabilities for each simple event. (i.e find $P(W), P(NW), P(I),$ and $P(O)$)

$P(W) = \frac{157}{420} = 0.3738$ $\quad$ $P(I) = \frac{121}{420} = \boxed{0.2881}$

$P(NW) = \frac{263}{420} = 0.6262$ $\quad$ $P(O) = \frac{299}{420} = \boxed{0.7119}$

(c) What is the probability that a student was "walking" AND was on the "inner ring road"?

$P(W \cap I) = \frac{55}{420} = \boxed{0.1310}$

(d) Are the events "walking" and "inner ring road" mutually exclusive? Why or why not?

$P(W \cap I) = 0.1310$

It is not equal to 0

$\boxed{\text{therefore they are not mutually exclusive}}$

(e) Are the events "walking" and "inner ring road" independent? Why or why not?

$P(W \cap I) \overset{?}{=} P(W) \cdot P(I)$

$0.1310 \overset{?}{=} 0.3738 \cdot 0.2881$

$0.1310 \neq 0.1077$

$\boxed{\therefore \text{ they are dependent}}$

(f) Given that a student was on the outer ring road, what is the probability that they were not walking?

$P(NW|O) = \frac{197}{299} = \boxed{0.6589}$

5. It is welcome week at UCI, and you made a list of 7 organizations that you are interested in joining, but only have time to visit 3 of them when you attend the involvement fair.

   (a) How many different orders can you visit 3 of the organizations in.

$$P_{n,r} = P_{7,3} = \frac{7!}{(7-3)!} = \frac{7 \cdot 6 \cdot 5 \cdot 4!}{4!} = \boxed{210 \text{ different orders}}$$

   (b) Suppose 2 of the organizations are only open in the morning, another 2 are only open in the afternoon and the last 3 are open all-day. How many different ways can you attend 5 of the organizations if you do the 2 morning organizations in the morning and the 2 afternoon organizations in the afternoon, and an all-day organization either first, in between the morning /afternoon organizations or at the end of the day?

$$3 \cdot \frac{2!}{(2-1)!} \cdot \frac{2!}{(2-1)!} \cdot \binom{3}{1} = \boxed{36 \text{ different ways}}$$

   (c) You need to leave the involvement fair because you feel unwell, but you come back the next day to learn that all of the organizations are open all day and so the **order in which you visit the organizations no longer matters**. There are also 3 new organizations that are outdoor based, e.g. hiking club, rock climbing club, snorkel club. You only have time to visit 8 organizations. How many possible combinations are there for you to visit 2 of the outdoor based organizations and 6 of the non-water based organizations?

$$\text{outdoor} = \binom{3}{2}$$

$$\text{non-water} = \binom{7}{6}$$

$$\text{total ways} = \binom{3}{2} \cdot \binom{7}{6} = \frac{3!}{2!(3-2)!} \cdot \frac{7!}{6!(7-6)!}$$

$$= \boxed{21 \text{ combinations}}$$

6. You want to put some posters up in your dorm. You have a wall that fits 8 posters but you brought 15 posters with you.
   (a) How many different posters combinations can you put up on your wall? (The locations of the posters do not matter.)

   $$C_{n,r} = C_{15,8} = \binom{15}{8} = \boxed{6435 \text{ combinations}}$$

   (b) You have 10 posters from concerts you've attended and 5 from your favorite tv shows. Out of the 8 posters that you can put up on your wall, you want 6 of concerts and 2 of tv shows. Now, how many possible combinations of posters can you put up?

   $$\binom{10}{6} \cdot \binom{5}{2} = \boxed{2100 \text{ combinations}}$$

   (c) After you spend some time focusing on school you realise you miss the good vibes of the concerts and want to have **at least** 7 of the posters be from concerts. How many different photo combinations can you put up now?

   Case 1: All 8 posters from concerts

   Case 2: 7 posters from concerts, 1 from TV shows

   $$\binom{5}{0}\binom{10}{8} + \binom{5}{1}\binom{10}{7} = 1 \cdot 45 + 5 \cdot 120 = \boxed{645 \text{ combinations}}$$

   case 1 + case 2

   (d) What is the probability that if you randomly chose a selection of 8 posters, that at least 7 of them would be concert posters?

   $$P(X \geq 7) = \frac{645}{6435} = \boxed{0.1002}$$

   (e) What is the probability if you randomly chose the posters you would get all of the concert posters?

   $$P(X = 8) = \frac{45}{6435} = \boxed{0.0070}$$