Exercise 1.6

1. Find the argument form for the following argument and determine whether it is valid. Can we conclude that the conclusion is true if the premises are true? If Socrates is human, then Socrates is mortal.

Socrates is human.

∴ Socrates is mortal.

Let p be "Socrates is human."

Let q be "Socrates is mortal."

p

$p \rightarrow q$

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$\therefore q$

Argument form: Modus ponens

It is valid.

Yes, the conclusion is true since the hypotheses are true.

3. What rule of inference is used in each of these arguments?

a) Alice is a mathematics major. Therefore, Alice is either a mathematics major or a computer science major.

Let p be "Alice is a mathematics major."

Let q be "Alice is a computer science major."

P

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$\therefore p \wedge q$

It uses Addition rule.

5. Use rules of inference to show that the hypotheses "Randy works hard," "If Randy works hard, then he is a dull boy," and "If Randy is a dull boy, then he will not get the job" imply the conclusion "Randy will not get the job."

Let p be "Randy works hard."

Let q be "Randy is a dull boy."

Let r be “Randy will get the job.”

$p \rightarrow q$

p

$\therefore q$ Modus Ponens

$q \rightarrow \neg r$

q

$\therefore \neg r$ Modus Ponens

7. What rules of inference are used in this famous argument? “All men are mortal. Socrates is a man. Therefore, Socrates is mortal.”

Let Mortal(x) be “x is mortal.”

Let Man(x) be “x is a man.”

Where the domain is all men.

$\forall x\, (\text{Man}(x) \rightarrow \text{Mortal}(x))$

$\therefore \text{Man}(\text{Socrates}) \rightarrow \text{Mortal}(\text{Socrates})$ Universal Instantiation

Man(Socrates)

$\therefore \text{Mortal}(\text{Socrates})$ Modus Ponens

Exercise 1.7

3. Show that the square of an even number is an even number using a direct proof.

Assume x is an even number,

$x = 2k$ for some integer k

$x^2 = (2k)^2$

$x^2 = 4k^2$

$x^2 = 2(2k^2)$

$x^2 = 2r$ for $r = 2k^2$

$\therefore$ x^2 is an even number

17. Show that if n is an integer and $n^3 + 5$ is odd, then n is even using

a) a proof by contraposition.

Let p be "$n^3 + 5$ is odd."

Let q be "n is even."

Assume n is odd,

$n = 2k + 1$ for some integer k

$n^3 + 5 = (2k + 1)^3 + 5$

$n^3 + 5 = 8k^3 + 12k^2 + 6k + 1 + 5$

$n^3 + 5 = 8k^3 + 12k^2 + 6k + 6$

$n^3 + 5 = 2(4k^3 + 6k^2 + 3k + 3)$

$n^3 + 5 = 2r$ for $r = 4k^3 + 6k^2 + 3k + 3$

$\therefore$ $n^3 + 5$ is even

Since $\neg p \rightarrow \neg q$ is true, $p \rightarrow q$ must also be true.

$\therefore$ if n is an integer and $n^3 + 5$ is odd, then n is even.

b) a proof by contradiction.

Assume $n^3 + 5$ is odd and n is odd,

$n = 2k + 1$ for some integer k

$n^3 + 5 = (2k + 1)^3 + 5$

$n^3 + 5 = 8k^3 + 12k^2 + 6k + 1 + 5$

$n^3 + 5 = 8k^3 + 12k^2 + 6k + 6$

$n^3 + 5 = 2(4k^3 + 6k^2 + 3k + 3)$

$n^3 + 5 = 2r$ for $r = 4k^3 + 6k^2 + 3k + 3$

$n^3 + 5$ is even.

This contradicts the assumption that $n^3 + 5$ is odd.

$\therefore$ if $n^3 + 5$ is odd, then n is even