CSCI 190 Discrete Mathematics Applied to Computer Science
Final Exam

Name : _____

Last 4 digits of your Student ID #: _____

Read these instructions before proceeding.

- Closed book. Closed notes. You can use calculator.

- You have **100 minutes** to complete this exam.

- No questions will be answered during the exam or immediately afterwards. Answer each question as best you can. Partial credit will be awarded for reasonable efforts. If a question contains an ambiguity or a misprint, then say so in your answer, providing the answer to a reasonable interpretation of the question; give your assumptions.

- Answer the problems on the blank spaces provided for each problem.

- **Box your answers**.

Q1 (11)	Q2 (12)	Q3 (12)	Q4 (8)	Q5 (12)	Q6 (8)	Q7 (6)	Q8 (6)	Q9 (6)	Q10 (6)	Q11 (4)	Q12 (5)	Q13 (4)	Total (100)

1. (11 pts)

 a) (3 pts) Write the converse of the following:

  If you are happy, then you will smile.

 b) (4 pts) Convert **$(9FA7)_{16}$** to base 4.

 c) (4 pts) A message has been **encrypted** using the function $f(x) = (x + 4) \bmod 26$.
  If the message in coded form is **NSC**, decode the message.

2. (12 pts)

 a) (5 pts) Use the Principle of Mathematical Induction to prove that
  $2 + 4 + 6 + 8 + \cdots + 2n = n(n+1)$ for all $n \geq 1$. Show all the steps

 b) (4 pts) Give a recursive definition with initial condition for the following function, square of
  n factorial.
  $f(n) = (n!)^3$, $n = 0, 1, 2, \ldots$.

c) **(3 pts)** In a certain lottery game you choose a set of seven numbers out of 40 numbers.
Find the probability that exactly one of your numbers match the seven winning numbers.

3. (12 pts) Determine whether the following binary relation is:
(1) reflexive, (2) symmetric, (3) antisymmetric, (4) transitive.
No justifications needed.

a) (4 pts) The relation **R** on Z where **aRb** means **a = b**.
Circle your answers.

R is	Reflexive?	Symmetric?	Antisymmetric?	Transitive?
	Yes or No	Yes or No	Yes or No	Yes or No

b) (4 pts) The relation **R** on the set of all people where **aRb** means that **a** is taller than **b**.
Circle your answers.

R is	Reflexive?	Symmetric?	Antisymmetric?	Transitive?
	Yes or No	Yes or No	Yes or No	Yes or No

c) (4 pts) If $M_R = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$,

determine if **R** is: (a) reflexive (b) symmetric (c) antisymmetric (d) transitive.
Circle your answers.

R is	Reflexive?	Symmetric?	Antisymmetric?	Transitive?
	Yes or No	Yes or No	Yes or No	Yes or No

4. (8 pts)

a) (4 pts) Suppose **R** is the relation on *N* where **aRb** means that **a** starts in the same digit in which **b** starts.
   Determine whether **R** is an **equivalence relation** on *N*. Justify your answer.

b) (4 pts) Suppose the relation **R** is defined on the set *Z* where **aRb** means that **ab < 0**.
   Determine whether **R** is an **equivalence relation** on *Z*. Justify your answer.

5. (12 pts)

a) (4 pts) Draw these four graphs. K_6, C_4, W_5 and $K_{4,5}$

b) (4 pts)

K_n has _____ edges and _____ vertices.

$K_{m,n}$ has _____ edges and _____ vertices.

W_n has _____ edges and _____ vertices.
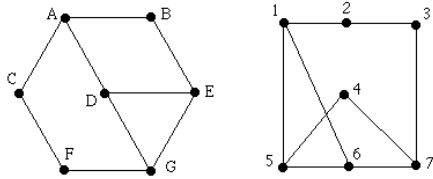
C_n has _____ edges and _____ vertices.

c) (4 pts) Draw the **digraph** with adjacency matrix
$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

6. (8 pts)

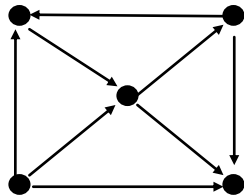a) (6 pts) Are these two graphs **isomorphic**?
If yes, give the mapping of vertices from the first graph to the second graph.
If no, explain why not.

b) (2 pts) Circle **Yes** or **No**. No justifications needed.

Determine whether the graph is **strongly connected**? Yes or No

Determine whether the graph is **weakly connected**.   Yes or No

7. (6 pts) Circle **TRUE** or **FALSE**. No justifications needed.

T / F   If T is a tree with 10 vertices, then there is a simple path in T of length 9.

T / F   Every tree is bipartite.
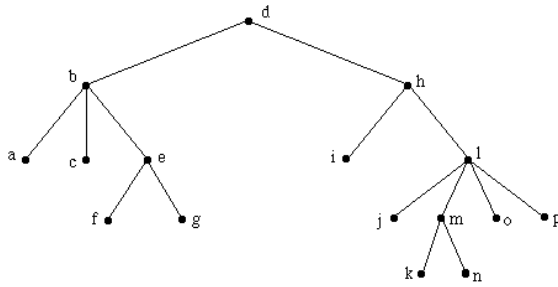
T / F   There is a tree with degrees 4, 3, 2, 2, 1, 1, 1, 1, 1.

T / F   There is a tree with degrees 3, 3, 3, 2, 1, 1, 1, 1.

T / F   If T is a tree with 30 vertices, the largest degree that any vertex can have is 29.

T / F   If two trees have the same number of vertices and the same degrees, then the two trees are isomorphic.

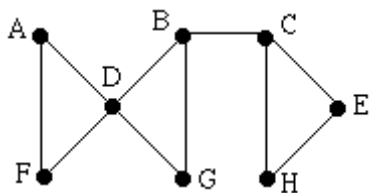**8. (6 pts)** Refer to the following tree.

a) (2 pts) Find the **preorder** traversal.

b) (2 pts) Find the **inorder** traversal.

c) (2 pts) Find the **postorder** traversal.

9. (6 pts) Refer to the following graph..

a) (3 pts) Using **alphabetical ordering**, find **a spanning tree** (starting from vertice **B**) for this graph by using DFS, **depth-first search**.

b) (3 pts) Using **alphabetical ordering**, find **a spanning tree** (starting from vertice **B**) for this graph by using BFS, **breadth-first search**.

10. (6 pts) Using a table to show that $F(x,y,z) = xyz + xy + x$ has a vaule of 1 if and only if variable **x** has a value of 1.

11. (4 pts) Find the **duals** of these Boolean expressions.

  a) (2 pts) $0 + x + y$

  b) (2 pts) $x \bar{y} z$

12. (5 pts) Draw a logic gate diagram for the Boolean function $F(x, y, z) = \overline{(x \bar{y})} + \bar{x} y z.$

13. (4 pts) Use **NOR** gates (only) to construct circuits with these outputs.

  a) (2 pts) $\bar{y}$

  b) (2 pts) $y z$