1) Domain : Customers going to a car dealer

Target : A list of cars that are available for purchase

Range : A list of cars that are actually bought by customers

Function f from customers to cars is well defined if every customer bought exactly one car.

f is one to one if every customer bought exactly one car. There may be more cars available for purchase

f is onto if all cars are sold to the customers. Some customers may bought more than one car,

f is bijection if every customer bought exacty one car and no car is unsold.
f^{-1} is in charge of each car.

2) $f(x) = 2x + 3$
$g(x) = 3x + 2$

$$f \circ g = f(g(x)) = f(3x+2) = 2(3x+2)+3$$
$$= 6x + 4 + 3$$
$$= 6x + 7$$

$$g \circ f = g(f(x)) = g(2x+3) = 3(2x+3) + 2$$
$$= 6x + 9 + 2$$
$$= 6x + 11$$

$$6x + 7 \neq 6x + 11$$

$\therefore$ they are not commutative

3) $A = \{1,2,3,4,5,6,7,8,9,10\}$
$B = \{A, B, C, D, E, F\}$
let $f: B \times B \to A \times A$
$\quad\quad 36 \quad\quad\quad 100$

a) No, because the domain has less elements than the target.

b) Yes, it is possible to be one to one since the domain has less elements than the target.

4) a) Domain : $(x \in \mathbb{Z} : x \geq 0)$

Range : $(x, k \in \mathbb{Z} : x = 2k$ and $k \geq 0$ and $x \geq 0\}$

b) Domain : $\{0,1\}^n, n \in \mathbb{Z}^+\}$

Range : $\{0,1,2,3,4,5,6,7\}$

c) Domain : $\{x \in \mathbb{Z}^+\}$

Range : $\{x \in \mathbb{Z} : x^2\}$

5) a) f is not a function since a bit string may contain multiple 0 bits.

b) f is a function since the number of 1 bit is equal to or greater than zero.

c) f is not a function since a bit string may contain any 1 bits.

b) $p(xyz) = xzx$

$q(xyz) = xzy$

$r(xyz) = yxz$

$v(xyz) = (r \circ q)(xyz)$

$v(xyz) = r(q(xyz))$

$= r(xzy)$

$= zxy$

$v^{-1}(xyz) = (q \circ r)(xyz)$

$= q(r(xyz))$

$= q(yxz)$

$= yzx$

$q^{-1}(xyz) = xzy$

$p^{-1}(xyz)$ does not exist