Graded for Honest Effort

1.7.3 a) "Someone did not get a large bonus" is a negation of "someone get a large bonus", and we can use the quantifier $\exists$ to express someone, we can translate it into logic as

$$\boxed{\exists x \ \neg B(x)}$$

b) We use the quantifier $\forall$ to express everyone. We can translate "Everyone got a large bonus" into logic as

$$\boxed{\forall x \ B(x)}$$

c) We use conjunction to express "even though" and "Sam did not get a large bonus" is a negation of "Sam got a large bonus". We can translate it into logic as

$$\boxed{\neg B(\text{Sam}) \land T(x)}$$

d) We use the quantifier $\exists$ to express "someone" and "not on the executive team" is a negation of "on the executive team" and finally we use conjunction to express "not on the excutive team and received large bonus"

$$\boxed{\exists x \ (\neg T(x) \land B(x))}$$

Graded for Honest Effort

1.7.3 e) We use the quantifier $\forall$ to express "Everyone", and being a executive team member implies he/she got a large bonus

$$\boxed{\forall x\,(T(x) \rightarrow B(x))}$$

1.7.7 b) In every row, we can find at least D(x) or N(x) is true, therefore

True

Among all the group members, they either missed the deadline or they are a new employee.

c) We can find that there exists a x that is both not D(x) and N(x), for example, Melanie is not D(x) and N(x), therefore

True

A group member did not miss the deadline and is a new empolyee.

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1.7.7 i) We find that only A1 is true on both B(x) and N(x) which means

False

Every group member missed the deadline if and only if he is a new employee.

1.8.3 d) Translate to logic

$\exists x\ (P(x) \land C(x))$

Apply negation

$\neg \exists x\ (P(x) \land C(x))$

$\equiv \forall x\ \neg(P(x) \land C(x))$ change quantifier to $\forall$

$\equiv \forall x\ (\neg P(x) \lor \neg C(x))$ change conjunction to disjunction

Translate to English

Every student showed up either without a pencil or without a calculator

Graded for Honest Effort

e) Translate to logic

$\exists x\ (P(x) \vee C(x))$

Apply negation

$\neg \exists x (P(x) \vee C(x))$

$\equiv \forall x \neg (P(x) \vee C(x))$ change the quantifier to $\forall$

$\equiv \forall x (\neg P(x) \wedge \neg C(x))$ change disjunction to conjunction

Translate to English

Every student showed up without a pencil and without a calculator.

f) Translate to logic

$\forall x\ (P(x) \wedge C(x))$

Apply negation

$\neg \forall x\ (P(x) \wedge C(x))$

$\exists x \neg (P(x) \wedge C(x))$ Change quantifier to $\exists$

$\exists x\ (\neg P(x) \vee \neg C(x))$ change conjunction to disjunction

Translate to English

Some students showed up either without a pencil or without a calculator.

Graded for Honest Effort

1.10.3 a) $\forall x \exists y\, P(x,y)$, $\exists x \forall y\, P(x,y)$

x \ y: P	a	b	c
a	F	F	F
b	T	T	T
c	F	F	T

$\forall x \exists y\, P(x,y)$ is false because $P(1,y)$ are all false. It needs at least one true to make the expression true
$\exists x \forall y\, P(x,y)$ is true because $P(2,y)$ are all true.

1.10.4 e) The reciprocal of every positive number less than one is greater than one

$R(x,y)$: x is the reciprorcal of y.

$G(z)$: z is greater than one.

$$\boxed{\forall x \exists y\, ((R(y,x) \land (x>0) \land (x<1)) \rightarrow G(y))}$$

If every real number x that is greater than and less than 1, then there is a real number y that is greater than 1.

Graded for Honest Effort

1.10.4 f) There is no smallest number.

$S(x, y)$: x is smaller than y.

$\exists x\ S(x)$: x is the smallest number.

$\boxed{\forall y \exists x\ (S(x, y))}$

For every real number y, there is a real number y

g) Every number other than 0 has a multiplicative inverse

$m(x)$: x has a multiplicative inverse

$\boxed{\forall x\ ((x \neq 0) \rightarrow m(x))}$

For every real number x, if x is not zero, it has a multiplicative inverse

Graded for Honest Effort

1.11.3 b) p: he studied for the test

q: he passed the test

$$\begin{array}{l} p \lor \neg q \\ q \\ \hline \therefore p \end{array}$$

p	q	¬q	p ∨ ¬q
T	T	F	T
T	F	T	T
F	T	F	F
F	F	T	T

The hypothesis are $p \lor \neg q$ and q, so we only look at the row when both are true which is row 1. The conclusion which is p is true therefore it is valid.

c) p: $\sqrt{2}$ is an irrational number

q: $2\sqrt{2}$ is an irrational number

$$\begin{array}{l} p \to q \\ q \\ \hline \therefore p \end{array}$$

p	q	p → q
T	T	T
T	F	F
F	T	T
F	F	T

The hypothesis are $p \to q$ and q so we only look at row 1 and 3, the conclusion at row 3 is False, therefore it is invalid.

Graded for Honest Effort and Feedback Given

1.8.4 b) $\neg \forall x (\neg P(x) \to Q(x))$
$\equiv \exists x (\neg P(x) \wedge \neg Q(x))$

$\neg \forall x (\neg P(x) \to Q(x))$	start
$\exists x \; \neg(\neg P(x) \to Q(x))$	DeMorgan's law
$\exists x \; \neg(\neg\neg P(x) \vee Q(x))$	Conditional identity
$\exists x \; \neg(P(x) \vee Q(x))$	Double negation law
$\exists x \; (\neg P(x) \wedge \neg Q(x))$	DeMorgan's law

$\therefore$ they are logically equivalent

1.9.4 d) $\exists x \, \forall y (P(x,y) \leftrightarrow P(y,x))$

apply negation

$\neg \exists x \, \forall y \, (P(x,y) \leftrightarrow P(y,x))$ start

$\forall x \, \exists y \, \neg(P(x,y) \leftrightarrow P(y,x))$ DeMorgan's law

$\forall x \, \exists y \, \neg((P(x,y) \to P(y,x)) \wedge (P(y,x) \to P(x,y)))$

conditional identity

$\forall x \, \exists y \, \neg((\neg P(x,y) \vee P(y,x)) \wedge (P(y,x) \to P(x,y)))$

conditional identity

$\forall x \, \exists y \, \neg((\neg P(x,y) \vee P(y,x)) \wedge (\neg P(y,x) \vee P(x,y)))$

conditional identity

$\forall x \, \exists y \, (\neg(\neg P(x,y) \vee P(y,x)) \vee \neg(\neg P(y,x) \vee P(x,y)))$

DeMorgan's law

$\forall x \, \exists y \, ((P(x,y) \wedge \neg P(y,x)) \vee \neg(\neg P(y,x) \vee P(x,y)))$

DeMorgan's law

$\forall x \, \exists y \, ((P(x,y) \wedge \neg P(y,x)) \vee (P(y,x) \wedge \neg P(x,y)))$

DeMorgan's law

Graded for Honest Effort and Feedback Given

1.10.7 c) There is at least one new employee who missed the deadline.

we can translate it into logic as

$$\boxed{\exists x\ (N(x) \land D(x))}$$

we use quantifier $\exists$ to express at least one, then we use conjunction to express a person is a new employee and missed the deadline.

d) Sam knows the phone number of everyone who missed the deadline.

we can translate it into logic as

$$\boxed{\forall x\ (D(x) \rightarrow P(Sam, x))}$$

we use quantifier $\forall$ to express everyone, and we use conditional statement to express that if you missed the deadline then Sam knows your phone number.

Graded for Honest Effort and Feedback Given

1.10.7 e) There is a new employee who knows everyone's phone number.
we can translate it into logic as

$$\boxed{\exists x \forall y \ (N(x) \land P(x,y))}$$

we use quantifier $\exists$ to express a person, and use quantifier $\forall$ to express everyone. Finally, we use conjunction to express there is a person who is a new employee and knows everyone's phone number.

f) Exactly one new employee missed the deadline.
we can translate it into logic as

$$\boxed{\exists x \ ((N(x) \land D(x)) \land \forall y \ ((x \neq y) \rightarrow \neg D(y)))}$$

we use quantifier $\exists$ to express there is one person, and this person is a new employee and missed the deadline. we use quantifier $\forall$ to express everyone and if they are the same person x then they didn't miss the deadline which means exactly one person missed the deadline.

Graded for Honest Effort and Feedback Given

1.10.10 x = a set of student at a university.
y = a set Math class offer at that university.
$T(x,y)$: student x has taken class y.

c) Every student has taken at least one class other than Math 101.

$$\boxed{\forall x \exists y \, ((y \neq \text{Math } 101) \wedge T(x,y))}$$

we use quantifier $\forall$ to express every student and $\exists$ to express at least one class, then we say that this class is not Math 101 and every student has taken this Math class.

d) There is a student who has taken every math class other than Math 101.

$$\boxed{\exists x \forall y \, ((y \neq \text{Math } 101) \rightarrow T(x,y))}$$

we use quantifier $\exists$ to express there is a student and $\forall$ to express every math class. Then we say that if every math class that is not math 101 then there is a student has taken that class.

Graded for Honest Effort and Feedback Given

1.10.10 f) Sam has taken exactly two math classes.

$$\boxed{\begin{array}{l}\exists a \exists b \forall y\, (T(Sam, a) \land T(Sam, b) \land \\ (((a \neq b) \land (a \neq y) \land (b \neq y)) \to \neg T(Sam, y)))\end{array}}$$

We say that Sam has taken class a and b, and if class a and b are not the same class, then Sam didn't take any other classes.

Graded for Honest Effort and Feedback Given

1.11.4 p: 4 is a prime number (False)
q: 5 is a prime number (True)

a)

$$\begin{array}{l} q \\ \underline{p \vee q} \\ \therefore p \end{array}$$

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

The conclusion is a false statement and the first row in truth table shows the argument is valid.

b)

$$\begin{array}{l} q \\ \underline{\neg p \wedge q} \\ \therefore p \end{array}$$

p	q	$\neg p$	$\neg p \wedge q$
T	T	F	F
T	F	F	F
F	T	T	T
F	F	T	F

The conclusion statement is true and the third row in truth table shows all the hypthoese are true but conclusion is false.