CSE276C-HW3-2023

Due November 14 end of day

1 Question 1

Consider the following differential equation over the interval $(0, 1]$:

$$\frac{dy}{dx} = \frac{1}{x^2(1-y)}$$

with $y(1) = -1$.

Often we use this function form to describe forces when navigating through a field of obstacles. In its general form this is called potential field navigation and borrows numerous results from physics.

(a) Obtain an exact analytical solution to the equation. Solve for y(0) (even though in theory the equation is not defined for x=0).

(b) Implement and use Euler's method to solve the differential equation numerically. Use a step size of 0.05. How accurate is your numerical solution?

(c) Implement and use a fourth-order Runge-Kutta method to solve the differential equation numerically. Again, use a step size of 0.05. Again, how accurate is your numerical solution?

(d) Implement and use a Richardson extrapolation to solve the equation, again with a step size of 0.05. How accurate is your solution compared to the analytical solution?

2 Question 2

We have multiple robots that can generate point clouds such as those coming from a RealSense camera. In many cases we want to use the robots to detect objects in its enviroment. We provide three data files:

- `Empty2.asc`, which contains data for an empty table
- `TableWithObjects2.asc`, which contains data for a cluttered table, and

- `CSE.asc`, which contains data of the CSE building viewed from the bear courtyard.

Each file has the point cloud data in a format where each line contains x_i y_i z_i. You can use `np.loadtxt` to load a pointcloud into a numpy array.

(a) Provide a method to estimate the plane parameter for the table. Test it both with the empty and cluttered table. Describe how you filter out the data from the objects. You have to be able to estimate the table parameters in the presence of clutter.

(b) Describe and show how the method can be generalized to extract all the dominant planes of the CSE building viewed in the outdoor environment.

2.1 Problem 1

1.a The analytical solution

$$\frac{dy}{dx} = \frac{1}{x^2(1-y)}$$

$$(1-y)dy = \frac{1}{x^2}dx$$

Integrate on both sides

$$y - \frac{y^2}{2} = \frac{1}{x} + C$$

The analytical solution is clearly $-\infty$
1.b Euler's Method - y(0) = -8.12493
1.c Fourth Order Runga Kutta - $y(0) = -6.01 * 10^{27}$
1.d Richardson exploration - $y(0) = -6.2 * 10^{27}$

Clearly the results vary significantly close to a singularity and it is important to analyze functions before you select an integration method

2.2 Problem 2

There are multiple methods to generate hypothesis, so as "simple" point clustering, or use of RANSAC. A strategy is to estimate plane parameters and the iterative cluster points that fit the plane and which are connected. Using this approach the normal for the table is

$$n = [0.0079 \quad 0.9115 \quad 0.41113]$$

The same value should be obtained for both table settings