

CH.8.1 Q.# 9
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Group In-Class Exercise (Week 9)

- A) Let a_n be the number of bit strings of length n that do not contain three consecutive 0's.
First way is to start with 1, that is, fix the first position. Then the remaining string will be of length $n-1$.
Another way for construction of strings is to start with 01, that is, fix the first two positions. Then the remaining string will be of length $n-2$.
There is also a way to fix the three positions, that is, the initial of strings is 001. And then choose the remaining string in such a way that it does not contain the three consecutive 0's.
The bit string definitely cannot start with 000. So, the last three cases are the only possible cases.

$$\text{For all } n \geq 3: a_n = a_{n-1} + a_{n-2} + a_{n-3}$$

- B) The initial conditions are $a_0 = 1$, $a_1 = 2$, and $a_2 = 4$. since all strings of length less than 3 satisfy the conditions.
- C) Compute a_3 through a_7 using the recurrence relation:

$$a_3 = a_2 + a_1 + a_0 = 4+2+1=7$$

$$a_4 = a_3 + a_2 + a_1 = 7+4+2 = 13$$

$$a_5 = a_4 + a_3 + a_2 = 13+7+4=24$$

$$a_6 = a_5 + a_4 + a_3 = 24+13+7=44$$

$$a_7 = a_6 + a_5 + a_4 = 44+24+13=81$$

Thus, there are 81 bit strings of length 7 that do not contain three consecutive 0's.