

Confidence Intervals - 1 Mean 1 Proportion

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Confidence Intervals

- Provides the **interval of likely values** for the population parameter
- We are ____% confident that the **population parameter** is captured within the interval.
- If we took m random samples from a population and created ____% confidence intervals for each of those m samples, then if m is large, ____% of those confidence intervals would contain the **population parameter**.
- In order to create a confidence interval for a population parameter, we use two things
 - We use the sample statistic as the best estimate of the population parameter.
 - We also need a measure of spread.
- The **sample statistic** **ESTIMATES** the **population parameter**.

General Formula for a Confidence Interval for a Parameter of Interest

Point Estimate \pm Margin of Error

$$(PE - ME, PE + ME)$$

- Point Estimate

a single point that estimates the parameter

sample statistic

the midpoint of your confidence interval

determines the location of the interval

- Margin of Error

an estimate of the variability of the population parameter

distance from the midpoint of the interval to the edge

critical value * spread

determines the precision of the interval

increasing the ME decreases precision of the interval

- critical value

accounts for the level of confidence

increasing confidence increases the width of the interval (decreases precision)

- spread

accounts for the variability

increasing the sample size decreases the width of the interval (increases precision)

Specific Formulas for Confidence Intervals

Parameter Description	Confidence Interval	Parameter Estimated
1 population proportion	$\hat{p} \pm Z \sqrt{\frac{\hat{p}\hat{q}}{n}}$	p
1 population mean (σ unknown)	$\bar{X} \pm t_{n-1} \frac{s}{\sqrt{n}}$	μ

Assumptions for Confidence Intervals

1 population proportion

- $n\hat{p} \geq 10$ and $n\hat{q} \geq 10$
- Sample size is less than 10% of the population size; if we are sampling is w/out replacement
- The sample can be regarded as a simple random sample from the population of interest.
- The data values are assumed to be independent of each other.

1 population mean

- We need to have a large enough sample size ($n \geq 30$). For $n < 30$ with extreme skewness or outliers, you cannot use this method.
- Sample size is less than 10% of the population size; if we are sampling is w/out replacement
- The sample can be regarded as a simple random sample from the population of interest.
- The data values are assumed to be independent of each other.

Ex: The drug Lipitor is meant to lower cholesterol levels. In a clinical trial of 875 randomly selected patients who received 12 mg doses of Lipitor daily, 53 reported a headache as a side effect.

- a. What is the point estimate for the true population proportion of Lipitor users who will experience a headache as a side effect?

- b. Verify that the requirements for constructing a confidence interval are satisfied.

- c. Construct a 90% confidence interval for the population proportion of Lipitor users who will report a headache as a side effect.

- d. Interpret your confidence interval.

Ex: Suppose that a random survey of 10 teenagers found that the average amount of time they spend on the Internet each day is 3.2 hours with a sample standard deviation of 0.78 hours.

- a. What assumptions must be made in order for a confidence interval to be valid?

- b. What are the point estimate and the standard error of the population average amount of time teenagers spend on the Internet each day?

- c. Assuming the necessary conditions are met, calculate a 98% confidence interval for the average amount of time a teenager spends on the Internet each day.

- d. Interpret your confidence interval in the context of the problem using a complete sentence.

EX: A study of 10,485 randomly selected 30-39 year old Americans conducted by the Center for Disease Control in 2000 found with 95% confidence that the true proportion of 30-39 Americans that are overweight is between 0.179 and 0.193.

a. Find the point estimate for the true proportion.

b. Find the margin of error.

Ex: The IRS is investigating the income obtained from tips in various types of services. In a small pilot study at a gourmet restaurant the agents randomly selected 30 charge-card receipts and computed the average tip to be \$11.20 with a standard deviation of \$1.798. A confidence interval for the average tip given by patrons of this restaurant is (\$10.7696 , \$11.6304)

a. The margin of error for the confidence interval is:

b. Based on the above confidence interval, determine the critical value used and the confidence level.