Exercise 5.1

3. Let P (n) be the statement that 12 + 22 + · · · + n2 = n(n + 1)(2n + 1)/6 for the positive integer n.

a) What is the statement P (1)?

12 = 1(1 + 1)(2(1) + 1)/6

b) Show that P(1) is true, completing the basis step of the proof.

12 = 1(1 + 1)(2(1) + 1)/6

1 = 1(2)(3)/6

1 = 6/6

1 = 1

∴ P(1) is true.

c) What is the inductive hypothesis?

d) What do you need to prove in the inductive step?

e) Complete the inductive step, identifying where you use the inductive hypothesis.

f ) Explain why these steps show that this formula is true whenever n is a positive integer.

Exercise 5.2

3. Let P (n) be the statement that a postage of n cents can be formed using just 3-cent stamps and 5-cent stamps. The parts of this exercise outline a strong induction proof that P (n) is true for n ≥ 8.

a) Show that the statements P (8), P (9), and P (10) are true, completing the basis step of the proof.

b) What is the inductive hypothesis of the proof?

c) What do you need to prove in the inductive step?

d) Complete the inductive step for k ≥ 10.

e) Explain why these steps show that this statement is true whenever n ≥ 8.

Exercise 5.3

7. Give a recursive definition of the sequence {an}, n = 1, 2, 3, . . . if

a) an = 6n.

Exercise 5.4

9. Give a recursive algorithm for finding the sum of the first n odd positive integers.