Exercise 6.1

1. There are 18 mathematics majors and 325 computer science majors at a college.

a) In how many ways can two representatives be picked so that one is a mathematics major and the other is a computer science major?

18 * 325 = 5850

b) In how many ways can one representative be picked who is either a mathematics major or a computer science major?

18 + 325 = 343

3. A multiple-choice test contains 10 questions. There are four possible answers for each question.

a) In how many ways can a student answer the questions on the test if the student answers every question?

4^{10} = 1028576

b) In how many ways can a student answer the questions on the test if the student can leave answers blank?

5^{10} = 9765625

7. How many different three-letter initials can people have?

26^3 = 17576

Exercise 6.2

1. Show that in any set of six classes, each meeting regularly once a week on a particular day of the week, there must be two that meet on the same day, assuming that no classes are held on weekends.

We have six classes but only five weekdays, by the pigeonhole principle, at least two days must be held on the same day.

3. A drawer contains a dozen brown socks and a dozen black socks, all unmatched. A man takes socks out at random in the dark.

a) How many socks must he take out to be sure that he has at least two socks of the same color?

He must take at least three socks to be sure he has at least two socks of the same color.

b) How many socks must he take out to be sure that he has at least two black socks?

He must take at least fourteen socks to be sure that he has at least two black socks.

Exercise 6.3

1. List all the permutations of {a, b, c}.

abc, acb, bac, bca, cab, cba

5. Find the value of each of these quantities.

a) P(6, 3)

$p(n, r) = n(n – 1)(n – 2)…(n – r + 1)$

$= 6(6 – 1)(6 – 2)$

$= 6 * 5 * 4$

$= 120$

9. How many possibilities are there for the win, place, and show (first, second, and third) positions in a horse race with 12 horses if all orders of finish are possible?

$p(n, r) = n(n – 1)(n – 2)…(n – r + 1)$

$= p(12, 3)$

$= 12(12 – 1)(12 – 2)$

$= 12(11)(10)$

$= 1320$

Exercise 6.4

5. How many terms are there in the expansion of $(x + y)^{100}$ after like terms are collected?

There are $n + 1$ term of $(x + y)^n$, so the expansion of $(x + y)^{100}$ has $100 + 1$ terms.

$\therefore$ the expansion of $(x + y)^{100}$ has 101 terms.

9. What is the coefficient of $x^{101}y^{99}$ in the expansion of $(2x - 3y)^{200}$?

$(2x + (-3y))^{200} = \sum_{j=99}^{200} \binom{200}{99} (2x)^{200-99} (-3y)^{99}$

$= \binom{200}{99} (2)^{101} (-3)^{99}$

$= C(200, 99) (2)^{101} (-3)^{99}$

$= -\frac{200!}{99!\ 1!} (2)^{101} (3)^{99}$

13. What is the row of Pascal's triangle containing the binomial coefficients $\binom{9}{k}$, $0 \leqslant k \leqslant 9$?

The 10th row of Pascal's triangle:

1 9 36 84 126 126 84 36 9 1