OBJECTIVES

1. Distinguish between discrete and continuous random variables
2. Determine a probability distribution for a discrete random variable
3. Describe the connection between probability distributions and populations
4. Construct a probability histogram for a discrete random variable
5. Compute the mean of a discrete random variable
6. Compute the variance and standard deviation of a discrete random variable

OBJECTIVE 1
DISTINGUISH BETWEEN DISCRETE AND CONTINUOUS RANDOM VARIABLES

If we roll a fair die, the possible outcomes are the numbers 1, 2, 3, 4, 5, and 6, and each of these numbers has probability 1/6. Rolling a die is a probability experiment whose outcomes are numbers. The outcome of such an experiment is called a **random variable**.

RANDOM VARIABLE: is a numerical outcome of a probability experiment

DISCRETE AND CONTINUOUS RANDOM VARIABLES

Discrete random variables are random variables whose possible values can be listed. Examples include:

The number comes up on the roll of a dice
The number of siblings a randomly chosen person has

Continuous random variables are random variables that can take on any value in an interval. Examples include:

The height of a randomly chosen college student
The amount of electricity used to light a randomly chosen classroom

OBJECTIVE 2
DETERMINE A PROBABILITY DISTRIBUTION FOR A DISCRETE RANDOM VARIABLE

A probability distribution for a discrete random variable specifies the probability for each possible value of the random variable.

PROPERTIES: $0 \le P(x) \le 1$ for every possible x

$\sum p(x) = 1$

EXAMPLE: Decide if the following is a probability distribution:

x	1	2	3	4
$P(x)$	0.25	0.65	−0.30	0.11

SOLUTION: This is not a probability distribution since $P(x=3) = -0.30$, which is not between 0 and 1

EXAMPLE: Decide if the following is a probability distribution:

x	−1	−0.5	0	0.5	1
$P(x)$	0.17	0.25	0.31	0.22	0.05

SOLUTION: Since all probability are between 0 and 1, and they add up to 1, this is a probability distribution

EXAMPLE: Decide if the following is a probability distribution:

x	1	10	100	1000
$P(x)$	1.02	0.31	0.90	0.43

SOLUTION: Because $P(x=1) = 1.02$ is not between 0 and 1, this is not a probability distribution

EXAMPLE: Four patients have made appointments to have their blood pressure checked at a clinic. Let X be the number of them that have high blood pressure. The probability distribution of X is

x	0	1	2	3	4
$P(x)$	0.23	0.41	0.27	0.08	0.01

(a) Find P(2 or 3)
(b) Find P(More than 1)
(c) Find P(At least 1)

mutually exclusive: cannot happen at the same time

SOLUTION: first notice that these events are mutually exclusive

a) $P(x=2 \text{ or } x=3) = P(x=2) + P(x=3)$ by the addition rule to mutually exclusive events

$= 0.27 + 0.08$

$= 0.35$

b) $P(x>1) = P(x=2 \text{ or } x=3 \text{ or } x=4)$
$= 0.27 + 0.08 + 0.01$
$= 0.36$

using complement rule,
$P(x>1) = 1 - P(x \le 1)$
$= 1 - (0.23 + 0.41)$
$= 0.36$

c) $P(x \ge 1) = 1 - P(x<1)$
$= 1 - P(x=0)$
$= 1 - 0.23 = 0.77$

OBJECTIVE 3
DESCRIBE THE CONNECTION BETWEEN PROBABILITY DISTRIBUTIONS AND POPULATIONS

PROBABILITY DISTRIBUTIONS AND POPULATIONS

Statisticians are interested in studying samples drawn from populations. Random variables are important because when an item is drawn from a population, the value observed is the value of a random variable. The probability distribution of the random variable tells how frequently we can expect each of the possible values of the random variable to turn up in the sample.

EXAMPLE: An airport parking facility contains 1000 parking spaces. Of these, 142 are covered long-term spaces that cost \$2.00 per hour, 378 are covered short-term spaces that cost \$4.50 per hour, 423 are uncovered long-term spaces that cost \$1.50 per hour, and 57 are uncovered short-term spaces that cost \$4.00 per hour. A parking space is selected at random. Let X represent the hourly parking fee for the randomly sampled space. Find the probability distribution of X.

SOLUTION: $x = \{\$2.00, \$4.50, \$1.50, \$4.00\}$

$P(x=\$2.00) = \frac{142}{1000} = 0.142$ $P(x=\$4.50) = \frac{378}{1000} = 0.378$

$P(x=\$1.50) = \frac{423}{1000} = 0.423$ $P(x=\$4.00) = \frac{57}{1000} = 0.057$

Probability distribution:

x	1.50	2.00	4.00	4.50
$P(x)$	0.423	0.142	0.057	0.378

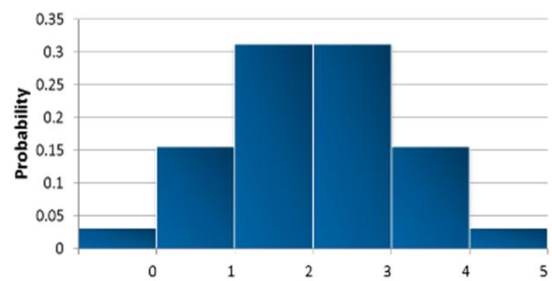
OBJECTIVE 4
CONSTRUCT A PROBABILITY HISTOGRAM FOR A DISCRETE RANDOM VARIABLE

PROBABILITY HISTOGRAMS

In an earlier chapter we learned to summarize the data in a sample with a histogram. We can represent discrete probability distributions with histograms as well. A histogram that represents a discrete probability distribution is called a probability histogram.

EXAMPLE: The following presents the probability distribution and histogram for the number of boys in a family of five children, using the assumption that boys and girls are equally likely and that births are independent events.

x	$P(x)$
0	0.03125
1	0.15625
2	0.31250
3	0.31250
4	0.15625
5	0.03125



OBJECTIVE 5
COMPUTE THE MEAN OF A DISCRETE RANDOM VARIABLE

MEAN OF A RANDOM VARIABLE

Recall that the mean is a measure of center. The mean of a random variable provides a measure of center for the probability distribution of a random variable.

MEAN OF A RANDOM VARIABLE:
The mean of a discrete random variable is defined as follows

$$\mu_X = \sum [x \cdot P(x)]$$

EXAMPLE: A computer monitor is composed of a very large number of points of light called pixels. It is not uncommon for a few of these pixels to be defective. Let X represent the number of defective pixels on a randomly chosen monitor. The probability distribution of X is as follows. Find the mean number of defective pixels.

x	0	1	2	3
$P(x)$	0.2	0.5	0.2	0.1

SOLUTION: The mean is $\mu_x = \sum[x \cdot P(x)] = 0(0.2) + 1(0.5) + 2(0.2) + 3(0.1)$
$= 1.2$

The mean number of defective pixels is 1.2

Note: The menu is a measure of center of the probability distribution

EXPECTED VALUE

There are many occasions on which people want to predict how much they are likely to gain or lose if they make a certain decision or take a certain action. Often, this is done by computing the mean of a random variable. In such situations, the mean is sometimes called the "**expected value**" and is denoted by $E(X)$. If the expected value is positive, it is an expected gain, and if it is negative, it is an expected loss.

EXAMPLE: A mineral economist estimated that a particular venture had probability 0.4 of a \$30 million loss, probability 0.5 of a \$20 million profit, and probability 0.1 of a \$40 million profit. Let X represent the profit. Find the probability distribution of the profit and the expected value of the profit. Does this venture represent an expected gain or an expected loss?

SOLUTION: The probability distribution of x is

(millions)

x	-30	20	40
$P(x)$	0.4	0.5	0.1

The expected value

$$E(x) = \sum[x \cdot P(x)] = (-30)(0.4) + (20)(0.5) + (40)(0.1)$$
$$= 2.0$$

There is an expected gain of \$ 2 million

OBJECTIVE 6
COMPUTE THE VARIANCE AND STANDARD DEVIATION OF A DISCRETE RANDOM VARIABLE

VARIANCE/STANDARD DEVIATION OF A RANDOM VARIABLE

The **variance** and **standard deviation** provide a measure of spread for the probability distribution of a random variable.

VARIANCE OF A RANDOM VARIABLE:

The variance of a discrete random variable x is given by

$$\sigma_x^2 = \Sigma[(x-\mu_x)^2 \cdot P(x)]$$

STANDARD DEVIATION OF A RANDOM VARIABLE:

The standard deviation of a discrete random variable is

$$\sigma_x = \sqrt{\sigma_x^2}$$

MEAN/STANDARD DEVIATION ON THE TI-84 PLUS

Step 1: Enter the values of the random variable into **L1** and the associated probabilities in **L2**.

Step 2: Press **STAT** and highlight the **CALC** menu and select **1-Var Stats**.

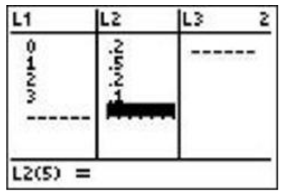
Step 3: Enter **L1** in the **List** field and **L2** in the **FreqList** and run the command.

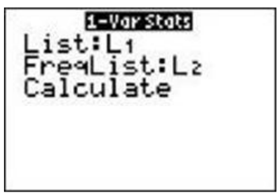
Note: If your calculator does not support Stat Wizards, enter L1 next to the 1-Var Stats command on the home screen and press enter to run the command

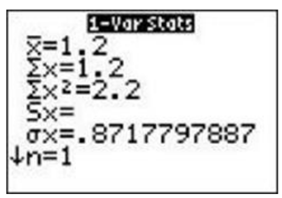
EXAMPLE:
Compute the mean and standard deviation of the following probability distribution using the TI-84 PLUS.

x	0	1	2	3
$P(x)$	0.2	0.5	0.2	0.1

SOLUTION:
We first enter values of the random variable and the associated probabilities into the data editor and then run the 1-Var Stats command. We find $\mu_X = 1.2$ and $\sigma_X = 0.872$.







YOU SHOULD KNOW …

- The difference between discrete and continuous random variables
- How to determine the probability distribution for a discrete random variable
- How to construct a probability distribution for a population
- How to construct a probability histogram
- How to compute the mean, variance, and standard deviation of a discrete random variable

OBJECTIVES

1. Determine whether a random variable is binomial
2. Determine the probability distribution of a binomial random variable
3. Compute binomial probabilities
4. Compute the mean and variance of a binomial random variable

${}_nC_r$ ${}_nC_x$

${}_3C_2 =$

OBJECTIVE 1
DETERMINE WHETHER A RANDOM VARIABLE IS BINOMIAL

Suppose that your favorite fast food chain is giving away a coupon with every purchase of a meal. Twenty percent of the coupons entitle you to a free hamburger, and the rest of them say "better luck next time." Ten of you order lunch at this restaurant.

What is the probability that three of you win a free hamburger? In general, if we let X be the number of people out of ten that win a free hamburger. What is the probability distribution of X? In this section, we will learn that X has a distribution called the **binomial distribution**, which is one of the most useful probability distributions.

In the problem just described, each time we examine a coupon, we call it a "trial," so there are 10 trials. When a coupon is good for a free hamburger, we will call it a "success." The random variable X represents the number of successes in 10 trials.

A random variable that represents the number of successes in a series of trials has a probability distribution called the **binomial distribution**. The conditions are:

- fixed number of trials are conducted
- there are two possible outcomes for each trial
  = "success" and "failure" (These are mutually exclusive outcomes)
- p(success) is the same on each trial
- trials are independent

The random variable x represents the numbers of success that occur

Notation: n = number of trials, p = probability of success

EXAMPLE: A fair coin is tossed ten times. Let X be the number of times the coin lands heads. Decide if this represents a binomial experiment.

SOLUTION: This is a binomial experiment
- each toss of a coin is a trial. The number of trials is fixed $n = 10$.
- two outcomes : heads and tails
- $P(\text{success}) = \frac{1}{2}$ (same in each trial)
- trials are independent since the outcome of one coin toss does not affect the other tosses.

EXAMPLE: Five basketball players each attempt a free throw. Let X be the number of free throws made.

SOLUTION: This is not a binomial experiment since the $p(\text{success} = \text{making a shot})$ differs from player to player

EXAMPLE: Ten cards are in a box. Five are red and five are green. Three of the cards are drawn at random. Let X be the number of red cards drawn.

SOLUTION: since the trials are not independent, this is not a binomial distribution

OBJECTIVE 2
DETERMINE THE PROBABILITY DISTRIBUTION OF A BINOMIAL RANDOM VARIABLE

Consider the binomial experiment of tossing 3 times a biased coin that has probability 0.6 of coming up heads. Let X be the number of heads that come up. If we want to compute $P(2)$, the probability that exactly 2 of the tosses are heads, there are 3 arrangements of two heads in three tosses: HHT, HTH, THH. The probability of HHT is P(HHT) = (0.6)(0.6)(0.4) = $(0.6)^2(0.4)$. Similarly, we find that P(HTH) = P(THH) = $(0.6)^2(0.4)$.

Now, $P(2) = P$(HHT *or* HTH *or* THH) = $3(0.6)^2(0.4)$, by the Addition Rule. Examining this result, we see the number 3 represents the number of arrangements of two successes (heads) and one failure (tails). In general, this number will be the number of arrangements of x successes in n trials, which is nCx. The number 0.6 is the success probability p which has an exponent of 2, the number of successes x. The number 0.4 is the failure probability $1 - p$ which has an exponent of 1, which is the number of failures, $n - x$.

BINOMIAL PROBABILITY DISTRIBUTION:

In general, for a binomial random variable x,

$$P(x) = nCx\, p^x (1-p)^{n-x}$$

The possible values of the random variable x are $0, 1, 2, 3, \ldots, n$.

OBJECTIVE 3
COMPUTE BINOMIAL PROBABILITIES

EXAMPLE: The Pew Research Center reported in June 2013 that approximately 30% of U.S. adults own a tablet computer such as an iPad, Samsung Galaxy Tab, or Kindle Fire. Suppose a simple random sample of 15 people is taken. Use the binomial probability distribution to find the following probabilities.

a) Find the probability that exactly four of the sampled people own a tablet computer.
b) Find the probability that fewer than three of the people own a tablet computer.
c) Find the probability that more than one person owns a tablet computer.
d) Find the probability that the number of people who own a tablet computer is between 1 and 4, inclusive.

$_nC_x\, p^x (1-p)^{n-x}$

SOLUTION:

a) $P(x=4) = {}_{15}C_4 (0.3)^4 (1-0.3)^{15-4} = {}_{15}C_4 (0.3)^4 (0.7)^{11} = 0.2186$

b) $P(x<3) = P(x=0 \text{ or } x=1 \text{ or } x=2)$

$= P(x=0) + P(x=1) + P(x=2)$

$= {}_{15}C_0 (0.3)^0 (0.7)^{15} + {}_{15}C_1 (0.3)^1 (0.7)^{14} + {}_{15}C_2 (0.3)^2 (0.7)^{13}$

$= 0.0047 + 0.0305 + 0.0916 = 0.1268$

c) $P(x>1) = 1 - P(x \leq 1)$ (by the complement rule)

$= 1 - P(x=0 \text{ or } x=1)$

$= 1 - (P(x=0) + P(x=1))$

$= 1 - (0.0047 + 0.0305)$

$= 0.9648$

d) $P(1 \leq x \leq 4) = P(x=1 \text{ or } x=2 \text{ or } x=3 \text{ or } x=4)$

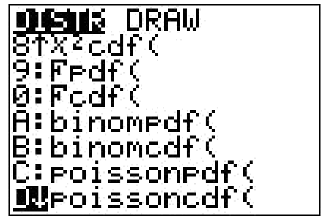
$= P(x=1) + P(x=2) + P(x=3) + P(x=4)$

$= 0.0305 + 0.0916 + 0.1700 + 0.2186$

$= 0.5107$

BINOMIAL PROBABILITIES ON THE TI-84 PLUS

In the TI-84 PLUS Calculator, there are two primary commands for computing binomial probabilities. These are **binompdf** and **binomcdf**. These commands are on the **DISTR** (distributions) menu accessed by pressing **2nd, VARS.**



The **binompdf** command is used when finding the probability that the binomial random variable X is *equal* to a specific value, x.

The **binomcdf** command is used when finding the probability that the binomial random variable X is *less than or equal* to a specified value, x.

binompdf $\Rightarrow P(X = K)$

To compute the probability that the random variable X equals the value x given the parameters n and p, use the binompdf command with the following format:

binompdf(n,p,x)

binomcdf $\Rightarrow P(X \leq K)$

To compute the probability that the random variable X is less than or equal to the value x given the parameters n and p, use the binomcdf command with the following format:

$n=15 \quad P = 0.30$

binomcdf(n,p,x)

a) $P(X=4) = \text{binompdf}(15, 0.30, 4) = 0.2186$

b) $P(X<3) = P(X \leq 2) = \text{binomcdf}(15, 0.30, 2) = 0.1268$

c) $P(X>1) = 1 - P(X \leq 1) = 1 - \text{binomcdf}(15, 0.30, 1) = 0.9648$

d) $P(1 \leq X \leq 4) = P(X \leq 4) - P(X<1) = P(X \leq 4) - P(X=0)$

$= \text{binomcdf}(15, 0.30, 4) - \text{binompdf}(15, 0.30, 0)$

$= 0.5107$

OBJECTIVE 4
COMPUTE THE MEAN AND VARIANCE OF A BINOMIAL RANDOM VARIABLE

MEAN, VARIANCE, AND STANDARD DEVIATION OF A BINOMIAL RANDOM VARIABLE

Let X be a binomial random variable with n trials and success probability p. The mean, variance, and standard deviation of X are:

MEAN: The mean of a binomial random variable is

$\mu_X = n \cdot p$

VARIANCE: The variance of a binomial random variable is

$$\sigma_x^2 = n \cdot p \cdot (1-p)$$

STANDARD DEVIATION: The standard deviation of a binomial random variable is

$$\sigma_x = \sqrt{\sigma_x^2} = \sqrt{n \cdot p \cdot (1-p)}$$

EXAMPLE: The probability that a new car of a certain model will require repairs during the warranty period is 0.15. A particular dealership sells 25 such cars. Let X be the number that will require repairs during the warranty period. Find the mean and standard deviation of X.

SOLUTION: Since this is binomial experiment with $n = 25$ and $p = 0.15$

The mean of $x = \mu_x = n \cdot p = 25(0.15) = 3.75$

the standard deviation of $x = \sigma_x = \sqrt{n \cdot p \cdot (1-p)}$

$$= \sqrt{25(0.15)(1-0.15)}$$

$$= 1.7854$$

YOU SHOULD KNOW …

- How to determine whether a random variable is binomial
- The notation for a binomial experiment
- How to determine the probability distribution of a binomial random variable
- How to compute binomial probabilities
- How to compute the mean and variance of a binomial random variable