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Instructions

Please read the following instructions carefully:

1. Please show all notation to recieve full credit.
2. Please verify that your scans are legible.
3. Please assign pages the the questions when submitting to gradescope.
4. This assignment is due via gradescope on the due date.

1. For endangered species, like the Eastern Lowland Gorilla, one aspect that interests conservationists and zoologists is the survival time, in months, of the females upon reaching sexual maturity. Survival time, in this example, is the technical term for saying "how much longer will a female Eastern Lowland Gorilla live after reaching sexual maturity?" One statistical model used to model survival times is the exponential distribution.

   (a) (2 points) Assume you have n independent observations from the following exponential pdf,

$$f(x) = \frac{1}{\beta}e^{\left(-\frac{1}{\beta}x\right)}$$

   Write out the likelihood function.

$$L(\theta) = \prod_{i=1}^{n} \frac{1}{\beta} e^{\left(-\frac{1}{\beta}x_i\right)} = \left(\frac{1}{\beta}e^{(-\frac{1}{\beta}x_1)}\right)\left(\frac{1}{\beta}e^{(-\frac{1}{\beta}x_2)}\right)\ldots\left(\frac{1}{\beta}e^{(-\frac{1}{\beta}x_n)}\right)$$

$$= \boxed{\left(\frac{1}{\beta}\right)^n e^{\frac{-\sum_{i=1}^{n} x_i}{\beta}}}$$

   (b) (2 points) Write out the log-likelihood equation.

$$\ell(\theta) = \ln\left(\left(\frac{1}{\beta}\right)^n e^{\frac{-\sum_{i=1}^{n} x_i}{\beta}}\right) = \cancel{n\ln(1)} - n\ln(\beta) + \cancel{\ln}(\cancel{e}^{-\frac{\sum_{i=1}^{n} x_i}{\beta}})$$

$$= \boxed{-n\ln(\beta) - \frac{\sum_{i=1}^{n} x_i}{\beta}}$$

   (c) (2 points) Take the partial derivative of the log-likelihood equation.

$$\frac{d}{d\beta}\ell(\theta) = \frac{d}{d\beta}\left(-n\ln(\beta) - \frac{\sum_{i=1}^{n} x_i}{\beta}\right)$$

$$= \boxed{-\frac{n}{\beta} + \frac{\sum_{i=1}^{n} x_i}{\beta^2}}$$

   (d) (2 points) Set the partial derivative equal to zero and solve for β.

$$-\frac{n}{\beta} + \frac{\sum_{i=1}^{n} x_i}{\beta^2} = 0$$

$$\beta^2\left(\frac{\sum_{i=1}^{n} x_i}{\beta^2}\right) = \left(\frac{n}{\beta}\right)\beta^2$$

$$\sum_{i=1}^{n} x_i = n\beta$$

We believe this is the maximum likehood estimator

$$\boxed{\frac{\sum_{i=1}^{n} x_i}{n} = \beta}$$

(e) (2 points) Find the second order derivative. Is the second derivative concave down at your estimated value.

$\frac{d}{d\theta}$ score < 0

$$\frac{d}{d\theta}\left(-\frac{n}{\beta}+\frac{\sum_{i=1}^{n} x_i}{\beta^2}\right) < 0$$

$$\left(\frac{\beta}{\beta}\right)\frac{n}{\beta^2}-\frac{2\sum_{i=1}^{n} x_i}{\beta^3} < 0$$

$$\frac{n\beta}{\beta^3}-\frac{2\sum_{i=1}^{n} x_i}{\beta^3} < 0$$

$$\frac{\cancel{n}\left(\frac{\sum_{i=1}^{n} x_i}{\cancel{n}}\right)}{\beta^3}-\frac{2\sum_{i=1}^{n} x_i}{\beta^3} < 0$$

$$\frac{\sum_{i=1}^{n} x_i}{\beta^3}-\frac{2\sum_{i=1}^{n} x_i}{\beta^3} < 0$$

$$\boxed{-\frac{\sum_{i=1}^{n} x_i}{\beta^3} < 0}$$

Yes, it concaves down

(f) (2 points) The data was collected on eight female Eastern Lowland Gorillas. The data recorded included the number of years the female gorillas lived after reaching maturity. The number of years recorded were: 33, 42, 39, 35, 34, 38, 40, and 32. What is the corresponding MLE for β, the survival time?

$$\beta = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{33+42+39+35+34+38+40+32}{8}$$

$$= \frac{293}{8}$$

$$= \boxed{36{,}625}$$