

## Problem 3 (A3)

a)

- ☐ Partial Order
- ☒ Strict Order
- ☐ Neither
- ☐ Total Order

b)

- ☐ Partial Order
- ☐ Strict Order
- ☒ Neither
- ☐ Total Order

c)

- ☒ Partial Order
- ☐ Strict Order
- ☐ Neither
- ☐ Total Order

### Explanation Boxes:

a)

Reflexive = It is not reflexive, If we have  $(5,3)$  is related to  $(5,5)$ , then we have  $(5-5) < (3-3) \rightarrow 0 < 0$ .

Since it is not reflexive, it is not a partial order.

Anti-reflexive = For any integers  $x, y$ ,  $(x, y)$  is related to  $(x, y)$ ,  $(x-x) < (y-y)$ ,  $\therefore$  it is anti-reflexive.

Transitive = For any integers  $f, g, h, i, j, k$ , if  $(f, g)$  is related to  $(h, i)$  where  $(f-h) < (g-i)$  and  $(h, i)$  is related to  $(j, k)$ , where  $(h-j) < (i-k)$ , then  $(f, g)$  is related to  $(j, k)$  where  $(f-j) < (g-k)$ ,  $\therefore$  it is transitive.

Anti-symmetric = Since it is anti-reflexive and transitive, then it is also anti-symmetric.

Since it is anti-reflexive and transitive, it is a strict order.

Total order = If we have  $(4,5)$  is related to  $(2,3)$ , where  $(4-2) < (5-3) \rightarrow 2 < 2$ , then  $(4,5)$  and  $(2,3)$  are not comparable  $\therefore$  it is not a total order.

b)

Reflexive = for  $x$  that is a subset of  $S$ ,  $x$  is related to  $x$ , then  $|x| = |x| + 1$ . It is false,  $\therefore$  it is not reflexive.

Anti-reflexive = for any  $x$  that is a subset of  $S$ ,  $|x| \neq |x| + 1$ ,  $\therefore$  it is anti-reflexive.

Transitive = for any  $x, y, z$  that are subsets of  $S$ , if  $|x| = |y| + 1$  and  $|y| = |z| + 1$  we have

$$|x| = |y| + 1$$

$$|x| = (|z| + 1) + 1$$

$$|x| = |z| + 2$$

$\therefore$  it is not transitive.

$$\{ \{ a, b, c, d \} \} \quad \{ a, b, c, d \}$$

c)

Reflexive: we know that any set is a subset of itself. For  $x \in P(A)$ ,  $x$  is related  $x$  because  $x$  is a subset of  $x$ ,  $\therefore$  it is reflexive.

Transitive: for  $x, y, z \in P(A)$ , if  $x$  is a subset of  $y$  and  $y$  is a subset of  $z$ , then  $x$  is a subset of  $z$ ,  $\therefore$  it is transitive.

Anti-symmetric: for  $x, y \in P(A)$ , if  $x$  is a subset of  $y$  and  $y$  is subset of  $x$ , then  $x = y$ ,  $\therefore$  it is anti-symmetric.

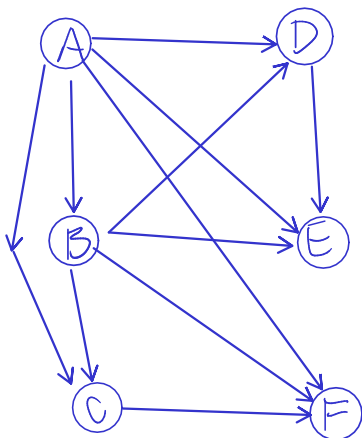
Since it is reflexive, transitive, and anti symmetric, it is a partial order.

Total order =  $\{1\}, \{2\} \in P(A)$ , but  $\{1\}$  is not a subset of  $\{2\}$  nor  $\{2\}$  is a subset of  $\{1\}$   $\therefore \{1\}$  and  $\{2\}$  are not comparable.  
 $\therefore$  it is not a total order.

## Problem 4 (A4)

a)

- ☐ Graph 1
- ☐ Graph 2
- ☐ Graph 3
- ☒ Graph 4



b)

- ☐ Sort 1
- ☒ Sort 2
- ☐ Sort 3
- ☒ Sort 4

