Problem 3 (A3)

a)

- ☐ Partial Order
- ☑ Strict Order
- ☐ Neither
- ☐ Total Order

b)

- ☐ Partial Order
- ☐ Strict Order
- ☑ Neither
- ☐ Total Order

c)

- ☑ Partial Order
- ☐ Strict Order
- ☐ Neither
- ☐ Total Order

Explanation Boxes:

a)

Reflexive = It is not reflexive, If we have (5,3) is related to (5,3), then we have $(5-5) < (3-3) \rightarrow 0 \not< 0$.

Since it is not reflexive, it is not a partial order.

Anti-reflexive : For any integers x, y, (x, y) is related to (x, y), $(x-x) \not< (y-y)$, ∴ it is anti-reflexive.

Transitive : For any integers f, g, h, i, j, k, if (f, g) is related to (h, i) where $(f-h) < (g-i)$ and (h, i) is related to (j, k), where $(h-j) < (i-k)$, then (f, g) is related to (j, k) where $(f-j) < (g-k)$, ∴ it is transitive.

Anti-symmetric : Since it is anti reflexive and transitive, then it is also anti symmetric ;

Since it is anti-reflexive and transitive, it is a strict order.

Total order : if we have (4,5) is related to (2,3), where $(4-2) < (5,3) \rightarrow 2 \not< 2$, then (4,5) and (2,3) are not comparable ∴ it is not a total order.

b)

Reflexive : for x that is a subset of S, x is related to x, then $|x| = |x| + 1$. It is false, ∴ it is not reflexive.

Anti-reflexive : for any x that is a subset of S. $|x| \neq |x| + 1$, ∴ it is anti-reflexive.

Transitive : for any x, y, z that are subsets of S, if $|x| = |y| + 1$ and $|y| = |z| + 1$ we have

$$|x| = |y| + 1$$
$$|x| = (|z| + 1) + 1$$
$$|x| = |z| + 2$$

∴ it is not transitive.

$\{\{a, b, c, d\}\}$ $\{a, b, c, d\}$

c)

Reflexive : we know that any set is a subset of itself. For $x \in P(A)$, x is related x because x is a subset of x ∴ it is reflexive.
Transitive : for $x, y, z \in P(A)$, if x is a subset of y and y is a subset of z, then x is a subset of z, ∴ it is transitive.
Anti-symmetric : for $x, y \in P(A)$, if x is a subset of y and y is subset of x, then $x = y$, ∴ it is anti-symmetric.
Since it is reflexive, transitive, and anti symmetric, it is a partial order.

Total order : $\{1\}, \{2\} \in P(A)$, but $\{1\}$ is not a subset of $\{2\}$ nor $\{2\}$ is a subset of $\{1\}$ ∴ $\{1\}$ and $\{2\}$ are not comparable.
∴ it is not a total order.

Problem 4 (A4)

a)

- ☐ Graph 1
- ☐ Graph 2
- ☐ Graph 3
- ☑ Graph 4

b)

- ☐ Sort 1
- ☑ Sort 2
- ☐ Sort 3
- ☑ Sort 4

(A, B, D, E / C, F)