Exercise 9.1

3. For each of these relations on the set {1, 2, 3, 4}, decide whether it is reflexive, whether it is symmetric, whether it is antisymmetric, and whether it is transitive.

a) {(2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)}

Transitive.

b) {(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)}

Reflexive, symmetric, transitive.

c) {(2, 4), (4, 2)}

Symmetric.

d) {(1, 2), (2, 3), (3, 4)}

Antisymmetric.

e) {(1, 1), (2, 2), (3, 3), (4, 4)}

Reflexive, symmetric, antisymmetric, transitive.

f) {(1, 3), (1, 4), (2, 3), (2, 4), (3, 1), (3, 4)}

None.

9. Show that the relation R = ∅ on the empty set S = ∅ is reflexive, symmetric, and transitive.

Since R is empty, R = {∅, ∅}, therefore R is reflexive, symmetric, and transitive.

Exercise 9.3

1. Represent each of these relations on {1, 2, 3} with a matrix (with the elements of this set listed in increasing order).

a) {(1, 1), (1, 2), (1, 3)}

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

3. List the ordered pairs in the relations on {1, 2, 3} corresponding to these matrices (where the rows and columns correspond to the integers listed in increasing order).

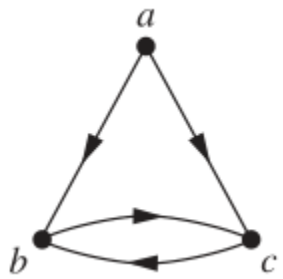
a) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

R = {(1, 1), (1, 3), (2, 2), (3, 1), (3, 3)}

7. Determine whether the relations represented by the matrices in Exercise 3 are reflexive, irreflexive, symmetric, antisymmetric, and/or transitive.

Reflexive, symmetric, and transitive.

23. list the ordered pairs in the relations represented by the directed graphs.



R = {(a, b), (a, c), (b, c), (c, b)}

Exercise 9.5

1. Which of these relations on {0, 1, 2, 3} are equivalence relations? Determine the properties of an equivalence relation that the others lack.

a) {(0, 0), (1, 1), (2, 2), (3, 3)}

Equivalence relation.

b) {(0, 0), (0, 2), (2, 0), (2, 2), (2, 3), (3, 2), (3, 3)}

It is lack of {1,1}, so it is not reflexive. It is not transitive since (0, 2) and (2, 3) are included but not (0,3).

c) {(0, 0), (1, 1), (1, 2), (2, 1), (2, 2), (3, 3)}

Equivalence relation.

d) {(0, 0), (1, 1), (1, 3), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)}

It is not transitive since (1, 3) and (3, 2) are included but not (1, 2).

e) {(0, 0), (0, 1), (0, 2), (1, 0), (1, 1), (1, 2), (2, 0), (2, 2), (3, 3)}

It is not symmetric since (1, 2) is included but not (2, 1). (1, 0) and (2, 0) are included but not (2, 1), so it is not transitive.