ICS 6B F23 Take Home Exam 5

Due: November 3th, 2023 at 11:59PM

Name: ______________________________

UCI NetID : | | | | | | | | |

(**alpha-numeric; NOT your student ID**)

- **Read** the instructions of each question carefully.
- **All problems will have a "What to show" section that will describe exactly what work is expected of you we solving the problem. Failure to meet the requirements of the "What to show" sections will result in a Not Yet. If you have questions about what to show please ask on Ed.**
- An answer where thought process is unclear will be given a grade of Not Yet
- Your submission should follow the template exactly. Any insertion, removal, or reordering of pages from the original template may result in readers not grading certain problems. In such an event you will receive "Not Yet" and no feedback on the problems in question.
- Place your answers in the boxed regions. Writing outside of the boxes will not be considered as part of your answers.
- This exam will cover the Outcomes from the F Learning Objective
- Please keep in mind of the academic honesty guidelines. This take-home exam is to be **completed individually, with no outside help**. You may use any resources from our class (ZyBooks and resources from Canvas), but you may not use any other online resources.
- You may choose to print the exam or use a digital editor for completing the exam. It is required that you use this PDF to complete your work. If you have no access to a printer or digital tools to fulfill the exam, feel free to reach out to the staffs regarding your concern.
- If you have any questions, please post a private Ed or attend available Office Hours. Note that we are not allowed to provide specific help to answering the exam questions.

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Problem 1.1 (F1)

What you need to show: For each part you just write an answer with any work or explanation. Make sure to include all domain and target elements on your arrow diagram.
Consider the function (Ted, Ross), (Barney, Joey), (Marshall, Chandler), (Lily, Monica), (Robin, Rachel) which has the following target: Ross, Joey, Chandler, Monica, Rachel, Phoebe.

a) Draw the arrow diagram of this function.

b) What is the Domain and range of this function?

c) What is the image of Barney?

d) What is the preimage of Rachel?

Problem 1.2 (F1)

What you need to show: For each function state whether it is well defined or not. If it is not well defined provide a counter example that violates one or both conditions for a function to be well defined. If it is well defined give a one sentence justification for each condition of how you know it holds for this function.

a) A famous particle physicist from CERN is asking for your help. They know you are a well respected function designer and want you to create a function that maps allowed (ie it has a Feynmann diagram) particle interactions to their Feynmann diagrams*. They explain to you that since each Feynmann diagram defines the incoming and outgoing particles it must correspond to one interactions. However they further explain that the intermediate steps of the diagram can vary from simple to extremely complicated resulting in each interaction having infinitely many Feynmann diagrams. Do you tell them that you can design such a function? (i.e. is the function well defined?)

b) Unfortunately you could not participate in the Petr Sunset Drop last Spring, but you need one of those stickers. To get one you wish to design a function that will map each of the 500 stickers to its current owner. Assume that none of the stickers have been lost (i.e. someone has them) and that no one have obtained a second one through trading (no one has more than 2 of them). Is it possible for you to design such a function? (i.e. is the function well defined?)

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Problem 2 (F2)

What you need to show: For each function state whether it is injective, surjective, bijective, or none of them (ONLY MARK ONE CHOICE). Give a few sentences of explanation of how you came to your answer.

a) Recall to when you were taking midterm 1 (sorry if that brings back bad memories). Consider the function that maps students in your lecture to the seats they took the exam in. For this problem assumed that everyone in your lecture was present during the exam, including the DSC students who took the exam elsewhere. Recall that there was empty seats during the exam. Is this function injective, surjective, or bijective?

b) Consider a function that maps a set A to an positive real number x such that x is the average of the cardinality of the elements in P(A). Is this function injective, surjective, or bijective?

Problem 3 (F3)

What you need to show: For each function state whether or not it has an inverse. If it does state the inverse and explain why it is an inverse of the function. If there is no inverse explain how you know there is no inverse.
For the following function determine if the function has an inverse. If it does then describe the inverse (some function functions may be their own inverses, if this is the case it is ok to say that it is its own inverse). If there is no inverse then explain why the the inverse cannot be well defined.

a) Function f: $\{0,1\}^{2k} \to \{0,1\}^{2k}$ for some $k \in \mathbb{Z}^+$ such that f breaks the bit string into 2 string from $\{0,1\}^k$, reverse them independently and put them back together again. For example if we take k to 3, f(110011) = 011110.

b) The bitwise xor function that for two bit strings of length n is as follows: The xor operator on one bit gives 1 if the bits are different and 0 if they are same. The bitwise xor function is the following $x \oplus y = (x_1 \oplus y_1, x_2 \oplus y_2, ..., x_n \oplus y_n)$. For example $101 \oplus 110 = 011$.

   Consider a function f: $\{0,1\}^n \to \{0,1\}^n$ for some $\{0,1\}^n$ such for some predefined length n bit string k, f maps a string a to $a \oplus k$.

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Problem 4 (F4)

What you need to show: Answer the question and provide a sentence or two of justification to support your answer. If the question ask what a function is then define the composite function as a single new function. Simply stating the steps of each member in the composition in the correct order will receive a Not Yet. If the function is not well defined then instead of answering the question state that is not well defined and explain what makes it not well defined.
Consider the following functions, where n and i $\in \mathbb{Z}^+$

$f_n : \{0.1\}^n \to \{0,1\}^n$ where f flips the value of all the even bits in the string. (Flipping bits means to change 1 to 0 or 0 to 1.) For example $f_5(00000) = 01010$

$g_i : \{0,1\}^{10} \to \{0,1\}^{10}$ where g rotates a string to the right by i. Rotating to the right by 1 means to remove the last bit and appending to the start of the string. Rotating to the right by $n > 1$ is achieved rotating to the right by 1, n times. For example $g_2(0110101101) = 0101101011$.

$h : \{0,1\}^5 \to \{0,1\}^{10}$ where h insert a duplicate of each bit immediately after that bit. For example h(01010) = 0011001100.

$k : \{0,1\}^{10} \to \{0,1\}^5$ where k removes all the odd bits of the string.

Answer the following questions. If at any point the function in the question not well defined then say so instead of answering the question.

a) What is the domain and range of the function $f_5 \circ k \circ f_{10}$?

b) What is the function $g_7 \circ f_{10} \circ g_3$?

c) What is the function $k \circ f_5 \circ g_1 \circ k$?

d) Is there an inverse to the function $k \circ g_1 \circ f_{10} \circ h$? If there is what is it. If there is no inverse then what would make the inverse not well defined.