CSCI 190 Discrete Mathematics Applied to Computer Science
Final Exam

Name : ______________________

Last 4 digits of your Student ID #: ______________________

Read these instructions before proceeding.

- Closed book. Closed notes. You can use calculator.
- You have **100 minutes** to complete this exam.
- No questions will be answered during the exam or immediately afterwards. Answer each question as best you can. Partial credit will be awarded for reasonable efforts. If a question contains an ambiguity or a misprint, then say so in your answer, providing the answer to a reasonable interpretation of the question; give your assumptions.
- Answer the problems on the blank spaces provided for each problem.
- **Box your answers**.

Q1 (11)	Q2 (12)	Q3 (12)	Q4 (8)	Q5 (12)	Q6 (8)	Q7 (6)	Q8 (6)	Q9 (6)	Q10 (6)	Q11 (4)	Q12 (5)	Q13 (4)	Total (100)

converse $q \to p$

1. *(11 pts)*

a) (3 pts) Write the converse of the following:

If you are happy, then you will smile.

If you smile, then you will be happy

b) (4 pts) Convert $(9FA7)_{16}$ to base 4.

3 2 1 0

$9 \cdot 16^3 + 16 \cdot 16^2 + 10 \cdot 16^1 + 7 \cdot 16^0 = 40871$

40871 mod 4 = 3

10217 mod 4 = 1

2554 mod 4 = 2

638 mod 4 = 2

159 mod 4 = 3

39 mod 4 = 3

9 mod 4 = 1

2

$(9F47)_{16} = (21332213)_4$

c) (4 pts) A message has been ***encrypted*** using the function f**(x) = (x + 4) mod 26**.

If the message in coded form is **NSC**, decode the message.

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

N = (13 − 4) mod 26 = J

S = (18 − 4) mod 26 = O

C = (2 − 4) mod 26 = Y

$\boxed{\text{JOY}}$

2. *(12 pts)*

a) (5 pts) Use the Principle of Mathematical Induction to prove that

2 + 4 + 6 + 8 +· · ·+ 2n = n(n+1)   for all n ≥ 1. Show all the steps

$n = 1$

$2(1) = 1(1+1)$

$2 = 2$

f(1) is true

Asume P(K) is true

$2+4+6+8+\dots+2K = K(K+1)$

$= K^2 + K$

show that f(K+1) is also true

$2+4+6+8+\dots+2K + 2(K+1) = K(K+1)(K+1+1)$

$2+4+6+8+\dots+2K + 2(K+1) = (2(1)+2(2)+2K) + 2(K+1)$

$= K(K+1) + 2(K+1)$

$= (K+1)(K+2)$

$= K^2 + 3K + 2$

$= (K^2 + 2K + 1) + K + 1$

$= (K+1)^2 + (K+1)$

∴ f(n) is true

for all $n \geq 1$

b) (4 pts) Give a recursive definition with initial condition for the following function, square of n factorial.

$f(n) = (n!)^3$, n = 0, 1, 2, ...

$a_0 = (0!)^3 = 1^3 = 1$

$a_1 = (1!)^3 = 1^3 = 1 = 1^3 \cdot a_0$

$a_2 = (2!)^3 = 2^3 = 8 = 2^3 \cdot a_1$

$a_3 = (3!)^3 = 6^3 = 218 = 3^3 \cdot a_2$

$a_4 = (4!)^3 = 24^3 = 13824 = 4^3 \cdot a_3$

$a_{n+1} = (n+1)^3 \cdot a_n$

∴ $a_{n+1} = (n+1)^3 \cdot a_n$

for $n \geq 0$ and

$a_0 = 1$

$$\frac{\binom{7}{x}\cdot\binom{33}{7-x}}{\binom{40}{7}} = \frac{C(7,x)\cdot C(33,7-x)}{C(40,7)}$$ Probability of winning with x matching numbers

c) **(3 pts)** In a certain lottery game you choose a set of seven numbers out of 40 numbers. Find the probability that exactly one x of your numbers match the seven winning numbers.

$$\frac{\binom{7}{1}\cdot\binom{33}{6}}{\binom{40}{7}} = \frac{\frac{7!}{1!6!}\cdot\frac{33!}{6!27!}}{\frac{40!}{7!33!}} = \frac{7\cdot\frac{33\cdot32\cdot31\cdot30\cdot29\cdot28}{6\cdot5\cdot4\cdot3\cdot2}}{\frac{40\cdot39\cdot38\cdot37\cdot36\cdot35\cdot34}{7\cdot6\cdot5\cdot4\cdot3\cdot2}}$$

$$= \frac{7752976}{18643560} = 0.4159$$

3. (12 pts) Determine whether the following binary relation is:
(1) reflexive, (2) symmetric, (3) antisymmetric, (4) transitive.
No justifications needed.

a) (4 pts) The relation ***R*** on Z where ***aRb*** means ***a = b***.
Circle your answers.

R is	Reflexive?	Symmetric?	Antisymmetric?	Transitive?
	Yes or No	Yes or No	Yes or No	Yes or No

b) (4 pts) The relation ***R*** on the set of all people where ***aRb*** means that ***a*** is taller than ***b***.
Circle your answers.

R is	Reflexive?	Symmetric?	Antisymmetric?	Transitive?
	Yes or No	Yes or No	Yes or No	Yes or No

c) (4 pts) If $\boldsymbol{M_R} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$,

determine if ***R*** is: (a) reflexive (b) symmetric (c) antisymmetric (d) transitive.
Circle your answers.

R is	Reflexive?	Symmetric?	Antisymmetric?	Transitive?
	Yes or No	Yes or No	Yes or No	Yes or No

4. (8 pts)

a) (4 pts)Suppose ***R*** is the relation on *N* where ***aRb*** means that ***a*** starts in the same digit in which ***b*** starts.

Determine whether ***R*** is an ***equivalence relation*** on *N*. Justify your answer.

Reflexivity: aRa both a starts in the same digit ∴ it is reflexive

Symmetry: aRb means a starts in the same digit as b
bRa means b starts in the same digit as a ∴ it is symmetric

Transitivity: aRb means a starts in the same digit as b
bRc means b starts in the same digit as c
so a starts in the same digit as c ∴ it is transitive

∴ R is an equivalence relation on N

b) (4 pts) Suppose the relation ***R*** is defined on the set *Z* where ***aRb*** means that $ab < 0$.

Determine whether ***R*** is an ***equivalence relation*** on *Z*. Justify your answer.

Reflexivity: aRa means $a \cdot a < 0$ but a^2 must be greater than or equal to zero ∴ it is not reflexive

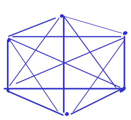
Since it is not reflexive

∴ R is not an equivalence relation on Z

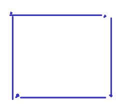
5. (12 pts)

a) (4 pts) Draw these four graphs. $\mathbf{K_6}$, $\mathbf{C_4}$, $\mathbf{W_5}$ and $\mathbf{K_{4,5}}$

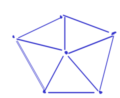
K_6



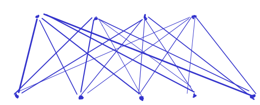
C_4



W_5



$K_{4,5}$



b) (4 pts)

K_n has $\frac{n(n-1)}{2} = 15$ edges and $n = 6$ vertices.

$K_{m,n}$ has $m \cdot n = 20$ edges and $m + n = 9$ vertices.

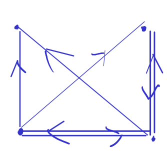
W_n has $2n = 10$ edges and $n+1 = 6$ vertices.

C_n has $n = 4$ edges and $n = 4$ vertices.

c) (4 pts) Draw the ***digraph*** with adjacency matrix

$$\begin{array}{c} \\ \end{array}\begin{bmatrix} 0&0&0&0 \\ 0&0&1&0 \\ 1&1&0&1 \\ 1&1&1&0 \end{bmatrix}\begin{array}{c} a \\ b \\ c \\ d \end{array}$$

a b

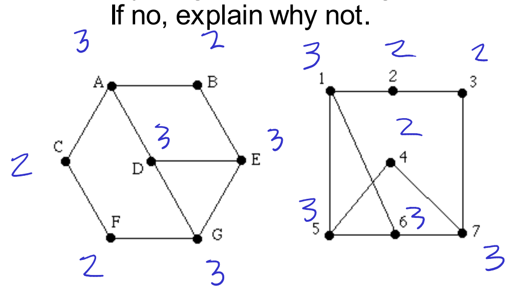


d c

6. (8 pts)

a) (6 pts) Are these two graphs ***isomorphic***?

If yes, give the mapping of vertices from the first graph to the second graph.
If no, explain why not.



A = 7 E = 5

C = 3 B = 4

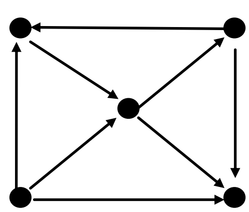
F = 2 D = 6

G = 1

b) (2 pts) Circle ***Yes*** or ***No***. No justifications needed.

Determine whether the graph is ***strongly connected***? Yes or No

Determine whether the graph is ***weakly connected***. Yes or No



7. (6 pts) Circle ***TRUE*** or ***FALSE***. No justifications needed.

T / F If T is a tree with 10 vertices, then there is a simple path in T of length 9.

T / F Every tree is bipartite.

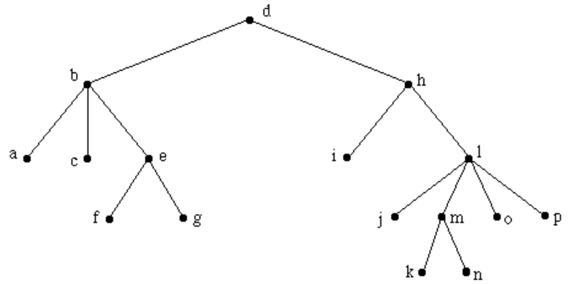
T / F There is a tree with degrees 4, 3, 2, 2, 1, 1, 1, 1, 1.

T / F There is a tree with degrees 3, 3, 3, 2, 1, 1, 1, 1.

T / F If T is a tree with 30 vertices, the largest degree that any vertex can have is 29.

T / F If two trees have the same number of vertices and the same degrees, then the two trees are isomorphic.

8. *(6 pts)* Refer to the following tree.



a) (2 pts) Find the ***preorder*** traversal.

root left right d b a c e f g h i l j m k n o p

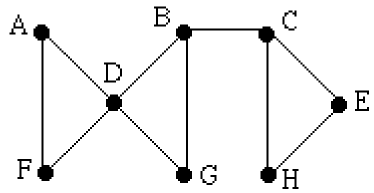
b) (2 pts) Find the ***inorder*** traversal.

left root right a b c f e g d i h j k m n l o p

c) (2 pts) Find the ***postorder*** traversal.

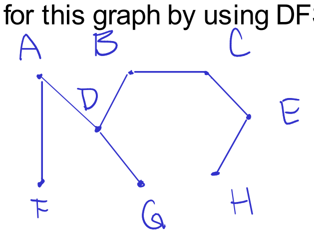
left right root a c f g e b i j k n m o p l h d

9. *(6 pts)* Refer to the following graph..



pre order

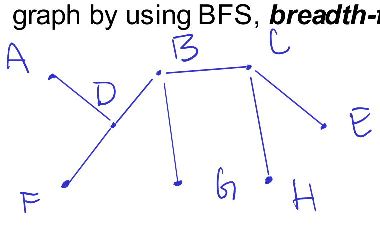
a) (3 pts) Using ***alphabetical ordering***, find ***a spanning tree*** (starting from vertice ***B***) for this graph by using DFS, ***depth-first search***.



B C E H D A F G

level order

b) (3 pts) Using ***alphabetical ordering***, find ***a spanning tree*** (starting from vertice ***B***) for this graph by using BFS, ***breadth-first search***.



B C D G E H A F

10. (6 pts) Using a table to show that ***F(x,y,z) = xyz + xy + x*** has a vaule of 1 if and only if variable ***x*** has a value of 1.

x	y	z	xy	xyz	$xyz + xy + x$
1	1	1	1	1	1
1	1	0	1	0	1
1	0	1	0	0	1
1	0	0	0	0	1
0	1	1	0	0	0
0	1	0	0	0	0
0	0	1	0	0	0
0	0	0	0	0	0

$\therefore$ $F(x,y,z) = xyz + xy + x$ has a value of 1 if and only if x has a value of 1 is true

11. (4 pts) Find the ***duals*** of these Boolean expressions.

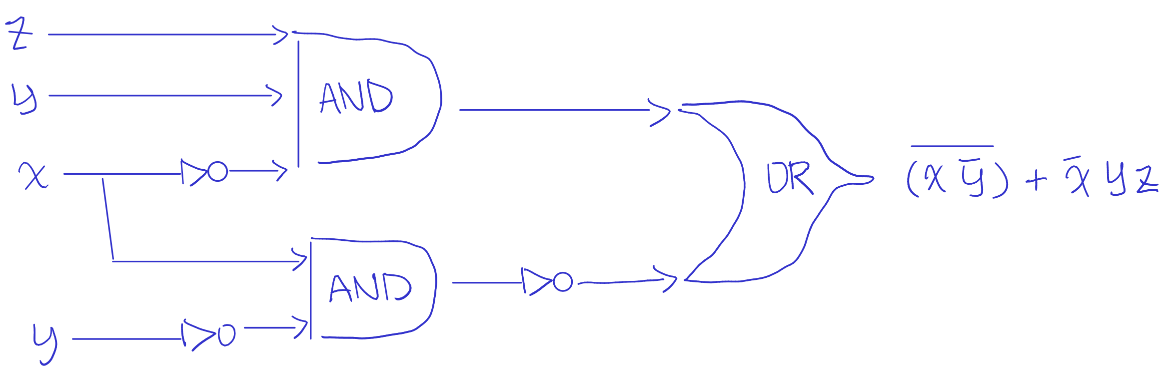
a) (2 pts) 0 + x + y

$1\,x\,y$

b) (2 pts) $x\,\overline{y}\,z$

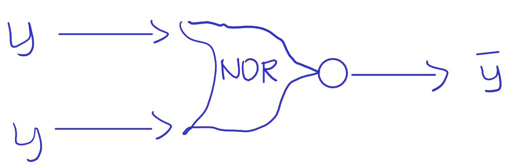
$x + \overline{y} + z$

12. (5 pts) Draw a logic gate diagram for the Boolean function **$F(x, y, z) = \overline{(x\ \overline{y})} + \overline{x}\,y\,z$.**



13. (4 pts) Use **NOR** gates (only) to construct circuits with these outputs.

a) (2 pts) $\overline{y}$



b) (2 pts) y z