

Ch. 1.4 G's. 1(a), 3(a), 5(a), 7(a)

Ch. 1.5 G's. 1(a), 3(a), 5(a), 9(a)(b)

H.W. (Bonus Point)

Ch. 1.4 Predicates and Quantifiers

1(a) 1. Let  $P(x)$  denote the statement " $x \leq 4$ ." What are these truth values?

a)  $P(0)$ , T, Since  $0 \leq 4$

3. Let  $Q(x, y)$  denote the statement " $x$  is the capital of  $y$ ." What are these truth values?

a)  $Q(\text{Denver}, \text{Colorado})$

$\Rightarrow$  "Denver is the capital of Colorado."

$\therefore$  This is true.

5. Let  $P(x)$  be the statement " $x$  spends more than five hours every weekday in class," where the domain for  $x$  consists of all students. Express each of these quantifications in English.

a)  $\exists x P(x)$

"There is a student who spends more than five hours every weekday in class."

7. Translate these statements into English, where  $C(x)$  is " $x$  is a comedian" and  $F(x)$  is " $x$  is funny" and the domain consists of all people.

a)  $\forall x (C(x) \rightarrow F(x))$

$\Rightarrow$  "For every  $x$ , if  $x$  is a comedian, then  $x$  is funny."

$\Rightarrow$  "Every comedian is funny."

## 1.5 Nested Quantifiers

1. Translate these statements into English, where the domain for each variable consists of all real numbers.

a)  $\forall x \exists y (x < y)$

=> for every real number  $x$  there exists a real number  $y$  such that  $x$  is less than  $y$ .

=> There is no largest real number, since there is always a larger number when we name any real number.

3. Let  $Q(x, y)$  be the statement " $x$  has sent an e-mail message to  $y$ ," where the domain for both  $x$  and  $y$  consists of all students in your class. Express each of these quantifications in English.

a)  $\exists x \exists y Q(x, y)$

=> There exist students  $x$  and  $y$  such that  $x$  has sent an e-mail message to  $y$ .

=> There is some students in your class who has sent an e-mail message to some student in your class.  
(students  $x$  &  $y$  can be the same student.)

5. Let  $W(x, y)$  mean that student  $x$  has visited website  $y$ , where the domain for  $x$  consists of all students in your school and the domain for  $y$  consists of all websites. Express each of these statements by a simple English sentence.

a)  $W(\text{Sarah Smith}, \text{www.att.com})$

=> Sarah Smith has visited website [www.att.com](http://www.att.com).

9. Let  $L(x, y)$  be the statement “ $x$  loves  $y$ ,” where the domain for both  $x$  and  $y$  consists of all people in the world. Use quantifiers to express each of these statements.

a) Everybody loves Jerry.

b) Everybody loves somebody.

$$a) L(\text{Everybody}, \text{Jerry})$$

$$\Rightarrow \forall x L(x, \text{Jerry})$$

$$b) \forall x \exists y L(x, y)$$