1.1 Propositional Logic

1. Which of these sentences are propositions? What are the truth values of those that are propositions?

a) Boston is the capital of Massachusetts.

This is a true proposition

d) $5 + 7 = 10$.

This is a false proposition.

truth => $5 + 7 = 12$

3. What is the negation of each of these propositions?

a) Mei has an MP3 player.

b) There is no pollution in New Jersey. $\sim(\sim p) = p$

c) $2 + 1 = 3$.

d) The summer in Maine is hot and sunny.

a) Mei does not have an MP3 player.

b) There is pollution in New Jersey.

c) $2 + 1 \neq 3$

d) It is not the case that the summer in Maine is hot and sunny.

=> The summer in Maine either not hot or it is not sunny.

$\sim(p \wedge q) = \sim p \vee \sim q$

=> Negation of p and q is either not p or not q.

9. Let p and q be the propositions "Swimming at the New Jersey shore is allowed" and "Sharks have been spotted near the shore," respectively. Express each of these compound propositions as an English sentence.

c) $\neg p \vee q$

c) Swimming at the New Jersey shore is not allowed, or sharks have been spotted near the shore.

1.2 Applications of Propositional Logic

1. You cannot edit a protected Wikipedia entry unless you are an administrator. Express your answer in terms of *e*: "You can edit a protected Wikipedia entry" and *a*: "You are an administrator."

The statement is $\neg a \rightarrow \neg e$.
$\equiv \neg\neg a \vee \neg e$
$\equiv a \vee \neg e$
$\equiv e \rightarrow a$

Any statement in the form of "not q unless $\neg p$" can be written as $p \rightarrow q$.
Given statement can be written as "$\neg e$ unless a".
=> Given statement can be written as "$\neg e$ unless $\neg\neg a$".
=> Given statement can be written as $\neg a \rightarrow \neg e$

$\Rightarrow$ If you can edit a protected Wikipedia entry, then you must be an administrator.

3. You can graduate only if you have completed the requirements of your major and you do not owe money to the university and you do not have an overdue library book. Express your answer in terms of *g*: "You can graduate," *m*: "You owe money to the university," *r*: "You have completed the requirements of your major," and *b*: "You have an overdue library book."

p only if q => $p \rightarrow q$

$g \rightarrow (r \wedge (\neg m) \wedge (\neg b))$

=> negation $\neg p$: not p
conjunction $p \wedge q$: p and q
Disconjunction $p \vee q$: p or q
Conditional statement $p \rightarrow q$: if p, then q.
Biconditional statement $p \leftrightarrow q$: p iff q
"p only if q" => "if p, then q".

"You can graduate only if you have completed the requirements of your major and you do not owe money to the university and you do not have an overdue library book".
Rewritting:
"g only if (r and not m and not b)
using symbols:

$g \rightarrow (r \wedge (\neg m) \wedge (\neg b))$

1.3 Propositional Equivalences

1. Use truth tables to verify these equivalences.

a) $p \wedge \mathbf{T} \equiv p$

p	$p \wedge T$	$p \vee q$	$p \wedge q$	$p \vee T$	$p \vee p$	$p \wedge p$
T	T	T	q	T	T	T
q	q	q	q	T	q	q

∴ by comparing the truth values of p and $p \wedge T$ the equivalence is $p \wedge T \equiv p$.

3. Use truth tables to verify the commutative laws

a) $p \vee q \equiv q \vee p$.

p	q	$p \vee q$	$q \vee p$
T	T	T	T
T	q	T	T
q	T	T	T
q	q	q	q

∴ the columns showing the truth values of the compound statements $p \vee q$ and $q \vee p$ are the same.

∴ $p \vee q \equiv q \vee p$

7. Use De Morgan's laws to find the negation of each of the following statements.

a) Jan is rich and happy.

=> conjunction "Jan is rich, and Jan is happy."

=> negation "Jan is not rich, or Jan is not happy."

☞ **9.** Show that each of these conditional statements is a tautology by using truth tables.

a) $(p \wedge q) \rightarrow p$

p	q	$p \wedge q$	$(p \wedge q) \rightarrow p$
T	T	T	T
T	q	q	T
q	T	q	T
q	q	q	T

A tautology is a formula which is always true for every value of its propositional variables.

∴ $(p \wedge q) \rightarrow p$ is a tautology.

11. Show that each conditional statement in Exercise 9 is a tautology without using truth tables.

A tautology is a formula which is always true for every value of its propositional variables.

☞ **9.** Show that each of these conditional statements is a tautology by using truth tables.

a) $(p \wedge q) \rightarrow p$

$(p \wedge q) \rightarrow p \equiv \neg(p \wedge q) \vee p$

$\equiv (\neg p \vee \neg q) \vee p$ De Morgan's laws

$\equiv (\neg q \vee \neg p) \vee p$ Commutative law

$\equiv \neg q \vee (\neg p \vee p)$ Associative law

$\equiv \neg q \vee T$ Negation law (w/ commutative law)

$\equiv T$ Domination law

$\therefore$ the conditional statement $(p \wedge q) \rightarrow p$ is a tautology.