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Instructions

Please read the following instructions carefully:

1. Please show all notation for probability statements.
2. Box your final answers.
3. Please verify that your scans are legible.
4. Please assign pages the the questions when submitting to gradescope.
5. This assignment is due via gradescope on the due date.

 $\mu = 20$ $n = 400$ $\bar{x} = 25$ $s = 8.3$ $\alpha = 0.05$

1. The video game company Square Enix is interested in determining if the average time spent playing Final Fantasy XIV among active players has increased following the release of a new game expansion in 2021. They have historical data from 2019, indicating that the average time spent playing the game was 20 hours per week. After randomly surveying 400 active players, they found that the mean time spent playing the game has now increased to 25 hours per week, with a sample standard deviation of 8.3 hours. The data science team aims to test whether there has been a significant increase in the average gaming time since 2019 due to the new expansion. Let μ represent the true mean daily gaming time after the expansion's release. Perform this test at a significance level of $\mu = 0.05$.

   (a) Set up the null and alternative hypothesis (using mathematical notation/numbers AND interpret them in context of the problem).

   $H_0: \mu = 20 \rightarrow$ the average gaming time since 2019 is the same.

   $H_A: \mu > 20 \rightarrow$ The average gaming time since 2019 has been increased.

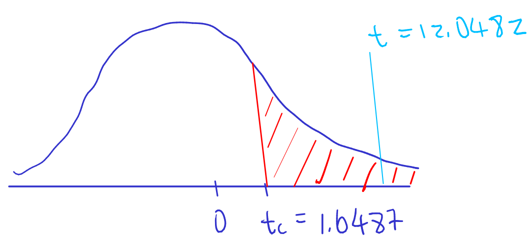
   (b) Calculate the test statistic.

   $$t = \frac{\bar{x} - \mu_0}{\left(\frac{s}{\sqrt{n}}\right)} = \frac{25 - 20}{\left(\frac{8.3}{\sqrt{400}}\right)} = \boxed{12.0482}$$

   (c) Calculate the critical value.

   $$t_c = qt(1 - \alpha, df = n - 1) = qt(0.95, 399) = \boxed{1.6487}$$

   (d) Draw a picture of the distribution of the test statistic under H_0. *Label and provide values for* the critical value and the test statistic, and shade the critical region.



   (e) Make and justify a statistical decision at the $\alpha = 0.05$ level and state your conclusion.

   Since the test statistic is more extreme than the critical value, we reject the null hypothesis at the alpha = 0.05 level.

   There is statistically significant evidence that the average gaming time since 2019 is greater than before due to the new expansion.

Population Mean for Paired Difference
Sigma Unknown

 $n = 28$ $\bar{x}_d = -65$ $s = 42$ $\alpha = 0.05$ $\mu_d = 0$

2. A researcher in the field of environmental science wants to investigate if a new policy aiming to reduce water pollution will lead to improved water quality in a local river over a 10-week period. Twenty-eight water samples are collected from various points along the river before and after implementing the new pollution control policy. The difference in water quality, measured in parts per million (ppm), is calculated, and the researcher is interested in determining if the new method has significantly reduced water pollution. Among the 28 samples, the average decrease in water pollution is 65 ppm, with a sample standard deviation of 42 ppm. Conduct a test at the $\alpha = 0.05$ level to assess the effectiveness of the new pollution control policy.

   (a) Set up the null and alternative hypothesis (using mathematical notation/numbers AND interpret them in context of the problem).

   $H_0: \mu_d = 0 \rightarrow$ there is no difference in water pollution levels after implementing the pollution control policy for 10 weeks on average.

   $H_a: \mu_d < 0 \rightarrow$ there is a decrease in water pollution levels after implementing the pollution control policy for 10 weeks on average.

   $\mu_d \rightarrow$ the true mean difference in water quality

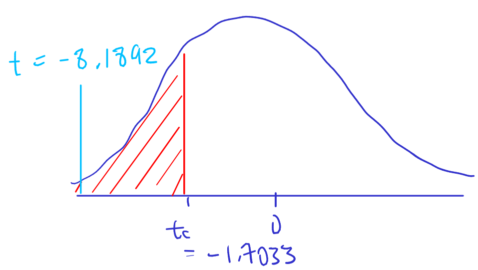
   (b) Calculate the test statistic.

   $$t = \frac{\bar{x}_d - \mu_d}{\frac{s_d}{\sqrt{n_d}}} = \frac{-65-0}{\frac{42}{\sqrt{28}}} = \boxed{-8.1892}$$

   (c) Calculate the critical value.

   $$t_c = qt(\alpha, df = n-1) = qt(0.05, 27) = \boxed{-1.7033}$$

   (d) Draw a picture of the distribution of the test statistic under H_0. *Label and provide values for* the critical value and the test statistic, and shade the critical region.



   (e) Make and justify a statistical decision at the $\alpha = 0.05$ level and state your conclusions in context of the problem.

   Since the test statistic is more extreme than the critical value, we reject the null hypothesis at the alpha = 0.05 level.
   There is statistically significant evidence that there is a decrease in waterpllution levels after implementing the pollution control policy for 10 weeks on average.

2 Population Mean

Group 1 (Strength)	$\alpha = 0.10$	Group 2 (Endurance)
$\mu_1 = 0$		$\mu_2 = 0$
$\bar{x}_1 = 2.1$		$\bar{x}_2 = 2.5$
$s_1 = 1.3$		$s_2 = 1.1$
$n_1 = 10$		$n_2 = 14$

3. A sports coach wants to compare the effects of two different training programs (strength-focused training vs. endurance-focused training) on the performance of track-and-field in 800-meter races. To investigate, the coach randomly selects 24 athletes and assigns 10 to the strength training group and 14 to the endurance training group. After 6 weeks, the coach measures the improvement in performance for each group. The average running time for the strength training group improved by 2.1 seconds with a standard deviation of 1.3 seconds. For the endurance training group, the average running time improved by 2.5 seconds with a standard deviation of 1.1 seconds. Let μ_1 be the true time improvement for the strength training group and μ_2 be the true time improvement for the endurance training group. Are these two means different? Perform a test at an $\alpha = 0.10$ level.

   (a) Set up the null and alternative hypothesis (using mathematical notation/numbers AND interpret them in context of the problem).

   $H_0: \mu_1 = \mu_2 \rightarrow$ the two training programs lead to the same time improvement on average.

   $H_A: \mu_1 \neq \mu_2 \rightarrow$ the two training programs lead to the difference time improvement on average.

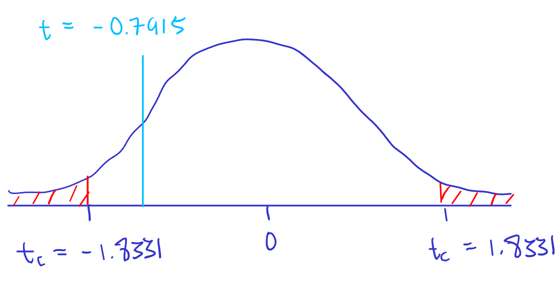
   (b) Calculate the test statistic.

   $$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(2.1 - 2.5) - 0}{\sqrt{\frac{1.3^2}{10} + \frac{1.1^2}{14}}} = \boxed{-0.7915}$$

   (c) Calculate the critical value.

   $$t_c = \pm\, qt\left(1 - \frac{\alpha}{2},\ df = n - 1\right) = \pm\, qt(0.95, 9) = \boxed{\pm 1.8331}$$

   (d) Draw a picture of the distribution of the test statistic under H_0. *Label and provide values for* the critical value and the test statistic, and shade the critical region.



(e) Make and justify a statistical decision at the $\alpha = 0.1$ level and state your conclusions in context of the problem.

Since the test statistic is not more extreme than the critical value, we fail to reject the null hypothesis at the alpha = 0.10 level.

There is not statistically significant evidence that the two training programs lead to the difference time improvement on average.

(f) Construct a **90%** confidence interval for the true difference in weight loss from the 2 different diet plans.

$$PE \pm ME = (\bar{x}_1 - \bar{x}_2) \pm t_c \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$= (2.1 - 2.5) \pm 1.8331 \sqrt{\frac{1.3^2}{10} + \frac{1.1^2}{14}}$$

$$= \boxed{(-1.3264,\ 0.5264)}$$

(g) Does this interval reaffirm your statistical decision from the hypothesis test? Explain.

Yes, since mu 1 - mu 2 equal to zero and it is captured within the confidence interval, it reaffirms the statistical decision from the hypothesis test.

Group 1 (Male) $\alpha = 0.01$ Group 2 (Female)

$n_1 = 300$ $n_2 = 250$

$x_1 = 180$ $x_2 = 135$

$\hat{p}_1 = \frac{180}{300} = 0.6$ $\hat{p}_2 = \frac{135}{250} = 0.54$

4. A market researcher aims to investigate the difference in smartphone preferences between men and women. They collect two samples: the first consists of 300 male smartphone users, and the second comprises 250 female smartphone users. Among the male participants, 180 use an Android smartphone, while among the female participants, 135 use an Android device. Let p_1 be the proportion of male smartphone users who use Android devices, and p_2 be the proportion of female smartphone users who use Android devices. Perform a test at the $\alpha = 0.01$ level to determine if there is a significant difference in smartphone preference between men and women.

find p-value ?

(a) Set up the null and alternative hypothesis (using mathematical notation/numbers AND interpret them in context of the problem).

$\hat{p}_1 - \hat{p}_2 = 0.6 - 0.54 = 0.06$ $\hat{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{180 + 135}{300 + 250} = 0.5727$

$H_0: p_1 = p_2 \longrightarrow p_1 - p_2 = 0$ smartphone preference between men and women are the same.

$H_A: p_1 \neq p_2 \longrightarrow$ smartphone preference between men and women are not the same.

(b) Calculate the test statistic.

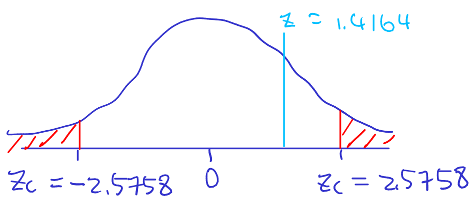
$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\hat{p}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} = \frac{0.06 - 0}{(0.5727)(1 - 0.5727)\left(\frac{1}{300} + \frac{1}{250}\right)}$$

$$= \boxed{1.4164}$$

(c) Calculate the critical value.

$z_c = \pm \text{qnorm}(1 - \frac{\alpha}{2}, 0, 1) = \pm \text{qnorm}(0.995, 0, 1) = \boxed{\pm 2.5758}$

(d) Draw a picture of the distribution of the test statistic under H_0. *Label and provide values for* the critical value and the test statistic, and shade the critical region.



(e) Make and justify a statistical decision at the $\alpha = 0.01$ level and state your conclusions in the context of the problem.

Since the test statistic is not more extreme than the critical value, we fail to reject the null hypothesis at the alpha = 0.01 level.

There is not statistically significant evidence that smartphone preference between men and women are not the same.

1 Population Proportion

 $p = 0.13$ $\quad x = 92$ $\quad n = 550$ $\quad \hat{p} = \frac{x}{n} = \frac{92}{550} = 0.1673$ $\quad \alpha = 0.05$

5. A wildlife conservation organization is interested in assessing the impact of its awareness campaign on the desert tortoise, an endangered species in Joshua Tree National Park. They previously conducted a survey in July 2021 and found that 13% of visitors were aware of the species' existence. They aim to investigate if this percentage has increased since then. Let p represent the true proportion of individuals who are aware of the endangered species after the campaign. To evaluate the campaign's effectiveness, they randomly surveyed 550 people in July 2022 and discovered that 92 of them are now aware of the species. They intend to perform a test at a significance level of $\alpha = 0.05$.

   (a) Set up the null and alternative hypothesis (using mathematical notation/numbers AND interpret them in context of the problem).

   $H_0: p_0 = 0.13 \longrightarrow$ the true proportion of individuals who are aware of the endangered species after the campaign is the same.

   $H_A: p_0 > 0.13 \longrightarrow$ the true proportion of individuals who are aware of the endangered species after the campaign has increased.

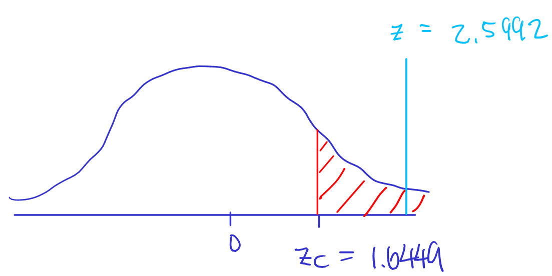
   (b) Calculate the test statistic.

   $$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{0.1673 - 0.13}{\sqrt{\frac{(0.13)(1-0.13)}{550}}} = \boxed{2.5992}$$

   (c) Calculate the critical value.

   $$z_c = \text{qnorm}(1-\alpha, 0, 1) = \text{qnorm}(0.95, 0, 1) = \boxed{1.6449}$$

   (d) Draw a picture of the distribution of the test statistic under H_0. *Label and provide values for* the critical value and the test statistic, and shade the critical region.



(e) Make and justify a statistical decision at the $\alpha = 0.05$ level and state your conclusions in context of the problem.

Since the test statistic is more extreme than the critical value, we reject the null hypothesis at the alpha = 0.05 level.

There is statistically significant evidence that the true proportion who are aware of endangered species after the campaign is greater than before the campaign.

(f) If we increased the sample size to be very large (>1000) but kept the sample proportion the same how (if at all) would you expect your decision of this hypothesis test to change?

We know that $z = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0 q_0}{n}}}$.

If we increase the sample size to be very large but kept the sample proportion the same it will make the denomiantor very small, therefore the test statistic will become larger. Since the test statistic is already more extreme than the critical value, it does not change the result of the hypothesis test.

6. For the multiple choice questions, circle the best answer.

a) In hypothesis testing, a Type 1 error occurs when:

   A. the null hypothesis is not rejected when the alternative hypothesis is true.
   B. the null hypothesis is rejected when the alternative hypothesis is true.
   C. the null hypothesis is not rejected when the null hypothesis is true.
   D. the null hypothesis is rejected when the null hypothesis is true.

b) In hypothesis testing, a Type 2 error occurs when:

   A. the null hypothesis is not rejected when the alternative hypothesis is true.
   B. the null hypothesis is not rejected when the null hypothesis is true.
   C. the null hypothesis is rejected when the alternative hypothesis is true.
   D. the null hypothesis is rejected when the null hypothesis is true.

c) In a hypothesis test, if the null hypothesis is actually false, what type of error could be made?

   A. Type 1.
   B. Type 2.
   C. Type 1 if it's a one-sided test and Type 2 if it's a two-sided test.
   D. Type 2 if it's a one-sided test and Type 1 if it's a two-sided test.

d) If we decide to reject the null hypothesis, which type of mistake could have been made when making this decision?

   A. Type 1.
   B. Type 2.
   C. Type 1 if it's a one-sided test and Type 2 if it's a two-sided test.
   D. Type 2 if it's a one-sided test and Type 1 if it's a two-sided test.

e) In an American criminal trial, the null hypothesis is that the defendant is innocent and the alternative hypothesis is that the defendant is guilty. Which of the following describes a Type 2 error for a criminal trial?

   A. A not guilty verdict for a person who is innocent
   B. A guilty verdict for a person who is innocent.
   C. A guilty verdict for a person who is not innocent.
   D. A not guilty verdict for a person who is guilty

f) A decrease in α in a statistical test for parameter θ, would make which of the following true? (select all that apply)

   A. Type I error decreases
   B. Type II error increases
   C. Power decreases
   D. $Var(\theta)$ decreases

Inversely related

g) The power of a statistical test increases when: (select all that apply)

   A. sample size is increased
   B. lowering α
   C. the value of the true parameter is closer to the null
   D. test sensitivity is decreased

h) If I performed a test with $\alpha = 0.05$, what is the probability that I correctly fail to reject the null hypothesis?

A. 0.05

B. 0.10

C. 0.95

D. Not enough information to answer the question.