Chapter 5 – 5.1 – 4, 6

4. Let P (n) be the statement that $1^3 + 2^3 +\cdots+ n^3 = (n(n + 1)/2)^2$ for the positive integer n.

a) What is the statement P (1)?

Substitute n = 1; P(1): $1^3 = (1(1 + 1)/2)^2 = 1$

b) Show that P (1) is true, completing the basic step of the proof.

L.H.S of P(1) = $1^3 = 1$

R.H.S of P(1):

$(1(1 + 1)/2)^2 = (1(2)/2)^2 = 1^2 = 1$

$\Rightarrow$ L.H.S of P(1) = R.H.S of P(1) Thus, P(1) is true.

c) What is the inductive hypothesis?

Inductive Hypothesis: Assume that for n=k, P(k) is true.

$1^3 + 2^3 +\cdots+ k^3 = (k(k + 1)/2)^2$ is true.

d) What do you need to prove in the inductive step?

We have to prove that for n = k+1,

P(k+1): $1^3 + 2^3 +\cdots+ k^3 + (k + 1)^3 = (\frac{(k+1)(k+2)}{2})^2$

e) Complete the inductive step, identifying where you use the inductive hypothesis.

L.H.S of P(k+1) = $1^3 + 2^3 +\cdots+ k^3 + (k + 1)^3$

$= (1^3 + 2^3 +\cdots+ k^3) + (k + 1)^3$

$= (\frac{k(k+1)}{2})^2 + (k + 1)^3$

$= (k + 1)^2(\frac{k^2}{4} + k + 1)$

$= (k + 1)^2(\frac{k^2 + 4k + 4}{4})$

$= (k + 1)^2(\frac{(k+2)^2}{4})$

$= (\frac{(k+1)(k+2)}{2})^2$

= R.H.S of P(k+1) $\Rightarrow$ L.H.S of P(k+1) = R.H.S of P(k+1)

f) Explain why these steps show that this formula is true whenever n is a positive integer

Prove that RHS = LHS thus, P(K+1) is true for all positive integers.

The statement is true for n =1.

Also, P(k) is true $\Rightarrow$ P(k+1) is also true.

So, by the principle of mathematical induction the statement is true for all natural numbers.

6. Prove that $1 \cdot 1! + 2 \cdot 2! + \cdots + n \cdot n! = (n + 1)! - 1$ whenever n is a positive integer

Basic: Step 1:
To check if the expression is true for n = 1.

$P(1) = 1.1! = (n+1)! - 1$
L.H.S = $1.1! = 1$
R.H.S = $(1+1)!-1 = 2! - 1 = 2 - 1 = 1$
$1 = 2! - 1$
$1 = 2 - 1$
$1 = 1$
Hence, the expression is true for n = 1.

Step 2:

Suppose the expression is true for n = k.
$P(k) = 1 \cdot 1! + 2 \cdot 2! + \cdots + k \cdot k! = (k + 1)! - 1$ ... (1)

Step 3:

To prove if the expression is true for n = k + 1.
Inductive: $\forall k \geq 1$; $P(k + 1) = 1 \cdot 1! + 2 \cdot 2! + \cdots + k \cdot k! + (k + 1).(k + 1)!$
From (1),
$P(k + 1) = ((k + 1)! - 1) + (k + 1).(k + 1)!$
$= (k + 1)! + (k + 1).(k + 1)! - 1$

$= (k + 1 + 1).(k + 1)! - 1$

$= (k + 2).(k + 1)! - 1$

$= (k + 2)! - 1 =((k+1)+1)! - 1$
P(k+1) is true.

Which is true according to the given expression.
L.H.S. = R.H.S.
Hence, the expression is true for all n.

5.2 – 2, 4

2. Use strong induction to show that all dominoes fall in an infinite arrangement of dominoes if you know that the first three dominoes fall and that when a domino falls, the domino three farther down in the arrangement also falls.

a) Basic step; P(1),P(2),P(3) IS ture, because first three dominoes

Inductive steps:
We know after ever 3 dominos and the next will fall asumm j >3
But we know K <=3 because every 3 dominos will fall

b) We want to prove that P(n) (i.e. all dominoes of n fall) is true for all positive integersn.

Base Case:
From the definition of the question we knoe that P(1), P(2) and P(3) is true

Inductive step:
Suppose that for all j <= k, P(j) holds (i.e. dominoes up to k fall), k >= 3
Let's show that P (k + 1) is true. Since k >=3, k - 2 is a positive integer less than or equal to k, so

Inductive hypothesis: we know that P(k-2) is true. We can infer from our hypothesis that P(k-2 is true). By the definition, we know that "three farther down in arrangement also falls" so P(k-2 + 3) = P(k+1) also falls.

4. Let P (n) be the statement that postage of n cents can be formed using just 4-cent stamps and 7-cent stamps. The parts of this exercise outline a strong induction proof that P (n) is true for n ≥ 18.

a) Show statements P (18), P (19), P (20), and P (21) are true, completing the basic step of the proof.
P(18) = 2*7+4
P(19) = 3*4+7
P(20) = 4*5
P(21)= 3*7

b) What is the inductive hypothesis of the proof?
18<=J<=K
And K>=21

c) What do you need to prove in the inductive step?

We can assume that P(k+1) for just 4 cents and 7 cents.

d) Complete the inductive step for $k \geq 21$.

For $k \geq 21$ we P(k-3) >= 18 is true, and we can have k-3 cents and add a 4 cent which k-3+4 = K+1 thus proves.

e) Explain why these steps show that this statement is true whenever $n \geq 18$.

Because we complete the basic steps and the inductive steps. thus for all n cents that include 4 and 7 cents $n \geq 18$ is true.

5.3 - 2, 4, 8

2. Find f(1), f(2), f(3), f(4), and f(5) if f(n) is defined recursively by f(0) = 3 and for n = 0,1,2,...

a) f(n+1)= −2f(n).

f(0) = 3

-2f(1) = -2*3 = -6

-2f(2) = -2*f(1) = -2*(-6) = 12

-2f(3) = -2*f(2) = -2*(12) = -24

-2f(4) = -2*f(3) = -2*(-24) = 48

-2f(5) = -2*f(4) = -2*(48) = -96

b) f(n+1)= 3f(n) + 7.

f(0) = 3

f(1): 3f(0) + 7 = 3*3 + 7 = 16

f(2): 3f(1) + 7 = 3*16 + 7 = 48 + 7 = 55

f(3): 3f(2) + 7 = 3*55 + 7 = 165 + 7 = 172

f(4): 3f(3) + 7 = 3*172 + 7 = 516 + 7 = 523

f(5): 3f(4) + 7 = 3*523 + 7 = 1569 + 7 = 1576

c) f(n+1)= $f(n)^2$ **− 2f(n) − 2.**

f(0) = 3

f(1): $f(0)^2$**+ 2f(0) - 2 =** $3^2 - 2 * 3 - 2$ **= 9-6-2 = 1**

f(2): $f(1)^2$**+ 2f(1) - 2 =** $1^2 - 2 * 1 - 2$ **= 1-2-2 = -3**

f(3): $f(2)^2$**+ 2f(2) - 2 =** $(-3)^2 - 2 * (-3) - 2$ **= 9+6-2 = 13**

f(4): $f(3)^2$**+ 2f(3) - 2 =** $13^2 - 2 * 13 - 2$ **= 169 - 26 - 2 = 141**

f(5): $f(4)^2$**+ 2f(4) - 2 =** $141^2 - 2 * 141 - 2$ **= 19881 - 282 - 2 = 19597**

d) f(n+1)= $3^{f(n)/3}$**.**

f(0) = 3

f(1) = $3^{f(0)/3}$**=** $3^{3/3}$**= 3**

f(2) = $3^{f(1)/3}$**=** $3^{3/3}$**= 3**

f(3) = $3^{f(2)/3}$**=** $3^{3/3}$**= 3**

f(4) = $3^{f(3)/3}$**=** $3^{3/3}$**= 3**

f(5) = $3^{f(4)/3}$**=** $3^{3/3}$**= 3**

4. Find f(2), f(3), f(4), and f(5) if f is defined recursively by f(0) = f(1) = 1 and for n = 1,2,...

a) f(n+1) = f(n) − f(n−1).
f(0) = f(1) = 1
f(2) = f(1) - f(1-1) = 1 - 1 = 0
f(3) = f(2) - f(2-1) = 0 - 1 = -1
f(4) = f(3) - f(3-1) = -1 - 0 = -1
f(5) = f(4) - f(4-1) = -1 + 1 = 0

b) f(n+1) = f(n)f(n−1).
f(0) = f(1) = 1
f(2) = f(1)f(1-1) = 1*1 = 1
f(3) = f(2)f(2-1) = 1*1 = 1
f(4) = f(3)f(3-1) = 1*1 = 1
f(5) = f(4)f(4-1) = 1*1 = 1

c) f(n+1) = $f(n)^2 + f(n-1)^3$
f(0) = f(1) = 1
f(2) = $f(1)^2 + f(1-1)^3$ **= 2**
f(3) = $f(2)^2 + f(2-1)^3$ **= 4+1 = 5**
f(4) = $f(3)^2 + f(3-1)^3$ **= 25 + 8 = 33**
f(5) = $f(4)^2 + f(4-1)^3$ **= 1214**

d) f(n+1) = f(n)/f(n−1).
f(0) = f(1) = 1
f(2) = $\frac{f(1)}{f(1-1)}$ **= 1**
f(3) = $\frac{f(2)}{f(2-1)}$ **= 1**
f(4) = $\frac{f(3)}{f(3-1)}$ **= 1**
f(5) = $\frac{f(4)}{f(4-1)}$ **= 1**

8. Give a recursive definition of the sequence {an}, n = 1,2,3,... if

a) a_n **= 4n−2.**

$\{a_n\}$**, n = 1,2,3 if**

a_n **= 4n−2.**

a_1 **= 4−2 = 2.**

a_2 **= 8−2 = 6.**

a_3 **= 12−2 = 10.**

n^{th}

a_{n+1} **= 4(n+1)−2 = 4n+4-2 = (4n-2)+4 =** $a_n + 4$

∴ sequence is $a_{n+1} = a_n + 4$ **when** $a_1 = 2$

b) $a_n = 1+(-1)^n$**.**

$a_1 = 1+(-1)^1 = 0$

$a_2 = 1+(-1)^2 = 2$

$a_3 = 1+(-1)^3 = 0$

$a_4 = 1+(-1)^4 = 2$

$a_{n+1} = 1+(-1)^{(n+1)} = 1+(-1)^n * (-1)^1 = 1 - (-1)^n = 1 - [1 + (-1)^n - 1] = 1 - [a_n - 1] = 2 - a_n$

$\therefore$ **sequence is** $a_{n+1} = 2 - a_n$ **when** $a_1 = 0$

c) $a_n = n(n+1)$**.**

$a_1 = 1(1+1) = 2$

$a_2 = 2(2+1) = 6$

$a_3 = 3(3+1) = 12$

$a_4 = 4(4+1) = 20$

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$a_{n+1} = (n+1)((n+1)+1) = (n+1)(n+2) = n(n+1) + 2(n+1) = a_n + 2(n+1)$

$\therefore$ **sequence is** $a_{n+1} = a_n + 2(n+1)$ **when** $a_1 = 2$

d) $a_n = n^2$**.**

$a_1 = 1^2 = 1$

$a_2 = 2^2 = 4$

$a_3 = 3^2 = 9$

$a_4 = 4^2 = 16$

$a_{n+1} = (n+1)^2 = n^2 + 2n + 1 = a_n + 2n + 1$

$\therefore$ **sequence is** $a_{n+1} = a_n + 2n + 1$ **when** $a_1 = 1$

5.4 – 2, 8

2. Trace Algorithm 1 when it is given n = 6 as input. That is, show all steps used by Algorithm 1 to find 6!, as is done in Example 1 to find 4!

n=0 return 1;
n=1 return 1*(1-1)!= 1
n=2 return 2*(2-1)! = 2*1!=2
n=3 return 3*(3-1)!=3*2!=6
n=4 return 4*(4-1)!=4*3!=24
n=5 return 5*(5-1)!=5*4!=120
n=6 return 6*(6-1)!=6*5!=720

8. Give a recursive algorithm for finding the sum of the first n positive integers

If n = 1 then return sum
Else
Sum = n - f(n-1)
End if;

6.1 – 2, 8, 30, 40, 44

2. An office building contains 27 floors and has 37 offices on each floor. How many offices are in the building?

27*37 = 999

8. How many different three-letter initials with none of the letters repeated can people have

26*25*24 = 15600ways

30. How many license plates can be made using either three uppercase English letters followed by three digits or four uppercase English letters followed by two digits?

26*26*26*10*10*10=17576000
26*26*26*26*10*10=45679600

40. How many subsets of a set with 100 elements have more than one element?

Let A be a subset with 100 elements

Then there will be

2^n *then* 2^{100} *and there will be more than one element thus* $2^{100} - 101$

44. How many ways are there to seat four of a group of ten people around a circular table where two seatings are considered the same when everyone has the same immediate left and immediate right neighbor?
10*9*8*7 = 5040,
And there are 4 ways to make an immediate left and immediate right neighbor thus
5040/4 = 1260ways

6.2 – 2, 4, 8, 18

2. Show that if there are 30 students in a class, then at least two have last names that begin with the same letter
Use the pigeonhole principle
There are n objects and k boxes there are 30 students as objects and 26 alphabets letters as boxes
Use the ceiling function. 30/26 = 1.15 = 2 thus, at least 2 students will have the same letter

4. A bowl contains 10 red balls and 10 blue balls. A woman selects balls at random without looking at them.
a) How many balls must she select to be sure of having at least three balls of the same color?
The ceiling function K/N=3 K/2 = 3 K= 6, but this is the ceiling function thus k = 5 thus 5/2 = 2.5 =3

b) How many balls must she select to be sure of having at least three blue balls?
Since there are 10 red balls thus at least select 13 balls in order to get three blue balls.

8. Show that if f is a function from S to T , where S and T are finite sets with |S| > |T |, then there are elements s1 and s2 in S such that f (s1) = f (s2), or in other words, f is not one-to-one.
Suppose S> T and T =n S>n+1 n + 1 or more elements are mapped on the elements there at least one pair of elements mapped on the function thus is not one to one function.

18. Suppose that there are nine students in a discrete mathematics class at a small college.
a) Show that the class must have at least five male students or at least five female students.
9/2 = 4.5 use the ceiling function, thus 5 males or 5 females.

b) Show that the class must have at least three male students or at least seven female students.

(0,9)(1,8)(2,7)(3,6)(4,5)(5,4)(6,3)(7,2)(8,1)(9,0)
As we can see there are at least 7 female or 3 males students

6.3 – 4, 6, 10, 12
4. Let S = {1, 2, 3, 4, 5}.
a) List all the 3-permutations of S.

P(5,3)= 5!/(5-3)! = 5!/2! = 60
123,132,213,231,312,321
124,142,214,241,412,421,
125,152,215,251,512,521
134,143,314,341,413,431
135,153,315,351,513,531,
145,154,415,451,514,541
234,243,324,342,423,432
235,253,325,352,523,532
245,254,425,452,524,542
345,354,435,453,534,543

b) List all the 3 combinations of S.

123, 124, 125, 134, 135, 145, 234, 235, 245, 345

6. Find the value of each of these quantities.

a) C(5, 1) = 5!/(5-1)! = 5
b) C(5, 3) = 5!/(5-3)! = 10
c) C(8, 4) =8!/(8-4)! = 70
d) C(8, 8)=8!(8-8)! = 1
e) C(8, 0) =8!/8!= 1
f) C(12, 6)=12!/(12-6) ! *6 ! =924

10. There are six different candidates for governor of a state. In how many different orders can the names of the candidates be printed on a ballot?

P（6, 6 ）= 6 ! /(6-6)! = 720

12. How many bit strings of length 12 contain

a) exactly three 1s?

$$\frac{12!}{3!(12-3)!} = \frac{12!}{3!9!} = \frac{12*11*10}{6} = \frac{1320}{6} = 220$$

b) at most three 1s?

$\frac{12!}{3!(12-3)!} + \frac{12!}{2!(12-2)!} + \frac{12!}{1!(12-1)!} + \frac{12!}{0!(12-0)!} = 220 + 66 + 12 + 1 = 299$

c) at least three 1s?

The total number of bit strings possible with length 12 is 2^{12} , as there are 12 blanks, and each blank has only two options, either 0 or 1. So the total number of options is 2^{12} .

Now, (number of strings with at least 3 1's) = (total number of strings) - (number of strings with at most 2 1's) =

$$\begin{aligned} 2^{12} - (b_0 + b_1 + b_2) &= 2^{12} - (\frac{12!}{0!(12-0)!} + \frac{12!}{1!(12-1)!} + \frac{12!}{2!(12-2)!}) \\ &= 2^{12} - (1 + 12 + 66) \\ &= 4096 - 79 \\ &= 4017 \end{aligned}$$

OR

$$\frac{12!}{12!(12-12)!} + \frac{12!}{11!(12-11)!} + \frac{12!}{10!(12-10)!} + \frac{12!}{9!(12-9)!} + \frac{12!}{8!(12-8)!} + \frac{12!}{7!(12-7)!} + \frac{12!}{6!(12-6)!} + \frac{12!}{5!(12-5)!}$$
$$+ \frac{12!}{4!(12-4)!} + \frac{12!}{3!(12-3)!} + \frac{12!}{2!(12-2)!} + \frac{12!}{1!(12-1)!} + \frac{12!}{0!(12-0)!} = 4017$$

d) an equal number of 0s and 1s?

(Number of strings with equal number of 0's and 1's) = (Number of strings with exactly 6 1's) = b_6 = 924

$\frac{12!}{6!(12-6)!} = \frac{12!}{6!6!} = \frac{12*11*10*9*8*7}{6*5*4*3*2*1} = 924$

6.4 – 2, 6, 8, 12

2. Find the expansion of (x + y)^5
a) using combinatorial reasoning, as in Example 1
(x+y)(x+y)(x+y)(x+y)(x+y)
(xx+xy+yx+yy)(xxx+xxy+xyx+yxx+xyy+yxy+yyx+yyy)
(x^2+2xy+y^2)(x^3+3x^2y+3y^2x+y^3
x^5+5x^4y+10x^3y^2+10x^2y^3+5xy^4+y^5

b) using the binomial theorem.
(x+y)^5

$$= \sum_{k=0}^{n=5} (\tfrac{n}{k})(1)x^{5-0}y^{0} + (5)x^{5-1}y^{1} + (10)x^{5-2}y^{2} + (10)x^{5-3}y^{3} + (5)x^{5-4}y^{4} + (1)x^{5-5}y^{5}$$

6. What is the coefficient of x^7 in $(1 + x)^{11}$?

6. By the binomial theorem the term involving x^7 in the expansion of $(1+x)^{11}$ is

$C(n,r) = \frac{n!}{r!(n-r)!}$

THE BINOMIAL THEOREM Let x and y be variables, and let n be a nonnegative integer. Then

$$(x+y)^n = \sum_{j=0}^{n} \binom{n}{j} x^{n-j} y^j = \binom{n}{0}x^n + \binom{n}{1}x^{n-1}y + \cdots + \binom{n}{n-1}xy^{n-1} + \binom{n}{n}y^n.$$

x^j in $(x+y)^n$

$$\binom{11}{7} 1^{(11-7)} x^7$$

therefore, the coefficient is $\binom{11}{7} 1^4 1^7$

$$= \binom{11}{7} = {}_{11}C_7 = \frac{11!}{7!(11-7)!} = \frac{11!}{7!\,4!} = \frac{11 \cdot 10 \cdot 9 \cdot 8}{4 \cdot 3 \cdot 2} = 330$$

8. What is the coefficient of x^8y^9 in the expansion of (3x + 2y)^17?

8. Using the binomial theorem, we see that the term involving x^8y^9 in the expansion of $(3x+2y)^{17}$.

THE BINOMIAL THEOREM Let x and y be variables, and let n be a nonnegative integer. Then

$$(x+y)^n = \sum_{j=0}^{n} \binom{n}{j} x^{n-j} y^j = \binom{n}{0}x^n + \binom{n}{1}x^{n-1}y + \cdots + \binom{n}{n-1}xy^{n-1} + \binom{n}{n}y^n.$$

$x\ y^j$

$$ {}_{17}C_9 \cdot (3x)^{17-9} \cdot (2y)^9$$

$$= {}_{17}C_9 \cdot 3^8 \cdot 2^9 \cdot x^8 \cdot y^9$$

$$= \frac{17!}{9!(17-9)!} \cdot 6561 \cdot 512 \cdot x^8 y^9$$

$$= 24310 \cdot 6561 \cdot 512 \cdot x^8 y^9$$

$$= 81662929920\, x^8 y^9$$

Thus, the required coefficient is 81662929920.

12. The row of Pascal's triangle containing the binomial coefficients $\binom{10}{k}$, $0 \leq k \leq 10$, is:

1 10 45 120 210 252 210 120 45 10 1

Use Pascal's identity to produce the row immediately following this row in Pascal's triangle.

12. $\binom{10}{k}$ $0 \leq k \leq 10$

$\binom{10+1}{k}$ $0 \leq k \leq 11$ $\binom{11}{0}\binom{11}{1}\binom{11}{2}\binom{11}{3}\binom{11}{4}\binom{11}{5}\binom{11}{6}\binom{11}{7}\binom{11}{8}\binom{11}{9}\binom{11}{10}\binom{11}{11}$

1 11 55 165 330 462 462 330 165 55 11 1

OR

The rows are just sums of consecutive terms of the previous row. First, get the original row and pad it with zeroes:
0 1 10 45 120 210 252 210 120 45 10 1 0
The next row is just:
(0+1) (1+10) (10+45) ... (45+10) (10+1) (1+0)
or,
1 11 55 165 330 462 462 330 165 55 11 1

7.1 – 2, 8, 30

2. What is the probability that a fair die comes up six when it is rolled?
1/6

8. What is the probability that a five-card poker hand contains the ace of hearts?
C (52, 5)
C (51,4)
$=\frac{C(52,5)}{C(51,4)}=\frac{5}{52}$

30. What is the probability that a player of a lottery wins the prize offered for correctly choosing five (but not six) numbers out of six integers chosen at random from the integers between 1 and 40, inclusive?
$\frac{C(6,\ 5)*c(34,1)}{C(40,6)} = 5.31 * 10^{-5}$

7.2 – 2, 6, 12, 24, 26

2. Find the probability of each outcome when a loaded die is rolled if a 3 is twice as likely to appear as each of the other five numbers on the die.

2p(3)+p(1)+p(2)+p(4)+p(5)+p(6)=$\frac{2}{7}$

6. What is the probability of these events when we randomly select a permutation of {1, 2, 3}?

a) 1 precedes 3. $\frac{3}{6} = \frac{1}{2}$

b) 3 precedes 1. $\frac{3}{6} = \frac{1}{2}$

c) 3 precedes 1 and 3 precedes 2. $\frac{2}{6} = \frac{1}{3}$

12. Suppose that E and F are events such that p(E) = 0.8 and p(F) = 0.6. Show that p(E ∪ F) ≥ 0.8 and p(E ∩ F) ≥ 0.4

p(E ∪ F) = 0.8
p(E ∪ F) = p(E)+p(F)- p(E ∩ F)
p(E)+p(F)- p(E ∩ F) <= 1
0.8+0.6- p(E ∩ F)<=1
1.4-1< = p(E ∩ F)
0.4< = p(E ∩ F)

24. What is the conditional probability that exactly four heads appear when a fair coin is flipped five times, given that the first flip came up tails?

$\frac{2}{2^5} = \frac{1}{16}$

26. Let E be the event that a randomly generated bit string of length three contains an odd number of 1s, and let F be the event that the string starts with 1. Are E and F independent?

If E and F are independent then P(E)*P(F) = P(E∩F)

P(E) = # of favorable outcomes / # of possible outcomes = $\frac{4}{8}$= $\frac{1}{2}$

4 of the 8 possible outcomes start with a 1 (111,110,101,100)

P(F) = # of favorable outcomes / # of possible outcomes = $\frac{4}{8} = \frac{1}{2}$

2 of the 8 possible outcomes contain an odd number of I's and start with a 1 (111,100)

P(E∩F) = # of favorable outcomes # of possible outcomes = $\frac{1}{4}$

So P(E)*P(F) = (1/2)*(1/2)= $\frac{1}{4}$ = P(E∩F). Thus, E and F are independent.

8.1 - 8

8. a) Find a recurrence relation for the number of bit strings of length n that contain three consecutive 0s.

First case, end in 1 then a_{n-1} then second 10 a_{n-2} , then third100 a_{n-3} then fourth 000 and there are $2^{a_n - 3}$. Then, $a_{n-1} + a_{n-2} + a_{n-3} + 2^{a_n - 3}$

b) What are the initial conditions?

$a_0 = 0\ a_1 = 0\ a_2 = 0\ a_3 = 2^0 = 1$

c) How many bit strings of length seven contain three consecutive 0s?

$a_0 = 0$

$a_1 = 0$

$a_2 = 0$

$a_3 = 1$

$a_4 = 2$

$a_5 = 4$

$a_6 = 8$

$a_7 = 47$

8.3 – 8, 10, 14, 16

8. Suppose that f (n) = 2f (n/2) + 3 when n is an even positive integer, and f (1) = 5. Find

a) f (2). = 2f(1)+3 = 2(5)+3=13

b) f (8). = 2f(4)+3 =2(29)+3 = 61

c) f (64). 64/2 = 32 32/ 2 =16 16 /2 =8 f(8)=61 f(16)=125 f(32)=253 f(64)=509

d) f (1024).f(64)= 509 f(128)=2045 f(256)=4093 f(512)=8109

10. Find f (n) when n = 2k, where f satisfies the recurrence relation f (n) = f (n/2) + 1 with f (1) = 1

$f(2^k) = f(\frac{2^k}{2}) + 1$

$f(2^k) = f(\frac{2^{k-1}}{2}) + 2$

$f(2^k) = f(\frac{2^{k-2}}{2}) + 3$

$f(2^k) = f(\frac{2^{k-(k-1)}}{2}) + 1 + k - 1$

Thus $n = 2^k\ is\ k + 1$

14. Suppose that there are n = 2k teams in an elimination tournament, where there are n/2 games in the first round, with the n/2 = 2k−1 winners playing in the second round, and so on. Develop a recurrence relation for the number of rounds in the tournament.

The number of terms $n = 2^k$

For k = 1 then $n = 2$ and f(2) = 1

K = 1 n = 4 $f(2^2) = 4$

k= 3 n = 4 $f(2^3) = 3$

Thus $f(n) = f(\frac{n}{2}) + 1$ n=2^k

16. Solve the recurrence relation for the number of rounds in the tournament described in Exercise 14

$f(n) = [f(\frac{2^{k-2}}{2}) + 1] + 2$

$f(n) = f(2^2) + (k-2)$

$f(n) = [f(\frac{2^1}{2}) + 1] + (k-1)$

$f(n) = f(2^0) + k = log_2 n$

Thus, k or $log_2 n$.