CSCI 190 Discrete Mathematics Applied to Computer Science
Exam 1

Name : ___________________________

Read these instructions before proceeding.

- Closed book. Closed notes. You can use calculator.
- You have **80 minutes** to complete this exam.
- No questions will be answered during the exam or immediately afterwards. Answer each question as best you can. Partial credit will be awarded for reasonable efforts. If a question contains an ambiguity or a misprint, then say so in your answer, providing the answer to a reasonable interpretation of the question; give your assumptions.
- Answer the problems on the blank spaces provided for each problem.
- **Box your answers**.

Q1 (6)	Q2 (7)	Q3 (7)	Q4 (6)	Q5 (6)	Q6 (8)	Q7 (4)	Q8 (4)	Q9 (4)	Q10 (4)	Q11 (6)	Q12 (6)	Q13 (5)	Q14 (6)	Q15 (5)	Q16 (5)	Q17 (6)	Q18 (5)	Total (100)

1. (6 pts) Determine whether the proposition is ***TRUE*** or ***FALSE***. No justifications needed.

a) **1 + 11 = 12 if and only if 2 + 2 = 22.** (2 pts)

b) **If it is raining, then it is raining.** (2 pts)

c) **If 2 > -1, then 7 = 9.** (2 pts)

2. (7 pts) Determine whether $(p \rightarrow q) \wedge (\neg p \rightarrow q) \equiv q$ using truth table.

3. (7 pts) Prove that $(\neg p \wedge (\neg q \rightarrow p)) \rightarrow q$ is a tautology using propositional equivalence and the laws of logic.

4. (6 pts) Write the contrapositive, converse, and inverse of the following:

If you give it a try, then you will be good.

a) **contrapositive** (2 pts)

b) **converse** (2 pts)

c) **inverse** (2 pts)

5. (6 pts) Suppose the variable x represents people, and
F(x): x is friendly T(x): x is tall A(x): x is angry.

Write the statement using these predicates and any needed quantifiers.

a) **Some people are not angry**. (3 pts)

b) **All tall people are friendly**. (3 pts)

6. (8 pts) Consider the following theorem:
"**if x and y are odd integers, then x + y is even**".
Give a direct proof of this theorem.

7. (4 pts) Draw two Venn diagrams for **A ∩ (B U C)** and **B ∩ (A U C).**
Are they the same?

8. (4 pts) determine whether the given set is the power set of some set. (Answer "**Yes**" or "**No**").
If the set is a power set, **give the set** of which it is a power set.

a) { Ø, {Ø}, {b}, {Ø, b}} (2 pts)

b) {{Ø}, {a}, {b}, {a,b}} (2 pts)

9. (4 pts) Just answer "***yes***" or "***no***" in the box. No justifications needed.

(a) Suppose $f:N \longrightarrow N$ has the rule $f(n) = 2n + 1$. Determine whether f is onto. (1 pts)

(b) Suppose $f:N \longrightarrow N$ has the rule $f(n) = 2n + 1$. Determine whether f is 1-1. (1 pts)

(c) Suppose $f:Z \longrightarrow Z$ has the rule $f(n) = 2n^2 - 1$. Determine whether f is 1-1. (1 pts)

(d) Suppose $f:Z \longrightarrow Z$ has the rule $f(n) = 2n^2 - 1$. Determine whether f is onto Z. (1 pts)

10. (4 pts) Find $\mathbf{a_n}$ (a formula that generates the following sequence $a_1, a_2, a_3 \ldots .$)

a) 20, 24, 28, 32, 36, . . . (2 pts)

b) -1, 2, -4, 8, -16, 32, . . . (2 pts)

11. (6 pts) Suppose $\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{1} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \mathbf{1} & \mathbf{0} \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} & \mathbf{0} \end{bmatrix}$

Find
(a) the ***join*** of A and B.

(b) the ***meet*** of A and B.

(c) the ***Boolean product*** of A and B.

12. (6 pts)
Show (step by step) how the binary search algorithm searches for **43** in the following list:
1 5 8 15 21 35 43.

13. (5 pts) Arrange the following functions in a list so each is ***big-O*** of the next one in the list. ***No justifications needed***.

$$n^3 + 7n^2 - 1, \quad \log n, \quad n^3, \quad n^4 \log n, \quad 2^n, \quad 1111$$

14. (6 pts)

(a) Give the ***best-case*** analysis of a linear search of a list of size n (counting the number of comparisons). (3 pts)

(b) Give the ***worst-case*** analysis of a linear search of a list of size n (counting the number of comparisons). (3 pts)

15. (5 pts) Prove or disprove: For all integers a, b, c, if $a|c$ and $b|c$, then $ab|c^2$.

16. (5 pts) Find the ***prime factorization*** of **6,600**.

17. (6 pts)

(a) Convert $(135)_{10}$ to base 2.. (3 pts)

(b) Convert $(1111000101)_2$ to base 16. (3 pts)

18. (5 pts) A message has been ***encrypted*** using the function f(x) = (x + 3) mod 26.

If the message in coded form is **UHRSHQ**, ***decode*** the message.