ICS 6B Sample Midterm 3

October 30th, 2023

Name: _____

UCI NetID : | | | | | | | | |

(**alpha-numeric; NOT your student ID**)

- Read the instructions of each question carefully.
- Place your answers in the boxed regions. Writing outside of the boxes will not be considered as part of your final answer.
- Your scratch work should stay on the exam booklet. This means no extra scratch paper– even if you have work on scratch paper, it won't be considered.
- Show your work. Where available, partial credit will be given on the work shown.
- Write your name and UCI NetID on every page.
- This test is intended to take 40 minutes.
- This test is closed-book. Calculators and other electronics are not allowed.
- Please keep in mind the academic honesty guidelines.
- Good luck!

Seat

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F.1: Defining functions

ChatGPT is a famous artificial intelligence application that operates like a chatbot. Consider a function f defined on a GPT-powered chatbot that takes in a query and outputs a response. For the following questions, **select** the choice that best fits the description of the mapping above.

1. f is...
   - ○ a well-defined function.
   - ○ not a well-defined function.
   - ○ None of the above.
2. The domain of f is...
   - ○ the set of all English words.
   - ○ the set of all English sentences.
   - ○ the union of the set of all English sentences to any positive integer power.
   - ○ the set of all English letters.
   - ○ the powerset of all English letters.
   - ○ None of the above.
3. f is well-defined or not well-defined because...
   - ○ f is a function who has only a single element in the range.
   - ○ f is a function that maps different elements in the domain to different elements in the target.
   - ○ f is a function that maps to each element of the target.
   - ○ f returns a different target element every time.
   - ○ f is a function that maps to at least one element of the target.
   - ○ None of the above.

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F.2: Function Types

Select the properties that best describes the function.

1. Consider a city consisting of an infinite grid of city blocks. Let $\{U, D, L, R\}$ represent the set of cardinal directions, and $\{U, D, L, R\}^* = \cup_{i=0}\{U, D, L, R\}^i$ represent a list of these cardinal directions of any length. consider the function $f : \mathbb{Z}^2 \to \{U, D, L, R\}^*$ that maps a location on the city grid to the *shortest* path to get back home, which is $(0, 0)$. If there are multiple such shortest paths, the function returns one of the shortest paths, and returns the same path for the same input.

   - ○ One-to-one, but not onto.
   - ○ Onto, but not one-to-one.
   - ○ Bijective
   - ○ None of the above

2. Nero is designing the latest booking software for making reservations at restaurants. Let the set of all people in a restaurant be S, where $|S| = n$. You consider the function $f : \{x : x \in \mathbb{Z}^+ \text{ and } x \leq n\} \to \mathcal{P}(S)$, the function that maps a reservation id (which is an integer) to the group of people that made the reservation.

   - ○ One-to-one, but not onto.
   - ○ Onto, but not one-to-one.
   - ○ Bijective
   - ○ None of the above

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F.3: Inverse of a Function

Consider $f : \mathbb{Z}^+ \to \mathbb{Z}^+$. Let $f(x) = x^2$.

1. Is this function invertible? ☐ Yes ☐ No
2. What is $f^{-1}(4)$? **Select** the option that best fits.
   - ○ 2
   - ○ -2
   - ○ 1
   - ○ Not invertible.
   - ○ None of the above.

Consider $g : \{0,1\} \to \{0,1\}$ where $g(x) = x \uparrow k$ for some predetermined bit k.

1. Is this function invertible? ☐ Yes ☐ No
2. What is $f^{-1}(1)$? **Select** the option that best fits.
   - ○ 0
   - ○ 1
   - ○ 10
   - ○ 01
   - ○ Not invertible.
   - ○ None of the above.

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F.4: Function Composition

Consider the following functions:

- $f : \{0,1\}^4 \to \{0,1\}^4$, where we flip the leftmost bit of the string.
- $g : \{0,1\}^4 \to \{0,1\}^4$ where you set the rightmost bit of the bitstring to 1.

1. Of the following choices, **select** the choice that best describes $f \circ g$.

   ○ f
   ○ g
   ○ $g \circ f$
   ○ $h : \{0,1\}^4 \to \{0,1\}^3$ where you remove the leftmost bit.
   ○ Not well-defined.
   ○ None of the above.

2. **Select** all the properties that apply.

   ☐ $f \circ g$ is one-ot-one.
   ☐ $f \circ g$ is onto.
   ☐ the domain of $f \circ g$ has the same cardinality as the target of $f \circ g$.
   ☐ None of the above.

B.1: Functions Representations

Evaluate the following functions if $x = 1$, $y = 1$, $z = 1$.

1. $(\overline{x}y\overline{z} + x\overline{y}\overline{z} + \overline{x}y\overline{z})(y + z)$ ☐

2. $\overline{(x + y)}(xy + \overline{x}\overline{z})\overline{(y + \overline{z})}$ ☐

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B.2: DNF and CNF Forms

You are given the table for a Boolean function $f(x, y, z)$.

x	y	z	$f(x, y, z)$
1	1	1	0
1	1	0	1
1	0	1	1
1	0	0	0
0	1	1	0
0	1	0	1
0	0	1	1
0	0	0	1

Find an equivalent expression in DNF form for $f(x, y, z)$. Write *only* your answer in the box.

Find an equivalent expression in CNF form for $\overline{f}(x, y, z)$, i.e., the negation of f. Write *only* your answer in the box.

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B.4: Satisfiability

Suppose you are a train scheduler for the famous Kaihon Star Rail. The company you work for has two types of trains: express (E), and standard (S). You know that the trains can be scheduled either for the AM (A), PM (P), or the midnight (M) time slots.
If the standard train is scheduled for the AM time slot, then it is expressed by the Boolean variable SA, etc. If the express train is scheduled for the AM time slot, then it is expressed by the Boolean variable EA.
You are told that

All trains should be scheduled for some time slot.

Express the above statement in Boolean logic. Write *only* your answer in the blank.

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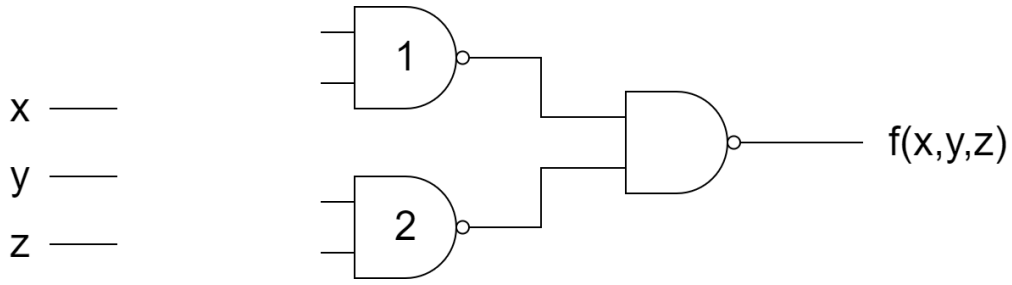
B.5: Circuit

You are given the following Boolean function.

x	y	z	$f(x, y, z)$
1	1	1	1
1	1	0	1
1	0	1	1
1	0	0	1
0	1	1	1
0	1	0	0
0	0	1	0
0	0	0	0



Consider the above circuit consisting of 3 NAND gates. For each gate, **select** the inputs that would complete the circuit.

Gate 1

- ○ x, x
- ○ x, y
- ○ y, y
- ○ y, z
- ○ z, z
- ○ z, x

Gate 2

- ○ x, x
- ○ x, y
- ○ y, y
- ○ y, z
- ○ z, z
- ○ z, x

(scratch space)