Problem 1 (P.1)

☑ Valid ☐ Invalid

a $\neg R(\text{Jake}) \wedge W(\text{Jake})$

b $W(\text{Susan}) \rightarrow A(\text{Jake})$

c $\forall x\ (R(x) \vee H(x))$

d $\therefore \exists x\ H(x)$

1	Jake is a particular element	element declaration
2	$\forall x\ (R(x) \vee H(x))$	hypothesis
3	$R(\text{Jake}) \vee H(\text{Jake})$	Universal instantiation (1,2)
4	$\neg R(\text{Jake}) \wedge W(\text{Jake})$	hypothesis
5	$\neg R(\text{Jake})$	simplification (4)
6	$H(\text{Jake})$	Disjunctive syllogism (3,5)
7	$\exists x\ H(x)$	Existential generalization (1,6)

Problem 2 (P.2)

a)

Assume that x and y are real numbers and $x > 46$ and $y > 64$

Therefore, $x + y > 110$.

b)

Assume that there exists a number cannot be written as a product of prime numbers

Problem 3 (P.3)

the proof jumps to the conclusion that x is the smallest number. The fact that x is the smallest number is the fact that needs to be prove. It is a example of circular reasoning.

Problem 4.1 (P.4)

Assuming:

Assume that n is an integer

WTP (What To Prove):

We will prove that $9n^2 + 3n + 6$ is divisible by 6 for any integer n.

Proof:

Since n is an integer, we have to consider two cases

Case 1: n is odd

Since n is odd, $n = 2k+1$ for some integer k

Plugging in $2k+1$ for n in $9n^2 + 3n + 6$, we get

$$\begin{aligned} 9n^2 + 3n + 6 &= 9(2k+1)^2 + 3(2k+1) + 6 \\ &= 9(4k^2 + 4k + 1) + (6k + 3) + 6 \\ &= 36k^2 + 36k + 9 + 6k + 3 + 6 \\ &= 36k^2 + 42k + 18 \\ &= 6(6k^2 + 7k + 3) \end{aligned}$$

Since k is an integer, $6k^2 + 7k + 3$ is also an integer.

Therefore, $9n^2 + 3n + 6$ is divisible by 6 when n is odd.

Case 2: n is even

Since n is even, $n = 2j$ for some integer j

Plugging in $2j$ for n in $9n^2 + 3n + 6$, we get

$$\begin{aligned} 9n^2 + 3n + 6 &= 9(2j)^2 + 3(2j) + 6 \\ &= 9(4j^2) + 6j + 6 \\ &= 36j^2 + 6j + 6 \\ &= 6(6j^2 + j + 1) \end{aligned}$$

Since j is an integer, $6j^2 + j + 1$ is also an integer.

Therefore, $9n^2 + 3n + 6$ is divisible by 6 when n is even.

Conclusion:

The two cases cover all possible integers. Therefore, $9n^2 + 3n + 6$ is divisible by 6 for any integer n.

Problem 4.2 (P.4)

Assuming:

Proof by contradiction: Assume that there are 982 freshmen which need to be assigned to 320 housing units such that there is no freshman with more than 2 roommates.

WTP (What To Prove):

We will prove that there will be a freshman with at least 3 roomates.

Proof:

Assume that there is no freshman with more than 2 roomates which means each housing unit has at most 3 freshmen. And we have 320 housing units, then we have

$$3 \cdot 320 = 960$$

which means we can accommodate at most 960 freshmen. However, we have a total of 982 freshmen which need to be assigned to 320 housing units. That means there will be a freshman with more than 2 roommates.

Conclusion:

The fact that there is no freshman with more than 2 roommates contradicts with the fact that there will be a freshman with at least 3 roommates.