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Problem 1.1 (Q.1)

a) ☑ True ☐ False

$\forall x\, ((P(x) \wedge R(x)) \rightarrow \neg Q(x))$

For all x, if x is an even number and x is a prime, then x is not a multiple of 3.

It is true. For all the positive even integers that are prime number they are not multiple of 3.

b) ☑ True ☐ False

$\exists x\, (\neg P(x) \wedge \neg Q(x) \wedge \neg R(x))$

There exists a x that is not an even number, not a multiple of 3, and not a prime number.

49 is an odd number, not a multiple of 3, and not a prime number. Therefore, it is true.

Problem 1.2 (Q.1)

a) ☐ True ☑ False ☐ Undetermined

$\forall x \forall y\, (P(x,y) \vee Q(x,y))$

P(x,y) is false when $x = y = 2$, we have $2^2 + 2^2 = 4 + 4 = 8$ which is greater than 4.

Q(x,y) is also false when $x = y = -1$, we have $-1 + (-1) = -2$ which is less than 0.

Since it says "for every x with every y", we only need one counterexample for each statement to prove them false.

Since both truth values are false, the statement is false.

b) ☐ True ☐ False ☑ Undetermined

$\exists x\, (P(x,y) \wedge Q(x,y) \wedge R(x,y))$

Since y is free variable, the statement is undetermined.

c) ☑ True ☐ False ☐ Undetermined $\exists x \forall y \, (P(x,y) \wedge R(x,y))$

when x is equal to zero, $P(x,y)$ is true because for every value y can be in the domain, y^2 can only be as large as 4.
when x is equal to zero, $R(x,y)$ is true because every number times zero, the result is zero.

Since both truth value are true, the statement is true.

Problem 1.3 (Q.1)

a) ☑ True ☐ False $\forall x \exists y \forall z \, (P(x,y) \rightarrow Q(y,z))$

$P(a,b)$ (false) $\rightarrow Q(b,a)$ (false)
$P(a,b)$ (false) $\rightarrow Q(b,b)$ (false)
$P(b,a)$ (false) $\rightarrow Q(a,a)$ (False)
$P(b,a)$ (false) $\rightarrow Q(a,b)$ (true)

since all the hypothese are false, all the statements are true

Since for every x there exists a y for every z is true, the statement is true

b) ☐ True ☑ False $\exists x \exists y \forall z \, (P(x,y) \wedge Q(y,z))$

$Q(y,z)$ is never true because there does not exist a y that all z is true.
$Q(a,a)$ = false $\leftarrow$ if this is true, then $\exists y \forall z \, Q(y,z)$ will be true.
$Q(a,b)$ = true
$Q(b,a)$ = false
$Q(b,b)$ = false

Since it is a conjunction and $\exists y \forall z \, Q(y,z)$ is false, the statement is false.

c) ☐ True ☑ False $\forall x \forall y \forall z \, (P(x,y) \vee \neg Q(y,z))$

$x=a, y=a, z=a$	$P(a,a)$ (True)
$x=a, y=a, z=b$	$P(a,a)$ (True)
$x=a, y=b, z=a$	$\neg Q(b,a)$ (true)
$x=a, y=b, z=b$	$\neg Q(b,b)$ (true)
$x=b, y=b, z=b$	$P(b,b)$ (true)
$x=b, y=b, z=a$	$P(b,b)$ (true)
$x=b, y=a, z=b$	$P(b,a)$ (false) $\vee \neg Q(a,b)$ (false)
$x=b, y=a, z=a$	$Q(a,a)$ (true)

when $x=b, y=a, z=b$, either predicates is true. Therefore, for every x, every y, and every z, they are true is a false statement.

Problem 2 (Q.2)

a)

$\forall x \, (P(x, Jim) \wedge \neg Q(x))$

Every member knows Jim won a science competition
and they do not received a special recognition certificate

b)

$\exists x \, (\neg R(x) \wedge \forall y \, (P(y, Jim)))$

$\exists x$: At least one member
$\neg R(x)$: is not a seasoned club member
$\wedge$: and
$\forall y$: every member
$P(y, Jim)$: knows that Jim won a science competition

c)

$\exists x \exists y \, (((x \neq y) \wedge Q(x) \wedge Q(y)) \longrightarrow (R(x) \wedge R(y)))$

$\exists x$: there is a member
$\exists y$: there is a member
$(x \neq y)$: indicates they are two distinct members
$(x \neq y) \wedge Q(x) \wedge Q(y)$: If x and y are two distict members and they received a special recognition certificate,
$\rightarrow (R(x) \wedge R(y))$: then x any y are seasoned members.

Problem 3 (Q.3)

a)

$\neg \forall x \exists y \, (P(x,y) \land Q(y) \to \exists z \, R(x,y,z))$	start
$\exists x \forall y \, \neg (P(x,y) \land Q(y) \to \exists z \, R(x,y,z))$	Demorgan's law
$\exists x \forall y \, \neg ((P(x,y) \land Q(y)) \to \exists z \, R(x,y,z))$	Associative law
$\exists x \forall y \, \neg (\neg (P(x,y) \land Q(y)) \lor \exists z \, R(x,y,z))$	Conditional identity
$\exists x \forall y \, (\neg\neg (P(x,y) \land Q(y)) \land \neg \exists z \, R(x,y,z))$	Demorgan's law
$\exists x \forall y \, ((P(x,y) \land Q(y)) \land \neg \exists z \, R(x,y,z))$	Double negation law
$\exists x \forall y \, ((P(x,y) \land Q(y)) \land \forall z \, \neg R(x,y,z))$	Demorgan's law

b)

$\neg \exists x \forall y \, ((P(x) \leftrightarrow Q(y)) \lor R(x,y))$	Start
$\forall x \exists y \, \neg ((P(x) \leftrightarrow Q(y)) \lor R(x,y))$	Demorgan's law
$\forall x \exists y \, \neg (((P(x) \to Q(y)) \land (Q(y) \to P(x))) \lor R(x,y))$	Conditional identity
$\forall x \exists y \, \neg (((\neg P(x) \lor Q(y)) \land (Q(y) \to P(x))) \lor R(x,y))$	Conditional identity
$\forall x \exists y \, \neg (((\neg P(x) \lor Q(y)) \land (\neg Q(y) \lor P(x))) \lor R(x,y))$	Conditional identity
$\forall x \exists y \, (\neg ((\neg P(x) \lor Q(y)) \land (\neg Q(y) \lor P(x))) \land \neg R(x,y))$	Demorgan's law
$\forall x \exists y \, ((\neg (\neg P(x) \lor Q(y)) \lor \neg (\neg Q(y) \lor P(x))) \land \neg R(x,y))$	Demorgan's law
$\forall x \exists y \, (((\neg\neg P(x) \land \neg Q(y)) \lor \neg (\neg Q(y) \lor P(x))) \land \neg R(x,y))$	Demorgan's law
$\forall x \exists y \, (((P(x) \land \neg Q(y)) \lor \neg (\neg Q(y) \lor P(x))) \land \neg R(x,y))$	Double negation law
$\forall x \exists y \, (((P(x) \land \neg Q(y)) \lor (\neg\neg Q(y) \land \neg P(x))) \land \neg R(x,y))$	DeMorgan's law
$\forall x \exists y \, (((P(x) \land \neg Q(y)) \lor (Q(y) \land \neg P(x))) \land \neg R(x,y))$	Double negation law