ICS 6B F23 Take Home Exam 3

Due: October 20th, 2023 at 11:59PM

Name: _____

UCI NetID : | | | | | | | | |

(**alpha-numeric; NOT your student ID**)

- **Read** the instructions of each question carefully.
- **All problems will have a "What to show" section that will describe exactly what work is expected of you we solving the problem. Failure to meet the requirements of the "What to show" sections will result in a Not Yet. If you have questions about what to show please ask on Ed.**
- An answer where thought process is unclear will be given a grade of Not Yet
- Your submission should follow the template exactly. Any insertion, removal, or reordering of pages from the original template may result in readers not grading certain problems. In such an event you will receive "Not Yet" and no feedback on the problems in question.
- Place your answers in the boxed regions. Writing outside of the boxes will not be considered as part of your answers.
- This exam will cover the Outcomes from the P Learning Objective
- Please keep in mind of the academic honesty guidelines. This take-home exam is to be **completed individually, with no outside help**. You may use any resources from our class (ZyBooks and resources from Canvas), but you may not use any other online resources.
- You may choose to print the exam or use a digital editor for completing the exam. It is required that you use this PDF to complete your work. If you have no access to a printer or digital tools to fulfill the exam, feel free to reach out to the staffs regarding your concern.
- If you have any questions, please post a private Ed or attend available Office Hours. Note that we are not allowed to provide specific help to answering the exam questions.

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Problem 1 (P1)

Using the predicates,

- A(x) : X got straight A's
- R(x) : x got a reward in their research
- H(x) : x graduated with honors
- W(x) : x became wealthy

Determine whether the argument is valid or invalid. Prove your answer using rules of inferences or a counterexample. If you are proving the argument valid, do so with the least number of hypotheses possible. Assume that both Jake and Susan are part of the domain of the problem.

What you need to show:

- If invalid, translate the argument to logic and provide a counter example (a set of truth values that make all hypotheses true but the conclusion false). Briefly (a sentence or two) explain why the counterexample makes the argument false.
- If valid, translate the argument to logic and provide a list of step where each step is an expression you derived to be true. Each step/line should have the following format (3 parts):
  - Step #
  - Logical expression that was derived to be true in this step
  - Justification (can be one of the following: "hypothesis" if the expression is a hypothesis of the argument, "variable declaration" if you are simply defining an arbitrary or specific variable, or rule/law name followed by the step numbers of the expressions the rule/law was applied too)

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a) Jake did not get a reward for research but he still became wealthy

b) Susan became wealthy only if Jake got straight A's

c) Everyone either got a reward for their research or graduated with honors

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d) $\therefore$ Someone graduated with honors

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Problem 2 (P2)

a) Consider that you were asked to prove the theorem, If for real numbers x and y, $x+y \leq 110$, then either $x \leq 46$ or $y \leq 64$ by contrapositive. State the assumption you would make to start the proof and the conclusion you would show to finish it.

What you need to show: Stating one sentence for the assumption and one sentence for the conclusion is sufficient.

b) Consider that you were ask to proved that all numbers can be written as a product of prime numbers. State the assumption you would make to start the proof if you were proving it by contradiction. (Note the proof itself is beyond 6B material, but stating the assumption is within what we have covered.)

What you need to show: Stating one sentence for the assumption is sufficient.

Problem 3 (P3)

Identify and explain the mistake in the proof below for the following statement. Note that pairwise distinctness is defined below in the assumptions for you.

For any pairwise distinct positive integers $x, y, z \in \mathbb{Z}^+$, there exists some integer m where m is equivalent to x, y, or z such that m is smaller than x, y, and z.

Assuming: x, y, z are pairwise distinct positive integers, i.e., $x \neq y, y \neq z, x \neq z$.

WTP: We will show that there exists an integer m among integers chosen that is smaller than x, y, and z.

Proof: Without loss of generality, let us assume x is the smallest integer among x, y, z, i.e., $x < y, x < z$. We can do this because whichever the smallest integer is of the three, we can just rename it to x and use its name for the integer that was originally x (essentially swapping their names).

Conclusion: Therefore, we have our smallest integer, which is x.

What you need to show: Identify the mistake. If there is more than 1 mistake present, find the earliest mistake in the proof. Then give a 1 or 2 sentence explanation why the logic is flawed.

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Problem 4.1 (P4)

Prove that the expression $9n^2 + 3n + 6$ is divisible by 6 for any integer n.

What you need to show: Follow the proof framework (it is outlined in on the answer sheet). Remember proofs are formal structures, so do not leave anything out.

- If you use a definition, you must state it.
- If you use a property such as integers being closed under addition, you must state it.
- If a variable appears in your proof it must have been defined previously, including specifying the type of value, e.g., integers, a real number, a person, etc.

Problem 4.2 (P4)

An incoming class of 982 freshman need to be assigned to 320 housing units. Prove that there will be a freshman with at least 3 roommates.

What you need to show: Follow the proof framework (it is outlined in on the answer sheet). Remember proofs are formal structures, so do not leave anything out.

- If you use a definition, you must state it.
- If you use a property such as integers being closed under addition, you must state it.
- If a variable appears in your proof it must have been defined previously, including specifying the type of value, e.g., integers, a real number, a person, etc.