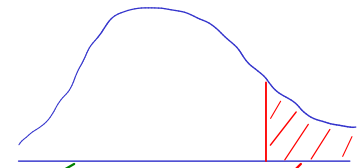
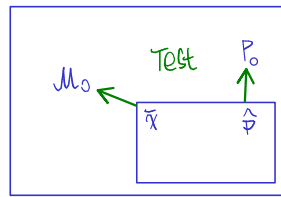


# Hypothesis Testing - 1 Mean 1 Proportion

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# Hypothesis Tests



- ASSESS the **evidence** provided by the data and to decide between two competing claims (hypotheses) about the population parameter

- The **critical value** gives us the first value for which we would **reject  $H_o$** .

Null Hypothesis

If the **test statistic** is further in the tails compared to the **critical value**, we **reject  $H_o$** .

If the **test statistic** is not further in the tails compared to the **critical value**, we **fail to reject  $H_o$** .

- The **significance level** gives the area of the **rejection region**

Common  $\alpha$  levels of rejection are  $\alpha = 0.10, 0.05, 0.01$ .

We use the significance level to find the critical value.

Set before we start our experiment.

This is the threshold that we can compare the p-value to.

If the p-value  $\leq \alpha$  we reject the null hypothesis.

If the p-value is low,  
we reject  $H_o$ .

- The **p-value** for the observed data measures the **weight of evidence** against the null hypothesis.

The **p-value** is the probability of seeing a value as extreme or more extreme than the one in our sample, **assuming the null hypothesis is true**.

The smaller the **p-value**, the more evidence we have against  $H_o$ .

The larger the **p-value**, the less evidence we have against  $H_o$ .

- In order to test the hypothesis for a population parameter, we use two things

We use the sample statistic as the best estimate of the population parameter.

We also need a measure of spread.

## Stating Hypotheses

- Null Hypothesis

This states what is generally believed to be the true population parameter.

$H_o : \text{parameter} = \text{hypothesized value}$

- Alternative Hypothesis

The “research hypothesis”. What we are trying to prove.

**Less** than test:  $H_o : \text{parameter} < \text{hypothesized value}$

**Greater** than test:  $H_o : \text{parameter} > \text{hypothesized value}$

**Different** than test:  $H_o : \text{parameter} \neq \text{hypothesized value}$

} One tailed

2 sided

## Draw a Picture

Use the sampling distribution to draw a picture of your hypothesis.

## Find the critical value

$q_{norm}$

Use the significance level to find the critical value ( $t_{crit}$  or  $z_{crit}$ ).

## Calculate the test statistic

$q_t$

Use the sampling distribution to calculate the test statistic ( $t_{TS}$  or  $z_{TS}$ ).

## Make a statistical decision and state your conclusion in context of the problem

- Since the test statistic IS/IS **NOT** more extreme than the critical value, we REJECT/**FAIL** TO REJECT the null hypothesis at the  $\alpha = \text{Value}$  level. There IS/IS **NOT** statistically significant evidence that **Context**
- Since the p-value = **Value**  $\leq$  vs  $>$  **Value** =  $\alpha$  we REJECT/FAIL TO REJECT the null hypothesis. There IS/IS NOT statistically significant evidence that **Context**
- The most important part is to use complete sentences to convey the context of the problem as well as the conclusion.

Hypothesis Alternative

$H_A \neq$

## Confidence Intervals and Two Sided Hypothesis Tests

- Another way to interpret a Confidence interval

The set of all values in which a 2 sided hypothesis test would fail to reject  $H_o$

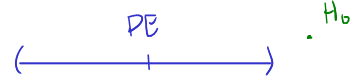
- Create a Confidence Interval

If the null value is INSIDE the confidence interval



Fail to Reject the null hypothesis

If the null value is OUTSIDE the confidence interval



Reject the null hypothesis

- Conduct a Hypothesis Test

If we Fail to Reject the null hypothesis

The null value is INSIDE the confidence interval

If we Reject the null hypothesis

The null value is OUTSIDE the confidence interval

- Since the claimed value IS NOT/IS captured within the interval we REJECT/FAIL TO REJECT the claim at the  $\alpha = \text{Value}$  level.

Parameter Description	Test Statistic	$H_o$ :
1 population proportion	$Z = \frac{\hat{p} - p_o}{\sqrt{\frac{p_o q_o}{n}}}$	$p = p_o$
1 population mean ( $\sigma$ unknown)	$t = \frac{\bar{X} - \mu_o}{\frac{s}{\sqrt{n}}}$ where $df = n - 1$	$\mu = \mu_o$

## Assumptions for Hypothesis Tests

### 1 population proportion

- $n\hat{p} \geq 10$  and  $n\hat{q} \geq 10$
- Sample size is less than 10% of the population size; if we are sampling is w/out replacement
- The sample can be regarded as a simple random sample from the population of interest.
- The data values are assumed to be independent of each other.

### 1 population mean

- We need to have a large enough sample size ( $n \geq 30$ ). For  $n < 30$  with extreme skewness or outliers, you cannot use this method.
- Sample size is less than 10% of the population size; if we are sampling is w/out replacement
- The sample can be regarded as a simple random sample from the population of interest.
- The data values are assumed to be independent of each other.

$$H_0: P = P_0$$

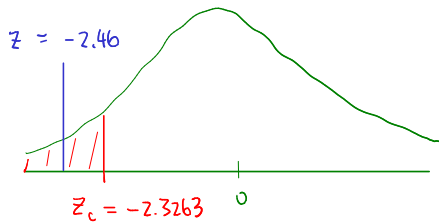
$$H_A: P < P_0$$

Ex: What is your decision?

Left Tailed Test:

Test Statistic:  $z = -2.46$

$\alpha = 0.01$



$$z_c = \text{qnorm}(0.01, 0, 1) = -2.3263$$

Are you in the rejection region?

Yes, since the test statistic is

more extreme than the critical

value we reject the null

hypothesis at the  $\alpha = 0.01$  level

$$P\text{value} = P(Z < -2.46)$$

$$= \text{pnorm}(-2.46, 0, 1)$$

$$= 0.0069$$

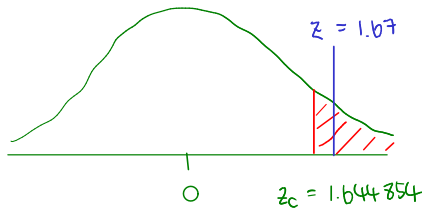
$$\text{Since } P\text{value} = 0.0069 \leq 0.01 = \alpha$$

we reject the null hypothesis

$H_0: P = P_0$  Right Tailed Test

$H_A: P > P_0$  Test Statistic:  $z = 1.67$

$\alpha = 0.05$



$$z_c = \text{qnorm}(1 - 0.05 = 0.95, 0, 1) = 1.644854$$

Since the test statistic is more extreme than the critical value, we reject the null hypothesis at the  $\alpha = 0.05$  level

$$P\text{value} = P(Z > 1.67) = 1 - P(Z < 1.67)$$

$$= 1 - \text{pnorm}(1.67, 0, 1) = 0.0475$$

$$\text{Since } P\text{value} = 0.0475 \leq 0.05 = \alpha$$

we reject the null hypothesis.

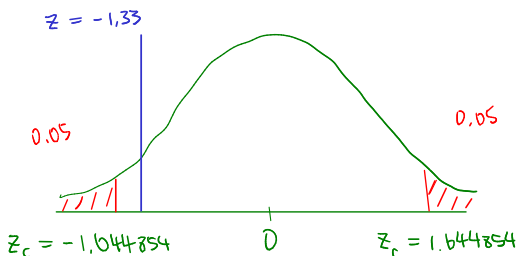
Two-Tailed Test

$H_0: P = P_0$

$H_A: P \neq P_0$

Test Statistic:  $z = -1.33$

$\alpha = 0.10$   $\frac{\alpha}{2} = 0.05$



$$z_c = \text{qnorm}\left(\frac{1 - 0.10}{2} = 0.05, 0, 1\right) = -1.644854$$

Since the test statistic is not more extreme than the critical value, we fail to reject the null hypothesis at the  $\alpha = 0.10$  level.

multiple by 2 because it is two-tailed.

$$P\text{value} = 2 P(Z < -1.33) = 2 \text{qnorm}(-1.33, 0, 1) = 0.1835$$

Since  $P\text{value} = 0.1835 > 0.10 = \alpha$ , we fail to reject the null hypothesis.

$$\text{If } z = +, \text{ then } P\text{value} = 2 (1 - P(Z < x))$$

FTR  $\longrightarrow$   $\left( \xrightarrow{\cdot H_0} \right)$

If  $\alpha = 0.05$  then it is a 95% CI

$\alpha = 0.10 = 90\%$  confidence interval

$$P_0 = 0.409 \quad x = 32 \quad n = 100 \quad \hat{p} = \frac{32}{100} = 0.32 \quad \alpha = 0.05$$

Ex: In a previous study, the National Highway Traffic Safety Administration reported that the proportion of traffic deaths attributable to alcohol was 40.9%. Out of 100 randomly selected traffic deaths this year, 32 were attributable to alcohol. Is there evidence that the true proportion of traffic deaths attributable to alcohol has decreased at the 0.05 significance level?

Step 1: Set up the null and alternative hypothesis

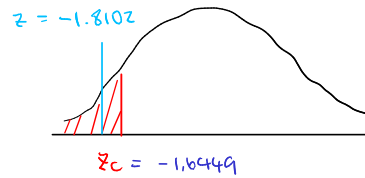
$$H_0 : P = P_0 = 0.409$$

$$H_A : P < P_0 = 0.409$$

The proportion of traffic deaths is less than 0.409

Step 2: Draw a picture

Assuming  $H_0$  is true



Step 3: Calculate the test statistic

$$z = \frac{P - P_0}{\sqrt{\frac{P_0(1-P_0)}{n}}} = \frac{0.32 - 0.409}{\sqrt{\frac{(0.409)(1-0.409)}{100}}} = -1.8102$$

Step 4: Calculate the critical value

$$z_c = \text{qnorm}(0.05, 0, 1) = -1.6449$$

Step 5: Make and justify a statistical decision at 0.05 level, and state your conclusions in context of the problem.

Since the test statistic is more extreme than the critical value, we reject the null hypothesis at the  $\alpha = 0.05$  level. There is statistically significant evidence that the proportion of traffic deaths attributable to alcohol is less than 0.409.

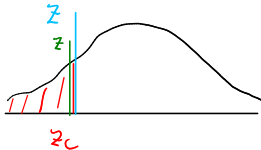
Ex: Suppose you calculate a test statistic and find that the p-value is 0.0511 and your level is 0.05. What is your decision? Do you feel comfortable reporting that? What if the p-value is 0.04999?

$p\text{-value} \leq \alpha$  we reject

$0.0511 > 0.05$  Fail to reject

sampling variability

statistically significant evidence



Ex: According to the Society of Human Resource Management, 19% of human resource professionals are concerned with losing their jobs. Suppose in a recent sample of 1000 employees, 200 said their biggest concern was losing their job.

a. Calculate a 95% confidence interval for the true population proportion  $p$  of employees who are concerned about losing their job.

$$\begin{aligned}
 n &= 1000 & \text{PE} \pm \text{ME} &= \hat{p} \pm z^* \sqrt{\frac{\hat{p}\hat{q}}{n}} \\
 x &= 200 & &= 0.20 \pm qnorm(0.95 + \frac{1-0.95}{2}) = 0.975, 0.1) \sqrt{\frac{(0.2)(0.8)}{1000}} \\
 \hat{p} &= \frac{200}{1000} = 0.20 \\
 p_0 &= 0.19 & &= (0.1752, 0.2248)
 \end{aligned}$$

b. Without conducting a hypothesis test, do you think there is evidence that the proportion of current employees who are concerned about losing their job is different? Explain in a full sentence.

$$\begin{aligned}
 H_a &= p \neq p_0 = 0.19 & \text{Interval: } & 0.1752 \quad 0.2248
 \end{aligned}$$

Since  $p_0 = 0.19$  is captured within the interval, we would fail to reject the claim at the  $\alpha = 0.05$  level.



$$p_0 = 0.85 \quad n = 125 \quad x = 97 \quad \hat{p} = \frac{97}{125} = 0.776 \quad \alpha = 0.10$$

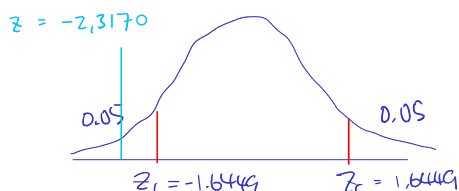
Ex: A local school board claims that 85% of its students score above 1200 on the SAT. A rival school board suspects the actual percentage is different. Of a random sample of 125 students who took the SAT, 97 of them scored above 1200 on the SAT. Test the claim at the 0.10 level.

Step 1: Set up the null and alternative hypothesis

$$H_0: p_0 = 0.85$$

$$H_a: p_0 \neq 0.85 \quad \text{The proportion of students who score above 1200 on the SAT is not 0.85.}$$

Step 2: Draw a picture



$$p\text{-value} = 2 * qnorm(-2.3170)$$

2 tailed

Step 3: Calculate the test statistic

Assume the Null hypothesis is true  $p = p_0$

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.776 - 0.85}{\sqrt{\frac{(0.85)(0.15)}{125}}} = -2.3170$$

Step 4: Calculate the critical value

$$Z_c = qnorm\left(1 - \frac{\alpha}{2}\right) = qnorm\left(1 - \frac{0.10}{2}\right) = qnorm(0.95) = \pm 1.644854$$

Step 5: Make and justify a statistical decision at 0.10 level, and state your conclusions in context of the problem.

Since the test statistic is more extreme than the critical value, we reject the null hypothesis at the  $\alpha$  at 0.10 level. There is statistically significant evidence that the proportion of students who score above 1200 on the SAT is not 0.85.

Would a 90% confidence interval contain the null proportion? Explain in a full sentence (verify by calculating the confidence interval)

( )  $\cdot H_0$

Since we rejected the claim at the  $\alpha = 0.10$  level, a 90% confidence interval would not capture the null value.

$$PE \pm ME = \hat{p} \pm z^* \sqrt{\frac{\hat{p}\hat{q}}{n}} = 0.776 \pm qnorm(0.95) \sqrt{\frac{(0.776)(0.224)}{125}} = (0.7147, 0.8373)$$

Reject  $H_0$   $\because p_0 = 0.85$  Outside

$$\theta = ?$$

$$\mu_0 = 21 \quad \bar{x} = 21.9 \quad s = 3.5 \quad n = 37 \quad \alpha = 0.01$$

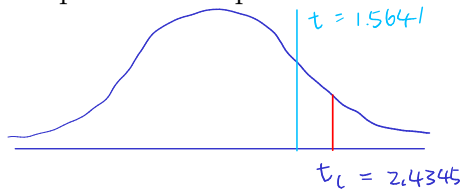
Ex: A feared predator in the ocean is the great white shark. It is known that the great white grows to an average length of 21 feet; however, one marine biologist feels that great whites of the coast off Bermuda grow much longer due to the type and amount of food they consume. Over a period of one year, the marine biologist used a capture/release procedure to measure the sharks. He captured 37 sharks; their average length was 21.9 feet and their standard deviation was 3.5 feet. Test the biologist's claim at the  $\alpha = 0.01$  level.

Step 1: Set up the null and alternative hypothesis

$$H_0: \mu = 21$$

$H_a: \mu > 21$  The average length of Bermuda great whites is greater than 21 feet.

Step 2: Draw a picture



$$p\text{-value} = 1 - pt(1.5641, 36)$$

Step 3: Calculate the test statistic Assume  $H_0$  is true

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{21.9 - 21}{\frac{3.5}{\sqrt{37}}} = 1.5641$$

Step 4: Calculate the critical value

$$t_c = qt(1 - \alpha, df = n - 1) = qt(0.99, 36) = 2.4345$$

Step 5: Make and justify a statistical decision at 0.01 level, and state your conclusions in context of the problem.

Since the test statistic is not more extreme than the critical value, we fail to reject the null hypothesis at the  $\alpha = 0.01$  level. There is not statistically significant evidence that the average length of Bermuda great whites is greater than 21 feet.