CSCI 190 Discrete Mathematics Applied to Computer Science
Final Exam

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Read these instructions before proceeding.

- Closed book. Closed notes. You can use calculator.
- You have **100 minutes** to complete this exam.
- No questions will be answered during the exam or immediately afterwards. Answer each question as best you can. Partial credit will be awarded for reasonable efforts. If a question contains an ambiguity or a misprint, then say so in your answer, providing the answer to a reasonable interpretation of the question; give your assumptions.
- Answer the problems on the blank spaces provided for each problem.
- **Box your answers.**

Q1 (11)	Q2 (12)	Q3 (12)	Q4 (8)	Q5 (12)	Q6 (8)	Q7 (6)	Q8 (6)	Q9 (6)	Q10 (6)	Q11 (4)	Q12 (5)	Q13 (4)	Total (100)

1. (11 pts) $q \to p$

a) (3 pts) Write the converse of the following:

If you are postive, then you will be sunny.

If you are sunny, then you are positive.

b) (4 pts) Convert $(9FA5)_{16}$ to base 4.

$9 \cdot 16^3 + 15 \cdot 16^2 + 10 \cdot 16^1 + 5 \cdot 16^0 = 40869$

$40869 \bmod 4 = 1$
$10217 \bmod 4 = 1$
$2554 \bmod 4 = 2$
$638 \bmod 4 = 2$
$159 \bmod 4 = 3$
$39 \bmod 4 = 3$
$9 \bmod 4 = 1$
$2 \bmod 4 = 2$

$(9FA5)_{16} = \boxed{(21332211)_4}$

c) (4 pts) A message has been ***encrypted*** using the function **f(x) = (x + 7) mod 26**.
If the message in coded form is **QVF**, decode the message.

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

$Q = (16 - 7) \bmod 26 = J$
$V = (21 - 7) \bmod 26 = O$
$F = (5 - 7) \bmod 26 = Y$

$\boxed{JOY}$

2. (12 pts)

a) (5 pts) Use the Principle of Mathematical Induction to prove that
$2 + 4 + 6 + 8 + \cdots + 2n = n(n+1)$ for all $n \geq 1$. Show all the steps

$n = 1$
$2(1) = 1(1+1)$
$2 = 2$
$f(1)$ is true
Assume $f(k)$ is true,
$2+4+6+\ldots+2k = k(k+1)$
$= k^2 + k$
show that $f(k+1)$ is also true.

$2+4+6+\ldots+2k + 2(k+1) = k(k+1) + 2(k+1)$
$= k^2 + k + 2k + 2$
$= (k^2 + 2k + 1) + (k+1)$
$= (k+1)^2 + (k+1)$

$\boxed{\therefore\ 2+4+6+8+\ldots+2n = n(n+1) \text{ is true for } n \geq 1}$

b) (4 pts) Give a recursive definition with initial condition for the following function.

.

$f(n) = n^{3n}$, n = 1, 2, 3,. . . .

$a_1 = 1^{3(1)} = 1^3 = 1$
$a_2 = 2^{3(2)} = 2^6 = 64$
$a_3 = 3^{3(3)} = 3^9 = 19683$
$a_4 = 4^{3(4)} = 4^{12} = 16777216$

My assumption is to use the previous function in the current function to create the recursive equation. In this function $f(n) = n^{3n}$, I cannot find a recursive relation.

c) **(3 pts)** In a certain lottery game you choose a set of seven numbers out of 38 numbers.
Find the probability that exactly one of your numbers match the seven winning numbers.

$$\frac{\binom{7}{1}\cdot\binom{31}{7-1}}{\binom{38}{7}} = \frac{\frac{7!}{1!\,6!}\cdot\frac{31!}{6!\,25!}}{\frac{38!}{7!\,31!}} = \frac{7\cdot\frac{31\cdot30\cdot29\cdot28\cdot27\cdot26}{6\cdot5\cdot4\cdot3\cdot2}}{\frac{38\cdot37\cdot36\cdot35\cdot34\cdot33\cdot32}{7\cdot6\cdot5\cdot4\cdot3\cdot2}}$$

$$= \frac{7\,(736281)}{12620256} = \boxed{0.4084}$$

3. (12 pts) Determine whether the following binary relation is:
(1) reflexive, (2) symmetric, (3) antisymmetric, (4) transitive.
No justifications needed.

a) (4 pts) The relation ***R*** on Z where ***aRb*** means ***a = b***.
Circle your answers.

R is	Reflexive?	Symmetric?	Antisymmetric?	Transitive?
	Yes or No	Yes or No	Yes or No	Yes or No

b) (4 pts) The relation ***R*** on the set of all people where ***aRb*** means that ***a*** is shorter than ***b***.
Circle your answers.

R is	Reflexive?	Symmetric?	Antisymmetric?	Transitive?
	Yes or No	Yes or No	Yes or No	Yes or No

c) (4 pts) If $M_R = \begin{bmatrix} 0&1&0&0 \\ 1&1&1&0 \\ 1&1&1&0 \\ 0&0&0&1 \end{bmatrix}$,

determine if ***R*** is: (a) reflexive (b) symmetric (c) antisymmetric (d) transitive.
Circle your answers.

R is	Reflexive?	Symmetric?	Antisymmetric?	Transitive?
	Yes or No	Yes or No	Yes or No	Yes or No

4. (8 pts)

a) (4 pts)Suppose ***R*** is the relation on *N* where ***aRb*** means that ***a*** ends in the same digit in which ***b*** ends.

Determine whether ***R*** is an ***equivalence relation*** on *N*. Justify your answer.

Reflexivity : aRa both a starts in the same digit ∴ it is reflexive.

Symmetry : aRb a starts in the same digit as b and b starts in the same
bRa digit as a ∴ it is symmetric

Transitivity aRb a starts in the same digit as b and b starts in the same
bRc digit as c then a starts in the same digit as c
∴ it is transitive.

$\boxed{\therefore R \text{ is an equivalence relation on } N}$

b) (4 pts) Suppose the relation ***R*** is defined on the set *Z* where ***aRb*** means that ***ab < 0***.

Determine whether ***R*** is an ***equivalence relation*** on *Z*. Justify your answer.

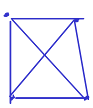
Reflexivity = aRa means $a \cdot a < 0$ but a^2 must be greater than or equal to zero ∴ it is not reflexive.

$\boxed{\therefore R \text{ is not an equivalence relation}}$

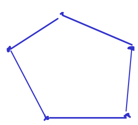
5. (12 pts)

a) (4 pts) Draw these four graphs. $\mathbf{K_4}$, $\mathbf{C_5}$, $\mathbf{W_4}$ and $\mathbf{K_{3,4}}$

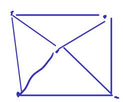
K_4



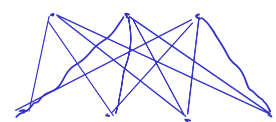
C_5



W_4



$K_{3,4}$



b) (4 pts)

$\boldsymbol{K_n}$ has $\underline{\frac{n(n-1)}{2} = 6}$ edges and $\underline{n = 4}$ vertices.

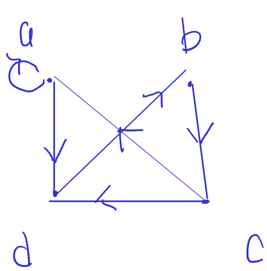
$\boldsymbol{K_{m,n}}$ has $\underline{m \cdot n = 12}$ edges and $\underline{m + n = 7}$ vertices.

$\boldsymbol{W_n}$ has $\underline{2n = 8}$ edges and $\underline{n + 1 = 5}$ vertices.

$\boldsymbol{C_n}$ has $\underline{n = 5}$ edges and $\underline{n = 5}$ vertices.

c) (4 pts) Draw the ***digraph*** with adjacency matrix

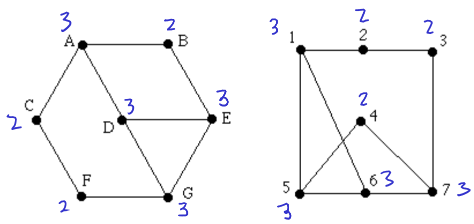
$$\begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

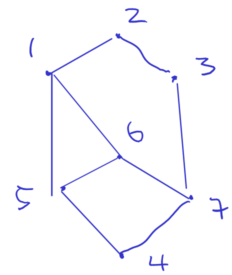


6. (8 pts)

a) (6 pts) Are these two graphs ***isomorphic***?

If yes, give the mapping of vertices from the first graph to the second graph.
If no, explain why not.





Yes, the graphs are isomorphic.

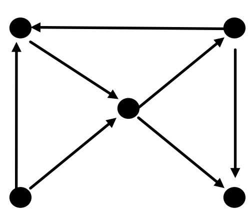
A = 7
C = 3
F = 2
G = 1

E = 5
B = 4
D = 6

b) (2 pts) Circle ***Yes*** or ***No***. No justifications needed.

Determine whether the graph is ***strongly connected***? Yes or No

Determine whether the graph is ***weakly connected***. Yes or No



7. (6 pts) Circle ***TRUE*** or ***FALSE***. No justifications needed.

T / F If T is a tree with 9 vertices, then there is a simple path in T of length 10.

T / F Every tree is bipartite.

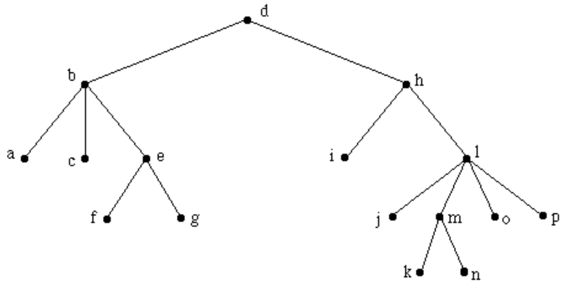
T / F There is a tree with degrees 4, 3, 6, 2, 2, 1, 1.

T / F There is a tree with degrees 1, 1, 3, 3, 3, 3.

T / F If T is a tree with 30 vertices, the largest degree that any vertex can have is 31.

T / F If two trees are isomorphic, then the two trees have the same number of vertices.

8. (6 pts) Refer to the following tree.



root left right a) (2 pts) Find the ***preorder*** traversal.

d b a c e f g h i l j m k n o p

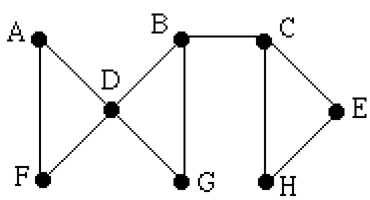
left root right b) (2 pts) Find the ***inorder*** traversal.

a b c f e g d i h j k m n l o p

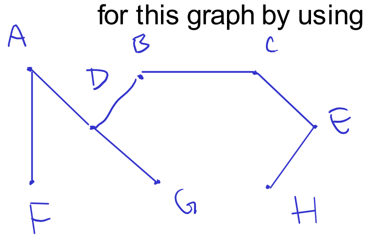
left right root c) (2 pts) Find the ***postorder*** traversal.

a c f g e b i j k n m o p l h d

9. (6 pts) Refer to the following graph..

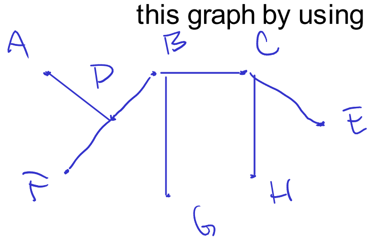


a) (3 pts) Using ***alphabetical ordering***, ***draw a spanning tree*** (starting from vertice ***B***) for this graph by using DFS, ***depth-first search***.



B C E H D A F G

b) (3 pts) Using ***alphabetical ordering***, ***draw a spanning tree*** (starting from vertice ***B***) for this graph by using BFS, ***breadth-first search***.



B C D G E H A F

10. (6 pts) Using a table to show that ***F(x,y,z) = xyz + xy + x*** has a vaule of 1 if and only if variable ***x*** has a value of 1.

x	y	z	xyz	xy	xyz + xy + x
1	1	1	1	1	1
1	1	0	0	1	1
1	0	1	0	0	1
1	0	0	0	0	1
0	1	1	0	0	0
0	1	0	0	0	0
0	0	1	0	0	0
0	0	0	0	0	0

∴ $F(x,y,z) = xyz + xy + x$ has value of 1 if and only if x has a value of 1 is true.

11. (4 pts) Find the ***duals*** of these Boolean expressions.

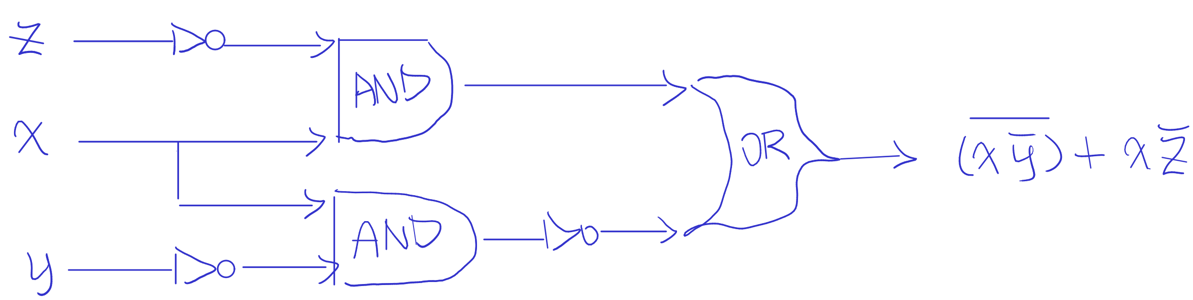
a) (2 pts) 0 + y + z

$1 * y * z$

b) (2 pts) $x\,\overline{y}\,z$

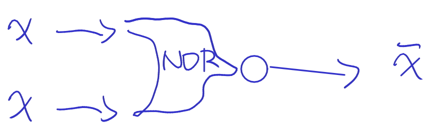
$x + \overline{y} + z$

12. (5 pts) Draw a logic gate diagram for the Boolean function **F(x, y, z) = $\overline{(x\ \overline{y})}$ + $x\,\overline{z}$.**



13. (4 pts) Use **NOR** gates (only) to construct circuits with these outputs.

a) (2 pts) $\overline{x}$



b) (2 pts) y z