1.1 - 2,4,6

2. Which of these are propositions? What are the truth values of those that are propositions?

a) Do not pass go.No, This is not a proposition; it’s a command.
b) What time is it? No, This is not a proposition; it’s a question.
c) There are no black flies in Maine. Yes, This is a proposition that is false. The truth value is: There are black flies in Maine.
d) 4 + x = 5. No, This is not a proposition; its truth value depends on the value of x.
e) The moon is made of green cheese. Yes, This is a proposition that is false. The truth value is: The moon is not made of green cheese.
f) $2n \geq 100$ .No, This is not a proposition; its truth value depends on the value of n .

4. What is the negation of each of these propositions?

a) Jennifer and Teja are friends.
Jennifer and Teja are not friends.

b) There are 13 items in a baker’s dozen.
There are not 13 items in a baker’s dozen.
or
The number of items in a baker’s dozen is not equal to 13.

c) Abby sent more than 100 text messages every day.
There exists a day on which Abby sent at most 100 text messages.

d) 121 is a perfect square.
121 is not a perfect square.

6. Suppose that SmartphoneA has 256 MB RAM and 32 GB ROM, and the resolution of its camera is 8 MP; Smartphone B has 288 MB RAM and 64 GB ROM, and the resolution of its camera is 4 MP; and Smartphone C has 128 MB RAM and 32 GB ROM, and the resolution of its camera is 5 MP. Determine the truth value of each of these propositions.

A	**B**	**C**
RAM:256MB	**RAM:288MB**	**RAM:128MB**
ROM:32GB	**ROM:64GB**	**ROM:32GB**
camera :8MP	**camera:4MP**	**camera:5MP**

a) Smartphone B has the most RAM of these three smartphones.
True, because 288 > 256 and 288 > 128.

b) Smartphone C has more ROM or a higher resolution camera than Smartphone B.
(32GB<64GB) V (5MP>4MP) => False or True, then is true
True, because C has 5 MP resolution compared to B’s 4 MP resolution. And only one of these conditions needs to be met because of the word or.

c) Smartphone B has more RAM, more ROM, and a higher resolution camera than Smartphone A.
(288>256) ^ (64>32) ^ (4mp<8mp) True and True and False, then is False.
False, because its resolution is not higher. All of the statements would have to be true for the conjunction to be true.

d) If Smartphone B has more RAM and more ROM than Smartphone C, then it also has a higher resolution camera.
((288mb>125mb) ^ (64gb>32gb)) → (4mp<5mp) True ^ True → False, then is False.
→ False, because the hypothesis of this conditional statement is true and the conclusion is false.

e) Smartphone A has more RAM than Smartphone B if and only if Smartphone B has more RAM than Smartphone A
(288mb>256MB) ↔ (256mb<288MB) true ↔ false then is false
False, because the first part of this biconditional statement is false and the second part is true.

1.2 - 2, 4

2. You can see the movie only if you are over 18 years old or you have the permission of a parent. Express your answer in terms of m: “You can see the movie,” e: “You are over 18 years old,” and p: “You have the permission of a parent.”

M → (E V P)
You can see the movie if you meet one of these conditions.
You can see the movie if You are over 18 years old or you have the permission of a parent.

4. To use the wireless network in the airport, you must pay the daily fee unless you are a subscriber to the service. Express your answer in terms of w: “You can use the wireless network in the airport,” d: “You pay the daily fee,” and s: “You are a subscriber to the service.”
The proposition in symbols is
W → (D V S)
If you can use the wireless network, then you pay the daily fee or you are a subscriber to the service

1.3 - 6, 8, 10, 12

6. Use a truth table to verify the first De Morgan law ¬(p ∧ q) ≡ ¬p ∨ ¬q

P	Q	P ^ Q	~(P ^ Q)	~P	~Q	~P v ~Q
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

THUS: ¬(p ∧ q) ≡ ¬p ∨ ¬q

8. Use De Morgan's laws to find the negation of each of the following statements.
a) Kwame will take a job in the industry or go to graduate school.
Kwame will not take a job in the industry and will not go to graduate school.

b) Yoshiko knows Java and calculus.
Yoshiko does not know Java or does not know calculus.

c) James is young and strong.
James is not young, or he is not strong.

d) Rita will move to Oregon or Washington
Rita will not move to Oregon and will not move to Washington.

10. Show that each of these conditional statements is a tautology by using truth tables.
a) [¬p ∧ (p ∨ q)] → q

P	Q	~P	P V Q	~P ^ (P V Q)	[¬p ∧ (p ∨ q)] → q
T	T	F	T	F	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	F	F	T

All entries in the last column are 'T'.
Hence, it is tautology.

b) $[(p \rightarrow q) \land (q \rightarrow r)] \rightarrow (p \rightarrow r)$

P	Q	R	P→Q	Q→R	(P→Q) ^(Q→R)	P→R	$[(p \rightarrow q) \land (q \rightarrow r)] \rightarrow (p \rightarrow r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

All entries in the last column are ‘T’.
Hence, it is tautology.

c) $[p \land (p \rightarrow q)] \rightarrow q$

P	Q	P→Q	P ^ (P→Q)	$[p \land (p \rightarrow q)] \rightarrow q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

All entries in the last column are ‘T’.
Hence, it is tautology.

d) $[(p \lor q) \land (p \rightarrow r) \land (q \rightarrow r)] \rightarrow r$

P	Q	R	P V Q	P→R	Q→R	$[(p \lor q) \land (p \rightarrow r) \land (q \rightarrow r)]$	$[(p \lor q) \land (p \rightarrow r) \land (q \rightarrow r)] \rightarrow r$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	T	T	T	T	T
T	F	F	T	F	T	F	T
F	T	T	T	T	T	T	T
F	T	F	T	T	F	F	T
F	F	T	F	T	T	F	T
F	F	F	F	T	T	F	T

All entries in the last column are 'T'.
Hence, it is tautology.

12. Show that each conditional statement in Exercise 10 is a tautology without using truth tables.

a) $[\neg p \land (p \lor q)] \rightarrow q$
Assume the hypothesis is true. Then p is false. Since $p \lor q$ is true, we conclude that q must be true.
$[\neg p \land (p \lor q)] \rightarrow q$
$\equiv \neg[\neg p \land (p \lor q)] \lor q$
$\equiv \neg\neg p \lor \neg(p \lor q)] \lor q$
$\equiv p \lor \neg(p \lor q) \lor q$
$\equiv (p \lor q) \lor \neg(p \lor q)$
$\equiv T$.

b) $[(p \rightarrow q) \land (q \rightarrow r)] \rightarrow (p \rightarrow r)$
We want to show that if the entire hypothesis is true, then the conclusion $p \rightarrow r$ is true. We need only show that if p is true, then r is true. Suppose p is true. Then by the first part of the hypothesis, we conclude that q is true. It now follows from the second part of the hypothesis that r is true, as desired.

c) $[p \land (p \to q)] \to q$ Assume the hypothesis is true. Then p is true, and since the second part of the hypothesis is true, we conclude that q is also true, as desired.
d) $[(p \lor q) \land (p \to r) \land (q \to r)] \to r$
Assume the hypothesis is true. Since the first part of the hypothesis is true, we know that either p or q is true. If p is true, then the second part of the hypothesis tells us that r is true; similarly, if q is true, then the third part of the hypothesis tells us that r is true. Thus in either case we conclude that r is true.

1.4 - 6, 8, 14

6. Let N (x) be the statement "x has visited North Dakota," where the domain consists of the students in your school. Express each of these quantifications in English.

a) $\exists x N(x)$

Some of the students in your school have visited North Dakota.

or

There exists a student in your school who has visited North Dakota.

b) $\forall x N(x)$ Every student in your school has visited North Dakota.

or All students in your school have visited North Dakota.

c) $\neg \exists x N(x)$ No student in your school has visited North Dakota.

or There does not exist a student in the school who has visited North Dakota.

d) $\exists x \neg N(x)$ Some student in your school has not visited North Dakota.

or There exists a student in the school who has not visited North Dakota.

e) $\neg \forall x N(x)$ Not all of the students in your school visited North Dakota

f) $\forall x \neg N(x)$ All of the students in your school have not visit North Dakota

8. Translate these statements into English, where R(x) is "x is a rabbit" and H (x) is "x hops" and the domain consists of all animals.

a) $\forall x(R(x) \to H(x))$

If an animal is a rabbit, then that animal hops.

=> every rabbit hops

b) $\forall x(R(x) \wedge H(x))$ Every animal is a rabbit and hops.

c) $\exists x(R(x) \rightarrow H(x))$ There exists an animal such that if it is a rabbit, then it hops.

d) $\exists x(R(x) \wedge H(x))$

There exists an animal that is a rabbit and hops.

=> Some hopping animals are rabbits.

14. Determine the truth value of each of these statements if the domain consists of all real numbers.

a) $\exists x(x^3 = -1)$ true, since (-1)^3 = -1 such that x = - 1

b) $\exists x(x^4 < x^2)$ false because when x = 1 or - 1 than (1)^4 = (1)^2

c) $\forall x((-x)2 = x2)$ true, this is one-to-one function

Since $(-x)2 = ((-1)x)2 = (-1)2\ x2 = x2$, we know that $\forall x((-x)2 = x2)$ is true.

d) $\forall x(2x > x)$ false because 2(-2)<(-2) when x = -2 ect.

Twice a positive number is larger than the number, but this inequality is not true for negative numbers or 0 . Therefore $\forall x(2x > x)$ is false.

1.5 - 2, 6, 10

2. Translate these statements into English, where the domain for each variable consists of all real numbers.

a) $\exists x \forall y(xy = y)$ There exists a real number x such that for every real number y , xy = y . This is asserting the existence of a multiplicative identity for the real numbers, and the statement is true, since we can take x = 1.

b) $\forall x \forall y(((x \geq 0) \wedge (y < 0)) \rightarrow (x - y > 0))$ for all real number x that greater than 0 and for all real number y that less than 0, then x - y greater than 0.

=> For every real number x and real number y , if x is nonnegative and y is negative, then the difference $x - y$ is positive.

=> A nonnegative number minus a negative number is positive. (True)

c) $\forall x \forall y \exists z(x = y + z)$

For all real numbers x and y, there are some numbers z that exist, such that the total of y plus z equal to x.

=> For every real number x and real number y , there exists a real number z such that $x = y + z$. This is a true statement, since we can take $z = x - y$ in each case.

6. Let C(x, y) mean that student x is enrolled in class y, where the domain for x consists of all students in your school and the domain for y consists of all classes being given at your school. Express each of these statements by a simple English sentence.

a) C(Randy Goldberg, CS 252) Randy Goldberg is enrolled in the class of CS252.

b) $\exists x C(x, \text{Math } 695)$ some of the students in your school have enrolled class math 659.

or Someone is enrolled in Math 695.

c) $\exists y C(\text{Carol Sitea}, y)$ Carol Sitea is enrolled in some of the classes.

d) $\exists x(C(x, \text{Math } 222) \wedge C(x, \text{CS } 252))$ Some of the students in your school is enrolled simultaneously in both class math 222 and cs 252.

e) $\exists x \exists y \forall z((x = y) \wedge (C(x, z) \rightarrow C(y, z)))$ There exist two distinct people, the second of whom is enrolled in every course that the first is enrolled in.

f) $\exists x \exists y \forall z((x = y) \wedge (C(x, z) \leftrightarrow C(y, z)))$ There exist two distinct people enrolled in exactly the same courses.

10. Let F (x, y) be the statement "x can fool y," where the domain consists of all people in the world. Use quantifiers to express each of these statements.

a) Everybody can fool Fred. $\forall x\, F(x, \text{Fred})$

b) Evelyn can fool everybody. $\forall y\, F(\text{Evelyn}, y)$

c) Everybody can fool somebody. $\forall x \exists y\, F(x,y)$

d) There is no one who can fool everybody. $\sim \exists x \forall y F(x,y)$

e) Everyone can be fooled by somebody. $\forall y \exists x\, F(x,y)$

f) No one can fool both Fred and Jerry. $\sim \exists x(F(x, \text{Fred}) \wedge F(x, \text{Jerry}))$

g) Nancy can fool exactly two people.

$\exists y1 \exists y2(F(\text{Nancy}, y1) \wedge F(\text{Nancy}, y2) \wedge y1 \neq y2 \wedge \forall y(F(\text{Nancy}, y) \rightarrow (y = y1 \vee y = y2))$

h) There is exactly one person whom everybody can fool.

$\exists y(\forall x F(x, y) \wedge \forall z(\forall x F(x, z)) \rightarrow z=y)$

i) No one can fool himself or herself. $\sim \exists x F(x,x)$

j) There is someone who can fool exactly one person besides himself or herself.

$\exists x \exists y(x \neq y\ F(x,y) \wedge \forall z((F(x, z)) \wedge\ z \neq x) \rightarrow z=y)$

1.6 - 2, 4, 8

2. Find the argument form for the following argument and determine whether it is valid. Can we conclude that the conclusion is true if the premises are true?

If George does not have eight legs, then he is not a spider.

George is a spider.

∴ George has eight legs.

Let If George does not has eight legs = P

George is not a spider. = Q

$P \rightarrow Q$

~Q

~P

The first statement is $p \rightarrow q$, where p is "George does not have eight legs" and q is "George is not a spider." The second statement is $\neg q$. The third is $\neg p$. Modus tollens is valid. We can therefore conclude that the conclusion of the argument (third statement) is true, given that the hypotheses (the first two statements) are true.

4. What rule of inference is used in each of these arguments?

a) Kangaroos live in Australia and are marsupials. Therefore, kangaroos are marsupials. Kangaroos live in Australia = P

Kangaroos are marsupials =Q

Q ^ P

∴Q

We have taken the conjunction of two propositions and asserted one of them.

b) It is either hotter than 100 degrees today, or the pollution is dangerous. It is less than 100 degrees outside today. Therefore, pollution is dangerous.

P= hotter than 100 degrees

Q= pollution is dangerous.

P v Q

~P

∴Q

We have taken the disjunction of two propositions and the negation of one of them, and asserted the other.

c) Linda is an excellent swimmer. If Linda is an excellent swimmer, then she can work as a lifeguard. Therefore, Linda can work as a lifeguard. modus ponens

P = Linda is an excellent swimmer

Q= then she can work as a lifeguard

$P \rightarrow Q$

P

∴Q

d) Steve will work at a computer company this summer. Therefore, this summer, Steve will work at a computer company or he will be a beach bum.

P = Steve will work at a computer company this summer

Q = he will be a beach bum

P

∴P V Q

Addition

e) If I work all night on this homework, then I can answer all the exercises. If I answer all the exercises, I will understand the material. Therefore, if I work all night on this homework, then I will understand the Material.

P = work all night on this homework

Q = I can answer all the exercises

R = I will understand the material

$P \rightarrow Q$

$Q \rightarrow R$

$\therefore P \rightarrow R$

Hypothetical Syllogism

8. What rules of inference are used in this argument? “No man is an island. Manhattan is an island. Therefore, Manhattan is not a man.”

First we use universal instantiation to conclude from “For all x, if x is a man, then x is not an island” the special case of interest, “If Manhattan is a man, then Manhattan is not an island.” Then we form the contrapositive (using also double negative): “If Manhattan is an island, then Manhattan is not a man.” Finally, we use modus ponens to conclude that Manhattan is not a man. Another way, we could apply modus tollens.

1.7 - 6, 18, 24, 28

6. Use a direct proof to show that the product of two odd numbers is odd.

Let n = 2k+1 is odd number

Then n* n = (2k+1)*(2k+1)

= 4k^2+2k +1

= 2(2k^2+2k) +1

Thus n * n is odd number

18. Prove that if n is an integer and 3n + 2 is even, then n is even using

a) a proof by contraposition.

Let n is odd means n = 2k+1

3n+2 = 3(2k+1)+2

= 6k+3+2

= 6k+4+1

$= 2(3k+2)+1$

Thus $3n+2 = 2(3k+2)+1$ is odd

But this is a contraposition, thus it is true. So it is odd.

b) a proof by contradiction.

Suppose that $3n+2$ is even and n is odd by contradiction.

Let n is odd means $n = 2k+1$

$3n+2 = 3(2k+1)+2$

$= 6k+3+2$

$= 6k+4+1$

$= 2(3k+2)+1$

Thus $3n+2 = 2(3k+2)+1$ is odd

But this contradicts our assumption.

OR Since $3n + 2$ is even, so is $3n$. If we add/subtract an odd number from an even number, we get an odd number, so $3n - n = 2n$ is odd. But this is obviously not true. Therefore our supposition was wrong, and the proof by contradiction is complete.

24. Show that at least three of any 25 days chosen must fall in the same month of the year.

We give a proof by contradiction.

If there were at most two days falling in the same month, then we could have at most $2 \cdot 12 = 24$ days, since there are 12 months. Since we have chosen 25 days, at least three of them must fall in the same month.

28. Prove that m2 = n2 if and only if m = n or m = −n.

There are two things to prove. For the "if" part, there are two cases.

If $m = n$, then of course $m2 = n2$

If $m = -n$, then $m2 = (-n)2 = (-1)2\ n2 = n2$.

For the "only if" part, we suppose that $m2 = n2$.

Putting everything on the left and factoring, we have $(m + n)(m - n) = 0$.

Now the only way that a product of two numbers can be zero is if one of them is zero. Therefore we conclude that either $m + n = 0$ in which case $m = -n$, or else $m - n = 0$ in which case $m = n$.

2.1 – 10, 16, 24, 32

10. Determine whether these statements are true or false.

a) $\varnothing \in \{\varnothing\}$ true

b) $\varnothing \in \{\varnothing,\{\varnothing\}\}$ true

c) $\{\varnothing\} \in \{\varnothing\}$ false

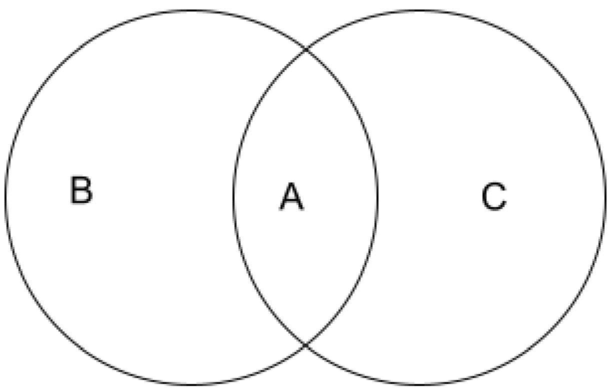
d) $\{\varnothing\} \in \{\{\varnothing\}\}$ true

e) $\{\varnothing\} \subset \{\varnothing,\{\varnothing\}\}$ true

f) $\{\{\varnothing\}\} \subset \{\varnothing,\{\varnothing\}\}$true

g) $\{\{\varnothing\}\} \subset \{\{\varnothing\},\{\varnothing\}\}$false

16. Use a Venn diagram to illustrate the relationships $A \subset B$ and $A \subset C$



24. Determine whether each of these sets is the power set of a set, where a and b are distinct elements.

a) $\varnothing = \{\varnothing,\{\varnothing\}\}$ not a power set

The power set of every set includes at least the empty set, so the power set cannot be empty. Thus, Ø is not the power set of any set.

b) $\{\varnothing,\{a\}\}$ This is the power set of $\{a\}$.

c) $\{\varnothing,\{a\},\{\varnothing, a\}\}$

This set has three elements. Since 3 is not a power of 2, this set cannot be the power set of any set.

d) {∅,{a},{b},{a, b}} This is the power set of {a, b}.

32. Let A = {a, b, c}, B = {x, y}, and C = {0, 1}. Find

a) A × B × C.

{ {a,x,0}, {a,x,1}, {a,y,0}, {a,y,1}, {b,x,0}, {b,x,1},

{b,y,0}, {b,y,1}, {c,x,0}, {c,x,1}, {c,y,0}, {c,y,1}}

b) C × B × A.

{ {0,x,a}, {0,x,b}, {0,x,c}, {0,y,a}, {0,y,b}, {0,y,c},

{1,x,a}, {1,x,b},{1,x,c},{1,y,a}, {1,y,b} {1,y,c}}

c) C × A × B.

{ {0,a,x}, {0,a,y}, {0,b,x}, {0,b,y}, {0,c,x}, {0,c,y},

{1,a,x}, {1,a,y},{1,b,x},{1,b,y}, {1,c,x} {1,c,y}}

d) B × B × B.

{ {x,x,x}, {x,x,y}, {x,y,x}, {x,y,y},

{y,x,x}, {y,x,y},{y,y,x},{y,y,y}}

2.2 – 2, 4, 26

2. Suppose that A is the set of sophomores at your school and B is the set of students in discrete mathematics at your school. Express each of these sets in terms of A and B.

a) the set of sophomores taking discrete mathematics in your school,

A ^ B

b) the set of sophomores at your school who are not taking discrete mathematics,

A ^ ~B

c) the set of students at your school who either are sophomores or are taking discrete mathematics

A V B

d) the set of students at your school who either are not sophomores or are not taking discrete mathematics

~A v ~B

4. Let A = {a, b, c, d, e} and B = {a, b, c, d, e, f, g, h}.Find

a) $A \cup B$.

{a,b,c,d,e,f,g,h}

b) $A \cap B$.

{a,b,c,d,e}

c) $A - B$.

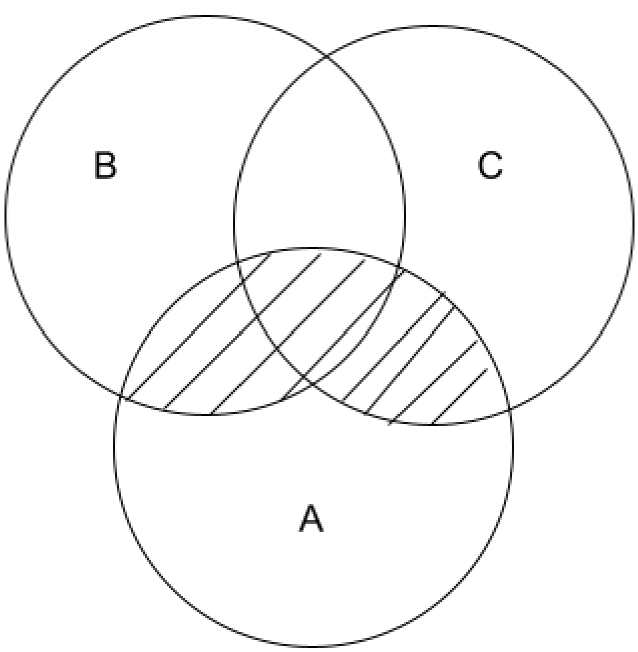
{∅}

d) $B - A$.

{f,g,h}

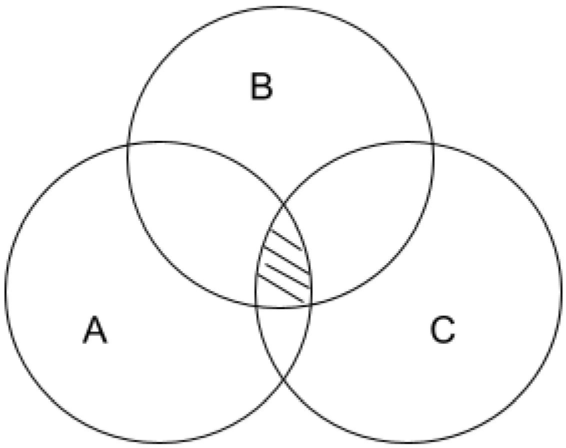
26. Draw the Venn diagrams for each of these combinations of the sets A, B, and C.

a) $A \cap (B \cup C)$



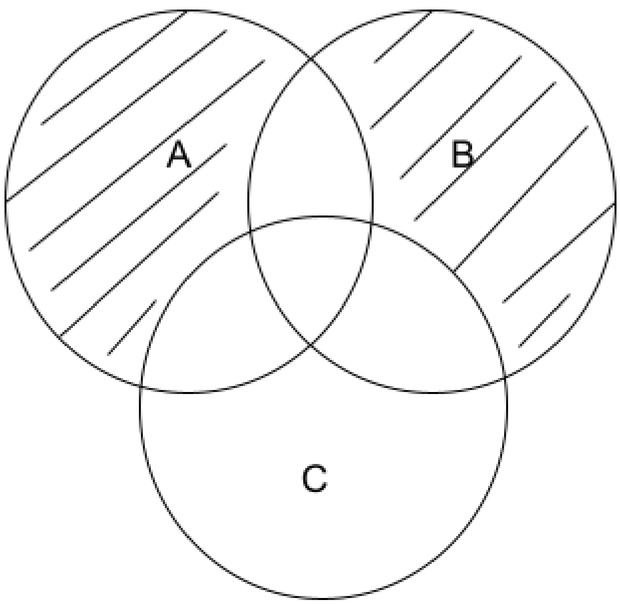
The lined area is $A \cap (B \cup C)$.

b) A ∩ B ∩ C



The lined area is A ∩ B ∩ C

c) (A − B) ∪ (A − C) ∪ (B − C)



The lined area is (A − B) ∪ (A − C) ∪ (B − C)

2.3 – 8, 10, 14, 20

8. Find these values.

a) $\lfloor 1.1 \rfloor$ = 1

b) $\lceil 1.1 \rceil$ =2

c) $\lfloor -0.1 \rfloor$ = -1

d) $\lceil -0.1 \rceil$ =0

e) $\lceil 2.99 \rceil$ = 3

f) $\lceil -2.99 \rceil$ = -2

g) $\lfloor \frac{1}{2} + \lceil \frac{1}{2} \rceil \rfloor$ =1

h) $\lceil \lfloor \frac{1}{2} \rfloor + \lceil \frac{1}{2} \rceil + \frac{1}{2} \rceil$ =2

10. Determine whether each of these functions from {a, b, c, d} to itself is one-to-one.

a) f (a) = b, f (b) = a, f (c) = c, f (d) = d This is one-to-one.

b) f (a) = b, f (b) = b, f (c) = d, f (d) = c

This is not one-to-one, since b is the image of both a and b .

c) f (a) = d, f (b) = b, f (c) = c, f (d) = d

This is not one-to-one, since d is the image of both a and d.

14. Determine whether f : Z × Z → Z is onto if

a) f (m, n) = 2m − n.

f(0, -x) = 2(0)-(-x)=x is onto

b) f (m, n) = m2 − n2. Is not onto

Two is not in the range. If m2 − n2 = (m − n)(m + n) = 2, then m and n must have the same parity (both even or both odd). In either case, both m − n and m + n are then even, so this expression is divisible by 4 and hence cannot equal 2 .

c) f (m, n) = m + n + 1. Yes onto

f (0, n − 1) = n for every integer n .

d) f (m, n) = |m|−|n|. Yes onto

To get negative values we set m = 0, and to achieve nonnegative values we set n = 0 .

e) f (m, n) = m2 − 4. Not onto. The range here is clearly a subset of the range in that part.

20. Give an example of a function from N to N that is

a) one-to-one but not onto. f (n) = n + 13

b) onto but not one-to-one. f (n) = 7n/56

c) both onto and one-to-one (but different from the identity function).

Let f (n) = n − 1 for even values of n , and f (n) = n + 1 for odd values of n . Thus we have f (1) = 2 , f (2) = 1 , f (3) = 4 , f (4) = 3 , and so on. This is just one function, even though its definition uses two formulae, depending on the parity of n.

d) neither one-to-one nor onto. f (n) = 15

2.4 – 2, 4, 6

2. What is the term a8 of the sequence {an} if an equals

a) $2^{n-1} = 2^{8-1} = 2^{7} = 128$

b) 7 = 7

c) $1 + (-1)^{n} = 1+(-1)^{8}=2$

d) $-(-2)^{n}= (-2)^{8} =-256$

4. What are the terms a0, a1, a2, and a3 of the sequence {an}, where an equals

a) $(-2)^{n}$ $a_0 = 1$ $a_1 = -2$ $a_2 = 4$ $a_3 = -8$

b) 3? $a_0 = 3$ $a_1 = 3$ $a_2 = 3$ $a_3 = 3$

c) $7 + 4^{n}$ $a_0 = 8$ $a_1 = 11$ $a_2 = 23$ $a_3 = 71$

d) $2^{n} + (-2)^{n}$ $a_0 = 2$ $a_1 = 0$ $a_2 = 8$ $a_3 = 0$

6. List the first 10 terms of each of these sequences.

a) the sequence obtained by starting with 10 and obtaining each term by subtracting 3 from the previous term

$a_1 = 10$ $a_2 = a_1 - 3 = 7$

$a_3 = a_2 - 3 = 4$ $a_4 = a_3 - 3 = 1$

$a_5 = -2$ $a_6 = -5$ $a_7 = -8$ $a_8 = -11$ $a_9 = -14$ $a_{10} = -17$

b) the sequence whose nth term is the sum of the first n positive integers

$a_1 = 1$ $a_2 = 1+3 = 4$ $a_3 = 1+3+5 = 9$ $a_4 = 1+3+5+7 = 16$

$a_5 = 25$ $a_6 = 36$ $a_7 = 49$ $a_8 = 64$ $a_9 = 81$ $a_{10} = 100$

c) the sequence whose nth term is 3n − 2n

$a_1 = 3^1 - 2^1 = 1$ $a_2 = 3^2 - 2^2 = 5$ $a_3 = 3^3 - 2^3 = 19$ ....

$a_4 = 65$ $a_5 = 211$ $a_6 = 665$ $a_7 = 2059$ $a_8 = 6305$ $a_9 = 19171$ $a_{10} = 58025$

d) the sequence whose nth term is √n

$a_1 = \sqrt{1} = \sqrt{1}$ $a_2 = \sqrt{2} = \sqrt{2}$ $a_3 = \sqrt{3} = \sqrt{3}$ $a_4 = \sqrt{4} = \sqrt{4}$ $a_5 = \sqrt{5} = \sqrt{5}$

$a_6 = \sqrt{6} = \sqrt{6}$ $a_7 = \sqrt{7} = \sqrt{7}$ $a_8 = \sqrt{8} = \sqrt{8}$ $a_9 = \sqrt{9} = \sqrt{9}$ $a_{10} = \sqrt{10} = \sqrt{10}$

e)the sequence whose first two terms are 1 and 5, and each succeeding term is the sum of the two previous terms

$a_1 = 1$ $a_2 = 5$ $a_3 = a_1 + a_2 = 6$ $a_4 = a_2 + a_3 = 11$

$a_5 = a_3 + a_4 = 17$ $a_6 = 28$ $a_7 = 45$ $a_8 = 73$ $a_9 = 118$ $a_{10} = 191$

f) the sequence whose nth term is the largest integer whose binary expansion (defined in Section 4.2) has n bits (Write your answer in decimal notation.)

$a_1 =$ $a_2 =$ $a_3 =$ $a_4 =$ $a_5 =$ $a_6 =$ $a_7 =$ $a_8 =$ $a_9 =$ $a_{10} =$

g) the sequence whose terms are constructed sequentially as follows: start with 1, then add 1, then multiply by 1, then add 2, then multiply by 2, and so on

$a_1 = 1$ $a_2 = 1 + 1 = 2$ $a_3 = 2 * 1 = 2$ $a_4 = 2 + 2 = 4$ $a_5 = 4 * 2 = 8$

$a_6 = 8 + 3 = 11$ $a_7 = 11 * 3 = 33$ $a_8 = 33 + 4 = 37$ $a_9 = 37 * 4 = 148$
$a_{10} = 148 + 5 = 153$

h) the sequence whose nth term is the largest integer k such that k! ≤ n

$a_1 = 1$ 1!<1 $a_2 = 2$ $2! < 2$ a_3 =2 2!<=3

a_4 =2 2!< 4 $a_5 = 2$ $2! < 5$ $a_6 = 3$ $3! = 6$ $a_7 = 3$ $3! < 7$

$a_8 = 3$ $3! < 8$ $a_9 = 3$ $3! < 9$ a_{10} =3 3!<10

2.6 – 2, 4, 26

2)a)

$$A+B=\begin{matrix} 1+(-1) & 0+3 & 4+5 \\ -1+2 & 2+2 & 2+(-3) \\ 0+2 & -2-3 & -3+0 \end{matrix} = \begin{matrix} 0 & 3 & 9 \\ 1 & 4 & -1 \\ 2 & -5 & -3 \end{matrix}$$

b)

$$\begin{matrix} -1-3 & 0+9 & 5-3 & 6+4 \\ -4+0 & -3-2 & 5-1 & -2+2 \end{matrix} = \begin{matrix} -4 & 9 & 2 & 10 \\ -4 & -5 & 4 & 0 \end{matrix}$$

2.6

2) $A + B = \begin{bmatrix} 1 & 0 & 4 \\ -1 & 2 & 2 \\ 0 & -2 & -3 \end{bmatrix} + \begin{bmatrix} -1 & 3 & 5 \\ 2 & 2 & -3 \\ 2 & -3 & 0 \end{bmatrix}$

$= \begin{bmatrix} 0 & 3 & 9 \\ 1 & 4 & -1 \\ 2 & -5 & -3 \end{bmatrix}$

$A + B = \begin{bmatrix} -1 & 0 & 5 & 6 \\ -4 & -3 & 5 & -2 \end{bmatrix} + \begin{bmatrix} -3 & 9 & -3 & 4 \\ 0 & -2 & -1 & 2 \end{bmatrix}$

$= \begin{bmatrix} -4 & 9 & 2 & 10 \\ -4 & -5 & 4 & 0 \end{bmatrix}$

4. Find the product **AB**, where

a) $\mathbf{A} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & -1 \\ -1 & 1 & 0 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 0 & 1 & -1 \\ 1 & -1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$.

b) $\mathbf{A} = \begin{bmatrix} 1 & -3 & 0 \\ 1 & 2 & 2 \\ 2 & 1 & -1 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 1 & -1 & 2 & 3 \\ -1 & 0 & 3 & -1 \\ -3 & -2 & 0 & 2 \end{bmatrix}$.

c) $\mathbf{A} = \begin{bmatrix} 0 & -1 \\ 7 & 2 \\ -4 & -3 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 4 & -1 & 2 & 3 & 0 \\ -2 & 0 & 3 & 4 & 1 \end{bmatrix}$.

a) $= \begin{pmatrix} 1\cdot 0 + 0\cdot 1 + 1\cdot(-1) & 1\cdot 1 + 0\cdot(-1) + 1\cdot 0 & 1\cdot(-1) + 0\cdot 0 + 1\cdot 1 \\ 0\cdot 0 + (-1)\cdot 1 + (-1)(-1) & 0\cdot 1 + (-1)(-1) + (-1)\cdot 0 & 0\cdot(-1) + (-1)\cdot 0 + -1\cdot 1 \\ -1\cdot 0 + 1\cdot 1 + 0\cdot(-1) & (-1)\cdot 1 + 1\cdot(-1) + 0\cdot 0 & -1\cdot -1 + 1\cdot 0 + 0\cdot 1 \end{pmatrix}$

$= \begin{bmatrix} -1 & 1 & 0 \\ 0 & 1 & -1 \\ 1 & -2 & 1 \end{bmatrix}$

b) $\begin{pmatrix} 1+3+0, & -1+0+0, & 2-9+0 & 3+3+0 \\ 1-2-6 & , -1+0+4 & , 2+6+0 & 3-2+4 \\ 2-1+3 & , -2+0+2 & , 4+3+0 & 6-1-2 \end{pmatrix}$

$= \begin{bmatrix} 4 & -1 & -7 & 6 \\ -7 & -5 & 8 & 5 \\ 4 & 0 & 7 & 3 \end{bmatrix}$

c) $\begin{pmatrix} 0+2 & 0+0 & -3 & -4 & -1 \\ 28-4 & -7+0 & 14+6 & 21+8 & 2 \\ -16+6 & 4 & -8-9 & -12-12 & -3 \end{pmatrix}$

$= \begin{bmatrix} 2 & 0 & -3 & -4 & -1 \\ 24 & -7 & 20 & 29 & 2 \\ -10 & 4 & -17 & -24 & -3 \end{bmatrix}$

26. Let

$$\mathbf{A} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

Find

a) $\mathbf{A} \vee \mathbf{B}$. **b)** $\mathbf{A} \wedge \mathbf{B}$. **c)** $\mathbf{A} \odot \mathbf{B}$.

$A \vee B = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ $\quad A \wedge B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

$A \odot B = \begin{bmatrix} (1\wedge 0)\vee(1\wedge 1) & (1\wedge 1)\vee(1\wedge 0) \\ (0\wedge 0)\vee(1\wedge 1) & (0\wedge 1)\vee(1\wedge 0) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

3.1 - 4, 12, 34

4. Describe an algorithm that takes as input a list of n integers and produces as output the largest difference obtained by subtracting an integer in the list from the one following it.

Max $\leqslant a_{n+1} - a_n$

Max $= a_{n+1} - a_n$
then return max

12. Describe an algorithm that uses only assignment statements that replaces the triple (x, y, z) with (y, z, x). What is the minimum number of assignment statements needed?

```
Procedure change (x,y,z integers)
    temp :=x
    x:=y
    z:=temp
Return (x,y,z)
```

34. Use the bubble sort to sort 6, 2, 3, 1, 5, 4, showing the lists obtained at each step.

Bubble sort
{6,2,3,1,5,4}
{2,6,3,1,5,4}
{2,3,6,1,5,4}
{2,3,1,6,5,4}
{2,3,1,5,6,4}
{2,3,1,5,4,6}
{2,1,3,5,4,6}
{1,2,3,5,4,6}
{1,2,3,4,5,6}

3.2 The Growth of Functions

2. Determine whether each of these functions is $O(x^2)$.

a) $f(x) = 17x + 11$ **b)** $f(x) = x^2 + 1000$
c) $f(x) = x \log x$ **d)** $f(x) = x^4/2$
e) $f(x) = 2^x$ **f)** $f(x) = \lfloor x \rfloor \cdot \lceil x \rceil$

a) $f(x) = 17x + 11$ Yes where $x \geq 11$
$= 17x + x$
$= 18x \leq 18x^2$ for $\forall x > 11$

$C = 18$ $k = 11$ $\therefore$ is $O(x^2)$

b) $f(x) = x^2 + 1000$
$\leq x^2 + x^2 = 2x^2$ $\forall x > 1000$ $C = 2$ & $k = 1000$
$\therefore$ Yes is $O(x^2)$

c) $f(x) = x \log x$

$\leq x \cdot x = x^2, \forall x$

$\log x < x, \forall x$

$C = 1 \quad k = 0 \quad \therefore$ Yes, is $O(x^2)$

d) $f(x) = x^4/2 \leq Cx^2$ for sufficiently large x.

$C \geq \frac{x^2}{2}$

NO Impossible for a constant to satisfy.

e) $f(x) = 2^x$

the largest growth is x.

$f(x) = 2^x$ is not $O(x^2)$

If 2^x were $O(x^2)$, then the fraction $\frac{2x}{x^2}$ would have to be bounded above by some constant C.

f) $f(x) = \lfloor x \rfloor \cdot \lceil x \rceil$

$\leq x(x+1) = x^2 + x$

$\leq x \cdot 2x = 2x^2$ for all $x > 1$.

$C = 2$ & $k = 1$.

24. Suppose that you have two different algorithms for solving a problem. To solve a problem of size n, the first algorithm uses exactly $n^2 2^n$ operations and the second algorithm uses exactly $n!$ operations. As n grows, which algorithm uses fewer operations?

The 1st algorithm uses fewer operations because $n^2 2^n$ is $O(n!)$ but $n!$ is not $O(n^2 2^n)$. In fact, the 2nd function overtakes the first function for good at $n = 8$, when $8^2 \cdot 2^8 = 16384$ and $8! = 40,320$.

$n^2 2^n$ is equal to the most dominating term in expansion. The running time of exponential function (2^n) is more than a polynomial function (n^2)

$n! = n(n-1)(n-2)\ldots$ it can be $O(n^n)$

The function $O(n^n) \geq O(n^2 2^n)$.

Thus, the first has less # than 2nd function.

3.3 – 2, 16, 18

2. Give a big-O estimate for the number additions used in this segment of an algorithm.

$t := 0$
for $i := 1$ **to** n
  for $j := 1$ **to** n
    $t := t + i + j$

$$\sum_{i=1}^{n}\sum_{j=1}^{n} 2 + \sum_{i=1}^{n} 1 = \sum_{i=1}^{n}(2n) + n$$

$$= n \cdot 2n = 2n^2$$

The number operation is $O(n^2)$

16. What is the largest n for which one can solve within a day using an algorithm that requires $f(n)$ bit operations, where each bit operation is carried out in 10^{-11} seconds, with these functions $f(n)$?

a) $\log n$ **b)** $1000n$ **c)** n^2
d) $1000n^2$ **e)** n^3 **f)** 2^n
g) 2^{2n} **h)** 2^{2^n}

a) $f(n) = \log n$

In a day there will be 10^5 seconds.
Total # of operations $= \frac{10^5}{10^{-11}}$
$= 10^{16}$

# of bit operation must less than 10^{16}

$f(n) = \log_2 n \leq 10^{16}$

$\log_2 n \leq 10^{16}$

$n \leq 2^{10^{16}}$

required largest value $= 2^{10^{16}}$

b) $f(n) = 1000n$

$f(n) \leq 10^{16}$

$1000n \leq 10^{16}$

$n \leq \frac{10^{16}}{1000}$

$n \leq 10^{13}$

Greatest value is 10^{13}

c) $f(n) = n^2$

$f(n) \leq 10^{16}$

$n^2 \leq 10^{16}$

$n \leq 10^{16 \cdot \frac{1}{2}}$

$n \leq 10^8$ Greatest value is 10^8

d) $f(n) = 1000n^2$

$$f(n) \le 10^{16}$$
$$n^2 \le \frac{10^{16}}{1000}$$
$$n^2 \le 10^{13}$$
$$n \le 10^{13 \cdot \frac{1}{2}} = 10^{\frac{13}{2}}$$ Greatest value is $10^{\frac{13}{2}}$

e) $f(n) = n^3$

$$n^3 \le 10^{16}$$
$$n \le 10^{\frac{16}{3}}$$ Greatest value is $10^{\frac{16}{3}}$

f) $f(n) = 2^n$

$$2^n \le 10^{16}$$
$$n \log 2 \le 16 \log 10$$
$$n \le \frac{16 \log 10}{\log 2}$$
$$n \le \log_2 10^{16}$$ Greatest value is $\log_2 10^{16}$

g) $f(n) = 2^{2^n}$

$$f(n) \le 10^{16}$$
$$2^{2^n} \le 10^{16}$$
$$2^n \log 2 \le 16 \log 10$$
$$2^n \le \frac{16 \log 10}{\log 2}$$
$$n \log 2 \le \frac{16 \log 10}{\log 2} \Rightarrow n \le \log_2 (\log_2 10^{16})$$

largest value $= \log_2 (\log_2 10^{16})$

H) $f(n) = 2^{2n}$

$$f(n) \le 10^{16}$$
$$2^{2n} \le 10^{16}$$
$$2n \log 2 \le 16 \log 10$$
$$n \le \frac{1}{2} \log_2 10^{16}$$ The largest value $= \frac{1}{2} \log_2 10^{16}$

18. How much time does an algorithm take to solve a problem of size n if this algorithm uses $2n^2 + 2^n$ operations, each requiring 10^{-9} seconds, with these values of n?

a) 10 **b)** 20 **c)** 50 **d)** 100

a) $n = 10s$

$(2(10)^2 + 2^{10}) \times 10^{-9} = 1.224 \times 10^{-6}$

b) $n = 20s$

$(2\cdot(20)^2 + 2^{20}) \cdot 10^{-9} = 1.05 \times 10^{-3}$

c) $n = 50s$

$(2\cdot(50)^2 + 2^{50}) \cdot 10^{-9} = 1.13 \times 10^{6}$

d) $n = 100s$

$(2\cdot(100)^2 + 2^{(100)}) \cdot 10^{-9} = 1.27 \times 10^{21}$

4.1 - 6, 10, 28

6. Show that if a, b, c, and d are integers, where $a \neq 0$, such that $a \mid c$ and $b \mid d$, then $ab \mid cd$.

under the hypotheses, we have $c = as$ and $d = bt$ for some s and t.

$cd = ab(st)$

$\Rightarrow ab \mid cd$, as desired.

10. What are the quotient and remainder when

a) 44 is divided by 8? $44/8 = 8 \cdot 5 + 4$

b) 777 is divided by 21? $777/21 = 21(37) + 0$

c) -123 is divided by 19? $-123/19 = -7(19) + 10$

d) -1 is divided by 23? $-1/23 = -23(1) + 22$

e) -2002 is divided by 87? $-2002/87 = 87(-24) + 86$

f) 0 is divided by 17? $0/17 = 17 \cdot 0 + 0$

g) 1,234,567 is divided by 1001? $1{,}234{,}567/1001 = 1001(1233) + 334$

h) -100 is divided by 101? $-100/101 = 101(-1) + 1$

28. Decide whether each of these integers is congruent to 3 modulo 7.

a) 37 **b)** 66

c) -17 **d)** -67

a) $37 - 3 \bmod 7 = 34 \bmod 7 = 6 \neq 0$,

37 is NOT congruent to 3 modulo 7.

$37 \not\equiv 3 \pmod 7$

b) $66 - 3 \bmod 7 = 63 \bmod 7 = 0$,

$66 \equiv 3 \pmod 7$ $\Rightarrow$ congruent

c) $-17 - 3 \bmod 7 = -20 \bmod 7 = 1 \neq 0$

-17 is NOT congruent to 3 modulo 7.

$-17 \not\equiv 3 \pmod 7$

d) $-67 - 3 \bmod 7 = -70 \bmod 7 = 0$

$-67 \equiv 3 \pmod 7$

-67 is congruent to 3 modulo 7.

4.2 Integer Representations and Algorithms

4.2 - 2, 4, 6, 8, 22

2. Convert the decimal expansion of each of these integers to a binary expansion.

a) 321 **b)** 1023 **c)** 100632

2|321
2|160
2|80
2|40
2|20
2|10
2|5
2|2
2|1
0

convert to binary
$(101000001)_2$

b)
2|1023
2|511
2|255
2|127
2|63
2|31
2|15
2|7
2|3
2|1
0

$(11\ 1111\ 1111)_2$

c)
2|100632
2|50316 0
2|25158 0
2|12579 0
2|6289 1
2|3144 1
2|1572 0
2|786 0
2|393 0
2|196 1
2|98 0
2|49 0 continue...

2|24 1
2|12 0
2|6 0
2|3 0
2|1 1
0 1

$(1\ 1000\ 1001\ 0001\ 1000)_2$

4. Convert the binary expansion of each of these integers to a decimal expansion.

a) $(1\ 1011)_2$ **b)** $(10\ 1011\ 0101)_2$
c) $(11\ 1011\ 1110)_2$ **d)** $(111\ 1100\ 0001\ 1111)_2$

a) 1 ⋮ 1011

$1+2+8+16=27$

b) 10 ⋮ 1011 ⋮ 0101

$1+4+16+32+128+512=693$

c) 11 ⋮ 1011 ⋮ 1110

$2+4+8+16+32+128+256+512=958$

d) 111 ⋮ 1100 ⋮ 0001 ⋮ 1111

$1+2+4+8+16+1024+2048+4096+8192+16384=31775$

6. Convert the binary expansion of each of these integers to an octal expansion.

a) $(1111\ 0111)_2$
b) $(1010\ 1010\ 1010)_2$
c) $(111\ 0111\ 0111\ 0111)_2$
d) $(101\ 0101\ 0101\ 0101)_2$

a) 011 ⋮ 110 ⋮ 111 $=(367)_8$
3 6 7

b) 101 ⋮ 010 ⋮ 101 ⋮ 010 $=(5252)_8$
5 2 5 2

c) 111 ⋮ 011 ⋮ 101 ⋮ 110 ⋮ 111 $=(73567)_8$
7 3 5 6 7

d) 101 ⋮ 010 ⋮ 101 ⋮ 010 ⋮ 101 $=(52525)_8$
5 2 5 2 5

8. Convert $(\text{BADFACED})_{16}$ from its hexadecimal expansion to its binary expansion.

B	A	D	F	A	C	E	D
11	10	13	15	10	12	14	13
1011	1010	1101	1111	1010	1100	1110	1101

$(1011\ 1010\ 1101\ 1111\ 1010\ 1100\ 1110\ 1101)_{32}$

22. Find the sum and product of each of these pairs of numbers. Express your answers as a base 3 expansion.

a) $(112)_3$, $(210)_3$
b) $(2112)_3$, $(12021)_3$
c) $(20001)_3$, $(1111)_3$
d) $(120021)_3$, $(2002)_3$

a) 112 + 210 = 1022 decimal : 14 + 21 = 35

112 · 210 = 101, 220 decimal : 14 · 21 = 294

b) 2112 + 12021 = 21210 decimal : 68 + 142 = 210

2112 · 12021 = 111, 020, 122 decimal : 68 · 142 = 9656

c) 20001 + 1111 = 21, 112 decimal : 163 + 40 = 203

20001 · 1111 = 22, 221, 111 decimal : 163 · 40 = 6520

d) 120021 + 2002 = 122, 100 dec. : 412 + 56 = 468

120021 · 2002 = 1, 011, 122, 112 dec.: 412 · 56 = 23,072

4.3 Primes and Greatest Common Divisors

4.3 - 2, 4, 16, 24, 26

2. Determine whether each of these integers is prime.

a) 19 **b)** 27
c) 93 **d)** 101
e) 107 **f)** 113

a) The prime # less than $\sqrt{19}$ are 2, 3.
19 does not divide 2 or 3.
$\therefore$ prime number

b) The prime # less than $\sqrt{27}$ are 2, 3, 5.
27 divisible by 3
$\therefore$ 27 is NOT prime.

c) The prime # less than $\sqrt{93}$ are 2, 3, 5, 7
93 does not divisible by any of these #.
$\therefore$ 93 is prime.

d) The prime # less than $\sqrt{101}$ are 2, 3, 5, 7.
101 does not divisible any
$\therefore$ 101 is prime.

e) The prime # less than $\sqrt{107}$ are 2, 3, 5, 7
107 does not divisible by any
$\therefore$ prime number

f) The prime # less than $\sqrt{113}$ are 2, 3, 5, 7
113 does not divisible
$\therefore$ prime number.

4. Find the prime factorization of each of these integers.

a) 39 **b)** 81 **c)** 101
d) 143 **e)** 289 **f)** 899

a)
$$\begin{array}{r|l} 3 & 39 \\ 13 & 13 \\ & 1 \end{array}$$

$3 \cdot 13 = 39$

$\boxed{1, 3, 13, 39.}$

b)
$$\begin{array}{r|l} 3 & 81 \\ 3 & 27 \\ 3 & 9 \\ & 3 \end{array}$$

$3^4 = 81$

$\boxed{1, 3, 81}$

c) $101 \overline{\smash{)}101}$ 1

$101 \cdot 1 = 101$

$\boxed{1 \quad 101}$

d) $11 \overline{\smash{)}143}$ 13

$11 \cdot 13 = 143$

$\boxed{1, 11, 13, 143}$

e) $17 \overline{\smash{)}289}$
$17 \overline{\smash{)}17}$ 1

$17^2 \cdot 1 = 289$

$\boxed{1, 17, 289}$

f) $29 \overline{\smash{)}899}$
$31 \overline{\smash{)}31}$ 1

$29 \cdot 31 = 899$

$\boxed{1, 29, 31, 899}$

16. Determine whether the integers in each of these sets are pairwise relatively prime.

a) 21, 34, 55
b) 14, 17, 85
c) 25, 41, 49, 64
d) 17, 18, 19, 23

pairwise relatively prime

a) $\gcd(21, 34) = 1$
$\gcd(34, 55) = 1$ is pairwise prime
$\gcd(21, 55) = 1$

b) $\gcd(14, 17) = 1$
$\gcd(14, 85) = 1$ is not.
$\gcd(17, 85) = 17$

c) $\gcd(25, 41) = 1$
$\gcd(25, 49) = 1$ is pairwise prime
$\gcd(25, 64) = 1$
$\gcd(41, 49) = 1$
$\gcd(49, 64) = 1$

d) gcd(17, 18) = 1
gcd(17, 19) = 1
gcd(17, 23) = 1
gcd(18, 19) = 1
gcd(18, 23) = 1
gcd(19, 23) = 1

is pairwise prime.

24. What are the greatest common divisors of these pairs of integers?

a) $2^2 \cdot 3^3 \cdot 5^5, 2^5 \cdot 3^3 \cdot 5^2$
b) $2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 13, 2^{11} \cdot 3^9 \cdot 11 \cdot 17^{14}$
c) $17, 17^{17}$
d) $2^2 \cdot 7, 5^3 \cdot 13$
e) $0, 5$
f) $2 \cdot 3 \cdot 5 \cdot 7, 2 \cdot 3 \cdot 5 \cdot 7$

a) $\gcd(2^2 \cdot 3^3 \cdot 5^5, 2^5 \cdot 3^3 \cdot 5^2)$
$= 2^2 \cdot 3^3 \cdot 5^2$

b) $\gcd(2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 13, 2^{11} \cdot 3^9 \cdot 11 \cdot 17^{14})$
$= 2 \cdot 3 \cdot 11$

c) $\gcd(17, 17^{17})$
$= 17$

d) $\gcd(2^2 \cdot 7, 5^3 \cdot 13)$
$= 1$

e) $\gcd(0, 5)$
$= 5$

f) $\gcd(2 \cdot 3 \cdot 5 \cdot 7, 2 \cdot 3 \cdot 5 \cdot 7)$
$= 2 \cdot 3 \cdot 5 \cdot 7$

26. What is the least common multiple of each pair in Exercise 24?

$\text{LCM}(2^2 \cdot 3^3 \cdot 5^5, 2^5 \cdot 3^3 \cdot 5^2) = 2^5 \cdot 3^3 \cdot 5^5$

$\text{LCM}(2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 13, 2^{11} \cdot 3^9 \cdot 11 \cdot 17^{14}) = 2^{(1,11)} = 2^{11}$
$3^{(1,9)} = 3^9$
$5^{(0,1)} = 5$
$7^{(0,1)} = 7$
$11^{(0,1)} = 11$
$13^{(0,1)} = 13$
$17^{(1,14)} = 17^{14}$

$2^{11} \cdot 3^9 \cdot 5 \cdot 7 \cdot 11 \cdot 13 \cdot 17^{14}$

$\text{LCM}(17, 17^{17}) = 17^{17}$

$\text{Lcm}(2^2 \cdot 7, 5^3 \cdot 13) = 2^{(1,2)} = 2^2$
$7^{(0,1)} = 7$
$5^{(1,3)} = 5^3$
$13^{(0,1)} = 13$

$2^2 \cdot 7 \cdot 5^3 \cdot 13$

$\text{Lcm}(0, 5) =$ undefined

(0 is not a positive integer)

$\text{Lcm}(2 \cdot 3 \cdot 5 \cdot 7, 2 \cdot 3 \cdot 5 \cdot 7) = 2^{(1,1)} = 2$
$3^{(1,1)} = 3$
$5^{(1,1)} = 5$
$7^{(1,1)} = 7$

$2 \cdot 3 \cdot 5 \cdot 7$

4.6 Cryptography

4.6 - 2, 4

2. Encrypt the message STOP POLLUTION by translating the letters into numbers, applying the given encryption function, and then translating the numbers back into letters.

a) $f(p) = (p + 4)$ **mod** 26
b) $f(p) = (p + 21)$ **mod** 26
c) $f(p) = (17p + 22)$ **mod** 26

a)

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

S (18+4) % 26 = 22 W
T (19+4) % 26 = 23 X
O (14+4) % 26 = 18 S
P (15+4) % 26 = 19 T

P (15+4) % 26 = 19 T
O (14+4) % 26 = 18 S
L (11+4) % 26 = 15 P
L (11+4) % 26 = 15 P
U (20+4) % 26 = 24 Y
T (19+4) % 26 = 23 X
I (8+4) % 26 = 12 M
O (14+4) % 26 = 18 S
N (13+4) % 26 = 17 R

a) WXST TSPPYXMSR

b) S (18+21) % 26 = 13 N
T (19+21) % 26 = 14 O
O (14+21) % 26 = 9 J
P (15+21) % 26 = 10 K

P (15+21) % 26 = 10 K
O (14+21) % 26 = 9 J
L (11+21) % 26 = 6 G
L (11+21) % 26 = 6 G
U (20+21) % 26 = 15 P
T (19+21) % 26 = 14 O
I (8+21) % 26 = 3 D
O (14+21) % 26 = 9 J
N (13+21) % 26 = 8 I

NOJK KJGGPODJI

c)

S	$(17 \cdot 18 + 22) \% 26$	$= 16$	Q
T	$(17 \cdot 19 + 22) \% 26$	$= 7$	H
O	$(17 \cdot 14 + 22) \% 26$	$= 0$	A
P	$(17 \cdot 15 + 22) \% 26$	$= 17$	R
P	$(17 \cdot 15 + 22) \% 26$	$= 17$	R
O	$(17 \cdot 14 + 22) \% 26$	$= 0$	A
L	$(17 \cdot 11 + 22) \% 26$	$= 1$	B
L	$(17 \cdot 11 + 22) \% 26$	$= 1$	B
U	$(17 \cdot 20 + 22) \% 26$	$= 24$	Y
T	$(17 \cdot 19 + 22) \% 26$	$= 7$	H
I	$(17 \cdot 8 + 22) \% 26$	$= 2$	C
O	$(17 \cdot 14 + 22) \% 26$	$= 0$	A
N	$(17 \cdot 13 + 22) \% 26$	$= 9$	J

QHAR RABBYHCAJ

4. Decrypt these messages that were encrypted using the Caesar cipher.

a) EOXH MHDQV
b) WHVW WRGDB
c) HDW GLP VXP

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

a) EOXH MHDQV

$f^{-1}(x) = (4-3)\ \%\ 26 = 1$ B
$= (14-3)\ \%\ 26 = 11$ L
$= (23-3)\ \%\ 26 = 20$ U
$= (7-3)\ \%\ 26 = 4$ E
$= (12-3)\ \%\ 26 = 9$ J
$= (7-3)\ \%\ 26 = 4$ E
$= (3-3)\ \%\ 26 = 0$ A
$= (16-3)\ \%\ 26 = 13$ N
$= (21-3)\ \%\ 26 = 18$ S

BLUE JEANS

b) WHVW WRGDB

$f^{-1}(x) = (22-3)\ \%\ 26 = 19$ T
$= (7-3)\ \%\ 26 = 4$ E
$= (21-3)\ \%\ 26 = 18$ S
$= (22-3)\ \%\ 26 = 19$ T
$= (22-3)\ \%\ 26 = 19$ T
$= (17-3)\ \%\ 26 = 14$ O
$= (6-3)\ \%\ 26 = 3$ D
$= (3-3)\ \%\ 26 = 0$ A
$= (1-3)\ \%\ 26 = 24$ Y

TEST TODAY

c) HDW GLP VXP

$f^{-1}(x) = (7-3)\ \%\ 26 = 4$ E
$= (3-3)\ \%\ 26 = 0$ A
$= (22-3)\ \%\ 26 = 19$ T
$= (6-3)\ \%\ 26 = 3$ D
$= (11-3)\ \%\ 26 = 8$ I
$= (15-3)\ \%\ 26 = 12$ M
$= (21-3)\ \%\ 26 = 18$ S
$= (23-3)\ \%\ 26 = 20$ U
$= (15-3)\ \%\ 26 = 12$ M

EAT DIM SUM