1) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a LT

Let $B = C = \{(1,0), (0,1)\}$

if $[T]_B^C = \begin{bmatrix} 3 & 5 \\ 7 & 9 \end{bmatrix}$

find $T(x,y)$

$[v]_C = [T]_B^C [v]_B$

$$= \begin{bmatrix} 3 & 5 \\ 7 & 9 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3x+5y \\ 7x+9y \end{bmatrix}$$

$\therefore \; T(x,y) = (3x+5y, 7x+9y)$

2) Let $T : \mathbb{R}^4 \to \mathbb{R}^3$ be LT defined by

$T(v) = Av$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 6 & 8 & 4 & 2 \\ 4 & 4 & -2 & -6 \end{bmatrix} \xRightarrow{\text{RREF}} \begin{bmatrix} 1 & 0 & -4 & -7 \\ 0 & 1 & 7/2 & 11/2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$c_1 = 4s + 7t$

$c_2 = -7/2\, s - 11/2\, t$

$c_3 = s$

$c_4 = t$

$$\begin{bmatrix} 4s+7t \\ -7/2\, s - 11/2\, t \\ s \\ t \end{bmatrix} = \begin{bmatrix} 4s \\ -7/2\, s \\ s \\ 0 \end{bmatrix} + \begin{bmatrix} 7t \\ -11/2\, t \\ 0 \\ t \end{bmatrix}$$

$$= s \begin{bmatrix} 4 \\ -7/2 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 7 \\ -11/2 \\ 0 \\ 1 \end{bmatrix}$$

$\text{Ker}\, T = \{(4, -7/2, 1, 0), (7, -11/2, 0, 1)\}$

$\text{Rng}\, T = \{(1,6,4), (2,8,4)\}$

3) Let $T: P_2 \to P_2$ be a LT defined by

$$T(a+bx+cx^2) = (a-b+c) + (3a+4b+2c)x + (2a+5b+c)x^2$$

find a basis for a) Ker T b) Rng T

To find the kernel

(a) solve $T(v) = \vec{0}$

(b) ① list the equation

② turn into matrix [ ]

③ do RREF on the matrix

(c) find the value of the variables

$$\begin{aligned} a-b+c &= 0 \\ 3a+4b+2c &= 0 \\ 2a+5b+c &= 0 \end{aligned} \Rightarrow \left[\begin{array}{ccc:c} 1 & -1 & 1 & 0 \\ 3 & 4 & 2 & 0 \\ 2 & 5 & 1 & 0 \end{array}\right] = \begin{bmatrix} 1 & 0 & \frac{6}{7} & 0 \\ 0 & 1 & -\frac{1}{7} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$a = -\frac{6}{7}t$, $b = \frac{1}{7}t$, $c = t$

$\text{Ker } T = \{(-\frac{6}{7}, \frac{1}{7}, 1)\}$

To find the Range

Separate the variables from $T(v)$

factor out the variables

set equation equal to $\vec{0}$

turn into matrix

do RREF

$\text{Rng } T = \{(1,3,2), (-1,4,5)\}$