Ivan Leung ihleung

Problem 1.1 (Q.1)

a) ☑ True ☐ False

$\forall x ((P(x) \wedge R(x)) \rightarrow \neg Q(x))$

For all x, if x is an even number and x is a prime, then x is not a multiple of 3.

It is true. For all the positive even integers that are prime number they are not multiple of 3.

b) ☑ True ☐ False

$\exists x (\neg P(x) \wedge \neg Q(x) \wedge \neg R(x))$

There exists a x that is not an even number, not a multiple of 3, and not a prime number.
49 is an odd number, not a multiple of 3, and not a prime number, therefore, it is true.

Problem 1.2 (Q.1)

$\forall x \forall y (P(x,y) \vee Q(x,y))$

a) ☐ True ☑ False ☐ Undetermined

$P(x,y)$ is false when $x = y = 2$, we have $2^2 + 2^2 = 4 + 4 = 8$ which is greater than 4,
$Q(x,y)$ is also false when $x = y = -1$, we have $-1 + (-1) = -2$ which is less than 0.
Since it says "for every x with every y", we only need one counterexample for each statement to prove them false.
Since both truth values are false, the statement is false.

b) ☐ True ☐ False ☑ Undetermined

$\exists x (P(x,y) \wedge Q(x,y) \wedge R(x,y))$

Since y is free variable, the statement is undetermined.

c) ☑ True ☐ False ☐ Undetermined $\exists x \forall y (P(x,y) \wedge R(x,y))$

When x is equal to zero, $P(x,y)$ is true because for every value y can be in the domain, y^2 can only be as large as 4.
When x is equal to zero, $R(x,y)$ is true because every number times zero, the result is zero.

Since both truth value are true, the statement is true.

Problem 1.3 (Q.1)

a) ☑ True ☐ False $\forall x \exists y \forall z (P(x,y) \rightarrow Q(y,z))$

$P(a,b)$ (false) $\rightarrow Q(b,a)$ (false)
$P(a,b)$ (false) $\rightarrow Q(b,b)$ (false)
$P(b,a)$ (false) $\rightarrow Q(a,a)$ (false)
$P(b,a)$ (false) $\rightarrow Q(a,b)$ (true)

} since all the hypothese are false, all the statements are true

Since for every x there exists a y for every z is true, the statement is true

b) ☐ True ☑ False $\exists x \exists y \forall z (P(x,y) \wedge Q(y,z))$

$Q(y,z)$ is never true because there does not exist a y that all z is true.
$Q(a,a)$ = false ← if this is true, then $\exists y \forall z Q(y,z)$ will be true.
$Q(a,b)$ = true
$Q(b,a)$ = false
$Q(b,b)$ = false

Since it is a conjunction and $\exists y \forall z Q(y,z)$ is false, the statement is false.

c) ☐ True ☑ False $\forall x \forall y \forall z (P(x,y) \vee \neg Q(y,z))$

$x=a, y=a, z=a$	$P(a,a)$ (true)
$x=a, y=a, z=b$	$P(a,a)$ (true)
$x=a, y=b, z=a$	$\neg Q(b,a)$ (true)
$x=a, y=b, z=b$	$\neg Q(b,b)$ (true)
$x=b, y=b, z=b$	$P(b,b)$ (true)
$x=b, y=b, z=a$	$P(b,b)$ (true)
$x=b, y=a, z=b$	$P(b,a)$ (false) $\vee \neg Q(a,b)$ (false)
$x=b, y=a, z=a$	$Q(a,a)$ (true)

when $x=b, y=a, z=b$, either predicates is true. Therefore, for every x, every y, and every z, they are true is a false statement.

Problem 2 (Q.2)

a)

$\forall x \; (\; P(x, Jim) \wedge \neg Q(x) \;)$

Every member knows Jim won a science competition
and they do not received a special recognition certificate

b)

$\exists x \; (\; \neg R(x) \; \wedge \; \forall y \; (P(y, Jim)) \;)$

$\exists x$: At least one member

$\neg R(x)$: is not a seasoned club member

$\wedge$: and

$\forall y$: every member

$P(y, Jim)$: knows that Jim won a science competition

c)

$\exists x \exists y \; (((x \neq y) \wedge (Q(x) \rightarrow R(x)) \wedge (Q(y) \rightarrow R(y))) \wedge \forall z \; ((z \neq x \wedge z \neq y) \rightarrow \neg (Q(z) \rightarrow R(z)))$

$\exists x \exists y$: there are two members

$(x \neq y)$: x and y are two distinct members

$\wedge (Q(x) \rightarrow R(x))$: and if x received a special recognition certificate then x is a seasoned club member

$\wedge (Q(x) \rightarrow R(x))$: and if y received a special recognition certificate then y is a seasoned club member

$\forall z \; ((z \neq x \wedge z \neq y) \rightarrow \neg (Q(z) \rightarrow R(z)))$: for every z that are not x or y member, they received a special recognition certificate or they are not seasoned club member.

Problem 3 (Q.3)

a)

$\neg \forall x \exists y (P(x,y) \wedge Q(y) \rightarrow \exists z R(x,y,z))$	start
$\exists x \neg \exists y (P(x,y) \wedge Q(y) \rightarrow \exists z R(x,y,z))$	Demorgan's law
$\exists x \forall y \neg (P(x,y) \wedge Q(y) \rightarrow \exists z R(x,y,z))$	Demorgan's law
$\exists x \forall y \neg (P(x,y) \wedge Q(y) \rightarrow \exists z R(x,y,z))$	Demorgan's law
$\exists x \forall y \neg ((P(x,y) \wedge Q(y)) \rightarrow \exists z R(x,y,z))$	Associative law
$\exists x \forall y \neg (\neg (P(x,y) \wedge Q(y)) \vee \exists z R(x,y,z))$	Conditional identity
$\exists x \forall y (\neg\neg (P(x,y) \wedge Q(y)) \wedge \neg \exists z R(x,y,z))$	Demorgan's law
$\exists x \forall y ((P(x,y) \wedge Q(y)) \wedge \neg \exists z R(x,y,z))$	Double negation law
$\exists x \forall y ((P(x,y) \wedge Q(y)) \wedge \forall z \neg R(x,y,z))$	Demorgan's law

b)

$\neg \exists x \forall y ((P(x) \leftrightarrow Q(y)) \vee R(x,y))$	start
$\forall x \exists y \neg ((P(x) \leftrightarrow Q(y)) \vee R(x,y))$	Demorgan's law
$\forall x \exists y \neg (((P(x) \rightarrow Q(y)) \wedge (Q(y) \rightarrow P(x))) \vee R(x,y))$	Conditional identity
$\forall x \exists y \neg (((\neg P(x) \vee Q(y)) \wedge (Q(y) \rightarrow P(x))) \vee R(x,y))$	Conditional identity
$\forall x \exists y \neg (((\neg P(x) \vee Q(y)) \wedge (\neg Q(y) \vee P(x))) \vee R(x,y))$	Conditional identity
$\forall x \exists y (\neg ((\neg P(x) \vee Q(y)) \wedge (\neg Q(y) \vee P(x))) \wedge \neg R(x,y))$	Demorgan's law
$\forall x \exists y ((\neg\neg (\neg P(x) \vee Q(y)) \vee \neg (\neg Q(y) \vee P(x))) \wedge \neg R(x,y))$	Demorgan's law
$\forall x \exists y (((\neg\neg p(x) \wedge \neg Q(y)) \vee \neg (\neg Q(y) \vee P(x))) \wedge \neg R(x,y))$	Demorgan's law
$\forall x \exists y (((p(x) \wedge \neg Q(y)) \vee \neg (\neg Q(y) \vee P(x))) \wedge \neg R(x,y))$	Double negation law
$\forall x \exists y (((P(x) \wedge \neg Q(y)) \vee (\neg\neg Q(y) \wedge \neg P(x))) \wedge \neg R(x,y))$	Demorgan's law
$\forall x \exists y (((P(x) \wedge \neg Q(y)) \vee (Q(y) \wedge \neg P(x))) \wedge \neg R(x,y))$	Double negation law