SECTION 8.1: CONFIDENCE INTERVALS FOR A POPULATION MEAN, σ KNOWN

OBJECTIVES μ σ P

1. Construct and interpret confidence intervals for a population mean when the population standard deviation is known
2. Find critical values for confidence intervals
3. Describe the relationship between the confidence level and the margin of error
4. Find the sample size necessary to obtain a confidence interval of a given width
5. Distinguish between confidence and probability

OBJECTIVE 1
CONSTRUCT AND INTERPRET CONFIDENCE INTERVALS FOR A POPULATION MEAN WHEN THE POPULATION STANDARD DEVIATION IS KNOWN

POINT ESTIMATE AND MARGIN OF ERROR

A simple random sample of 100 fourth-graders is selected to take part in a new experimental approach to teach reading. At the end of the program, the students are given a standardized reading test. On the basis of past results, it is known that scores on this test have a population standard deviation of σ = 15.

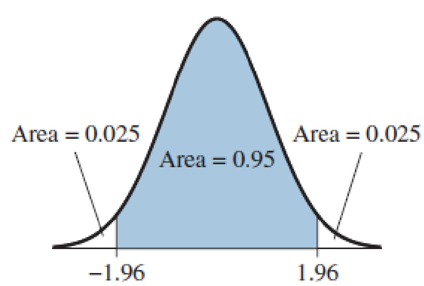
The sample mean score for the 100 students was $\bar{x}$ = 67.30. The administrators want to estimate what the mean score would be if the entire population of fourth-graders in the district had enrolled in the program. The best estimate for the population mean is the sample mean, $\bar{x}$ = 67.30. The sample mean is a **point estimate**, because it is a single number.

It is very unlikely that $\bar{x}$ = 67.30 is exactly equal to the population mean, μ, of all fourth-graders. Therefore, in order for the estimate to be useful, we must describe how close it is likely to be. For example, if we think that it could be off by 10 points, we would estimate μ with the interval 57.30 < μ < 77.30, which could be written 67.30 $\pm$ 10. The plus-or-minus number is called the **margin of error**.

We need to determine how large to make the margin of error so that the interval is likely to contain the population mean. To do this, we use the sampling distribution of $\bar{x}$. Because the sample size is large ($n >$ 30), the Central Limit Theorem tells us that the sampling distribution of $\bar{x}$ is approximately normal with mean μ and standard error $\frac{\sigma}{\sqrt{n}}$.

For the teaching reading example, the standard error is $\frac{\sigma}{\sqrt{n}} = \frac{15}{\sqrt{100}} = 1.5$.

We now construct a 95% confidence interval for μ. Begin with a normal curve and find the z-scores that bound the middle 95% of the area under the curve. The z-scores are 1.96 and –1.96. The value 1.96 is called the **critical value**. To obtain the margin of error, multiply the critical value by the standard error.



Margin of error = (Critical value)·(Standard error) = $(1.96)(1.5) = 2.94$.

A 95% confidence interval for μ is therefore

$\bar{x} - 2.94 < \mu < \bar{x} + 2.94$

$67.30 - 2.94 < \mu < 67.30 + 2.94$

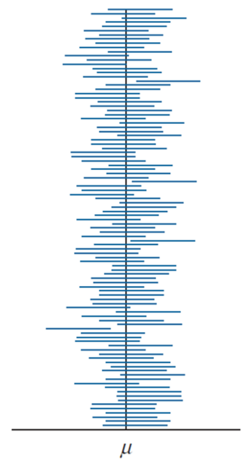
$64.36 < \mu < 70.24$

we are 95% confidence that the population mean is between 64.36 and 70.24

CONFIDENCE LEVEL

Based on the sample of 100 fourth-graders using the new approach to teaching reading, a 95% confidence interval for the mean score was constructed. **95%** is the **confidence level** for the confidence interval. The confidence level measures the success rate of the method used to construct the confidence interval.

If we were to draw many samples and use each one to construct a confidence interval, then in the long run, the percentage of confidence intervals that cover the true value of μ would be equal to the confidence interval.



TERMINOLOGY

A point estimate is a single number that is used to estimate the value of an unknown parameter

A confidence interval is an interval that is used to estimate the value of a parameter

The interval level is a percentage between 0% and 100% that measures the success rate of the method used to construct the confidence interval

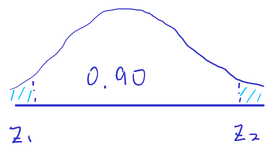
The margin of error is completed by multiplying the critical value by the standard error

OBJECTIVE 2
FIND CRITICAL VALUES FOR CONFIDENCE INTERVALS

Although 95% is the most commonly used confidence level, sometimes we will want to construct a confidence interval with a different level. We can construct a confidence interval with any confidence level between 0% and 100% by finding the appropriate critical value for that level.

EXAMPLE: A large sample has mean $\bar{x} = 7.1$ and standard error $\frac{\sigma}{\sqrt{n}} = 2.3$. Construct a 90% confidence interval for the population mean μ.

SOLUTION: Critical value :



$\frac{1-0.90}{2}$

z_2 = invNorm $(1-0.05, 0, 1)$

$= 1.645$

margin of error : (critical value)(standard error) $= 1.645(2.3) = 3.8$

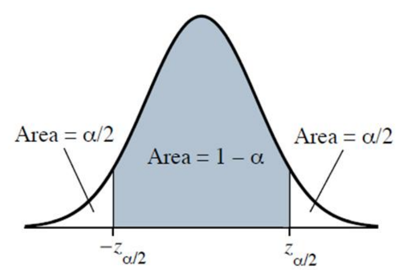
A 90% of confidence level

$3.3 < \mu < 10.9$

We are 90% confident that the population mean is between 3.3 and 10.9

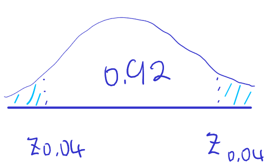
z_α NOTATION

it is useful to learn a notation for a z-score with a given area to its right. If $1-\alpha$ is the confidence level, then the critical value is $z_{\alpha/2}$ because the area under the standard normal curve between $-z_{\alpha/2}$ and $z_{\alpha/2}$ is $1-\alpha$. These z-scores can be found using technology.



EXAMPLE: Find the critical value $z_{\alpha/2}$ for a 92% confidence interval.

SOLUTION:



confidence level = 92%

$1-\alpha = 0.92$

$\alpha = 0.08$

$\alpha/2 = 0.04$

$z_{0.04}$ = invNorm $(1-0.04, 0, 1,$ left$)$

$= 1.75$

ASSUMPTIONS

The method described for confidence interval requires us to assume that the population standard deviation σ is known. In practice, σ is not known. We make this assumption because it allows us to use the familiar normal distribution. We will learn how to construct confidence intervals when σ is unknown in the next section. Other assumptions for the method described here for constructing confidence intervals are:

Assumptions:

1) We have a simple random sample (SRS)
2) the sample size is large ($n > 30$) or the population is approximately normal

EXAMPLE: A machine that fills cereal boxes is supposed to put 20 ounces of cereal in each box. A simple random sample of 6 boxes is found to contain a sample mean of 20.25 ounces of cereal. It is known from past experience that the fill weights are normally distributed with a standard deviation of 0.2 ounce. Construct a 90% confidence interval for the mean fill weight.

SOLUTION: Assumption: SRS ✓
normally distributed ✓

Confidence interval: $\bar{x} - m < \mu < \bar{x} + m$ where $m = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$

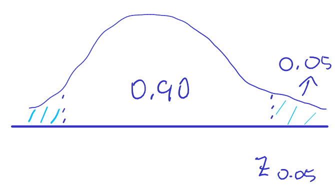
point estimate: $\bar{x} = 20.25$

critical value: $z_{0.05} = \text{invNorm}(0.95, 0, 1)$ $\qquad \frac{1 - 0.90}{2} = 0.05$

$= 1.645$

margin of error: $m = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} = (1.645)\left(\frac{0.2}{\sqrt{6}}\right)$

$= 0.13431$



Now, the confidence interval is

$20.25 - 0.1343 < \mu < 20.25 + 0.13431$

$20.12 < \mu < 20.38$

we are 90% confident that the mean weight μ is between 20.12 and 20.38

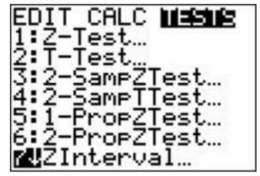
Calculator check: Z interval

$\sigma = 0.2$ $\qquad n = 6$

$\bar{x} = 20.25$ $\qquad$ C-level: 0.90

CONFIDENCE INTERVALS ON THE TI-84 PLUS

The Zinterval command constructs confidence intervals when the population standard deviation σ is known. This command is accessed by pressing STAT and highlighting the TESTS menu.

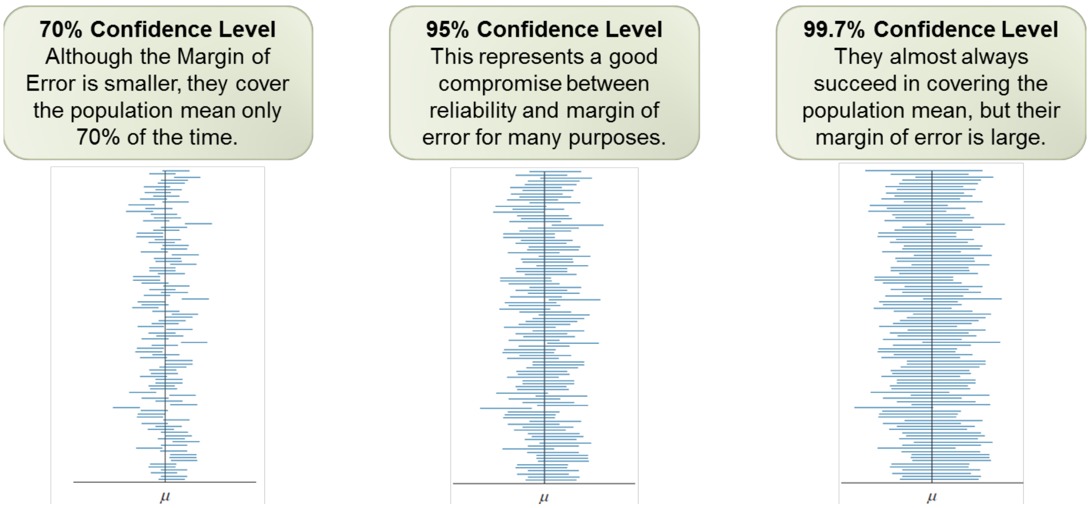


If the summary statistics are given the Stats option should be selected for the input option.

If the raw sample data are given, the Data option should be selected.

OBJECTIVE 3
DESCRIBE THE RELATIONSHIP BETWEEN THE CONFIDENCE LEVEL AND THE MARGIN OF ERROR

If we want to be more confident that our interval contains the true value, we must increase the critical value, which increases the margin of error. There is a trade-off. We would rather have a higher level of confidence than a lower level, but we would also rather have a smaller margin of error than a larger one.



MEASURING THE SUCCESS RATE OF THE METHOD

The diagram presents 100 different 95% confidence intervals. When we construct a confidence interval with level 95%, we are getting a look at one of these confidence intervals.

We don't get to see any of the other confidence intervals, nor do we get to see the vertical line that indicates where the true value μ is. We cannot be sure whether we got one of the confidence intervals that covers μ, or whether we were unlucky enough to get one of the unsuccessful ones.

What we do know is that our confidence interval was constructed by a method that succeeds 95% of the time. The confidence level describes the success rate of the method used to construct a confidence interval, not the success of any particular interval.

OBJECTIVE 4
FIND THE SAMPLE SIZE NECESSARY TO OBTAIN A CONFIDENCE INTERVAL OF A GIVEN WIDTH

We can make the margin of error smaller if we are willing to reduce our level of confidence, but we can also reduce the margin of error by increasing the sample size. If we let m represent the margin of error, then m = $z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$.

Using algebra, we may rewrite this formula as $n = \left(\frac{z_{\alpha/2} \cdot \theta}{m}\right)^2$ which represents the minimum sample size needed to achieve the desired margin of error m.

If the value of n given by the formula is not a whole number, round it up to the nearest whole number. By rounding up we can be sure that the margin of error is no greater than the desired value m.

EXAMPLE: Scientists want to estimate the mean weight of mice after they have been fed a special diet. From previous studies, it is known that the weight is normally distributed with standard deviation 3 grams. How many mice must be weighed so that a 95% confidence interval will have a margin of error of 0.5 grams?

SOLUTION: Since we want 95 % confidence interval, we use

$z_{\alpha/2} = z_{0.025} = 1.96$. We also know that $\theta = 3$, and $m = 0.5$

Implies $\Rightarrow n = \left(\frac{z_{\alpha/2} \cdot \theta}{m}\right)^2 = \left(\frac{1.96 \cdot 3}{0.15}\right)^2$

$= 138.30 \approx 139$

$z_{0.025}$ = invNorm (1 - 0.025, 0, 1, left)

= 1.96

We must weigh 139 mice in order to obtain a 95 % confidence interval with a margin of error of 0.5 grams

OBJECTIVE 5

DISTINGUISH BETWEEN CONFIDENCE AND PROBABILITY

Suppose that a 90% confidence interval for the mean weight of cereal boxes was computed to be $20.12 < \mu < 20.38$. It is tempting to say that the **probability** is 90% that μ is between 20.12 and 20.38.

The term "probability" refers to random events, which can come out differently when experiments are repeated. The numbers 20.12 and 20.38 are fixed, not random. The population mean is also fixed, even if we do not know precisely what value it is. The population mean weight is either between 20.12 and 20.38 or it is not. Therefore we say that we have 90% **confidence** that the population mean is in this interval.

On the other hand, let's say that we are discussing a **method** used to construct a 90% confidence interval. The method will succeed in covering the population mean 90% of the time, and fail the other 10% of the time. Therefore it is correct to say that a **method** for constructing a 90% confidence interval has **probability** 90% of covering the population mean.

YOU SHOULD KNOW …

- How to construct and interpret confidence intervals for a population mean when the population standard deviation is known
- How to find critical values for confidence intervals
- How to describe the relationship between the confidence level and the margin of error
- How to find the sample size necessary to obtain a confidence interval of a given width
- The difference between confidence and probability

OBJECTIVES

1. Describe the properties of the Student's t distribution
2. Construct confidence intervals for a population mean when the population standard deviation is unknown

OBJECTIVE 1
DESCRIBE THE PROPERTIES OF THE STUDENT'S t DISTRIBUTION

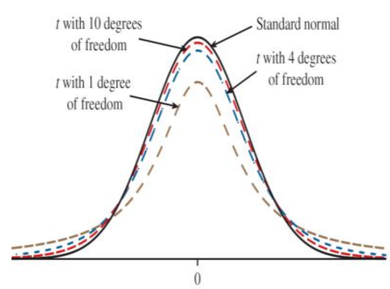
STUDENT'S t DISTRIBUTION

When constructing a confidence interval where we know the population standard deviation σ, the confidence interval is $\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$. The critical value is $z_{\alpha/2}$ because the quantity $\frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$ has a normal distribution.

It is rare that we would know the value of σ while needing to estimate the value of μ. In practice, it is more common that σ is unknown. When we don't know the value of σ, we may replace it with the sample standard deviation s. However, we cannot then use $z_{\alpha/2}$ as the critical value, because the quantity $\frac{\bar{x}-\mu}{s/\sqrt{n}}$ does not have a normal distribution. The distribution of this quantity is called the **Student's t distribution**.

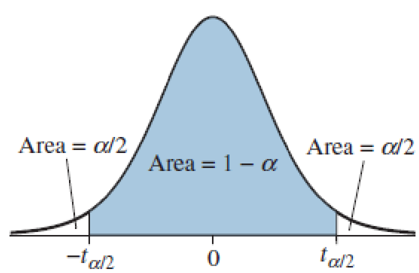
There are actually many different Student's t distributions and they are distinguished by a quantity called the **degrees of freedom**. When using the Student's t distribution to construct a confidence interval for a population mean, the number of degrees of freedom is 1 less than the sample size. $(df = n-1)$

Student's t distributions are symmetric and unimodal, just like the normal distribution. However, they are more spread out. The reason is that s is, on the average, a bit smaller than σ. Also, since s is random, whereas σ is constant, replacing σ with s increases the spread. When the number of degrees of freedom is small, the tendency to be more spread out is more pronounced. When the number of degrees of freedom is large, s tends to be close to σ, so the t distribution is very close to the normal distribution.



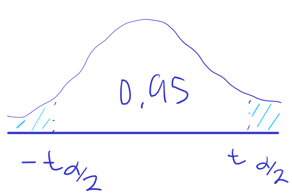
THE CRITICAL VALUE $t_{\alpha/2}$

To find the critical value for a confidence interval, let $1-\alpha$ be the confidence level. The critical value is then $t_{\alpha/2}$, because the area under the Student's t distribution between $-t_{\alpha/2}$ and $t_{\alpha/2}$ is $1-\alpha$.



EXAMPLE: A simple random sample of size 10 is drawn from a normal population. Find the critical value $t_{\alpha/2}$ for a 95% confidence interval.

SOLUTION:



$\frac{1-0.95}{2} = 0.025$ $\quad df = n-1 \Rightarrow df = 10-1 = 9$

$t_{0.025} = invT(1-0.025, 9)$

$= 2.2622$

Note : t = invT(area to the left , df)

ASSUMPTIONS

1. SRS

2. The sample size is large ($n > 30$), or the population is approximately normal

When the sample size is small ($n \leq 30$), we must check to determine whether the sample comes from a population that is approximately normal. A simple method is to draw a dotplots or boxplot of the sample. If there are no outliers, and if the sample is not strongly skewed, then it is reasonable to assume the population is approximately normal and it is appropriate to construct a confidence interval using the Student's t distribution.

OBJECTIVE 2
CONSTRUCT CONFIDENCE INTERVALS FOR A POPULATION MEAN WHEN THE POPULATION STANDARD DEVIATION IS UNKNOWN

If the assumptions are satisfied, the confidence interval for μ when σ is unknown is found using the following steps:

Step 1: Compute the sample mean $\bar{x}$ and sample standard deviation, s, if they are not given.

Step 2: Find the number of degrees of freedom $n - 1$ and the critical value $t_{\alpha/2}$.

Step 3: Compute the standard error $s/\sqrt{n}$ and multiply it by the critical value to obtain the margin of error $t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$.

Step 4: Construct the confidence interval: $\bar{x} \pm t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$. $\bar{x} - m < \mu < \bar{x} + m$

Step 5: Interpret the result.

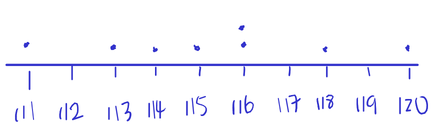
EXAMPLE 1: A food chemist analyzed the calorie content for a popular type of chocolate cookie. Following are the numbers of calories in a sample of eight cookies.

113, 114, 111, 116, 115, 120, 118, 116

Find a 98% confidence interval for the mean number of calories in this type of cookie.

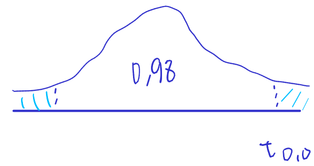
SOLUTION: Check assumption : SRS ✓

since $n = 8 \leq 30$, we must check if the population is approximately normal



Based on the dot plot, this shows no evidence of strong skewness and no outliers, and therefore we can use the normal distribution

$\bar{x} = 115.375 \quad s = 2.8253 \quad n = 8$



$df = n - 1 = 7$

$t_{\alpha/2} = t_{0.01} = \text{invT}(0.99, 7) = 2.998)$

$m = t_{\alpha/2} \cdot \frac{\alpha}{\sqrt{5}} = 2.998 \left(\frac{2.8253}{\sqrt{8}} \right) = 2.9947$

confidence interval: $\bar{x} - m < \mu < \bar{x} + m$

$115 - 2.9947 < \mu < 115 + 2.9947$

$112.4 < \mu < 118.4$

We are 98 % confident that the mean number of calories per cookie is between 112.4 and 118.4

Calculator check: T interval

List: L1 freq: 1 C-level: 0.98

always 1

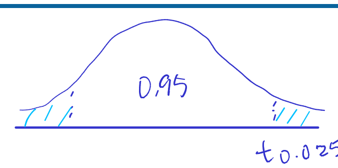
confidence interval

$(112.38, 118.37)$

EXAMPLE 2: A sample of 123 people aged 18–22 reported the number of hours they spent on the Internet in an average week. The sample mean was 8.20 hours, with a sample standard deviation of 9.84 hours. Assume this is a simple random sample from the population of people aged 18–22 in the U.S. Construct a 95% confidence interval for μ, the population mean number of hours per week spent on the Internet by people aged 18–22 in the U.S.

SOLUTION: Check assumption: SRS ✓

$n = 123 > 30$ ✓



The sample size is large enough

$\bar{x} = 8.20 \quad s = 9.84 \quad n = 123 \quad df = n - 1 = 122$

$t_{\alpha/2} = t_{0.025} = \text{invT}(0.975, 122) = 1.9800$

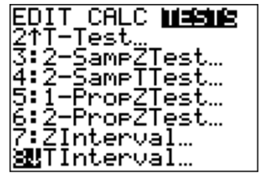
$m = t_{\alpha/2} \cdot \frac{s}{\sqrt{n}} = (1.98)\left(\frac{9.84}{\sqrt{123}}\right) = 1.7567$

Confidence interval

$\bar{x} - m < \mu < \bar{x} + m$

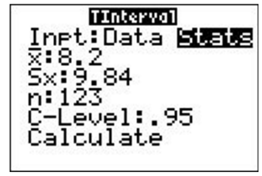
CONFIDENCE INTERVALS ON THE TI-84 PLUS

The Tinterval command constructs confidence intervals when the population standard deviation σ is unknown. This command is accessed by pressing STAT and highlighting the TESTS menu.



If the summary statistics are given the Stats option should be selected for the input option.

If the raw sample data are given, the Data option should be selected.



YOU SHOULD KNOW …

- The properties of the Student's t distribution
- Why we must determine whether the sample comes from a population that is approximately normal when the sample size is small ($n \leq 30$)
- How to construct and interpret confidence intervals for a population mean when the population standard deviation is unknown

OBJECTIVES

1. Construct a confidence interval for a population proportion
2. Find the sample size necessary to obtain a confidence interval of a given width
3. Describe a method for constructing confidence intervals with small samples

OBJECTIVE 1
CONSTRUCT A CONFIDENCE INTERVAL FOR A POPULATION PROPORTION

The music organization Little Kids Rock surveyed 517 music teachers, and 403 of them said that video games like Guitar Hero and Rock Band, in which players try to play music in time with a video image, have a positive effect on music education.

Assuming these teachers to be a random sample of U.S. music teachers, we would like to construct a **confidence interval for the proportion** of music teachers who believe that music video games have a positive effect on music classrooms.

Notation:

p is the population proportion of individuals who are in a specific category

x is the number of individuals in the sample who are in a specific category

n is the sample size

$\hat{p}$ is the sample proportion of individuals who are in the specific category

$\hat{p}$ is defined as $\hat{p} = \frac{x}{n}$

CONFIDENCE INTERVAL

To construct a confidence interval, we need a point estimate. The point estimate for the population proportion p is:

Point estimate = $\hat{p} = \frac{x}{n}$

We also need the standard error of $\hat{p}$. By the Central Limit Theorem for Proportions, we have:

Standard error of $\hat{p}$ = $\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

To compute the margin of error, we multiply the standard error by the critical value:

Margin of error = $z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

The confidence interval is:

$\hat{p} - m < p < \hat{p} + m$

ASSUMPTIONS

The method for constructing a confidence interval for a population proportion requires that the sampling distribution be approximately normal. The following assumptions ensure this:

1) SRS
2) The population is at least 20 times as large as the sample ($n \le 5\%$ of the population)
3) The items in the population are divided into two categories
4) The sample must contain at least 10 individuals in each categories

EXAMPLE: In a survey of 517 music teachers, 403 said that the video games Guitar Hero and Rock Band have a positive effect on music education. Construct a 95% confidence interval for the proportion of music teachers who believe that these video games have a positive effect.

SOLUTION: Assumption : SRS

The population of music is at least 20 times as large as the sample $n = 517$

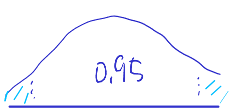
Two categories → 403 teachers who believe
→ $517 - 403 = 114$ who do not believe

and there are more than 10 in each category

point estimate : $\hat{p} = \frac{x}{n} = \frac{403}{517} = 0.779497$

margin of error : $m = z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 1.96\sqrt{\frac{(0.779497)(1-0.779497)}{517}}$

$= 0.035738$



$z_{\alpha/2} = z_{0.025} =$ invNorm(0.975,0,1)

Confidence Interval : $\hat{p} - m < p < \hat{p} + m$

$0.744 < p < 0.815$

We are 95% confident that the proportion of music teachers who believe that video game have a positive effect is between 0.744 and 0.815

Calculator check :

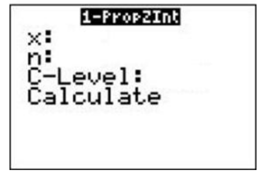
|STAT| → |TEST|

1 - propZInt
x : 403
n : 517
C-level : 0.95

confidence Interval
(0.744 , 0.815)

CONFIDENCE INTERVALS ON THE TI-84 PLUS

The **1-PropZInt** command constructs confidence intervals for the population proportion. This command is accessed by pressing **STAT** and highlighting the **TESTS** menu.



Enter the values of x, n, and the confidence level.

OBJECTIVE 2
FIND THE SAMPLE SIZE NECESSARY TO OBTAIN A CONFIDENCE INTERVAL OF A GIVEN WIDTH

If we wish to make the margin of error of a confidence interval smaller while keeping the confidence level the same, we can do this by making the sample size larger. Let m represent the margin of error: $m = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$.

By rewriting this formula and solving for n, we have $n = \hat{p}(1-\hat{p})\left(\frac{z_{\alpha/2}}{n}\right)^2$. This is the minimum sample size needed to attain a margin of error of size m. If the value of n is not a whole number, round up to the nearest whole number.

In order to use this formula, we need a value for m and $\hat{p}$. We can set the value of m, but we don't know ahead of time what $\hat{p}$ is going to be.

There are two ways to determine a value for $\hat{p}$.

1. Use a value that is available from a previously drawn sample.
2. To assume that $\hat{p}$ = 0.5, which makes the margin of error as large as possible for any sample size. In this case, the formula simplifies to $n = 0.25\left(\frac{z_{\alpha/2}}{m}\right)^2$. → $\hat{p}$ is unknown

EXAMPLE: In a survey of 517 music teachers, 403 said that the video games Guitar Hero and Rock Band have a positive effect on music education. Estimate the sample size needed so that a 95% confidence interval will have a margin of error of 0.03.

SOLUTION: We know that $\hat{p} = \frac{x}{n} = \frac{403}{517} = 0.779497$ and $m = 0.03$

The necessary sample size is

$$n = \hat{p}(1-\hat{p})\left(\frac{z_{\alpha/2}}{m}\right)^2$$
$$= (0.779497)(1-0.779497)\left(\frac{1.96}{0.03}\right)^2$$
$$= 733.67 \approx 734$$

The sample size needed is 734

EXAMPLE: We plan to sample music teachers in order to construct a 95% confidence interval for the proportion who believe that listening to hip-hop music has a positive effect on music education. We have no value of $\hat{p}$ available. Estimate the sample size needed so that a 95% confidence interval will have a margin of error of 0.03.

SOLUTION: Since $\hat{p}$ is unknown, we use $\hat{p} = 0.50$. Thus,

$$n = (0.50)(1-0.50)\left(\frac{z_{\alpha/2}}{m}\right)^2$$

$$n = 0.25\left(\frac{1.96}{0.03}\right)^2 = 1067.1 \approx 1068$$

The sample size needed is 1068

YOU SHOULD KNOW …

- How to construct and interpret confidence intervals for a population proportion
- How to find the sample size necessary to obtain a given confidence interval for a population proportion of a given width where:
    - An estimate of $\hat{p}$ exists
    - No estimate of $\hat{p}$ exists
- How to construct confidence intervals for a population proportion with small samples

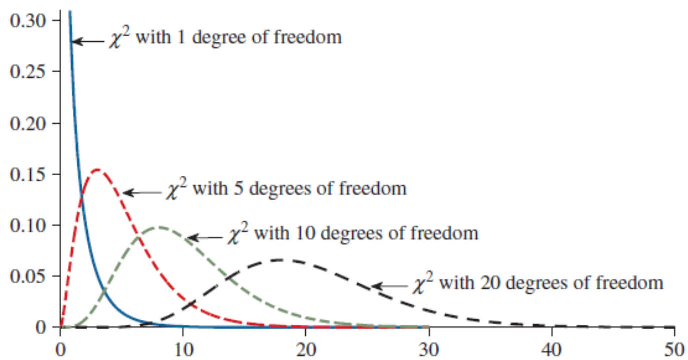
SECTION 8.4: CONFIDENCE INTERVALS FOR A STANDARD DEVIATION

OBJECTIVES

1. Find critical values of the chi-square distribution
2. Construct confidence intervals for the variance and standard deviation of a normal distribution

OBJECTIVE 1
FIND CRITICAL VALUES OF THE CHI-SQUARE DISTRIBUTION

When the population is normal, it is possible to construct confidence intervals for the standard deviation or variance. These confidence intervals are based on a distribution known as the **chi-square distribution**, denoted χ^2. There are actually many different chi-square distributions, each with a different number of degrees of freedom. The figure below shows several examples of chi-square distributions.



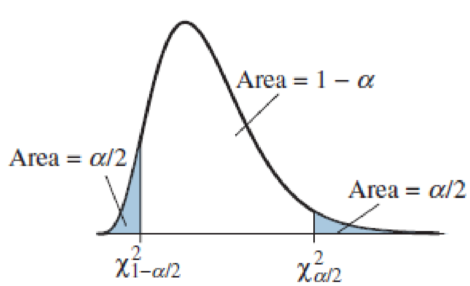
There are two important characteristics of the chi-square distribution

1. The chi-square distribution are not symmetric. They are skewed to the right

2. Values of the χ^2 statistic are always greater or equal to 0. They are never negative.

CRITICAL VALUES

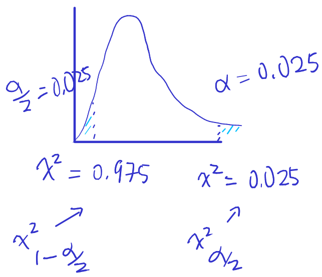
The critical values for a level $100(1-\alpha)\%$ confidence interval are the values that contain the middle $100(1-\alpha)\%$ of the area under the curve between them.



The notation for the critical values tells how much area is to the **right** of the critical value. For a level $1-\alpha$ confidence interval, the critical values are denoted $\chi^2_{1-\alpha/2}$ and $\chi^2_{\alpha/2}$.

EXAMPLE: Find the critical values for a 95% confidence interval using the chi-square distribution with 10 degrees of freedom.

SOLUTION:



$\frac{1-0.95}{2} = 0.025$

$\chi^2_{1-\alpha/2} = \chi^2_{0.975} = 3.247$

$\chi^2_{\alpha/2} = \chi^2_{0.05} = 20.483$

degrees of f	area	0.975	0.05
⋮			
10		3.247	20.483

OBJECTIVE 2
CONSTRUCT CONFIDENCE INTERVALS FOR THE VARIANCE AND STANDARD DEVIATION OF A NORMAL DISTRIBUTION

Let s^2 be the sample variance from a simple random sample of size n from a normal population

A level $100(1-\alpha)\%$ confidence interval for the population variance σ^2 is

$$\frac{(n-1)s^2}{\chi^2_{\alpha/2}} < \theta^2 < \frac{(n-1)s^2}{\chi^2_{1-\alpha/2}}$$

A level $100(1-\alpha)\%$ confidence interval for the population standard deviation σ is

$$\sqrt{\frac{(n-1)s^2}{\chi^2_{\alpha/2}}} < \theta < \sqrt{\frac{(n-1)s^2}{\chi^2_{\alpha/2}}}$$

Note: the critical values are taken from a chi-square distribution with $n-1$ degree of freedom

EXAMPLE: The compressive strengths of seven concrete blocks, in pounds per square inch, are measured, with the following results.

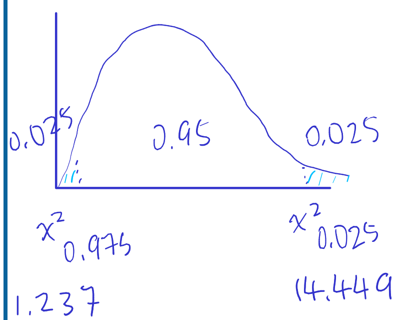
1989.9	1993.8	2074.5	2070.5
2070.9	2033.6	1939.6	

Assume these values are a simple random sample from a normal population. Construct a 95% confidence interval for the population standard deviation.

SOLUTION: Using TI-84, we find $s = 51.9602$. So $s^2 = 2699.8624$

$df = 7 - 1 = 6$

$\chi^2_{0.975} = 1.237$ and $\chi^2_{0.025} = 14.449$



$$\text{Lower bound} = \sqrt{\frac{(n-1)s^2}{\chi^2_{\alpha/2}}} = \sqrt{\frac{(7-1)(2699.8624)}{14.449}} = 33.4832$$

$$\text{Upper bound} = \sqrt{\frac{(n-1)s^2}{1-\chi^2_{\alpha/2}}} = \sqrt{\frac{(7-1)(2699.8624)}{1.237}} = 114.4357$$

Confidence Interval:

$33.48 < \theta < 114.44$

We are 95% confident that the population standard deviation of the strength of the concrete blocks is between

33.48 and 114.44 lb/in^2.

JUSTIFICATION OF THE METHOD

Confidence intervals for the variance of a normal distribution are based on the fact that when a sample of size n is drawn from a normal distribution, the quantity $(n-1)s^2/\sigma^2$ follows a chi-square distribution with $n-1$ degrees of freedom. Therefore, for a proportion $1-\alpha$ of all possible samples,

$$\chi^2_{1-\alpha/2} < \frac{(n-1)s^2}{\sigma^2} < \chi^2_{\alpha/2}$$

Through algebraic manipulation, we can solve for σ^2, to obtain a level $100(1-\alpha)\%$ confidence interval for σ^2:

$$\frac{(n-1)s^2}{\chi^2_{\alpha/2}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{1-\alpha/2}}$$

CAUTION!

The methods of this section apply only for samples drawn from a normal distribution. If the distribution differs even slightly from normal, these methods should not be used.

YOU SHOULD KNOW …

- How to find critical values of the chi-square distribution.
- How to construct and interpret confidence intervals for the variance and standard deviation of a normal distribution.

OBJECTIVES

1. Determine which method to use when constructing a confidence interval

OBJECTIVE 1
DETERMINE WHICH METHOD TO USE WHEN CONSTRUCTING A CONFIDENCE INTERVAL

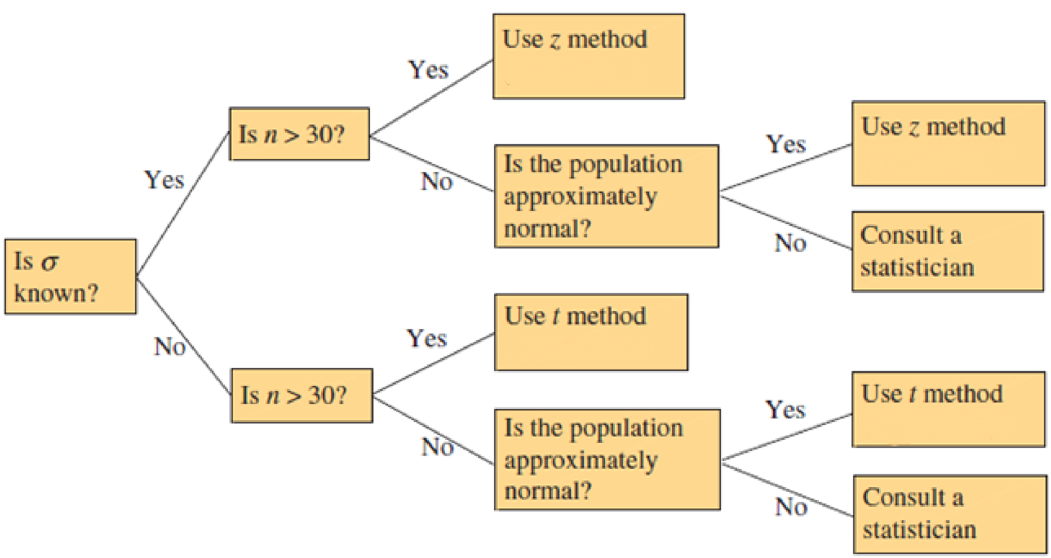
One of the challenges in constructing a confidence interval is to determine which method to use. The first step is to determine **which type** of parameter we are estimating.

There are three types of parameters for which we have learned to construct confidence intervals:

- Population mean : μ (z or t distribution)
- Population proportion : P (z - distribution)
- Population standard deviation : θ or variance : θ^2 (χ^2 - distribution)

ESTIMATING THE POPULATION MEAN μ

If you are estimating the population mean μ, there are two methods for constructing a confidence interval: the z-method and the t method. To determine which method to use, we must determine whether the population standard deviation is known, whether the population is approximately normal, and whether the sample size is large ($n > 30$). The diagram below can help you with the correct choice.



YOU SHOULD KNOW …

- How to determine which method to use when constructing a confidence interval