

- 1) $E(x) = x$ exercises regularly
 $M(x) = x$ has good muscle strength
 $L(x) = x$ has low blood pressure

$$\forall x (E(x) \rightarrow M(x))$$

$$\forall x (E(x) \rightarrow L(x))$$

$$E(\text{Juanita})$$

$$L(\text{Juanita})$$

$$\therefore \exists x (M(x) \wedge L(x))$$

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|---|-------------------------------------|----------------------------------|
| 1 | $\forall x (E(x) \rightarrow M(x))$ | hypothesis |
| 2 | $\forall x (E(x) \rightarrow L(x))$ | hypothesis |
| 3 | Juanita exercises regularly | hypothesis |
| 4 | $E(c) \rightarrow M(c)$ | universal instantiation (1,3) |
| 5 | $E(c) \rightarrow L(c)$ | universal instantiation (2,3) |
| 6 | $M(c)$ | modus Ponens (3,4) |
| 7 | $L(c)$ | modus Ponens (3,5) |
| 8 | $M(c) \wedge L(c)$ | conjunction (6,7) |
| 9 | $\exists x (M(x) \wedge L(x))$ | Existential generalization (3,8) |

True

If we replace $L(c)$ with "Juanita has low blood pressure"

the statement is invalid because we can't use modus ponens

2) The proof used the assumption $\forall y (y \leq x)$ which assume there is a largest number. However, there is no largest number. Therefore, the proof is invalid.

3) By the definition of fast squaring trick for 2-digit numbers ends in 5 is its first digit $x \cdot (x+1)$ and write 25 at the end.

Assume that x is an integer that is between one to nine inclusive.

The two digit number that ends in 5 are
15, 25, 35, ..., 95

we can write it in algebra form as

$$x \cdot 10 + 5$$

then we square it

$$\text{we got } (10x + 5)^2$$

By the square trick definition
the square of the two digit number that ends in 5 is

$$(x)(x+1) \cdot 100 + 25$$

$$\text{then we got } 100x(x+1) + 25$$

to find out if they are equal

$$\begin{array}{lcl} (10x+5)^2 & & 100x(x+1)+25 \\ = 100x^2+100x+25 & & = 100x^2+100x+25 \end{array}$$

Therefore, they are equal and
the fast square trick is true.

4) let x, y, z be real numbers

$$\text{then } |x-y| \leq |x-z| + |y-z|$$

Case 1 = $y \leq x$

$$|x-y| = x-y$$

$$|x-z| = x-z$$

$$|y-z| = z-y$$

$$x-y \leq x-z + z-y$$

$$x-y \leq x-y$$

It is always true.

Case 2 = $y > x$

$$|x-y| = y-x$$

$$|x-z| = x-z$$

$$|y-z| = y-z$$

$$y-x \leq x-z + y-z$$

$$y-x - x + z - y + z \leq 0$$

$$-2x + 2z \leq 0$$

Since $y > x$, it is always true.

In both cases, $|x-y| \leq |x-z| + |y-z|$ is true, therefore the claim is true.

5) Song(x) = x is a song
Remove(x, y) = remove word y from song x
Add(x, y) = add word y to song x

$\forall x (\text{Remove}(x, w) \rightarrow \neg \text{Song}(x)) \rightarrow \forall x (\text{Add}(x, w) \rightarrow \neg \text{Song}(x))$

I would say removing a word is modifying a song
and if removing a word make it not a song
then adding a word is also modifying a song
which also makes it not a song.