Ch. 10.1 GRAPHS AND GRAPH MODELS

Q3, 5, 7, 9

Q3: This is a simple graph; the edges are undirected, and there are no parallel edges or loops.

Q5: This is a pseudograph; the edges are undirected, but there are loops and parallel edges.

Q7: This is a directed graph; the edges are directed, but there are no parallel edges. (Loops and antiparallel edges-see the solution to Exercise ld for a definition-are allowed in a directed graph.)

Q9: This is a directed multigraph; the edges are directed, and there is a set of parallel edges.

Ch. 10.2 GRAPH TERMINOLOGY AND SPECIAL TYPES OF GRAPHS

Q3, 7

Q3: There are 9 vertices here, and 12 edges.

The degree of each vertex is the number of edges incident to it.

Thus deg(a) = 3, deg(b) = 2, deg(c) = 4, deg(d) = 0 (and hence d is isolated), deg(e) = 6, deg(!) = 0 (and hence f is isolated), deg(g) = 4, deg(h) = 2, and deg(i) = 3. Note that the sum of the degrees is 3 + 2 + 4 + 0 + 6 + 0 + 4 + 2 + 3 = 24, which is twice the number of edges.

Q7: This directed graph has 4 vertices and 7 edges.

The in-degree of vertex a is deg- (a) = 3 since there are 3 edges with a as their terminal vertex; its out-degree is deg+(a) = 1 since only the loop has a as its initial vertex. Similarly we have deg-(b) = 1, deg+(b) = 2, deg-(c) = 2, deg+(c) = 1, deg-(d) = 1, and deg+ (d) = 3 . As a check we see that the sum of the in-degrees and the sum of the out-degrees are equal (both are equal to 7).

Ch. 10.3 Representing Graphs and Graph Isomorphism

Q1, 3, 13

Q1:

Vertex	Adjacent vertices
a	b,c,d
b	a,d
c	a,d
d	a,b,c

Q3:

Initial vertex	Terminal vertices
a	a,b,c,d
b	d
c	a,b
d	b,c,d

Q13:

If there are k parallel edges between vertices i and j, then we put the number k into the (i,j)th entry of the matrix. There is only one pair of parallel edges.

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 2 \\ 1 & 1 & 0 & 1 \\ 0 & 2 & 1 & 0 \end{bmatrix}$$

Ch. 10.4 Connectivity

Q1(a)(b), 11

Q1:

a) This is a path of length 4, but it is not simple, since edge {b, c} is used twice. It is not a circuit, since it ends at a different vertex from the one at which it began.

b) This is not a path, since there is no edge from c to a.

Q11:

a) Notice that there is no path from a to any other vertex, because both edges involving a are directed toward a. Therefore the graph is not strongly connected. However, the underlying undirected graph is clearly connected, so this graph is weakly connected.

b) Notice that there is no path from c to any other vertex, because both edges involving c are directed toward c. Therefore the graph is not strongly connected. However, the underlying undirected graph is clearly connected, so this graph is weakly connected.

c) The underlying undirected graph is clearly not connected (one component has vertices b, f, and e), so this graph is neither strongly nor weakly connected.

Ch. 10.5 Euler and Hamilton Paths

Q1

Since there are four vertices of odd degree (a, b, c, and e) and $4 > 2$, this graph has neither an Euler circuit nor an Euler path.