3-1 Inference in a chain

(a) $p(Y_1 | X_1) = \sum_x P(Y_1, X_0 = x | X_1)$

$= \sum_x P(X_0 = x | X_1) \cdot P(Y_1 | X_0 = x, X_1)$ (By product rule)

$= \sum_x P(X_0 = x) \cdot P(Y_1 | X_0 = x, X_1)$ (By conditional independence)

(b) $p(Y_1) = \sum_x P(Y_1, X_0 = x)$

$= \sum_{x'} \sum_x P(Y_1, X_0 = x, X_1 = x')$

$= \sum_{x'} \sum_x P(X_0 = x) \cdot P(X_1 = x' | X_0 = x) \cdot P(Y_1 | X_0 = x, X_1 = x')$

$= \sum_{x'} \sum_x P(X_0 = x) \cdot P(X_1 = x') \cdot P(Y_1 | X_0 = x, X_1 = x')$

(c)

$p(X_n | Y_1, Y_2, \ldots Y_{n-1}) = p(X_n)$

(d) $p(Y_n | X_n, Y_1, Y_2, \ldots Y_{n-1}) = \sum_x p(Y_n, X_{n-1} = x | X_n, Y_1, Y_2, \cdots Y_{n-1})$ (marginalization)

$= \sum_x \frac{P(Y_n, X_{n-1} = x, X_n, Y_1, Y_2 \cdots Y_{n-1})}{P(X_n, Y_1, Y_2, \ldots Y_{n-1})}$

$= \sum_x P(X_{n-1} = x | X_n, Y_1, Y_2 \ldots Y_{n-1}) \cdot P(Y_n | X_{n-1} = x, X_n, Y_1, Y_2 \ldots, Y_{n-1})$

$= \sum_x P(X_{n-1} = x | Y_1, Y_2 \cdots Y_{n-1}) \cdot P(Y_n | X_{n-1} = x, X_n)$ #

(e) $P(Y_n | Y_1, Y_2, \dots Y_{n-1}) = \sum_{x} P(Y_n, X_{n-1}=x | Y_1, Y_2, \dots Y_{n-1})$

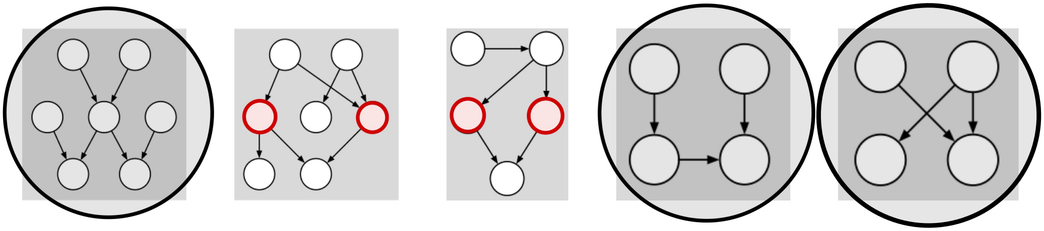
$= \sum_{x} P(X_{n-1}=x | Y_1, Y_2 \dots Y_{n-1}) \cdot P(Y_n | X_{n-1}=x, Y_1, Y_2, \dots Y_{n-1})$

$= \sum_{x'} \sum_{x} P(X_{n-1}=x | Y_1, Y_2 \cdots Y_{n-1}) \cdot P(Y_n, X=x' | X_{n-1}=x, Y_1, Y_2, \dots Y_{n-1})$

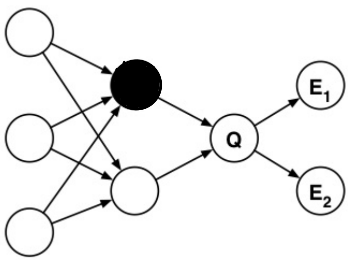
$= \sum_{x'} \sum_{x} P(X_{n-1}=x | Y_1, Y_2 \dots Y_{n-1}) \cdot P(X_n = x' | X_{n-1}=x) \cdot P(Y_n | X_n=x', X_{n-1}=x, Y_1, Y_2 \cdots Y_{n-1})$

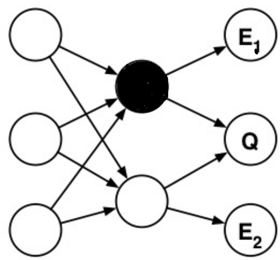
$= \sum_{x'} \sum_{x} P(X_{n-1}=x | Y_1, Y_2 \dots Y_{n-1}) \cdot P(Y_n | X_n=x', X_{n-1}=x) \cdot P(X_n = x')$ #

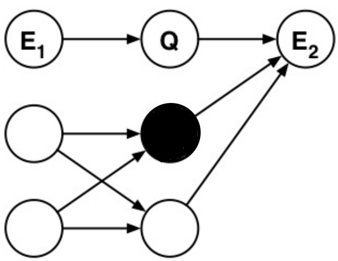
3-2 Node clustering and polytrees

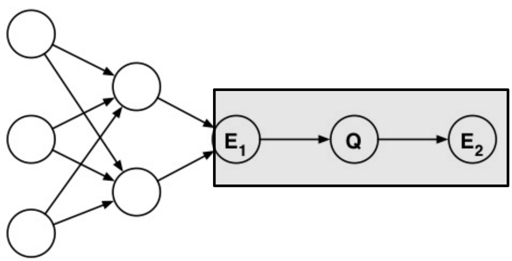


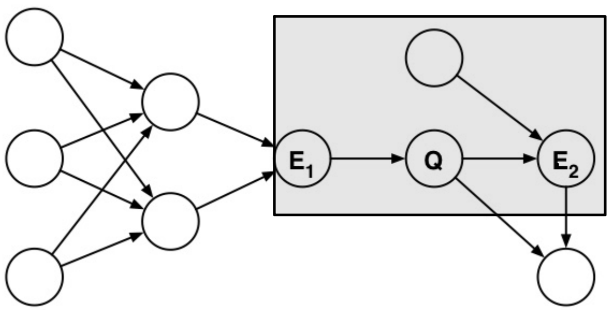
3-3 Cutsets and Polytrees











3-4 Even more inference

(a)

$$P(B|A,C,D) = \frac{P(A,B,C,D)}{P(A,C,D)}$$

$$= \frac{P(C)\cdot P(B|C)\cdot P(D|B,C)\cdot P(A|D,B,C)}{P(A,C,D)}$$

$$= \frac{P(C)\cdot P(B)\cdot P(D|B,C)\cdot P(A|B)}{P(A,C,D)}$$

$$= \frac{P(C)\cdot P(B)\cdot P(D|B,C)\cdot \frac{P(B|A)}{P(B)}}{P(A,C,D)}$$

$$= \frac{P(C)\cdot P(D|B,C)\cdot P(B|A)}{P(A,C,D)}$$

$$= \frac{P(C)\cdot P(D|B,C)\cdot P(B|A)}{\sum_b P(A,B=b,C,D)}$$

$$= \frac{P(C)\cdot P(D|B,C)\quad P(B|A)}{\sum_b P(C)\cdot P(D|B=b,C)\cdot P(B=b|A)}$$

$$= \frac{P(D|B,C)\cdot P(B|A)}{\sum_b P(D|B=b,C)\cdot P(B=b|A)}$$

(b)

The nodes A, C and D from the Markov blanket of B

Hence $P(B|A,C,D,E,F) = P(B|A,C,D)$

$$= \frac{P(D|B,C)\cdot P(B|A)}{\sum_b \cdot P(D|B=b,C)\cdot P(B=b|A)}$$

(c)

$$P(B,E,F|A,C,D) = \frac{P(B,E,F,A,C,D)}{P(A,C,D)}$$

$$= \frac{P(A)\cdot P(C|A)\cdot P(D|A,C)\cdot P(B|A,C,D)\cdot P(E|A,B,C,D)\cdot P(F|A,B,C,D,E)}{P(A)\cdot P(C|A)\cdot P(D|A,C)}$$

$$= P(B|A,C,D)\,P(E|A,B,C,D)\cdot P(F|A,B,C,D,E)$$

$$= P(B|A,C,D)\,P(E|C)\cdot P(F|A) \;\#$$

3-5 More likelihood weighting

(a)

$x_i \sim P(X)$

$q_i \sim P(Q)$

$y_i \sim P(Y \mid X = x_i, Q = q_i)$

$z_i \sim P(Z \mid Q = q_i)$

$$p(Q=q \mid E=e) \approx \frac{\sum_{i=1}^{N} 1(q, q_i)\, P(e \mid y, z)}{\sum_{i=1}^{N} P(e \mid y, z)}$$

(b)

$q_{1i} \sim P(Q_1)$

$x_i \sim P(X)$

$z_i \sim P(Z \mid X = x_i, Y = y_i)$

$q_{2i} \sim p(Q_2 \mid Z)$

$$p(Q_1 = q_1, Q_2 = q_2 \mid E_1 = e_1, E_2 = e_2) \cong \frac{\sum_{i=1}^{N} 1(q_1, q_{1i})\, 1(q_2, q_{2i})\, P(e_1 \mid q_{1i}, x_i)\, P(e_2 \mid e_1, z_i)}{\sum_{i=1}^{N} P(e_1 \mid q_{1i}, x_i)\, P(e_2 \mid e_1, z_i)}$$