CSCI 190 Discrete Mathematics Applied to Computer Science
Exam 2

Name : ______________________

Last 4 digits of your Student ID #: ______________________

Read these instructions before proceeding.

- Closed book. Closed notes. You can use calculator.
- You have **80 minutes** to complete this exam.
- No questions will be answered during the exam or immediately afterwards. Answer each question as best you can. Partial credit will be awarded for reasonable efforts. If a question contains an ambiguity or a misprint, then say so in your answer, providing the answer to a reasonable interpretation of the question; give your assumptions.
- Answer the problems on the blank spaces provided for each problem.
- **Box your answers**.

Q1 (8)	Q2 (8)	Q3 (8)	Q4 (8)	Q5 (6)	Q6 (12)	Q7 (10)	Q8 (8)	Q9 (8)	Q10 (8)	Q11 (8)	Q12 (8)	Total (100)

1. (8 pts)
Use the Principle of Mathematical Induction to prove that
$1 + 4 + 16 + 64 + \cdots + 4^n = (4^{n+1} - 1)/3$ for all $n \geq 0$.

2. (8 pts) **Give a recursive definition with initial condition(s).**

a) The function $f(n) = (2n)!$, $n = 0, 1, 2, \ldots$ (4 pts)

b) The Fibonacci numbers **1, 1, 2, 3, 5, 8, 13, ...** starting from f(0) (4 pts)

3. (8 pts)

a) **Find f(2) and f(3) if** $f(n) = 3f(n - 1) + 5$, $f(0) = 1$. (4 pts)

b) **Find f(8) if** $f(n) = 2f(n/2) + 2$, $f(1) = 2$. (4 pts)

4 (8 pts)

a) **Consider a bit string of length 15. How many begin with 100 and end with 010?** (4 pts)

b) **How many ways are there to seat 7 people around a circular table where two seatings are considered the same when everyone has the same immediate left and immediate right neighbor?** (4 pts)

5. (6 pts)
Explain how the Pigeonhole Principle can be used to show that among any 21 integers, at least three must have the same last digit.

6. (12 pts)

a) **How many ways are there to select 4 students from a class of 30 to serve on a committee?** (4 pts)

b) **How many ways are there to select 5 students from a class of 35 to hold six different executive positions on a committee?** (4 pts)

c) **How many bit strings of length 14 have equal numbers of 0's and 1's?** (4 pts)

7. (10 pts)

a) **Use the Pascal's Tringle to expand $(x + y)^5$.** Show your work. (5 pts)

b) **Find the coefficient of x^4y^6 in the expansion of $(4x - 2y)^{10}$.** (5 pts)

8. (8 pts)
(a) **What is the probability that a card chosen from an ordinary deck of 52 cards is a Queen or Jack?.** (4 pts)

(b) **What is the probability that a fair coin lands Heads exactly 3 times out of 4 flips?** (4 pts)

9. (8 pts) Suppose you have a class with 30 students — 10 freshmen, 12 sophomores, and 8 juniors.

a) **You pick three students at random, one at a time. What is the probability that all three are junior?** (4 pts)

b) **You pick two students at random, one at a time. What is the probability that the second student is a freshman, given that the first is a sophomore?** (4 pts)

10. (8 pts)

a) **In a certain lottery game you choose a set of six numbers out of 33 numbers. Find the probability that none of your numbers match the six winning numbers.** (4 pts)

b) **An experiment consists of picking at random a bit string of length five. Consider the following events:**

E_1: the bit string chosen begins with 0;
E_2: the bit string chosen ends with 1.

Determine whether E_1 and E_2 are independent. Show your work. (4 pts)

11. (8 pts) Three coins are tossed.

a) **List the elements in the sample space.** (4 pts)

b) **Find the probability that at least two heads show.** (4 pts)

12. (8 pts)

Describe the following sequence recursively. Include initial condition of $\mathbf{a_1}$.

a_n **= the number of bit strings of length n with at least two consecutive zeros.**

a) $\mathbf{a_1}$ **=** ____________________________________ (2 pts)

b) $\mathbf{a_2}$ **=** ____________________________________ (2 pts)

c) $\mathbf{a_n}$ **=** ____________________________________ (4 pts)