Homework 3 Fall 2023

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Instructions

Please read the following instructions carefully:

1. Please show all notation for probability statements.
2. Box your final answers.
3. Please verify that your scans are legible.
4. Please assign pages for the questions when submitting to gradescope.
5. This assignment is due via gradescope on the due date.

1. State yes/no in each part, and explain in a sentence or two (or write down a relevant formula).

   (a) Consider flipping a coin once, is the probability of a head on the first flip independent of the probability of tail on the first flip? You can assume that they have equal probability.

   Yes. The probability of flipping a head or a tail are constant and independent of each other.

   (b) Consider flipping a coin and rolling a die. Is the probability of rolling a 5 independent from flipping a coin and getting a head?

   Yes. Flipping a coin does not affect rolling a die at all.

   (c) Consider a situation where you flip a coin 20 times. The probability of flipping the coin and getting a head is 0.5. It is fair to assume that the probability of flipping a head in one trial does not affect the probability of flipping a head in another trial. Does the total number of heads follow a binomial distribution? Why or why not?

   Yes. The probability is either head or tail (succes or fail) and they are independent. It has fixed number of trails

   (d) Continuing from the part above, make all the same assumptions. However, this time you can flip the coin infinitely many times. Does the total number heads follow a binomial distribution? Why or why not?

   No, the number of trails has to be a fixed number

   (e) Say you flip a coin 3 times. The probability of a head on the first trial is 0.5, the probability of a head on the second trial is 0.7, and the probability of a head on the third trial is 0.3. Does the total number of heads follow a binomial distribution?

   No. The probability should be constant.

   (f) Consider a situation where you roll a die once. Therefore, our sample space is $\{1, 2, 3, 4, 5, 6\}$. Assume that each side has an equal probability. Could you solve for the probability of rolling a 2 or rolling a 4 or rolling a 6 (the union of these three events)?

   Yes, $P(x=2) + P(x=4) + P(x=6)$

   $= \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{1}{2}$

2. You are in charge of selecting a few of the 50 new designs of the Peter the Anteater stickers submitted at the end of a quarter. The probability of a sticker design being selected is 0.08, and this probability remains constant. We can assume that the probability of a sticker design being selected is independent of whether another sticker design is selected.

   Let X be the number of Peter the Anteater sticker designs selected at the end of the quarter.

   (a) What is the distribution of X?

   $$\boxed{X \sim \text{Binomial}(50, 0.08)}$$

   (b) Write the pmf $f(x)$ and describe its parameters.

   $$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$$
   $$= \boxed{\binom{50}{x} 0.08^x (1 - 0.08)^{50-x}}$$

   (c) What key assumptions about the newly designed Peter the Anteater stickers are needed to determine this distribution?

   The probability of a sticker design being selected is independent.

   The probability of a sticker design being selected is constant at 0.08

   (d) What is the expected number of Peter the Anteater sticker designs selected at the end of a quarter? Interpret this number in a sentence.

   The expected number of Peter the Anteater sticker designs selected at the end of a quarter is the number of new designs times the probability which is 4.

   (e) What is the probability that *exactly 7* Peter the Anteater sticker designs are selected at the end of the quarter? Round your answer to two decimal places.

   $$P(X = 7) = \text{dnorm}(7, 50, 0.08)$$
   $$= \boxed{0.06}$$

(f) For a smooth mass production of the new Peter the Anteater stickers, it is ideal that **at most 5** sticker designs are selected. What is the probability that there will be a smooth mass production of these stickers at the end of the quarter? (Round to 2 decimal places or write an integer as necessary.)

$P(X \leq 5) = \text{pbinom}(5, 50, 0.08)$

$= \boxed{0.68}$

(g) Suppose you can select these sticker designs on a rolling basis throughout the quarter. Now you become curious about the number of Peter the Anteater sticker designs submitted until a sticker design is selected. Let Y represent this quantity. In theory, there could be an infinite number of sticker designs submitted before a design is selected. The probability of a sticker design being selected remains the same, $p = 0.08$, and the same assumptions of independence and constant probability apply. What is the distribution of Y, and explain why you believe that is the distribution of Y.

$\boxed{Y \sim \text{Geometric}(0.08)}$

With the same assumptions of independence and constant probability but with an infinite number of trials.

(h) Using the scenario from part (g), what is the probability that it takes 30 sticker design submissions until the first sticker design is selected. Round your answer to 4 decimal places.

$P(X = 30) = \text{dgeom}(30 - 1, 0.08)$

$= \boxed{0.0071}$

3. Suppose that at a manufacturer, the probability of a laptop having a manufacturing defect is $\frac{1}{100}$. We assume that this probability is constant and independent. You want to model X, the number of laptops manufactured until a manufacturing defect is found.

   (a) What distribution does X follow? What assumptions do you need to make to say that X has this distribution?

   $$\boxed{X \sim \text{Geometric}\left(\frac{1}{100}\right)}$$

   The probability is constant and independent.

   The number of trials is infinite.

   (b) Compute the expected number of manufactured laptops until you find the first manufacturing defect (Make sure to identify the units).

   $$E(X) = \frac{1}{\frac{1}{100}} = \boxed{100 \text{ laptops}}$$

   (c) What is the variance and standard deviation of the number of manufactured laptops until you find the first manufacturing defect? Provide the appropriate units and answer rounded to 2 decimal places.

   $$VAR(X) = \frac{1-p}{p^2} = \frac{1-\frac{1}{100}}{\left(\frac{1}{100}\right)^2} = \frac{0.99}{0.01^2} = \boxed{9900 \text{ trials}^2}$$

   $$\sigma = \sqrt{VAR(X)} = \sqrt{9900} = \boxed{99.4987 \text{ trials}}$$

   (d) What is the probability that you find the first manufacturing defect on the 85^{th} laptop manufactured? Round to 4 decimal places if necessary.

   $$P(X = 85) = \text{dgeom}(85-1, 0.01)$$

   $$= \boxed{0.0043}$$

(e) What is the probability that the number of laptops manufactured until you find the first manufacturing defect is greater than or equal to 15 laptops? Round your answer to 4 decimal places.

$$P(x \geq 15) = 1 - P(x \leq 14)$$
$$= 1 - \text{pgeom}(14-1, 0.01)$$
$$= \boxed{0.8687}$$

(f) Now suppose that you are interested in the number of manufacturing defects that you will find for the next 40 manufactured laptops. The probability of finding a manufacturing defect is still $\frac{1}{100}$ and remains fixed.

Use R to find the probability that you will find 2 manufacturing defects in the next 40 manufactured laptops and state the function used. Round to 4 decimal places.

$$\boxed{Y \sim \text{Binomial}(40, 0.01)}$$

$$P(Y=2) = \binom{40}{2} 0.01^2 (1-0.01)^{40-2}$$
$$= \boxed{0.0532}$$

dbinom(2, 40, 0.01)

4. Consider a situation where a customer service hotline receives 500 calls per day from different customers. The probability of a call disconnecting is 0.03. We assume that for any customer calling the hotline, the probability of the call disconnecting is constant and independent.

   (a) Let X be the number of calls that get disconnected. What kind of distribution does X follow? Interpret the parameters of this distribution.

   $\boxed{X \sim \text{Binomial}(500, 0.03)}$

   The parameters are 500 calls per day and probability of the call disconnecting.

   (b) What assumptions do you need to make to say that X has this distribution?

   We have a fixed number of trials.

   The probability of the call disconnecting is constant and independent.

   (c) What is the probability that 50 calls get disconnected? Round to 4 decimal places (use scientific notation).

   $P(X = 50) = \text{dbinom}(50, 500, 0.03)$

   $= \boxed{1.8526 \times 10^{-13}}$

(d) What is the probability that *at least* 18 calls get disconnected per day? Round to 4 decimal places if necessary.

$$P(X \geq 18) = 1 - P(X \leq 17)$$
$$= 1 - \text{pbinom}(17, 500, 0.03)$$
$$= \boxed{0.2485}$$

(e) What is the probability that *less than* 15 calls get disconnected out of the 500 calls received per day? Round to 4 decimal places if necessary.

$$P(X < 15) = P(X \leq 14)$$
$$= \text{pbinom}(14, 500, 0.03)$$
$$= \boxed{0.4641}$$

5. One day in Las Vegas, you decide to play on the slot machines at a casino for the entire day. The probability of you getting a jackpot is 0.07. You decide to play on every slot machine once until you get a jackpot.

   (a) Let Y be the number of slot machines you play on until you get a jackpot. What kind of distribution does Y follow? What would be its expected value and variance? Round to 4 decimal places.

$$\boxed{Y \sim \text{Geometric}(0.07)}$$

$$E(Y) = \frac{1}{0.07} = \boxed{14.2857}$$

$$VAR(Y) = \frac{1-0.07}{0.07^2} = \boxed{189.7959}$$

   (b) If you do finally get a jackpot, you win \$100. If you spend \$2 on each slot machine, would you expect to make a profit from getting a jackpot? What is the variance of this profit?

To make a profit, we need to win at 49^{th} play or earlier.

$$\frac{100}{2} - 1 = 50 - 1 = 49$$

And we are expected to win at 14.2857 play.
I would say it is very likely to make a profit.

$$VAR(Y) = \boxed{189.7959}$$

   (c) You only have \$30 to spend on the slot machines. What is the probability that you will not get a jackpot before running out of money? Round your answer to 4 decimal places.

$$y = \frac{30}{2} = 15$$

$$P(Y > 15) = 1 - P(Y \leq 14)$$

$$= 1 - \text{pgeom}(14, 0.07)$$

$$= \boxed{0.3367}$$

(d) Assume that you have unlimited funds and can go play on as many slot machines as you want. You play on 70 slot machines in one day. Let X be the number of times you get a jackpot out of the 70 slot machines.

What is the distribution of X?

$$\boxed{X \sim \text{Binomial}(70, 0.07)}$$

(e) 1) Out of the 70 slot machines, how many times would you expect to get a jackpot?
2) What is the variance of the number of times you get a jackpot? Round to 2 decimal places.

1) $E(X) = np = 70(0.07) = \boxed{4.9}$

2) $VAR(X) = np(1-p)$

$= E(X)(1-0.07)$

$= 4.9(0.93)$

$= \boxed{4.557}$

6. As you study for Stats 67, you occasionally look out the window of your dorm and observe the number of times the Anteater bus stops at the bus stop in front of your dorm. You notice that on average there are 6 buses that stop by during your 2-hour study session. Let X be the number of buses that stop by in one hour.

   (a) Assume that the expected value of X is a constant rate per hour, and the number of buses that stop by per hour is independent of any other hour. What is the distribution of X? Justify your answer.

   $\boxed{X \sim \text{Poisson}\left(\frac{6}{2}\right)}$

   we have the number of events in a time interval. The expected value is constant per hour and the time interval is independent.

   (b) Using the distribution from part (a), what is the Mean and Standard Deviation of the number of buses that stop by in a given hour. Round to 4 decimal places.

   $$\text{mean} = \lambda = \frac{6}{2} = 3 \ \frac{\text{buses}}{1 \text{ hrs}}$$

   $$\sigma = \sqrt{VAR(X)} = \sqrt{E(X)} = \sqrt{\lambda} = \sqrt{3} = \boxed{1.7321}$$

   (c) Using the distribution from part (a), what is the probability that you observe at least 3 buses stopping by in a given hour? Round to 4 decimal places.

   $$P(X \geq 3) = 1 - P(X \leq 2)$$
   $$= 1 - \text{ppois}(2, 3)$$
   $$= \boxed{0.5768}$$

   (d) Find the probability of observing at least 6 buses stopping by in a 3-hour study session and the probability of observing exactly 3 buses stopping by in a 3-hour study session. Round to 4 decimal places.

   $$\lambda_y = 9 \ \frac{\text{buses}}{3 \text{hrs}} \qquad Y \sim \text{Poisson}(9)$$

   $$P(Y \geq 6) = 1 - P(Y \leq 5)$$
   $$= 1 - \text{ppois}(5, 9)$$
   $$= \boxed{0.8843}$$

   $$P(X = 3) = \text{dpois}(3, 9)$$
   $$= \boxed{0.0150}$$