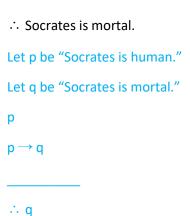
Exercise 1.6

1. Find the argument form for the following argument and determine whether it is valid. Can we conclude that the conclusion is true if the premises are true? If Socrates is human, then Socrates is mortal.

Socrates is human.



Argument form: Modus ponens

It is valid.

Yes, the conclusion is true since the hypotheses are true.

- 3. What rule of inference is used in each of these arguments?
- a) Alice is a mathematics major. Therefore, Alice is either a mathematics major or a computer science major.

Let p be "Alice is a mathematics major."

Let q be "Alice is a computer science major."

Р

 \therefore p \land q

It uses Addition rule.

5. Use rules of inference to show that the hypotheses "Randy works hard," "If Randy works hard, then he is a dull boy," and "If Randy is a dull boy, then he will not get the job" imply the conclusion "Randy will not get the job."

Let p be "Randy works hard."

Let q be "Randy is a dull boy."

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Let r be "Randy will get the job."
p \rightarrow q
∴ q Modus Ponens
q \rightarrow \neg r
q
∴ ¬ r Modus Ponens
7. What rules of inference are used in this famous argument? "All men are mortal. Socrates is a man.
Therefore, Socrates is mortal."
Let Mortal(x) be "x is mortal."
Let Man(x) be "x is a man."
Where the domain is all men.
\forall x (Man(x) \rightarrow Mortal(x))
∴ Man(Socrates) → Mortal(Socrates) Universal Instantiation
Man(Socrates)
∴ Mortal(Socrates) Modus Ponens
Exercise 1.7
3. Show that the square of an even number is an even number using a direct proof.
Assume x is an even number,
       x = 2k for some integer k
       x^2 = (2k)^2
       x^2 = 4k^2
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 $x^2 = 2(2k^2)$

$$x^2 = 2r$$
 for $r = 2k^2$

 \therefore x² is an even number

17. Show that if n is an integer and $n^3 + 5$ is odd, then n is even using a) a proof by contraposition.

Let p be "
$$n^3 + 5$$
 is odd."

Let q be "n is even."

Assume n is odd,

$$n = 2k + 1 \qquad \text{for some integer k}$$

$$n^3 + 5 = (2k + 1)^3 + 5$$

$$n^3 + 5 = 8k^3 + 12k^2 + 6k + 1 + 5$$

$$n^3 + 5 = 8k^3 + 12k^2 + 6k + 6$$

$$n^3 + 5 = 2(4k^3 + 6k^2 + 3k + 3)$$

$$n^3 + 5 = 2r \qquad \text{for } r = 4k^3 + 6k^2 + 3k + 3$$

 \therefore n³ + 5 is even

Since $\neg p \rightarrow \neg q$ is true, $p \rightarrow q$ must also be true.

 \therefore if n is an integer and $n^3 + 5$ is odd, then n is even.

b) a proof by contradiction.

Assume $n^3 + 5$ is odd and n is odd,

$$n = 2k + 1 \qquad \text{for some integer k}$$

$$n^3 + 5 = (2k + 1)^3 + 5$$

$$n^3 + 5 = 8k^3 + 12k^2 + 6k + 1 + 5$$

$$n^3 + 5 = 8k^3 + 12k^2 + 6k + 6$$

$$n^3 + 5 = 2(4k^3 + 6k^2 + 3k + 3)$$

$$n^3 + 5 = 2r \qquad \text{for } r = 4k^3 + 6k^2 + 3k + 3$$

$$n^3 + 5 \text{ is even.}$$

This contradicts the assumption that $n^3 + 5$ is odd.

 \therefore if $n^3 + 5$ is odd, then n is even