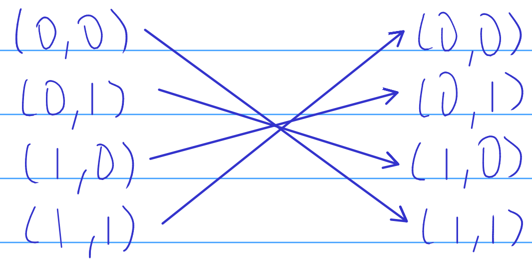
Honest Effort

4.1.4 a) Domain of f = $\{(0,0), (0,1), (1,0), (1,1)\}$

b)



c) Range of f = $\{(0,0), (0,1), (1,0), (1,1)\}$

4.2.1 a) $f(2.2) = \lfloor 2.2 + \frac{1}{2} \rfloor = \lfloor 2.7 \rfloor = 2$

$f(2.9) = \lfloor 2.9 + \frac{1}{2} \rfloor = \lfloor 3.4 \rfloor = 3$

$f(2.5) = \lfloor 2.5 + \frac{1}{2} \rfloor = \lfloor 3 \rfloor = 3$

$f(3) = \lfloor 3 + \frac{1}{2} \rfloor = \lfloor 3.5 \rfloor = 3$

b) For each value of x, $f(x)$ is equal to the largest integer y such that y is less than or equal to x plus $\frac{1}{2}$.

c) For each value of x, $g(x)$ is equal to the smallest integer y such that y is greater than or equal to x minus $\frac{1}{2}$.

4.2.2 a) $\left\lceil \frac{24}{5} \right\rceil = \lceil 4.8 \rceil = 5$ boxes

b) $\left\lfloor \frac{y}{8} \right\rfloor$ boxes

4.2.4 b) If n is an odd integer, then $\left\lfloor \frac{n}{2} \right\rfloor = \frac{n-1}{2}$

Suppose n is an odd integer, then $n = 2K+1$ for some integer K.

$$\left\lfloor \frac{n}{2} \right\rfloor = \left\lfloor \frac{2K+1}{2} \right\rfloor = \left\lfloor \frac{2K}{2} \right\rfloor + \left\lfloor \frac{1}{2} \right\rfloor = \lfloor K \rfloor + 0 = K$$

$$\frac{n-1}{2} = \frac{(2K+1)-1}{2} = \frac{2K}{2} = K$$

$\therefore$ True

4.3.2 i) $f: \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}$, $f(x,y) = \left(\left\lceil \frac{x}{5} \right\rceil, 5y-2 \right)$

One to One: False, $f(2,1) = (1,3)$ and $f(3,1) = (1,3)$
Onto: True

k) $f: \mathbb{Z}^+ \times \mathbb{Z}^+ \longrightarrow \mathbb{Z}^+$, $f(x,y) = 2^x + y$

One to One : True

Onto : False, there does not exist an positive integer such that $2^x + y = 1$

Since 1 is the smallest positive integer then $2^1 + 1 = 3$

4.3.6 b) Let $c \cdot f$ be defined as $(c \cdot f)(x)$, where f is a bijection and $c \neq 0$.

We want to show that if $(c \cdot f)(x_1) = (c \cdot f)(x_2)$ then $x_1 = x_2$

Assume that
$$(c \cdot f)(x_1) = (c \cdot f)(x_2)$$
$$c \cdot f(x_1) = c \cdot f(x_2)$$
$$f(x_1) = f(x_2)$$

Since f is a bijection, then if $f(x_1) = f(x_2)$ then $x_1 = x_2$

$\therefore$ $c \cdot f$ is injective.

To prove that $c \cdot f$ is surjective, we need to show that for every y in the target of $c \cdot f$, there exist an x in the domain such that $(c \cdot f)(x) = y$

Suppose y is a real number.

Since $f(x)$ is a bijection, then
$$f(x) = \frac{y}{c} \quad \text{for some integer } c$$

$$c\, f(x) = \cancel{c}\, \frac{y}{\cancel{c}} = y$$

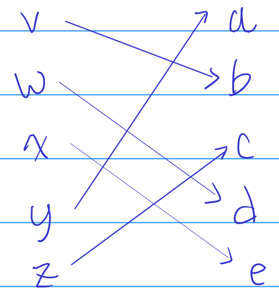
Since $c\, f(x) = y$

$\therefore$ $c f(x)$ is surjective

$\therefore$ if f is a bijection then $c \cdot f$ is also a bijection for $c \neq 0$

4.4.1 a) f^{-1} is not well-defined. $f^{-1}(w) = b$ and c

b)



c) f^{-1} is not well-defined. The element y does not map to any element.

4.5.2 b) $(f \circ h)(52)$

$$h(25) = \left\lceil \frac{25}{5} \right\rceil = 5$$

$$f(h(25)) = f(5) = 5^2 = 25$$

d) $h \circ f$

$$f(x) = x^2$$
$$h(f(x)) = \left\lceil \frac{x^2}{5} \right\rceil$$

Honest Effort and Feeback Given

4.3.4 d) $f : \{1,0\}^3 \longrightarrow \{0,1\}^4$

f is one to one.
f is not onto because the target has more elements than the domain.

g) $A = \{1, 2, 3, \ldots, 8\}$, $B = \{1\}$

f is one to one
f is onto

4.4.2 b) It is not well-defined

e) $f^{-1}(Y) = A - Y$

4.5.6 b) $(g \circ h)(010)$

$h(010) = 010$

$g(h(010)) = g(010) = 010$

e) Range of $g \circ f = \{001, 011, 101, 111\}$