Ch. 1.6 Rules of Inference.

1. Find the argument form for the following argument and determine whether it is valid. Can we conclude that the conclusion is true if the premises are true?

p q

If Socrates is human, then Socrates is mortal.
Socrates is human.

$\therefore$ Socrates is mortal.

p = Socrates is human.
q = Socrates is mortal.

first statement	$p \to q$	hypotheses are true.
second statement	p	Modus ponens is valid
third statement	$\therefore q$	therefore, we can conclude that the conclusion of the argument is true.

$\therefore$ yes, because all premises are true.

3. What rule of inference is used in each of these arguments?

p

a) Alice is a mathematics major. Therefore, Alice is either a mathematics major or a computer science major.

q = Alice is a computer science major.

Rule of Inference	Tautology	
$$\begin{array}{l} p \\ \hline \therefore p \vee q \end{array}$$	$p \to (p \vee q)$	Addition Rule

5. Use rules of inference to show that the hypotheses "Randy works hard," "If Randy works hard, then he is a dull boy," and "If Randy is a dull boy, then he will not get the job" imply the conclusion "Randy will not get the job."

p q

r = Randy will get the job.

$$\begin{array}{ll} p & \text{hypothesis} \\ p \to q & \text{hypothesis} \\ \hline \therefore q & \text{Modus ponens} \\ q \to \neg r & \text{hypothesis} \\ \hline \therefore \neg r & \text{Modus ponens} \end{array}$$

7. What rules of inference are used in this famous argument? "All men are mortal. Socrates is a man. Therefore, Socrates is mortal."

Rule of Inference

$$\frac{\forall x P(x)}{\therefore P(c)} \quad \text{universal instantiation}$$

"for all x, if x is a man, then x is mortal."

"if Socrates is a man, then Socrates is mortal."

use modus ponens to conclude

=> "Socrates is mortal."

Step	Reason
1. $\forall x\,(Man(x) \to Mortal(x))$	premise
2. $Man(Socrates) \to Mortal(Socrates)$	UI from (1)
3 $Man(Socrates)$	premise
4. $Mortal(Socrates)$	MP from (2) and (3)

1.7 Introduction to Proofs

3. Show that the square of an even number is an even number using a direct proof.

prove square of an even number is an even number.

Assume x is an even number.

Let $x = 2k$ for some integer k.

$\Rightarrow x^2 = (2k)^2 = 4k^2 = 2(2k^2)$ $\quad 2k^2 \in \mathbb{Z}$

$\therefore$ square of an even number is an even number since we have written x^2 as 2 times an integer.

17. Show that if n is an integer and $\overset{p}{n^3 + 5 \text{ is odd}}$, then $\overset{q}{n \text{ is even}}$ using

a) a proof by contraposition. $\neg q \rightarrow \neg p \Leftrightarrow p \rightarrow q$

b) a proof by contradiction.

a) By Contraposition, $n \in \mathbb{Z}$

$(a+b)^3 = a^3 + 3a^2b + 3b^2a + b^3$

suppose n is odd,

$\Rightarrow n = 2k+1$ for some integer k.

$\Rightarrow n^3 + 5 = (2k+1)^3 + 5$

$= (2k)^3 + 3(2k)^2(1) + 3(1)^2(2k) + 1^3 + 5$

$= 8k^3 + 12k^2 + 6k + 6$

$= 2(4k^3 + 6k^2 + 3k + 3)$

$4k^3 + 6k^2 + 3k + 3 \in \mathbb{Z}$

$\Rightarrow n^3 + 5$ is even since $n^3 + 5$ is two times some integer.

$\therefore n^3 + 5$ is odd, then n is even.

Continue ...

b) By contradiction,

$p \to q$

$p \wedge \sim q$

Assume n^3+5 is odd and n is odd.

$\Rightarrow n = 2k+1$ for some integer k.

$\Rightarrow n^3+5 = (2k+1)^3+5$

$= (2k)^3 + 3(2k)^2(1) + 3(1)^2(2k) + 1^3 + 5$

$(a+b)^3 = a^3 + 3a^2b + 3b^2a + b^3$

$= 8k^3 + 12k^2 + 6k + 6$

$= 2(4k^3 + 6k^2 + 3k + 3)$

$4k^3 + 6k^2 + 3k + 3 \in \mathbb{Z}$

$\Rightarrow n^3+5$ is even which contradict the assumption.

$\therefore n^3+5$ is odd, then n is even.