CSCI 190 Discrete Mathematics Applied to Computer Science Exam 1

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Read these instructions before proceeding.

- Closed book. Closed notes. You can use calculator.
- You have **80 minutes** to complete this exam.
- No questions will be answered during the exam or immediately afterwards. Answer each question as best you can. Partial credit will be awarded for reasonable efforts. If a question contains an ambiguity or a misprint, then say so in your answer, providing the answer to a reasonable interpretation of the question; give your assumptions.
- Answer the problems on the blank spaces provided for each problem.
- **Box your answers**.

Q1 (6)	Q2 (7)	Q3 (7)	Q4 (6)	Q5 (6)	Q6 (8)	Q7 (4)	Q8 (4)	Q9 (4)	Q10 (4)	Q11 (6)	Q12 (6)	Q13 (5)	Q14 (6)	Q15 (5)	Q16 (5)	Q17 (6)	Q18 (5)	Total (100)

1. *(6 pts)* Determine whether the proposition is ***TRUE*** or ***FALSE***. No justifications needed.

$\boxed{F}$ a) **1 + 4 = 3 if and only if 3 + 3 = 6.** (2 pts)

$\boxed{T}$ b) **If it is raining, then it is raining.** (2 pts)

$\boxed{F}$ c) **If 10 > 3, then 3 > 4.** (2 pts)

2. *(7 pts)* Determine whether **(p → ¬q) ≡ (q → ¬p)** using truth table.

p	q	¬p	¬q	p -> ¬q		q -> ¬p
T	T	F	F	F		F
T	F	F	T	T	≡	T
F	T	T	F	T		T
F	F	T	T	T		T

$\boxed{\therefore \text{ yes, } (p \to \neg q) \equiv (q \to \neg p)}$

3. *(7 pts)* Prove that **(¬p ∧ (¬q → p)) → q** is a tautology using propositional equivalence and the laws of logic. Give names of laws used in all steps.

$(\neg p \land (\neg(\neg q) \lor p)) \to q$ logical equivalent

$(\neg p \land (q \lor p)) \to q$ double negation law

$\neg(\neg p \land (q \lor p)) \lor q$ logical equivalent

$(\neg(\neg p) \lor \neg(q \lor p)) \lor q$ DeMorgan

$(p \lor \neg(q \lor p)) \lor q$ double negation law

$(p \lor (\neg q \land \neg p)) \lor q$ DeMorgan

$((p \lor \neg q) \land (p \lor \neg p)) \lor q$ Distribution

$((p \lor \neg q) \land T) \lor q$ Negation law

$(p \lor \neg q) \lor q$ Identity Law

Continue...

$p \lor (\neg q \lor q)$ Associative law

$p \lor T$ Negation law

T Domination law

$\boxed{(\neg p \land (\neg q \to p)) \to q \text{ is a tautology}}$

4. (6 pts) Write the contrapositive, converse, and inverse of the following:

If you try attend the class, then you will be happy.

p (attend the class) $\qquad$ q (be happy)

a) **contrapositive** (2 pts)

$\sim q \rightarrow \sim p$

If you are not happy, then you will not try attend the class.

b) **converse** (2 pts)

$q \rightarrow p$

If you will be happy, then you try attend the class.

c) **inverse** (2 pts)

$\sim p \rightarrow \sim q$

If you don't try attend the class, then you will not be happy.

5. (6 pts) Suppose the variable x represents people, and
F(x): x is friendly $\qquad$ T(x): x is tall $\qquad$ A(x): x is angry.

Write the statement using these predicates and any needed quantifiers.

a) **All people are not angry**. (3 pts)

$\forall x \sim A(x)$

b) **Some tall people are friendly**. (3 pts)

$\exists x\,(T(x) \rightarrow F(x))$

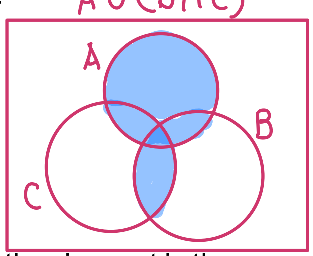
6. (8 pts) Consider the following theorem:
"**if x and y are odd integers, then 2x + 2y is even**".
Give a direct proof of this theorem.

Let x and y be odd integers.
Then $x = 2k+1$ for some integer k
& $y = 2l+1$ for some integer l

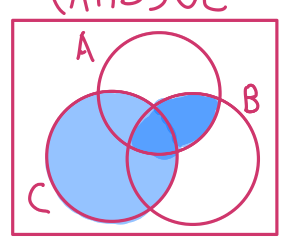
$\Rightarrow 2x + 2y = 2(2k+1) + 2(2l+1)$
$= 4k + 2 + 4l + 2 = 4k + 4l + 4$
$= 2(2k + 2l + 2)$ which is even.

7. (4 pts) Draw two Venn diagrams for **A U (B ∩ C)** and **(A ∩ B) U C**.
Are they the same?

A ∪ (B ∩ C)



(A ∩ B) ∪ C



∴ No, they are not same.

8. (4 pts) determine whether the given set is the power set of some set. (Answer "**Yes**" or "**No**").
If the set is a power set, **give the set** of which it is a power set.

a) {Ø, {b}, {Ø}, {b,Ø}} (2 pts)

Yes, the power set is $\{b, \emptyset\}$

b) {{Ø}, {{a,b}}} (2 pts)

No, there is no empty contain in this set.

9. (4 pts) Just answer "***yes***" or "***no***" in the box. No justifications needed.

$\boxed{\text{Yes}}$ (a) Suppose f: Z ⟶ Z has the rule f(n) = n + 7. Determine whether f is 1-1. (1 pts)

$\boxed{\text{Yes}}$ (b) Suppose f: Z ⟶ Z has the rule f(n) = n + 7. Determine whether f is onto. (1 pts)

$\boxed{\text{NO}}$ (c) Suppose f:N ⟶ N has the rule f(n) = n² + 1. Determine whether f is 1-1. (1 pts)

$\boxed{\text{NO}}$ (d) Suppose f:N ⟶ N has the rule f(n) = n² + 1. Determine whether f is onto. (1 pts)

10. (4 pts) Find $\mathbf{a_n}$ (a formula that generates the following sequence a_1, a_2, a_3 . . . .)

a) 23, 26, 29, 32, 35, . . . (2 pts) $\boxed{a_n = 20 + 3n}$

b) -5, 5, -5, 5, -5, 5, . . . (2 pts) $\boxed{a_n = (-1)^n \cdot 5}$

11. (6 pts) Suppose **A =** $\begin{bmatrix} 1&0&1 \\ 0&1&1 \\ 1&1&0 \end{bmatrix}$ and **B =** $\begin{bmatrix} 0&1&0 \\ 0&1&1 \\ 1&0&0 \end{bmatrix}$

Find
(a) the ***join*** of A and B.

$A \vee B$

$$\begin{bmatrix} 1\vee 0 & 0\vee 1 & 1\vee 0 \\ 0\vee 0 & 1\vee 1 & 1\vee 1 \\ 1\vee 1 & 1\vee 0 & 0\vee 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

(b) the ***meet*** of A and B.

$A \wedge B$

$$\begin{bmatrix} 1\wedge 0 & 0\wedge 1 & 1\wedge 0 \\ 0\wedge 0 & 1\wedge 1 & 1\wedge 1 \\ 1\wedge 1 & 1\wedge 0 & 0\wedge 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

(c) the ***Boolean product*** of A and B.

$$A \odot B = \begin{bmatrix} \overset{0 \vee 0 \vee 1}{(1\wedge 0)\vee(0\wedge 0)\vee(1\wedge 1)} & \overset{1 \vee 0 \vee 0}{(1\wedge 1)\vee(0\wedge 1)\vee(1\wedge 0)} & \overset{0 \vee 0 \vee 0}{(1\wedge 0)\vee(0\wedge 1)\vee(1\wedge 0)} \\ \overset{0 \vee 0 \vee 1}{(0\wedge 0)\vee(0\wedge 0)\vee(1\wedge 1)} & \overset{0 \vee 1 \vee 0}{(0\wedge 1)\vee(1\wedge 1)\vee(1\wedge 0)} & \overset{0 \vee 1 \vee 0}{(0\wedge 0)\vee(1\wedge 1)\vee(1\wedge 0)} \\ \overset{0 \vee 0 \vee 0}{(1\wedge 0)\vee(1\wedge 0)\vee(0\wedge 1)} & \overset{1 \vee 1 \vee 0}{(1\wedge 1)\vee(1\wedge 1)\vee(0\wedge 0)} & \overset{0 \vee 1 \vee 0}{(1\wedge 0)\vee(1\wedge 1)\vee(0\wedge 0)} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

12. (6 pts)
Show (step by step) how the binary search algorithm searches for **8** in the following list:

a = 2 3 8 15 21 25 57.
index 0 1 2 3 4 5 6

function binary search (A, n, T) is
  S := 0
  R := n-1
 while S ≤ R do
  m := floor ((S+R)/2)
  if A [m] < T then
   S := m+1
 else if A [m] > T then
   R := m-1
 else Return m.

Let $i = 0$ & L = array length $-1 \sim 6$

Let $\frac{i+L}{2} = \frac{0+6}{2} = 3$ = middle ⟸ middle index

1st iteration a[middle] = a[3] = 15 ≠ 8

Let i = middle −1 = 3 − 1 = 2

2nd iteration check 0 ~ 2 ⟹ from this we see the mid # is 1

3rd iteration check a[1] = 3, 8 > 3; 2 ~ 2, $\frac{2+2}{2} = 2$, a[2] = 8, 8 = 8 $\boxed{\checkmark}$

$n^3+3n^2+11 < n^3+3n^3+11n^3 = 15n^3$

$C=15$

$k=1$

13. (5 pts) Arrange the following functions in a list so each is ***big-O*** of the next one in the list. ***No justifications needed***.

$n^3 + 3n^2 + 11$, $\log n$, $100n$, $n^3 \log n$, 2^n

Ascending Order

$\log n,\ 100n,\ n^3+3n^2+11,\ n^3\log n,\ 2^n$

14. (6 pts)

(a) Give the ***best-case*** analysis of a lineary search of a list of size n (counting the number of comparisons). (3 pts) If n = 10, list = [50, 4, 6, 8, 10, 1, 11, 70, 66, 33]

element to search = 50, 50 is the 1^{st} of this list.

The # of comparison in case of the best case = 1 $\therefore O(1)$

(b) Give the ***worst-case*** analysis of a lineary search of a list of size n (counting the number of comparisons). (3 pts) If in the same scenario, element to search is 33. Then we compare each element till the last element.

hence, the # of comparison is n. $\therefore O(n)$

15. (5 pts) Prove or disprove: For all integers a, b, c, if **a|c** and **a|d**, then **a²|cd**.

$a|c \Rightarrow c = at \Rightarrow cd = at \cdot ar$

$a|d \Rightarrow d = ar$

$cd = a^2(tr)$ [tr = s since product of two integers is again an integer.]

Thus, $cd = a^2 s$

By def. of divisibility, $\therefore a^2 | cd$.

Hence, for all integers a, b, c, d, if a|c and a|d, then $a^2|cd$. ∎

16. (5 pts) Find the ***prime factorization*** of **1,100**.

2	1100
2	550
5	275
5	55
	11

$\Rightarrow 2^2 \cdot 5^2 \cdot 11$

2	71	1
2	35	0
2	17	0
2	8	0
2	4	1
2	2	1
	1	1

17. (6 pts)

(a) Convert $(\mathbf{71})_{10}$ to base 2.. (3 pts) $\boxed{(1000111)_2}$

(b) Convert $(\mathbf{11011111001})_2$ to base 16. (3 pts) $\boxed{6F9}$

6 F 9

18. (5 pts) A message has been ***encrypted*** using the function f(x) = (x + 7) mod 26.

If the message in coded form is **LEHJASF**, ***decode*** the message.

A	0
B	1
C	2
D	3
E	4
F	5
G	6
H	7
I	8
J	9
K	10
L	11
M	12
N	13
O	14
P	15
Q	16
R	17
S	18
T	19
U	20
V	21
W	22
X	23
Y	24
Z	25

LEHJASF $f(x) = (x+7) \bmod 26$

L = (11 − 7) mod 26 = 4 → E

E = (4 − 7) mod 26 = 23 → X

H = (7 − 7) mod 26 = 0 → A

J = (9 − 7) mod 26 = 2 → C

A = (0 − 7) mod 26 = 19 → T

S = (18 − 7) mod 26 = 11 → L

F = (5 − 7) mod 26 = 24 → Y

⇒ EXACTLY