CSCI 190 Discrete Mathematics Applied to Computer Science
Exam 1

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Read these instructions before proceeding.

- Closed book. Closed notes. You can use calculator.
- You have **80 minutes** to complete this exam.
- No questions will be answered during the exam or immediately afterwards. Answer each question as best you can. Partial credit will be awarded for reasonable efforts. If a question contains an ambiguity or a misprint, then say so in your answer, providing the answer to a reasonable interpretation of the question; give your assumptions.
- Answer the problems on the blank spaces provided for each problem.
- **Box your answers.**

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	Q14	Q15	Q16	Q17	Q18	Total
(6)	(7)	(7)	(6)	(6)	(8)	(4)	(4)	(4)	(4)	(6)	(6)	(5)	(6)	(5)	(5)	(6)	(5)	(100)

1. (6 pts) Determine whether the proposition is ***TRUE*** or ***FALSE***. No justifications needed.

T F

$\boxed{F}$ a) **1 + 11 = 12 if and only if 2 + 2 = 22.** (2 pts)

T T

$\boxed{T}$ b) **If it is raining, then it is raining.** (2 pts)

T F

$\boxed{F}$ c) **If 2 > -1, then 7 = 9.** (2 pts)

2. (7 pts) Determine whether **(p → q) ∧ (¬p → q) ≡ q** using truth table.

p	q	¬p	p → q	¬p → q	(p → q) ∧ (¬p → q)
T	T	F	T	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	F	T	T	F	F

$\boxed{\therefore (p \to q) \land (\neg p \to q) \equiv q}$

3. (7 pts) Prove that **(¬p ∧ (¬q → p)) → q** is a tautology using propositional equivalence and the laws of logic.

$(\neg p \land (\neg q \to p)) \to q$

$(\neg p \land (\neg(\neg q \lor) p)) \to q$

$(\neg p \land (q \lor p)) \to q$

$\neg(\neg p \land (q \lor p)) \lor q$

$\neg(\neg p) \lor \neg(q \lor p) \lor q$

$p \lor \neg(q \lor p) \lor q$

$(p \lor (\neg q \land \neg p)) \lor q$

$((p \lor \neg q) \land (p \land \neg p)) \lor q$

$((p \lor \neg q) \land T) \lor q$

$(p \lor \neg q) \lor q$

$p \lor (\neg q \lor q)$

$p \lor T$

T

$\boxed{\therefore \text{ it is a tautology}}$

4. *(6 pts)* Write the contrapositive, converse, and inverse of the following:

If you give it a try, then you will be good.
p q

a) **contrapositive** (2 pts)
$\neg q \rightarrow \neg p$

If you are not good, then you will not give it a try.

b) **converse** (2 pts)
$q \rightarrow p$

If you are good, then you will give it a try

c) **inverse** (2 pts)
$\neg p \rightarrow \neg q$

If you do not give it a try, then you will not be good

5. *(6 pts)* Suppose the variable x represents people, and
F(x): x is friendly   T(x): x is tall   A(x): x is angry.

Write the statement using these predicates and any needed quantifiers.

a) **Some people are not angry**. (3 pts)

$\exists x \quad \neg A(x)$

b) **All tall people are friendly**. (3 pts)

$\forall x \ (T(x) \rightarrow F(x))$

6. *(8 pts)* Consider the following theorem:
"**if x and y are odd integers, then x + y is even**".
Give a direct proof of this theorem.

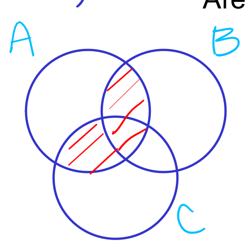
suppose x and y are odd integers
then $x = 2k+1$ for some integer k
and $y = 2t+1$ for some integer t

$x+y = 2k+1+2t+1$ $\quad x+y = 2(k+t+1)$

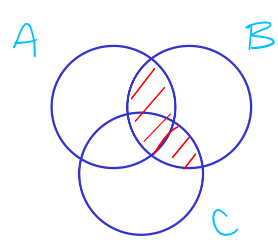
$x+y = 2k+2t+2$ $\quad \boxed{\therefore\ x+y \text{ is even}}$

7. *(4 pts)* Draw two Venn diagrams for **A ∩ (B U C)** and **B ∩ (A U C).**
Are they the same?

A ∩ (B ∪ C)



B ∩ (A ∪ C)



8. *(4 pts)* determine whether the given set is the power set of some set. (Answer "**Yes**" or "**No**").
If the set is a power set, **give the set** of which it is a power set.

a) { Ø, {Ø}, {b}, {Ø, b}} (2 pts)

Yes, { Ø, b }

b) {{Ø}, {a}, {b}, {a,b}} (2 pts)

No

9. (4 pts) Just answer "***yes***" or "***no***" in the box. No justifications needed.

Yes (a) Suppose f:N ⟶ N has the rule f(n) = 2n + 1. Determine whether f is onto. (1 pts)

Yes (b) Suppose f:N ⟶ N has the rule f(n) = 2n + 1. Determine whether f is 1-1. (1 pts)

No (c) Suppose f:Z ⟶ Z has the rule f(n) = $2n^2 - 1$. Determine whether f is 1-1. (1 pts)

No (d) Suppose f:Z ⟶ Z has the rule f(n) = $2n^2 - 1$. Determine whether f is onto Z. (1 pts)

10. (4 pts) Find $\mathbf{a_n}$ (a formula that generates the following sequence a_1, a_2, a_3 . . . .)

a) 20, 24, 28, 32, 36, . . . (2 pts) $a_n = 16 + 4n$

b) -1, 2, -4, 8, -16, 32, . . . (2 pts) $a_n = (-1)^n \cdot 2^{(n-1)}$

11. (6 pts) Suppose **A =** $\begin{bmatrix} 0&0&1 \\ 0&1&1 \\ 1&1&0 \end{bmatrix}$ and **B =** $\begin{bmatrix} 0&1&0 \\ 0&1&1 \\ 1&0&0 \end{bmatrix}$

Find

(a) the ***join*** of A and B.

A ∨ B or

$$A \vee B = \begin{bmatrix} 0\vee 0 & 0\vee 1 & 1\vee 0 \\ 0\vee 0 & 1\vee 1 & 1\vee 1 \\ 1\vee 1 & 1\vee 0 & 0\vee 0 \end{bmatrix} = \begin{bmatrix} 0&1&1 \\ 0&1&1 \\ 1&1&0 \end{bmatrix}$$

(b) the ***meet*** of A and B.

A ∧ B and

$$A \wedge B = \begin{bmatrix} 0\wedge 0 & 0\wedge 1 & 1\wedge 0 \\ 0\wedge 0 & 1\wedge 1 & 1\wedge 1 \\ 1\wedge 1 & 1\wedge 0 & 0\wedge 0 \end{bmatrix} = \begin{bmatrix} 0&0&0 \\ 0&1&1 \\ 1&0&0 \end{bmatrix}$$

(c) the ***Boolean product*** of A and B.

$$A \odot B = \begin{bmatrix} (0\wedge 0)\vee(0\wedge 0)\vee(1\wedge 1) & (0\wedge 1)\vee(0\wedge 1)\vee(1\wedge 0) & (0\wedge 0)\vee(0\wedge 1)\vee(1\wedge 0) \\ (0\wedge 0)\vee(1\wedge 0)\vee(1\wedge 1) & (0\wedge 1)\vee(1\wedge 1)\vee(1\wedge 0) & (0\wedge 0)\vee(1\wedge 1)\vee(1\wedge 0) \\ (1\wedge 0)\vee(1\wedge 0)\vee(0\wedge 1) & (1\wedge 1)\vee(1\wedge 1)\vee(0\wedge 0) & (1\wedge 0)\vee(1\wedge 1)\vee(0\wedge 0) \end{bmatrix}$$

$$= \begin{bmatrix} 0\vee 0\vee 1 & 0\vee 0\vee 0 & 0\vee 0\vee 0 \\ 0\vee 0\vee 1 & 0\vee 1\vee 0 & 0\vee 1\vee 0 \\ 0\vee 0\vee 0 & 1\vee 1\vee 0 & 0\vee 1\vee 0 \end{bmatrix} = \begin{bmatrix} 1&0&0 \\ 1&1&1 \\ 0&1&1 \end{bmatrix}$$

12. (6 pts)

Show (step by step) how the binary search algorithm searches for **43** in the following list:

let a = 1 5 8 15 21 35 43.

index = 0 1 2 3 4 5 6

start = 0

end = 6

middle = $\frac{\text{start} + \text{end}}{2}$

while start ≤ end

1st iteration middle = $\frac{0+6}{2} = 3$

$a_3 = 15$, $15 < 43$

start = middle + 1 = 4

2nd iteration middle = $\frac{4+6}{2} = 5$

$a_5 = 35$, $35 < 43$

start = middle + 1 = 6

3rd iteration middle = $\frac{6+6}{2} = 6$

$a_6 = 43$

$\boxed{\therefore\ 43 \text{ is found at index } 6.}$

13. (5 pts) Arrange the following functions in a list so each is ***big-O*** of the next one in the list. ***No justifications needed***.

$n^3 + 7n^2 - 1$, $\log n$, n^3, $n^4 \log n$, 2^n, 1111

$\boxed{1111 \rightarrow \log(n) \rightarrow n^3 \rightarrow n^3+7n^2-1 \rightarrow n^4 \log(n) \rightarrow 2^n}$

14. (6 pts)

(a) Give the ***best-case*** analysis of a linear search of a list of size n (counting the number of comparisons). (3 pts) The best-case of a linear search of a list of size n is that the search term is the first one in the list.

$\boxed{\therefore \text{ it is } 1.}$

(b) Give the ***worst-case*** analysis of a linear search of a list of size n (counting the number of comparisons). (3 pts) The worst-case of a linear search of a list of size n is that the search term is the last one in the list.

$\boxed{\therefore \text{ it is } n.}$

15. (5 pts) Prove or disprove: For all integers a, b, c, if **a|c** and **b|c**, then **ab|c²**.

suppose $a|c \Rightarrow c = at$ for some integer t

and $b|c \Rightarrow c = bk$ for some integer k

$ab|c^2 \Rightarrow c^2 = at \cdot bk$

$c^2 = ab \cdot tk$

$\boxed{\therefore ab|c^2}$

16. (5 pts) Find the ***prime factorization*** of **6,600**.

2 | 6600
2 | 3300
2 | 1650
5 | 825
165

5 | 165
3 | 33
11

$\boxed{\therefore 6600 = 2^3 \cdot 3 \cdot 5^2 \cdot 11}$

17. (6 pts)

(a) Convert $(\mathbf{135})_{10}$ to base 2.. (3 pts)

135
−128
7
−4
3
−2
1
−1

1 0 0 0 0 1 1 1
128 64 32 16 8 4 2 1

$\boxed{(10000111)_2}$

(b) Convert $(\mathbf{11|1100|0101})_2$ to base 16. (3 pts)

0011 1100 0101
3 C 5

$\boxed{(3C5)_{16}}$

9
a 10
b 11
c 12
d 13
e 14
f 15

18. (5 pts) A message has been ***encrypted*** using the function f(x) = (x + 3) mod 26.

If the message in coded form is **UHRSHQ**, ***decode*** the message.

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

U = (20 − 3) mod 26 = R
H = (7 − 3) mod 26 = E
R = (17 − 3) mod 26 = O
S = (18 − 3) mod 26 = P
H = (7 − 3) mod 26 = E
Q = (16 − 3) mod 26 = N

$\boxed{\therefore \text{ the message is REOPEN.}}$