Graded for Honest Effort

1.12.5 c) c : I will buy a new car

h : I will buy a new house

j : I am going to get a job

$$\begin{array}{l} (c \wedge h) \rightarrow j \\ \neg j \\ \hline \therefore \neg c \end{array}$$

1	$(c \wedge h) \rightarrow j$	hypothesis	
2	$\neg j$	hypothesis	
3	$\neg(c \wedge h)$	Modus tollens	(1, 2)
4	$\neg c \vee \neg h$	DeMorgan's law	
	$\therefore \neg c$		

c	h	$\neg c$	$\neg h$	$\neg c \vee \neg h$	
T	T	F	F	F	
T	F	F	T	T	← Invalid
F	T	T	F	T	
F	F	T	T	T	

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1.12.5 d) c : I will buy a new car

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$$\begin{array}{l} (c \wedge h) \rightarrow j \\ \neg j \\ h \\ \hline \therefore \neg c \end{array}$$

1	$(c \wedge h) \rightarrow j$	hypothesis
2	$\neg j$	hypothesis
3	$\neg(c \wedge h)$	modus tollens (1, 2)
4	$\neg c \vee \neg h$	Demorgan's law
5	$\neg h \vee \neg c$	Commutative law
6	h	hypothesis
$\therefore$	$\neg c$	Disjunctive syllogism (5, 6)

Valid

Graded for Honest Effort

1.13.1 a) $P(x)$: x practices hard

$B(x)$: x plays badly

$$\forall x\,(P(x) \vee B(x))$$
$$\exists x\,(\neg P(x))$$
$$\therefore \exists x\,(B(x))$$

D.S.

1	$\forall x\,(P(x) \vee B(x))$	hypothesis
2	$\exists x\,(\neg P(x))$	hypothesis
3	$c \wedge \neg P(c)$	existential instantiation (2)
4	c is a particular element	simplification (3)
5	$P(c) \vee B(c)$	universal instantiation
6	$\neg P(c)$	simplification (3)
7	$B(c)$	disjunctive syllogism
$\therefore$	$\exists x\, B(c)$	existential generalization (4, 7)

Valid

Graded for Honest Effort

1.13.4 b) $\exists x\, Q(x) \wedge \exists x\, P(x)$

$\therefore \exists x\, (P(x) \wedge Q(x))$

	$Q(x)$	$P(x)$
a	T	F
b	F	T

There exists a "a" that Q(a) is true and there exists a "b" that P(x) is true but "a" is false for P(x) and "b" is false for Q(x).
Therefore it is invalid.

Graded for Honest Effort

2.2.1 a) False, we need to proof the statement holds for all possibles cases within the domain because an universal statement is a general statement that claims to be true for all possible cases.

2.2.5 b) Suppose x is a positive integer such that

$$x = 1 + 2 + \dots + (x-1)$$

when $x = 3$

then $1 + 2 = 3$

$\therefore$ there exist a positive integer x that is equal to the sum of all the positive integers less than x

d) Suppose c is a integer and d is a integer such that $7c + 5d = 1$

we can use algebra to find c and d

$$7c + 5d = 1$$

$$7c = 1 - 5d$$

Now, we plug in an arbitrary integer for d to solve for c until we find that c is an integer

Suppose $d = 10$

$$7c = 1 - 5(10)$$

$$7c = 1 - 50$$

$$7c = -49$$

$$c = -7$$

$\therefore$ when $c = -7$, $d = 10$ such that $7c + 5d = 1$

Graded for Honest Effort

2.3.2 a) In the proof, integer k is being used for both kw and ky where y should use a different integer variable.

b) Instead of having $xz = m \cdot wy$, it should have

$$xz = (kw)(jy)$$

c) It is missing the step where it shows

$$xz = (kw)(jy)$$
$$xz = (kj)(wy)$$

d) I do not find any mistakes in the proof.

Graded for Honest Effort

2.4.3 b) If x is a real number and $x \leq 3$ then
$12 - 7x + x^2 \geq 0$

Assume x is a real number and $x \leq 3$
we shall prove that $12 - 7x + x^2 \geq 0$

$$12 - 7x + x^2 = 0$$
$$(x-4)(x-3) = 0$$
$$x = 4 \quad , \quad x = 3$$

$$x < 3 \quad , \quad 2^2 - 7(2) + 12 = 2$$

Therefore, for x values less than or equal to 3,
$12 - 7x + x^2 \geq 0$

2.4.4 m) If x, y and z are integers and $x \mid (y+z)$,
then $x \mid y$ or $x \mid z$

If $x = 3$, $y = 4$, $z = 5$
$3 \mid 9$ but $3 \nmid 4$ and $3 \nmid 5$

Therefore, the statement is false.

Graded for Honest Effort and Feedback Given

1.13.4 a) $\dfrac{\exists x\,(P(x) \wedge Q(x))}{\therefore\ \exists x\, P(x) \wedge \exists x\, Q(x)}$

1	$\exists x\,(P(x) \wedge Q(x))$	hypothesis
2	c is a particular element $\wedge$ $(P(c) \wedge Q(c))$	existential instantiation
3	$P(c) \wedge Q(c)$	simplification (2)
4	c is a particular element	simplification (2)
5	$P(c)$	simplification (3)
6	$\exists x\, P(x)$	existential generalization (4,5)
7	$Q(c)$	simplification (3)
8	$\exists x\, Q(x)$	existential generalization (4,7)
$\therefore$	$\exists x\, P(x) \wedge \exists x\, Q(x)$	Conjunction (6,8)

Valid

2.4.4 k) If x, y, and z are integers and $xy \mid z$, then $x \mid z$ and $y \mid z$

Assume x, y and z are integers and $xy \mid z$, we prove that $x \mid z$ and $y \mid z$

Since $xy \mid z$, then $z = kxy$ for some integer k. Since k and x are integers then kx is also an integer since z equal y times an integer, which means y divides z. Similarly, since k and y are integers, then ky is also an integer. Since z equal to x times an integer, which means x divide z.

Therefore, $x \mid z$ and $y \mid z$. ■

Graded for Honest Effort and Feedback Given

2.5.3 c) For every pair of real numbers x and y, if x is rational and xy is irrational, then y is irrational.

Assume that x and y are real numbers and y is rational number and prove that x is irrational or xy is rational.

Since y is rational, $y = \frac{c}{d}$, where c and d are integers and $d \neq 0$.

Since x is rational, $x = \frac{e}{f}$, where e and f are integers and $f \neq 0$.

$xy = \frac{e}{f} \cdot \frac{c}{d} = \frac{ec}{fd}$, where ec and fd are integers and $fd \neq 0$.

Therefore, if y is rational, then xy is rational or x is irrational.

Graded for Honest Effort and Feedback Given

2.6.6 d) There is no smallest integer.

Assume that there is a smallest integer x. Since x is an integer, $x-1$ is also an integer. However, $x-1$ is less than x, it contradicts the assumption there is a smallest integer x.

∎

Graded for Honest Effort and Feedback Given

2.7.3 d) For real numbers x and y, $|x+y| \leq |x| + |y|$.
You can use the fact proven in the previous problem that for any real number z, $z \leq |z|$ and $-z \leq |z|$.

Case 1: $x \geq 0$, $y \geq 0$

since x and y are both positive
$|x+y| = x+y$ and $|x| + |y| = x+y$
$x+y \leq x+y$

Therefore, it holds.

Case 2: $x < 0$, $y \geq 0$

since x is negative and y is positive,
$|x+y| = -(x+y)$ and $|x| + |y| = -x+y$
$-(x+y) \leq -x+y$
$-x-y \leq -x+y$
$-y \leq y$

Since y is positive, it is always true.

Case 3: $x \geq 0$, $y < 0$

since x is positive and y is negative
$|x+y| = -(x+y)$ and $|x| + |y| = x-y$

$$-(x+y) \leq x-y$$
$$-x-y \leq x-y$$
$$-x \leq x$$

Since x is positive, it is always true.

Case 4: $x < 0$, $y < 0$

Since both x and y are negative,
$|x+y| = -(x+y)$ and $|x|+|y| = -x+(-y)$
$$-(x+y) \leq -x+(-y)$$
$$-x-y \leq -x-y$$

Therefore, it holds

Since all 4 cases hold,
therefore $|x+y| \leq |x|+|y|$ is true