CSCI 190 Discrete Mathematics Applied to Computer Science
Final Exam

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Last 4 digits of your Student ID #: 3160

Read these instructions before proceeding.

- Closed book. Closed notes. You can use calculator.
- You have **100 minutes** to complete this exam.
- No questions will be answered during the exam or immediately afterwards. Answer each question as best you can. Partial credit will be awarded for reasonable efforts. If a question contains an ambiguity or a misprint, then say so in your answer, providing the answer to a reasonable interpretation of the question; give your assumptions.
- Answer the problems on the blank spaces provided for each problem.
- **Box your answers.**

Q1 (11)	Q2 (12)	Q3 (12)	Q4 (8)	Q5 (12)	Q6 (8)	Q7 (6)	Q8 (6)	Q9 (6)	Q10 (6)	Q11 (4)	Q12 (5)	Q13 (4)	Total (100)

1. (11 pts)

a) (3 pts) Write the converse of the following:

If you are happy, then you will smile.

$p \to q \overset{\text{converse}}{\Rightarrow} q \to p$

if you will smile, then you are happy.

b) (4 pts) Convert $(9F6)_{16}$ to base 4.

from hexadecimal [base 16] to quaternary [base 4]

A	0
B	1
C	2
D	3
E	4
F	5
G	6
H	7
I	8
J	9
K	10
L	11
M	12
N	13
O	14
P	15
Q	16
R	17
S	18
T	19
U	20
V	21
W	22
X	23
Y	24
Z	25

$(9F6)_{16} = 9 \cdot 16^2 + f \cdot 16^1 + 6 \cdot 16^0 = 2550_{10}$

$\Rightarrow$ convert from 2550_{10} to quaternary

$2550/4 = 637 \ldots 2$ $\quad 39/4 = 9 \ldots 3$

$637/4 = 159 \ldots 1$ $\quad 9/4 = 2 \ldots 1$

$159/4 = 39 \ldots 3$ $\quad 2/4 = 0 \ldots 2$

$\Rightarrow 213312_4$

c) (4 pts) A message has been ***encrypted*** using the function f**(x) = (x + 4) mod 26**. If the message in coded form is **NSC**, decode the message.

$N = (13 - 4) \bmod 26 = 9 \bmod 26 = J$

$S = (18 - 4) \bmod 26 = 14 \bmod 26 = O$

$C = (2 - 4) \bmod 26 = 24 \bmod 26 = Y$

2. (12 pts)

a) (5 pts) Use the Principle of Mathematical Induction to prove that

$2 + 4 + 6 + 8 + \cdots + 2n = n(n+1)$ for all $n \geq 1$. Show all the steps

① we use induction on n.

② Base Case:

for $n = 1$

$p(1)$ is true

$2(1) \overset{?}{=} 1 \cdot (1+1)$

$2 \overset{\checkmark}{=} 2$

Hence, $s(1)$ is true.

③ Inductive Step:

we assume that $p(k)$ is true for an arbitrary nonnegative integer k.

we assume $2+4+6+8+\cdots+2k = k(k+1) = k^2+k$

$p(k)$ is true $\Rightarrow p(k+1)$ is also true.

$p(k+1) = \underbrace{2+4+6+8+\cdots+2k}_{k(k+1)} + 2(k+1) = k(k+1) + 2 \cdot (k+1)$

$= (k+2)(k+1)$

$= k^2 + k + 2k + 2$

$= k^2 + 2k + 1 + 1 + k$

$= (k+1)^2 + (k+1)$

L.H.S. = R.H.S.

$\therefore 2+4+6+8+\cdots+2n = n(n+1)$ is true for all positive value of n. By induction, $P(n)$ is true.

b) (4 pts) Give a recursive definition with initial condition for the following function, square of n factorial.

$f(n) = (n!)^3$, $n = 0, 1, 2, \ldots.$

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initial condition: $f(0) = (0!)^3 = 1^3 = 1$

$f(1) = (1!)^3 = 1$

$f(2) = (2!)^3 = 8$

$f(3) = (3!)^3 = 216$

$f(4) = (4!)^3 = 13824$

$\Rightarrow f(1) = 1 = 1^3 \cdot f(0)$

$f(2) = 8 = 2^3 \cdot f(1)$

$f(3) = 216 = 3^3 \cdot f(2)$

$f(4) = 13824 = 4^3 \cdot f(3)$

Recursive definition:

$f(n) = (n^3) f(n-1)$

$_nC_k = \frac{n!}{k!(n-k)!}$

c) **(3 pts)** In a certain lottery game you choose a set of seven numbers out of 40 numbers. Find the probability that exactly one of your numbers match the seven winning numbers.

$_{40}C_7$ => # of possible outcomes of the lottery drawing

=> exactly 1 of your #s' match the 7 winng #s.

& => 6 of the 33 losing #s.

$$\frac{_7C_1 \cdot {}_{33}C_6}{_{40}C_7} = \frac{\frac{7!}{1!(7-1)!} \cdot \frac{33!}{6!(33-6)!}}{\frac{40!}{7!(40-7)!}} = \frac{7 \cdot 1107568}{18643560} = \frac{7752976}{18643560} \approx 41.59\% \text{ or } \approx 41.6\%$$

3. *(12 pts)* Determine whether the following binary relation is:
(1) reflexive, (2) symmetric, (3) antisymmetric, (4) transitive.
No justifications needed.

a) (6 pts) The relation ***R*** on the set of all people where ***aRb*** means that ***a*** is taller than ***b***.
Circle your answers.

R is	Reflexive?	Symmetric?	Antisymmetric?	Transitive?
	Yes or No	Yes or No	Yes or No	Yes or No

b) (6 pts) If $\boldsymbol{M_R} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$,

determine if ***R*** is: (a) reflexive (b) symmetric (c) antisymmetric (d) transitive.
Circle your answers.

R is	Reflexive?	Symmetric?	Antisymmetric?	Transitive?
	Yes or No	Yes or No	Yes or No	Yes or No

Solution :

since all conditions hold .

∴ the relation is equivalence relation .

4. (8 pts)

a) (4 pts)Suppose ***R*** is the relation on *N* where ***aRb*** means that ***a*** starts in the same digit in which ***b*** starts.

Determine whether ***R*** is an ***equivalence relation*** on *N*. Justify your answer.

Reflexivity : aRa since a and a start from the same digit, so the relation is reflexive . ∴ Reflexive .

Symmetry : aRb is a and b start from the same digit . It can also be stated as b and a start from same digit . so bRa and the relation is symmetric . ∴ symmetric.

Transitivity : aRb is a and b starts from the same digit, bRc is b and c start from the same digit therefore, a & c starts from the same digit and aRc ∴ Transitivity

b) (4 pts) Suppose the relation ***R*** is defined on the set *Z* where ***aRb*** means that ***ab < 0***.

Determine whether ***R*** is an ***equivalence relation*** on *Z*. Justify your answer.

Reflexivity : aRa => $a \cdot a < 0$ but $a \cdot a = a^2$ is square of an integer which will always be positive ∴ inequality . Not Reflexive.

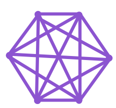
Symmetry : aRb => $a \cdot b < 0$ is same as bRa => $b \cdot a < 0$ is symmetric.

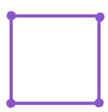
Transitivity : aRb => $ab < 0$ & bRc => $bc < 0$ but aRc does't hold. ∴ not transitivity.

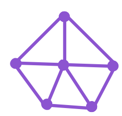
Since it is neither reflexive, nor transitive. ∴ NOT equivalence relation.

5. (12 pts)

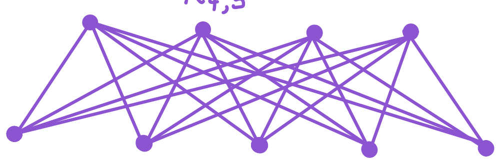
a) (4 pts) Draw these four graphs. $\mathbf{K_6}$, $\mathbf{C_4}$, $\mathbf{W_5}$ and $\mathbf{K_{4,5}}$

K_6 = 

C_4 = 

W_5 = 

$K_{4,5}$



b) (4 pts)

$C(n,2) = {}_nC_2$

$\boldsymbol{K_n}$ has $\frac{n(n-1)}{2} = 15$ edges and $n = 6$ vertices.

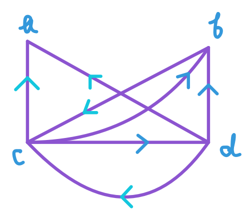
$\boldsymbol{K_{m,n}}$ has $m \cdot n = 20$ edges and $m + n = 9$ vertices.

$\boldsymbol{W_n}$ has $2 \cdot n = 10$ edges and $n + 1 = 6$ vertices.

$\boldsymbol{C_n}$ has $n = 4$ edges and $n = 4$ vertices.

c) (4 pts) Draw the ***digraph*** with adjacency matrix

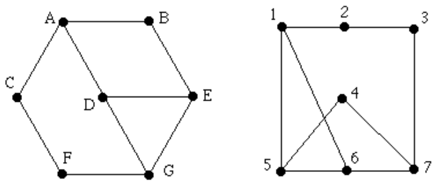
$$\begin{array}{c|c} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$



6. (8 pts)

a) (6 pts) Are these two graphs ***isomorphic***?

If yes, give the mapping of vertices from the first graph to the second graph.
If no, explain why not.



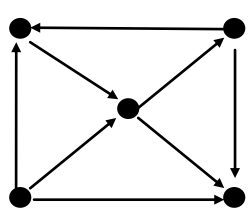
yes, the given graphs are isomorphic.
Mapping of the vertices from the 1st graph to the 2nd graph.

A = 7
B = 4
C = 3
D = 6
E = 5
F = 2
G = 1

b) (2 pts) Circle ***Yes*** or ***No***. No justifications needed.

Determine whether the graph is ***strongly connected***? Yes or No

Determine whether the graph is ***weakly connected***. Yes or No



7. (6 pts) Circle ***TRUE*** or ***FALSE***. No justifications needed.

T / F If T is a tree with 10 vertices, then there is a simple path in T of length 9.

T / F Every tree is bipartite.

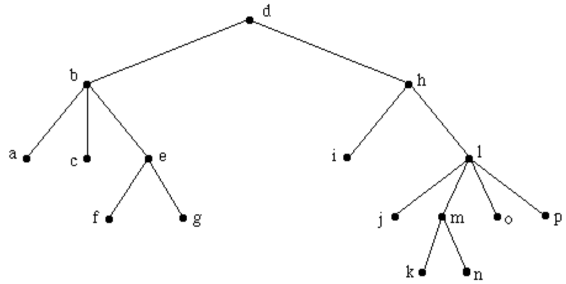
T / F There is a tree with degrees 4, 3, 2, 2, 1, 1, 1, 1, 1.

T / F There is a tree with degrees 3, 3, 3, 2, 1, 1, 1, 1.

T / F If T is a tree with 30 vertices, the largest degree that any vertex can have is 29.

T / F If two trees have the same number of vertices and the same degrees, then the two trees are isomorphic.

8. (6 pts) Refer to the following tree.



a) (2 pts) Find the ***preorder*** traversal.

d b a c e f g h i l j m k n o p

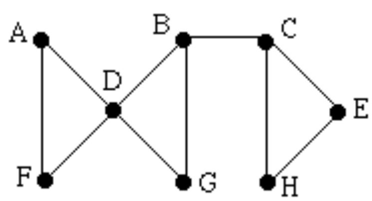
b) (2 pts) Find the ***inorder*** traversal.

a b c f e g d i h j k m n l o p

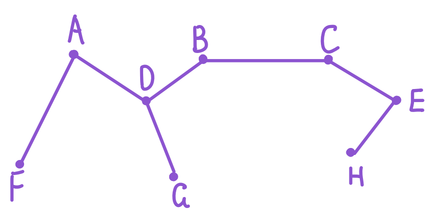
c) (2 pts) Find the ***postorder*** traversal.

a c f g e b i j k n m o p l h d

9. (6 pts) Refer to the following graph..

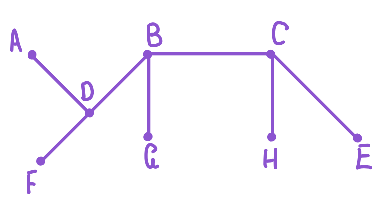


a) (3 pts) Using ***alphabetical ordering***, find ***a spanning tree*** (starting from vertice ***B***) for this graph by using DFS, ***depth-first search***.



ORDER:
BCEHDAFG

b) (3 pts) Using ***alphabetical ordering***, find ***a spanning tree*** (starting from vertice ***B***) for this graph by using BFS, ***breadth-first search***.



ORDER:
BCDGEHAF

10. (6 pts) Using a table to show that ***F(x,y,z) = xyz + xy + x*** has a vaule of 1 if and only if variable ***x*** has a value of 1.

x	y	z	xyz	xy	x	$xyz+xy+x$
1	1	1	1	1	1	1
1	1	0	0	1	1	1
1	0	1	0	0	1	1
1	0	0	0	0	1	1
0	1	1	0	0	0	0
0	1	0	0	0	0	0
0	0	1	0	0	0	0
0	0	0	0	0	0	0

11. (4 pts) Find the ***duals*** of these Boolean expressions.

a) (2 pts) $0 + x + y$

$= 1 \cdot x \cdot y$

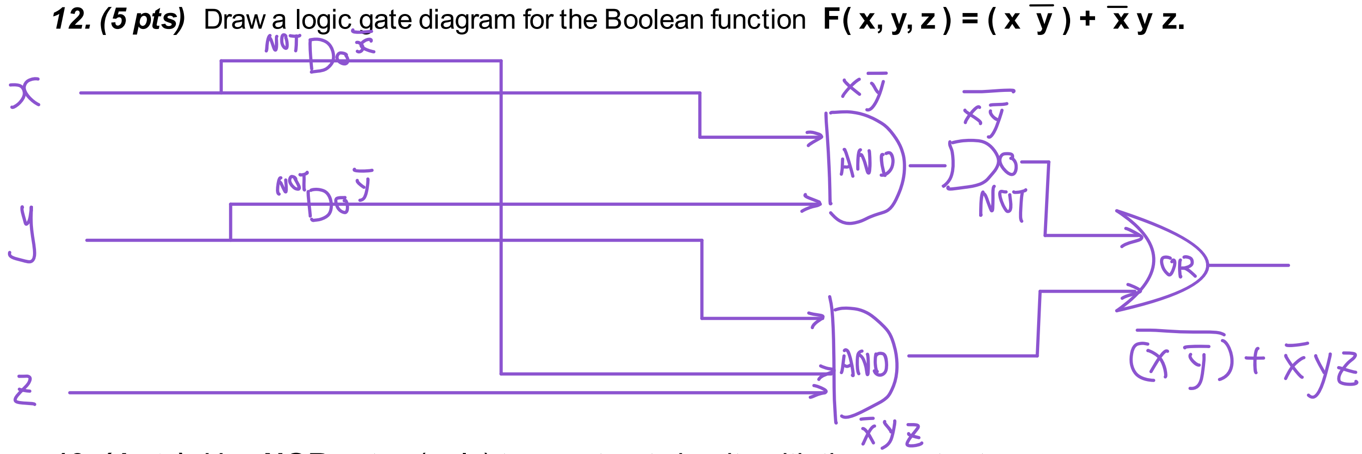
b) (2 pts) $x\,\overline{y}\,z$

$= x + \bar{y} + z$

Duality

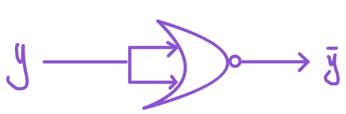
Interchanging . signs & + signs & interchanging 0s and 1s.

12. (5 pts) Draw a logic gate diagram for the Boolean function **F(x, y, z) = $\overline{(x\ \overline{y})}$ + $\overline{x}$ y z.**



13. (4 pts) Use **NOR** gates (only) to construct circuits with these outputs.

a) (2 pts) $\overline{y}$



b) (2 pts) y z