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HW 7 4.7

Given the coordinate matrix of x relative to a (nonstandard) basis B for Rn, find the coordinate matrix of x relative to the standard basis

7.) B = {(1,0,1),(1,1,0),(0,1,1)},

$$[x]_B = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$$

$2(1,0,1) + 3(1,1,0) + 1(0,1,1)$

$= (2,0,2) + (3,3,0) + (0,1,1)$

$= (5,4,3)$

Find the coordinate matrix of x in Rn relative to the basis B'

13.) B' = {(8,11,0),(7,0,10),(1,4,6)}, x = (3,19,2)

Suppose $c_1(8,11,0) + c_2(7,0,10) + c_3(1,4,6) = (3,19,2)$

$(8c_1, 11c_1, 0) + (7c_2, 0, 10c_2) + (c_3, 4c_3, 6c_3) = (3,19,2)$

$(8c_1 + 7c_2 + c_3,\ 11c_1 + 0 + 4c_3,\ 0 + 10c_2 + 6c_3) = (3,19,2)$

$$\begin{aligned} 8c_1 + 7c_2 + c_3 &= 3 \\ 11c_1 + 0 + 4c_3 &= 19 \\ 0 + 10c_2 + 6c_3 &= 2 \end{aligned} \Rightarrow \left[\begin{array}{ccc:c} 8 & 7 & 1 & 3 \\ 11 & 0 & 4 & 19 \\ 0 & 10 & 6 & 2 \end{array}\right] \overset{RREF}{\Rightarrow} \left[\begin{array}{ccc:c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$$\therefore [x]_B = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

Find the transition matrix from B to B'

19) B = {(2,4),(-1,3)}, B' = {(1,0),(0,1)}

$$[B'\ B] \Rightarrow \begin{bmatrix} 1 & 0 & 2 & -1 \\ 0 & 1 & 4 & 3 \end{bmatrix}$$

$$P^{-1} = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$$

21.) B = {(-1,0,0),(0,1,0),(0,0,-1)}, B' = {(0,0,2),(1,4,0),(5,0,2)}

$$[B' \quad B] = \begin{bmatrix} 0 & 1 & 5 & -1 & 0 & 0 \\ 0 & 4 & 0 & 0 & 1 & 0 \\ 2 & 0 & 2 & 0 & 0 & -1 \end{bmatrix} \xRightarrow{\text{RREF}} \begin{bmatrix} 1 & 0 & 0 & 0.2 & 0.05 & -0.5 \\ 0 & 1 & 0 & 0 & 0.25 & 0 \\ 0 & 0 & 1 & -0.2 & -0.05 & 0 \end{bmatrix}$$

$$P^{-1} = \begin{bmatrix} 0.2 & 0.05 & -0.5 \\ 0 & 0.25 & 0 \\ -0.2 & -0.05 & 0 \end{bmatrix}$$

23.) B = {(3,4,0),(-2,-1,1),(1,0,-3)}, B' = {(1,0,0),(0,1,0),(0,0,1)}

$$[B' \ B] = \begin{bmatrix} 1 & 0 & 0 & 3 & -2 & 1 \\ 0 & 1 & 0 & 4 & -1 & 0 \\ 0 & 0 & 1 & 0 & 1 & -3 \end{bmatrix}$$

$$P^{-1} = \begin{bmatrix} 3 & -2 & 1 \\ 4 & -1 & 0 \\ 0 & 1 & -3 \end{bmatrix}$$

(a) Find the transition matrix from B to B', (b) find the transition matrix from B' to B, (c) verify that the two transition matrices are inverses of each other, and (d) find the coordinate matrix [x]B, given the coordinate matrix [x]B'

37.) B = {(1,3),(-2,-2)}, B' = {(-12,0),(-4,4)}, [x]B' = $\begin{bmatrix} -1 \\ 3 \end{bmatrix}$

a.) $$[B' \quad B] = \begin{bmatrix} -12 & -4 & 1 & -2 \\ 0 & 4 & 3 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\frac{1}{3} & \frac{1}{3} \\ 0 & 1 & \frac{3}{4} & -\frac{1}{2} \end{bmatrix}$$

$$P^{-1} = \begin{bmatrix} -\frac{1}{3} & \frac{1}{3} \\ \frac{3}{4} & -\frac{1}{2} \end{bmatrix}$$

b.) $$[B \quad B'] = \begin{bmatrix} 1 & -2 & -12 & -4 \\ 3 & -2 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 6 & 4 \\ 0 & 1 & 9 & 4 \end{bmatrix}$$

$$P = \begin{bmatrix} 6 & 4 \\ 9 & 4 \end{bmatrix}$$

c.) $$(P^{-1})^{-1} = \begin{bmatrix} -\frac{1}{3} & \frac{1}{3} & 1 & 0 \\ \frac{3}{4} & -\frac{1}{2} & 0 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 4 & 1 & 0 \\ 9 & 4 & 0 & 1 \end{bmatrix}$$

$$(P)^{-1} = \begin{bmatrix} 6 & 4 & 1 & 0 \\ 9 & 4 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\frac{1}{3} & \frac{1}{3} \\ 0 & 1 & \frac{3}{4} & -\frac{1}{2} \end{bmatrix}$$

d.) $[B' \text{ to } B] \cdot [x]_{B'}$

$$= \begin{bmatrix} 6 & 4 \\ 9 & 4 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \end{bmatrix} = \begin{bmatrix} -6+12 \\ -9+12 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$$

Find the coordinate matrix of p relative to the standard basis for P3

45.) $p = 1 + 5x - 2x^2 + x^3$

$$\begin{bmatrix} 1 \\ 5 \\ -2 \\ 1 \end{bmatrix}$$

47.) $p = 13 + 114x + 3x^2$

$$\begin{bmatrix} 13 \\ 114 \\ 3 \\ 0 \end{bmatrix}$$

Find the coordinate matrix of X relative to the standard basis for M3,1

49.) $X = \begin{bmatrix} 0 \\ 3 \\ 2 \end{bmatrix}$

$$\begin{bmatrix} 0 \\ 3 \\ 2 \end{bmatrix}$$

5.2

Show that the function defines an inner product on R^2, where u = (u1,u2) and v = (v1,v2))

3) $\langle u, v \rangle = \frac{1}{2} u_1 v_1 + \frac{1}{4} u_2 v_2$

Positive Definite $\langle u,u \rangle = \langle (u_1,u_2), (u_1,u_2) \rangle = \frac{1}{2} u_1^2 + \frac{1}{4} u_2^2 \geq 0$ and $\frac{1}{2} u_1^2 + \frac{1}{4} u_2^2 = 0$ iff $(u_1,u_2) = (0,0)$

Symmetry $\langle u,v \rangle = \langle (u_1,u_2),(v_1,v_2) \rangle = \frac{1}{2} u_1 v_1 + \frac{1}{4} u_2 v_2$

$\langle v,u \rangle = \langle (v_1,v_2), (u_1,u_2) \rangle = \frac{1}{2} v_1 u_1 + \frac{1}{4} v_2 u_2$

Linearity $\langle u_1+u_2, v \rangle$ $\langle u_1, v \rangle + \langle u_2, v \rangle$

$= \langle (a,b) + (c,d), (e,f) \rangle$ $= \langle (a,b), (e,f) \rangle + \langle (c,d), (e,f) \rangle$

$= \langle (a+c, b+d), (e,f) \rangle$ $= \frac{1}{2} ae + \frac{1}{4} bf + \frac{1}{2} ce + \frac{1}{4} df$

$= \frac{1}{2} (a+c) e + \frac{1}{4} (b+d) f$ $= \frac{1}{2} ae + \frac{1}{2} ce + \frac{1}{4} bf + \frac{1}{4} df$

$= \frac{1}{2} (a+c) e + \frac{1}{4} (b+d) f$

$\therefore$ Yes

Show that the function defines an inner product on R^3, where u = (u1,u2,u3) and v = (v1,v2,v3)

7) $\langle u, v \rangle = 4 u_1 v_1 + 3 u_2 v_2 + 2 u_3 v_3$

Positive Definite $\langle u_1, u_1 \rangle = \langle (u_1,u_2,u_3), (u_1,u_2,u_3) \rangle = 4u_1^2 + 3u_2^2 + 2u_3^2 \geq 0$

and $4u_1^2 + 3u_2^2 + 2u_3^2 = 0$ iff $(u_1,u_2,u_3) = (0,0,0)$

Symmetry $\langle u,v \rangle = \langle (u_1,u_2,u_3)(v_1,v_2,v_3) \rangle = 4u_1v_1 + 3u_2v_2 + 2u_3v_3$

$\langle v,u \rangle = \langle (v_1,v_2,v_3),(u_1,u_2,u_3) \rangle = 4 v_1 u_1 + 3 v_2 u_2 + 2 v_3 u_3$

Linearity $\langle u+v, w \rangle = \langle (a,b,c) + (e,f,g), (h,i,j) \rangle$

$= \langle (a+e, b+f, c+g), (h,i,j) \rangle$

$= 4(a+e)h + 3(b+f)i + 2(c+g)j$

$\langle u,w \rangle + \langle v,w \rangle = \langle (a,b,c), (h,i,j) \rangle + \langle (e,f,g), (h,i,j) \rangle$

$= 4ah + 3bi + 2cj + 4eh + 3fi + 2gj$

$= 4(ah + eh) + 3(bi + fi) + 2(ci + gi)$

$= 4(a+e)h + 3(b+f)i + 2(c+g)i$

Show that the function does not define an inner product on R^3, where u = (u1,u2) and v = (v1,v2)

9.) $\langle u,v \rangle = u_1 v_1$

$\langle (0,1),(0,1) \rangle = 0$ but $(0,1) \neq (0,0)$

$\therefore$ does not define

11.) $\langle u,v \rangle = u_1^2 v_1^2 - u_2^2 v_2^2$

$\langle (0,1),(0,1) \rangle = 0^2 0^2 - 1^2 1^2 = 0-1 = -1 \neq 0$

$\therefore$ does not define

Find (a) <u,v>, (b) ||u||, (c) ||v||, and d(u,v) for the given inner product defined by R^n

21.) $u = (0,7,2)$, $v = (9,-3,-2)$, $\langle u,v \rangle = u \cdot v$

a.) $\langle u,v \rangle = \langle (0,7,2),(9,-3,-2) \rangle = (0,-21,-4) = 0+(-21)+(-4) = -25$

b.) $\|u\| = \sqrt{\langle u,u \rangle} = \sqrt{\langle (0,7,2),(0,7,2) \rangle} = \sqrt{(0,49,4)} = \sqrt{0+49+4} = \sqrt{53}$

c.) $\|v\| = \sqrt{\langle v,v \rangle} = \sqrt{\langle (9,-3,-2),(9,-3,-2) \rangle} = \sqrt{(81,9,4)} = \sqrt{81+9+4} = \sqrt{94}$

d.) $d(u,v) = \|u-v\| = \sqrt{\langle u-v,u-v \rangle} = \sqrt{\langle (0,7,2)-(9,-3,-2),(0,7,2)-(9,-3,-2) \rangle} = \sqrt{\langle (-9,10,4),(-9,10,4) \rangle}$

$= \sqrt{(81,100,16)} = \sqrt{81+100+16} = \sqrt{197}$

23.) $u = (8,0,-8)$, $v = (8,3,16)$, $\langle u,v \rangle = 2u_1v_1 + 3u_2v_2 + u_3v_3$

a.) $\langle u,v \rangle = 2(8)(8) + 3(0)(3) + (-8)(16) = 128 + 0 - 128 = 0$

b.) $\|u\| = \sqrt{\langle u,u \rangle} = \sqrt{2(8)(8) + 0 + (-8)(-8)} = \sqrt{128+64} = \sqrt{192} = 8\sqrt{3}$

c.) $\|v\| = \sqrt{\langle v,v \rangle} = \sqrt{2(8)(8) + 3(3)(3) + (16)(16)} = \sqrt{128+27+256} = \sqrt{411}$

d.) $d(u,v) = \|u-v\| = \sqrt{\langle u-v,u-v \rangle} = \sqrt{\langle (0,-3,-24),(0,-3,-24) \rangle}$ $\quad u-v = (0,-3,-24)$

$= \sqrt{2(0)(0) + 3(-3)(-3) + (-24)(-24)}$

$= \sqrt{0+27+576} = \sqrt{603} = 3\sqrt{67}$

Find (a) <p,q>, (b) ||p||, (c) ||q||, and (d) d(p,q) for the polynomials in P2 using the inner product <p,q> = a0b0 + a1b1 + a2b2

37.) $p(x) = 1 + x^2 \,, \quad q(x) = 1 - x^2$

a.) $\langle p, q \rangle = (1)(1) + (0)(0) + (1)(-1) = 0$

b.) $\|p\| = \sqrt{\langle p,p \rangle} = \sqrt{(1)(1) + 0 + (1)(1)} = \sqrt{2}$

c.) $\|q\| = \sqrt{\langle q,q \rangle} = \sqrt{(1)(1) + 0\,(-1)(-1)} = \sqrt{2}$

d.) $d(p,q) = \|p-q\| = \sqrt{\langle p-q,\, p-q \rangle} = \sqrt{\langle 1+x^2-(1-x^2),\, 1+x^2-(1-x^2) \rangle} = \sqrt{\langle 2x^2, 2x^2 \rangle}$

$= \sqrt{0 + 0 + (2)(2)} = \sqrt{4} = 2$

Use the function f and g in C[-1,1] to find (a) <f,g>, (b) ||f||, (c) ||g||, and (d) d(f,g) for the inner product

$$\langle f,g \rangle = \int_{-1}^{1} f(x)\, g(x)\, dx$$

41.) a.) $f(x) = x \,, \quad g(x) = e^x$

$$\begin{array}{ccc} x & & e^x \\ & \xrightarrow{+} & e^x \\ 1 & & \\ & \xrightarrow{-} & e^x \\ 0 & & \end{array}$$

$\langle f, g \rangle = \int_{-1}^{1} x e^x \, dx$

$= x e^x - e^x \Big|_{-1}^{1}$

$= e - e \quad - \left( (-1)e^{-1} - e^{-1} \right)$

$= 2e^{-1} = \frac{2}{e}$

b.) $\|f\| = \sqrt{\langle f,f \rangle} = \sqrt{\int_{-1}^{1} x \, x \, dx}$

$= \sqrt{\int_{-1}^{1} x^2 \, dx}$

$= \sqrt{\frac{x^3}{3} \Big|_{-1}^{1}}$

$= \sqrt{\frac{1}{3} - \left(-\frac{1}{3}\right)} = \sqrt{\frac{2}{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{6}}{3}$

c.) $\|g\| = \sqrt{\langle g,g \rangle} = \sqrt{\int_{-1}^{1} e^x \, e^x \, dx}$

$u = 2x$

$= \sqrt{\int_{-1}^{1} e^{2x} \, dx}$

$du = 2\,dx$

$= \sqrt{\int_{-1}^{1} \frac{1}{2} e^u \, du}$

$\frac{du}{2} = dx$

$= \sqrt{\frac{1}{2} \int_{-1}^{1} e^u \, du}$

$= \sqrt{\frac{1}{2} \left( e^u \Big|_{-1}^{1} \right)}$

$= \sqrt{\frac{1}{2} \left( e^{2x} \Big|_{-1}^{1} \right)}$

$= \sqrt{\frac{1}{2} \left( e^2 - e^{-2} \right)}$

$= \sqrt{\frac{e^2}{2} - \frac{1}{2e^2}}$

d.) $d(f,g) = \|f-g\| = \sqrt{\langle f-g, f-g\rangle}$

$= \sqrt{\langle x-e^x, x-e^x\rangle}$

$= \sqrt{\int_{-1}^{1} (x-e^x)^2\, dx}$

$= \sqrt{\int_{-1}^{1} (x^2 - 2xe^x + e^{2x})\, dx}$

$= \sqrt{\frac{x^3}{3} - 2(xe^x - e^x) + \frac{1}{2}e^{2x} \Big|_{-1}^{1}}$

$= \sqrt{\frac{1}{3} - 2(e-e) + \frac{1}{2}e^2 - \left(-\frac{1}{3} - 2(-e^{-1} - e^{-1}) + \frac{1}{2}e^{-2}\right)}$

$= \sqrt{\frac{1}{3} + \frac{1}{2}e^2 + \frac{1}{3} - \frac{4}{e} - \frac{1}{2e^2}}$

$= \sqrt{\frac{e^2}{2} + \frac{2}{3} - \frac{4}{e} - \frac{1}{2e^2}}$

Find the angle $\theta$ between the vectors

45.) $u = (-4,3)$, $v = (0,5)$, $\langle u,v\rangle = 3u_1v_1 + u_2v_2$

a.) $\langle p,q\rangle = 3(-4)(0) + (3)(5) = 15$

b.) $\|u\| = \sqrt{\langle u,u\rangle} = \sqrt{3(-4)(-4)+(3)(3)} = \sqrt{48+9} = \sqrt{57}$

c.) $\|v\| = \sqrt{\langle v,v\rangle} = \sqrt{3(0)(0)+(5)(5)} = \sqrt{25} = 5$

$\theta = \cos^{-1}\left(\frac{\langle p,q\rangle}{\|u\|\,\|v\|}\right) = \cos^{-1}\left(\frac{15}{5\sqrt{57}}\right) \approx 66.59^\circ$

49.) $p(x) = 1 - x + x^2$, $q(x) = 1 + x + x^2$, $\langle p,q\rangle = a_0b_0 + a_1b_1 + a_2b_2$

a.) $\langle p,q\rangle = (1)(1) + (-1)(1) + (1)(1) = 1$

b.) $\|p\| = \sqrt{\langle p,p\rangle} = \sqrt{(1)(1)+(-1)(-1)+(1)(1)} = \sqrt{3}$

c.) $\|q\| = \sqrt{\langle q,q\rangle} = \sqrt{(1)(1)+(1)(1)+(1)(1)} = \sqrt{3}$

$\theta = \cos^{-1}\left(\frac{\langle p,q\rangle}{\|p\|\,\|q\|}\right) \approx 70.53^\circ$

Show that f and g are orthogonal in the inner product space C[a,b] with inner product

$\langle f, g \rangle = \int_a^b f(x)\, g(x)\, dx$

65.) C[-pi/2,pi/2], f(x) = cos(x), g(x) = sin(x)

show $\langle f, g \rangle = 0$

$$\langle f, g \rangle = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x \sin x \, dx \qquad u = \sin x$$

$$= \int_{-\pi/2}^{\pi/2} u \, du \qquad du = \cos x \, dx$$

$$= \frac{u^2}{2} \Big|_{-\pi/2}^{\pi/2} \qquad \frac{du}{\cos x} = dx$$

$$= \frac{\sin^2 x}{2} \Big|_{-\pi/2}^{\pi/2}$$

$$= \frac{\sin^2(\frac{\pi}{2})}{2} - \frac{\sin^2(-\frac{\pi}{2})}{2}$$

$$= \frac{(1)^2}{2} - \frac{(-1)^2}{2}$$

$$= 0$$

$\therefore$ f and g are orthogonal

5.3

(a) determine whether the set of vectors in R^n is orthogonal, (b) if the set is orthogonal, then determine whether it is also orthonormal, and (c) determine whether the set is a basis for R^n

5 $\{\overset{v_1}{(4,-1,1)},\overset{v_2}{(-1,0,4)},\overset{v_3}{(-4,-17,-1)}\}$

a.) $\langle v_1, v_2 \rangle = \langle (4,-1,1),(-1,0,4) \rangle = (-4,0,4) = -4+4=0$

$\langle v_2, v_3 \rangle = \langle (-1,0,4),(-4,-17,-1) \rangle = (4,0,-4) = 4+(-4)=0$

$\langle v_1, v_3 \rangle = \langle (4,-1,1),(-4,-17,-1) \rangle = (-16,17,-1) = -16+17+(-1) = 0$

$\therefore$ Yes

b.) $\|v_1\|^2 = \sqrt{\langle v_1, v_1 \rangle}^2 = \langle (4,-1,1),(4,-1,1) \rangle = (16,1,1) = 16+1+1 = 18 \neq 1$

$\therefore$ No

c.) It is orthogonal

$\therefore$ It is a basis for $\mathbb{R}^3$

11.) $\{\overset{v_1}{(\frac{\sqrt{2}}{2},0,0,\frac{\sqrt{2}}{2})}, \overset{v_2}{(0,\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2},0)}, \overset{v_3}{(-\frac{1}{2},\frac{1}{2},-\frac{1}{2},\frac{1}{2})}\}$

a.) $\langle v_1, v_2 \rangle = \langle (\frac{\sqrt{2}}{2},0,0,\frac{\sqrt{2}}{2}),(0,\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2},0) \rangle = (0,0,0,0) = 0$

$\langle v_2, v_3 \rangle = \langle (0,\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2},0),(-\frac{1}{2},\frac{1}{2},-\frac{1}{2},\frac{1}{2}) \rangle = (0,\frac{\sqrt{2}}{4},-\frac{\sqrt{2}}{4},0) = \frac{\sqrt{2}}{4}-\frac{\sqrt{2}}{4} = 0$

$\langle v_1, v_3 \rangle = \langle (\frac{\sqrt{2}}{2},0,0,\frac{\sqrt{2}}{2}),(-\frac{1}{2},\frac{1}{2},-\frac{1}{2},\frac{1}{2}) \rangle = (-\frac{\sqrt{2}}{4},0,0,\frac{\sqrt{2}}{4}) = -\frac{\sqrt{2}}{4}+\frac{\sqrt{2}}{4} = 0$

$\therefore$ Yes

b.) $\|v_1\|^2 = \sqrt{\langle v_1, v_1 \rangle}^2 = \langle (\frac{\sqrt{2}}{2},0,0,\frac{\sqrt{2}}{2}),(\frac{\sqrt{2}}{2},0,0,\frac{\sqrt{2}}{2}) \rangle = (\frac{2}{4},0,0,\frac{2}{4}) = \frac{1}{2}+\frac{1}{2} = 1$

$\|v_2\|^2 = \sqrt{\langle v_2, v_2 \rangle}^2 = \langle (0,\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2},0),(0,\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2},0) \rangle = (0,\frac{2}{4},\frac{2}{4},0) = \frac{1}{2}+\frac{1}{2} = 1$

$\|v_3\|^2 = \sqrt{\langle v_3, v_3 \rangle}^2 = \langle (-\frac{1}{2},\frac{1}{2},-\frac{1}{2},\frac{1}{2}),(-\frac{1}{2},\frac{1}{2},-\frac{1}{2},\frac{1}{2}) \rangle = (\frac{1}{4},\frac{1}{4},\frac{1}{4},\frac{1}{4}) = \frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4} = 1$

$\therefore$ Yes

c.) $v_1, v_2, v_3 \notin \mathbb{R}^3$

$\therefore$ No

(a) show that the set of vectors in R^n is orthogonal, and (b) normalize the set to produce an orthonormal set

15.) $\{ \underset{v_1}{(\sqrt{3}, \sqrt{3}, \sqrt{3})}, \underset{v_2}{(-\sqrt{2}, 0, \sqrt{2})} \}$

a.) show $\langle v_1, v_2 \rangle = 0$

$\langle v_1, v_2 \rangle = \langle (\sqrt{3}, \sqrt{3}, \sqrt{3}), (-\sqrt{2}, 0, \sqrt{2}) \rangle = (-\sqrt{6}, 0, \sqrt{6}) = -\sqrt{6} + \sqrt{6} = 0$

$\therefore$ Yes

b.) $w_1 = \dfrac{v_1}{\|v_1\|} = \frac{1}{3}(\sqrt{3}, \sqrt{3}, \sqrt{3}) = (\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3})$

$w_2 = \dfrac{v_2}{\|v_2\|} = \frac{1}{2}(-\sqrt{2}, 0, \sqrt{2}) = (-\frac{\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2})$

$\{ (\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}), (-\frac{\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2}) \}$

$\|v_1\| = \sqrt{\langle (\sqrt{3}, \sqrt{3}, \sqrt{3}), (\sqrt{3}, \sqrt{3}, \sqrt{3}) \rangle}$

$= \sqrt{(3, 3, 3)} = \sqrt{3+3+3}$

$= \sqrt{9} = 3$

$\|v_2\| = \sqrt{\langle (-\sqrt{2}, 0, \sqrt{2}), (-\sqrt{2}, 0, \sqrt{2}) \rangle}$

$= \sqrt{(2, 0, 2)}$

$= \sqrt{2+2} = \sqrt{4} = 2$