HW 8 5.2

Find the orthogonal projection of f onto g. Use the inner product in C[a,b]

$\langle f,g\rangle = \int_a^b f(x)\,g(x)\,dx$

79.) $C[0,1]$, $f(x) = x$, $g(x) = e^x$

$\text{proj}_g f = \dfrac{\langle f,g\rangle}{\langle g,g\rangle} g$

$\langle f,g\rangle = \int_0^1 x e^x\,dx = xe^x - e^x \Big|_0^1 = e - e - (-1) = 1$

$x \quad + \quad e^x$
$1 \quad - \quad e^x$
$0 \quad e$

$\langle g,g\rangle = \int_0^1 e^x e^x\,dx = \int_0^1 e^{2x}\,dx = \int_0^1 e^u \frac{du}{2} = \frac{1}{2}\int_0^1 e^u\,du = \frac{1}{2}(e^u\Big|_0^1$

$= \frac{1}{2}(e^{2x}\Big|_0^1 = \frac{1}{2}e^2 - \frac{1}{2}$

$u = 2x$

$\text{proj}_g f = \dfrac{1}{\frac{1}{2}e^2 - \frac{1}{2}} e^x = \dfrac{2e^x}{e^2-1}$

$du = 2\,dx$

$\frac{du}{2} = dx$

83.) $C[-\pi,\pi]$, $f(x) = x$, $g(x) = \sin 2x$

$\text{proj}_g f = \dfrac{\langle f,g\rangle}{\langle g,g\rangle} g$

$\langle f,g\rangle = \int_{-\pi}^{\pi} x \sin 2x = (2x\cos 2x + 4\sin 2x)\Big|_{-\pi}^{\pi}$

$x \quad + \quad \sin 2x$
$1 \quad - \quad 2\cos 2x$
$0 \quad -4\sin 2x$

$= 2\pi(1) + 0 - (2(-\pi)(1) + 0)$

$= 2\pi + 2\pi = 4\pi$

$\langle g,g\rangle = \int_{-\pi}^{\pi} \sin^2 2x = \int_{-\pi}^{\pi} \frac{1}{2}\sin^2 u\,du = \frac{1}{2}\int_{-\pi}^{\pi} \sin^2 u\,du$

$2x = u$

$= \frac{1}{2}\left(-\frac{1}{2}\sin u \cos u\,du + \frac{1}{2}\int du\right)\Big|_{-\pi}^{\pi}$

$2\,dx = du$

$= -\frac{1}{4}\sin u \cos u + \frac{u}{4}\Big|_{-\pi}^{\pi} = \left(-\frac{1}{4}\sin 2x \cos 2x + \frac{x}{2}\right)\Big|_{-\pi}^{\pi}$

$dx = \frac{du}{2}$

$= 0 + \frac{\pi}{2} - \left(0 - \frac{\pi}{2}\right) = \pi$

$\text{proj}_g f = \dfrac{4\pi}{\pi} \sin 2x = 4\sin 2x$

5.3

(a) dtermine whether the set of vectors in R^n is orthogonal, (b) if the set is orthogonal, then determine whether it is also orthonormal, and (c) determine whether the set is basis for R^n

5.) {(4,-1,1),(-1,0,4),(-4,-17,-1)}

a) $\langle v_1, v_2 \rangle = \langle (4,-1,1), (-1,0,4) \rangle = (-4,0,4) = 0$

$\langle v_1, v_3 \rangle = \langle (4,-1,1), (-4,-17,-1) \rangle = (-16,17,-1) = 0$

$\langle v_2, v_3 \rangle = \langle (-1,0,4), (-4,-17,-1) \rangle = (4,0,-4) = 0$

$\therefore$ It is orthogonal

b) $\|v_1\|^2 = \sqrt{\langle v_1, v_1 \rangle}^2 = \langle (4,-1,1), (4,-1,1) \rangle = (16,1,1) = 18 \neq 1$

$\therefore$ It is not orthonormal

c) It is orthogonal

$\therefore$ It is a basis for $\mathbb{R}^3$

Find the coordinate matrix of w relative to the orthonormal basis B in R^n

19.) w = (1,2), B = $\left\{ \underset{v_1}{\left(-\frac{2\sqrt{13}}{13}, \frac{3\sqrt{13}}{13} \right)}, \underset{v_2}{\left(\frac{3\sqrt{13}}{13}, \frac{2\sqrt{13}}{13} \right)} \right\}$

$w\, v_1 = (1,2) \cdot \left(-\frac{2\sqrt{13}}{13}, \frac{3\sqrt{13}}{13} \right) = \left(-\frac{2\sqrt{13}}{13}, \frac{6\sqrt{13}}{13} \right) = \frac{4\sqrt{13}}{13}$

$w\, v_2 = (1,2) \cdot \left(\frac{3\sqrt{13}}{13}, \frac{2\sqrt{13}}{13} \right) = \left(\frac{3\sqrt{13}}{13}, \frac{4\sqrt{13}}{13} \right) = \frac{7\sqrt{13}}{13}$

$[w]_B = \begin{bmatrix} \frac{4\sqrt{13}}{13} \\ \frac{7\sqrt{13}}{13} \end{bmatrix}$

23.) w = (5,10,15), B = $\left\{ \underset{v_1}{\left(\frac{3}{5}, \frac{4}{5}, 0 \right)}, \underset{v_2}{\left(-\frac{4}{5}, \frac{3}{5}, 0 \right)}, \underset{v_3}{(0,0,1)} \right\}$

$w\, v_1 = \langle (5,10,15), (\frac{3}{5}, \frac{4}{5}, 0) \rangle = (3,8,0) = 11$

$w\, v_2 = \langle (5,10,15), (-\frac{4}{5}, \frac{3}{5}, 0) \rangle = (-4,6,0) = 2$

$w\, v_3 = \langle (5,10,15), (0,0,1) \rangle = (0,0,15) = 15$

$[w]_B = \begin{bmatrix} 11 \\ 2 \\ 15 \end{bmatrix}$

apply the Gram-Schmidt orthonoramlizatoin process to transform the given basis for R^n into an orthonormal basis. Use the vector in the order in which they are given

33.) $B = \{(\overset{v_1}{0,1,1}), (\overset{v_2}{1,1,0}), (\overset{v_3}{1,0,1})\}$

$$w_1 = v_1 = (0,1,1)$$

$$w_2 = v_2 - \frac{\langle v_2, w_1\rangle}{\langle w_1, w_1\rangle} w_1$$

$$\langle v_2, w_1\rangle = \langle (1,1,0),(0,1,1)\rangle = (0,1,0) = 1$$

$$\langle w_1, w_1\rangle = \langle (0,1,1),(0,1,1)\rangle = (0,1,1) = 2$$

$$w_2 = (1,1,0) - \frac{1}{2}(0,1,1) = (1,1,0) - (0,\tfrac{1}{2},\tfrac{1}{2}) = (1,\tfrac{1}{2},-\tfrac{1}{2})$$

$$w_3 = v_3 - \frac{\langle v_3, w_1\rangle}{\langle w_1, w_1\rangle} w_1 - \frac{\langle v_3, w_2\rangle}{\langle w_2, w_2\rangle} w_2$$

$$\langle v_3, w_1\rangle = \langle (1,0,1),(0,1,1)\rangle = (0,0,1) = 1$$

$$\langle w_1, w_1\rangle = 2$$

$$\langle v_3, w_2\rangle = \langle (1,0,1),(1,\tfrac{1}{2},-\tfrac{1}{2})\rangle = (1,0,-\tfrac{1}{2}) = \tfrac{1}{2}$$

$$\langle w_2, w_2\rangle = \langle (1,\tfrac{1}{2},-\tfrac{1}{2}),(1,\tfrac{1}{2},-\tfrac{1}{2})\rangle = (1,\tfrac{1}{4},\tfrac{1}{4}) = \tfrac{3}{2}$$

$$w_3 = (1,0,1) - \frac{1}{2}(0,1,1) - \frac{\frac{1}{2}}{\frac{3}{2}}(1,\tfrac{1}{2},-\tfrac{1}{2}) = (1,0,1) - (0,\tfrac{1}{2},\tfrac{1}{2}) - (\tfrac{1}{3},\tfrac{1}{6},-\tfrac{1}{6}) = (1,-\tfrac{1}{2},\tfrac{1}{2}) - (\tfrac{1}{3},\tfrac{1}{6},-\tfrac{1}{6}) = (\tfrac{2}{3},-\tfrac{2}{3},\tfrac{2}{3})$$

$$\|w_1\| = \sqrt{\langle (0,1,1),(0,1,1)\rangle} = \sqrt{(0,1,1)} = \sqrt{2}$$

$$\|w_2\| = \sqrt{\langle (1,\tfrac{1}{2},-\tfrac{1}{2}),(1,\tfrac{1}{2},-\tfrac{1}{2})\rangle} = \sqrt{(1,\tfrac{1}{4},\tfrac{1}{4})} = \frac{\sqrt{3}}{\sqrt{2}}$$

$$\|w_3\| = \sqrt{\langle (\tfrac{2}{3},-\tfrac{2}{3},\tfrac{2}{3}),(\tfrac{2}{3},-\tfrac{2}{3},\tfrac{2}{3})\rangle} = \sqrt{(\tfrac{4}{9},\tfrac{4}{9},\tfrac{4}{9})} = \sqrt{\tfrac{4}{3}} = \frac{2}{\sqrt{3}}$$

orthonormal basis $= \left\{\frac{w_1}{\|w_1\|}, \frac{w_2}{\|w_2\|}, \frac{w_3}{\|w_3\|}\right\} = \left\{\frac{1}{\sqrt{2}}(0,1,1), \frac{\sqrt{2}}{\sqrt{3}}(1,\tfrac{1}{2},-\tfrac{1}{2}), \frac{\sqrt{3}}{2}(\tfrac{2}{3},-\tfrac{2}{3},\tfrac{2}{3})\right\} = \left\{(0,\tfrac{1}{\sqrt{2}},\tfrac{1}{\sqrt{2}}), (\tfrac{\sqrt{2}}{\sqrt{3}},\tfrac{\sqrt{2}}{2\sqrt{3}},-\tfrac{\sqrt{2}}{2\sqrt{3}}), (\tfrac{\sqrt{3}}{3},-\tfrac{\sqrt{3}}{3},\tfrac{\sqrt{3}}{3})\right\}$

apply the Gram-Schmidt orthonoramlization process to transform the given basis for a subspace of R^n into an orthonormal basis for the subspace. Use the vectors in the order in which they are given

37.) $B = \{(\overset{v_1}{3,4,0}), (\overset{v_2}{2,0,0})\}$

$$w_1 = v_1 = (3,4,0)$$

$$w_2 = v_2 - \frac{\langle v_2, w_1\rangle}{\langle w_1, w_1\rangle} w_1$$

$$\langle v_2, w_1\rangle = \langle (2,0,0),(3,4,0)\rangle = (6,0,0) = 6$$

$\langle w_1, w_1\rangle = \langle (3,4,0),(3,4,0)\rangle = (9,16,0) = 25$

$w_2 = (2,0,0) - \frac{6}{25}(3,4,0) = (2,0,0) - (\frac{18}{25}, \frac{24}{25}, 0) = (\frac{32}{25}, -\frac{24}{25}, 0)$

$\|w_1\| = \sqrt{\langle (3,4,0),(3,4,0)\rangle} = \sqrt{(9,16,0)} = \sqrt{25} = 5$

$\|w_2\| = \sqrt{\langle (\frac{32}{25}, -\frac{24}{25}, 0), (\frac{32}{25}, -\frac{24}{25}, 0)\rangle} = \sqrt{(\frac{1024}{625}, \frac{576}{625}, 0)} = \sqrt{\frac{1600}{625}} = \sqrt{\frac{64}{25}} = \frac{8}{5}$

orthonormal basis $= \{\frac{w_1}{\|w_1\|}, \frac{w_2}{\|w_2\|}\} = \{\frac{1}{5}(3,4,0), \frac{5}{8}(\frac{32}{25}, \frac{-24}{25}, 0)\} = \{(\frac{3}{5}, \frac{4}{5}, 0), (\frac{4}{5}, \frac{-3}{5}, 0)\}$

39.) $B = \{\overset{v_1}{(1,2,-1,0)}, \overset{v_2}{(2,2,0,1)}, \overset{v_3}{(1,1,-1,0)}\}$

$w_1 = v_1 = (1,2,-1,0)$

$w_2 = v_2 - \frac{\langle v_2, w_1\rangle}{\langle w_1, w_1\rangle} w_1$ $\quad \langle v_2, w_1\rangle = \langle (2,2,0,1),(1,2,-1,0)\rangle = (2,4,0,0) = 6$

$\langle w_1, w_1\rangle = \langle (1,2,-1,0),(1,2,-1,0)\rangle = (1,4,1,0) = 6$

$w_2 = (2,2,0,1) - \frac{6}{6}(1,2,-1,0) = (2,2,0,1) - (1,2,-1,0) = (1,0,1,1)$

$w_3 = v_3 - \frac{\langle v_3, w_1\rangle}{\langle w_1, w_1\rangle} w_1 - \frac{\langle v_3, w_2\rangle}{\langle w_2, w_2\rangle} w_2$ $\quad \langle v_3, w_1\rangle = \langle (1,1,-1,0),(1,2,-1,0)\rangle = (1,2,1,0) = 4$

$\langle w_1, w_1\rangle = 6$

$\langle v_3, w_2\rangle = \langle (1,1,-1,0),(1,0,1,1)\rangle = (1,0,-1,0) = 0$

$\langle w_2, w_2\rangle = \langle (1,0,1,1),(1,0,1,1)\rangle = (1,0,1,1) = 3$

$w_3 = (1,1,-1,0) - \frac{4}{6}(1,2,-1,0) - \frac{0}{3}(1,0,1,1) = (1,1,-1,0) - (\frac{2}{3}, \frac{4}{3}, -\frac{2}{3}, 0) = (\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, 0)$

$\|w_1\| = \sqrt{\langle (1,2,-1,0),(1,2,-1,0)\rangle} = \sqrt{(1,4,1,0)} = \sqrt{6}$

$\|w_2\| = \sqrt{\langle (1,0,1,1),(1,0,1,1)\rangle} = \sqrt{(1,0,1,1)} = \sqrt{3}$

$\|w_3\| = \sqrt{\langle (\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, 0)\rangle} = \sqrt{(\frac{1}{9}, \frac{1}{9}, \frac{1}{9}, 0)} = \sqrt{\frac{3}{9}} = \sqrt{\frac{1}{3}} = \frac{1}{\sqrt{3}}$

orthonormal basis $= \{\frac{w_1}{\|w_1\|}, \frac{w_2}{\|w_2\|}, \frac{w_3}{\|w_3\|}\} = \{\frac{1}{\sqrt{6}}(1,2,-1,0), \frac{1}{\sqrt{3}}(1,0,1,1), \sqrt{3}(\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, 0)\}$

$= \{(\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, 0), (\frac{1}{\sqrt{3}}, 0, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}), (\frac{\sqrt{3}}{3}, -\frac{\sqrt{3}}{3}, -\frac{\sqrt{3}}{3}, 0)\}$

$= \{(\frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{3}, -\frac{\sqrt{6}}{6}, 0), (\frac{\sqrt{3}}{3}, 0, \frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}), (\frac{\sqrt{3}}{3}, -\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}, 0)\}$

Let B = {1,x,x^2} be a basis for P2 with the inner product

$\langle p, q \rangle = \int_{-1}^{1} p(x)\, q(x)\, dx$

47) $\langle x, x \rangle = \frac{2}{3}$

$\langle x, x \rangle = \int_{-1}^{1} x^2\, dx = 2\int_{0}^{1} x^2\, dx = 2\left(\frac{x^3}{3}\right)\Big|_0^1 = 2\left(\frac{1}{3}\right) = \frac{2}{3}$

63.) Proof: Let {u1,u2,...,un} be an orthonormal basis for R^n. Prove that $\|v\|^2 = |v \cdot u_1|^2 + |v \cdot u_2|^2 + \ldots + |v \cdot u_n|^2$ for any vector v in R^n. This equation is Parseval's equality.

$v = [(v, u_1)u_1 + (v, u_2)u_2 + \ldots + \ldots (v, u_n)u_n]$

$\|v\|^2 = \sqrt{v \cdot v}^{\,2}$

$= v \cdot v$

$= [(v, u_1)u_1 + (v, u_2)u_2 + \ldots + (v, u_n)u_n] \cdot v$

$= [(v, u_1)(v, u_1) + (v, u_2)(v, u_2) + \ldots + (v, u_n)(v, u_n)]$

$= (v \cdot u_1)^2 + (v \cdot u_2)^2 + \ldots + (v \cdot u_n)^2$

$= \|v \cdot u_1\|^2 + \|v \cdot u_2\|^2 + \ldots + \|v \cdot u_n\|^2$

5.3

(a) determine whether the set of vectors in R^n is orthogonal, (b) if the set is orthogonal, then determine whether it is also orthonormal, and (c) determine whether the set is a basis for R^n

9) $\{\overset{v_1}{(2,5,-3)},\overset{v_2}{(4,2,6)}\}$

a.) $\langle v_1, v_2\rangle = \langle (2,5,-3),(4,2,6)\rangle = (8,10,-18) = 0$

$\therefore$ Yes

b.) $\|v_1\|^2 = \sqrt{\langle v_1, v_1\rangle}^2 = \langle (2,5,-3),(2,5,-3)\rangle = (4,25,9) = 38 \neq 1$

$\therefore$ No

c.) No

11.) $\{\overset{v_1}{(\frac{\sqrt{2}}{2},0,0,\frac{\sqrt{2}}{2})}, \overset{v_2}{(0,\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2},0)}, \overset{v_3}{(-\frac{1}{2},\frac{1}{2},-\frac{1}{2},\frac{1}{2})}\}$

a.) $\langle v_1, v_2\rangle = \langle (\frac{\sqrt{2}}{2},0,0,\frac{\sqrt{2}}{2}),(0,\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2},0)\rangle = (0,0,0,0) = 0$

$\langle v_1, v_3\rangle = \langle (\frac{\sqrt{2}}{2},0,0,\frac{\sqrt{2}}{2}),(-\frac{1}{2},\frac{1}{2},-\frac{1}{2},\frac{1}{2})\rangle = (-\frac{\sqrt{2}}{4},0,0,\frac{\sqrt{2}}{4}) = 0$

$\langle v_2, v_3\rangle = \langle (0,\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2},0),(-\frac{1}{2},\frac{1}{2},-\frac{1}{2},\frac{1}{2})\rangle = (0,\frac{\sqrt{2}}{4},-\frac{\sqrt{2}}{4},0) = 0$

$\therefore$ Yes

b.) $\|v_1\|^2 = \langle v_1, v_1\rangle = \langle (\frac{\sqrt{2}}{2},0,0,\frac{\sqrt{2}}{2}),(\frac{\sqrt{2}}{2},0,0,\frac{\sqrt{2}}{2})\rangle = (\frac{2}{4},0,0,\frac{2}{4}) = 1$

$\|v_2\|^2 = \langle v_2, v_2\rangle = \langle (0,\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2},0),(0,\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2},0)\rangle = (0,\frac{2}{4},\frac{2}{4},0) = 1$

$\|v_3\|^2 = \langle v_3, v_3\rangle = \langle (-\frac{1}{2},\frac{1}{2},-\frac{1}{2},\frac{1}{2}),(-\frac{1}{2},\frac{1}{2},-\frac{1}{2},\frac{1}{2})\rangle = (\frac{1}{4},\frac{1}{4},\frac{1}{4},\frac{1}{4}) = 1$

$\therefore$ Yes

c.) No

(a) show that the set of vectors in R^n is orthogonal, and (b) normalize the set to produce an orthonormal set

13) $\{\overset{v_1}{(-1,3)},\overset{v_2}{(12,4)}\}$

a) $\langle v_1,v_2\rangle = \langle(-1,3),(12,4)\rangle = (-12,12) = 0$

$\therefore$ Yes

b) $w_1 = \frac{v_1}{\|v_1\|} = \frac{1}{\sqrt{10}}(-1,3) = (-\frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}}) = (-\frac{\sqrt{10}}{10}, \frac{3\sqrt{10}}{10})$ $\qquad \|v_1\| = \sqrt{\langle(-1,3),(-1,3)\rangle} = \sqrt{(1,9)} = \sqrt{10}$

$w_2 = \frac{v_2}{\|v_2\|} = \frac{1}{4\sqrt{10}}(12,4) = (\frac{3}{\sqrt{10}}, \frac{1}{\sqrt{10}}) = (\frac{3\sqrt{10}}{10}, \frac{\sqrt{10}}{10})$ $\qquad \|v_2\| = \sqrt{\langle(12,4)(12,4)\rangle} = \sqrt{(144,16)} = \sqrt{160}$

$= 4\sqrt{10}$

find the coordinate matrix of w relative to the orthonormal basis B in R^n

21) w = (2,-2,1)

$B = \{\overset{v_1}{(\frac{\sqrt{10}}{10}, 0, \frac{3\sqrt{10}}{10})}, \overset{v_2}{(0,1,0)}, \overset{v_3}{(-\frac{3\sqrt{10}}{10}, 0, \frac{\sqrt{10}}{10})}\}$

$w \cdot v_1 = (2,-2,1)\cdot(\frac{\sqrt{10}}{10}, 0, \frac{3\sqrt{10}}{10}) = (\frac{2\sqrt{10}}{10}, 0, \frac{3\sqrt{10}}{10}) = \frac{\sqrt{10}}{2}$

$w \cdot v_2 = (2,-2,1)\cdot(0,1,0) = (0,-2,0) = -2$

$w \cdot v_3 = (2,-2,1)\cdot(-\frac{3\sqrt{10}}{10}, 0, \frac{\sqrt{10}}{10}) = (-\frac{6\sqrt{10}}{10}, 0, \frac{\sqrt{10}}{10}) = \frac{-5\sqrt{10}}{10} = \frac{-\sqrt{10}}{2}$

$$[w]_B = \begin{bmatrix} \frac{\sqrt{10}}{2} \\ -2 \\ \frac{-\sqrt{10}}{2} \end{bmatrix}$$

23) W = (5,10,15), $B = \{\overset{v_1}{(\frac{3}{5}, \frac{4}{5}, 0)}, \overset{v_2}{(-\frac{4}{5}, \frac{3}{5}, 0)}, \overset{v_3}{(0,0,1)}\}$

$w\, v_1 = (5,10,15)(\frac{3}{5}, \frac{4}{5}, 0) = (3,8,0) = 11$

$w\, v_2 = (5,10,15)(-\frac{4}{5}, \frac{3}{5}, 0) = (-4,6,0) = 2$

$w\, v_3 = (5,10,15)(0,0,1) = (0,0,15) = 15$

$$[w]_B = \begin{bmatrix} 11 \\ 2 \\ 15 \end{bmatrix}$$

apply the Gram-Schmidt orthonormalization process to transform the given basis for R^n into an orthonormal basis. Use the vectors in the order in which they are given.

29.) $B = \{\overset{v_1}{(2,1,-2)},\overset{v_2}{(1,2,2)},\overset{v_3}{(2,-2,1)}\}$

$w_1 = v_1 = (2,1,-2)$

$w_2 = v_2 - \frac{\langle v_2, w_1\rangle}{\langle w_1, w_1\rangle} w_1$

$\langle v_2, w_1\rangle = \langle (1,2,2),(2,1,-2)\rangle = (2,2,-4) = 0$

$\langle w_1, w_1\rangle = \langle (2,1,-2),(2,1,-2)\rangle = (4,1,4) = 9$

$w_2 = (1,2,2) - \frac{0}{9}(2,1,-2) = (1,2,2)$

$w_3 = v_3 - \frac{\langle v_3, w_1\rangle}{\langle w_1, w_1\rangle} w_1 - \frac{\langle v_3, w_2\rangle}{\langle w_2, w_2\rangle} w_2$

$\langle v_3, w_1\rangle = \langle (2,-2,1),(2,1,-2)\rangle = (4,-2,-2) = 0$

$\langle w_1, w_1\rangle = 9$

$\langle v_3, w_2\rangle = \langle (2,-2,1),(1,2,2)\rangle = (2,-4,2) = 0$

$\langle w_2, w_2\rangle = \langle (1,2,2),(1,2,2)\rangle = (1,4,4) = 9$

$w_3 = (2,-2,1) - \frac{0}{9}(2,1,-2) - \frac{0}{9}(1,2,2) = (2,-2,1)$

$\|w_1\| = \sqrt{9} = 3$

$\|w_2\| = \sqrt{9} = 3$

$\|w_3\| = \sqrt{\langle (2,-2,1),(2,-2,1)\rangle} = \sqrt{(4,4,1)} = \sqrt{9} = 3$

orthonormal basis $= \left\{\frac{w_1}{\|w_1\|}, \frac{w_2}{\|w_2\|}, \frac{w_3}{\|w_3\|}\right\} = \left\{\frac{1}{3}(2,1,-2), \frac{1}{3}(1,2,2), \frac{1}{3}(2,-2,1)\right\} = \left\{\left(\frac{2}{3},\frac{1}{3},-\frac{2}{3}\right),\left(\frac{1}{3},\frac{2}{3},\frac{2}{3}\right),\left(\frac{2}{3},-\frac{2}{3},\frac{1}{3}\right)\right\}$

31.) $B = \{\overset{v_1}{(4,-3,0)},\overset{v_2}{(1,2,0)},\overset{v_3}{(0,0,4)}\}$

$w_1 = v_1 = (4,-3,0)$

$w_2 = v_2 - \frac{\langle v_2, w_1\rangle}{\langle w_1, w_1\rangle} w_1$

$\langle v_2, w_1\rangle = \langle (1,2,0),(4,-3,0)\rangle = (4,-6,0) = -2$

$\langle w_1, w_1\rangle = \langle (4,-3,0),(4,-3,0)\rangle = (16,9,0) = 25$

$w_2 = (1,2,0) - \left(\frac{-2}{25}\right)(4,-3,0) = (1,2,0) + \left(\frac{8}{25}, \frac{-6}{25}, 0\right) = \left(\frac{33}{25}, \frac{44}{25}, 0\right)$

$w_3 = v_3 - \frac{\langle v_3, w_1\rangle}{\langle w_1, w_1\rangle} w_1 - \frac{\langle v_3, w_2\rangle}{\langle w_2, w_2\rangle} w_2$

$\langle v_3, w_1\rangle = \langle (0,0,4),(4,-3,0)\rangle = (0,0,0) = 0$

$\langle w_1, w_1\rangle = 25$

$\langle v_3, w_2\rangle = \langle (0,0,4),\left(\frac{33}{25},\frac{44}{25},0\right)\rangle = (0,0,0) = 0$

$\langle w_2, w_2\rangle = \langle \left(\frac{33}{25},\frac{44}{25},0\right),\left(\frac{33}{25},\frac{44}{25},0\right)\rangle = \left(\frac{1089}{625}, \frac{1936}{625}, 0\right) = \frac{121}{25}$

$w_3 = (0,0,4) - \frac{0}{25}(4,-3,0) - \frac{0}{\frac{121}{25}}\left(\frac{33}{25},\frac{44}{25},0\right) = (0,0,4)$

$\|w_1\| = \sqrt{25} = 5$, $\|w_2\| = \sqrt{\frac{121}{25}} = \frac{11}{5}$, $\|w_3\| = \sqrt{\langle (0,0,4),(0,0,4)\rangle} = \sqrt{(0,0,16)} = \sqrt{16} = 4$

orthonormal basis $= \left\{\frac{w_1}{\|w_1\|}, \frac{w_2}{\|w_2\|}, \frac{w_3}{\|w_3\|}\right\} = \left\{\frac{1}{5}(4,-3,0), \frac{5}{11}\left(\frac{33}{25},\frac{44}{25},0\right), \frac{1}{4}(0,0,4)\right\} = \left\{\left(\frac{4}{5},-\frac{3}{5},0\right),\left(\frac{3}{5},\frac{4}{5},0\right),(0,0,1)\right\}$

apply the Gram-Schmidt orthonormalization process to transform the given basis for a subspace of Rn into an orthonormal basis for the subspace. Use the vectors in the order in which they are given.

35) B = {(-8,3,5)} v_1

$w_1 = v_1 = (-8,3,5)$

$\|w_1\| = \sqrt{\langle w_1, w_1\rangle} = \sqrt{\langle(-8,3,5),(-8,3,5)\rangle} = \sqrt{(64,9,25)} = \sqrt{98} = 7\sqrt{2}$

orthonormal basis $= \{\frac{1}{7\sqrt{2}}(-8,3,5)\} = \{(\frac{-8}{7\sqrt{2}}, \frac{3}{7\sqrt{2}}, \frac{5}{7\sqrt{2}})\} = \{(\frac{-4\sqrt{2}}{7}, \frac{3\sqrt{2}}{14}, \frac{5\sqrt{2}}{14})\}$

37) B = {(3,4,0),(2,0,0)} v_1 v_2

$w_1 = v_1 = (3,4,0)$

$w_2 = v_2 - \frac{\langle v_2, w_1\rangle}{\langle w_1, w_1\rangle} w_1$ $\qquad \langle v_2, w_1\rangle = \langle(2,0,0),(3,4,0)\rangle = (6,0,0) = 6$

$\langle w_1, w_1\rangle = \langle(3,4,0),(3,4,0)\rangle = (9,16,0) = 25$

$w_2 = (2,0,0) - \frac{6}{25}(3,4,0) = (2,0,0) - (\frac{18}{25}, \frac{24}{25}, 0) = (\frac{32}{25}, -\frac{24}{25}, 0)$

$\|w_1\| = \sqrt{\langle(3,4,0),(3,4,0)\rangle} = \sqrt{25} = 5$

$\|w_2\| = \sqrt{\langle(\frac{32}{25}, -\frac{24}{25}, 0),(\frac{32}{25}, -\frac{24}{25}, 0)\rangle} = \sqrt{(\frac{1024}{625}, \frac{576}{625}, 0)} = \sqrt{\frac{1600}{625}} = \sqrt{\frac{64}{25}} = \frac{8}{5}$

orthonormal basis $= \{\frac{1}{5}(3,4,0), \frac{5}{8}(\frac{32}{25}, -\frac{24}{25}, 0)\} = \{(\frac{3}{5}, \frac{4}{5}, 0), (\frac{4}{5}, -\frac{3}{5}, 0)\}$

let p(x) = a0 + a1x + a2x^2 and q(x) = b0 + b1x + b2x^2 be vectors in P2 with ⟨ p, q⟩ = a0b0 + a1b1 + a2b2. Determine whether the polynomials form an orthonormal set, and if not, apply the Gram-Schmidt orthonormalization process to form an orthonormal set.

57) $\{\overset{v_1}{1}, \overset{v_2}{x}, \overset{v_3}{x^2}\}$ $\qquad$ Let $= B = \{\overset{v_1}{(1,0,0)}, \overset{v_2}{(0,1,0)}, \overset{v_3}{(0,0,1)}\}$

$v_1 = (1,0,0)$ $\qquad \langle v_1, v_2\rangle = 1(0) + 0(1) + (0)(0) = 0$

$v_2 = (0,1,0)$ $\qquad \langle v_1, v_3\rangle = 1(0) + (0)(0) + (0)(1) = 0$

$v_3 = (0,0,1)$ $\qquad \langle v_2, v_3\rangle = (0)(0) + (1)(0) + (0)(1) = 0$

$\|v_1\|^2 = \sqrt{\langle(1,0,0),(1,0,0)\rangle}^2 = (1,0,0) = 1$

$\|v_2\|^2 = \sqrt{\langle(0,1,0),(0,1,0)\rangle}^2 = (0,1,0) = 1$

$\|v_3\|^2 = \sqrt{\langle(0,0,1),(0,0,1)\rangle}^2 = (0,0,1) = 1$

Orthonormal

58) $\{-1 + x^2,\ -1 + x\}$ $B = \{ \overset{v_1}{(-1,0,1)}, \overset{v_2}{(-1,1,0)} \}$

$\langle v_1, v_2 \rangle = \langle (-1,0,1), (-1,1,0) \rangle = (1,0,0) = 1$

$w_1 = v_1 = (-1,0,1)$

$w_2 = v_2 - \frac{\langle v_2, w_1 \rangle}{\langle w_1, w_1 \rangle} w_1$ $\langle v_2, w_1 \rangle = \langle (-1,1,0), (-1,0,1) \rangle = (1,0,0) = 1$

$\langle w_1, w_1 \rangle = \langle (-1,0,1), (-1,0,1) \rangle = (1,0,1) = 2$

$w_2 = (-1,1,0) - \frac{1}{2}(-1,0,1) = (-1,1,0) - (-\frac{1}{2}, 0, \frac{1}{2}) = (-\frac{1}{2}, 1, -\frac{1}{2})$

$\|w_1\| = \sqrt{\langle (-1,0,1), (-1,0,1) \rangle} = \sqrt{2}$

$\|w_2\| = \sqrt{\langle (-\frac{1}{2}, 1, -\frac{1}{2}), (-\frac{1}{2}, 1, -\frac{1}{2}) \rangle} = \sqrt{(\frac{1}{4}, 1, \frac{1}{4})} = \sqrt{\frac{3}{2}}$

orthonormal $= \{ \frac{w_1}{\|w_1\|}, \frac{w_2}{\|w_2\|} \} = \{ \frac{1}{\sqrt{2}}(-1,0,1), \sqrt{\frac{2}{3}}(-\frac{1}{2}, 1, -\frac{1}{2}) \}$

$= \{ \frac{\sqrt{2}}{2}(-1 + x^2), \frac{\sqrt{6}}{3}(-\frac{1}{2} + x - \frac{1}{2}x^2) \}$

$= \{ \frac{\sqrt{2}}{2}(-1 + x^2), -\frac{\sqrt{6}}{6}(1 - 2x + x^2) \}$