Exercise 5.1

3. Let P (n) be the statement that $1^2 + 2^2 + \cdots + n^2 = n(n + 1)(2n + 1)/6$ for the positive integer n.

a) What is the statement P (1)?

n = 1, p(1)

$1^2 = 1(1 + 1)(2(1) + 1)/6$

$1^2 = 1(2)(3)/6$

b) Show that P(1) is true, completing the basis step of the proof.

$1^2 = 1(2)(3)/6$

$1 = 6/6$

$1 = 1$

∴ P(1) is true.

c) What is the inductive hypothesis?

$1^2 + 2^2 + \cdots + k^2 = k(k + 1)(2k + 1)/6$

d) What do you need to prove in the inductive step?

We show that for every positive k, $p(k) \rightarrow p(k + 1)$ is true.

$1^2 + 2^2 + \cdots + k^2 + (k + 1)^2 = (k + 1)(k + 2)(2k + 3)/6$

e) Complete the inductive step, identifying where you use the inductive hypothesis.

$$1^2 + 2^2 + \cdots + k^2 + (k + 1)^2 = (1^2 + 2^2 + \cdots + k^2) + (k + 1)^2$$

$$= \frac{k(k + 1)(2k + 1)}{6} + (k + 1)^2$$

$$= \frac{k + 1}{6}(k(2k + 1) + 6(k + 1)$$

$$= \frac{k + 1}{6}(2K^2 + K + 6K + 6)$$

$$= \frac{k + 1}{6}(2K^2 + 7K + 6)$$

$$= \frac{k + 1}{6}(K + 2)(2K + 3)$$

$$= \frac{(k + 1)(K+2)(2K+3)}{6}$$

f) Explain why these steps show that this formula is true whenever n is a positive integer.

We have completed both the basis step and the inductive step, so by the principle of mathematical induction, the statement is true for every positive integer n.

Exercise 5.2

3. Let P (n) be the statement that a postage of n cents can be formed using just 3-cent stamps and 5-cent stamps. The parts of this exercise outline a strong induction proof that P (n) is true for $n \geqslant 8$.

a) Show that the statements P (8), P (9), and P (10) are true, completing the basis step of the proof.

p(8) is true because 8 cents of postage can be formed by one 3-cent stamp and one 5-cent stamp.

p(9) is true because 9 cents of postage can be formed by three 3-cent stamps.

p(10) is true because 10 cents of postage can be formed by two 5-cent stamps.

b) What is the inductive hypothesis of the proof?

We can form j cents of postage for all j with $8 \leq j \leq k$, where k is greater than 10.

c) What do you need to prove in the inductive step?

We can form k + 1 cents of postage using just 3-cent stamps and 5-cent stamps.

d) Complete the inductive step for $k \geqslant 10$.

Because $k \geqslant 10$, we know that P $(k - 2)$ is true so that we can form $k-2$ cents of postage. By putting one more 3-cent stamp on the envelope, we have formed k + 1 cents of postage.

e) Explain why these steps show that this statement is true whenever $n \geqslant 8$.

We have completed both the basis step and the inductive step, so by the principle of strong induction, the statement is true for every integer $n \geqslant 8$

Exercise 5.3

7. Give a recursive definition of the sequence $\{a_n\}$, n = 1, 2, 3, . . . if

a) $a_n = 6n$.

$a_1 = 6(1) = 6$

$a_2 = 6(2) = 12$

$a_3 = 6(3) = 18$

$a_n + 1 = 6(n + 1) = 6n + 6 = a_n + 6$

$\therefore\ a_n + 1 = a_n + 6$ for $n \geqslant 1$ and $a_1 = 6$

Exercise 5.4

9. Give a recursive algorithm for finding the sum of the first n odd positive integers.

Procedure sum of odd number (n: positive integer)

If n = 1, return n

Else return (2n – 1) + sum of odd number (n – 1)