Maximum Liklihood Estimator

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Estimating Parameters

Suppose $X_1, \ldots, X_n$ is a random sample from a continuous pdf $f_X(x)$ whose unknown parameter is θ.

The question is, how should we use the data to approximate θ?

Method of Maximum Likelihood

The likelihood function $L(\theta)$ is the product of the pdf $f_X(x;\theta)$ evaluated at the n data points. That is,

$$L(\theta) = \prod_{i=1}^{n} f_X(x_i;\theta)$$

L is a function of θ; it should not be considered a function of the $x_i's$.

MLE

Let $X_1, \ldots, X_n$ is a random sample from a continuous pdf $f_X(x;\theta)$ and let $L(\theta)$ be the corresponding likelihood function. Suppose $L(\hat{\theta}) \geq L(\theta)$ for all possible values of θ. The $\hat{\theta}$ is called the *maximum likelihood estimate (MLE) for* θ

Note

Finding $\hat{\theta}$ that maximizes a likelihood function is basically an exercise in Calculus. Since $\ln L(\theta)$ increases with $L(\theta)$, the same $\hat{\theta}$ that maximizes the ln of the likelihood function maximizes the likelihood function.

Maximum Likelihood Steps

1. Find $L(\theta)$
2. Find $\ln L(\theta)$
3. Calculate the Score Function $= \dfrac{\partial \ln L(\theta)}{\partial \theta}$
4. Find the Score equation by setting the Score Function $\equiv 0$
5. Solve for the parameter
6. Check that the second derivative of the $\ln L(\theta)$ is negative, that is $\dfrac{\partial^2 \ln L(\theta)}{\partial \theta^2} < 0$
7. Check the support

Ex: Suppose $x_1, \ldots, x_n$ is a set of n observations representing a Bernoulli probability model.

- Find the MLE of p.
- Assume you see the data $\{0, 1, 0, 0, 1, 1, 0, 0, 1, 0\}$ from Bernoulli trials Use the Maximum Liklihood Estimator to estimate the true population parameter p.

EX: Suppose $x_1, \ldots, x_n$ are i.i.d. random variables with density function:

$$f_X(x;\sigma) = \frac{1}{2\sigma} e^{-\frac{|x|}{\sigma}}$$

Find the maximum liklihood estimate of σ