3.2 1. b) c)

In Exercises 1–14, to establish a big-O relationship, find witnesses C and k such that $|f(x)| \leq C|g(x)|$ whenever $x > k$.

1. Determine whether each of these functions is $O(x)$.

b) $f(x) = 3x + 7$

1 b) Yes

$|3x+7| \leq |4x| = 4|x| \quad \forall x > 7$

$C = 4$ & $k = 7$

c) $f(x) = x^2 + x + 1$

No.

There is no constant C s.t. $|x^2+x+1| \leq C|x|$ for all sufficiently large x.

3.3 3, 19 a)

3. Give a big-O estimate for the number of operations, where an operation is a comparison or a multiplication, used in this segment of an algorithm (ignoring comparisons used to test the conditions in the **for** loops, where $a_1, a_2, \ldots, a_n$ are positive real numbers).

```
m := 0
for i := 1 to n
    for j := i + 1 to n
        m := max(a_i a_j, m)
```

$\Longrightarrow$ Executed roughly $n^2/2$ times.

$\Longrightarrow$ The number of operations is $O(n^2)$

$m := 0$
for $i := 1$ to n
for $j := i+1$ to n.

$1+2+3+\ldots+(n-2)+(n-1)$

$= \frac{n(n-1)}{2}$ the number of total iterations

There are 2 operations per loop, i.e. comparison & multiplication, so the iteration is $2 \cdot \frac{n(n-1)}{2}$

$= n^2 - n$

$\Rightarrow f(n) = n^2 - n$

$\Rightarrow n^2 - n \leq n^2$ for $n > 1$

Hence, the algorithm is $O(n^2)$ with $C = 1$ & $k = 1$.

19. How much time does an algorithm using 2^{50} operations need if each operation takes these amounts of time?

the number of operations.

a) 10^{-6} s

$2^{50} \cdot 10^{-6}$

$\approx 1{,}125{,}899.907$ seconds $\cdot \frac{1 \text{ min}}{60 \text{ sec.}} \cdot \frac{1 \text{ HR.}}{60 \text{ min.}} \cdot \frac{1 \text{ day}}{24 \text{ HRS.}} \cdot \frac{1 \text{ month}}{30 \text{ days}} \cdot \frac{1 \text{ year}}{12 \text{ mon.}}$

approximate

≈ 36 years.

OR I can also use 365 days

I will get the same result.