ch. 7.1 1, 3, 24(a), 33

1. What is the probability that a card selected at random from a standard deck of 52 cards is an ace?

4 of the deck of cards are aces.

$\frac{4}{52} = \frac{1}{13} \approx 7.7\%$

3. What is the probability that a randomly selected integer chosen from the first 100 positive integers is odd?

Among the first 100 positive integers, there are exactly 50 odd.

$50/100 = \frac{1}{2}$

24. Find the probability of winning a lottery by selecting the correct six integers, where the order in which these integers are selected does not matter, from the positive integers not exceeding

a) 30.

$${}_nC_r = \frac{n!}{(n-r)!\,r!}$$

$$\frac{1}{\binom{30}{6}} = \frac{1}{{}_{30}C_6} = \frac{1}{\frac{30!}{(30-6)!\,6!}} = \frac{24!\,6!}{30!} = \frac{\cancel{24!}\,6!}{30\cdot 29\cdot 28\cdot 27\cdot 26\cdot 25\cdot \cancel{24!}}$$

$$= \frac{6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{30\cdot 29\cdot 28\cdot 27\cdot 26\cdot 25}$$

$$\approx 0.000001684$$

33. What is the probability that Abby, Barry, and Sylvia win the first, second, and third prizes, respectively, in a drawing if 200 people enter a contest and

a) no one can win more than one prize.
b) winning more than one prize is allowed.

a) There are $200 \cdot 199 \cdot 198$ equally likely outcomes of the drawings only one of these do Abby, Barry and Sylvia win the 1st, 2nd, 3rd prizes, respectively.

$$\frac{1}{200 \cdot 199 \cdot 198} = \frac{1}{7880400}$$

b) There are $200 \cdot 200 \cdot 200$ equally likely outcomes of the drawings only one of these do Abby, Barry and Sylvia win the 1st, 2nd, 3rd prizes, respectively.

$$\frac{1}{200 \cdot 200 \cdot 200} = \frac{1}{8{,}000{,}000}$$

7.2 Probability Theory

Ch. 7.2 1, 3, 23

1. What probability should be assigned to the outcome of heads when a biased coin is tossed, if heads is three times as likely to come up as tails? What probability should be assigned to the outcome of tails?

Given: $P(H) = 3P(T)$

$P(H) + P(T) = 1$

$3P(T) + P(T) = 1$

$4P(T) = 1$

$P(T) = \frac{1}{4}$ ⟶ probability of tail

$P(H) + \frac{1}{4} = 1$

$P(H) = 1 - \frac{1}{4}$

$P(H) = \frac{3}{4}$ ⟶ probability of heads

3. Find the probability of each outcome when a biased die is rolled, if rolling a 2 or rolling a 4 is three times as likely as rolling each of the other four numbers on the die and it is equally likely to roll a 2 or a 4.

let denote by t the probability that a 2 or 4 appears.

$(1-t)$ is the probability that some other number appear.

$t = 3(1-t)$

$t = 3 - 3t$

$4t = 3$

$t = \frac{3}{4}$

we assume from the statement of the problem that 2 and 4 are equally likely.

=> each of them must have probability 3/8.

=> each of other numbers (1, 3, 5, or 6) must have probability $\frac{(1-t)}{4}$

$\Rightarrow \frac{(1-\frac{3}{4})}{4} = \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16}$

23. What is the conditional probability that exactly four heads appear when a fair coin is flipped five times, given that the first flip came up heads?

There are 16 equally likely outcomes of flipping a fair coin five times in which the first flip comes up heads

H H H H T
H H H T H
H H T H H
H T H H H

=> By the definition of conditional probability:

$$\frac{4}{16} = \frac{1}{4}$$