Logic is the study of formal reasoning.

A proposition is a statement that is either true or false.

Conjunction

p ^ q

Both p and q must be true to output true

Disjunction

p v q

Either p or q is true to output true

Negation

¬ p

Reverse p’s truth value

Conditional statements

p → q

Converse:

q → p

Contrapositive:

¬q → ¬p

Inverse:

¬p → ¬q

False only if p is true and q is false

Tautology

p ↔ q

Evaluates for true for every truth assignment

Contradiction

Evaluates for false for every truth assignment

Logically equivalent

De Morgan’s Law

Conditional statements

P

Predicate: a logical statement whose truth value is a function of one or more variables

Universal quantifier

For all x, P(x) is true for every possible value.

∀x P(x)

It is a proposition.

Existential quantifier

There exists an x, such that p(x)

∃x P(x)

The logical expression is a proposition if all the variables are bound.

Nested quantifier

A logical expression with more than one quantifier that binds different variables in the same predicate is said to have nested quantifiers.

Existential player: tries to make the expression true.

Universal player: tries to make the expression false.

Dis 1

Reading books is sufficient for reading philosophy. False

If you can read books, then you can read philosophy.

Reading philosophy is sufficient for reading books. True

Drinking tea is necessary for one to be smart. False

If you drink tea, then you are smart.

Beer tastes good when you’ve worked a hard day.

If beer tastes good, then you’ve worked a hard day.

If you’ve worked a hard day, then beer tastes good. True

Homework is fun only when it is not graded.

If homework is fun, then it is not graded.

Markers don’t have ink unless you’ve tried them out yourself.

To express everyone else

M(x, y) indicates whether x sent an email to y.

∀x ∀y ((x ≠ y) → M(x, y))

To express uniqueness

L(x) indicates where or not x came late to the meeting.

∃x (L(x) ^ ∀y ((x ≠ y) → ¬L(y)))

A hypothesis is true when p → q is true and p v q is true.

A valid argument is a guarantee that the conclusion is whenever all the hypotheses are true.