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Instructions

Please read the following instructions carefully:

1. Please show all notation for probability statements.
2. Box your final answers.
3. Please verify that your scans are legible.
4. Please assign pages the the questions when submitting to gradescope.
5. This assignment is due via gradescope on the due date.

1. The amount of time Anna the Anteater spends on TikTok per day, in hours, follows a normal distribution with $\mu = 2$ hours and $\sigma = 0.25$ hours.

   (a) What is the probability that Anna the Anteater spends between 2 and 3 hours on TikTok (over the course of one day)? Write out the solution in integral form and use the following R code to compute the numerical answer: $pnorm(3, 2, 0.25) - pnorm(2, 2, 0.25)$.

$$\begin{aligned} P(2<X<3) &= P(X<3) - P(X<2) \\ &= \int_{-\infty}^{3} f(3)\,dx - \int_{-\infty}^{2} f(2)\,dx \\ &= pnorm(3,2,0.25) - pnorm(2,2,0.25) = \boxed{0.5000} \end{aligned}$$

   (b) How many hours (over the course of one day) would Anna the Anteater need to spend on TikTok to be in the top 4% of her daily viewing time? Write out the equation we would need to solve in order to answer this question, then use the $qnorm()$ function in R to find this amount of time.

$$z_c = qnorm(0.96, 0, 1) = 1.750686$$

$$\begin{aligned} x_c = z_c\,\sigma + \mu &= 1.750686(0.25) + 2 \\ &= qnorm(0.96, 2, 0.25) = \boxed{2.4377} \end{aligned}$$

   (c) What is the probability that Anna will **not** spend more than 1.5 hours on TikTok (over the course of one day)?

$$\begin{aligned} P(X \leq 1.5) &= \int_{-\infty}^{1.5} f(1.5)\,dx \\ &= pnorm(1.5, 2, 0.25) \\ &= \boxed{0.0228} \end{aligned}$$

   (d) Some students at USC, are known to spend 3 hours a day on TikTok. What is the probability that Anna spends more time on TikTok than these students (over the course of one day)?

$$\begin{aligned} P(X>3) &= 1 - P(X \leq 3) \\ &= 1 - \int_{-\infty}^{3} f(3)\,dx \\ &= 1 - pnorm(3, 2, 0.25) \\ &= \boxed{3.1671 \times 10^{-5}} \end{aligned}$$

 $X \sim$ Normal (

2. The number of hours before your phone runs out of battery can be described by a random variable X, with distribution function $f(x) = \dfrac{c}{3x^3}$. And let $\mathbb{S}_X = [1, \infty)$. Assume X is a continuous variable.

   (a) For what value of c is $f(x)$ a valid density?

$$\int_1^\infty \frac{c}{3x^3}\,dx = \frac{c}{3}\int_1^\infty \frac{1}{x^3}\,dx = \frac{c}{3}\left(-\frac{1}{2x^2}\right)\Big|_1^\infty$$
$$= \frac{c}{3}\left(\left(-\frac{1}{2(\infty)^2}\right) - \left(-\frac{1}{2(1)^2}\right)\right)$$
$$= \frac{c}{3}\left(0 + \frac{1}{2}\right) = \frac{c}{6}$$

$$\frac{c}{6} = 1$$
$$c = \boxed{6}$$

   (b) What is the average length of time before your phone battery dies?

$$E(X) = \int_1^\infty x\left(\frac{6}{3x^3}\right)dx = \frac{\cancel{6}^{\,2}}{\cancel{3}}\int_1^\infty \frac{\cancel{x}}{3x^{\cancel{3}\,2}}\,dx = 2\int_1^\infty \frac{1}{3x^2}\,dx = 2\left(-\frac{1}{x}\right)\Big|_1^\infty$$
$$= 2\left(-\frac{1}{\infty} - \left(-\frac{1}{1}\right)\right) = 2(0+1) = \boxed{2 \text{ hours}}$$

   (c) What is the probability that your phone goes exactly 2 hours without running out of battery?

$$P(X = 2) = \boxed{0}$$

   (d) What is the probability that your phone battery lasts between a half hour and 3 hours?

$$P(0.5 < X < 3) = P(1 < X < 3)$$
$$= \int_1^3 \frac{6}{3x^3}\,dx$$
$$= \left(-\frac{1}{x^2}\right)\Big|_1^3$$
$$= \left(-\frac{1}{9}\right) - \left(-\frac{1}{1}\right)$$
$$= -\frac{1}{9} + \frac{9}{9}$$
$$= \boxed{\frac{8}{9}}$$

 $X \sim \text{Uniform}(4,7)$

3. Let X be a continuous random variable. Let $f(x) = c(x-2)^2$ and $\mathbb{S}_X = [4, 7]$.

(a) What value of c will make $f(x)$ a valid density? Leave as a fraction.

$$\int_4^7 c(x-2)^2\,dx = c\int_4^7 (x^2-4x+4)\,dx = c\left(\int_4^7 x^2\,dx - \int_4^7 4x\,dx + \int_4^7 4\,dx\right)$$

$$= c\left(\frac{x^3}{3} - \frac{4x^2}{2} + 4x\right)\Big|_4^7 = c\left(\frac{7^3}{3} - \frac{4(7)^2}{2} + 4(7) - \left(\frac{4^3}{3} - \frac{4(4)^2}{2} + 4(4)\right)\right)$$

$$= c\left(\frac{343}{3} - 98 + 28 - \frac{64}{3} + 32 - 16\right)$$

$$= c\left(\frac{279}{3} - 54\right) = 39c$$

$39c = 1$

$c = \boxed{\frac{1}{39}}$

(b) What is $P(X = 6)$?

$P(X=6) = \boxed{0}$

(c) What is $P(X > 7)$?

$P(X>7) = \boxed{0}$

(d) Find $E(X)$.

$$E(X) = \int x \int_4^7 \frac{1}{39}(x-2)^2\,dx$$

$$= \frac{1}{39}\int_4^7 x(x^2-4x+4)\,dx$$

$$= \frac{1}{39}\int_4^7 (x^3-4x^2+4x)\,dx$$

$$= \frac{1}{39}\left(\frac{x^4}{4} - \frac{4x^3}{3} + \frac{4x^2}{2}\right)\Big|_4^7$$

$$= \frac{1}{39}\left(\frac{7^4}{4} - \frac{4(7)^3}{3} + 2(7)^2 - \left(\frac{4^4}{4} - \frac{4(4)^3}{3} + 2(4)^2\right)\right)$$

$$= \frac{1}{39}\left(\frac{2401}{4} - \frac{1372}{3} + 98 - \frac{256}{4} + \frac{256}{3} - 32\right)$$

$$= \frac{1}{39}\left(\frac{2145}{4} - \frac{1116}{3} + 66\right) = \boxed{5.9038}$$

(e) What is $P(5 < X < 6)$?

$$P(5<X<6) = \int_5^6 \frac{1}{39}(x-2)^2\,dx$$

$$= \frac{1}{39}\int_5^6 (x^2-4x+4)\,dx$$

$$= \frac{1}{39}\left(\frac{x^3}{3} - \frac{4x^2}{2} + 4x\right)\Big|_5^6 = \frac{1}{39}\left(\frac{6^3}{3} - \frac{4(6)^2}{2} + 4(6) - \left(\frac{5^3}{3} - \frac{4(5)^2}{2} + 4(5)\right)\right) = \boxed{0.3162}$$

$P(5<X<6) = P(X<6) - P(X<5)$

$= \text{punif}(6,4,7) - \text{punif}(5,4,7)$

4. Anna the Anteater usually spends between 0 and 35 minutes walking to class. More specifically, the time it takes her to walk to class follows a normal distribution with a mean of $\mu = 15$ minutes and variance $\sigma^2 = 36$ minutes.

   $\theta = 6$ minutes

   (a) Write out the integral form of the probability it will take Anna between 25 and 35 minutes to walk to class, then use the pnorm function in R to find this probability.

   $$P(25 < x < 35) = P(x < 35) - P(x < 25)$$
   $$= \int_{25}^{35} \frac{1}{\theta\sqrt{2\pi}} e^{-(x-\mu)^2/2\theta^2}\, dx$$
   $$= \text{pnorm}(35, 15, 6) - \text{pnorm}(25, 15, 6)$$
   $$= \boxed{0.0474}$$

   (b) What is the probability that it will take Anna less than 10 minutes to talk to class?

   $$P(x < 10) = \text{pnorm}(10, 15, 6)$$
   $$= \boxed{0.2023}$$

   (c) What is the probability that it will take Anna longer than 18 minutes to walk to class?

   $$P(x > 18) = 1 - P(x \leq 18)$$
   $$= 1 - \text{pnorm}(18, 15, 6)$$
   $$= \boxed{0.3085}$$

   (d) What is the walking time that would put Anna's walk to class in the shortest 1% of all her walks to class?

   $1\% = 0.01$

   $$x_c = \text{qnorm}(0.01, 15, 6)$$
   $$= \boxed{1.0419 \text{ mins}}$$

5. You are in the library trying to study for your upcoming Stats 67 quiz and phones keep making noise. The time until a phone makes noise, in minutes, follows an exponential distribution with parameter $\lambda = \frac{1}{11}$.

(a) What is the probability that no phones will make noise in the first 10 minutes of studying?

$P(X > 10) = 1 - P(\leq 10) = 1 - \text{pexp}(10, \frac{1}{11}) = \boxed{0.4029}$

(b) What is the expected time until the first noise from a phone?

$E(X) = \frac{1}{\lambda} = \frac{1}{(\frac{1}{11})} = \boxed{11 \text{ mins}}$

(c) Surprisingly, after half an hour in the library, no phones have made noise, what is the probability that this is still the case 40 minutes into studying at the library?

$P(X > 40 \mid X > 30) = P(X > 10) = 1 - P(X \leq 10)$

$= 1 - \text{pexp}(10, \frac{1}{11})$

$= \boxed{0.4029}$

(d) Suppose that it is a particularly bad day for phones going off in the library and it is very difficult to concentrate with all of the noise. You decide that if three more phones ring you will give up trying to study at the library and go home. How long do you expect to wait until you go home? Assume that each phone in the library rings independently and the time between rings is identically distributed.

$E(T) = \frac{3}{\lambda} = \frac{3}{(\frac{1}{11})} = \boxed{33 \text{ mins}}$

6. Each day you go to Coco's Pizza in Newport to buy a slice of pizza. At Coco's they do not pre-slice the pizza and you are allowed to buy any percentage of a pizza that you like. For example, you could ask for a slice that is 20% of the pizza. Your favorite pizza is pepperoni and they cook 5 of these pizzas each day. When you go to Coco's, the amount of pepperoni pizza left follows a uniform distribution with bounds 0 and 5.

   (a) What is the probability that there is at least 1 whole pepperoni pizza left on a given day?

   $$P(X \geq 1) = 1 - P(X \leq 1)$$
   $$= 1 - \frac{1-0}{5-0}$$
   $$= 1 - \text{punif}(1,0,5)$$
   $$= \boxed{0.8}$$

   (b) What is the expected amount of pepperoni pizza that will be left each day when you visit Coco's?

   $$E(X) = \frac{b+a}{2} = \frac{5-0}{2} = \boxed{2.5 \text{ pizza}}$$

   (c) One day you decide you want to buy 0.4 of a pepperoni pizza, what is the probability that there is not enough pizza left for you to do this?

   $$P(X \leq 0.4) = \frac{0.4-0}{5-0}$$
   $$= \text{punif}(0.4, 0, 5)$$
   $$= \boxed{0.08}$$

   Now suppose your friend Anna the Anteater also goes to Coco's Pizza Parlor and likes to buy the cheese pizza. The amount left of cheese pizza follows a uniform distribution with bounds 1.2 and 4. For questions (d) and (e) assume that the amount of each type of pizza is independent.

   (d) What is the probability that there are more than 2 pizzas left of exactly 1 of the types of pizza? (E.g. 2+ pepperoni but 2- cheese, or 2+ cheese but 2- pepperoni.)

   A = there are more than 2 pepperoni pizzas

   B = there are more than 2 cheese pizzas

$$P(A \cap B^c) + P(A^c \cap B)$$
$$= \text{punif}(2,0,5) \cdot (1 - \text{punif}(2,1.2,4)) + (1 - \text{punif}(2,0,5)) \cdot \text{punif}(2,1.2,4))$$
$$= \boxed{0.4571}$$

 $P(A \cup B)^c = P(A^c \cap B^c)$

(e) What is the probability that there are at least two whole pizzas left for at least one type of pizza?

P(at least 2 pepperoni or at least 2 cheese)

= 1 − P(less than 2 full pizzas for each type)

= 1 − (P(peperoni < 2) · P(cheese < 2))

= 1 − (punif(2,0,5) · punif(2,1.2,4))

= $\boxed{0.8857}$

7. You visit the on campus Starbucks. The waiting time in line can be modelled as an exponential distribution where the average time in line is 22 minutes.

   (a) What is the parameter of the Exponential distribution? What is it equal to in this problem?

   $X \sim \text{Exponential}\left(\lambda = \frac{1}{22} \frac{\text{person}}{\text{mins}}\right)$

   The parameter λ is each person's expected wait time is 22 minutes.

   (b) What is the probability that you visit the on campus Starbucks and you wait in line between 15 and 30 minutes?

   $$P(15 < X < 30) = P(X < 30) - P(X < 15)$$
   $$= \text{pexp}(30, \tfrac{1}{22}) - \text{pexp}(15, \tfrac{1}{22})$$
   $$= \boxed{0.25}$$

   (c) You have been waiting in line for 12 minutes, your total time waiting in line cannot be more than 35 minutes or you will be late for your next class. What is the probability that you are late for your class?

   $$P(X > 35 \mid X > 12) = P(X > (35 - 12))$$
   $$= P(X > 23)$$
   $$= 1 - P(X \leq 23)$$
   $$= 1 - \text{pexp}(23, \tfrac{1}{22})$$
   $$= \boxed{0.3515}$$