ICS 6B F23 Take Home Exam 5 Redo

Due: November 9th, 2023 at 11:59PM

Name: ____

UCI NetID : | | | | | | | | |

(**alpha-numeric; NOT your student ID**)

- **Read** the instructions of each question carefully.
- **All problems will have a "What to show" section that will describe exactly what work is expected of you we solving the problem. Failure to meet the requirements of the "What to show" sections will result in a Not Yet. If you have questions about what to show please ask on Ed.**
- An answer where thought process is unclear will be given a grade of Not Yet
- Your submission should follow the template exactly. Any insertion, removal, or reordering of pages from the original template may result in readers not grading certain problems. In such an event you will receive "Not Yet" and no feedback on the problems in question.
- Place your answers in the boxed regions. Writing outside of the boxes will not be considered as part of your answers.
- This exam will cover the Outcomes from the F Learning Objective
- Please keep in mind of the academic honesty guidelines. This take-home exam is to be **completed individually, with no outside help**. You may use any resources from our class (ZyBooks and resources from Canvas), but you may not use any other online resources.
- You may choose to print the exam or use a digital editor for completing the exam. It is required that you use this PDF to complete your work. If you have no access to a printer or digital tools to fulfill the exam, feel free to reach out to the staffs regarding your concern.
- If you have any questions, please post a private Ed or attend available Office Hours. Note that we are not allowed to provide specific help to answering the exam questions.

Problem 1.1 (F1)

What you need to show: For each part you just write an answer with any work or explanation. Make sure to include all domain and target elements on your arrow diagram. Consider the function T = {(Lawson, Joseph), (Mandy, Joseph), (Jonathan, Wen), (Quang, Anson)} which has the following target: {Joseph, Ahn, Anson, Wen}.

a) Draw the arrow diagram of T.

b) What is the Domain and range of T?

c) What is the T(Mandy)?

d) What element of the domain gets mapped to Anson by T?

Problem 1.2 (F1)

What you need to show: For each function state whether it is well defined or not. If it is not well-defined, provide a counter example that violates one or both conditions for a function to be well defined. If it is well defined give a one sentence justification for each condition of how you know it holds for this function.

a) Let G = {A+, A, A-, B+, B, B-, C+, C, C-, D+, D, D-, F}, and S be the set of all students in ICS 6B last quarter (which had 100s of students). Assume all students received a grade.
The function f: $G \to S$: f maps a grade to all the students that received that grade.

b) Let F be the set of all functions.
The function g: $F \to F$: g maps a function to its inverse.

Problem 2 (F2)

What you need to show: For each function state whether it is bijective, or not bijective (ONLY MARK ONE CHOICE). If it is bijective provide 1 brief explanation to show that it is onto and another brief explanation to show it is one-to-one. If it is not bijective then provide a counterexample or a brief explanation that shows it isn't one-to-one or onto (you may choose one property, no need to do both of them).

a) Let P be the set of all propositional logic expressions.
The function f: $P^2 \to P$: f maps the pair of expressions (p, q) to the expression $(p) \wedge (q)$. Assume that two expressions are different if they are symbolically different, even if they are logically equivalent. For example $(r \wedge p) \wedge q$ is a different expression than $r \wedge p \wedge q$. Similarly $p \wedge q$ is consider to be a different expression than $q \wedge p$. Is this function bijective?

b) Let M = $\{x \in \mathbb{Z} : -50 \leq x \leq 50\}$.
The function g: $\{0, 1\}^{50} \to M$: The output of g is computed by starting at 0 and walking through the input string, you add 1 whenever you see a 1 and subtract 1 whenever you see a 0. Is this function bijective?

Problem 3 (F3)

What you need to show: For each function state whether or not it has an inverse. If it does, state the inverse and explain why it is an inverse of the function. If there is no inverse explain how you know there is no inverse.
For the following function determine if the function has an inverse. If it does then describe the inverse (some functions may be their own inverses, if this is the case it is ok to say that it is its own inverse). If there is no inverse then explain why the the inverse cannot be well defined.

a) The function X: $\{0, 1\}^{2n} \to \{0, 1\}^n$ for some $n \in \mathbb{Z}^+$: the ith bit of the output of X is ((2i - 1)th bit of the input) $\oplus$ ((2i)th bit of the input). Assume the string are 1 indexed (ie the first digit is the first bit, not the zeroth bit). For example (for n=2) X(1101) = 01, since $1 \oplus 1 = 0$ (1st and 2nd bits), and $0 \oplus 1 = 1$ (3rd and 4th bits)

b) The function Y: $\{0, 1\}^5 \to \{0, 1\}^5$: Y first replaces the last bit with the first bit then if the original last bit was 1, Y complements the first bit. For example Y(10010) = 10011, Y(10011) = 00011, and Y(00011) = 10010

Problem 4 (F4)

What you need to show: Answer the question and provide a sentence or two of justification to support your answer. If the question asks what a function is then define the composite function as a single new function. Defining your function as a composition of functions instead of as a single function is not allowed, and will receive a not yet. If the function is not well defined then instead of answering the question state that is not well defined and explain what makes it not well defined.

NOTE: If you are defining the composition as a single function I am looking for one piece of key information that shows why that composition can be simplified in the proposed way. For example on the original take-home exam 5 this key piece of information for part b was that the first rotation shifted the original odd bits into the even spots to be flipped. For part d the key information was that k dropped all bits that were changed leaving just the original string.

Consider the following functions, where n and $i \in \mathbb{Z}^+$.

$f_n : \{0.1\}^n \to \{0,1\}^n$ where f flips the value of all the even bits in the string. (Flipping bits means to change 1 to 0 or 0 to 1.) For example $f_5(00000) = 01010$.

$g_i : \{0,1\}^{10} \to \{0,1\}^{10}$ where g rotates a string to the right by i. Rotating to the right by 1 means to remove the last bit and append it to the start of the string. Rotating to the right by $n > 1$ is achieved rotating to the right by 1, n times. For example, $g_2(0110101101) = 0101101011$.

$h : \{0,1\}^5 \to \{0,1\}^{10}$ where h inserts a duplicate of each bit immediately after that bit. For example, h(01010) = 0011001100.

$k : \{0,1\}^{10} \to \{0,1\}^5$ where k removes all the odd bits of the string.

Answer the following questions. If at any point the function in the question not well defined then say so instead of answering the question.

a) What is the domain and target of the function $g_{10} \circ h$?

b) What is the function $h \circ f_{10} \circ g_3$?

c) What is the function $k \circ f_{10} \circ g_{10} \circ h$?

d) Is there an inverse to the function $h \circ f_5 \circ k \circ f_{10}$? If there is, **describe and define** the inverse function. If there is no inverse, then **explain** what would make the inverse not well defined.