

o 9.1 – 2, 4, 10

Ping Ju

2. a) List all the ordered pairs in the relation $R = \{(a, b) \mid a \text{ divides } b\}$ on the set $\{1, 2, 3, 4, 5, 6\}$.

1|1 1|2 1|3 1|4 1|5 1|6 2|4 2|6 3|3 3|6 2|2 3|3 4|4 5|5 6|6

$\Rightarrow (1,1) (1,2) (1,3) (1,4) (1,5) (1,6) (2,4) (2,6) (3,3) (3,6) (2,2) (4,4) (5,5) (6,6)$

b) Display this relation graphically.

$R = (1,1) (1,2) (1,3) (1,4) (1,5) (1,6) (2,4) (2,6) (3,3) (3,6) (2,2) (4,4) (5,5) (6,6)$

9.1



c)

2c)

R	1	2	3	4	5	6
1	x	x	x	x	x	x
2		x		x		x
3			x			x
4				x		
5					x	
6						x

4. Determine whether the relation R on the set of all people is reflexive, symmetric, antisymmetric, and/or transitive, where $(a, b) \in R$ if and only if

a) a is taller than b .

Let a, b in A $(a, b) \in R$

a is taller than b

b is not taller than a

$(b, a) \notin R$

Thus R is not symmetric

Let $a, b, c \in A$ $(a, b), (b, c) \in R$
 a is taller than b b is taller than c
 a is taller than c
 $(a, c) \in R$
 Thus transitive

b) a and b were born on the same day.
 Let $a, b, c \in A$ $(a, b), (b, c) \in R$
 a and b were born on the same day
 b and c were born on the same day
 $(a, c) \in R$
 Thus transitive

c) a has the same first name as b .
 Let $a, b \in A$ and $(a, b) \in R$
 a and b have the same first name
 b and a have the same first name
 $(b, a) \in R$
 Thus symmetric

d) a and b have a common grandparent
 Let $a, b \in A$ and $(a, b) \in R$
 a and b have the common grandparent
 b and a have the common grandparent
 $(b, a) \in R$
 Thus symmetric

10. Give an example of a relation on a set that is
 a)

The empty set on $\{a\}$ (vacuously symmetric and antisymmetric)

both symmetric and antisymmetric.

$A = \{1, 2, 3, 4\}$

$R = \{(1, 1), (2, 2), (3, 3), (4, 4)\}$

A relation R on a set A is symmetric if $(a, b) \in R$ then $(b, a) \in R$ for all $a, b \in A$

A relation R on a set A is antisymmetric if $(a, b) \in R$ then $a = b$ all $a, b \in A$

For $1 \in A$ $(1, 1)$

$R(1, 1)$

Also $1 = 1$

Thus hold

b) neither symmetric nor antisymmetric.

$\{(a,b), (b,a), (a,c)\}$ on $\{a,b,c\}$

$A = \{1,2,3\}$

$R = \{(1,2), (1,3), (1,4)\}$

A relation R on a set A is symmetric if $(a,b) \in R$ then $(b,a) \in R$ for all $a,b \in A$

A relation R on a set A is antisymmetric if $(a,b) \in R$ then $a=b$ all $a,b \in A$

$1,2 \in A$ $(1,2) \in R$ but $(2,1) \notin R$

Also $(1,3), (3,1) \in R$ but $1 \neq 3$

o 9.3 – 2, 4, 8, 24

2)

9.3

2. Represent each of these relations on $\{1, 2, 3, 4\}$ with a matrix (with the elements of this set listed in increasing order).

a) $\{(1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4)\}$

b) $\{(1, 1), (1, 4), (2, 2), (3, 3), (4, 1)\}$

c) $\{(1, 2), (1, 3), (1, 4), (2, 1), (2, 3), (2, 4), (3, 1), (3, 2), (3, 4), (4, 1), (4, 2), (4, 3)\}$

d) $\{(2, 4), (3, 1), (3, 2), (3, 4)\}$

a)
$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

b)
$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

c)
$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

d)
$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

4)

a) since the $(1,1)$ th entry is 1, $(1,1)$ is in the relation.

Since $(1,3)$ th is a 0, $(1,3)$ is not in the relation. Continuing in this way, the relation contains

$(1,1), (1,2), (1,4), (2,1), (2,3), (3,2), (3,3), (3,4), (4,1), (4,3), (4,4)$

b) $(1,1) (1,2) (1,3) (2,2) (3,3) (3,4) (4,1) (1,4)$

c) $(1,2) (1,4) (2,1) (2,3) (3,2) (3,4) (4,1) (4,3)$

8)

a) Since some 1's and some 0's on the main diagonal, this relation is neither reflexive nor irreflexive. Since the matrix is symmetric, the relation is symmetric. The relation is not antisymmetric by looking at the positions (1,2) and (2,1).

Lastly, the relation is not transitive; for example, the 1's in positions (1,2) and (2,3) would require a 1 in position (1,3) if the relation were to be transitive.

b) Since there are all 1's on the main diagonal, this relation is reflexive and not irreflexive. Since the matrix is not symmetric, the relation is not symmetric. The relation is antisymmetric since there are never two 1's symmetrically placed with respect to the main diagonal. Lastly, the Boolean square of this matrix is not itself, so the relation is not transitive.

c) Since there are all 0's on the main diagonal, this relation is not reflexive but is irreflexive. Since the matrix is symmetric, the relation is symmetric. The relation is not antisymmetric — look at positions (1,2) and (2,1). Lastly, the Boolean square of this matrix has 1 in position (1,1) so the relation is not transitive.

24. Give an example of an asymmetric relation on the set of all people.

There are 3 edges and three loops in graphs with a,b,c

An edge is initiated at vertex a (a,c)

An edge is initiated at vertex a (b,a)

An edge is initiated at vertex a (b,c)

$R = \{(a,a)(a,c)(b,a)(b,b)(b,c)(c,c)\}$

o 9.5 – 2

2.

a) This is an equivalence relation.

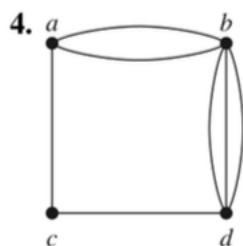
b) This is an equivalence relation.

c) This is not an equivalence relation since it need not be transitive.

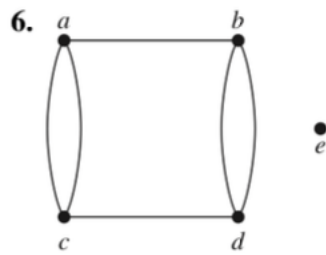
d) This is not an equivalence relation since it is clearly not transitive.

e) This is not transitive.

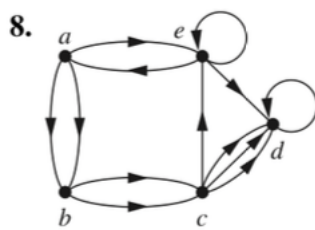
o 10.1 – 4, 6, 8, 18, 32



4) This is a multigraph; the edges are undirected, and there are no loops, but there are parallel edges.



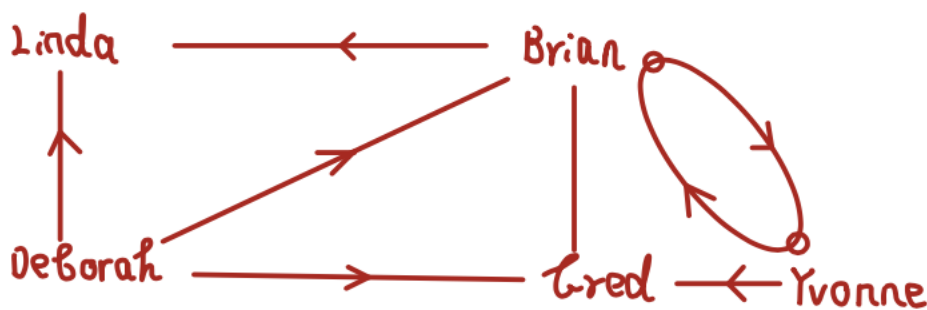
6) This is a multigraph; the edges are undirected, and there are no loops, but there are parallel edges.



8) This is directed multigraph; the edges are directed, and there are parallel edges.

18. Who can influence Fred and whom can Fred influence in the influence graph in Example 2? Fred influences Brian since there is an edge from Fred to Brian. Yvonne and Deborah influence Fred since there are edges from these vertices to Fred.

18. Who can influence Fred and whom can Fred influence in the influence graph in Example 2?



32.

The model says that the statements for which there are edges to S_6 must be executed before S_6 , namely the statements S_1 , S_2 , S_3 , and S_4 .

o 10.2 – 6, 14, 20, 58

6) Show that the sum, over the set of people at a party, of the number of people a person has shaken hands with, is even. Assume that no one shakes his or her own hand.

Consider there are five persons a, b, c, d, e in party

Suoops a and b b and c c and d c and e and a and e have shaken hands

Number of people a has shaken hands is equal to 2 (b, and e)

Number of people b has shaken hands is equal to 3 (a, c, and d)

Number of people c has shaken hands is equal to 3 (b, d, and e)

Number of people d has shaken hands is equal to 2 (b, and c)

Number of people e has shaken hands is equal to 2 (a, and c)

$$2+3+3+2+2=12$$

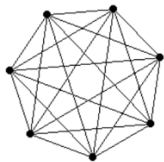
Even

14. What does the degree of a vertex in the Hollywood graph represent? What does the neighborhood of a vertex represent? What do the isolated and pendant vertices represent?

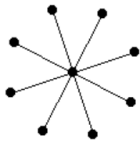
The neighborhood is each of the actors that a provide actor has worked with an isolated vertex is an actor who has not worked with any of the other actors A pendant actor is an actor who has only worked with one other actor

20.

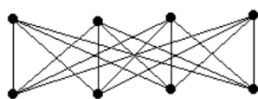
a) This graph has 7 vertices, with an edge joining each pair of distinct vertices.



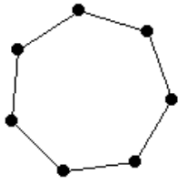
b) This graph is the complete bipartite graph on parts of size 1 and 8; we have put the part of size 1 in the middle.



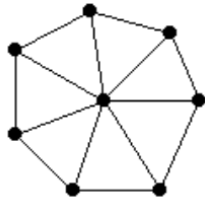
c) This is the complete bipartite graph with 4 vertices in each part.



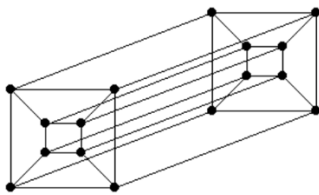
d) This is the 7-cycle



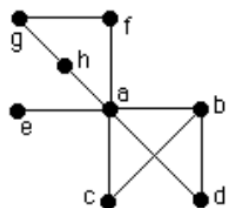
e) the 7-wheel is the 7-cycle with an extra vertex joined to the other 7 vertices.



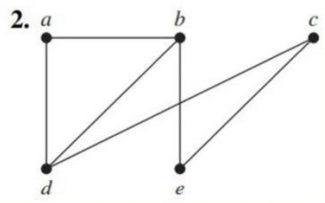
f) We take two copies of Q_3 and join corresponding vertices



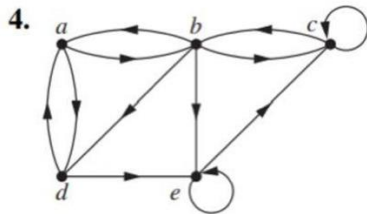
58. The union is shown here. The only common vertex is a, so we have reoriented the drawing so that the pieces will not overlap.



o 10.3 – 2, 4, 6, 8, 12, 14, 26, 38, 42

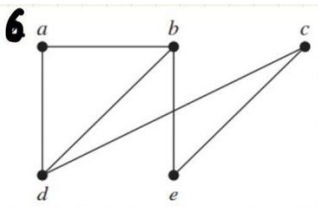


Vertex	Adjacent Vertices
a	b, d
b	a, d, e
c	d, e
d	a, b, c
e	b, c



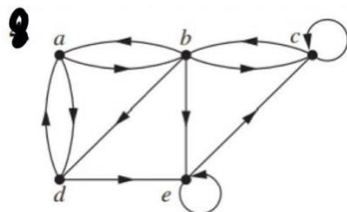
Initial Vertex	Terminal Vertices
a	b, d
b	a, c, d, e
c	b, c
d	a, e
e	c, e

6.



$$\begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

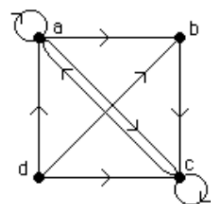
8.



$$\begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

12.

This graph is directed, since the matrix is not symmetric



14.

$$\begin{bmatrix} 0 & 3 & 0 & 1 \\ 3 & 0 & 1 & 0 \\ 0 & 1 & 0 & 3 \\ 1 & 0 & 3 & 0 \end{bmatrix}$$

26. Each column represents an edge; the two 1's in the column are in the rows for the endpoints of the edge.

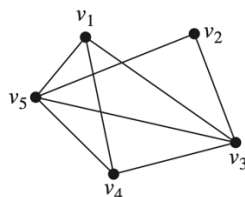
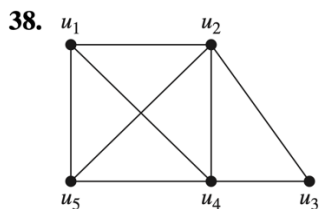
$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

Exercise 1

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

Exercise 2

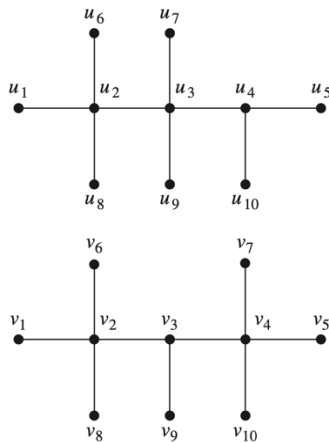
38.



These two graphs are isomorphic. Each consists of a K_4 with a fifth vertex adjacent to two of the vertices in the K_4 . Many isomorphisms are possible. $f(u_1) = v_1$, $f(u_2) = v_3$, $f(u_3) = v_2$, $f(u_4) = v_5$, and $f(u_5) = v_4$,

42.

42.



These graphs are not isomorphic since the degrees of the vertices are not the same.

10.4 Connectivity

10.4 – 2, 8, 14

2) a) This is a path of length 4, but it is not a circuit since it ends at a vertex other than the one at which it began. It is simple since no edges are repeated.

b) This is a path of length 4 which is a circuit. It is not simple since it uses an edge more than once.

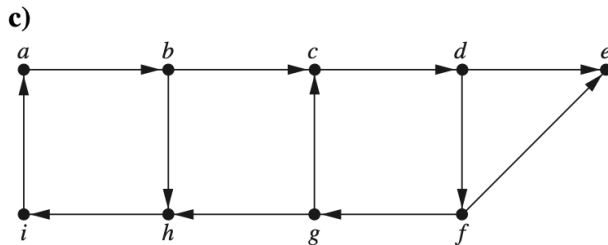
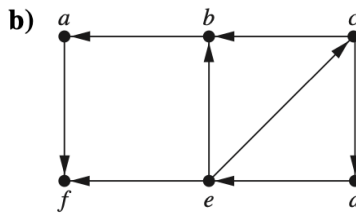
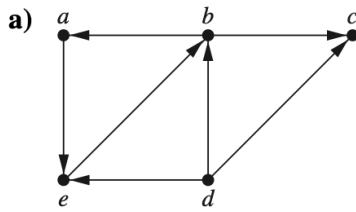
c) This is not a path since there is no edge from d to b.

d) This is not a path since there is no edge from b to d.

8). What do the connected components of a collaboration graph represent?

A connected component of a collaboration graph represents a maximal set of people with the property that for any two of them, we can find a string of joint works that takes us from one to the other. The word maximal implies that nobody else can be added to this set of people without destroying this property.

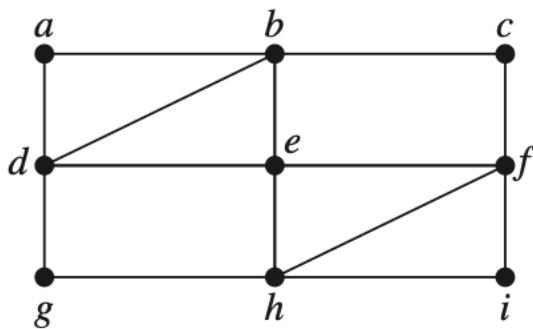
14. Find the strongly connected components of each of these graphs.



- a) The cycle baeb guarantees that these three vertices are in one strongly connected component. Since there is no path from c to any other vertex, and there is no path from any other vertex to d, these two vertices are in strong components by themselves. Therefore the strongly connected components are {a,b,e} {c} and {d}
- b) The cycle cdec guarantees that these three vertices are in one strongly connected component. The vertices a,b, and f are in strong components by themselves since there are no paths both to and from each of these to every other vertex. Therefore the strongly connected components are {a} {b} {c,d,e} and {f}
- c) The cycle abcd fghia guarantees that these eight vertices are in one strongly connected component. Since there is no path from e to any other vertex, this vertex is in a strong component by itself. Therefore the strongly connect components are {a,b,c,d,f,g,h,i} and {e}

10.5 Euler and Hamilton Paths

2.



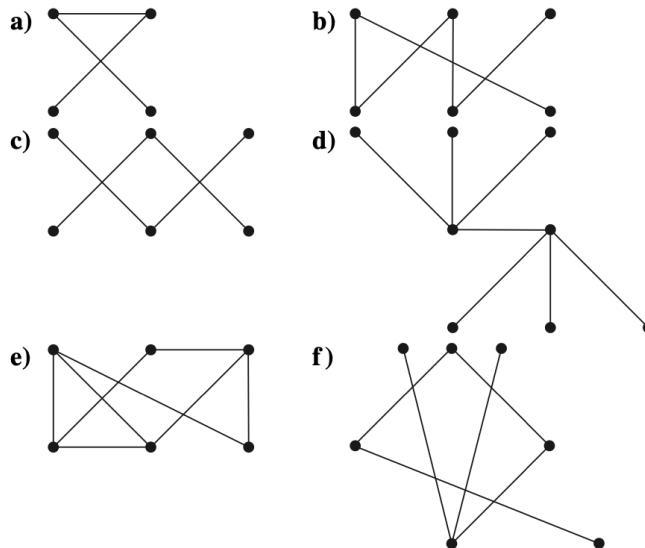
All the vertex degrees are even, so this is an Euler circuit. We can find one by trails and error, or by using Algorithm 1. One such circuit is a, b, c, f, i, h, g, d, e, h, f, e, b, d, a

11.1 Introduction to Trees

o 11.1 – 2, 4, 6, 10, 18, 20

2. Which of these graphs are trees?

2. Which of these graphs are trees?



- a) This is a tree since it is connected and has no simple circuits
- b) This is a tree since it is connected and has no simple circuits
- c) This is not a tree, since it is not connected
- d) This is a tree since it is connected and has no simple circuits
- e) This is not a tree since it has a simple circuit
- f) This is a tree since it is connected and has no simple circuits

6. Is the rooted tree in Exercise 4 a full m -ary tree for some positive integer m ?

This is not a full m -ary tree for any m . It is an m -ary tree for all $m \geq 3$, since each vertex has at most 3 children, but since some vertices have 3 children, while others have 1 or 2, it is not full for any m .

10.

10. Draw the subtree of the tree in Exercise 4 that is rooted at

a) a . b) c . c) e .

- a) The subtree rooted at a is the entire tree since a is the root.
- b) The subtree rooted at c consists of just the vertex c .
- c) The subtree rooted at e consists of e , j , and k , and the edges ej and ek .

18. How many vertices does a full 5-ary tree with 100 internal vertices have?

By Theorem 4(ii),

$$M = 5, j = 100$$

$$n = mj + 1 = 5 \cdot 100 + 1 = 501$$

Thus full-5 ary tree with 100 internal vertices has 501 vertices

20. How many leaves does a full 3-ary tree with 100 vertices have?

By Theorem 4(i),

$$m = 3, n = 100$$

$$l = \frac{(m - 1)n + 1}{m}$$

$$l = \frac{2 \cdot 100 + 1}{3} = \frac{201}{3} \text{ leaves} = 67 \text{ leaves}$$

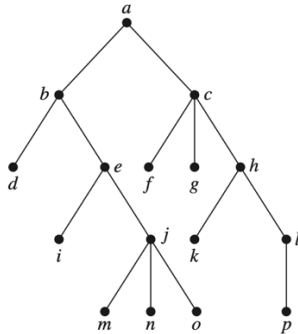
Tree Traversal

o 11.3 – 8, 12, 14, 16, 22, 24

For number 7, in preorder, the root comes first, then the left subtree in preorder, then the right subtree in preorder. Thus the preorder is a , followed by the vertices of the left subtree (the one rooted at b) in preorder, then c . Recursively, the preorder in the subtree rooted at b is b , followed by d , followed by the vertices in the subtree rooted at e in preorder, namely e, f, g . Putting this all together, we obtain the answer a, b, d, e, f, g, c .

8)

8.

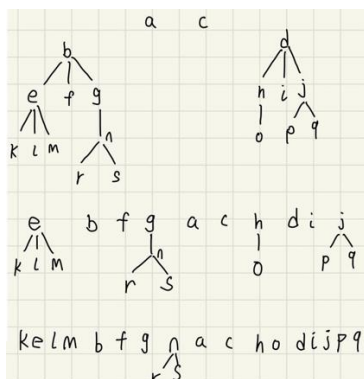


The only difference from number 7 is that some vertices have more than two children: after listing such a vertex, we list the vertices of its subtrees, in preorder, from left to right. The answer is $a, b, d, e, i, j, m, n, o, c, f, g, h, k, l, p$

12. In which order are the vertices of the ordered rooted tree in Exercise 9 visited using an inorder traversal?

For problem 11, Inorder traversal requires that the left-most subtree be traversed first, then the root, then the remaining subtrees (if any) from left to right. Applying this principle, we see that the list must start with the left subtree in inorder. To find this, we need to start with *its* left subtree, namely d . Next comes the root of that subtree, namely b , and then the right subtree in inorder. This is i , followed by the root e , followed by the subtree rooted at j in inorder. This latter listing is m, j, n, o . We continue in this manner, ultimately obtaining: $d, b, i, e, m, j, n, o, a, f, c, g, k, h, p, l$.

This is similar to 11, the answer is $k, e, l, m, b, f, r, n, s, g, a, c, o, h, d, i, p, j, q$.



14. In which order are the vertices of the ordered rooted tree in Exercise 8 visited using a postorder traversal?

The procedure is the same as in Exercise 13, except that some vertices have more than two children here: before listing such a vertex, we list the vertices of its subtrees, in postorder, from left to right. The answer is d,i,m,n,o,j,e,b,f,g,k,p,l,h,c,a.

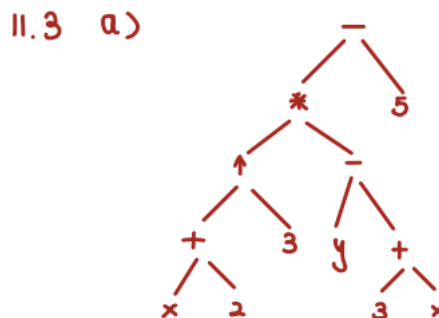
16.

16. a) Represent the expression $((x + 2) \uparrow 3) * (y - (3 + x)) - 5$ using a binary tree.

Write this expression in

- b)** prefix notation.
- c)** postfix notation.
- d)** infix notation.

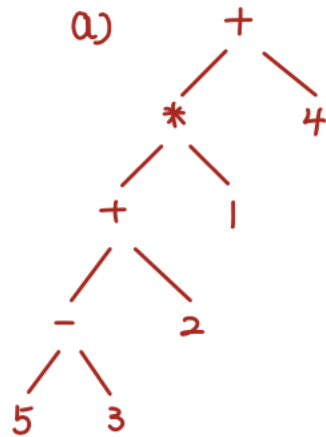
- a) We build the tree from the top down while analyzing the expression by identifying the outermost operation at each stage. The outermost operation in this expression is the final subtraction. Therefore the tree has $-$ at its root, with the two operands as the subtrees at the root. The right operand is clearly 5, so the right child of the root is 5. The left operand is the result of a multiplication, so the left subtree has $*$ as its root. We continue recursively in this way until the entire tree is constructed.



- b) We can read off the answer from the picture we have just drawn simply by listing the vertices of the tree in preorder: first list the root, then the left subtree in preorder, then the right subtree in preorder. Therefore the answer is $- * \uparrow + x 2 3 - y + 3 x 5$.
- c) We can read off the answer from the picture we have just drawn simply by listing the vertices of the tree in postorder: $x 2 + 3 \uparrow y 3 x + - * 5 -$.
- d) The infix expression is just the given expression, fully parenthesized: $((((x+2) \uparrow 3) * (y - (3+x))) - 5)$. This corresponds to traversing the tree inorder, putting in a left parenthesis whenever we go down to a left child and putting in a right parenthesis whenever we come up from a right child.

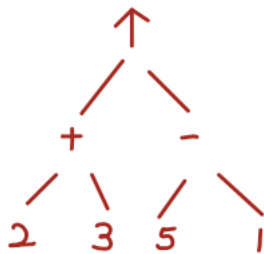
22) a) + * + - 5 3 2 1 4 → In infix form: (((5-3)+2)*1)+4

11.3) 22)



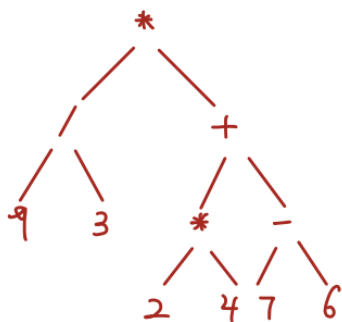
b) The infix expressions are therefore $((2+3) \uparrow (5-1))$

b)



c) $((9/3)*((2*4)+(7-6)))$

c)



24.

$$a) 5(2 \ 1 \ -) - 3 \ 1 \ 4 \ + \ + \ * = (5 \ 1 \ -) \ 3 \ 1 \ 4 \ + \ + \ * = 4 \ 3 \ (1 \ 4 \ +) \ + \ * = 4 \ (3 \ 5 \ +) \ * = (4 \ 8 \ *) = 32$$

$$b) (9 \ 3 \ /) \ 5 \ + \ 7 \ 2 \ - \ * = (3 \ 5 \ +) \ 7 \ 2 \ - \ * = 8 \ (7 \ 2 \ -) \ * = (8 \ 5 \ *) = 40$$

$$c) (3 \ 2 \ *) \ 2 \ \uparrow \ 5 \ 3 \ - \ 8 \ 4 \ / \ * \ - = (6 \ 2 \ \uparrow) \ 5 \ 3 \ - \ 8 \ 4 \ / \ * \ - = 36(5 \ 3 \ -) \ 8 \ 4 \ / \ * \ - = 36 \ 2(8 \ 4 \ /) \ * \ - = 36 \ (2 \ 2 \ *) \ - = (36 \ 4 \ -) = 32$$

o 11.4 Spanning Trees – 4, 8, 10, 14, 16

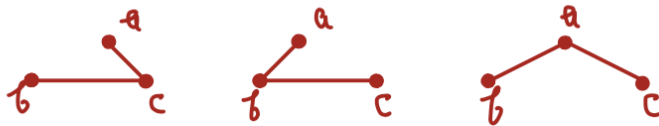
4. We can remove these edges to produce a spanning tree.

{a,i}, {b,i}, {b,j}, {c,d}, {c,j}, {d,e}, {e,j} {f,i}, {f,j} and {g,i}.

8. We can remove any one of the three edges to produce a spanning tree. The tree are therefore the ones below:

Graph of the tree:

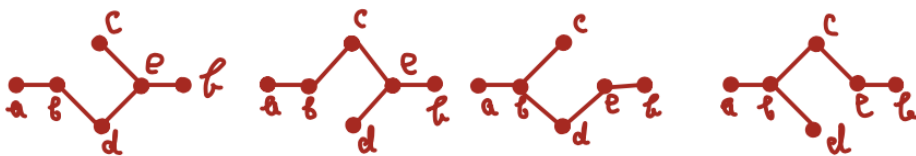
11.4) 8)



10. We can remove any one of the four edges in the middle square to produce a spanning tree.

Graph of the tree:

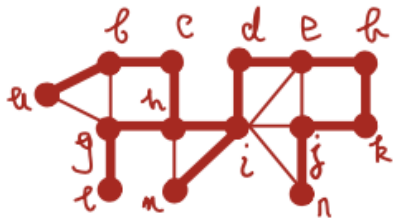
11.4) 10)



14. The tree is shown in heavy lines. It is produced by starting at a and continuing as far as possible without backtracking, choosing the first unused vertex (in alphabetical order) at each point. When the path reaches vertex l, we need to back track. Backtracking to h, we can then form the path all the way to n without further backtracking. Finally we backtrack to vertex i to pick up vertex m.

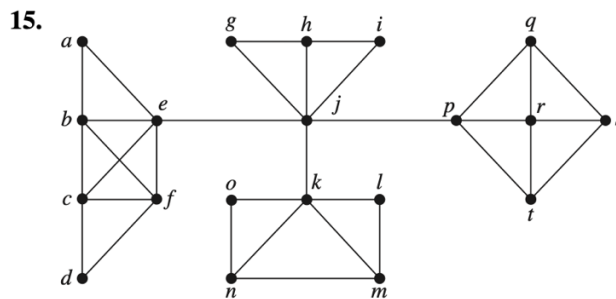
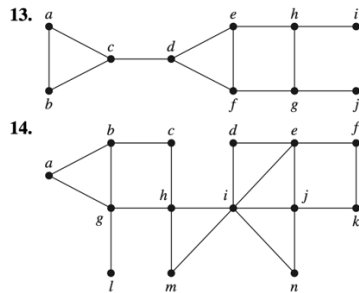
Graph of the tree: continue ...

11.4) 14)



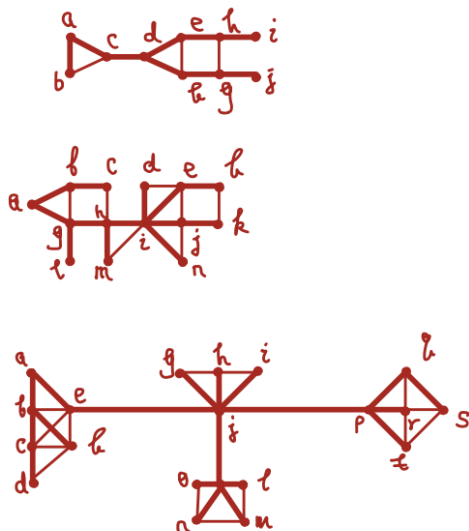
16.

16. Use breadth-first search to produce a spanning tree for each of the simple graphs in Exercises 13–15. Choose a as the root of each spanning tree.



Graphs:

11.4) 16)



o 12.1 – 2, 6, 10, 12, 24, 28

2. a) since $x * 1 = x$, the only solution is $x=0$

b) since $0+0 = 0$ and $1+1 = 1$, the only solution is $x = 0$

c) since the equation holds for all x , there are two solutions $x = 0$ and $x = 1$

d) since either x or \bar{x} must be 0, no matter what x is, there are NO solutions

6. next page

6) a)

x	y	z	\bar{z}
1	1	1	0
1	1	0	1
1	0	1	0
1	0	0	1
0	1	1	0
0	1	0	1
0	0	1	0
0	0	0	1

b)

x	y	z	\bar{x}	$\bar{x}y$	\bar{y}	$\bar{y}z$	$\bar{x}y + \bar{y}z$
1	1	1	0	0	0	0	0
1	1	0	0	0	0	0	0
1	0	1	0	0	1	0	0
1	0	0	0	0	1	0	0
0	1	1	1	1	0	0	1
0	1	0	1	1	0	0	1
0	0	1	1	0	1	1	1
0	0	0	1	0	1	0	0

c)

x	y	z	\bar{y}	$x\bar{y}z$	$x\bar{y}\bar{z}$	$\bar{x}yz$	$x\bar{y}z + \bar{x}yz$
1	1	1	0	0	0	0	0
1	1	0	0	0	0	0	0
1	0	1	1	1	0	0	1
1	0	0	1	0	1	0	1
0	1	1	0	0	0	1	1
0	1	0	0	0	0	0	0
0	0	1	1	0	0	0	0
0	0	0	1	0	0	0	0

d)

x	y	z	\bar{x}	\bar{y}	\bar{z}	xz	$\bar{x}\bar{z}$	$xz + \bar{x}\bar{z}$	$\bar{y}(xz + \bar{x}\bar{z})$
1	1	1	0	0	0	1	0	1	0
1	1	0	0	0	1	0	0	0	0
1	0	1	0	1	0	1	0	1	0
1	0	0	0	1	1	0	0	0	0
0	1	1	1	0	0	0	1	1	0
0	1	0	1	0	1	0	1	1	0
0	0	1	1	1	0	0	0	0	0
0	0	0	1	1	1	0	0	0	0

10. There are 2^{2^n} different Boolean functions of degree n , so the answer is
 $2^{2^7} = 2^{128} \approx 3.4 \times 10^{38}$

12. The only way for the sum to have the value 1 is for one of the summands to have the value 1, since $0+0+0=0$.

Each summand is 1 iff the two variables in the product making up the summand are both 1.

24. a) since $0 \oplus 0 = 0$ and $1 \oplus 0 = 1$, this expression simplifies to x

b) since $0 \oplus 1 = 1$ and $1 \oplus 1 = 0$, this expression simplifies to \bar{x}

c) looking at the definition, we see that $x \oplus x = 0$ for all x

d) Always equals to 1

28. In each case we simply change each 0 to a 1 and vice versa, and change all the sums to products and vice versa

a) xy b) $\bar{x} + \bar{y}$ c) $(x+y+z)(\bar{x} + \bar{y} + \bar{z})$ d) $(x+\bar{z})(x+1)(\bar{x}+0)$

o 12.2 – 2, 4, 6

2.

a) We can rewrite this as $F(x,y) = \bar{x} * 1 + \bar{y} * 1 = \bar{x}(y + \bar{y}) + y(x + \bar{x})$

Using commutative and idempotent laws to simplify this to

$$\bar{x}y + \bar{x}\bar{y} + xy$$

b) This is already in sum-of-products form

c) We need to write the sum of all products $\Rightarrow xy + x\bar{y} + \bar{x}y + \bar{x}\bar{y}$

d) $F(x,y) = 1 \cdot \bar{y} = (x + \bar{x})\bar{y} = x\bar{y} + \bar{x}\bar{y}$.

4.

$$4) a) \bar{x}yz + \bar{x}y\bar{z} + \bar{x}\bar{y}z + \bar{x}\bar{y}\bar{z}$$

$$b) x\bar{y}z + x\bar{y}\bar{z} + \bar{x}yz + \bar{x}y\bar{z} + \bar{x}\bar{y}z + \bar{x}\bar{y}\bar{z}$$

$$c) \bar{x}\bar{y}z + \bar{x}\bar{y}\bar{z}$$

$$d) x\bar{y}\bar{z} + x\bar{y}z + x\bar{y}\bar{z} + \bar{x}yz + \bar{x}y\bar{z} + \bar{x}\bar{y}z + \bar{x}\bar{y}\bar{z}$$

b) $1 + 5 + 10 = 16$ terms

$x_1, x_2, x_3, x_4, x_5 +$

$x_1, x_2, x_3, x_4, \bar{x}_5 + x_1, x_2, x_3, \bar{x}_4, x_5 + x_1, x_2, \bar{x}_3, x_4, x_5 + x_1, \bar{x}_2, x_3, x_4, x_5 + \bar{x}_1, x_2, x_3, x_4, x_5$
 $+ x_1, x_2, x_3, \bar{x}_4, \bar{x}_5 + x_1, x_2, \bar{x}_3, x_4, \bar{x}_5 + x_1, x_2, \bar{x}_3, \bar{x}_4, x_5 + x_1, \bar{x}_2, x_3, x_4, \bar{x}_5 + x_1, \bar{x}_2, x_3, \bar{x}_4, x_5$
 $+ x_1, \bar{x}_2, \bar{x}_3, x_4, x_5 + \bar{x}_1, x_2, x_3, x_4, \bar{x}_5 + \bar{x}_1, x_2, x_3, \bar{x}_4, x_5 + \bar{x}_1, \bar{x}_2, x_3, x_4, x_5 + \bar{x}_1, \bar{x}_2, x_3, \bar{x}_4, x_5$

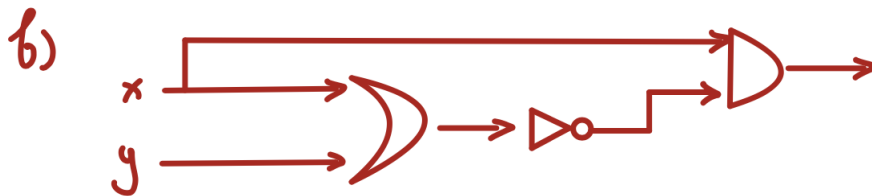
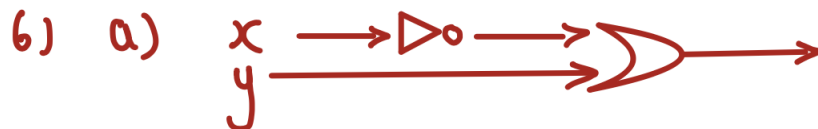
o 12.3 – 2, 4, 6, 16(a)(b)(c)

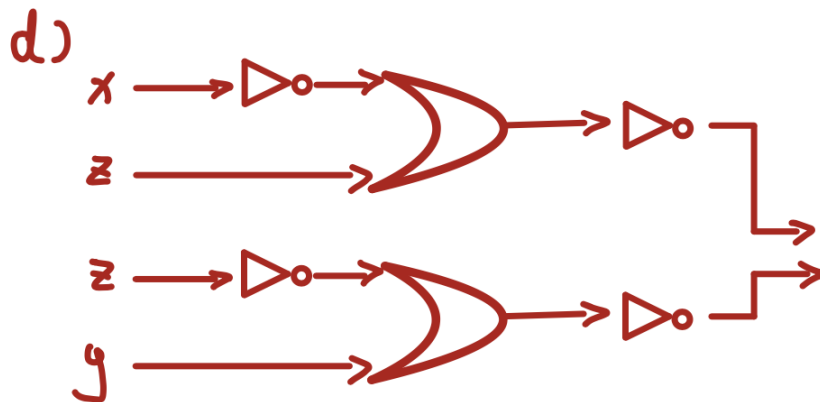
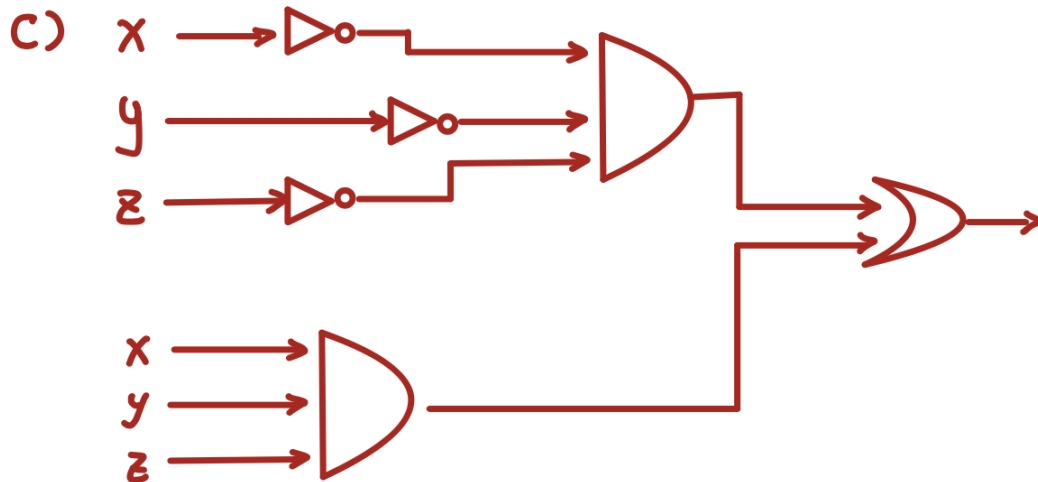
12.3 Logic Gates

2) The input to the AND gate are \bar{x} and \bar{y} . The output is then passed through the inverter. Therefore the final output is $\overline{(\bar{x}\bar{y})}$.

4) Output: $\overline{(\bar{x}yz)}(\bar{x} + y + \bar{z})$

6.

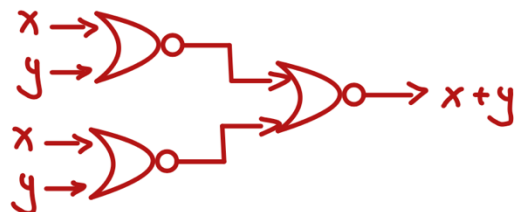




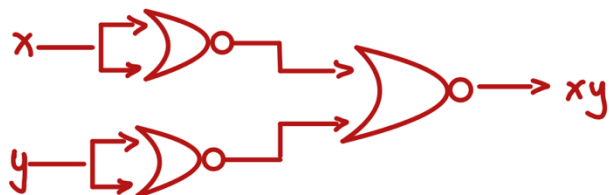
16) a) w/ a NOR gate in place of a NAND gate, since
 $\bar{x} = x \mid x = x \downarrow x$



b) since $x + y = (x \downarrow y) \downarrow (x \downarrow y)$



c) $xy = (x \downarrow x) \downarrow (y \downarrow y)$



d) we use the representation $x \oplus y = \overline{(x+y)(\overline{xy})}$
 $= \overline{(\overline{x+y} + xy)}$
 $= (x \downarrow y) \downarrow (xy)$
 $= (x \downarrow y) \downarrow ((x \downarrow x) \downarrow (y \downarrow y)).$

