CSCI 190 Discrete Mathematics Applied to Computer Science
Final Exam

Name : ______________________

Last 4 digits of your Student ID #: ______________________

Read these instructions before proceeding.

- Closed book. Closed notes. You can use calculator.
- You have **100 minutes** to complete this exam.
- No questions will be answered during the exam or immediately afterwards. Answer each question as best you can. Partial credit will be awarded for reasonable efforts. If a question contains an ambiguity or a misprint, then say so in your answer, providing the answer to a reasonable interpretation of the question; give your assumptions.
- Answer the problems on the blank spaces provided for each problem.
- **Box your answers**.

Q1 (11)	Q2 (12)	Q3 (12)	Q4 (8)	Q5 (12)	Q6 (8)	Q7 (6)	Q8 (6)	Q9 (6)	Q10 (6)	Q11 (4)	Q12 (5)	Q13 (4)	Total (100)

$p \to q$

contrapositive $\sim q \to \sim p$

converse $q \to p$

Inverse $\sim p \to \sim q$

1. (11 pts)

a) (3 pts) Write the converse of the following:

If you are happy, then you will smile.

If you smile, the you are happy

b) (4 pts) Convert $\mathbf{(9FA7)_{16}}$ to base 4.

$9 \cdot 16^3 + F \cdot 16^2 + A \cdot 16^1 + 7 \cdot 16^0 = 36864 + 3840 + 160 + 7$
$= 40871$

40871 mod 4 = 3
10217 mod 4 = 1
2554 mod 4 = 2

638 mod 4 = 2
159 mod 4 = 3
39 mod 4 = 3

9 mod 4 = 1
2

$(9FA7)_{16} = (21332213)_4$

c) (4 pts) A message has been ***encrypted*** using the function f**(x) = (x + 4) mod 26**.
If the message in coded form is **NSC**, decode the message.

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

$(N-4) \bmod 26 = J$
$(S-4) \bmod 26 = O$
$(C-4) \bmod 26 = Y$

JOY

2. (12 pts)

a) (5 pts) Use the Principle of Mathematical Induction to prove that
$2 + 4 + 6 + 8 + \cdots + 2n = n(n+1)$ for all $n \geq 1$. Show all the steps

$n = 1$

$2(1) = 1(1+1)$
$2 = 2$

$f(1)$ is true

Assume that $f(K)$ is true,
show that $f(K+1)$ is also true.

$2+4+6+8+\ldots 2K + 2(K+1) = K(K+1)(K+1+1)$

$2+4+6+8+\ldots 2K + 2(K+1) = (2(1)+2(2)+2(3)+2(4)+2K)+2(K+1)$
$= K(K+1) + 2(K+1)$
$= (K+2)(K+1)$
$= K^2 + 2K + 1 + K + 1$

b) (4 pts) Give a recursive definition with initial condition for the following function, square of n factorial.

$f(n) = (n!)^3$, $n = 0, 1, 2, \ldots.$

$a_0 = (0!)^3 = 1^3 = 1$
$a_1 = (1!)^3 = 1^3 = 1 = 1^3 \cdot f(0)$
$a_2 = (2!)^3 = 2^3 = 8 = 2^3 \cdot f(1)$
$a_3 = (3!)^3 = 6^3 = 216 = 3^3 \cdot f(2)$
$a_{n+1} = ((n+1)!)^3 = (n+1)^3 \cdot f(n)$

$\therefore a_{n+1} = (n+1)^3 \cdot f(n)$
for $n \geq 0$ and $a_0 = 1$

40

7

Probability of winning with x matching numbers.

$$\frac{\binom{7}{x}\cdot\binom{33}{7-x}}{\binom{40}{7}} = \frac{C(7,x)\cdot C(33,7-x)}{C(40,7)}$$

c) **(3 pts)** In a certain lottery game you choose a set of seven numbers out of 40 numbers. Find the probability that exactly one of your numbers match the seven winning numbers.

$$\frac{\binom{7}{1}\cdot\binom{33}{7-1}}{\binom{40}{7}} = \frac{\frac{7!}{1!\,6!}\cdot\frac{33!}{6!\,27!}}{\frac{40!}{7!\,33!}} = \frac{7\cdot\frac{\overset{11}{\cancel{33}}\cdot\overset{4}{\cancel{32}}\cdot31\cdot\cancel{30}\cdot29\cdot28}{\cancel{6}\cdot\cancel{5}\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}}}{\frac{\overset{5}{\cancel{40}}\cdot39\cdot38\cdot37\cdot\overset{2}{\cancel{36}}\cdot\cancel{35}\cdot34}{\cancel{7}\cdot\cancel{6}\cdot\cancel{5}\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot1}}$$

$$= \frac{7752976}{18643560} = 0.4159$$

3. (12 pts) Determine whether the following binary relation is:
(1) reflexive, (2) symmetric, (3) antisymmetric, (4) transitive.
No justifications needed.

a) (4 pts) The relation ***R*** on Z where ***aRb*** means ***a = b***.
Circle your answers.

R is	Reflexive?	Symmetric?	Antisymmetric?	Transitive?
	Yes or No	Yes or No	Yes or No	Yes or No

b) (4 pts) The relation ***R*** on the set of all people where ***aRb*** means that ***a*** is taller than ***b***.
Circle your answers.

R is	Reflexive?	Symmetric?	Antisymmetric?	Transitive?
	Yes or No	Yes or No	Yes or No	Yes or No

c) (4 pts) If $M_R = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$,

determine if ***R*** is: (a) reflexive (b) symmetric (c) antisymmetric (d) transitive.
Circle your answers.

diagonal = 1

R is	Reflexive?	Symmetric?	Antisymmetric?	Transitive?
	Yes or No	Yes or No	Yes or No	Yes or No

4. (8 pts)

a) (4 pts) Suppose ***R*** is the relation on *N* where ***aRb*** means that ***a*** starts in the same digit in which ***b*** starts.

Determine whether ***R*** is an ***equivalence relation*** on *N*. Justify your answer.

Reflexivity : aRa since a and a start from the same digit $\therefore$ reflexive

Symmetry : aRb is a and b start from the same digit and in bRa b and a also start from the same digit $\therefore$ symmetric

Transitivity : aRb is a and b start from the same digit
bRc is b and c start from the same digit
so aRc is also true $\therefore$ transitive

$\therefore$ R is an equivalence relation

b) (4 pts) Suppose the relation ***R*** is defined on the set *Z* where ***aRb*** means that ***ab < 0***.

Determine whether ***R*** is an ***equivalence relation*** on *Z*. Justify your answer.

Reflexivity : $aRa \equiv a \cdot a < 0$ but $a^2 \geq 0$ $\therefore$ Not reflexive

Symmetry : aRb is $ab < 0$ and bRa is $ba < 0$ $\therefore$ symmetric

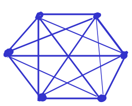
Transitivity : aRb is $ab < 0$ and bRc is $bc < 0$ but aRc $ac < 0$ is not always true $\therefore$ Not transitive

$\therefore$ R is not an equivalence relation

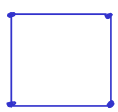
5. (12 pts)

a) (4 pts) Draw these four graphs. $\mathbf{K_6}$, $\mathbf{C_4}$, $\mathbf{W_5}$ and $\mathbf{K_{4,5}}$

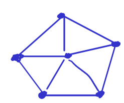
K_6



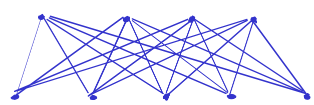
C_4



W_5



$K_{4,5}$



b) (4 pts)

K_n has $\frac{n(n-1)}{2} = 15$ edges and $n = 6$ vertices.

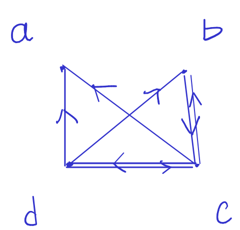
$K_{m,n}$ has $m \cdot n = 20$ edges and $m + n = 9$ vertices.

W_n has $2n = 10$ edges and $n = 5$ vertices.

C_n has $n = 4$ edges and $n = 4$ vertices.

c) (4 pts) Draw the ***digraph*** with adjacency matrix

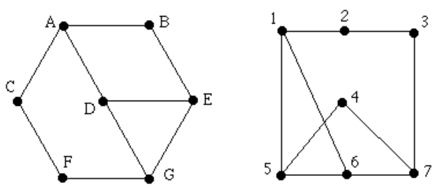
$$\begin{array}{cc} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} & \begin{array}{c} a \\ b \\ c \\ d \end{array} \end{array}$$

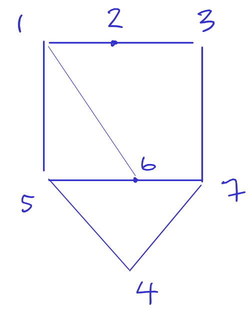


6. (8 pts)

a) (6 pts) Are these two graphs ***isomorphic***?

If yes, give the mapping of vertices from the first graph to the second graph.
If no, explain why not.





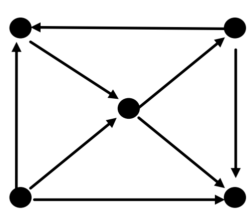
A = 7
B = 4
E = 5
G = 1
F = 2

C = 3
D = 6

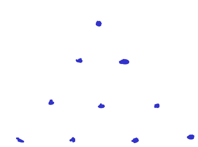
b) (2 pts) Circle ***Yes*** or ***No***. No justifications needed.

Determine whether the graph is ***strongly connected***? Yes or No

Determine whether the graph is ***weakly connected***. Yes or No



All strongly connected graph are also weakly connected.



7. (6 pts) Circle ***TRUE*** or ***FALSE***. No justifications needed.

T / F If T is a tree with 10 vertices, then there is a simple path in T of length 9.

T / F Every tree is bipartite. ← always true

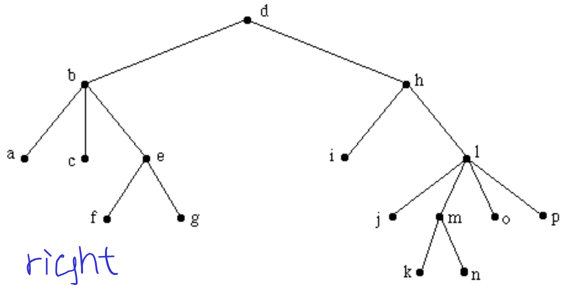
T / F There is a tree with degrees 4, 3, 2, 2, 1, 1, 1, 1, 1.

T / F There is a tree with degrees 3, 3, 3, 2, 1, 1, 1, 1.

T / F If T is a tree with 30 vertices, the largest degree that any vertex can have is 29.

T / F If two trees have the same number of vertices and the same degrees, then the two trees are isomorphic.

8. *(6 pts)* Refer to the following tree.



root left right

a) (2 pts) Find the ***preorder*** traversal.

d b a c e f g h i l j m k n o p

left root right

b) (2 pts) Find the ***inorder*** traversal.

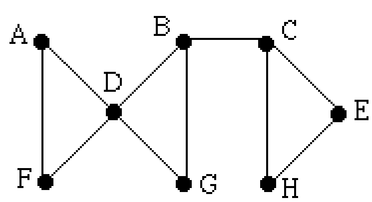
a b c f e g d i h j k m n l o p

left right root

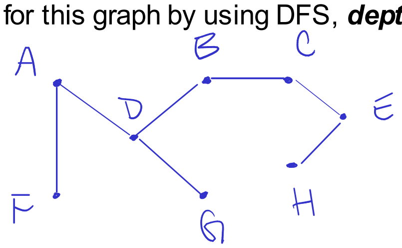
c) (2 pts) Find the ***postorder*** traversal.

a c f g e b i j k n m o p l h d

9. *(6 pts)* Refer to the following graph..

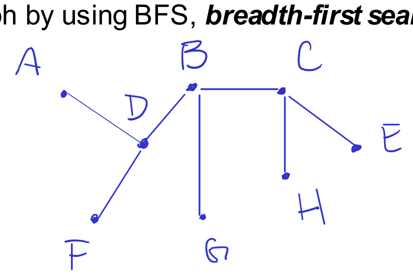


a) (3 pts) Using ***alphabetical ordering***, find ***a spanning tree*** (starting from vertice ***B***) for this graph by using DFS, ***depth-first search***. preorder



B C E H D A F G

b) (3 pts) Using ***alphabetical ordering***, find ***a spanning tree*** (starting from vertice ***B***) for this graph by using BFS, ***breadth-first search***. level order



B C D G E H A F

10. (6 pts) Using a table to show that ***F(x,y,z) = xyz + xy + x*** has a vaule of 1 if and only if variable ***x*** has a value of 1.

x	y	z	xyz	xy	$xyz+xy+x$
1	1	1	1	1	1
1	1	0	0	1	1
1	0	1	0	0	1
1	0	0	0	0	1
0	1	1	0	0	0
0	1	0	0	0	0
0	0	1	0	0	0
0	0	0	0	0	0

$\therefore F(x,y,z) = xyz + xy + x$ has a value of 1 if and only if variable x has a value of 1.

11. (4 pts) Find the ***duals*** of these Boolean expressions.

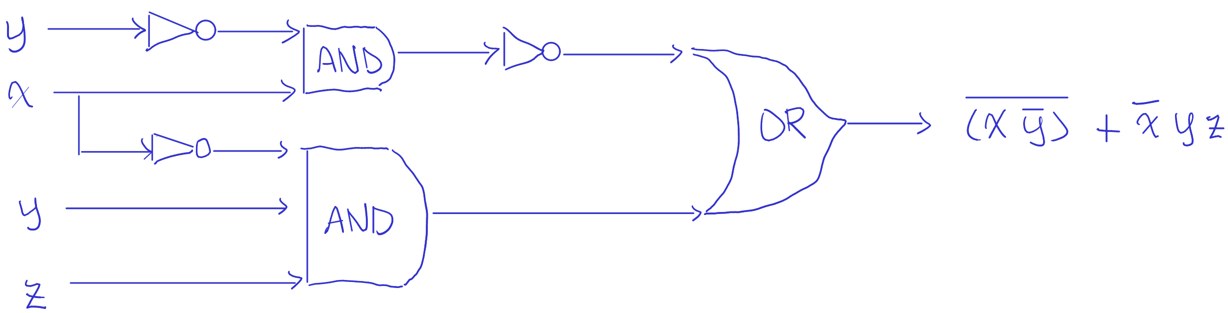
a) (2 pts) $0 + x + y$

$1 \cdot x \cdot y$

b) (2 pts) $x\,\overline{y}\,z$

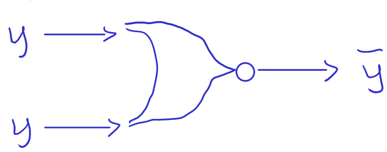
$x + \overline{y} + z$

12. (5 pts) Draw a logic gate diagram for the Boolean function **$F(x, y, z) = \overline{(x\,\overline{y})} + \overline{x}\,y\,z$.**



13. (4 pts) Use **NOR** gates (only) to construct circuits with these outputs.

a) (2 pts) $\overline{y}$



b) (2 pts) y z